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TRANSPARENCIES

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These are lecture notes, intended for distribution to participants.

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Theory of collisionless reconnection in high temperature laboratory plasmas

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- reconnection
- high temperature plasmas.

— ideal MHD

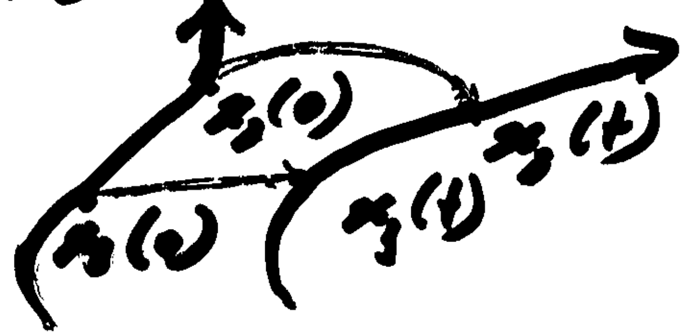
$$\frac{d}{dt}\left(\vec{dl} \times \vec{B}\right) = 0$$

if $dl \times B = 0$ at $t=0$

$\frac{d}{dt}$ along fluid veloc.

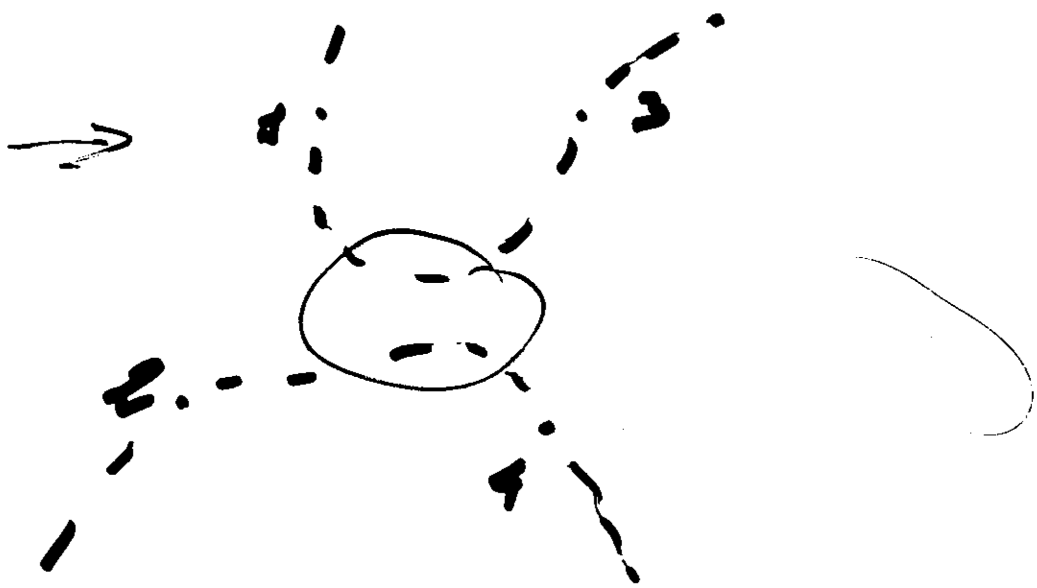
⇒ if x_1, x_2 are connected by a field line at $t=0$

a $t \neq 0$ ∃ field line: $x_1(t)$ and $x_2(t)$ are connected



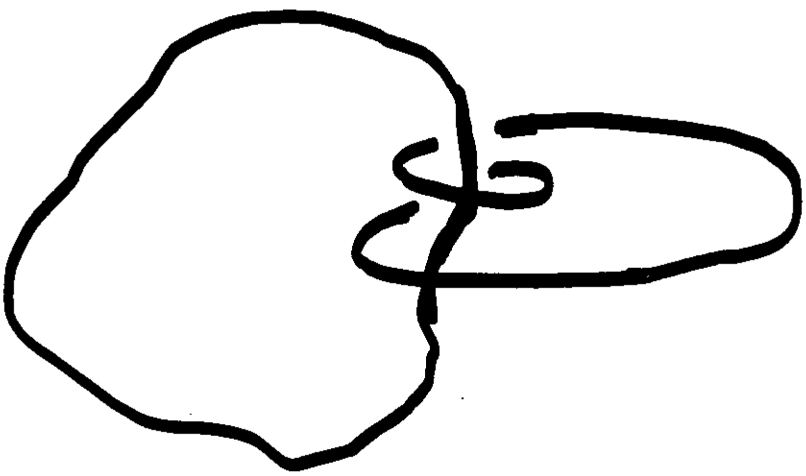
reconnection

fast localised violation of $\frac{d}{dt}(dl \times B) = 0$



- non conservation of magnetic flux
- transformation of magnetic energy into other forms of energy
- change of the magnetic topology

$A \cdot B$ linking number



- dissipation
- irreversibility
- fractional exponents of $\tau_A/\tau_R \ll 1$

$D_\eta \sim \frac{c^2 \eta}{4\pi}$ $\tau_A \sim L/c_A$ $\tau_R \sim L^2/D_\eta$

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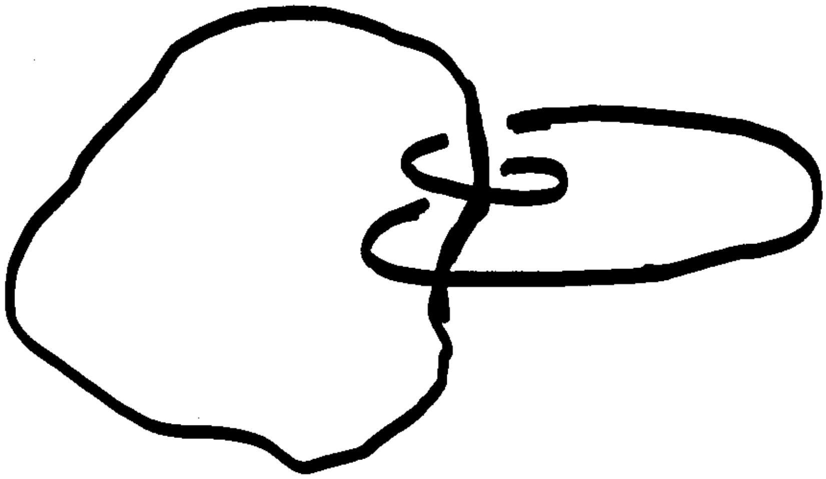

- dissipation
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$D_\eta \sim \frac{c^2\eta}{4\pi}$ $\tau_A \sim L/c_A$ $\tau_R \sim L^2/D_\eta$

$$E + \frac{v}{c} \times B = \eta J + \cdots - \frac{m_e}{e}\frac{\partial v}{\partial t} + \cdots$$

resistivity — electron inertia

[other effects can be important*]

$$\eta \sim \frac{\nu}{\omega_p^2} \qquad J \sim nev \sim \frac{\omega_p^2 m_e v}{4\pi e}$$

$$\eta J \Big/ \frac{m_e}{e}\frac{\partial v}{\partial t} \sim \nu/\omega$$

$$\omega \sim \nu^\alpha \qquad \alpha < 1$$

$$\sim \nu^{(1-\alpha)} \rightarrow 0 \quad \text{if } \nu \rightarrow 0$$

---

*Hall term does not count $v \rightarrow v_e$, $\frac{d}{dt} \rightarrow \frac{d}{dt_e}$

$\nabla p/n$ does not count if $p = p(n)$

– anisotropy ⇒ Kinetic effects

→ reconnection of magnetic field lines can occur on time scales fast enough for dissipative effects to be unimportant (at least the JJ term)

→ magnetic energy can be transformed (apparently in a reversible way) in other forms of energy

→ Hamiltonian reconnection
no dissipation is present or at least it does not play an important role

is this magnetic reconnection?

yes because of the formation of current (and vorticity) layers.

$$B_e = B - \frac{m_e c}{e} \nabla \times v$$

$$\frac{d}{dt}\left(B_e \times dl\right) = 0$$

magnetic energy goes into
ordered kinetic electron energy

—

this problem survives in a
kinetic treatment
Vlasov eq. is Hamilt.

—

\# B reconnects B_e does
not –
definition of reconnection
more important
does the conservation of B_e
impede the reconnection of B?
No – current layers

– 2 D discussion

2-D allows for a simple visualization of the reconnection process and for a ~~an~~ simpler analysis of the dynamics

standard picture

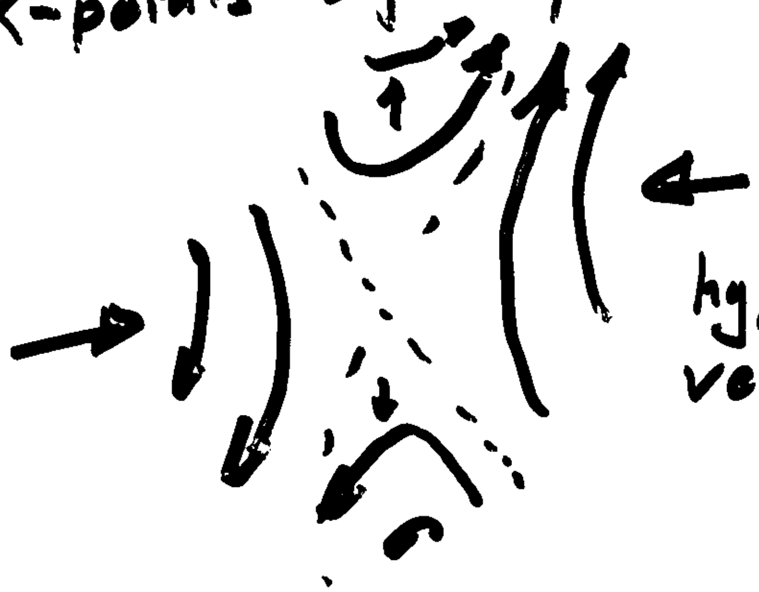
$\vec{B}_{plane} = \vec{B}(x,y)$

x, y and time

$B_z(x,y)$

constant

X-points of $\vec{B}_{plane}$



neutral line is a degenerate case

–

current density around the X point

simplest 2-D geometry

$$\vec{B} = B_0\left(\vec{e}_z + \vec{e}_z \times \nabla \psi\right)$$

$\psi = \psi(x, y, t)$ z-comp. vect. pot.

$$\vec{E} = -\nabla \phi + \frac{B_0}{c}\frac{\partial \psi}{\partial t}\vec{e}_z$$

$\phi = \phi(x, y, t)$ electrostatic potential

→ equations

continuity eq for the electrons

momentum balance

with $\frac{\partial}{\partial z} = 0$

It is relatively simple to include in a two fluid description (electron fluid - ion fluid) non MHD effects in order to model collisionless reconnection

- first we include electron inertia

- . . .

- goal full kinetic treatment of the electron response

-

in order to illustrate
the effect of electron inertia
first we consider the simplest
case cold electrons

continuity eq.

$$\frac{\partial n}{\partial t} + \nabla \cdot n v = 0$$

$$m_e n \left(\frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -en \left(E + \frac{v}{c} \times B \right)$$

electron inertia counts only
in the parallel direction
velocity in the x-y plane

$$v_\perp = \frac{c}{B_0} \vec{e}_z \times \nabla \phi + v_z \vec{e}_z \times \nabla \psi$$

$$n = n_0(x) \left[1 + \tilde{n}(x, t) \right]$$

$$\frac{\partial}{\partial t}\ln n + \frac{c}{B_0}[\phi, \ln n] - \frac{cB_0}{4\pi n e}[\psi, J] = 0$$

$J = \nabla^2\psi$

$$[A, B] = e_z \cdot \nabla A \times \nabla B = \frac{\partial}{\partial x}\frac{\partial}{\partial y} - \frac{\partial}{\partial y}\frac{\partial}{\partial x}$$

- rescaled

$$\frac{\partial}{\partial t}\ln n + [\phi, \ln n] - [\psi, J] = 0$$

parallel momentum balance

$$\frac{\partial \psi_e}{\partial t} + [\phi, \psi_e] = 0$$

$$\psi_e = \psi - d_e^2 \nabla_\perp^2 \psi$$

$d_e = c/\omega_p$

let us examine these eq.

$$\frac{\partial \psi_e}{\partial t} + [\phi, \psi_e] = 0$$

ψ_e is conserved

$$\frac{d}{dt} \psi_e = 0$$

true for all functions of ψ_e

in Euler hydro-vorticity eq.

$$\frac{d}{dt} F(\psi_e) = 0$$

$$\int dx\,dy \; F(\psi_e) = \text{const}$$

if electron inertia is neglected - conservation of the magnetic flux ψ

$$\frac{\partial}{\partial t} \ln n + [\phi, \ln n] = [\psi, J]$$
$$= [\psi_e, J]$$

$$\frac{d}{dt} \ln n \neq 0$$

$$\int dx\, dy \ln n \; G(\psi_e) = \text{const}$$

— Ion eq.

$$\ln n = \ln n(\phi)$$

cold ions
MHD-type solution

$$\ln n = \rho_s^2 \nabla_\perp^2 \Phi \qquad \phi = \frac{e\varphi}{T_e}$$

vorticity

$$= \frac{m_i c^2 e}{(e B_0)^2} \nabla_\perp^2 \varphi$$

← polirization drift

This model tells us something about the reconnection process can occur

ψ_e is conserved

ψ should reconnect

— linear theory – tearing mode –

the current is limited not by resistive dissipation but by the work done against electron inertia –

– if ψ is smooth $d_e^2 \nabla^2 \psi$ is small –

current layers are formed where $\psi_e \neq \psi$ decoupled

$\psi \sim d_e^2 \nabla^2 \psi$ are of the same order

– more general set of eq- where electrons are not taken to be cold

pressure [and gyroviscosity] included

momentum balance is changed

(simpl. isothermal plasma)

$$m_e n \left(\frac{\partial v}{\partial t} \cdots \right) = -en \left(E \right.$$

$$- \nabla n T_e - \nabla \cdot \Pi$$

$$\hookrightarrow T_e \nabla n$$

$$\underline{v} = \cdots\cdots \quad - \frac{c T}{e B_0} e_z \times \nabla \ln n$$

$$\frac{\partial \psi_e}{\partial t} + [\phi, \psi_e] + [\psi, \ln n] = 0$$

compressibility of electrons along field lines –

quasi neutrality

ions move $\bar{E}\times B$ drift
polarisation
drift

electrons move $\bar{E}\times B$ drift
pressure gradient
drift (* density
divergence free in
this geometry)
move along total
field lines

quasi neutrality requires that
since there is no net current
in the $\bar{E}\times B$ drifts
the contribution of the
parallel compressibility of the
electrons counts in Ohm's law

when $\rho_s^2 \nabla^2 \phi \sim \phi$

i.e. finally if $\rho_s \gtrsim d_e$

ρ_s = gyroradius with ion mass
and electron temp.

more formal point of view –

set of eq. that extend
RMHD (2-D incompressible)
in a simple geometry (no curvature)
but include d_e, ρ_s
(outside RMHD)

$$\frac{\partial \psi_e}{\partial t} = a\left[\phi, \psi_e\right] + a\left[\psi, \ln \frac{n}{n_0}\right]$$

$$\frac{\partial}{\partial t} \ln \frac{n}{n_0} + a\left[\phi, \ln \frac{n}{n_0}\right] - \frac{a}{\beta_e}\left[\psi, J\right]$$

$a = cT_e/(eB_0)$
$\beta_e = 4\pi n_0 T_e / B_0^2$

$\ln \frac{n}{n_0} = \rho_s^2 \nabla^2 \phi$ – the ion response can be treated in a more general form – see notes –

ψ_e is no longer conserved

new invariants are

$$\left.\begin{array}{l}\psi_e \pm \beta_0^{1/2} d_e \ln n/n_0 \\ \psi_e \pm \beta_0^{1/2} d_e \ell_s^2 \nabla^2 \phi\end{array}\right\} \equiv G_\pm$$

$$\frac{d}{dt_+} G_+ = 0 \qquad \frac{\partial}{\partial t} G_+ + [\phi_+, G_+] = 0$$

$$\frac{d}{dt_-} G_- = 0 \qquad \frac{\partial}{\partial t} G_- + [\phi_-, G_-] = 0$$

$$\phi_\pm = \phi - \ln \frac{n}{n_0} \pm \frac{d_e}{\beta_0^{1/2}} \nabla_\perp^2 \psi$$

– different forms of $\phi_\pm$ can be given

$$\phi_+ \to \phi_+ + \alpha G_+$$

$$\int F_+(G_+)\, dx\, dy = \text{const}$$

$$\int F_-(G_-)\, dx\, dy = \text{const}$$

let us put order in all these different forms of the equations.
diff. invariants - etc -

Hamiltonian ??

- energy is conserved
  no dissipation

- we thus have an energy density but where is the Hamiltonian

→ it can be shown that —

$$\frac{\partial \varphi_i}{\partial t} = \{\varphi_i, H\}$$

$\varphi_i = G_\pm$ in our case

$\{\ ,\ \}$ non canonical Poisson brackets – i.e. no p's or q's canonically conjugated are used

$$\{F, G\} = -\{G, F\}$$

$$\{F, GH\} = \{F, G\}H + G\{F, H\}$$

$$\{F, \{G, H\}\} + \{G, \{H, F\}\} + \{H, \{F, G\}\} = 0$$ Jacobi identity

H = hamiltonian functional

$$H = \frac{1}{2}\int dx\,dy\,\left[|\nabla_\perp \psi|^2 + d_e^2 J^2 + \beta_e \ln^2 \frac{n}{n_0} + \beta_e \ell_s^2 |\nabla_\perp \phi|^2 \right]$$

$B_\perp^2$ / $m_e v_e^2$ ...

thermal energy of the electrons , ion kinetic energy

invariants

$$\{A, B\} = 0 \qquad \forall B$$

Kernel of the Poisson brackets

$$A = F_+(G_+) + F_-(G_-)$$

two infinite sets of invariants - Casimirs

no X-points

(area) can be formed in G_+ and G_-

consequences for collisionless reconnection

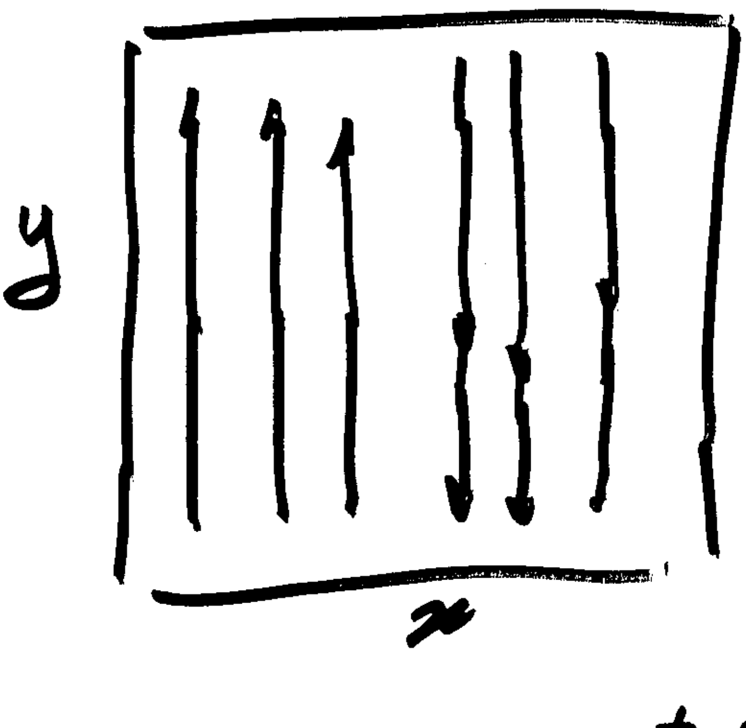
simulations

$\rho_s << d_e$ $\quad$ ψ_e is conserved

$\rho_s \gtrsim d_e$ $\quad$ $G_\pm$ are conserved

what are the dynamical consequences of these different conservations?

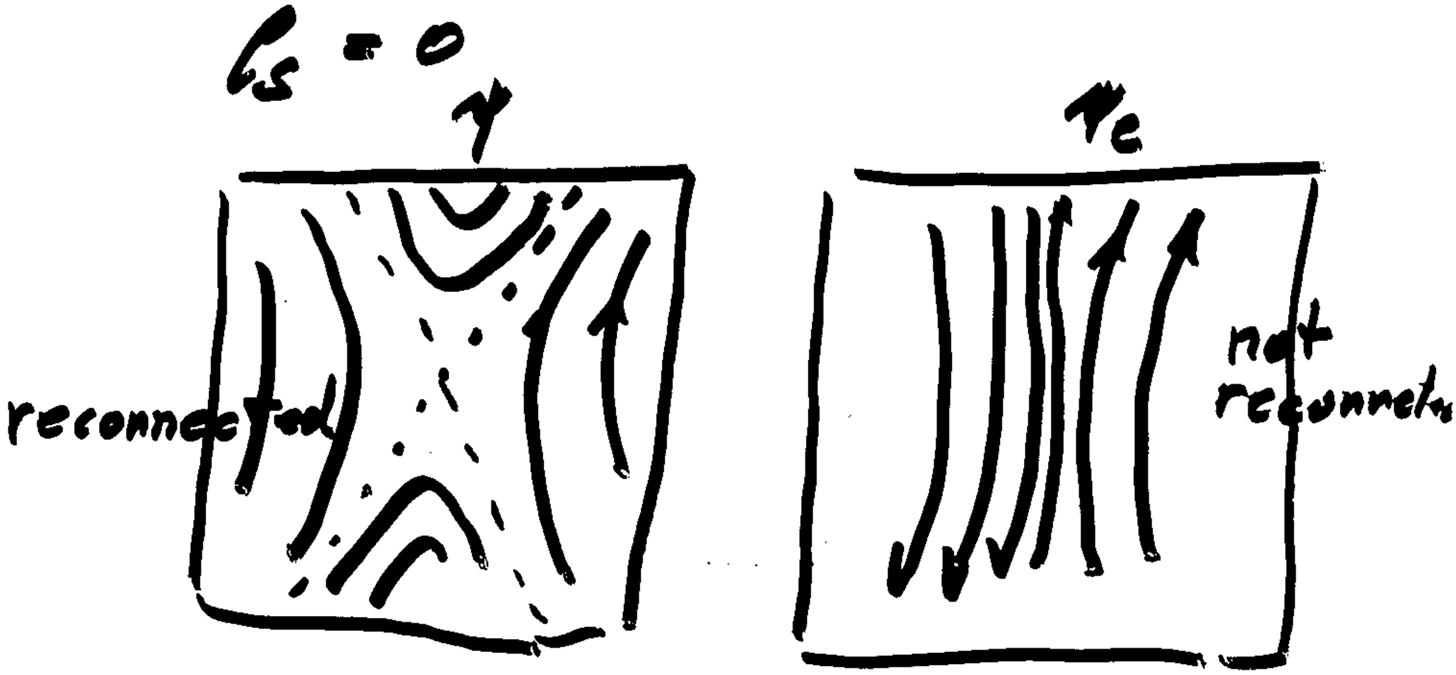
periodic box

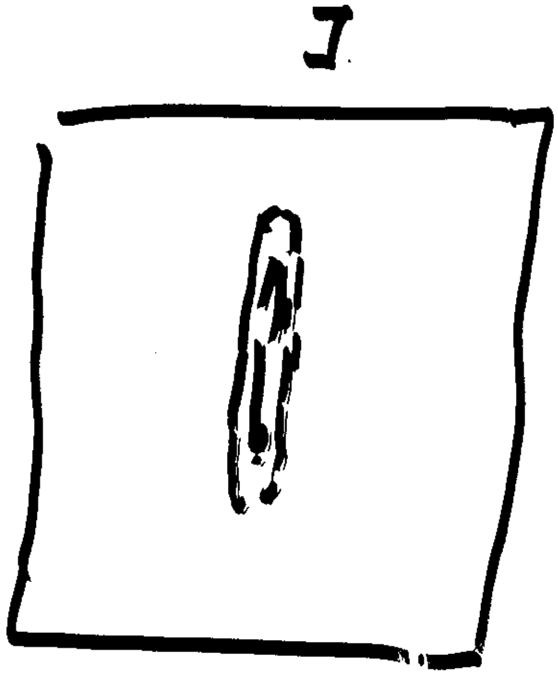


$\psi \sim \sin \frac{\pi x}{L}$

linearly unstable against a single mode –

destabilized by current gradient + electron inertia





J is localised in a bar shaped structure which effectively disconnects ψ_e and ψ

smaller and smaller spatial scales are formed

(in the regime considered large Δ' the mode continues to exponentiate)

Conclusions –

- models for collisionless reconnection

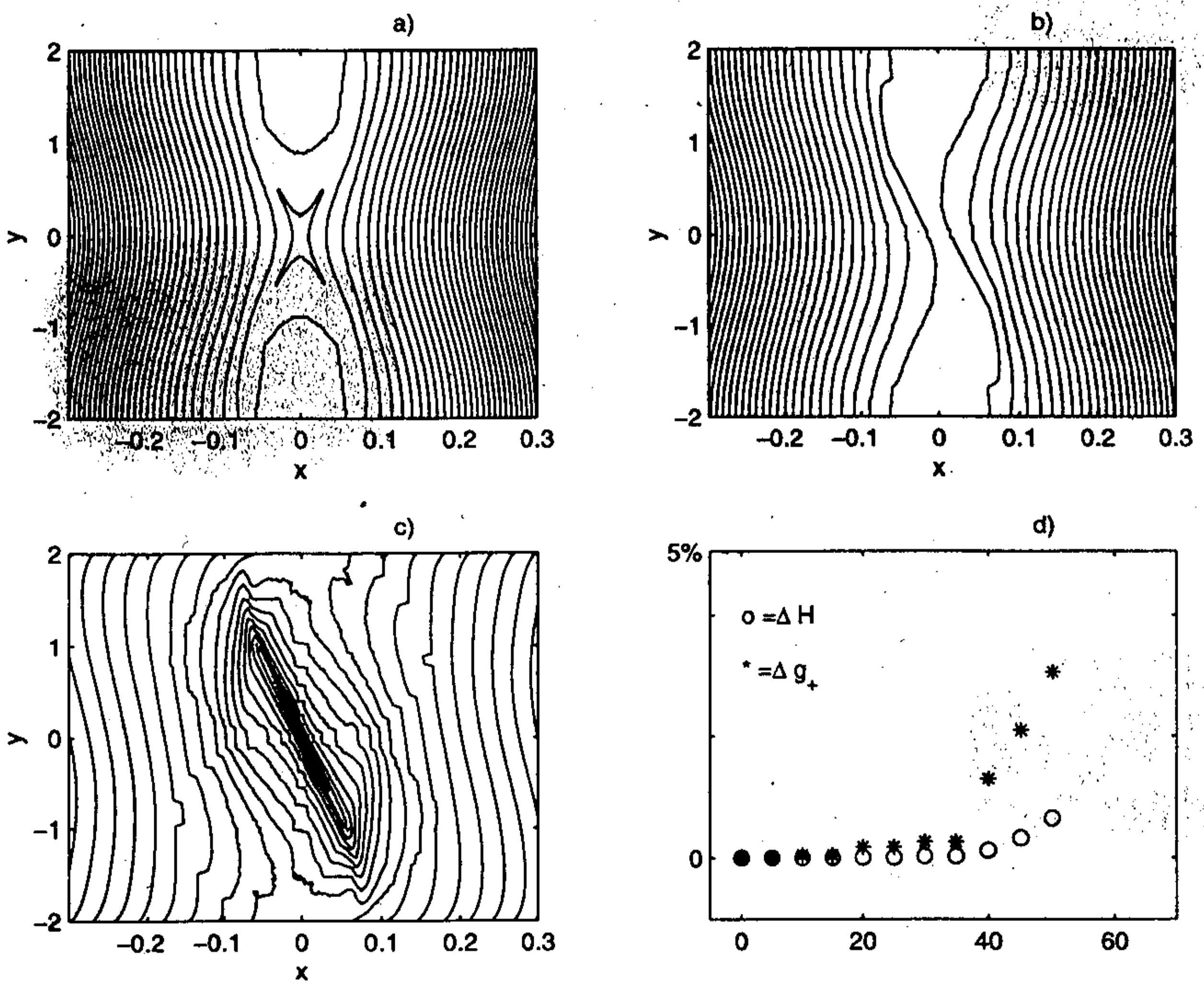
- reconnection is made possible by the formation of current and vorticity layers

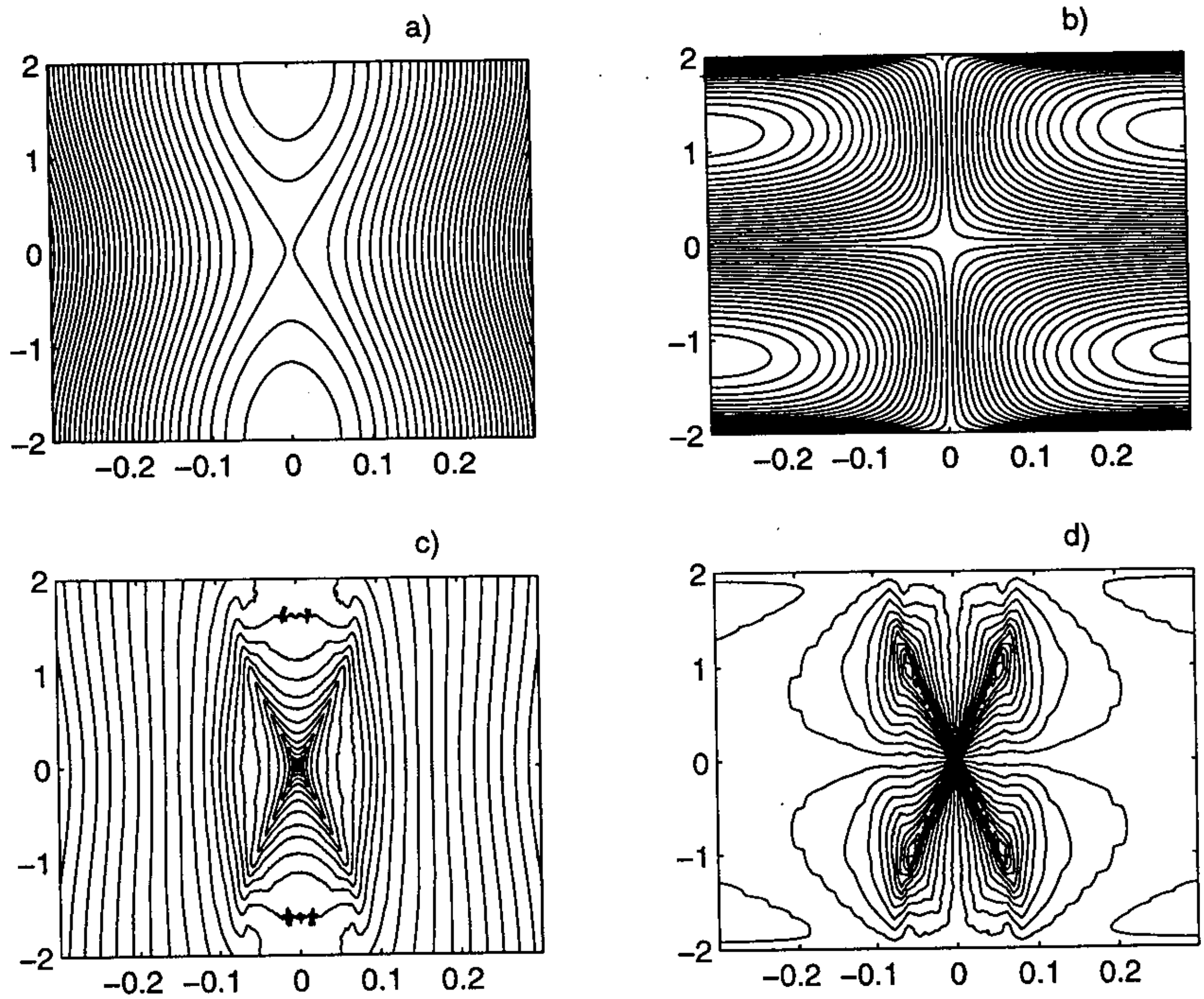
  current layer disconnects ψ from ψ_e

  vorticity layer disconnects ψ_e from $G_\perp$

Problems – smaller and smaller spatial scales – large electron velocity and ion vorticity gradients – e.g. electron viscosity could regularize the current density –

electron kinetic treatment
large localized currents
two stream instability
slide away instability
...
— it would be interesting to include these effects by respecting the Hamiltonian structure of Vlasov eq.


a)
b)
c)
d)
y
x
2
1
0
−1
−2
−0.2
−0.1
0
0.1
0.2
0.3
5%
o =Δ H
* =Δ g_+
20
40
60



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