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AUTUMN COLLEGE ON PLASMA PHYSICS

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Relations Between Eletromagnetic Plasmas and Stellar Systems

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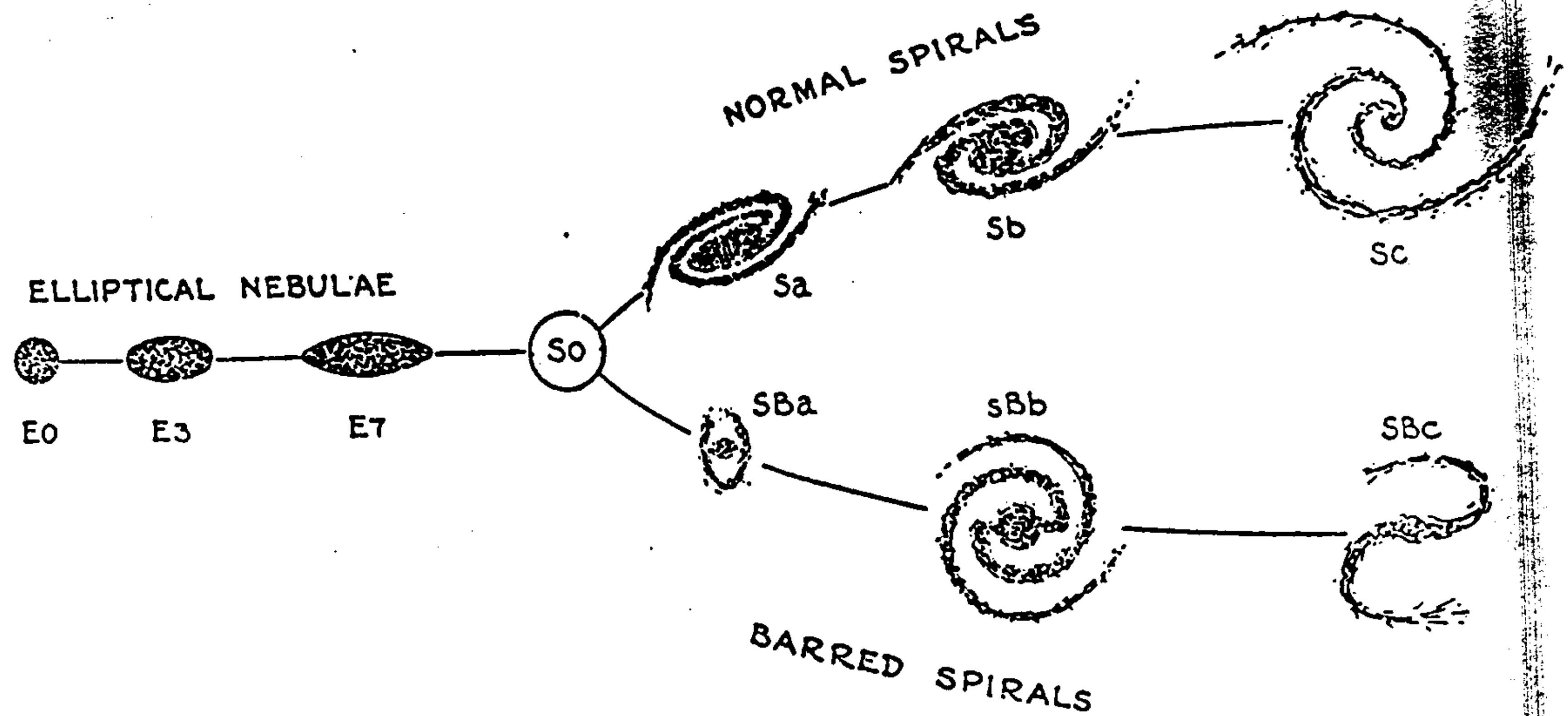
These are lecture notes, intended for distribution to participants.

I A

BASIC PHENOMENOLOGY

"grand design" vs "flocculent"



D. Elmegreen Ap.J. Suppl. 47 (1981)

M81

M51

M74

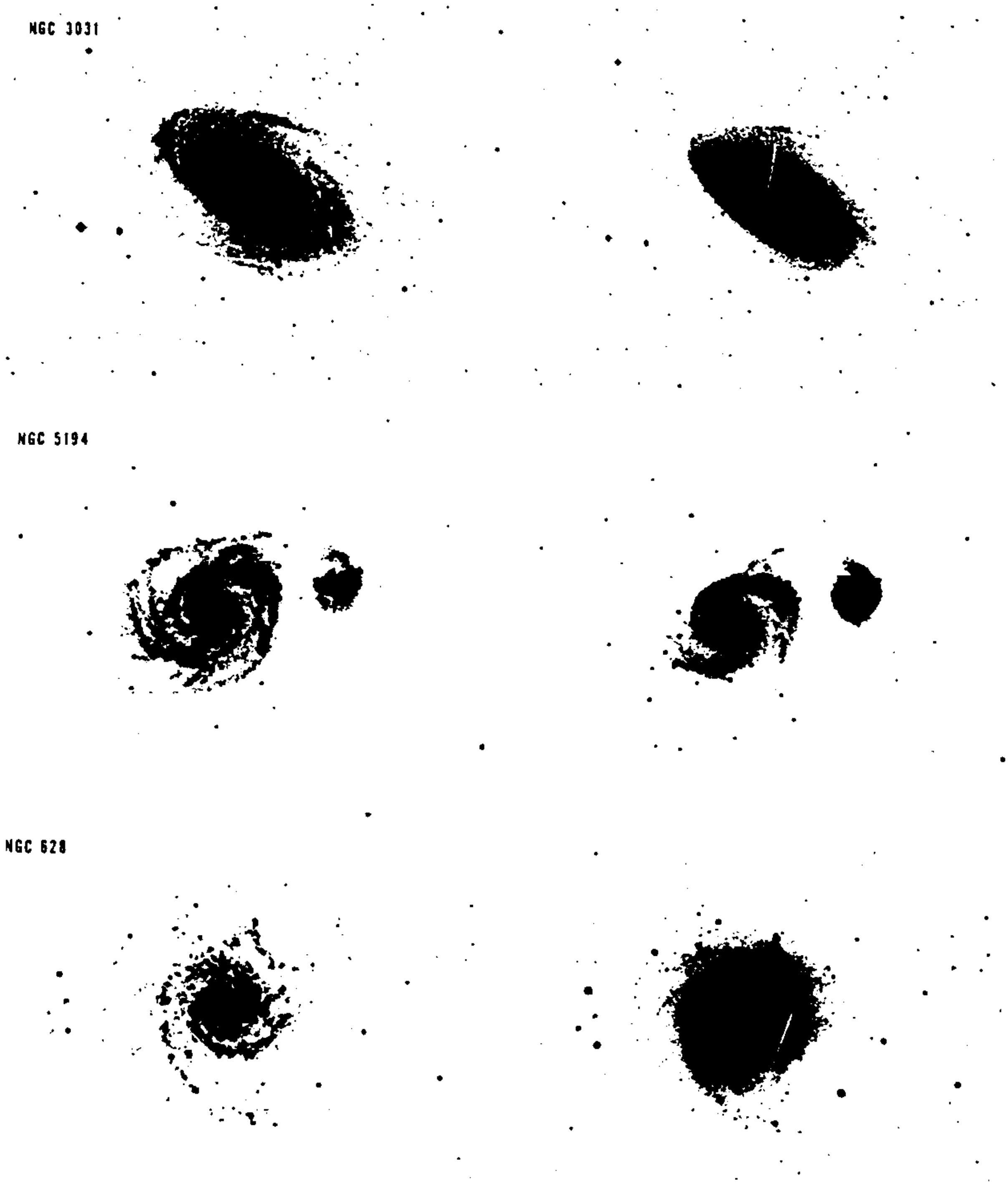


FIG. 1.—All of the following figures show the blue (103a-0 + GG13) photographs on the left, and the near-infrared (IV-N + Wr 88A) on the right. NGC 3031: north is to the right. NGC 5194: N is to the right. NGC 628: N is to the top.

ELMEGREEN (*see* page 230)

SCALES

$M \sim 10^{9} - 10^{12}\ M_\odot$ $\qquad (1 M_\odot \sim 2\times10^{33}\,g)$

$L \sim 1 - 100$ kpc $\qquad (1\,\mathrm{kpc} \sim 3\times10^{21}\,\mathrm{cm})$

T $\qquad t_{dyn} \sim 10^{8} - 10^{9}$ yrs

$t_{Hubble} \sim 10^{10}$ yrs

$\tau_{**} \sim 10^{11} - 10^{12}$ yrs $\qquad$ star-star relaxation

$\tau_{gas} \sim 10^{7}$ yrs $\qquad$ cloud-cloud collisions

Milky Way (Our Galaxy)

$M \sim 2\times10^{11}\ M_\odot$ $\qquad \sigma_\odot \sim 50\ M_\odot/pc^2$ *

$\rho_\odot \sim 0.1\ M_\odot/pc^3$

$R_\odot \sim 8$ kpc $\qquad$ distance of the Sun from galactic center

$V_{rot} \sim 220$ ~~kpc~~ k/sec

$T_\odot \sim 200\times10^{6}$ yrs $\qquad \dfrac{t_{Hubble}}{T_\odot} \approx 60$ *

$\varepsilon_B \sim \varepsilon_{cr} \sim \varepsilon_{BB} \sim 1\,\mathrm{eV/cm^3}\ ;\ \frac{1}{2}\rho V^2 \sim 1\,\mathrm{keV/cm^3}$

Habing et al (1985) IRAS

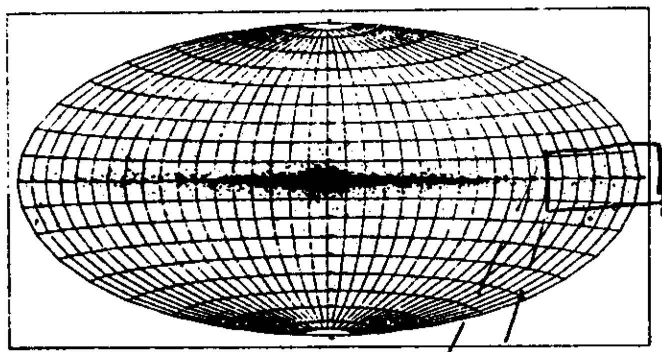
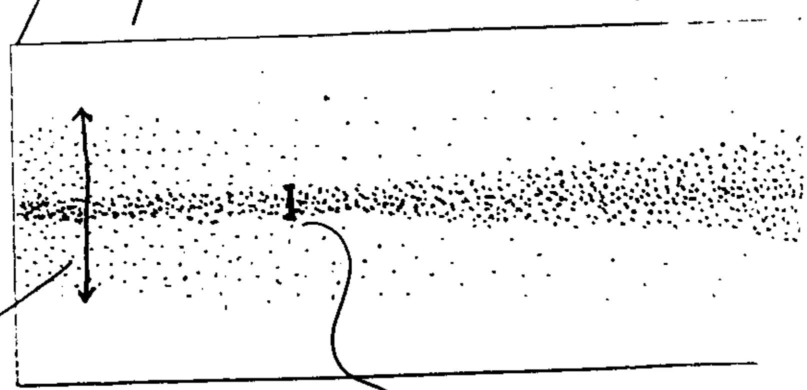


Figure 2. The distribution of sources in the sky with flux densities at 12 μm, $f_\nu(12)$, satisfying $1 < f_\nu(12) < 5$ Jy and $0.5 < f_\nu(12)/f_\nu(25) < 1.5$. The grid is plotted in Galactic coordinates in an Aitoff projection and the Galactic center is in the middle of the picture.

sketch of 3-D structure of a galaxy disk



thick optical disk
(stars)

thin cold component
(gas)
HI, molecules

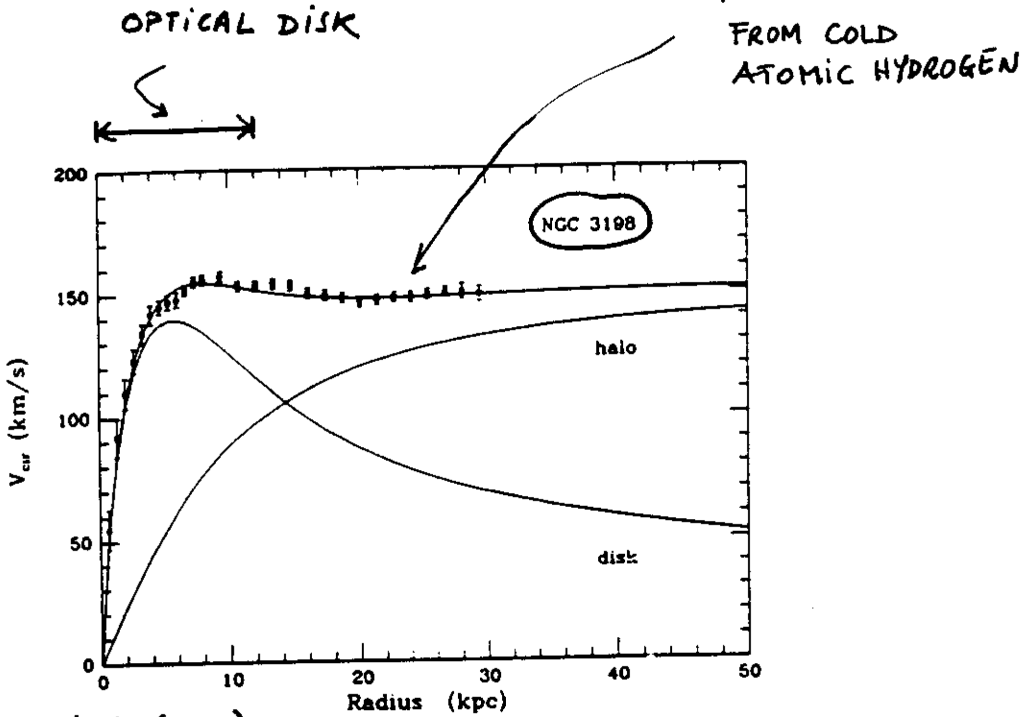
OUR PERCEPTION OF THE BASIC STATE OF GALAXIES HAS CONSIDERABLY CHANGED IN THE LAST TWO DECADES AS A RESULT OF THREE MAJOR SURPRISES:

1. FLAT ROTATION CURVES &
   MASSIVE DARK HALOS (SPIRALS)

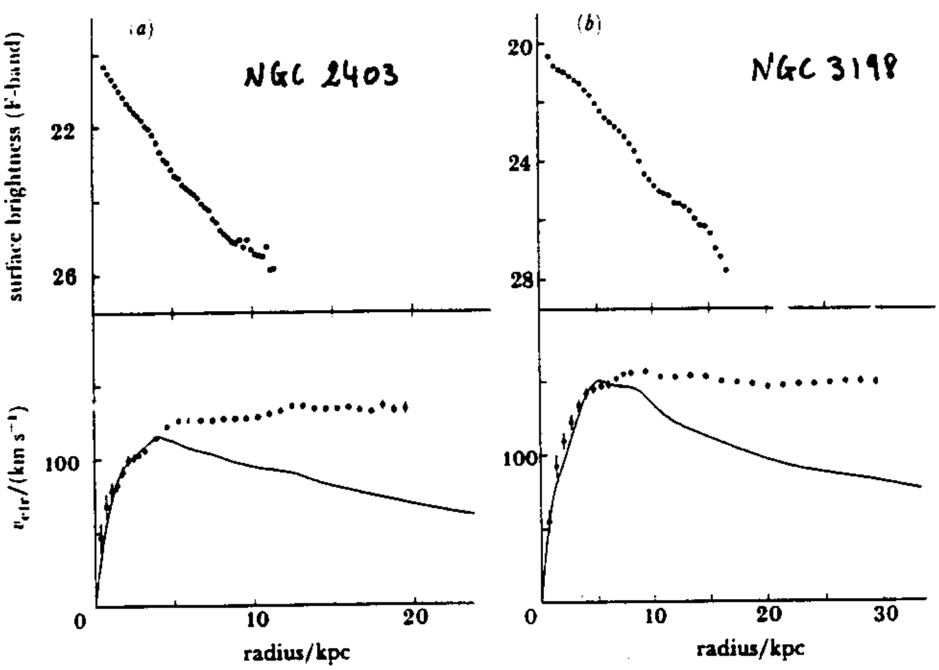
2. LOW ROTATION IN MANY ELLIPTICALS,
   DOMINATED BY (ANISOTROPIC) PRESSURE

3. COHERENT STRUCTURES IN THE UNDERLYING
   STELLAR MASS DISTRIBUTION PROBED BY
   NEAR-INFRARED IMAGING ($\approx 2\mu$)

③



van Albada et al (1985)
, Bahcall, Begeman, Sancisi



van Albada & Sancisi (1986)

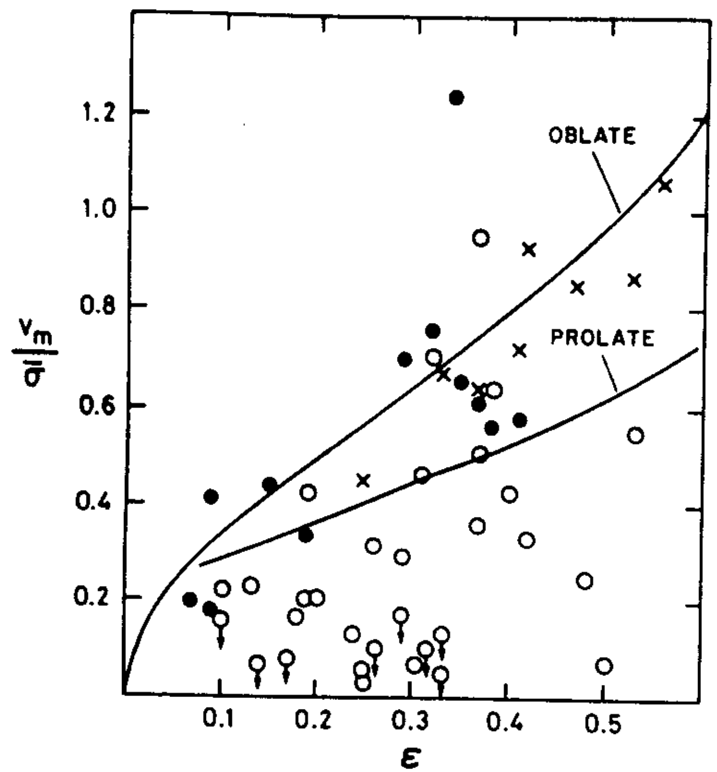


Figure 1: Peak rotation velocity plotted against velocity dispersion; open circles are luminous ellipticals ($M_B < -20.5$), filled circles are lower luminosity ellipticals and crosses are the bulges of spirals [this figure adapted from DEFIS]. The mean line for oblate isotropic galaxies and the median line for prolate isotropic galaxies are shown.

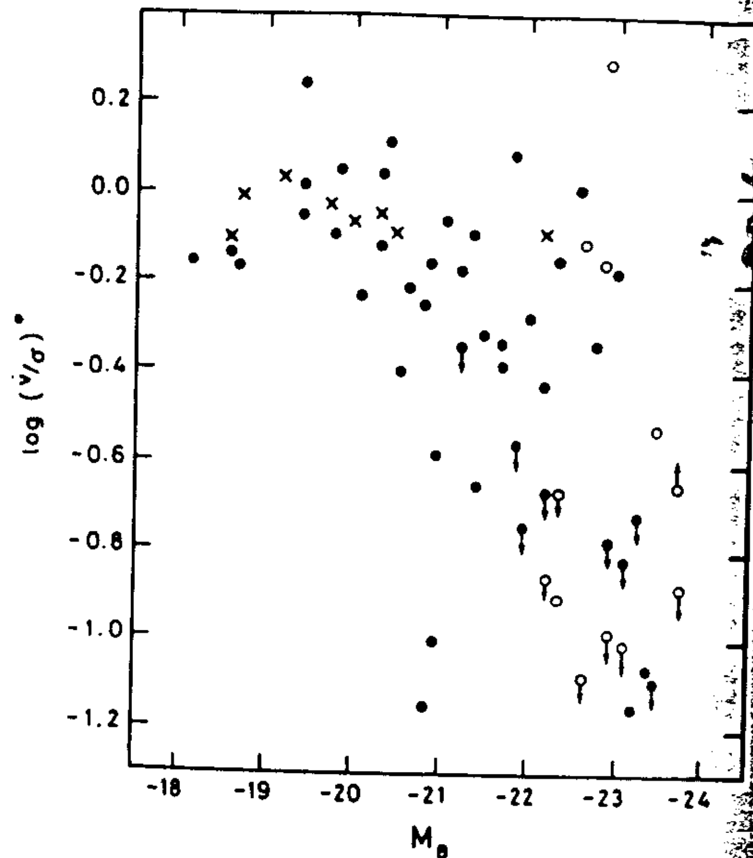


Figure 2: Log $(v/\sigma)^*$, the value of v/σ normalized to the value expected for oblate isotropic galaxies, plotted against absolute magnitude. This plot is taken from DEFIS with the cD and brightest cluster galaxies of Tonry (1984, 1985) added as open circles, filled circles are ellipticals and crosses are bulges.

DEFIS = Davies et al Ap. J. 266 (1983)

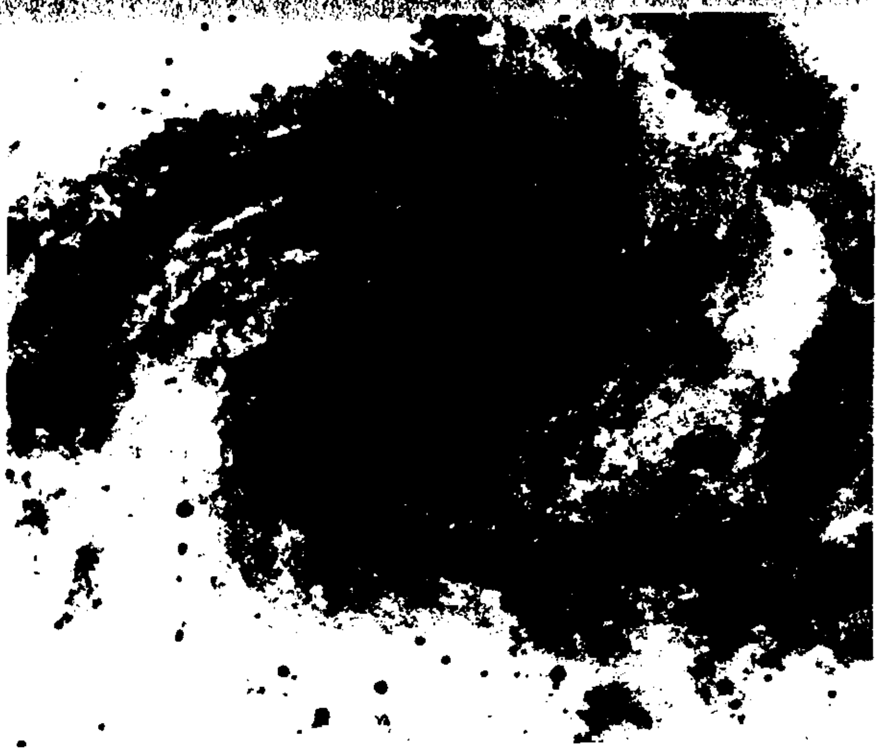


Figure 1: *A 90-minute blue exposure (GG385+IIIa-J) of NGC 2997 made by S. Laustsen at the 3.6-m prime focus in 1977.*

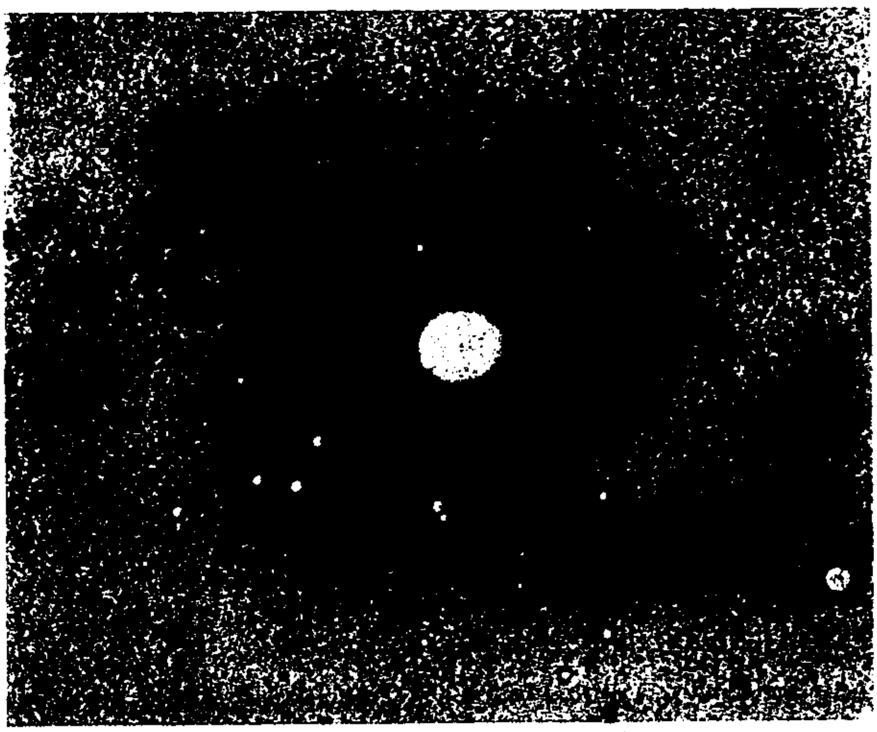


Figure 3: *Mosaic of NGC 2997 in K′ consisting of 6 fields each with a total of 15 minutes exposure taken at the 2.2-m with IRAC2.*

→ see Block, Bertin, et al 1994

I B

BASIC CONCEPTS AND MODELS

I. Galaxies as gravitational plasmas

... interesting problems of plasma physics in the

This is not at all a new way of looking at stellar systems. In fact, the work of Jeans (1915), with the goal of describing the motion of a large collection of stars under the influence of their own gravity forces, and the work of Chandrasekhar (1942), in setting out the equations for the effects of collisions in a stellar system, *precede* much of the work related to the Vlasov equation (1945) and to the various transport processes in electromagnetic plasmas. Thus it is no surprise that many scientists have contributed, either explicitly or indirectly, to both fields of e.m. and gravitational plasmas. As an example, the topic of*weakly collisional stellar systems* is nicely introduced in a book by L. Spitzer ("Dynamical evolution of globular clusters", 1987).

To be sure, the field of stellar dynamics is much less diversified than that of e.m. plasmas. This is probably more the result of the several cancellations related to the fact that $q/m = 1$ for the gravitational charge, which limits the number of relevant frequency windows, rather than the effect of the absence of the complexity of the full set of Maxwell's equations (with classical gravity, we only need to keep track of the Poisson equation).

The study of *galactic dynamics* usually proceeds through the use of some *model equations*, which turn out to be very rich in quantitative predictions. Most papers deal with the study of a one-component distribution function subject to the coupled Vlasov-Poisson equations or of fluid equations supplemented by a barotropic equation of state for a single gravitational fluid. A major problem is the adequacy of the adopted model equations in describing a specific, complex physical system, such as a galaxy.

COMPARISON WITH ELECTROMAGNETIC PLASMAS:

A CONCISE DICTIONARY

SELF-GRAVITATING SYSTEMS	ELECTROMAGNETIC PLASMAS
NEWTON $1/r$	COULOMB $1/r$
ROTATION Ω (10^8 yrs)	MAGNETIC FIELD B 1.3 10^4 Gauss
CORIOLIS $2\Omega \times v$	LORENTZ $\frac{q}{c} v \times B$
EPICYCLIC OSCILLATIONS (10^8 yrs $\approx 3\times10^{15}$ sec)	LARMOR (CYCLOTRON) GYRATIONS $\omega_{ce} \sim 2\times10^{11}$ rad sec^{-1}
JEANS λ_J (1 kpc $\approx 3\times10^{21}$ cm)	DEBYE $\lambda_D \sim 2\times10^{-3}$ cm
$4\pi G\rho$	PLASMA FREQUENCY $\frac{4\pi e^2 n}{m_e}$ $\omega_{pe} \sim 2\times10^{12}$ rad sec^{-1}
10^8 yrs $\approx 3\times10^{15}$ sec DYNAMICAL TIME SCALE ROTATION PERIOD / CROSSING TIME $t_{cr} = GM^{5/2}/\lvert 2E\rvert^{3/2}$	ALFVEN TIME SCALE 10^{-5} sec
ROTATION SUPPORTED SYSTEMS (DISKS: SPIRALS, PLANETARY RINGS) $t = \kappa_{ord}/\lvert W\rvert$	MAGNETICALLY CONFINED PLASMAS β
STARS IN GALAXIES ARE ESSENTIALLY A COLLISIONLESS COMPONENT $\tau_R >$ Hubble time	HIGH TEMPERATURE PLASMAS

Igniton-type plasma

$n \sim 10^{15}$ cm^{-3}

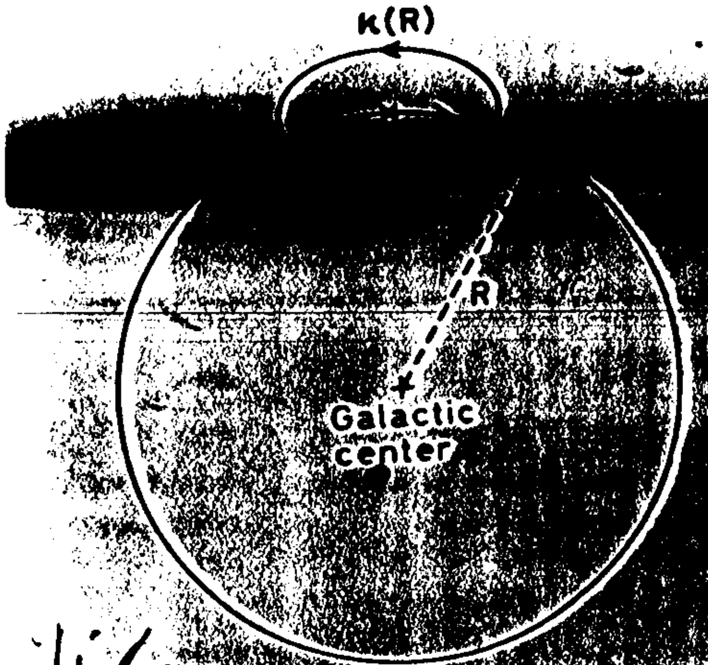
$T \sim 10$ keV

$B \sim 1.3$ 10^4 Gauss

$L \sim 1$ m

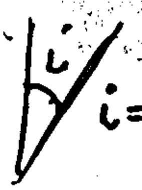
$\nu_e \sim 3\times10^4$ sec^{-1}

$\frac{4}{3}\pi n \lambda_D^3 \sim 5$ 10^7

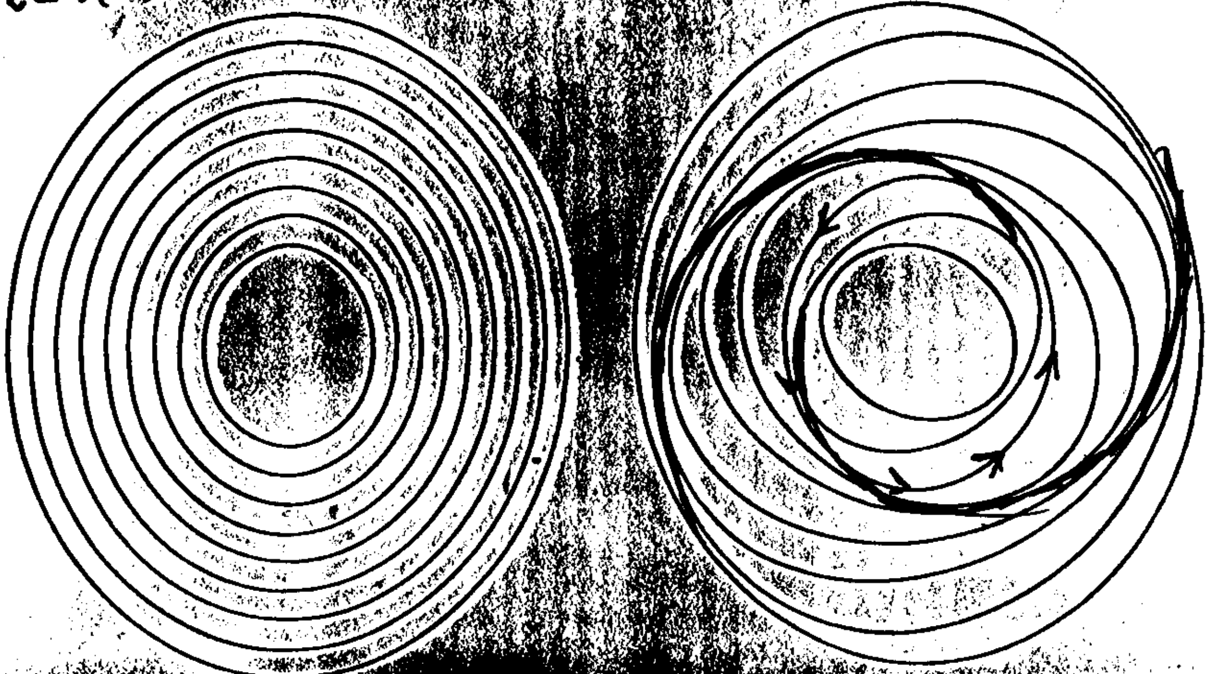


Kinematic waves
(no self-gravity,
no random motions)

B. LINDBLAD



$i = i(r)$



$$\nu = -1 \Rightarrow |\Omega - \Omega_p| = \frac{\kappa}{2}$$

$t = \pi/\omega_0$

top view

III (a) ORBITS

1) TRAPPING AT RESONANCES
(see Contopoulos 1973 ; Lynden-Bell 1973)

AND DE-TRAPPING ......

2) GUIDING CENTER ORBITS

AND STELLAR HYDRODYNAMICS
(Berman & Mark 1979)

Integrability ....
Chaos ....

Guiding center equations:

$$\begin{cases} \dot{r}_0 = -\dfrac{2\Omega}{r_0\kappa^2}\dfrac{\partial H_c}{\partial \phi_0} - \dfrac{r_0}{\kappa^2}\dfrac{\partial \Omega^2}{\partial t} \\ \dot{\phi}_0 = \dfrac{2\Omega}{r_0\kappa^2}\dfrac{\partial H_c}{\partial r_0} \end{cases}$$

$$H_c = \frac{1}{2} r_0^2 \Omega^2 + \Phi - r_0^2 \Omega \Omega_p$$

$$\Omega^2 \equiv \frac{1}{r_0}\frac{\partial \Phi}{\partial r_0} \quad ; \quad \kappa^2 \equiv 4\Omega^2\left[1 + \frac{1}{2}\frac{r_0}{\Omega}\frac{\partial \Omega}{\partial r_0}\right]$$

$$\mu = \frac{H - H_c}{\kappa^2} .$$ adiabatic invariant

$\Rightarrow$ "double-adiabatic theory" for STELLAR HYDRODYNAMICS (with anisotropic pressure tensor)

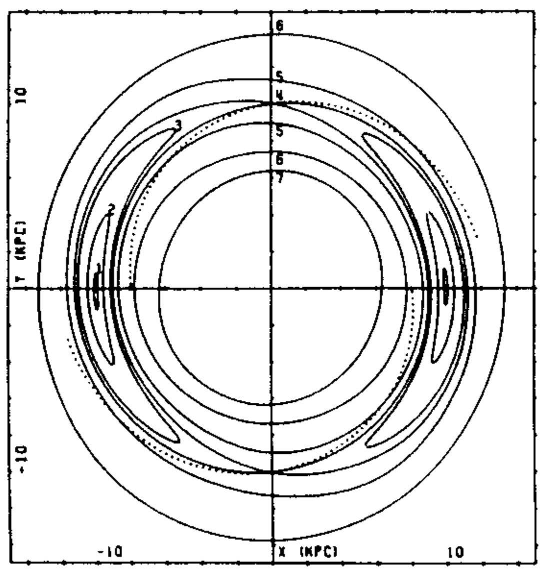
Berman & Mark 1979

(see Chew, Goldberger, and Low 1956)

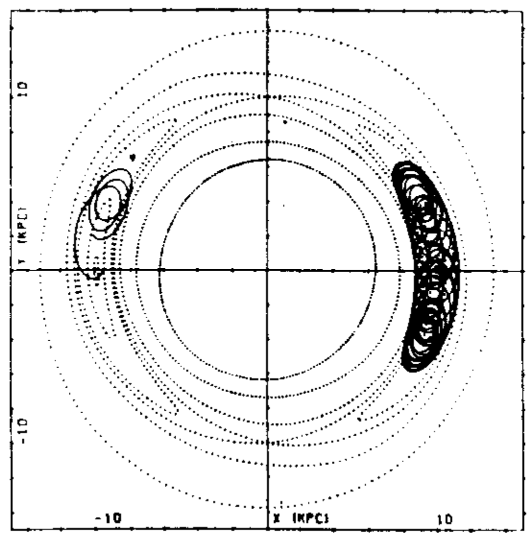
Duffing's equation :

$$\frac{d^2\xi}{d\tau^2} + \xi\left(1 - \frac{\xi^2}{6}\right) = a\cos(\omega\tau)$$

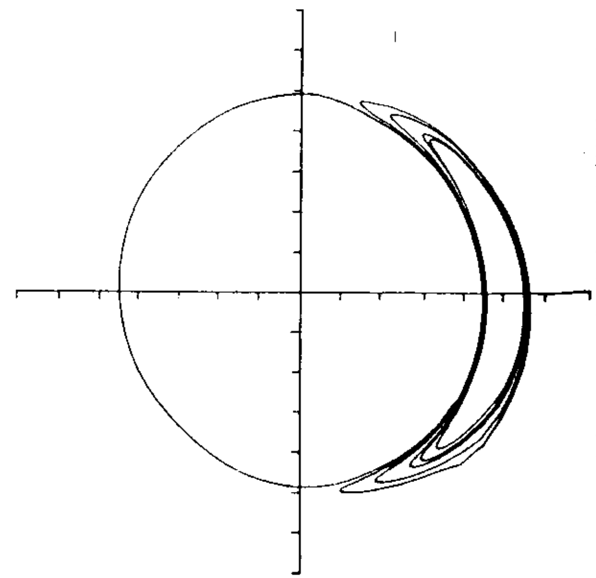
$$a_{crit} \approx 0.115$$



Berman & Mark 1979



Contopoulos 1973



Bertin, Coppi, Taroni 1977

Coppi, Minardi, Schram 1972
Coppi, Taroni 1974

CORОTATION

Berman 1975

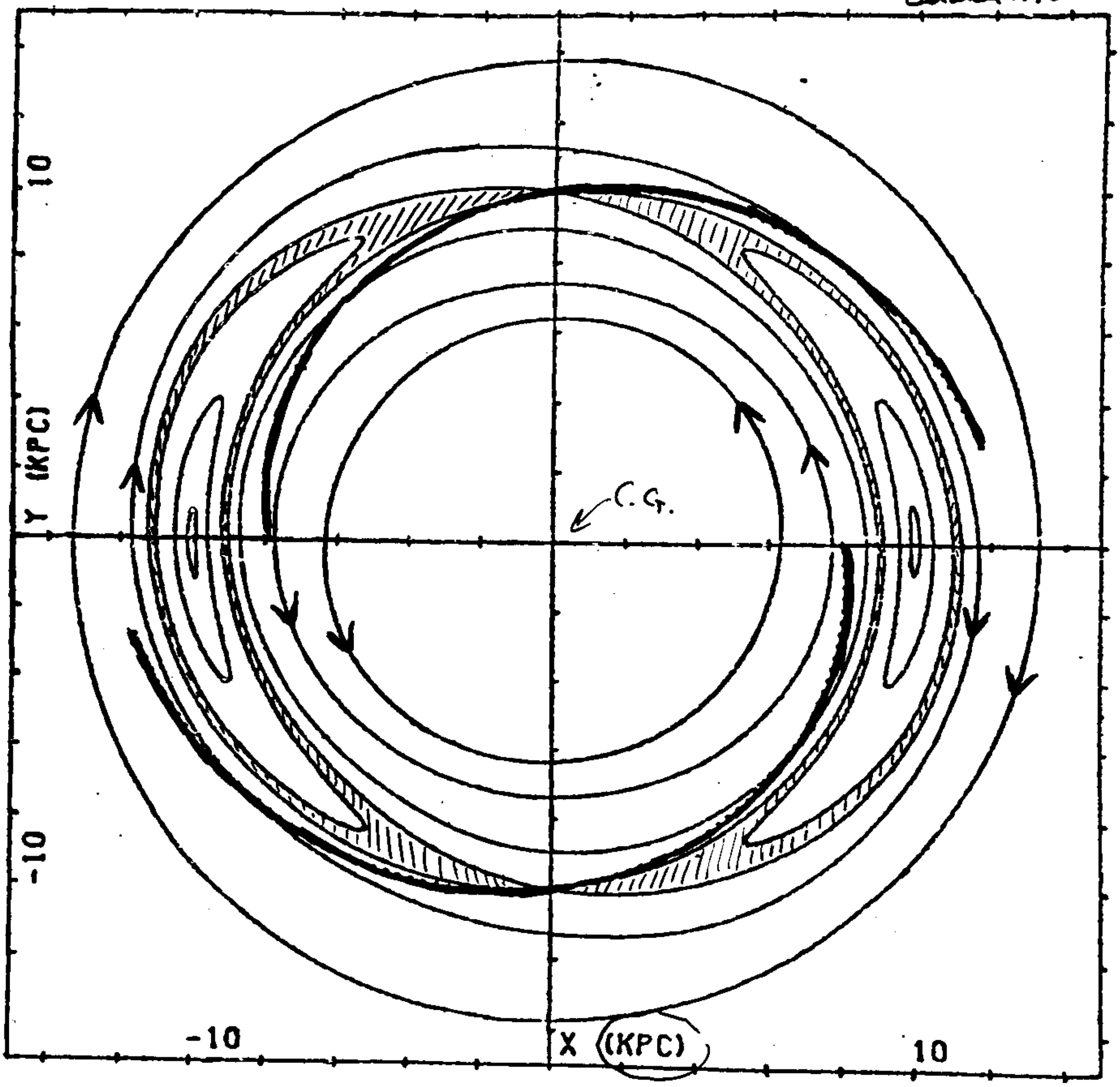


Fig. 17

Ω_p

$\underline{B} \leftrightarrow \underline{\Omega}$

$1/r$

* density minima
potential maxima

2. FIRST PHYSICAL MODELS AND BASIC EQUATIONS

- FOR SEVERAL PURPOSES, A FIRST USEFUL MODEL OF A GALAXY IS THAT OF A COLLECTION OF N >>> 1 STARS IN MUTUAL GRAVITATIONAL INTERACTION. $N \approx 10^{11}$
  (IN REALITY, THE DISSIPATIVE INTERSTELLAR MEDIUM PLAYS A CRUCIAL ROLE IN THE DYNAMICS OF THE DISK) $M_{gas} \ll M$

- COLLISIONALITY: LARGE STELLAR SYSTEMS ARE GENERALLY COLLISIONLESS AND FAR FROM THERMODYNAMICAL EQUILIBRIUM !
  (-------> MUCH IN LINE WITH THE CASE OF HIGH TEMPERATURE PLASMAS)
  Chandrasekhar

- TWO LIMIT DESCRIPTIONS USED:

1) CONTINUUM LIMIT: ONE-PARTICLE DISTRIBUTION FUNCTION f (x,v,t) GOVERNED BY VLASOV + POISSON (MEAN FIELD THEORY)
   Jeans

2) N-BODY SIMULATIONS
   (BUT EACH 'PARTICLE' REPRESENTS AT LEAST 100,000 STARS OF THE REAL SYSTEM ! ---> INHOMOGENEITIES ??? FLUCTUATIONS ??? RELAXATION ??? RESONANCES ???)

- FLUID MODELS
  → cf . classical ellipsoids
  (Maclaurin, Jacobi, Riemann, Poincaré ....)

$$\tau_R = \frac{v^3}{8\pi G^2 m^2 n \ln\Lambda}$$

$$\frac{\tau_R}{\tau_{cr}} \sim \frac{N}{8\pi \ln N}$$

CONTINUUM DESCRIPTION

Simplest description is in terms of a one-component system, characterized by one-star distribution function f and mean potential Φ

$$f = f(\underline{x}, \underline{v}, t)$$

$$\Phi = \Phi(\underline{x}, t)$$

under the Hamiltonian (per unit mass)

$$H = \frac{1}{2} v^2 + \Phi$$

$$\begin{cases} \dfrac{Df}{Dt} \equiv \dfrac{\partial f}{\partial t} + \{f, H\} = 0 & \text{Vlasov (collisionless Boltzmann)} \\[2ex] \nabla^2 \Phi = 4\pi G \int f \, d^3 v & \text{Poisson} \end{cases}$$

(comparison with observations usually assumes M/L = constant)

ABSENCE OF THERMODYNAMICAL EQUILIBRIUM

In many cases the observed stellar system have a clear signature of <u>pressure anisotropy</u> (solar vicinity, ellipticals not flattened by rotation).

$\Longrightarrow$ $f = f(E)$ is not viable!

$\Longrightarrow$ "thermodynamical equilibrium" not reached!

$$\bar{v} = \frac{1}{\rho} \int v f \, d^3 v$$

$$p_{ij} = \int (v_i - \bar{v}_i)(v_j - \bar{v}_j) f \, d^3 v$$

$$f = f\left(\tfrac{1}{2} v^2 + \Phi\right) \Rightarrow \bar{v} = 0 \quad ; \quad p_{ij} = p \, \delta_{ij}$$

BASIC EQUATIONS OF THE ONE-COMPONENT FLUID MODEL

$$\frac{\partial \sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\sigma u) + \frac{1}{r}\frac{\partial}{\partial \theta}(\sigma v) = 0$$ mass

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{\partial}{\partial r}(\Phi + h)$$ momentum

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{uv}{r} = -\frac{1}{r}\frac{\partial}{\partial \theta}(\Phi + h)$$ momentum

$$\Rightarrow dh = c^2 \frac{d\sigma}{\sigma}$$ state

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \sigma \delta(z) + 4\pi G \rho_{BH}$$ Poisson

Linearize for density waves (ρ_{BH} kept fixed in a first approximation) $\exp(i\underline{\omega} t - im\theta)$

$$\begin{cases} \mathcal{L}(h_1 + \Phi_1) = -\mathcal{C} h_1 & \text{Euler + continuity} \\ \Phi_1 = \mathcal{P}(\sigma_1) = \mathcal{P}\left(\frac{\sigma}{c^2} h_1\right) & \text{Poisson} \end{cases}$$ integrodifferential

where

$$\mathcal{L} = \frac{d^2}{dr^2} + A\frac{d}{dr} + B$$

$$A = -\frac{d}{dr}\left\{\ln\left[\frac{(1-\nu^2)\kappa^2}{r\sigma}\right]\right\}$$

$$B = -\frac{m^2}{r^2} - \frac{4m\Omega}{r\kappa}\underline{\underline{\frac{1}{(1-\nu^2)}}}\frac{d\nu}{dr} + \frac{2m\Omega}{r\kappa\underline{\nu}}\frac{d}{dr}\left[\ln\left(\frac{\kappa^2}{\sigma\Omega}\right)\right]$$

$$\mathcal{C} = -\frac{\kappa^2}{c^2}(1-\nu^2)$$

$\nu = 0, \pm 1$ resonances!

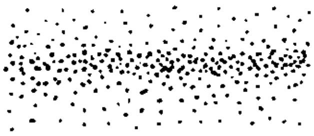
$$\nu = \frac{\omega - m\Omega}{\kappa}$$

$$\Rightarrow \mathcal{P}(\sigma_1) = -2\pi G \int_0^\infty K_m(r,r')\sigma_1(r')\,dr'$$

cf Pannatoni 1983

IAU 146

modeling
("dynamical window")



2 COMPONENT
("STARS" + "GAS")
FINITE THICKNESS
2 (FT)

1 COMPONENT
ZERO THICKNESS

1 (ZT)

c_*, σ_*, z_* c_g, σ_g, z_g

(same V)

"ACTIVE DISK MASS"

$c\ ,\ \sigma$

$Q = \frac{ck}{\pi G \sigma}$ "EFFECTIVE Q"

EQUILIBRIUM AND STABILITY

(SYMMETRY AND SYMMETRY-BREAKING)

- **MODELING OBSERVED STRUCTURES***
  (see spiral structure)

  - specific structures
  - less symmetric states

  ⇒ cf. Peierls J. Phys A 24 (1991) 5273

- **FORMATION**
  (how did we get to the current state? in the large class of equilibrium models that can be constructed, can we identify those that are violently unstable and rule them out as unrealistic?)

- **RESPONSE TO EXTERNAL DRIVING**
  (e.g., by tidal interaction)

- **TRANSPORT PROCESSES AND LONG TERM EVOLUTION**
  (---> very hard, even for the cases of electromagnetic plasmas where a large body of experimental data is available !) cf. Khoruzhii

.... also: study of "dynamical mechanisms"]

* symmetric systems are easier to handle, but do not exist.....

II. Equilibrium and stability

ELLIPTICALS

$\Downarrow$

Axisymmetry?
Triaxiality?
A significant fraction of E's may be well represented by a spherical basic state

SPIRALS

$\Downarrow$

Basic state generally well represented by axisymmetric disk embedded in a spheroidal bulge-halo

$$2K_{random} + 2K_{ordered} + W = 0$$

$$\frac{K_{random}}{K_{ordered}} \gtrsim 1$$

$$\left(\frac{K_{random}}{K_{ordered}}\right)_{disk} \ll 1$$

even in many flat systems!

Ellipticals are hot, pressure dominated equilibria

Complex orbital structure, many orbits cross the whole stellar system

Disks are cool, rotation supported equilibria

"Simple" orbital structure, most orbits are confined to thin annuli

M_* / M_{dark} ?

(dark matter is likely to have more diffuse distribution)

M_{disk} / M_{dark} ?

JEANS

$$\rho_0 \frac{\partial \underline{u}_1}{\partial t} = -\underline{\nabla} p_1 - \rho_0 \underline{\nabla} \Phi_1$$ momentum (1)

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \underline{\nabla} \underline{u}_1 = 0$$ mass (2)

$$\nabla^2 \Phi_1 = 4\pi G \rho_1$$ potential (3)

$$p_1 = c_s^2 \rho_1$$ state (4)

ρ_0, c_s^2 are "equilibrium" quantities. Take $\underline{\nabla}\cdot(1)$ and eliminate $\underline{u}_1$, Φ_1, p_1 using (2)-(4), and get

$$\rho_0 \frac{\partial}{\partial t}\left(\underline{\nabla}\cdot\underline{u}_1\right) = \left[-\frac{\partial^2 \rho_1}{\partial t^2} = -c_s^2 \nabla^2 \rho_1 - (4\pi G \rho_0)\rho_1 \right]$$

which is a wave equation modified by a "mass" term $\omega_p^2 = 4\pi G\rho_0$ $\quad \exp\left[i(\omega t - \underline{k}\cdot\underline{x})\right]$

The corresponding <u>DISPERSION RELATION</u> reads

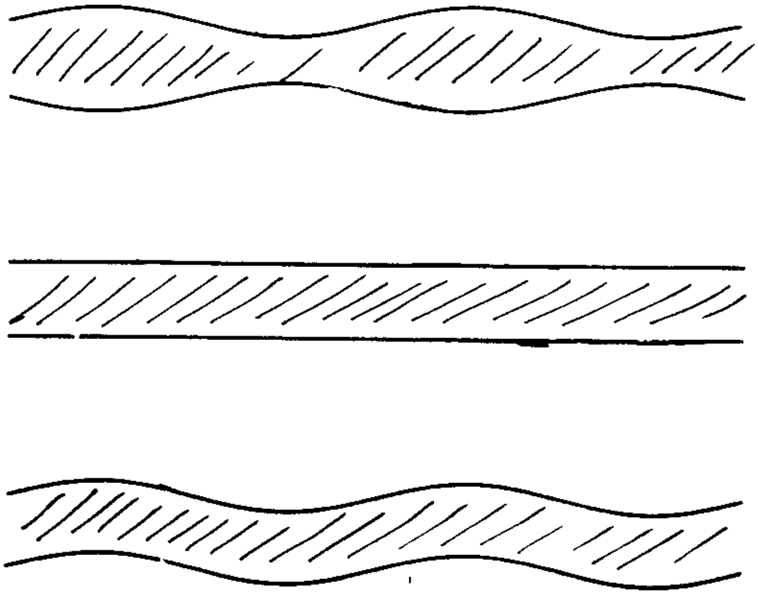
$$\boxed{\omega^2 = k^2 c_s^2 - 4\pi G \rho_0}$$

In the limit $G \to 0$ we recover <u>sound waves</u>.
In the limit $c_s^2 \to 0$ we recover <u>free fall kinematic limit</u>.

PARITY CLASSIFICATION OF MODES OF A FLAT LAYER

- magnetic reconnection

EVEN (DENSITY)



$z \to -z$ symmetry of basic state

- shear-Alfvén waves

ODD (BENDING)

one-component fluid model, zero-thickness, density waves $\exp\left[i\left(\omega t - m\theta + \int k\,dr\right)\right]$, with ← even

$|rk| \gg 1$:

$$(\omega - m\Omega)^2 = \kappa^2 + c_s^2 k^2 - 2\pi G\sigma|k|$$

$\boxed{\text{odd: } +2\pi G\sigma|k|}$

"Doppler shift" — rotation — sound — (Jeans) destabilizing Term

note: surface waves!

FOR $Q \equiv \dfrac{c_s\kappa}{\pi G\sigma} > 1$, LOCAL STABILITY FOR ALL k VALUES!

WARM DISKS ARE (LOCALLY) JEANS-STABLE!

WARP

R. SANCISI

Figure 1. Two channel maps of neutral hydrogen receding and approaching with almost maximum rotational speed, superposed on a photograph of NGC 5907. The photograph is from two stacked IIIa-J plates taken by Van der Kruit and Bosma with the 48-inch telescope of the Hale Observatories. The contours show the distribution of beam-averaged brightness temperature at 2 (≈ 3σ, dashed), 4, 8, 12, 16, 20 K. The velocity channels at and near the systemic velocity are not shown; this explains the gap in the radio map towards the centre. The velocities are heliocentric. The beamwidth at half-power (hatched ellipse) is 51" x 61". North is at the top, east at left.

cf. Bertin & Coppi (1985)

from Jokipii & Thomas (1981) Ap.J.

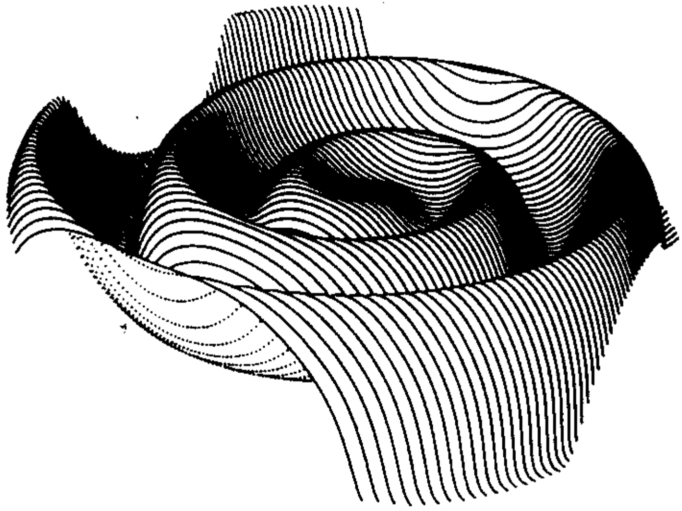


FIG. 2.—A representation of the heliospheric current sheet given by eq. (1) as seen by an observer 30° above the equatorial plane and 75 AU from the Sun. The variable $\alpha = 15°$, $\Omega_{\odot} = 2.9 \times 10^{-6}$, $V_w = 4 \times 10^7$ and the figure is 25 AU across.

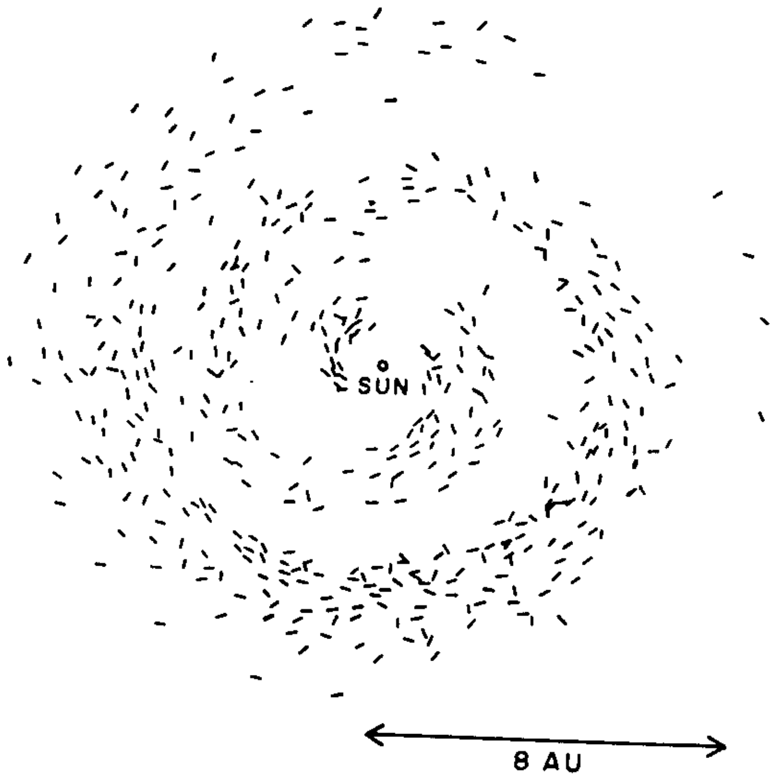


FIG. 3.—Daily averages of the *Pioneer 10* magnetic field data projected on the X-Y plane and plotted as unit vectors. Only periods during which the field was inward (toward the Sun) are plotted. The coordinate system is corotating with the Sun with a rotation period of 25.4 days.

also: Alfvén
"ballerina effect" (1977)

III (c) BENDING WAVES

$$-2\pi G\sigma|k| \longrightarrow +2\pi G\sigma|k|$$

density waves   bending waves

So, bending waves and modes are generally stable

Hunter & Toomre 1969; Kulsrud, Mark, Caruso 1971

Mark 1971

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BENDING WAVES AND CURRENT DISK MODEL FOR THE HELIOSPHERE

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ABSTRACT

Axisymmetric current sheets, where the magnetic field direction is inverted, are shown to be subject to bending waves. In the limit where the effects of magnetic reconnection can be neglected, the modes that are found can be identified as a new type of shear-Alfvén waves. The presence of a wind, i.e., of a plasma streaming motion, in the current sheet is taken into account, and the relevance of these waves to the observation of magnetic polarity sectors in the solar wind and to other astrophysical environments, such as planetary magnetospheres, is suggested.

Subject headings: plasmas — Sun: solar wind

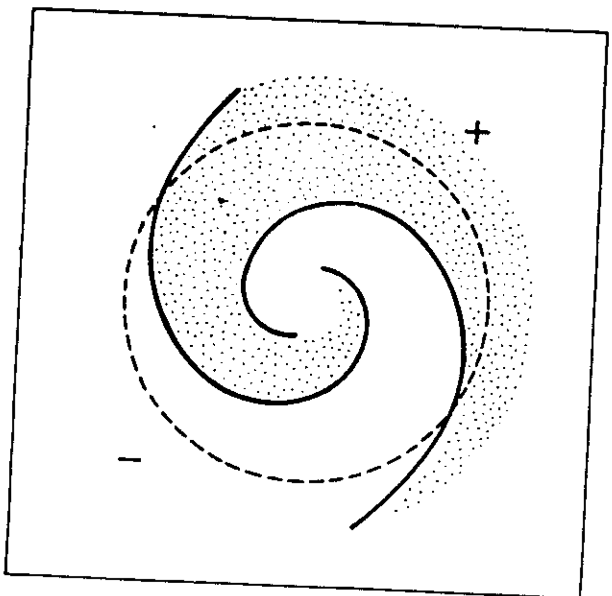
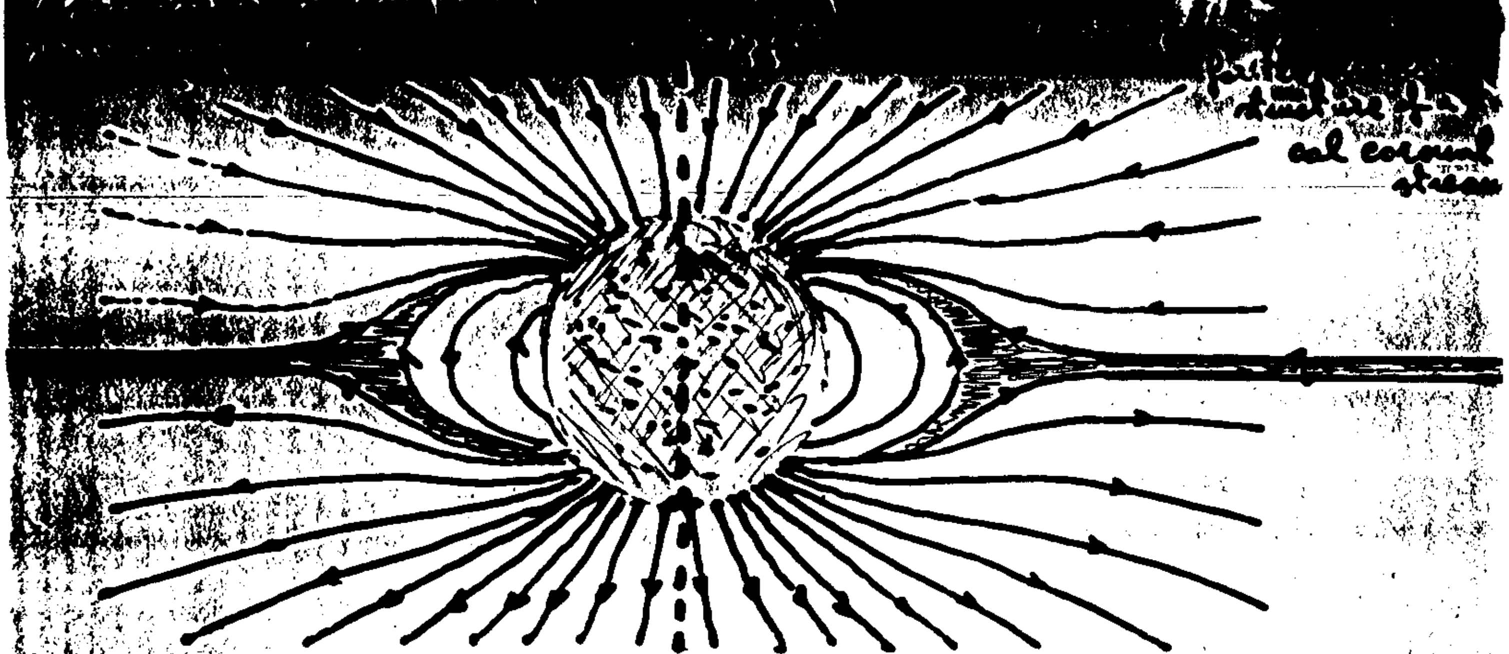


FIG. 1.—Top view of an $m = 1$ bending wave. In this example, which could represent a configuration with two magnetic sectors, the lines of nodes (*solid lines*) have a spiral configuration. An observer in the "equatorial" plane along the dashed circle would see two polarity reversals while crossing the lines of nodes. In the shaded region the current sheet is bent upward, and the observer would be below it. In the other region the current sheet is bent downward, and the observer would be above it.

$$\xi_z = \tilde{\xi} \exp(-i\omega t + im\phi)$$

$m=1 \leftrightarrow$
2 SOLAR SECTOR.

... axisymmetric ... dipole field added to the ... neutral sheet and ...



from H. Schultz (1973)

PNEUMAN & KOPP (1971)

Realistic axisymmetric magnetic field configuration including the effect of the solar wind flow and of the presence of a neutral sheet