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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4-SMR 1012 - 20

AUTUMN COLLEGE ON PLASMA PHYSICS

13 October - 7 November 1997

Models of Elliptical Galaxies as Partially Relaxed Stellar Systems

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Bibliography

Bertin, G., Stiavelli, M. 1993, Reports on Progress in Physics, 56, 493-556

These are lecture notes, intended for distribution to participants.

III A

ELLIPTICAL GALAXIES AS HOT STELLAR SYSTEMS

- Homogeneity
- ellipsoidal figures
- Solid body rotation

S. CHANDRASEKHAR

$\sigma \sim 400$ km/s

V_r (?) $\sim$ 50 km/s

$\frac{\sigma^2}{V^2}$

Ω

(5)

s. Substituting for A_1
ions (36), we recover

$1-e^2$). (6)

ical with the equation
ı the basis of Newton's

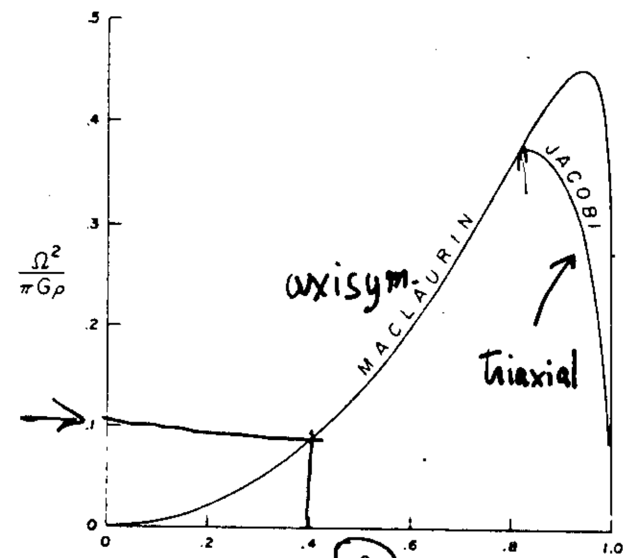


FIG. 5. The square of the angular velocity (in the unit $\pi G\rho$) along the Maclaurin and the Jacobian sequences. The abscissa, in both cases, is the eccentricity of the (1, 3)-section.

$e^2 = 1 - \frac{a_3^2}{a_1^2}$

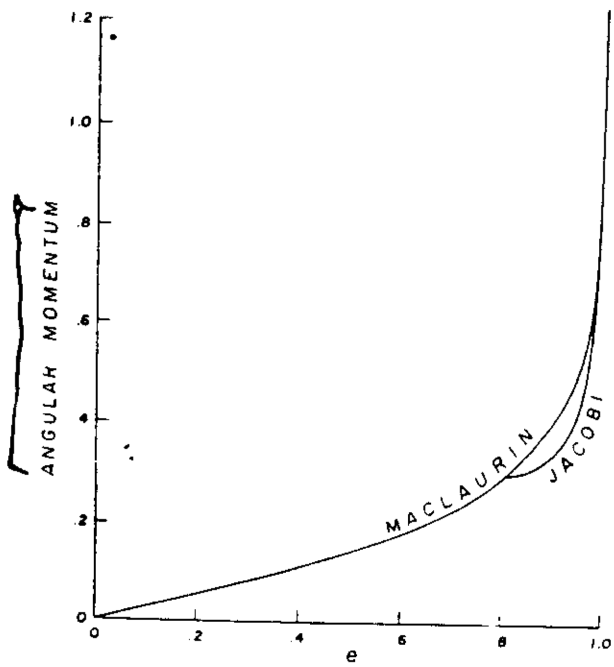


FIG. 6. The angular momentum (in the unit $(GM^3\bar{a})^{\frac{1}{2}}$) along the Maclaurin and the Jacobian sequences. The abscissa in both cases is the eccentricity of the (1, 3-)section.

* nuclei

TWO MAIN MESSAGES FROM OBSERVATIONS:

(1) Ellipticals have a luminosity profile that is well fitted by the <u>empirical</u> <u>$R^{1/4}$ law</u> proposed by de Vaucouleurs (<u>1948</u>, 1953)

$$\mu = \mu_e + 8.325 \left[(r/r_e)^{1/4} - 1 \right]$$

i.e.

$$\log I(r) = \log I_0 - 3.33 \, (r/r_e)^{1/4}$$

r_e = half-luminosity radius

<u>10 magnitudes</u>

(2) In contrast to spirals, ellipticals are mainly <u>"pressure" supported</u> (Bertola & Capaccioli <u>1975</u>, ...)

$$W + \underbrace{2K_{random}} + \cancel{2K_{ordered}} = 0$$

(little is known on their 3-D structure)

ELLIPTICALS

2) low rotation *

Bertola and Capaccioli (1975)

Illingworth (1977)

V/σ

→ pressure anisotropy

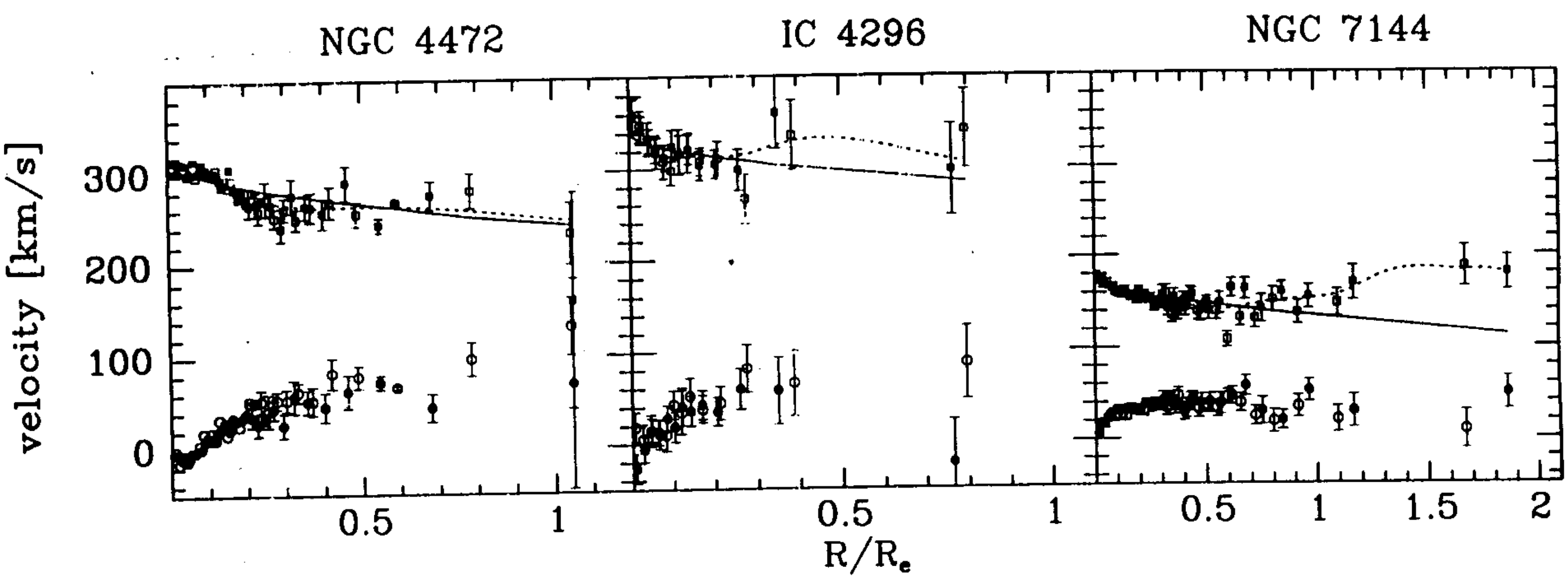
Binney (1978, 1982)

("radial" pressure outside)

van Albada (1982)

Note : most of the available kinematical data on the stars are so far restricted to $R < R_e$

Saglia et al ApJ 403 (1993) 567



EQUILIBRIUM

(f, Φ)

$$\begin{cases} \{f, H\} = 0 & (1) \\ \nabla^2 \Phi = 4\pi G \int f \, d^3v & (2) \end{cases}$$

$$\Phi = \Phi(\underline{x})$$

$f = f$ (Integrals of the motion associated with $\Phi(\underline{x})$)

Usually, one starts out from a given symmetry† for Φ. Then, the isolating integrals are identified. A combination of integrals of the motion is taken for f. (1) ✓

THE MODEL IS THEN IDENTIFIED BY SOLVING THE NON-LINEAR SELF-CONSISTENCY EQUATION FOR Φ

(2) ✓

† spatial (e.g. spherical, axial, ..) or dynamical (e.g. Staeckel)

A NOTE OF CAUTION

various attempts at deriving phase space properties directly from observations, i.e. at "measuring" pressure anisotropy

but: data sample projected quantities on a limited radial range with sizable errors

(cf. attempts at "measuring" Q in disk.

① direct inversion $\rho \to f$ (actually, f_{even})

Eddington (1916), Merritt (1985)

de Jonghe & de Zeeuw (1987) insist on exact models

- obviously non unique
- often leads to negative f!

② Σ, σ + hydrodynamics $\Rightarrow$ pressure anisotropy
(spherical symmetry)

Binney Mamon (1982) Tonry (1983)

③ Linear programming, "maximum entropy", etc

Schwarzschild (1979)

Pfenniger (1984), Newton & Binney (1984), Richstone & Tremaine (1984, 1986)

de Jonghe (1989)

BUT INVERSION IS UNSTABLE

AND OFTEN LEADS TO UNREALISTIC MODELS! (dynamically unstable models)

see application by Caldwell, Kirshner, Richstone (1986) on N7097

Inversion $(\rho \to f)$

Take $\Phi = \Phi(r)$ as given ; $\rho(r) = \frac{1}{4\pi G} \nabla^2 \Phi$

Can we invert :

$$\rho(r) = \int f\,(E, J^2)\, d^3 v \qquad ?$$

Example

"Abel" inversion for $f = f(E)$ (isotropy imposed)

$$f(E) = \frac{1}{2\pi^2\sqrt{2}} \left[\int_0^E \frac{d^2\rho}{d\Phi^2} \frac{d\Phi}{\sqrt{E-\Phi}} + \frac{1}{\sqrt{|E|}} \left(\frac{d\rho}{d\Phi}\right)_{\Phi=0} \right]$$

(Eddington, 1916)

BUT : (i) this method is not predictive

(ii) method sometimes fails (e.g. f derived is not positive definite)

(iii) leads to irregularities in phase space, i.e. unrealistic f

JEANS THEOREM:

$f = f$ (integrals of the motion)
E

Triaxial models

(VERY DIFFICULT

necessary ?

OBLATE MODELS

initialize N-body
"lucky" asymptotes

SLIGHTLY
OBLATE MODELS
$\sim$ TvA 1982

(E, J_z, I_3)

Bertin and Stiavelli 1984

$$f_\infty = A(-E)^{3/2} \exp\left[-aE - cJ^2/2\right] \qquad E \leq 0$$

$$= 0 \qquad E > 0$$

3 parameters:
2 scales
→ 1 PARAMETER EQ. SEQUENCE

SPHERICAL MODELS

(EASY, TOO MANY)

$F(E, J^2)$

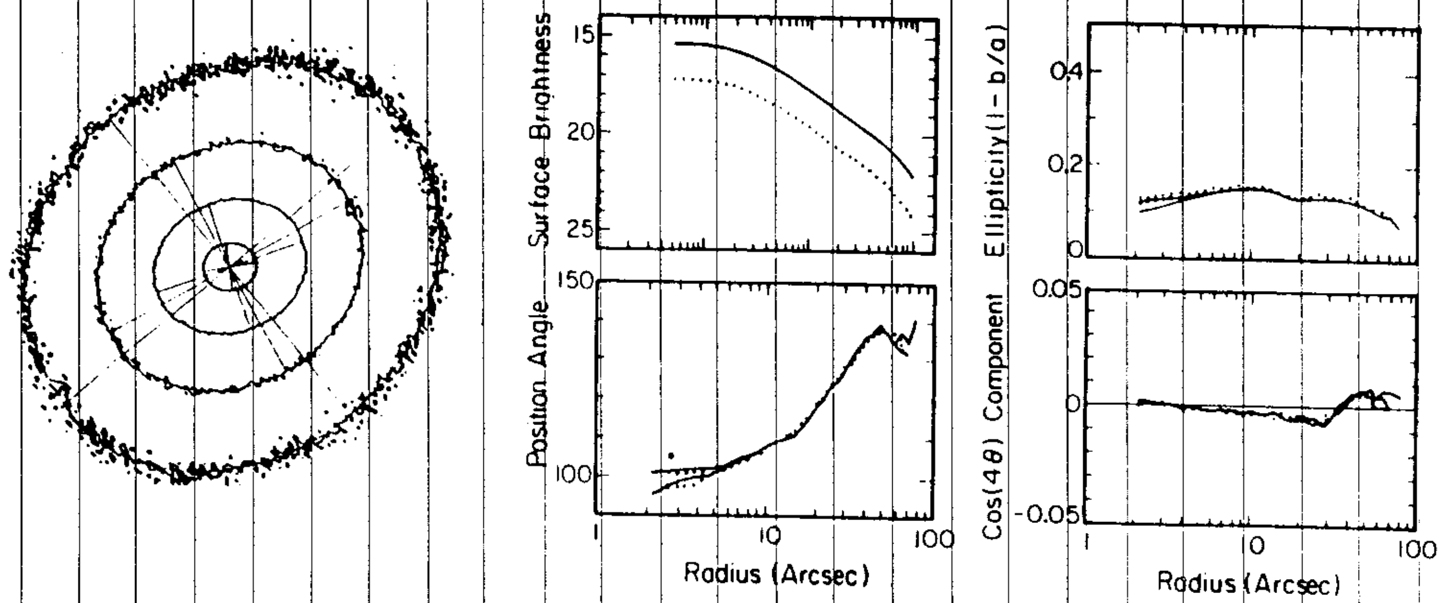


Figure 2. a) Contour map with major and minor axes overlaid and b) surface photometry diagrams for NGC 1549. The solid lines refer to the R band, dotted lines to B.

JEDRZEJEWSKI 1986

III B

PARTIAL RELAXATION AND THE $R^{1/4}$ LUMINOSITY LAW

INTRODUCE f ON A PHYSICAL BASIS

T. van Albada (1982) : collisionless collapse of star clouds , $N \sim$ a few 10^3

$t = 0$ $\quad K_{ord} = 0$

$\qquad K_{random} \sim 0$

$t > 0$ $\quad$ collapse

$t \sim 2\text{-}3\ \tau_{ff} \rightarrow 2K_{random} + W \sim 0$

collisionless violent relaxation!

partial relaxation!

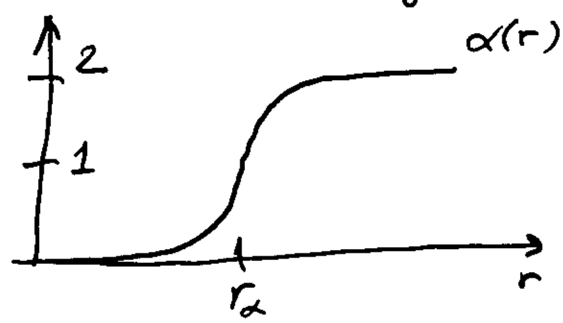
Results:

(i) fairly round , partially relaxed systems

(ii) rather "realistic" density distributions

(iii) pressure tensor: isotropic at $r < r_\alpha$,
$\sim$ radial at large radii

$$\alpha(r) = 2 - \frac{\langle v_\theta^2 \rangle + \langle v_\varphi^2 \rangle}{\langle v_r^2 \rangle}$$



T. van Albada   MNRAS (1982) 201, 9[illegible]

$$\alpha = 2 - \frac{\langle v_\perp^2 \rangle}{\langle v_r^2 \rangle}$$

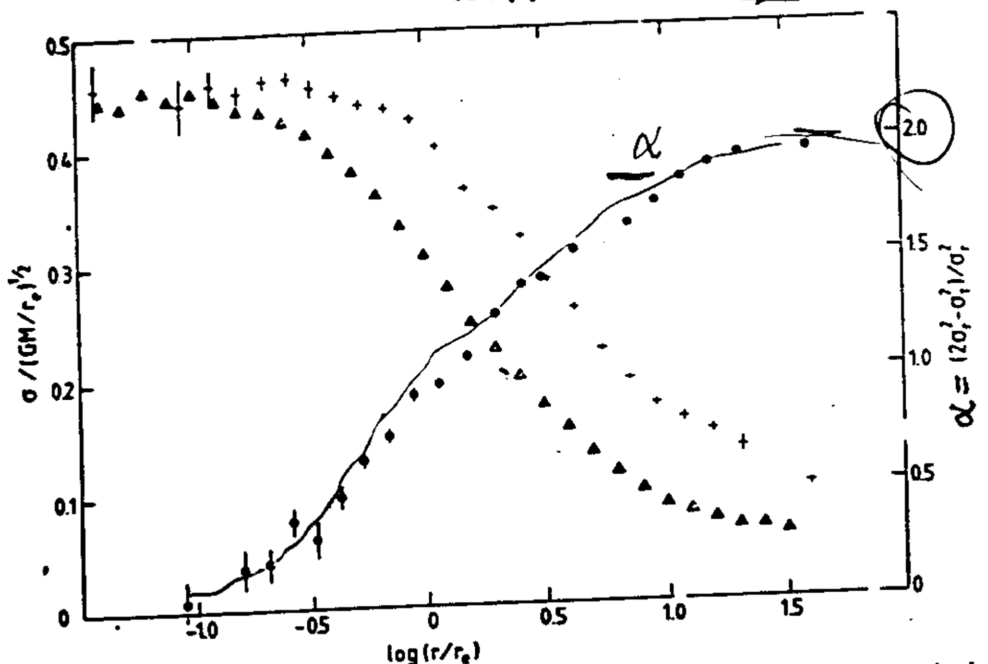


Figure . Velocity dispersion and anisotropy as a function of radius for model C3. Crosses: dispersion in radial component of velocity (averaged over spherical shells). Triangles: velocity dispersion as measured along line of-sight (averaged over circular rings and three perpendicular viewing directions). The vertical scale on the left applies to these two quantities. The velocity anisotropy, inidcated by filled circles with error bars, is measured by the vertical scale on the right.

α → 2
MEANS
"RADIAL"
PRESSURE
ONLY

fig. 1

main assumption

slightly oblate

First attempt:

can I construct $f = f(E, J_z^2)$ for an axisymmetric stellar system with $\alpha = \alpha(r)$ of the type found by van Albada in his simulations?

NO!

$$E = \frac{1}{2}\left(v_r^2 + v_\theta^2 + v_\varphi^2\right) + \Phi$$

$$J_z^2 = r^2 \sin^2\theta \, v_\varphi^2$$

$$\Rightarrow \quad \langle v_r^2 \rangle = \langle v_\theta^2 \rangle \quad \Rightarrow$$

$$\alpha(r) = 2 - \frac{\langle v_\theta^2 \rangle + \langle v_\varphi^2 \rangle}{\langle v_r^2 \rangle} = 1 - \frac{\langle v_\varphi^2 \rangle}{\langle v_r^2 \rangle} < 1$$

$\Rightarrow$ WE NEED A THIRD INTEGRAL!

From what is available in the literature, systems with 3 isolating integrals of the motion (Stäckel) are characterized by $\rho \sim r^{-4}$ at large radii....

EDDINGTON POTENTIAL (→ Lynden-Bell 1962)

(STÄCKEL)

$r, \theta, \varphi \longrightarrow \lambda, \mu, \varphi$ coordinates

spherical spheroidal

λ, μ are solutions of $(x^2+y^2)/(\tau^2-\beta^2) + z^2/\tau^2 = 1$; $\beta \le \lambda < +\infty$, $-\beta \le \mu \le \beta$

$$r \gg \beta \longrightarrow \begin{cases} \lambda \sim r \\ \mu \sim \beta\cos\theta \end{cases}$$

$$\phi_{EDDINGTON} = \frac{\zeta(\lambda) - \eta(\mu)}{\lambda^2-\mu^2} \quad ; \quad \zeta, \eta \text{ arbitrary functions}$$

ISOLATING INTEGRALS:

E, J_z, I_3

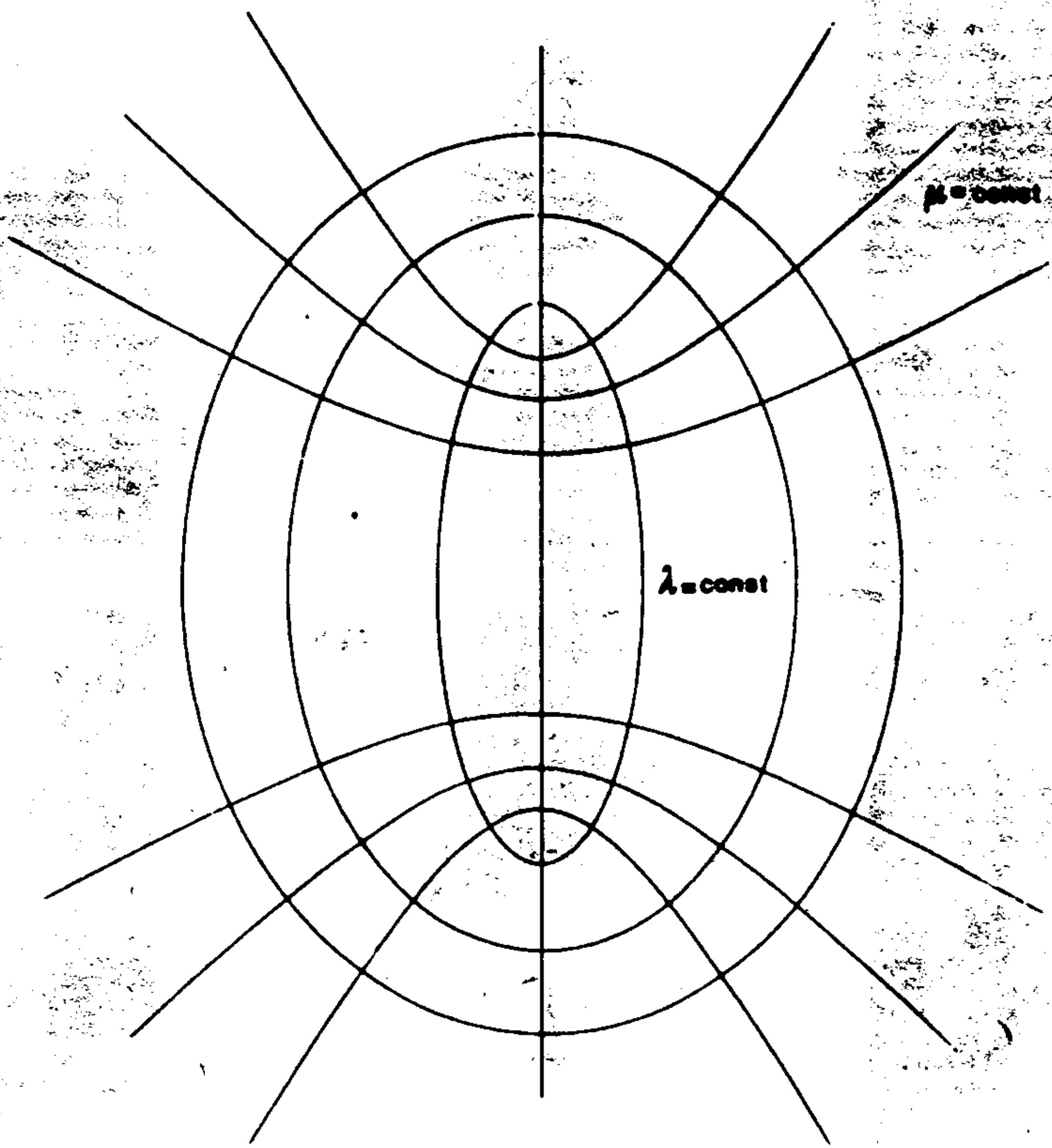
$$I_3 = \frac{1}{2}\left[(\mu^2-\beta^2)v_\lambda^2 + (\lambda^2-\beta^2)v_\mu^2 + (\lambda^2+\mu^2-2\beta^2)v_\varphi^2 + \frac{(\mu^2-\beta^2)\zeta(\lambda) - (\lambda^2-\beta^2)\eta(\mu)}{(\lambda^2-\mu^2)}\right]$$

$$\lambda \gg \beta \implies \begin{cases} \phi_{EDD} \sim \dfrac{\zeta(r) - \eta(\beta\cos\theta)}{r^2} \\ I_3 \sim \dfrac{J^2}{2} - \eta(\beta\cos\theta) \end{cases}$$

$\Delta\phi_{EDD} = 4\pi G \rho_{EDD}$

$\phi_{EDD} \implies \rho \sim r^{-4}$ at large radii

[ϕ_{EDD} at large radii ρ ($\neq 0$ at large radii) $\implies \rho \sim r^{-4}$ at large radii]



μ=const
λ=const

A distribution function with the desired properties:

Bertin Stiavelli 1984 AA 137, 26

$$f_\infty = A\,(-E)^{3/2} \exp\left[-aE - b\left(J_z^2/2\right) - c\,I_3\right] \qquad E \leq 0$$

$$= 0 \qquad E > 0$$

$$E = \frac{1}{2}v^2 + \Phi \qquad , \qquad J_z^2 = r^2 \sin^2\theta \, v_\varphi^2$$

$$I_3 \simeq \frac{J^2}{2} + \eta(\cos\theta) \qquad \text{(at large radii)}$$

$$\left[\ \nabla^2 \Phi = 4\pi G \int f_\infty \, d^3v\ \right]$$

SOME ASYMPTOTIC RELATIONS

for values of r such that $\left(-a\,\Phi(r)\right)\left(1+\frac{cr^2}{a}\right) \gg 1$

(Typically $r \gtrsim r_\alpha$) can be obtained by saddle-point integration over velocity space

$$\rho \sim \frac{3\sqrt{2}\,\pi c}{16 G \gamma} \frac{\exp(-c\eta)\;\Phi^2}{\sqrt{\left(1+\frac{cr^2}{a}\right)\left[1+\frac{cr^2}{a}\left(1+\frac{b}{c}\sin^2\theta\right)\right]}}\, I_0(a\Phi)$$

$$I_\ell(a\Phi) = \frac{8}{3\pi}\int_{-1}^{1} dx\, \exp\left[-a\Phi\left(1-x^2\right)\right]\left(1-x^2\right)^{3/2} x^\ell$$

- $\langle v_r^2 \rangle \sim -\frac{1}{3}\,\Phi\left[\frac{6 I_2(a\Phi)}{I_0(a\Phi)}\right]$

- $\langle v_\theta^2 \rangle \sim \left(a + cr^2\right)^{-1}$

- $\langle v_\varphi^2 \rangle \sim \left(a + cr^2 + br^2\sin^2\theta\right)^{-1}$

- $e(r) \sim \left|\frac{2\,b/c}{4-p}\right|^{1/2} \frac{1}{\left|d\ln\rho^{1/2}/d\ln r\right|^{1/2}}$

- $\beta^2 = -\frac{a}{c}\left[\frac{b/c}{4-p}\right]\frac{p}{2\Psi}$

Spherical limit

$$f_\infty = A(-E)^{3/2} \exp\left[-aE - c\frac{J^2}{2}\right] \qquad E \le 0$$

$$= 0 \qquad E > 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right) = 4\pi G \int f_\infty \, d^3v$$

NON-LINEAR ODE, SOL'N GIVES GLOBAL MODEL. FULL PHASE SPACE + INHOMOGENEITIES!

b.c.'s : $\frac{d\Phi}{dr} = 0$ at $r = 0$

$\Phi \sim -\frac{GM}{r}$ at $r \to \infty$

2 scales , 1 dimensionless parameter

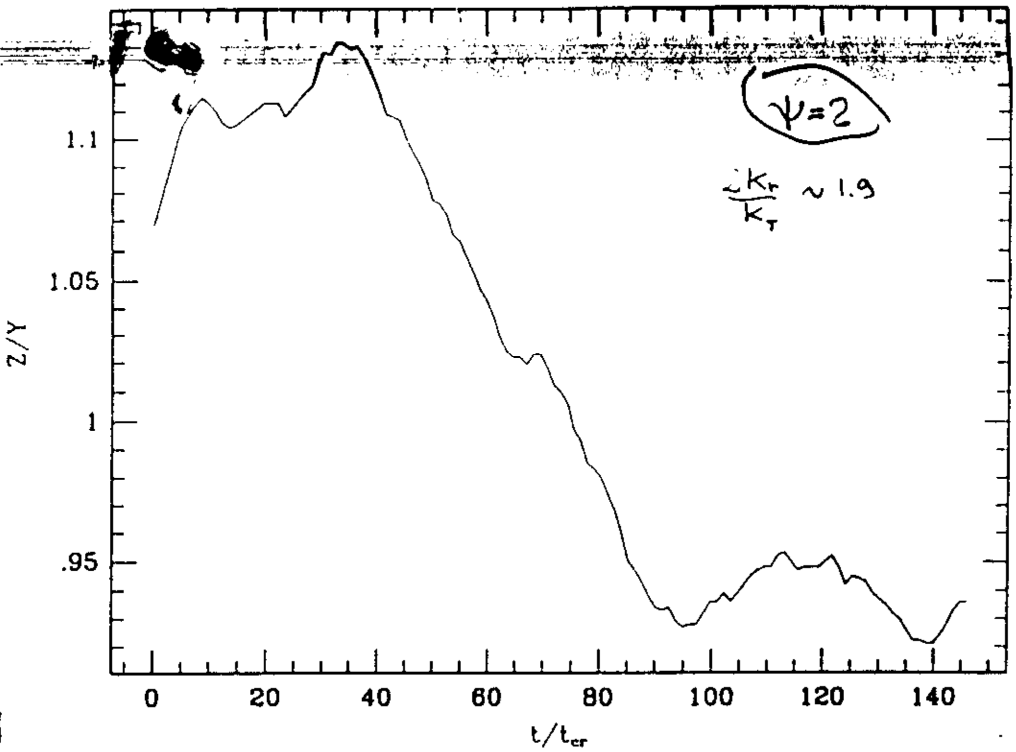
$$\gamma \equiv \frac{ac}{4\pi G A}$$

or

$$\Psi \equiv -a\Phi(r=0)$$

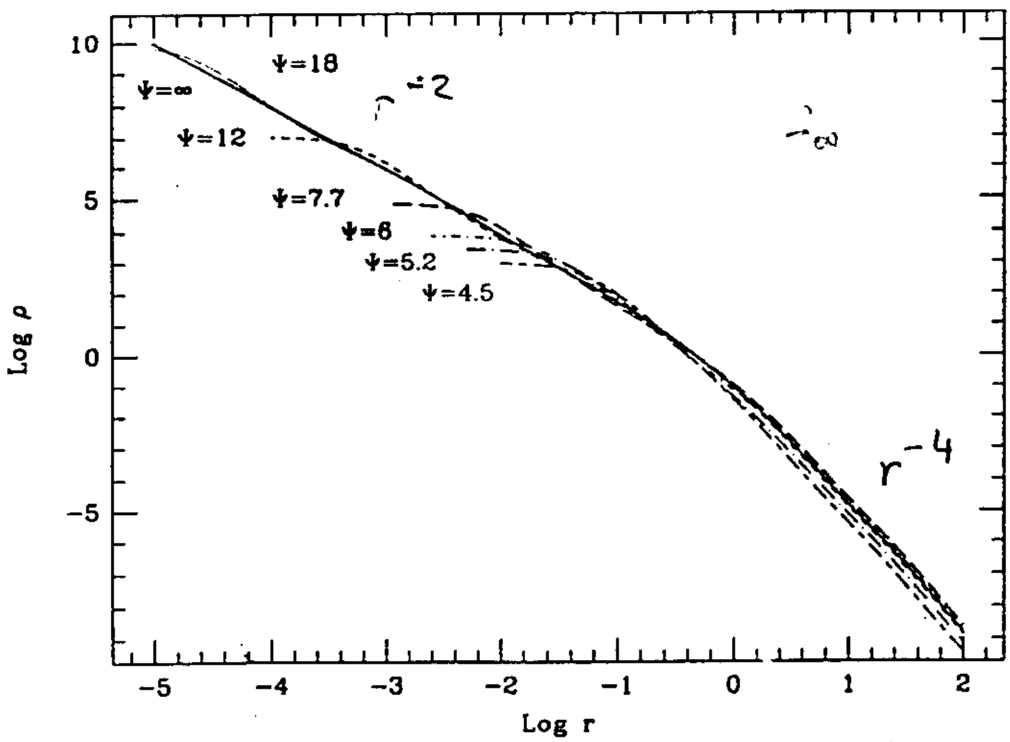
CONSTANT M/L ratio
→ luminosity profile
+ velocity dispersion profile

(i)



$\Sigma |z_i| / \Sigma |y_i|$

Bertin & Stiavelli, Astrophys. J. ~~in press~~ 1989



$\Psi \gtrsim 5.2 \;,\; \frac{2K_r}{K_T} \approx 1.1$

SOME LIMITING CASES FOR f_∞ - MODELS

1) SMALL Ψ $\qquad (\Psi < 2)$

$\Psi \equiv -a\,\Phi(r=0)$ $\qquad$ (unstable)

$$\rho_\infty \sim \rho_p = \frac{1}{(1+r^2)^2}$$

$$\Phi_p = -2\pi G \arctan r / r$$

$$q \equiv r_M |W| / GM^2 \to 0.35$$

also not far from isochrone model (Hénon 1959)

$$\Phi_I = -\frac{1}{1+\sqrt{1+r^2}}$$

2) LARGE Ψ $\qquad (\Psi > 7)$

$$\rho_\infty \sim \rho_J = \frac{1}{r^2(1+r)^2}$$

$$\Phi_J = 4\pi G \ln[r/(1+r)]$$

$\Psi > 7$, f_∞-models change only in the innermost nucleus,

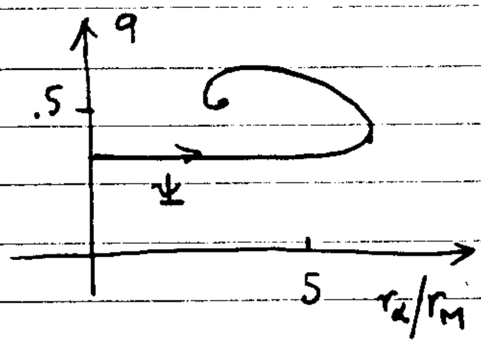
$\Rightarrow$ $R^{1/4}$ law

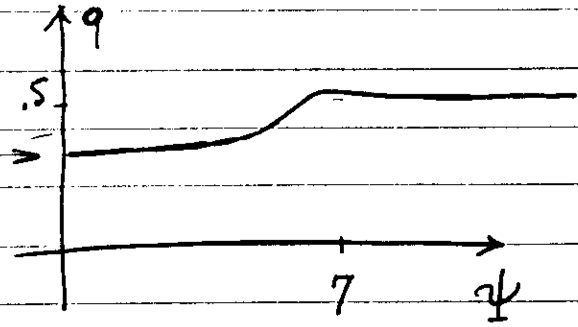
3) NEGATIVE TEMPERATURE MODELS $\qquad (\Psi < 0)$

argued to be physically interesting by Merritt, Tremaine & Johnstone (1989)
?? unstable !!

$$f = A(-E)^{3/2} \exp\left[-aE - cJ^2/2\right]$$

one parameter eq. sequence (two scales + $\gamma = \frac{ac}{4\pi G A}$)





$$\Longrightarrow \quad \Psi \equiv -a\Phi(0)$$

depth of the central potential well

$$q \equiv \frac{|W| r_M}{GM^2}$$ "form factor"

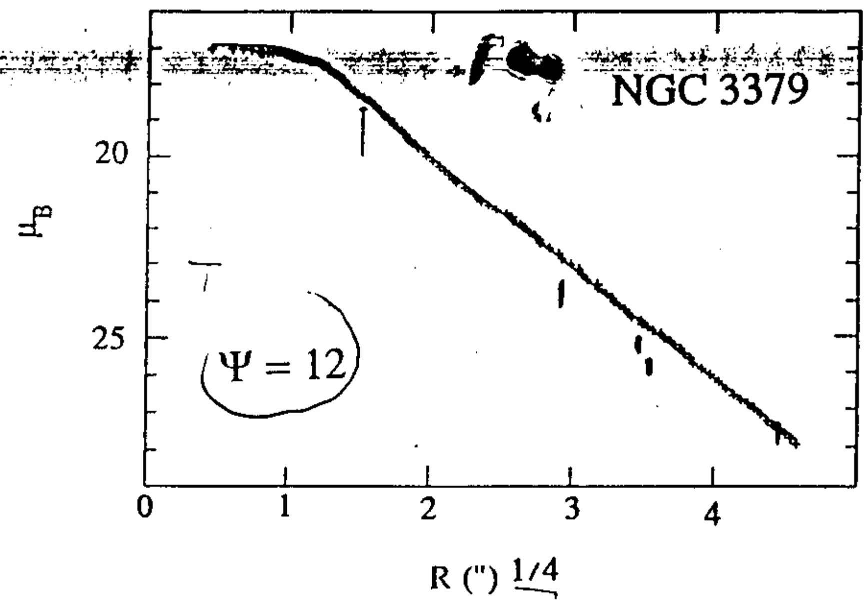
$r_\alpha =$ "anisotropy radius"

$r_M =$ "half-mass radius"

- For $\Psi \gtrsim 7$ models change only innermost "nuclear" structure
- For a given value of Ψ the whole phase space, i.e. all observable profiles are specified
- For $\Psi \gtrsim 7$ essentially nothing changes, except in the nucleus

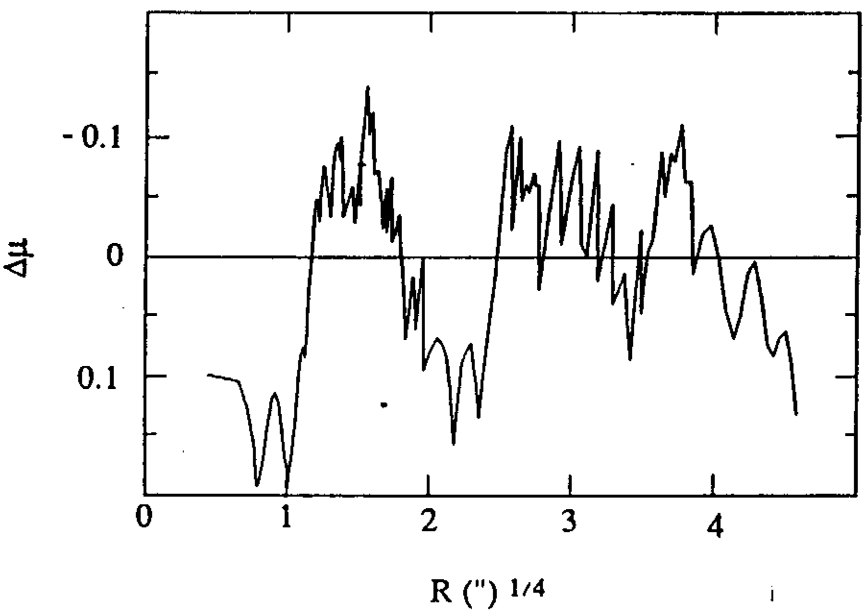
A "FIT" OVER ELEVEN MAGNITUDES

see Bertin, Saglia, Stiavelli: Ap. J. 330 (1988)



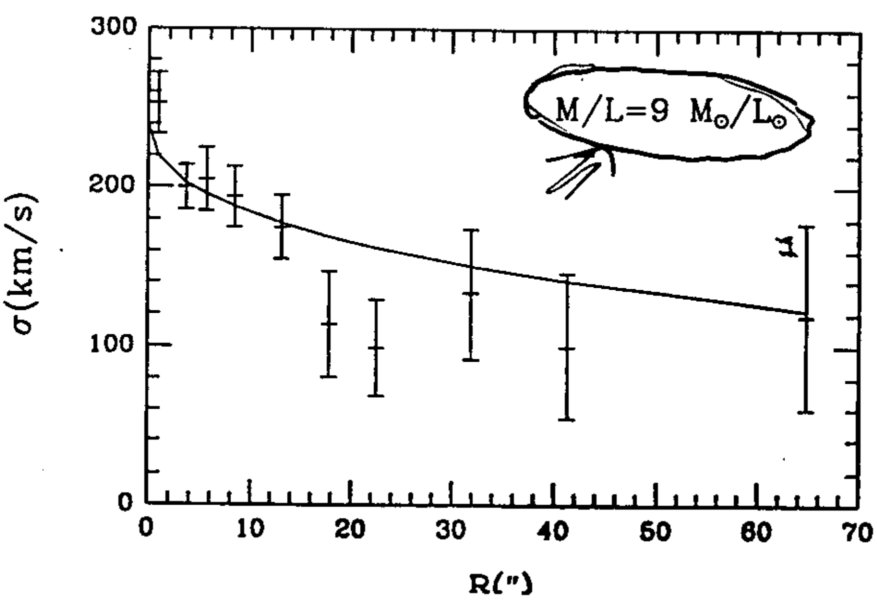
μ_B from de Vaucouleurs & Capaccioli (1979)

$M/L = \text{cost}$



$\Delta\mu = \mu_B - \mu_\infty$

μ_∞ from $f_\infty = A(-E)^{3/2}\exp(-aE - cJ_z^2/2)$

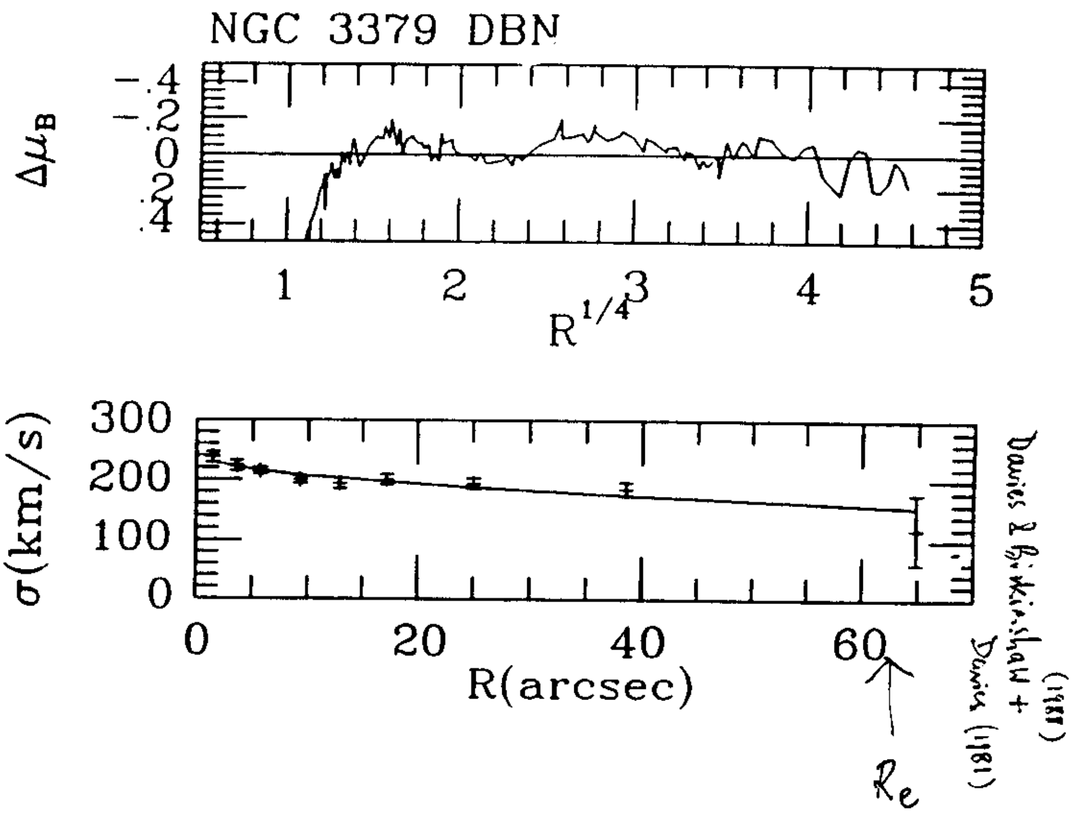


16 Mpc

σ from Davies (1981)

Saglia, Bertin, Stiavelli

SBS 92



$R_e \approx 4.8\,\mathrm{kpc}$

Statistical mechanics (Stiavelli & Bertin MNRAS (1987) 229)

Use E, J^2 partition

Argue

$$S = -\int N(E,J^2)\ln N(E,J^2)\,dE\,dJ^2$$

then

$$= -\int f(E,J^2)\ln\left[f(E,J^2)\,T_r(E,J^2)\right] d^3p\,d^3q \;+\text{const}$$

extremize S under conservation of:

$$E_{TOT} = \frac{2}{3}\,m\int E\,f(E,J^2)\,d^3p\,d^3q \qquad \text{total energy}$$

$$N = \int f(E,J^2)\,d^3p\,d^3q \qquad \text{total number of stars}$$

$$N_J = 4\pi^2\int f(E,J^2)\,T_r(E,J^2)\,dE \qquad \text{for } J^2 > J_0^2$$

detailed angular momentum ("large radii")

INCOMPLETE VIOLENT RELAXATION

$$\Rightarrow\quad f = A\,\frac{1}{T_r(E,J^2)}\exp\left[-aE - c_J\right]$$

at large radii $T_r \to |E|^{-3/2}$ (Keplerian)

by requiring N_J to match the distribution N_J° of an initially uniform Maxwellian distribution* we get $c_J = cJ^2/2$

$$\rightarrow\quad f = f_\infty = A\,(-E)^{3/2}\exp\left[-aE - \frac{cJ^2}{2}\right]$$

$$N_J = A'\int_{E_{min}}^{0}\exp(-aE)\,dE\,\exp(-c_J) \sim \exp(-cJ^2/2)/J^2 \propto N_J^\circ$$

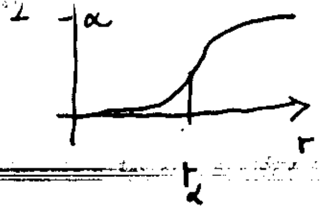
$$E_{min} \sim -1/J^2$$

III (e) STABILITY OF COLLISIONLESS SPHERICAL STELLAR SYSTEMS

$$\frac{2K_r}{K_T} \gtrsim 1.7 \pm 0.25$$

condition for $l=2$
"radial orbit instability"

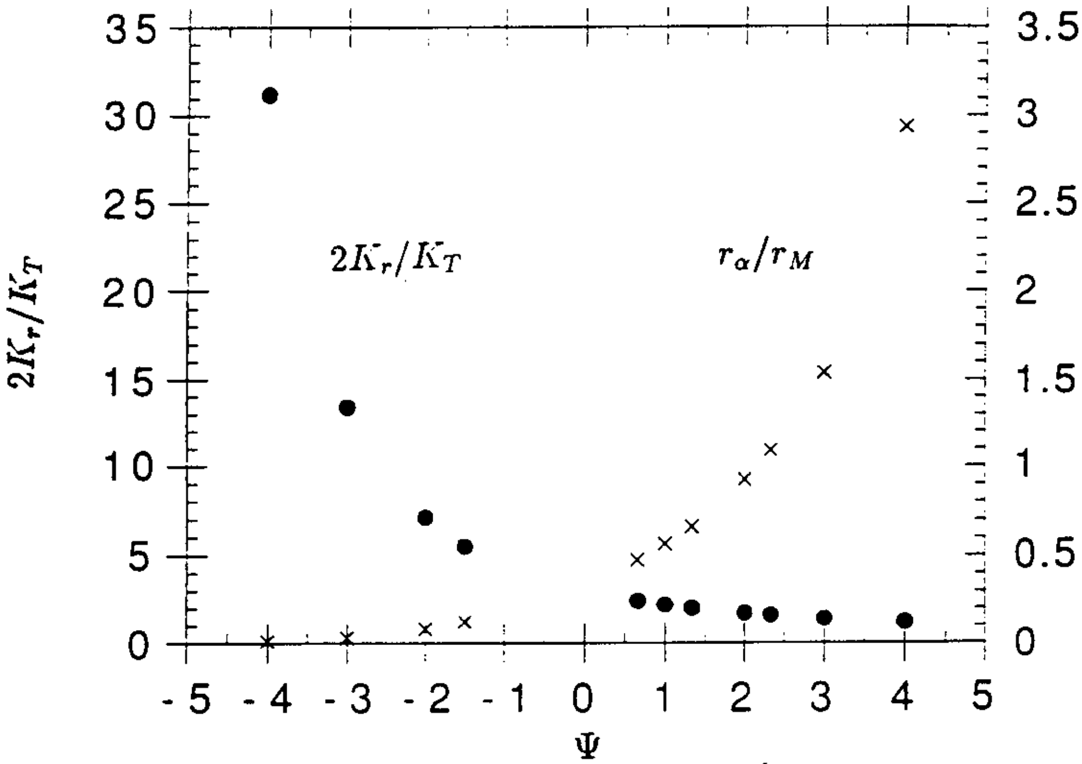
Polyachenko & Shukhman 1981
see Fridman & Polyachenko 1984



$$\alpha(r) = 2 - \frac{\langle v_\theta^2 \rangle + \langle v_\varphi^2 \rangle}{\langle v_r^2 \rangle}$$

$$\alpha(r_\alpha) = 1$$

global measures of radial anisotropy



"negative temperature" "positive temperature"

$$\Psi = -a\,\Phi(0)$$

Fig. 3

CLASSIFICATION OF MODES OF A SPHERICAL COLLISIONLESS STELLAR SYSTEM

Y^{l}_{m} ...... just as solar oscillations ?

NOT QUITE!

Phase space is NOT "spherically symmetric"

$$f_0 = f_0(r, v_r, v_\perp)$$

subtle!

STABILITY (Bertin, Pegoraro, Rubini, Vesperini 1994)

2

$$f_0 = f_0(E, J^2)$$

Frequencies of the basic equilibrium state

$$dE = \Omega_\theta \, dJ + \Omega_r \, dJ_r$$
$$J_r = \oint v_r \, dr = \oint \sqrt{2(E - \Phi_{eff}(J; r))} \, dr$$
$$\Omega_r = 2\pi / T_r(E, J^2)$$
$$T_r = \oint dr / v_r$$
$$\Omega_\theta = \frac{1}{T_r} \oint \frac{J}{r^2} \frac{dr}{v_r}$$

$$\left(\Omega_r^2 \rightarrow \kappa^2 = 4\Omega^2 \left[1 + \frac{1}{2} \frac{d\ln\Omega}{d\ln r} \right] \qquad \Omega_\theta^2 \rightarrow \Omega^2 = \frac{1}{r} \frac{d\Phi}{dr} \right)$$

for quasi-circular orbits

$$\tilde{\Omega}(E, J, r) = \frac{J^2}{r^2} - \Omega_\theta(E, J)$$

Linear stability analysis

"adiabatic response"

$$h_1 \equiv f_1 - \Phi_1 \frac{\partial f_0}{\partial E}$$

$$\left\{ \omega + i\sigma \left[2(E - \Phi_{eff}(J; r)) \right]^{1/2} \frac{\partial}{\partial r} + s \frac{J}{r^2} \right\} \hat{h}_s^\ell = \left(-\omega \frac{\partial f_0}{\partial E} + s \frac{\partial f_0}{\partial J} \right) A_s^{(\ell)} \Phi^\ell$$

Vlasov

$$h_0^\ell = \sum_{s'} A_{s'}^{(\ell)} \hat{h}_{s'}^c \qquad A^{(2)} \equiv \frac{1}{4} \left(\sqrt{6}, -2, \sqrt{6} \right)$$

$$\left[\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) - \frac{\ell(\ell+1)}{r^2} \right] \Phi^\ell = 4\pi G \int \left(h_0^\ell + \Phi^\ell \frac{\partial f_0}{\partial E} \right) d^3 v$$

Poisson

non-adiabatic response

adiabatic response

$s: -\ell, -\ell+2, \dots, +\ell$; ℓ: spherical harmonics

$r \to 0 \quad \Phi^\ell \sim r^\ell$; $r \to \infty \quad \Phi^\ell \sim r^{-(\ell+1)}$ $(\ell \geq 2$