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AUTUMN COLLEGE ON PLASMA PHYSICS

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TOKAMAK EDGE TURBULENCE

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These are lecture notes, intended for distribution to participants.

TOKAMAK EDGE TURBULENCE

Outline

1. introduction
2. equations and model
3. example: Hasegawa-Wakatani system
4. general aspects of turbulence
5. drive mechanisms
6. complete example from our research

Introduction

Transport in fusion devices

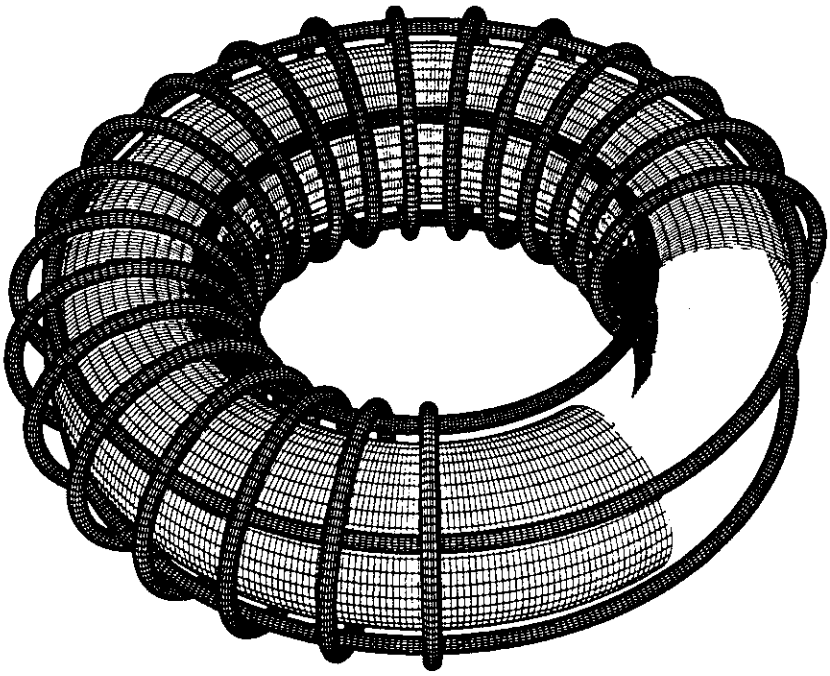
- magnetic fusion devices: plasma is confined by magnetic fields and currents flowing in the plasma

  $$\nabla p = \mathbf{j} \times \mathbf{B}.$$

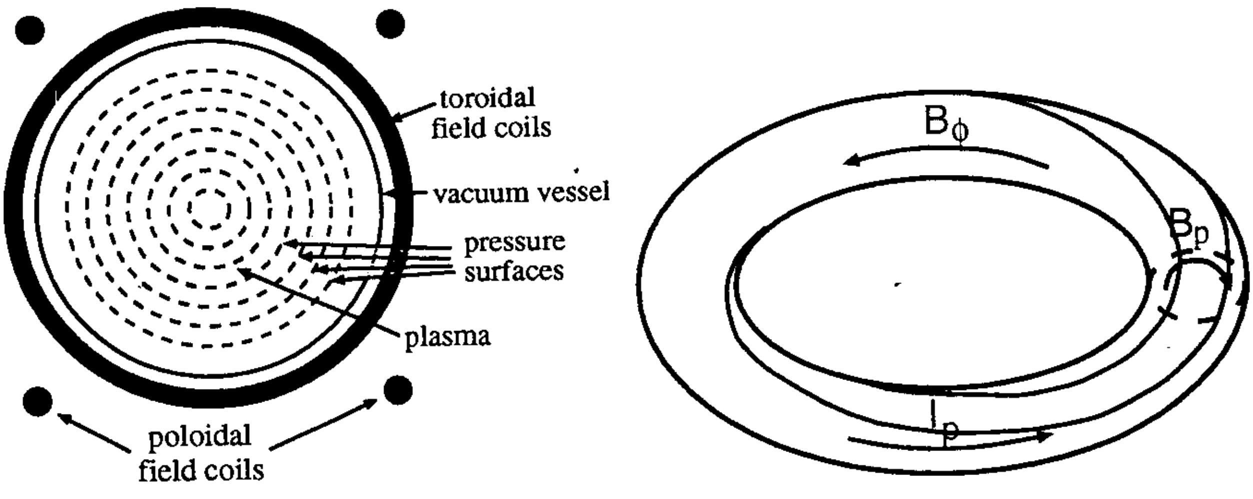
  pressure gradient balanced by Lorentz's force, no force down the pressure gradient
  ⇒ volume element stays on surface with constant pressure

- however: *there is transport along the pressure gradient*
  - collisions: particles from adjacent volume elements are exchanged
    too small to explain experimentally observed transport rates
  - turbulence: the plasma is not in static quiescent equilibrium; small scale fluctuations lead to mixing

schematic plot of a tokamak



schematic plot of a tokamak



- toroidal plasma current generates poloidal magnetic field
- magnetic fields lines stay on pressure surfaces

Turbulent transport

How can fluctuations lead to transport?

- particles move on circles in plane perpendicular to the magnetic field (Lorentz's force), move freely along the magnetic field
- in equilibrium magnetic field lines stay on surface with constant pressure

$$\nabla p = \mathbf{j} \times \mathbf{B}$$

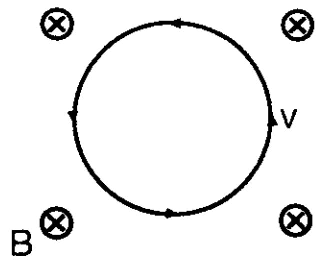
- magnetic turbulence: small fluctuating component of **B** parallel to the pressure gradient: particles can flow from one pressure surface to another
- electrostatic turbulence: in the presence of an electric field, particles drift in the plane perpendicular to the magnetic field. Particles can move from one pressure surface to another, magnetic topology is not affected

Motion of charged particles in constant magnetic field

equation of motion

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B}) + \mathbf{F}$$

- no constraint due to magnetic field in direction along **B**
- in plane perpendicular to **B**:
  - $\mathbf{F} = 0$: force $\mathbf{v} \times \mathbf{B}$ always perpendicular to $\mathbf{v}$; circular orbit



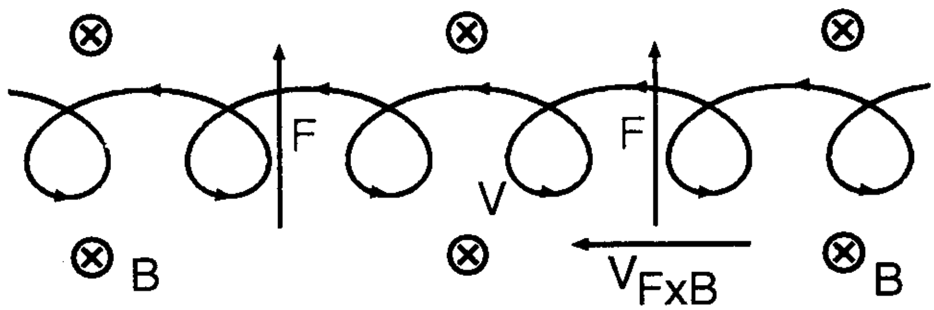
– $\mathbf{F} \neq 0$: split $\mathbf{v}$ into $\mathbf{v}_{drift} + \mathbf{v}_{gyro}$

$$m\frac{d\mathbf{v}_{gyro}}{dt} = q(\mathbf{v}_{gyro} \times \mathbf{B})$$
$$0 = q(\mathbf{v}_{drift} \times \mathbf{B}) + \mathbf{F}$$

first equation: circular orbit in moving frame
second equation:

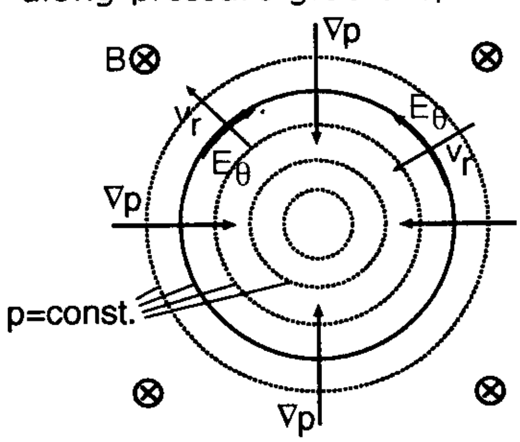
$$\mathbf{v}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

drift motion perpendicular to **B** *and perpendicular to* **F**



Electrostatic transport

- no static electric field in homogeneous magnetic field (remark: weak electric field in a torus)
- no net velocity along pressure gradient, since $\nabla \times \mathbf{E} = 0$



$$\int_p n_0 v_r \, ds \sim n_0 \int_p E_\theta \, ds = 0$$

- *need correlated fluctuations of potential and density*

Where do fluctuations come from ?

- equilibrium is unstable
- electrons and ions move into different directions: charge separation, electric fields
- instability saturates at finite amplitude due to nonlinear effects
- stationary turbulence

Model and Equations

Equations

- description must distinguish between ions and electrons to allow for charge separation: single fluid MHD not sufficient ⇒ easiest extension: two fluid description
- why is two fluid description reasonable ?
  ion-ion and electron-electron energy exchange due to collisions much more efficient than between species with different mass ⇒ thermal equilibrium within one species is reached much faster than within total system
  each species can be in thermal equilibrium with well defined temperature, but electron and ion temperature may be different

- Braginskii two fluid equations:
  - for each species (ions and electrons): continuity equation, momentum balance equation, energy balance equation (describes evolution of temparature)
  - can be derived from kinetic description with phase space distribution function $f(\mathbf{x}, \mathbf{v})$ by taking moments in velocity space:

$$
\begin{aligned}
n(\mathbf{x}, t) &= \int f(\mathbf{x}, \mathbf{v})\, d\mathbf{v} \\
\mathbf{V}(\mathbf{x}, t) &= \frac{1}{n} \int \mathbf{v} f(\mathbf{x}, \mathbf{v})\, d\mathbf{v} \\
T(\mathbf{x}, t) &= \frac{1}{n} \int \frac{m}{3} (\mathbf{v} - \mathbf{V})^2 d\mathbf{v}
\end{aligned}
$$

  - here: introduce main aspects in a more intuitive way

- isothermal limit: ion and electron temperature are constants (on each pressure surface) ⇒ two equations for each species
- continuity equation (conservation of total particle number):

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} = 0$$

- momentum conservation

$$mn\frac{d\mathbf{v}}{dt} = qn\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right] - \nabla p + \mathbf{R}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

(compare Euler/ Navier-Stokes equations in hydromechanics)

dynamics along the magnetic field

- neglect electron mass

$$\begin{aligned} m_i n_i \frac{d}{dt}\mathbf{v}_\parallel &= -en\nabla_\parallel \phi - \nabla_\parallel p_i + R_{ie} \\ 0 &= +en\nabla_\parallel \phi - \nabla_\parallel p_e + R_{ei} \end{aligned}$$

- R_{ei}, R_{ie}: collisional momentum exchange
  conservation of total momentum requires $R_{ei} = -R_{ie}$
  friction must be proportional to velocity difference $v_{\parallel i} - v_{\parallel e} = j_\parallel / en$:

$$R_{ei} = -R_{ie} = ne\eta_\parallel j_\parallel, \qquad \textit{Spitzer resistivity } \eta_\parallel \sim T_e^{-3/2}$$

-

$$m_i n_i \frac{dv_\parallel}{dt} = -\nabla_\parallel (p_i + p_e) \quad \text{parallel equation of motion}$$

$$\eta_\parallel j_\parallel = -\nabla_\parallel \phi + \frac{1}{en}\nabla_\parallel p_e \quad \text{generalized Ohm's law}$$

simplified dynamics perpendicular to B

equations still contain the fast gyro motion; restrict now to time scales that are slow compared to the gyro motion

to lowest order: neglect inertia term d/dt

$$0 = qn\left[\mathbf{E} + \mathbf{v}_0 \times \mathbf{B}\right] - \nabla p$$

this yields

$$\mathbf{v}_0 = \mathbf{v}_E + \mathbf{v}_d$$

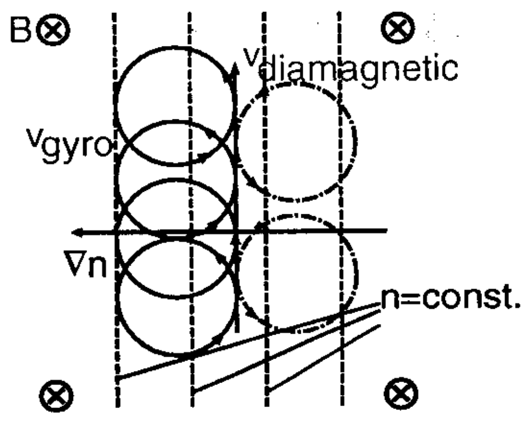
$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \qquad \mathbf{E} \times \mathbf{B}\text{-}drift$$

$$\mathbf{v}_d = \frac{\mathbf{B} \times \nabla_\perp p}{qnB^2} \qquad diamagnetic\ drift$$

note: $\mathbf{E} \times \mathbf{B}$-drift depends neither on mass nor on charge, identical for ions and electrons ⇒ no charge separation; diamagnetic drift has opposite sign for ions and electrons

physical meaning of diamagnetic drift

- particles move on circular orbits
- in system with homogeneous density and temperature different contributions average to zero; no net fluid velocity
- in system with density (pressure) gradient net contribution remains in the direction perpendicular to the density (pressure) gradient



polarization drift

first order correction from equation of motion

$$mn\frac{d\mathbf{v}}{dt} = qn\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right] - \nabla p$$

since $d/dt \ll 1$ approximate $\mathbf{v}$ on the left hand side by the lowest order solution $\mathbf{v}_0 \equiv \mathbf{v}_E + \mathbf{v}_{di}$:

$$mn\frac{d\mathbf{v}_0}{dt} = qn\left[\mathbf{E} + (\mathbf{v}_0 + \mathbf{v}_1)\times\mathbf{B}\right] - \nabla p$$

with

$$0 = qn\left[\mathbf{E} + \mathbf{v}_0\times\mathbf{B}\right] - \nabla p$$

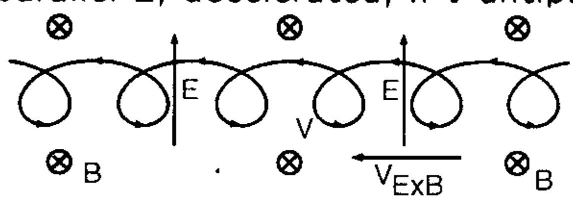
we obtain

$$\mathbf{v}_{pol} \equiv \mathbf{v}_1 = \frac{1}{\omega_c}\frac{d}{dt}\mathbf{b}\times\mathbf{v}_0 = -\frac{1}{\omega_c B}\frac{d}{dt}\left(\nabla_\perp\phi + \frac{1}{qn}\nabla_\perp p\right), \quad \omega_c = \frac{eB}{m}$$

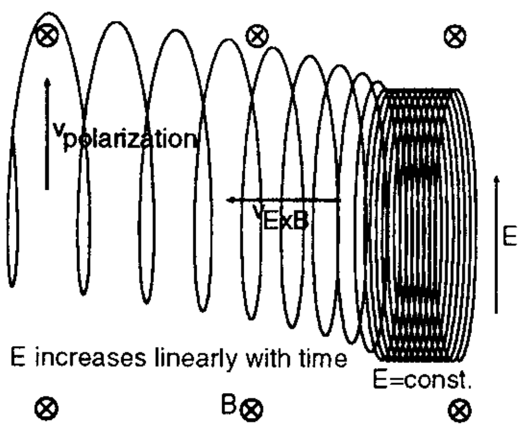
polarization drift only for ions important, since gyro frequency $\omega_{ce} \gg \omega_{ci}$

physical meaning of polarization drift

- motion in constant electric field ($\mathbf{E}\times\mathbf{B}$-drift): particle is accelerated, if $\mathbf{v}$ parallel $\mathbf{E}$; decelerated, if $\mathbf{v}$ antiparallel $\mathbf{E}$



- slowly increasing electric field: acceleration and deceleration phases become increasingly strong, drift in the direction of the electric field



drift reduced equations

insert drift velocities into continuity equations

- ions

$$\frac{\partial n_i}{\partial t} + \nabla \cdot n_i(\mathbf{v}_E + \mathbf{v}_{di} + \mathbf{v}_{pol} + \mathbf{v}_{\|}) = 0$$

- electrons

$$\frac{\partial n_e}{\partial t} + \nabla \cdot n_e(\mathbf{v}_E + \mathbf{v}_{de} + \mathbf{v}_{\|,e}) = 0, \qquad e n_e \mathbf{v}_{\|,e} = e n_i \mathbf{v}_{\|} - \mathbf{j}_{\|}$$

- electrostatic potential

$$\nabla^2 \phi = -(n_i - n_e) e/\epsilon_0$$

Debye-shielding — how large is $n_i - n_e$?

- Poisson equation (fourier transformed)

$$\nabla^2 \phi = -\frac{e(n_e - n_i)}{\epsilon_0} \qquad \Rightarrow \qquad -k^2 \phi = -\frac{e(n_e - n_i)}{\epsilon_0}$$

- electron density in thermal equilibrium

$$n_e = n_0 \exp\left\{\frac{e\phi}{T_e}\right\} \simeq n_0 \left(1 + \frac{e\phi}{T_e}\right) = n_0 + \frac{n_e - n_i}{k^2 \lambda_{debye}^2} \qquad \lambda_{debye} = \left(\frac{\epsilon_0 T_e}{n_0 e^2}\right)^{1/2}$$

$$\frac{n_e - n_i}{n_e - n_0} = k^2 \lambda_{debye}^2 \ll 1$$

- on large scales $1/k \gg \lambda_{debye}$ the difference between electron and ion density is negligible (quasi-neutrality)

$$n_e = n_i$$

this implies *not*, that there is no electric field

final equations

- electron continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot n(\mathbf{v}_E + \mathbf{v}_{de}) + \nabla \cdot n v_\parallel - \nabla_\parallel j_\parallel / e = 0$$

- subtract ion and electron continuity equation: determines electrostatic potential ϕ, since $n_e = n_i$

$$\nabla \cdot n\mathbf{v}_{pol} + \nabla \cdot n(\mathbf{v}_{di} - \mathbf{v}_{de}) + \nabla_\parallel j_\parallel / e = 0$$

- parallel motion and parallel Ohm's law

$$m_i n \frac{dv_\parallel}{dt} = -\nabla_\parallel (p_i + p_e)\,, \qquad \eta_\parallel j_\parallel = -\nabla_\parallel \phi + \frac{1}{en}\nabla_\parallel p_e$$

$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla$ total time derivative including convection

final equations in homogeneous magnetic field

- in homogeneous magnetic field

$$\nabla \cdot n\mathbf{v}_E = \mathbf{v}_E \cdot \nabla n + n \nabla \cdot \mathbf{v}_E$$

$$\nabla \cdot \mathbf{v}_E = \frac{1}{B} \nabla \cdot \mathbf{b} \times \nabla \phi = -\frac{1}{B} \mathbf{b} \cdot \nabla \times \nabla \phi = 0$$

$$\nabla \cdot n\mathbf{v}_d = \frac{1}{qB} \nabla \cdot \mathbf{b} \times \nabla p = 0$$

- equations

$$\frac{dn}{dt} + \nabla_\parallel \cdot n v_\parallel - \nabla_\parallel \frac{j_\parallel}{e} = 0$$

$$\frac{1}{B\omega_{ci}} \nabla \cdot n \frac{d}{dt} \left(\nabla_\perp \phi + \frac{1}{qn} \nabla_\perp p_i \right) - \nabla_\parallel \frac{j_\parallel}{e} = 0$$

discussion of the equations

simplest case (Hasegawa-Wakatani):
homogeneous magnetic field, $T_e \gg T_i \simeq 0$, neglect parallel ion motion, linearize except for $\mathbf{v}_E \cdot \nabla$ convection

$$\frac{n}{B\omega_{ci}} \frac{d}{dt} \nabla_\perp^2 \phi = \frac{1}{e} \nabla_\| j_\| \;\;,\;\; \frac{dn}{dt} = \frac{1}{e} \nabla_\| j_\|$$

- different motion of ions and electrons (polarization drift $\leftrightarrow$ parallel current): charge separation, electric field, $\mathbf{E} \times \mathbf{B}$-drift
- parallel current: fluctuating magnetic field
- $\mathbf{E} \times \mathbf{B}$-drift is dominant velocity component, $d/dt = \partial/\partial t + \mathbf{v}_E \cdot \nabla$ yields convective nonlinearity as in hydromechanics $\Rightarrow$ turbulence
- **key ingredient for every model of plasma turbulence**

connection to 2D hydromechanics

equation for ϕ in the limit $\nabla_\| = 0$:

$$\frac{d}{dt} \nabla_\perp^2 \phi = 0 \,, \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{e}_\| \times \nabla \phi \cdot \nabla$$

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incompressible ($\nabla \cdot \mathbf{v} = 0$) 2D hydromechanics:

$$\frac{d}{dt} \rho \mathbf{v} = 0, \;\; \frac{d\rho}{dt} = 0, \;\; \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

after some calculation: $\mathbf{v} = \mathbf{e}_\| \times \nabla \phi$

$$\frac{d}{dt} \nabla_\perp^2 \phi = 0 \,, \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{e}_\| \times \nabla \phi \cdot \nabla$$

dynamics in plane perpendicular to the magnetic field very similar to two dimensional hydromechanic
expect similar behaviour $\Rightarrow$ turbulence

Example: Hasegawa-Wakatani system

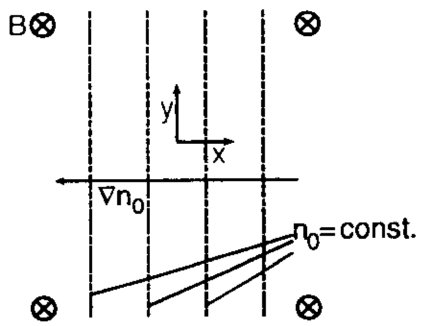
example: Hasegawa-Wakatani system

homogeneous magnetic field along **z**, cold ions, constant electron temperature, background density n_0 with gradient along **x**

$$\frac{n_0}{B\omega_{ci}}\frac{d}{dt}\nabla_\perp^2\phi = \frac{d\tilde{n}}{dt} + \mathbf{v}_E\cdot\nabla n_0\,, \qquad \mathbf{v}_E\cdot\nabla n_0 = -\frac{1}{B}\frac{\partial\phi}{\partial y}\frac{\partial n_0}{\partial x}$$

$$\frac{d\tilde{n}}{dt} + \mathbf{v}_E\cdot\nabla n_0 = \frac{1}{e}\nabla_\parallel j_\parallel\,, \qquad \eta_\parallel j_\parallel = -\nabla_\parallel\phi + \frac{T_e}{en_0}\nabla_\parallel\tilde{n}$$

for numerical simulations add dissipation at short scales, for example $\sim\nabla_\perp^8\phi$, similar for n



linear properties: drift waves

- calculate linear frequencies: ansatz exp$\{i\omega t\}$
- restrict to limit $\eta_{\|} \ll 1$: parallel Ohm's law yields

$$\frac{\tilde{n}}{n_0} = \frac{e\phi}{T_e}$$

adiabatic limit; physical meaning: electrons thermalize along the fields lines, follow linearized Boltzmann distribution:

$$n_0 + \tilde{n} = n_0 \left(1 + \frac{e\phi}{T_e}\right) = n_0 \mathrm{exp} \left\{\frac{e\phi}{T_e}\right\}$$

- insert into vorticity equation:

$$-\frac{i\omega k_\perp^2 \phi}{B\omega_{ci}} = i\omega \frac{e\phi}{T_e} - \frac{1}{n_0}\frac{\partial n_0}{\partial x}\frac{ik_y\phi}{B}$$

- dispersion relation

$$\omega = \frac{\omega_{*e}}{1 + k_\perp^2 \rho_s^2}$$

with the drift frequency

$$\omega_{*e} = -\frac{T_e}{m_i\omega_{ci}}\frac{k_y}{L_n}, \quad \frac{1}{L_n} = -\frac{1}{n_0}\frac{\partial n_0}{\partial x}$$

and the gyro-radius of an ion at electron thermal energy

$$\rho_s^2 = \frac{T_e}{m_i \omega_{ci}^2}$$

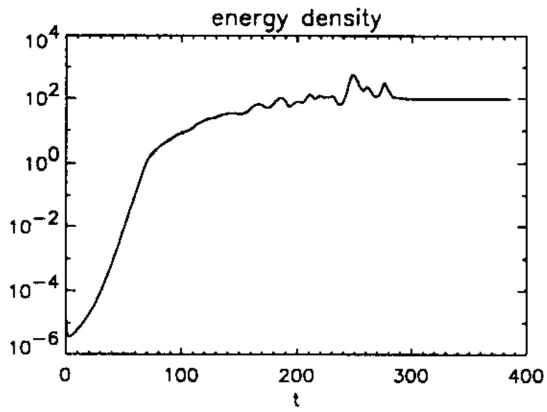
- at long wave length: $\omega \sim k_y$
wave with constant phase velocity, drift-wave; drift-waves are present in most systems used for studying plasma turbulence and transport; widely used name: **drift-wave turbulence**

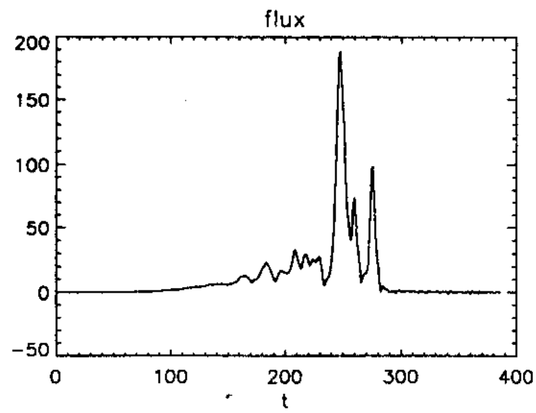
numerical simulation: algorithm

how can we solve the equations numerically?

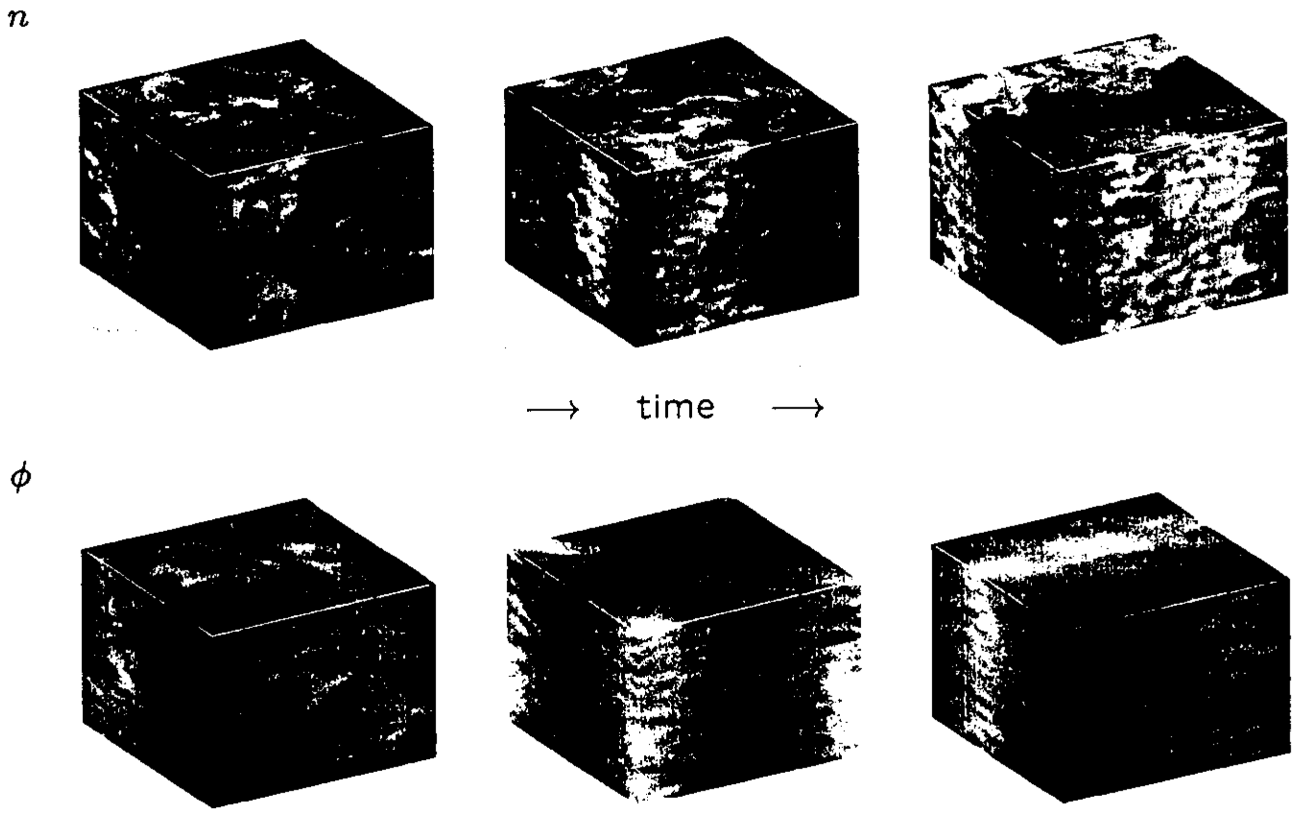
- take Fourier transform of the equations in all three space dimensions:
$$k_\perp^2 \frac{\partial \phi}{\partial t} = f_\phi\,, \qquad \frac{\partial n}{\partial t} = f_n$$
f_ϕ and f_n contain spatial derivatives of n and ϕ, but no time derivatives; in fourier space spatial derivatives are substituted by $\partial/\partial x \to ik_x$, etc.
- substitute time derivative by finite difference, example:
$$\frac{\partial n}{\partial t} \approx \frac{n(t+\Delta t) - n(t)}{\Delta t}$$
- with $n(t)$ and $\phi(t)$ given we can compute n and ϕ at the advanced time $t + \Delta t$; repeated application of this algorithm yields complete time evolution
- start from some low amplitude noise and look what happens ...

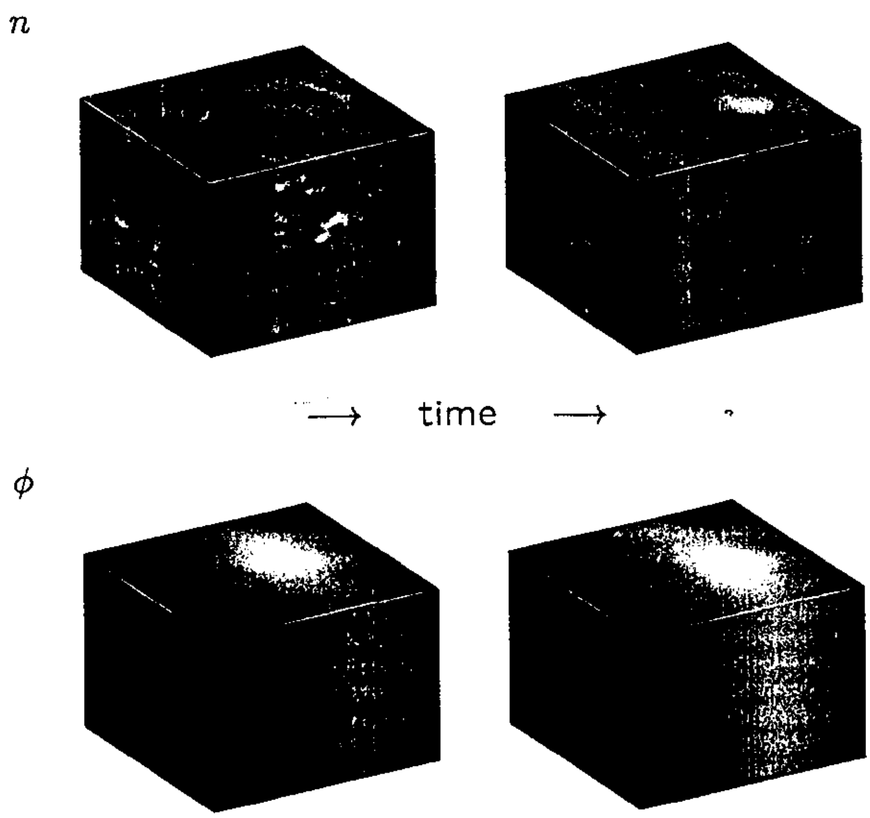
numerical simulation





- define turbulent energy to measure fluctuation level
- linear phase at the beginning: fluctuation level is small, nonlinear correction is negligible; fluctuation level grows exponentially following linear instability (complete linear analysis shows linear instability due to finite $\eta_\|$)
- nonlinear saturation; in general stationary turbulence, here: evolution into state with strong sheared flow


n
→ time →
ϕ


n
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ϕ

turbulent energy

The equations

$$\frac{n_0}{B\omega_{ci}}\frac{d}{dt}\nabla_\perp^2\phi = \frac{d\tilde{n}}{dt} + \mathbf{v}_E\cdot\nabla n_0\,, \qquad \mathbf{v}_E\cdot\nabla n_0 = -\frac{1}{B}\frac{\partial\phi}{\partial y}\frac{\partial n_0}{\partial x}$$

$$\frac{d\tilde{n}}{dt} + \mathbf{v}_E\cdot\nabla n_0 = \frac{1}{e}\nabla_\parallel j_\parallel\,, \qquad \eta_\parallel j_\parallel = -\nabla_\parallel\phi + \frac{T_e}{en_0}\nabla_\parallel\tilde{n}$$

imply the evolution of the turbulent energy

$$\frac{1}{2}\frac{d}{dt}\int_V \underbrace{\left\{\rho_s^2\left|\nabla_\perp\frac{e\phi}{T_e}\right|^2 + \left(\frac{\tilde{n}}{n_0}\right)^2\right\}}_{\text{turbulent energy}} dV = \int_V \left\{\underbrace{\frac{\rho_s^2\omega_{ci}}{L_n}\frac{\tilde{n}}{n_0}\frac{\partial}{\partial y}\frac{e\phi}{T_e}}_{\text{drive}} - \underbrace{\frac{\eta_\parallel}{T_e n_0}j_\parallel^2}_{\text{dissipation}}\right\} dV$$

- drive due to background gradient
- ohmic dissipation
- *dominant damping due to additional viscous terms at small scales*

nonlinear saturation

how does nonlinear saturation work ?

- linear instability drives turbulence: potential energy is extracted from the density gradient, converted into fluctuation energy (kinetic energy of flows and potential energy)
- measure rate, with which energy is dumped into fluctuations: remains large even after saturation, when the energy content does not grow further
- **there must be a strong mechanism, which damps the fluctuations**; there is dissipation at small scales, however energy is injected at large scales
- **how does energy transport from large to small scales occur ?**

General aspects of turbulence

general aspects of turbulence

2D incompressible hydrodynamics with viscosity

$$\frac{d}{dt}\nabla_\perp^2\phi = \nu\nabla_\perp^4\phi$$

energy balance (periodic boundary conditions)

$$\int_V dV\,\left\{\phi\frac{d}{dt}\nabla_\perp^2\phi\right\} = -\int_V dV\,\left\{\nabla_\perp\phi\frac{\partial}{\partial t}\nabla_\perp\phi\right\} + \int_V dV\,\left\{\phi\mathbf{z}\times\nabla_\perp\phi\cdot\nabla\nabla_\perp^2\phi\right\} =$$

$$= -\frac{1}{2}\frac{d}{dt}\int_V dV\,(\nabla_\perp\phi)^2$$

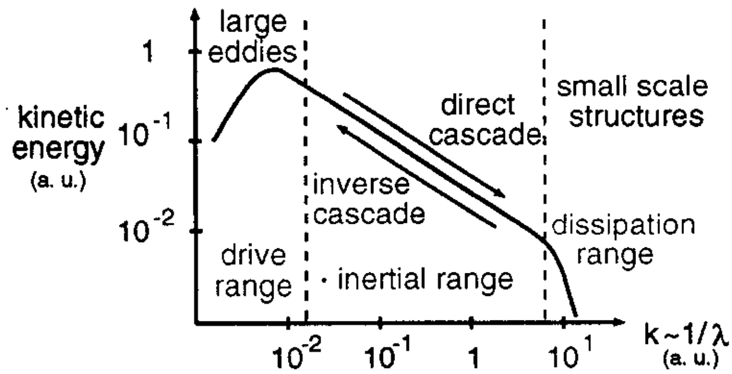
since $\phi\mathbf{z}\times\nabla_\perp\phi\cdot\nabla\nabla_\perp^2\phi = \nabla\cdot(\phi\mathbf{z}\times\nabla\phi\nabla_\perp^2\phi)$

$$\int_V dV\,\left\{\phi\nabla_\perp^4\phi\right\} = \int_V dV\,\left\{\left(\nabla_\perp^2\phi\right)^2\right\}$$

$$\frac{1}{2}\frac{d}{dt}\int_V dV\,(\nabla_\perp\phi)^2 = -\nu\int_V dV(\nabla_\perp^2\phi)^2$$

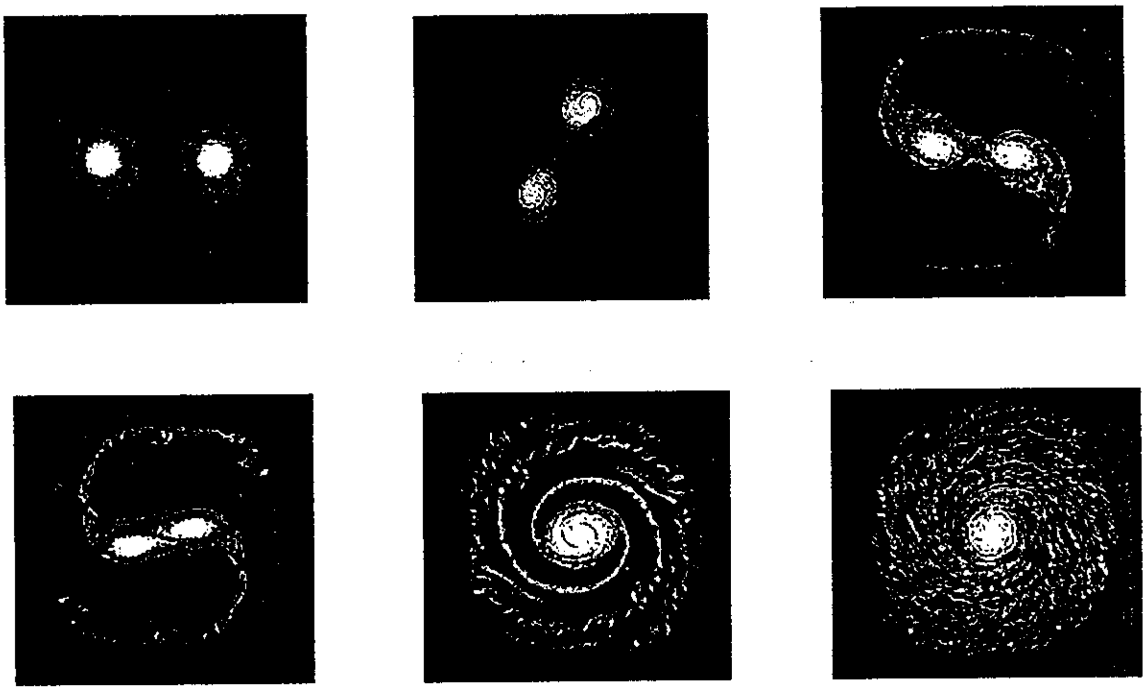
- kinetic energy $\int(\nabla_\perp\phi)^2 = \int \mathbf{v}^2$ is conserved except for viscous terms: dissipation at small scales (where $\nabla_\perp^2$ is large)
- without drive: decaying turbulence, kinetic energy decreases continously
- stationary turbulence: need energy *source*; in plasma turbulence this is usually provided by an instability mechanism at large scales
- energy is transported nonlinearly to small scales; convection itself conserves kinetic energy

energy flux in turbulent system



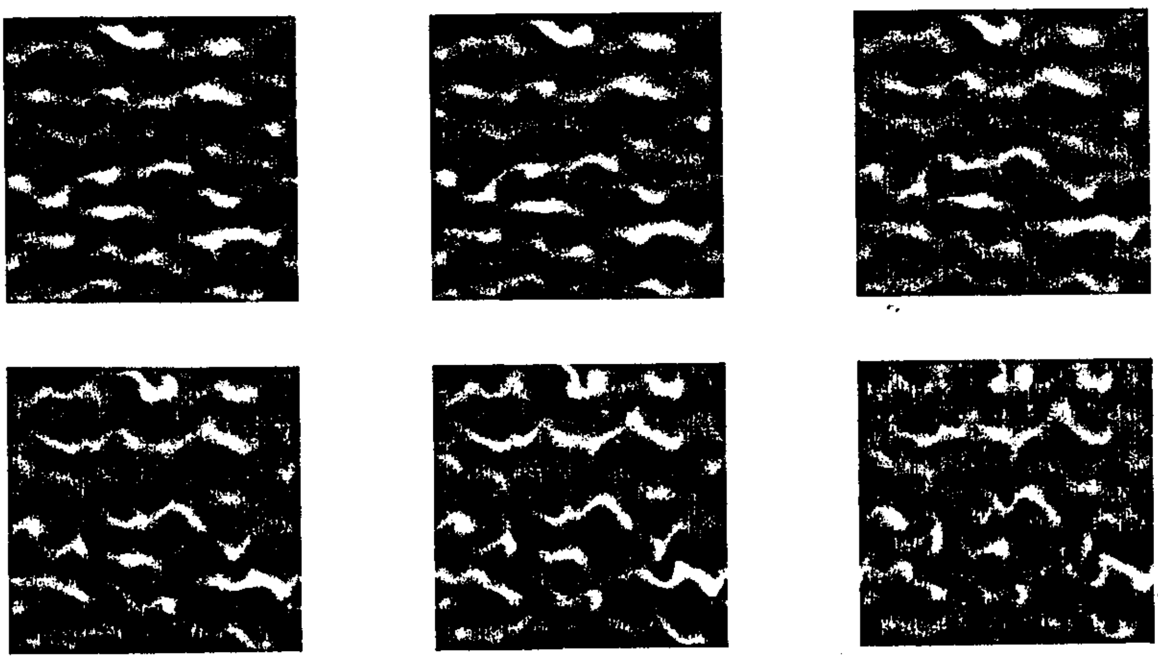
- large scales: drive by instability
- small scales: kinetic energy is transformed into heat (viscosity)
- in between: transfer by direct and inverse cascades, inertial range
  - direct cascade: from large scales to small scales (break-up of large scale structures, e. g. Kelvin-Helmholtz instability)
  - inverse cascade: from small scales to large scales (e. g. merging of vorticies)

coalescence of vortices

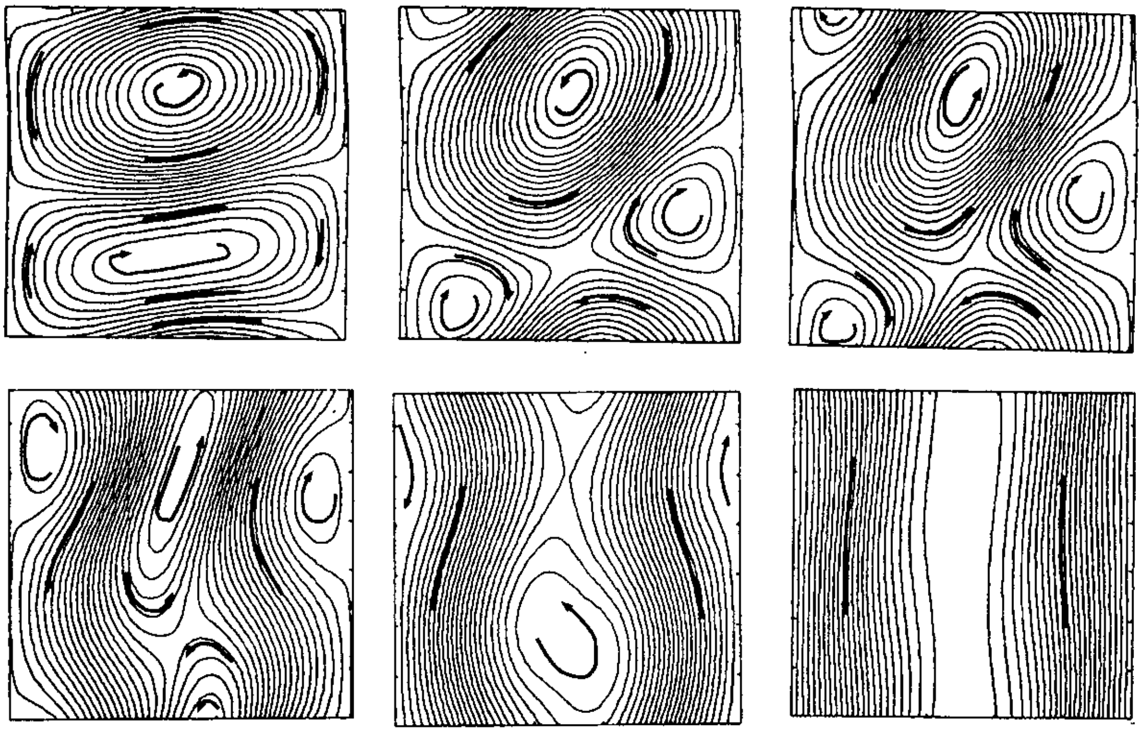


D. Porter, P. R. Woodward, W. Yang, Q. Mei, Nonlinear Astrophysical Fluid Dynamics **617**, 234 (1990)

Kelvin-Helmholtz instability



generation of sheared flow



J. M. Finn, J. F. Drake, P. N. Guzdar, Phys. Fluids B **4**, 2758 (1992)

Drive mechanisms

drive mechanisms

- instability mechanism extracts energy from background gradients and drives turbulence at large scale
- saturation based on nonlinear $\mathbf{E} \times \mathbf{B}$ convection and transfer to small scales similar in most systems exhibiting (small scale) turbulence
- drive mechanism sets characteristic scale length L_{drive} (wave length of the unstable mode) and time scale t_{drive} (growth rate)
  allows rough estimate of turbulent diffusion rates:
  $$D \sim \frac{L_{drive}^2}{t_{drive}} \left[\frac{m^2}{s}\right]$$
  actual transport rate must be determined by nonlinear simulation !
- **drive mechanism is most important distinguishing feature for different types of turbulence (in the small scale regime which is relevant for transport)**

complete set of equations

- continuity $$\frac{\partial n}{\partial t} + \nabla \cdot n(\mathbf{v}_E + \mathbf{v}_{di} + \mathbf{v}_{pol} + \mathbf{v}_{\parallel i}) = 0$$
- vorticity $$\nabla \cdot n\mathbf{v}_{pol} + \frac{1}{e}\nabla_{\parallel} j_{\parallel} + \nabla \cdot n(\mathbf{v}_{di} - \mathbf{v}_{de}) = 0$$
- ion temperature $$\frac{3}{2} n \frac{dT_i}{dt} - T_i \frac{dn}{dt} = 0$$
- electron temperature $$\frac{3}{2} n \frac{dT_e}{dt} - T_e \frac{dn}{dt} - \nabla_{\parallel} \kappa_{\parallel} \nabla_{\parallel} T_e - \frac{0.71}{e} T_e \nabla_{\parallel} j_{\parallel} = 0$$
- parallel motion $$m_i n \frac{d}{dt} \mathbf{v}_{\parallel i} = -\nabla_{\parallel}(p_i + p_e)$$
- generalized Ohm's law (ψ = parallel component of vector potential) $$\eta_{\parallel} j_{\parallel} = -\nabla_{\parallel}\phi + \frac{\partial \psi}{\partial t} + \frac{1}{en}\nabla_{\parallel} p_e + \frac{0.71}{e}\nabla_{\parallel} T_e$$

$$\mathbf{v}_{pol} = \frac{\mathbf{B}}{B\omega_{ci}} \times \frac{d}{dt}(\mathbf{v}_E + \mathbf{v}_{di})$$ generalized polarization drift

resistive g-mode

- general force **F** (gravitation) leads to drift

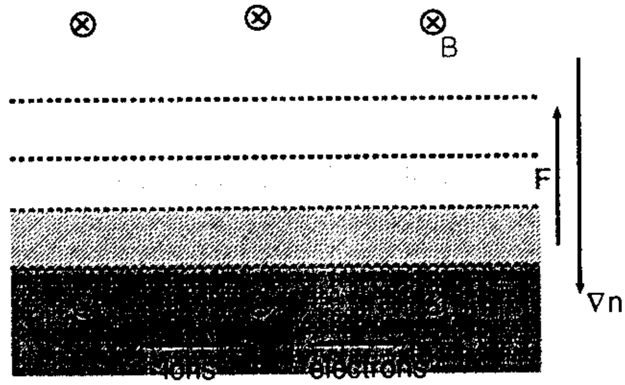
$$\mathbf{v}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

  different for electrons and ions!

- if drift-velocity is along density gradient:
  region with high ion density is shifted into region with low electron density (or vice versa)
  charge separation ⇒ electric field ⇒ $\mathbf{E} \times \mathbf{B}$-drift
- **F** parallel density gradient: stable equilibrium
- **F** anti-parallel density gradient: unstable equilibrium

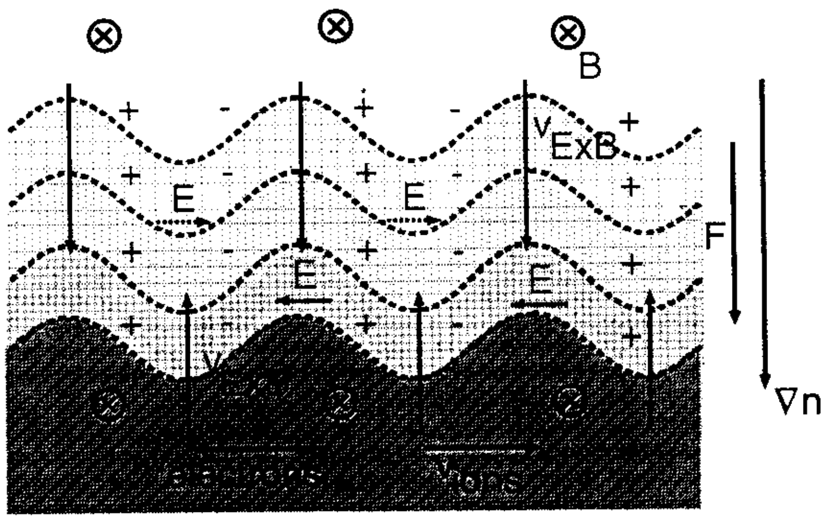
resistive g-mode

equilibrium



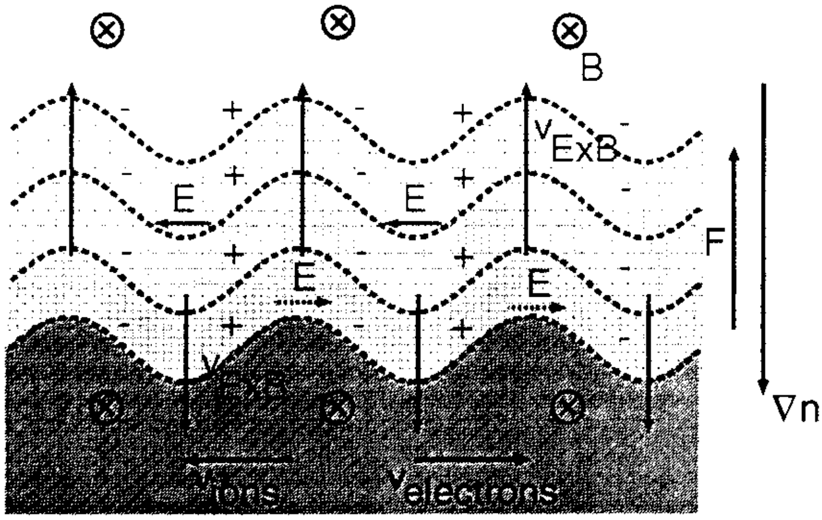
resistive g-mode

stable equilibrium: perturbation is damped



resistive g-mode

unstable equilibrium: perturbation grows



resistive ballooning mode

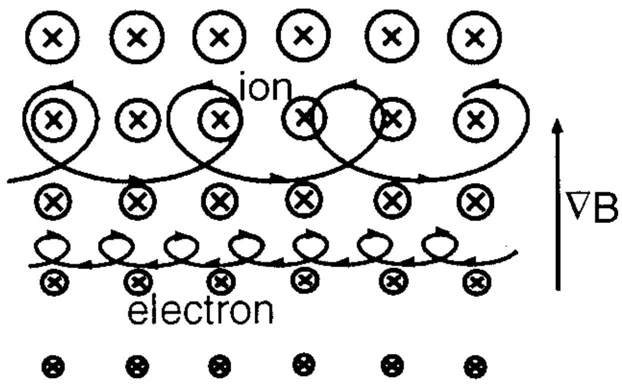
- in realistic system resistive **g**-mode is negligible, because gravitation is too weak
- however: very similar mechanism in a torus; gravitational force is substituted by grad-B drift, effective gravity is antiparallel to ∇B
- unstable at outside of the torus, stable at inside of the torus: mode must be localized at outside of the torus
- $\mathbf{E} \times \mathbf{B}$ convection of density gradient is mechanism to tap source of energy
- why is resistivity required ?

$$\oint \phi \mathbf{v}_E \cdot \mathbf{r} \sim \oint \phi \frac{\partial \phi}{\partial \theta} = 0$$

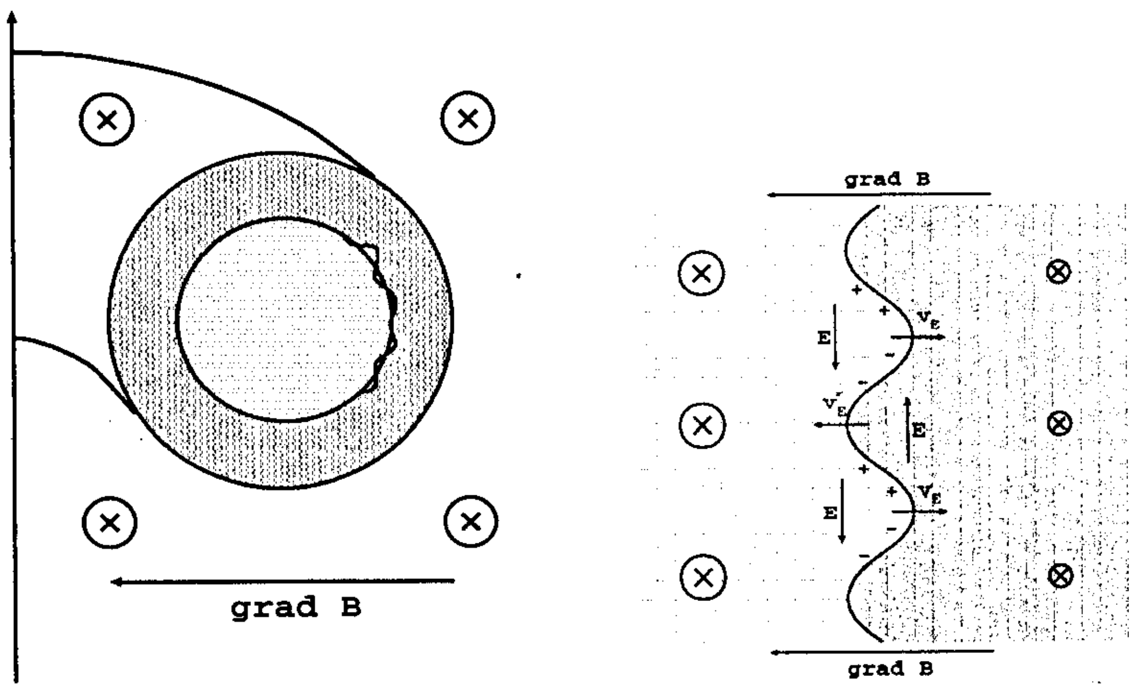
no drive if $\tilde{n} = \phi$ (adiabatic limit) $\Rightarrow$ resistivity is essential

grad-B drift

- spatially varying magnetic field: gyro-radius changes during orbit
- drift depends on gyro-radius and velocity of the particle: grad-B drift depends linearly on kinetic energy $\simeq$ temperature



ballooning instability



grad-B drift: ions move upwards, electrons move downwards

M. N. Rosenbluth, C. L. Longmire, Annals of Physics 1, 120 (1957)

ballooning instability

- basic equations

$$\frac{n_0}{B\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2\phi + \frac{2T_e}{eBR}\frac{\partial \tilde{n}}{\partial y} - \frac{1}{e}\nabla_\parallel j_\parallel = 0$$

$$\frac{\partial \tilde{n}}{\partial t} + \frac{n_0}{BL_n}\frac{\partial \phi}{\partial y} = 0$$

- curvature drive evaluated at outside of the torus
- parallel spread connects favourable and unfavourable curvature location
- parallel spread negligible (short wavelength limit):

$$\gamma = c_s\left(\frac{2}{RL_n}\right)^{1/2}, \qquad c_s^2 = \frac{T_e}{m_i}$$

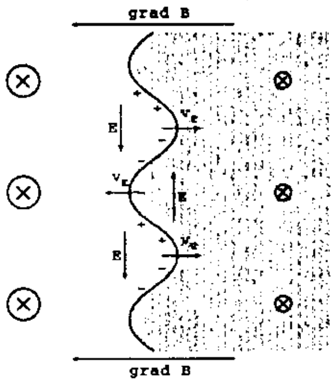
ideal ballooning growth rate

toroidal η_i mode

- resistive ballooning mode taps density gradient: only possible, if $\tilde{n} \neq \phi$ in resistive plasma; impossible in hot core plasma, where resistivity is very small ($\eta_\parallel \sim T_e^{-3/2}$)
- grad-B drift is proportional to the temperature: charge separation due to temperature gradient $\Rightarrow$ resistive **g** instability works also for temperature gradient
- instability taps ion temperature gradient, does not depend on $\tilde{n} \neq \phi$
- mode localized at the outside of the torus; localization along the field line determined by ion sound propgation speed
- carefull analysis shows: stability depends on

$$\frac{1}{n}\frac{dn}{dr}\Big/\frac{1}{T_i}\frac{dT_i}{dr} \equiv \eta_i$$

- **widely believed to be responsible for transport in plasma core**



grad-B drift:
ions move upwards,
electrons move downwards

- **resistive ballooning mode: high/low density**
  electrons and ions move into opposite direction; ions from low density area and electrons from high density area appear at the same place and vice versa ⇒ charge
- η_i **mode: high/low temperature**
  particles with high temperature are displaced faster than particles with low temperature; creates regions with increased ion density and reduced electron density (and vice versa) ⇒ charge

---

toroidal η_i mode

- basic equations

$$\frac{n_0}{B\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2\left(\phi+\frac{\tilde{p}_i}{en_0}\right)+\frac{2}{eBR}\frac{\partial\tilde{p}_i}{\partial y}-\frac{1}{e}\nabla_\| j_\| = 0$$

$$\frac{\partial\tilde{n}}{\partial t}-\frac{1}{e}\nabla_\| j_\| = 0\,, \qquad \frac{\partial\tilde{p}_i}{\partial t}+\frac{p_{i0}}{BL_{p_i}}\frac{\partial\phi}{\partial y} = 0$$

- low resistivity (adiabatic electrons):

$$\eta_\| j_\| = -\nabla_\|\phi+\frac{T_e}{en_0}\nabla_\|\tilde{n} = 0 \quad\Rightarrow\quad \tilde{n} = \frac{en_0}{T_e}\phi$$

- dispersion relation

$$\left(1+k_y^2\rho_s^2\right)\gamma^2 - i\omega_{*i}k_y^2\rho_s^2\gamma - \omega_{*i}\omega_{curv} = 0,$$

$$\omega_{*i} \equiv \frac{k_yT_i}{eBL_p}, \quad \omega_{curv} \equiv \frac{2k_yT_e}{eBR}$$

slab η_i mode

besides the toroidal η_i mode there is a solution, which does not depend on ∇B

- start with locally increased density
- homogeneous system (no pressure/density gradients):
  pressure rises $\Rightarrow$ excites sound wave, which carries away perturbation along field lines $\Rightarrow$ stable
- inhomogeneous system: density perturbation causes electric potential and **E** $\times$ **B**-drift
- radial **E** $\times$ **B** drift convects low density and temperature plasma into region of increased density: pressure *drops*
- **non-oscillating, unstable parallel sound wave**
- complete analysis shows again dependence on η_i

slab η_i mode

- basic equations

$$\frac{\partial \tilde{n}}{\partial t} + \frac{n_0}{BL_n}\frac{\partial \phi}{\partial y} + n_0 \nabla_{\parallel} v_{\parallel} = 0$$

$$\frac{\partial \tilde{T}_i}{\partial t} + \frac{T_{i0}}{BL_{T_i}}\frac{\partial \phi}{\partial y} + \frac{2}{3} T_{i0} \nabla_{\parallel} v_{\parallel} = 0$$

$$\frac{\partial}{\partial t} v_{\parallel} = -\frac{1}{m_i} \nabla_{\parallel} \tilde{T}_i$$

- assume growthrate much smaller than drift wave frequency: neglect time derivative in continuity equation

$$\frac{1}{BL_n}\frac{\partial \phi}{\partial y} = -\nabla_{\parallel} v_{\parallel}$$

- final equation

$$\frac{\partial^2 \tilde{T}_i}{\partial t^2} + \frac{T_{i0}}{m_i}\left(\eta_i - \frac{2}{3}\right) \nabla_{\parallel}^2 \tilde{T}_i = 0$$

sound wave if $\eta_i < 2/3$*; unstable if* $\eta_i > 2/3$

nonlinear drift wave instability

- basic equations (Hasegawa-Wakatani)

$$\frac{n_0}{B\omega_{ci}}\frac{d}{dt}\nabla_\perp^2\phi + \frac{1}{e\eta_\|}\left(\nabla_\|^2\phi - \frac{T_e}{en_0}\nabla_\|^2\tilde{n}\right) = 0$$

$$\frac{d\tilde{n}}{dt} + \frac{n_0}{BL_n}\frac{\partial\phi}{\partial y} + \frac{1}{e\eta_\|}\left(\nabla_\|^2\phi - \frac{T_e}{en_0}\nabla_\|^2\tilde{n}\right) = 0$$

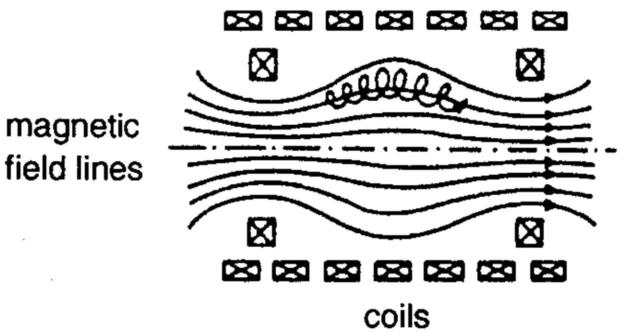
- linearly stable in sheared magnetic field
- algebraic growth of $\tilde{n}$ for modes with $\nabla_\| = 0$
- nonlinear coupling involves full spectrum

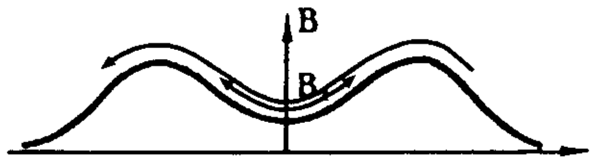
trapped electron mode

- adiabatic limit: no friction, electrons are always in thermal equilibrium along field lines, $\tilde{n} \sim \phi \Rightarrow$ no net particle transport $\Rightarrow$ $\mathbf{E} \times \mathbf{B}$ drift cannot extract energy from density gradient
- trapped electrons: fraction of electrons is trapped in the region of low magnetic field $\Rightarrow$ cannot follow potential fluctuation, since motion along the magnetic field is restricted $\Rightarrow \tilde{n}_{trapped} \not\sim \phi$
- $\mathbf{E} \times \mathbf{B}$ drift extracts energy from density gradient of trapped particles

particle trapping

- gyro-motion of charged particle causes loop current perpendicular to the magnetic field ⇒ magnetic moment μ ⇒ potential energy $W = \mu B$
- if the parallel kinetic energy of the particle is lower than $W_{max} = \mu B_{max}$ the particle becomes trapped
- trapping depends on ratio of perpendicular kinetic energy ($\sim \mu$) and parallel kinetic energy





drive mechanisms (summary)

resistive instabilities play a role only at the plasma edge, since $\eta_{\|} \sim T_e^{-3/2}$

- instabilities important in the plasma core
  - toroidal η_i mode
  - trapped electron mode
  - (slab η_i mode)
- instabilities important at the plasma edge
  - resistive ballooning mode
  - nonlinear drift wave instability
  - toroidal η_i mode

Max-Planck-Institut
für Plasmaphysik

Three dimensional simulations of Tokamak Edge Turbulence

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Max-Planck-Institut für Plasmaphysik,
EURATOM Association, Germany

J. F. Drake, B. N. Rogers
Institute for Plasma Research,
University of Maryland, USA

OVERVIEW (1)

3D fluid simulations of plasma-edge turbulence and transport

- complete two-fluid Braginskii equations
- drift approximation
- selfconsistent nonadiabatic electron response (resistive plasma) electrostatic potential and density evolve independently
- selfconsistent electron and ion temperature dynamics
- field line curvature, magnetic shear

OVERVIEW (2)

MAIN PHYSICS EFFECTS

- drift-wave turbulence
- resistive ballooning modes
- slab and toroidal η_i modes
- nonlinear drift-wave instability
- reduced MHD (this talk: electrostatic limit)

electromagnetic effects: B. N. Rogers, J. F. Drake, Phys. Rev. Lett. **79**, 229 (1997); B. D. Scott, Plasma Physics Control. Fusion, to appear

EQUATIONS (1)

- two fluid Braginskii equations, remove fast gyro time scale:

$$\frac{\partial}{\partial t} \ll \omega_{ci}$$

- derive drift velocities from momentum equations

	ions	electrons
$\perp \vec{B}$	$\vec{E} \times \vec{B}$-drift $\vec{v}_E$ diamagn. drift $\vec{v}_{di}$ polarization drift: $\vec{v}_{pol} = -\frac{c}{\omega_{ci} B} \frac{d}{dt} \left(\nabla \phi + \frac{1}{en} \nabla p_i \right)$	$\vec{v}_E$ $\vec{v}_{de}$
$\parallel \vec{B}$	ion sound waves	Ohm's law

EQUATIONS (2)

- continuity $\frac{\partial n}{\partial t} + \nabla \cdot n(\vec{v}_E + \vec{v}_{di} + \vec{v}_{pol} + \vec{v}_{\|i}) = 0$
- vorticity $\nabla \cdot n\vec{v}_{pol} + \frac{1}{e}\nabla_{\|} j_{\|} + \nabla \cdot n(\vec{v}_{di} - \vec{v}_{de}) = 0$
- ion temperature $\frac{3}{2} n \frac{dT_i}{dt} - T_i \frac{dn}{dt} = 0$
- electron temperature

$$\frac{3}{2} n \frac{dT_e}{dt} - T_e \frac{dn}{dt} - \nabla_{\|} \kappa_{\|} \nabla_{\|} T_e - \frac{0.71 T_e}{e} \nabla_{\|} j_{\|} = 0$$

- parallel motion $m_i n \frac{d}{dt} \vec{v}_{\|i} = -\nabla_{\|}(p_i + p_e)$

....

$$\eta_{\|} j_{\|} = -\nabla_{\|}\phi + \frac{1}{en}\nabla_{\|} p_e + \frac{0.71}{e}\nabla_{\|} T_e \ , \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}_E \cdot \nabla$$

INSTABILITY-MECHANISMS

drive mechanisms for turbulence:

- resistive ballooning
- nonlinear drift-wave instability
- toroidal η_i mode

which parameters determine the dominant instability ?

- space and time scales
- behaviour of η_i-mode in resistive plasma

BALLOONING vs. η_i MODE

- simplified equations ($\tau = T_i/T_e$):

$$-k_\perp^2 \rho_s^2 \gamma(\phi + p_i) + i\omega_{curv}(n + p_i) + D_{\parallel e}\nabla_\parallel^2(\phi - n) = 0$$
$$\gamma n + i\omega_{*e}\phi + D_{\parallel e}\nabla_\parallel^2(\phi - n) = 0$$
$$\gamma p_i + i\omega_{*i}(1+\eta_i)\phi = \tfrac{5}{3}\left(\gamma\tau n + i\omega_{*i}\phi\right)$$

- $D_{\parallel e}\nabla_\parallel^2$ small: resistive ballooning

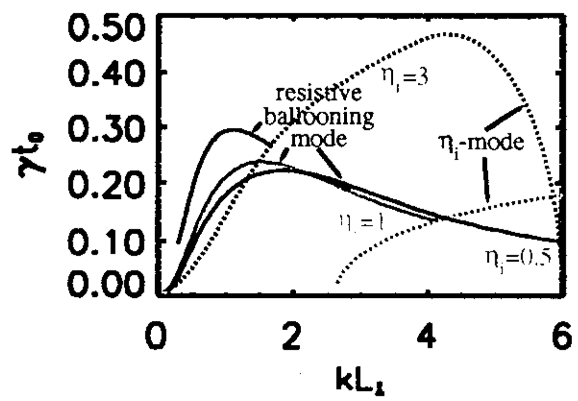
$$\gamma(\gamma - i\omega_{*i})k_\perp^2\rho_s^2 = [\omega_{*e} + \omega_{*i}(1+\eta_i)]\omega_{curv}$$

- $D_{\parallel e}\nabla_\parallel^2$ large (adiabatic electrons): toroidal η_i mode

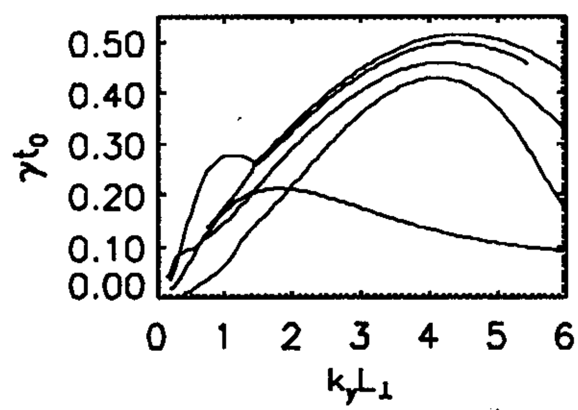
$$\gamma^2 + i\gamma[\omega_{*e} - \omega_{curv}(1 + \tau 5/3)] - \omega_{*i}\omega_{curv}(\eta_i - 2/3) = 0 \quad (k_\perp^2\rho_s^2 \ll 1)$$

BALLOONING vs. η_i mode

without soundwave



with soundwave



RESISTIVE BALLOONING (1)

SCALING

- basic equations for resistive ballooning

$$-k_\perp^2 \rho_s^2 \gamma \phi + i\omega_{curv} n + D_{\parallel e} \nabla_\parallel^2 (\phi - n) = 0$$

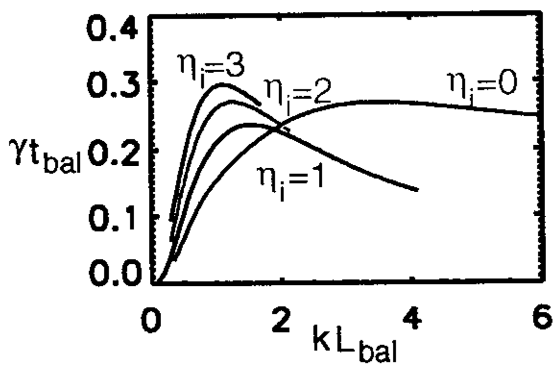
$$\gamma n + i\omega_{*e} \phi = 0$$

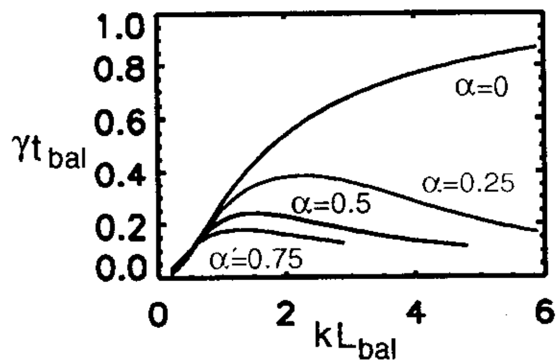
- large $k_\perp$: $\nabla_\parallel$ negligible, ideal growth rate $\gamma_{bal} = c_s/(RL_n)^{1/2}$
  time scale: $t_{bal} \sim 1/\gamma_{bal}$
- smaller $k_\perp$: $\nabla_\parallel$ important, weak ballooning
  balance leads to perpendicular scale length

$$L_{bal} = 2\pi q_a \left(\frac{\nu_{ei} R \rho_s}{2\omega_{ce}} \right)^{1/2} \left(\frac{2R}{L_n} \right)^{1/4}$$

RESISTIVE BALLOONING (2)

RESULTS FROM EIGENVALUE CODES



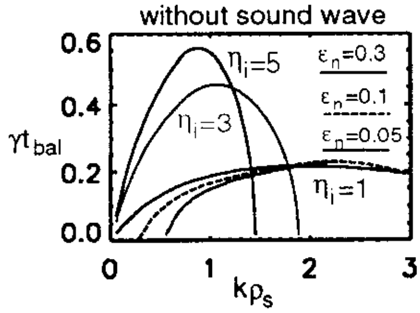


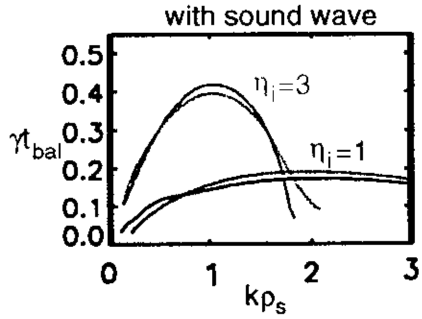
- dominant parameter $\alpha_d = \omega_{*e}/\gamma_{bal}$
- small $\alpha_d \leq 0.5$: strong ballooning
- large $\alpha_d \geq 1$: stabilization by diamagnetic effects
- ∇T_i: stabilizes at short scales, destabilizes at long scales

η_i MODE

RESULTS FROM EIGENVALUE CODES

- growth rate independent of plasma resistivity
- ρ_s natural collisionless scale length; however smallest unstable $k_\perp \ll 1/\rho_s$ depending on η_i and L_n/R
- parallel sound wave effects weak for typical plasma edge parameters
- mode with peak growth rate localized at top/bottom of the torus, not at the outside





NONLINEAR INSTABILITY

basic equations: resistive drift-wave system (Hasegawa-Wakatani)

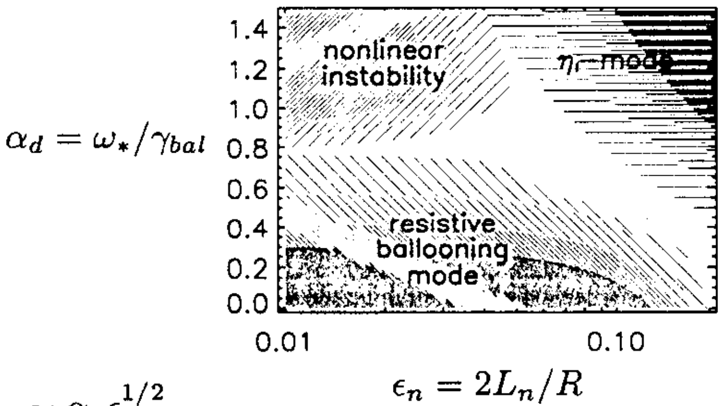
$$\frac{d}{dt}\nabla_\perp^2\phi + \nabla_\parallel^2(\phi - n) = 0$$

$$\frac{dn}{dt} + \frac{\partial\phi}{\partial y} + \nabla_\parallel^2(\phi - n) = 0$$

- linearly stable in sheared magnetic field
- nonlinear simulations and analytic calculations demonstrate existence of nonlinear drive mechanism, self sustained turbulence
- basic feature: algebraic growth of density perturbation n for modes with small $\nabla_\parallel$, nonlinear coupling drives ϕ
- requires resistive, nonadiabatic electron response; scale length similar to resistive ballooning

PARAMETER SPACE

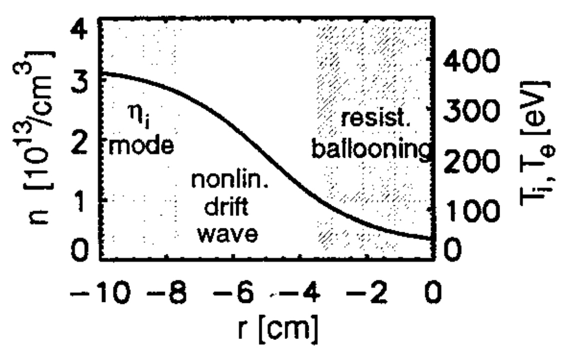
WHICH INSTABILITY DOMINATES ?

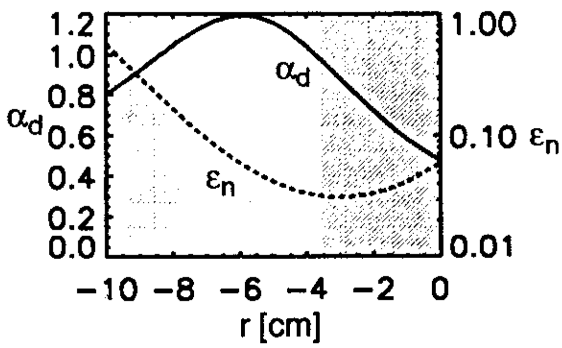


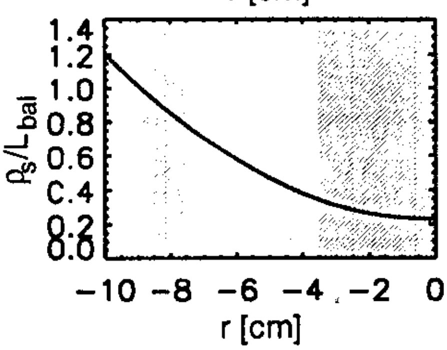
- $\rho_s / L_{bal} \sim \alpha_d \epsilon_n^{1/2}$
- $\rho_s > L_{bal}$: η_i mode dominates over resistive instabilities
- $\rho_s < L_{bal}$: $\gamma_{bal} > \omega_*$: resistive ballooning
  $\gamma_{bal} < \omega_*$: nonlinear drift wave instability

TYPICAL EDGE PROFILE

$R = 165\,cm,\ B = 2.2\,T,$
$Z_{eff} = 2.5,\ q_a = 3.5$

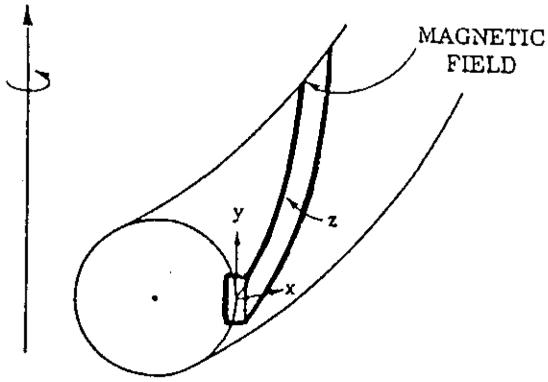






NONLINEAR SIMULATIONS

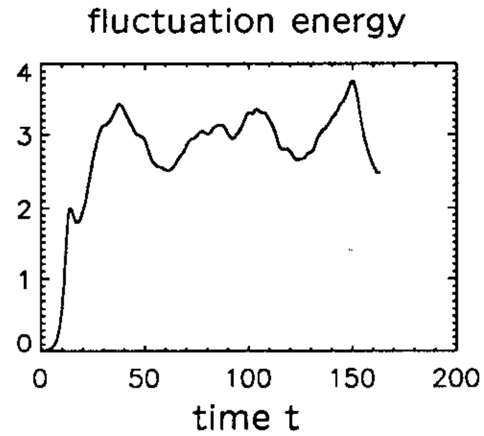
IPP

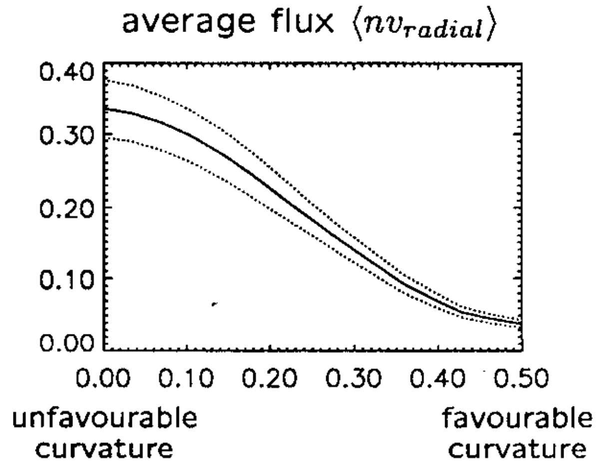


- helical flux tube system: size along $\vec{B} = 3 \times 2\pi qR$, perpendicular to $\vec{B} = 12 - 25 \times L_{bal} \simeq 6 - 15\,cm$
- variables split into background part n_0 and fluctuation $\tilde{n}$, allowing for periodic boundary conditions in $\tilde{n}$
- typical resolution about $90 \times 90 \times 90$
- code running on CRAY J90 and T3E (ported by R. Tisma, IPP)

BALLOONING REGIME

IPP

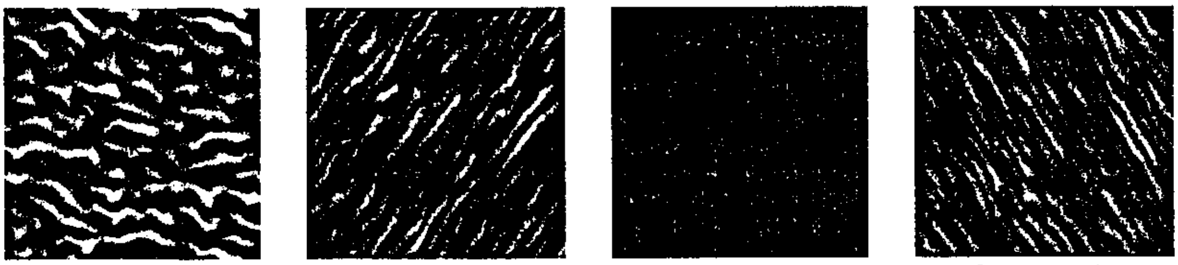




- start from low level random noise
- after initial linear phase stationary turbulence
- time averaged particle and heat fluxes

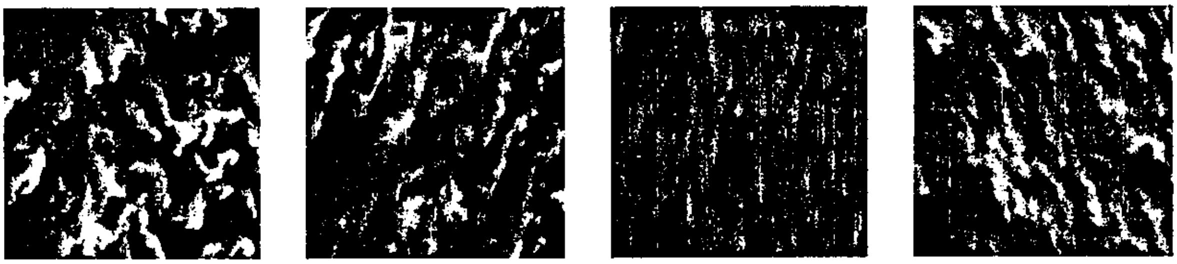
BALLOONING REGIME

LINEAR PHASE



$\theta = 0°$ $\theta = 90°$ $\theta = 180°$ $\theta = 270°$

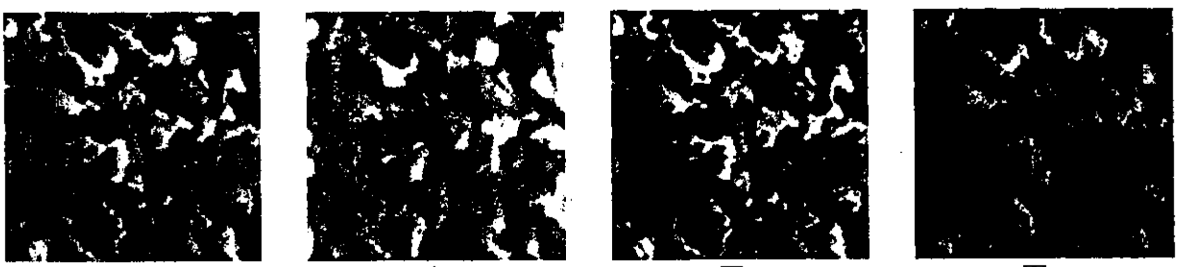
NONLINEAR PHASE



$\theta = 0°$ $\theta = 90°$ $\theta = 180°$ $\theta = 270°$

BALLOONING REGIME

UNFAVOURABLE CURVATURE



n ϕ T_i T_e

FAVOURABLE CURVATURE

n ϕ T_i T_e

BALLOONING REGIME: SUMMARY

- typically for high collisionality, moderate gradients (OH/L-Mode parameters)
- strong linear ballooning instability at low field side, driven by ion pressure and electron density gradient; ∇T_e drive suppressed by parallel heat conduction
- turbulence and transport peaked at low field side
- T_e convected passively
- anomalous diffusion coefficients D, χ_i, χ_e
  - independent of ∇T_e
  - $\eta_e = \eta_i = 1$:

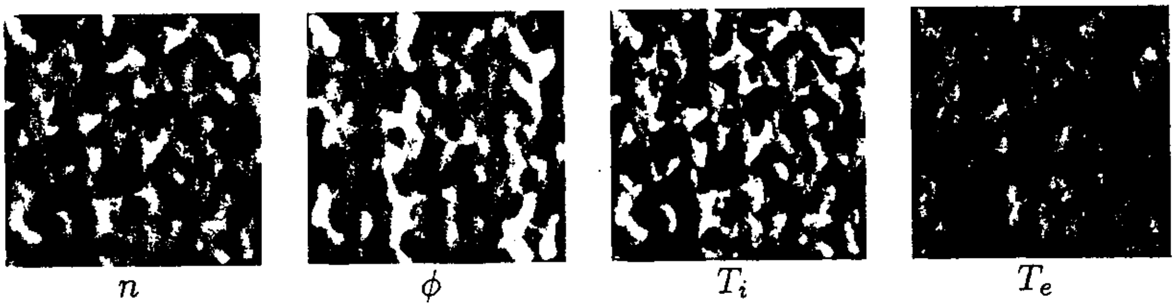
$$D \simeq \chi_i \simeq \chi_e \simeq \frac{1}{10} \frac{L_{bal}^2}{t_{bal}}$$

DRIFT-WAVE REGIME: SUMMARY

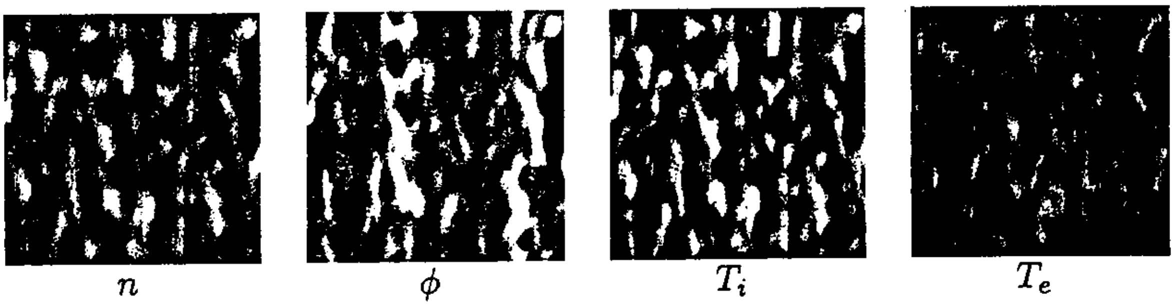
- typically for moderate collisionality and steep gradients
- ballooning drive weak
- turbulence sustained by nonlinear drift-wave instability, if electrons sufficiently nonadiabatic:
  - driven by electron density gradient
  - isotropic structures in poloidal plane, weak inside/outside asymmetry
- if electrons become more adiabatic, stabilization of nonlinear mechanism and transition to η_i mode turbulence

DRIFT-WAVE REGIME

LOW FIELD SIDE



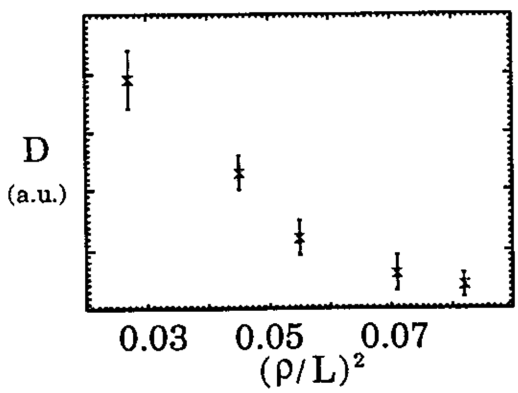
HIGH FIELD SIDE



DRIFT-WAVE REGIME

TRANSPORT SCALING

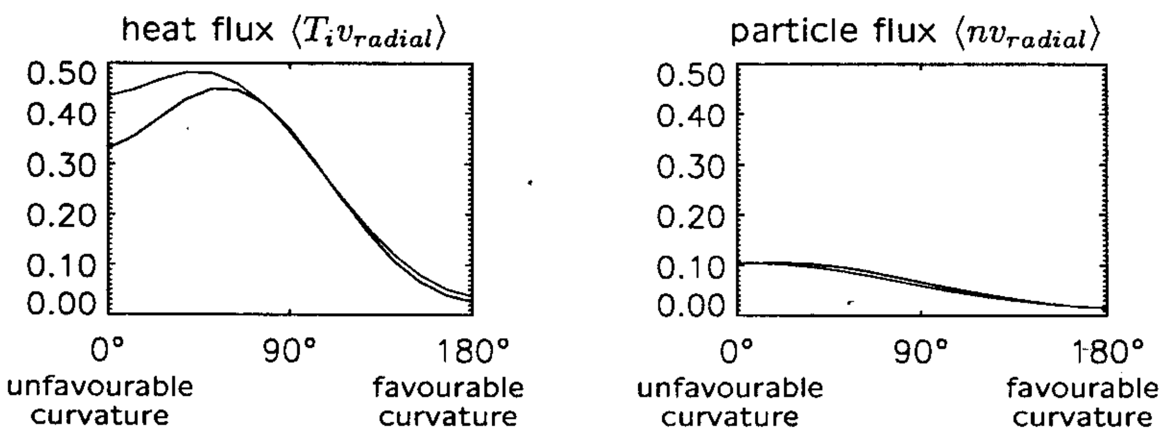
- particle and heat diffusion coefficients D, χ_e, and χ_i independent of η_e: no ∇T_e contribution to drive of turbulence (suppressed by parallel heat conduction)
- perpendicular scale size $\simeq L_{bal}$ (for $\alpha_d \simeq 1$)
- strong reduction of transport due to parallel diffusion: parameter ρ_s / L_{bal}



η_i MODE TURBULENCE: SUMMARY

- typically for low collisionality
- perpendicular scale size and transport rates do not depend on collisionality
- ion heat flux dominates: $\chi_i \gg D \gg \chi_e$
- peak transport and peak fluctuation rate *not* at low field side, but close to top/bottom of the torus; location of fastest growing eigen mode
- perpendicular scale length much larger than for resistive ballooning and nonlinear drift wave turbulence

η_i-MODE TURBULENCE



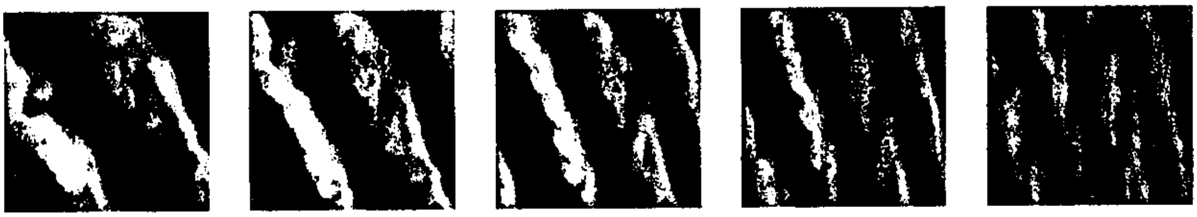
- with parallel sound wave
- without parallel sound wave

- weak influence of parallel sound wave
- peak transport *not* at outside of the torus
- ion heat flux dominates

η_i-MODE TURBULENCE

MODE STRUCTURE ALONG FIELD LINE (T_i)

$\theta = -144°$ $\theta = -108°$ $\theta = -72°$ $\theta = -36°$ $\theta = 0°$



$\theta = 36°$ $\theta = 72°$ $\theta = 108°$ $\theta = 144°$ $\theta = 180°$

η_i-MODE REGIME

LOW FIELD SIDE

n ϕ T_i T_e

HIGH FIELD SIDE

n ϕ T_i T_e

SUMMARY

three different instability mechanisms in plasma-edge turbulence

- resistive ballooning regime (outermost edge)
  - turbulence strongly peaked at outside of the torus
  - $\gamma_{bal} > \omega_*$, resistive scale size $L_{bal} \gg \rho_s$
- nonlinear drift wave regime (intermediate regime)
  - $\gamma_{bal} < \omega_*$; linearly stable, turbulence nonlinearly sustained
  - not related to field line curvature
- η_i mode regime (low collisionality)
  - no dependence on plasma resistivity
  - turbulence driven by unfavourable curvature, however peak fluctuation level not at outside of the torus

OUTLOOK

- magnetic fluctuations
  - magnetic effects lead to great amplification of turbulence in resistive ballooning regime (B. N. Rogers, J. F. Drake; PRL 79, p.229); regime with large diamagnetic drifts, temperature effects?
  - amplification of nonlinear drift wave instability in cold ion limit (B. Scott, to appear); result for $T_i = T_e$?
- generation and stability of sheared flow
  - no significant sheared flow in cold ion limit, unstable sheared flow for $T_i = T_e$ (to be considered as part of the turbulence)
  - in some simulations with strong magnetic effects complete stabilization of turbulence observed; to be further explored