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LECTURES ON BASIC PLASMA THEORY

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These are lecture notes, intended for distribution to participants.

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LECTURE 1

Plasma and its Parameters. Gas Approximation. The Simplest Plasma Models. - The Model of Independent Particles

* 1 What is the "PLASMA"

The first incomplete definition of " PLASMA " was given by I. Langmuir in 1923. According to this definition " Plasma is a shine gas consisting of electrons, several types of ions and neutral atoms and molecules ". But people saw the plasma and moreover used it, more than thousand and thousand years before I. Langmuir. It is obvious that the first who saw the plasma was the God. Creating the Earth and Water and Sky, he noticed that everything was at dark and sad " let to be light." Then he noticed the Sun in the sky. Of course it was the solar plasma. But this phenomenon occurs outside of people's understanding for many thousand centuries. Moreover they did not suspect that they dealt with real plasma when they observed the lightning and even used it.

The first mention about ionized gas of particules was done by O. Heaviside when he predicted more than 100 years ago that around the Earth at the altitudes of 300-400 km there exists a layer of sufficiently high density ionized gas, which reflects the radiowaves. This layer is known as ionosphere. O. Heaviside not only predicted the existence of ionospheric F-layer but also gave explanation : "the origin of this layer is the atmospheric gas ionization by the ultraviolet Solar radiation". According to the modern representation the concentration of charged particles exceeds $(n_e \sim n_i) \sim (1 \text{ to } 3)\cdot 10^{6}$ cm^{-3} , their temperature $T_e \sim (1\text{-}2)\cdot 10^{3}$ K and $T_i/T_e \sim 0{,}3$. At the same time the concentration of neutrals n_0 is $\sim 10^9$ cm^{-3} and their temperature $T_0 \sim 200K$ or the ionization degree $\sim n_e/(n_e+n_0) < 10^{-3}$ (weak ionized gas). The Earth magnetic field at this altitude is $B_0 \sim 0.5$ Gauss and therefore the pressure ratio will be $\beta = 8\pi (n_e+n_i) Te/ B_0^2 \sim 10^{-4} << 1$. Thus for the F-Layer the electron Langmuir frequency $\omega_{Le} = \sqrt{4\pi e^2 n_e / m} \sim 8\cdot 10^7 s^{-1}$, whereas the electron collision frequency $\nu_e \sim 3\cdot 10^3$ s^{-1} which provides the stable radiocommunication on the Earth in the range of radiowave lengths $20m < \lambda < 2000m$.

Inspite of very important role of the ionospheric F-Layer for mankind the regular investigations of the parameters and properties of this plasma were begun only in the 60-s when the rockets and atmospheric probes appeared. Much before, the properties of ionized gas or plasma were investigated in the laboratory experiments when the physicists tried to create artificial plasma.

The most significant achievement in this way was received by I. Langmuir at the beginning of 20-s. He introduced the conception of plasma as a gas of charged particles and neutral atoms and molecules, their concentrations n_e, n_i and n_0 and temperatures T_e, T_i and T_0. Besides he discovered in the gas discharge plasmas the high frequency oscillations with phase velocity much larger than electron thermal velocity, not depending on the masses of ions and neutrals . Moreover he measured their frequency $\omega = \omega_{Le} = \sqrt{4\pi e^2 n_e / m}$, that is known as Langmuir electron frequency. I. Langmuir described also the low frequency oscillations in the gas discharge plasmas with linear dispersion dependence $\omega = k\cdot v_s$ (like sound waves). The phase velocity of such waves v_s is much less than electron thermal velocity and is of the order of ion thermal velocity. I. Langmuir was sure that these waves represent the usual sound waves and used hydrodynamical expression for their description:

$$v_s = \sqrt{\gamma P/\rho} = \sqrt{\gamma (T_e + T_i)/M} \tag{1}$$

Here $\gamma = c_p/c_v$, as it was supposed by I. Langmuir. Unfortunately this assumption is incorrect and only in 1954 the correct expression for v_s was obtained by G. Gordeev, who revealed the physical sense of low frequency oscillations in this type of plasmas.

Many types of gas discharge plasmas are known today. They are created by different types of ionizing radiation: microwave and optical discharge plasmas, radiofrequency and direct current gas discharges, electron and ion beam discharges and etc. They have very large applications in physics and technology: light sources and current commutators, plasma-chemical and nuclear fusion reactors, plasma accelerators and "*thrusters*" are based on gas discharges. Therefore, the great interest of scientists and engineers in the plasma physics and technology is natural.

The parameters of gas discharge plasmas are numerous in a wide ranges. Thus in the neon lamps $n_e \sim n_i \sim 10^{11}$ to 10^{13} cm^{-3}, $n_0 \sim 3 \cdot 10^{13}$ to 10^{16} cm^{-3} (or $P_0 \sim 10^{-3}$ to 1 torr), $T_e \sim 10^4$ to 10^5K (or $T_e \sim 1$ to 10ev) and $T_i \sim 1000$K (or $T_i \sim 0.1$ ev). In other words plasma in the neon lamps usually is a low density weakly ionized and highly nonisothermal with $T_e >> T_i$. On the other hand, in the MHD energy convertors the plasma has high density and is practically completely ionized ($n_e \sim n_i \sim 10^{18}$ cm^{-3}, $n_0 \sim 10^{19}$ cm^{-3}), and very low temperature $T_e \sim T_i \sim T_0 \sim 0.2$ to 0.3 ev. At the opposite of the ionospheric plasma, which is practically unbounded, the laboratory gas discharge plasmas are essentially bounded, their sizes don't exceed several centimeters or decimeters (plasma in MHD convertors)

The thermonuclear plasma deserves a specific attention, the plasma is very hot in the thermonuclear devices. The idea of magnetic plasma confinement and initiation fusion reactions in a hot plasma was proposed at the beginning of 50-s by A. Sakharov and I. Tamm in USSR and L. Spitzer in USA. There exist different types of thermonuclear plasma reactors. The most popular are tokamaks, toroidal magnetic confinement (mirror) systems. Plasma in the thermonuclear devices must be very hot, $T_e \sim T_i \sim 10^8$ K~10 kev, and at plasma density $n \sim 10^{14}$ cm^{-3} the confinement time, according to the Lawson's criterium, is

$$n\tau > 10^{14} \tag{2}$$

which leads to $\tau > 1$s. The strength of magnetic field $B_0 \sim 40$ to 50 KGauss which provides the fulfillment of the inequality $\beta = (8\pi nT)/B_0^2 < 10^{-2} << 1$.

The very interesting alternative method of initiation fusion reactions is the so-called inertial confinement and heating of solid target (d-T tablets). The confinement time of such dense and hot plasma ($n \sim 10^{23}$ cm^{-3} ,$T \sim 10$ kev) is less than inertial time, or $\tau \leq a/v_s$, where $v_s \sim 10^8$ cm/s is the sound velocity and a is the target radius.Taking into account Lawson's criterium one can estimate $\tau \sim 10^{-9}$ s = 1 ns and $a < v_s\tau \sim 0.1$cm. The energy input necessary for heating of such a target plasma is about $Q = 4/3(\pi a^3 nT) \sim 1$ MJ and heating source power $P_W \sim 10^{15}$ W.

In 1964 Soviet physicists N. Basov and O. Krokhin proposed to use the very powerfull laser radiation as a source for target heating and initiating the thermonuclear reactions. The another method of heating was proposed in 1970 by H. Winterberg (USA) and E. Zavoskiy (USSR), they proposed to use for this aim a short pulse ($\tau<10^{-7}$s) of very powerfull relativistic electron beam with $P_b \cong 10^{14}$ W and Q > 10MJ.

In connection with the thermonuclear plasmas it must be mentioned that the sun and stars are natural thermonuclear reactors. In the inner part of stars, plasma is very hot T ~ 10 to $1000 \cdot 10^6$ K and very dense n ~ 10^{24} to 10^{26} cm^{-3}, whereas on their surfaces T ~ 10^4 K and density 1 to 10 cm^{-3}. Investigations of the stars plasmas and inter planetary plasmas is the main goal of Astrophysics.

In conclusion let us discuss the parameters of solid state plasmas in metals and semiconductors. In solid state the real particles are placed in a periodical field of lattice (crystalline) and therefore one can say about fermion type excitation with positive (holes) or negative (electrons) carriers. There arises complicated energetical structure in which the effective masses of carriers are determined as $m_{\pm} = (\partial^2\varepsilon_{\pm}(P)/\partial P^2)^{-1}$, where $\varepsilon_{\pm}(P)$ are the energy spectrum of carriers in the conductive zones. Usually in a semiconductor $m_{+} \sim m$ (is of the order of the real electron mass) whereas $m_{-} \sim 0.1$ to 0.01 m. At the same time, in metals exist only negative carriers with $m_{-} \sim m$, and the wide band of conducting zone is practically infinite. For description of conducting media usually, the conception of carriers is used: "electron-hole" plasma in semiconductors and purely electron plasma in metals. However it must be done very accurate.

Thus from above discussion it is seen that plasma is very wide spread in nature, more than 99% substances of the Universe exists in a plasma state. Therefore, it is natural that a plasma is very often considered as the 4-th aggregate state of matter.

* 2 Plasma as a gas of charged particles

Below we will consider plasma as a gas of charged particles. What does that mean? For clarifying this problem the interactions between the plasma particles must be considered. Let us begin from neutral particles - the problem already has been investigated by great L. Boltzmann. He understood that the interaction between them is very strong, but they interact only on very short distances. Therefore he imagined them as a hard spherical balls with radius a ~ 10^{-7} to 10^{-8} cm. The potential of neutral particles interaction then can be written as

$$U(r) = \begin{cases} \infty & \text{if } r \le a \\ 0 & \text{if } r \ge a \end{cases} \tag{3}$$

Inspite of very strong interaction, if the density of neutrals is sufficiently small , the following inequality takes place

$$\eta_0 = a/r_{av} = a n_0^{1/3} << 1 \tag{4}$$

then the motion of neutral particles is practically free, they interact to each other very seldom and in the first approximation we can neglect this interaction completely.

In the second approximation we can take into account the interaction between the particles as a small correction to the free motion. Thus the inequality (4) represents a condition of validity of gas approximation for the neutral component of plasma.It is obvious that the condition (4) is valid also for the interaction of charged particles with neutrals.
Quite another physical meaning has the condition for validity of gas approximation for the interaction between the charged particles of a plasma. The Coulomb interaction is a long range one and therefore the gas approximation is valid if the potential energy of charged particles interaction is small in comparison with their kinetic energy (freedom energy). In other words, gas approximation is valid if :

(5)

$$\eta_1 = U_\alpha(r_{av})/\langle\varepsilon_\alpha\rangle \sim (e^2 n_\alpha^{1/3})/\langle\varepsilon_\alpha\rangle \quad \ll 1$$

Here $n_\alpha \sim n_{e,i}$, $T_\alpha = T_{e,i}$, $m_\alpha = m_{e,i}$ and

(6)

$$\langle\varepsilon_\alpha\rangle = \begin{cases} T_\alpha & \text{if } T_\alpha > \varepsilon_{F\alpha} = ((3\pi^2)^{2/3}\eta^2 n_\alpha^{2/3})/2m_\alpha \\ \varepsilon_{F\alpha} & \text{if } \varepsilon_{F\alpha} > T_\alpha \end{cases}$$

Here $n \sim n_{e,i}$,$T=T_{e,i}$, $m= m_{e,i}$.

The condition (5) was firstly formulated in 1937 by L. Landau
It must be noted that for a nondegenerate, $T_\alpha > \varepsilon_{F\alpha}$ isothermal, $T \sim T_e \sim T_i$, and neutral plasma, $n \sim n_e \sim n_i$, the conditions (4) and (5) are similar in the sense that with increasing of particles density (n_o or $n_{e,i}$) plasma becomes more and more nonideal. At the same time, for a degenerate case, $\varepsilon_{Fe,i} > T_{e,i}$, the physical sense of (5) is opposite to (4) and corresponds to the fact that when n increases then the plasma becomes more and more ideal. This follows from dependence $\varepsilon_F \sim n^{2/3}$ which leads to $\eta_1 \sim n^{-1/3}$. Thus the more dense degenerate plasma in metals occurs to be more ideal.
Another difference between the conditions (4) and (5) follows from comparing the ratio of interaction ranges for charged and neutral particles to the average distances between them. In agreement to (4) , this ratio is small. In this case, relation (5) has quite opposite meaning. For convincing this let us consider the potential of a point charged particle q located at r=0 in the nondegenerate plasma :

(7)

$$\Delta\phi = 4\pi q\ \delta(r) + 4\pi e\{n_e e^{e\phi/T_e} - n_i e^{-e\phi/T_i}\}$$

For simplicity electron and ion charges in a plasma are supposed to be equal and opposite, $e_i = -e$, and consequently their densities $n_e=n_i=n$. Then from (7) under the conditions $|e\phi| \ll T_e, T_i$, follows:

(8)

$$\phi = (qe^{-r/D})/r, \quad D = \{\Sigma_{e,i}\ 4\pi e_\alpha^2 n_\alpha/T_\alpha\}^{-1/2} \text{ Debye length}$$

and $r_{D\alpha} = (T_\alpha/(4\pi e_\alpha^2 n_\alpha))^{1/2}$ are the Debye lengths of electrons and ions, $\alpha = e,i$.

It is easy to understand that D characterizes the Coulomb interaction range of charged particles in a plasma. Therefore for the ratio of this range to the average distance between the particles one obtains (9):

$$D\, n^{1/3} \sim \sqrt{T / e^2 n^{1/3}} \sim 1/\eta_i^{1/2} >> 1 \tag{9}$$

This inequality is opposite to (4) and it means that the average distance between charged particles in gaseous plasma is much less than the interaction range, or a large number of charged particles must exist in a Debye sphere. It is easy to show that this statement takes place in the case of degenerate plasma too.

It must be noticed that a plasma can be considered not only as a simple totality of charged particles but as a medium if its size is much larger than Debye length. Moreover only under this condition the Debye length has a physical meaning.

In conclusion let us make some estimations of conditions (4) and (5) for different plasmas. First of all we must notice that for a ~ 10^{-7} to 10^{-8} cm, from (4) follows the validity of gaseous approximation for neutrals $n_0 \leq 10^{21}$ to 10^{22} cm^{-3}. This means that gaseous approximation for neutrals is valid up to hundreds atmospheric pressure. It is obvious that for the usual gases this condition is satisfied with great supply.

Another situation takes place for charged components of plasmas and for condition (5). For ionospheric plasma in F-Layer where $n_{e,i} \sim 10^6$ to $10^7 cm^{-3}$, and $T_e \sim 10^4 K$, $T_i \sim 10^3 K$ we have $\eta_1 \leq 10^{-4} << 1$, or this plasma is highly ideal. Analogical situation takes place in the laboratory gas discharge plasmas with $n_e \sim 10^{11}$ to 10^{14} cm^{-3} and $T_e \sim 10^4$ to $10^5 K$ where $\eta_1 \leq 10^{-2}$ to $10^{-4} << 1$. At the same time in the high density plasmas of MHD convertors and light sources usually $n \sim 10^{13}$ to 10^{19} cm^{-3} and $T_e \leq 1$ to $5\ 10^4$ K. Therefore $\eta_1 \sim 0.1$ to 0.5 which means that, in such plasmas, nonideal effects are essential.

For thermonuclear plasmas in the magnetic confinement devices $n \sim 10^{14}$ cm^{-3} and $T \sim 10^8 K$ what means that $\eta_1 \leq 10^{-5} << 1$, whereas for inertial fusion plasma with $n \sim 10^{23} cm^{-3}$ and $T \sim 10^8 K$ we have $\eta_1 \sim 0{,}01$. In the last case the slightly nonideal effects must be taken into account.

Finally we will say some words about solid states plasmas. In a good conducting metals as copper $n_e \sim 5 \cdot 10^{22}$ cm^{-3}, and therefore electrons are degenerate, $\varepsilon_{Fe} \sim 1ev$ and $\eta_1 \sim 0.2$, and they can be approximately considered as a weakly nonideal gas. But for the most metals $n_e \leq 10^{22}$ cm^{-3}, and $\eta \geq 1$, which means that the electrons in such metals represent liquid, so-called degenerate Fermi-liquid. In semiconductors, carriers parameters are varied in a very wide range and therefore different situations are possible. Below we restrict ourselves in consideration of only gaseous plasma.

* 3 The single particle model - Its achievements and failures

It is obvious that the most consistent description of plasma properties was reached by using the kinetic description. But historically the early plasma models were much more simple and despite of this, they gave quite good results, in a good agreement with experiments. However sometimes such models were applied to problems outside of the frameworks of models. Then some disappointments arised , which stimulated the development and improvement of other models until the perfect kinetic description was proposed by A. Vlasov.

Below we will follow this historical process of the development of plasma physics and begin our consideration from the simplest model - the model of independent particles. This model firstly proposed by I. Langmuir consists of Newton equations of electron and ion motions which are completed with Maxwell equations. This model was very fruitfull for investigating the propagation of radiowaves through the ionospheric plasma, as it was shown by V.Ginzburg, before the second world war. The equations of motions in the model of independent particles look as

(10)

$$d\mathbf{v_e}/dt = e/m\{ \mathbf{E} + 1/c[\mathbf{v_e}\times\mathbf{B}] \} - \nu_e \mathbf{v_e}$$

$$d\mathbf{v_i}/dt = + e_i / M\{ \mathbf{E} + 1/c[\mathbf{v_i}\times\mathbf{B}] - \nu_i \mathbf{v_i}$$

Here $\mathbf{v_e}$ and $\mathbf{v_i}$ are the electron and ion velocities, $\nu_e = \nu_{ei} + \nu_{e0}$ and $\nu_i = \nu_{ie} + \nu_{i0}$ their collision frequencies, for which the following equality takes place $m\nu_{ei} = M\nu_{ie}$. The electric **E** and magnetic **B** fields must be finite because they determined the Lorentz force acting on a test charge q :

(11)

$$\mathbf{F} = q [\mathbf{E} + 1/c[\mathbf{v}\times\mathbf{B}]]$$

These quantities satisfy the Maxwell equations :

(12)

$$\mathrm{div}\, \mathbf{E} = 4\pi\rho = \Sigma\, 4\pi en , \ \mathrm{div}\, \mathbf{B} = 0$$

$$\mathrm{rot}\, \mathbf{E} = -1/c[\partial\mathbf{B}/\partial t], \ \mathrm{rot}\, \mathbf{B} = 1/c[\partial\mathbf{E}/\partial t] + 4\pi\mathbf{j}/c = 1/c\, [\partial \mathbf{E}/\partial t] + 4\pi/c\, \Sigma\, env$$

moreover for each components of charged particles the continuity equation is satisfied (α= e,i)

(13)

$$\partial n_\alpha /\partial t + \mathrm{div}\, n_\alpha \mathbf{v}_\alpha = 0$$

Thus in accordance with (10) the motions of charged particles are defined by the electric and magnetic field **E** and **B** and at the same time these fields themselves are determined by the charged particles motions. Thus we have selfconsistence connection between the particles motions and electromagnetic fields. This idea of selfconsistence was proposed by I. Langmuir at the beginning of 20-s. However it has remained still misunderstandable for many scientists up to day.

From the equations of particles motions (10) only one vector quantity must be defined - the current density **j** which appears in the field equations (12) as an external source. As about charge density ρ, this quantity can be easily defined by using the continuity equation (13). Taking in consideration that the magnetic field **B** also can be expressed in terms of electric field **E** we conclude that the problem of any plasma model is the calculation of induced current density

(14)

$$j_i = \Sigma\, en\mathbf{v_i} = \sigma_{ij}(\mathbf{E})\, \mathbf{E}_j$$

This relation in general represents nonlinear Ohm's law and σij (**E**) is nonlinear operator of plasma conductivity.

Instead of **j** and ρ one can introduce the induction vector **D** by the following relation

$$\partial \mathbf{D}/\partial t = \partial \mathbf{E}/\partial t + 4\pi \mathbf{j} \quad (15)$$

Using this relation the field equations (12) take the form

$$\text{div } \mathbf{D} = 4\pi\rho_0, \qquad \text{div } \mathbf{B} = 0 \quad (16)$$

$$\text{rot } \mathbf{E} = -1/c(\partial \mathbf{B}/\partial t), \quad \text{rot } \mathbf{B} = 1/c(\partial \mathbf{D}/\partial t) + 4\pi \mathbf{j_0}/c$$

These equations differ from (12), they take into consideration not only the induced current and charges densities $\mathbf{j}$ and ρ but also the external sources $\mathbf{j_0}$ and ρ_0. These equations in addition to the equations of motions (10) represent the complete system of the simplest plasma model - the model of independent particles. The validity limits of this model can be estimated by considering some basic linear problems. In linear approximation and in the absence of external magnetic field $\mathbf{B_0}$ and the sources $\mathbf{j_0}$ and ρ_0 the solutions of the equation (10) can be presented as $e^{-(i\omega t + i\mathbf{k}\cdot\mathbf{r})}$. Then one can easily obtain

$$\mathbf{v_e} = (ie\mathbf{E})/[m(\omega + i\nu_e)] \ , \ \mathbf{v_i} = (ie_i\mathbf{E})/[M(\omega + i\nu_i)] \quad (17)$$

which leads to the following expression for induced current density

$$\mathbf{j} = \Sigma_\alpha e_\alpha n_\alpha \mathbf{v}_\alpha = \Sigma_\alpha (ie_\alpha{}^2 n_\alpha \mathbf{E})/[m_\alpha(\omega + i\nu_\alpha)] = \sigma \mathbf{E} \quad (18)$$

Thus for plasma conductivity we have

$$\sigma_{ij} = \sigma\delta_{ij} \ , \ \sigma = \Sigma \ (ie^2 n/m[\omega + i\nu]) \quad (19)$$

(we omit the summation index α). Using the definition (15) one can introduce the dielectric permittivity

$$D_i = \varepsilon_{ij}E_j \ , \quad \varepsilon_{ij} = \delta_{ij} + 4\pi i \ \sigma_{ij}/\omega \quad (20)$$

For the isotropic plasma we will get the following result

$$\varepsilon_{ij} = \varepsilon(\omega)\delta_{ij} \quad \varepsilon(\omega) = 1 + 4\pi i\sigma(\omega)/\omega = 1 - \Sigma\omega_L{}^2/\omega(\omega + i\nu) \quad (21)$$

where $\omega_{L\alpha} = \sqrt{4\pi \, e_\alpha{}^2 n_\alpha / m_\alpha}$ is the Langmuir frequency of charged particles of type α.

Now we can verify the validity of the model of independent particles using the relations (16) to (20). First of all let us check the static limit

$$\sigma(0) = \Sigma \ e^2 n/m\nu \sim e^2 n_e/m\nu_e, \quad \varepsilon = 1 + 4\pi i\sigma(0)/\omega \quad (22)$$

Here $\sigma(0)$ is the static conductivity of isotropic plasma. In weakly ionized plasma $\nu_e = \nu_{e0}$ and the expression becomes correct not only qualitatively but also quantitavely. At the same time, in completely ionized plasma, or more exactly, when :

$\nu_{e0} << \nu_{ei} = 4/3 \sqrt{2\pi/m} * e^2 e_i{}^2 n_i / T_e{}^{3/2}$ (the electron-ion effective collision frequency), the expression (21) occurs approximately 2 times less than the correct expression known as L. Spitzer's formula for static conductivity of plasma :

(23)

$$\sigma_{sp} = 1{,}96\, e^2 n_e / m\nu_{ei}$$

The very important conclusion which follows from (21) is that the independent particles model explains quantitatively the propagation of high frequency transverse electromagnetic waves in an isotropic plasma. From the Maxwell equations in the abscence of external sources $\mathbf{j}_0$ and ρ_0 follows the dispersion equation for such waves .

(24)

$$k^2c^2 = \omega^2\varepsilon(\omega)$$

Using the relations (20)-(21) and supposing $\omega >> \nu e$, it's easy to find the solution of this equation in the high frequency limit [$\omega \rightarrow \omega + i\delta$]

(25)

$$\omega^2 = \omega_{Le}{}^2 + k^2c^2, \quad \delta = -\,\nu_e\omega_{Le}{}^2/\omega^2$$

With regard to the low frequency limit ($\omega << \nu e$) from (24) it follows the well-known expression for the penetration depth of the normalous skin-effect,

(26)

$$\lambda_{sk} = 1/\Im m k = c/\sqrt{(2\pi\sigma\omega)}$$

This expression is correct for completely ionized plasma as well as for weakly ionized. Of course this statement must take into account the above remark about the static conductivity of plasma (see (22) and (23)).

More essential seems to be that the formula (26) is correct only if $\nu_e > v_{Te}/\lambda_{sk}$. In the opposite limit as it was shown by A. Pippard in 1948, the anomalous skin-effect takes place which can not be described in the model of independent particles. This phenomena will be studied below in the next lectures.

Finally the most penalizing failure of the model of independent particles was exposed in the description of longitudinal oscillations of plasma. From the field equations (16) taking in consideration (21) we get the following dispersion equation for such oscillations

(27)

$$\varepsilon(\omega) = 0$$

The solution of this equation in high frequency range ($\omega >> \nu_e$) looks as ($\omega \rightarrow \omega + i\delta$)

(28)

$$\omega^2 = \omega_{Le}{}^2\ , \quad \delta = -\nu_e/2$$

Plasma longitudinal oscillations firstly was investigated by I. Langmuir in 1926.Moreover he firstly obtained these formulas and gave their physical interpretation.

However I. Langmuir was also the first who noticed that the discussed model is limited. This model can not explain the existence of low frequency oscillations with spectrum $\omega = kv_s$ discussed above, which were called by I. Langmuir as ion sound waves. And finally this model leads to the obviously absurd result for the problem of static potential for a rest point charged particle in a plasma. In order to show this let us consider a point particle with varying charge

$\rho_0 = qe^{-i\omega t}\delta(r)$. From field equations (16) we find the following expressions for static field and its potential

$$\mathbf{E} = -\nabla \Phi, \quad \Phi(\mathbf{r}) = qe^{-i\omega t}/r\varepsilon(\omega) \tag{29}$$

where $\varepsilon(\omega)$ is given by the expression (21). If now we take the static limit $\omega \to 0$ we obtain obviously absurd result : $\Phi(r) \to 0$ because $\varepsilon(\omega) \to \infty$ when $\omega \to 0$. Thus in the low frequency limit the model of independent particles is absolutely incorrect for description of isotropic plasma.

* 4 The properties of magnetoactive plasmas in the model of independent particles

Despite above mentioned failures of the independent particles model for description of isotropic plasma, let us now apply this model to the magnetoactive plasmas. Remind that this model occurs to be very successful for the problems of radiowaves propagation in the earth ionosphere. But the ionospheric plasma is magnetoactive and therefore below we will consider the properties of such plasmas.

We suppose that external magnetic field $\mathbf{B_0}$ is parallel to **OZ** axis. Then in the linear approximation for perturbations from the equations (10) one can easily find small **v** and use the relation $\mathbf{j} = \Sigma en\mathbf{v}$ to calculate the induced current density which leads to the following expression for dielectric permittivity:

$$\varepsilon_{ij}(\omega) = \begin{pmatrix} \varepsilon_\perp & ig & 0 \\ -ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{//} \end{pmatrix} \tag{30}$$

$\varepsilon_\perp = 1+\Sigma[\omega_L{}^2(\omega+i\nu)/\omega(\Omega^2-(\omega+i\nu)^2)$, $g = \Sigma\omega_L{}^2\Omega/\omega(\Omega^2-(\omega+i\nu)^2)$, $\varepsilon_{//} = 1-\Sigma\omega_L{}^2/(\omega(\omega+i\nu))$

where $\Omega = eB_0/mc$ is the Larmor frequency for charged particles rotation around the magnetic field $\mathbf{B_0}$. By using this tensor of dielectric permittivity the above mentioned success in the analysis of radiowave propagation through the ionospheric plasma was achieved by V. Ginzburg. Below we will not discuss these triumphal results. On the contrary we will consider the problems for which this tensor and more generally the independent particles model is'nt correct.

Let us begin from dispersion equation which can be easily got from the field equations :

$$|k^2\delta_{ij}-k_ik_j - (\omega^2/c^2)\varepsilon_{ij}(\omega)| = \tag{31}$$

$$k^2(k_\perp{}^2\varepsilon_\perp + k_{//}{}^2\varepsilon_{//}) - \omega^2/c^2[(\varepsilon_\perp{}^2-g^2-\varepsilon_\perp\varepsilon_{//})k_\perp{}^2+2k^2\varepsilon_\perp\varepsilon_{//}] + (\omega^4/c^4)\varepsilon_{//}(\varepsilon_\perp{}^2-g^2) = 0$$

Here $k_\perp = k\sin\theta$ and $k_{//} = k\cos\theta$ respectively represent the transversal and longitudinal components of the wave vector **k** and θ is the angle between **k** and $\mathbf{B_0}$

In general the equation (31) is very complicated and the solutions $\omega(k)$ are impossible to find analytically. At the same time, it can be solved very easily as an equation relative to

$$k(\omega) = (\omega/c)n(\omega)$$

We find the following solution

$$n_{(1,2)}^2(\omega) = \left(-B \pm \sqrt{B^2 - 4AC}\right)/2A,$$

$$A=\varepsilon_\perp \sin^2\theta+\varepsilon_{//}\cos^2\theta,\ C=\varepsilon_{//}(\varepsilon_\perp{}^2-g^2)\ B=-\varepsilon_\perp\varepsilon_{//}(1+\cos^2\theta)-(\varepsilon_\perp{}^2-g^2)\sin^2\theta \quad (32)$$

The quantities $n_{1,2}$ are called as the complex reflection coefficients $n_1(\omega)$ for ordinary waves and $n_2(\omega)$ for extraordinary waves.

Just using the relations (31) and (32) and taking into account the expressions (30) the radiowave propagation (their reflection and absorption) in the F-layer of ionospheric plasma as a function of the angle θ were explained. On the figure 1 there are presented the dependences $n_{1,2}{}^2(\omega)$ for $\theta\neq 0,\pi/2$ and $\omega_{Le}{}^2 > \Omega_e{}^2$ (as it takes place in F-layer where $\omega_{Le}\cong 10^8 s^{-1}$ and $\Omega_e\cong 10^7 s\text{-}1$). Moreover (31) and (32) give the good quantitative explanation for not only high frequency ($\omega>>\Omega_e$), but also low frequency ones in the range $\Omega_i<<\omega<<\Omega_e$. These formulas predict the existence of the transverse waves, in the low-frequency range with spectrum

$$\omega = (k^2c^2\ |\Omega_e|\cos\theta)/\omega_{Le}{}^2 \quad (33)$$

Such waves were really observed in the ionospheric plasma and they were called as whistlers. The model of independent particles appears to be significantly less successful in explaining the low frequency waves in the range $\omega<<\Omega_i$. From dispersion equation (31) in this frequency range one can obtain the spectra of two branches of low frequency waves

$$\omega_1{}^2 = k_{//}{}^2 v_A{}^2/(1+v_A{}^2/c^2),\quad \omega_2{}^2 = k^2 v_A{}^2/(1+v_A{}^2/c^2) \quad (34)$$

where $v_A = B_0/\sqrt{4\pi nM}$ is called Alfven velocity. The first branch corresponds to the purely transverse waves and is well known as Alfven waves. They are predicted theoretically by H. Alfven in the framework of MHD equations and were really observed in ionospheric plasma.

As for the second branch then the theoretical spectrum (34) differs from the experimental observed. In experiments there exist two branches of low frequency waves with significant longitudinal field components instead of one branch. Besides the phase velocities of both depend on the plasma temperature, which is completely ignored in the model of independent particles. This fact indicated to the serious difficulties of the model. However the main difficulty of the independent particles model was clarified when static potential of a point charged particle in the magnetoactive plasma was considered. The result of this consideration, using the field equations (16), leads to the following formula for the field potential

$$\Phi(r) = q/2\pi^2 \int dk(e^{ikr})/(k_\perp{}^2\varepsilon_\perp(\omega)+k_{//}{}^2\varepsilon_{//}(\omega)) \underset{\lim \omega\to 0}{\to} 0 \quad (35)$$

where $\varepsilon_\perp(\omega)$ and $\varepsilon_{//}(\omega)$ are given by (30). Thus in magnetoactive plasma the static potential of point charged particle tends to zero. This result coincides with the result of the isotropic plasma and also seems perfectly absurd.

Thus from the above consideration we can conclude that the model of independent particles is quite satisfactory for description of fast and high-frequency processes in a plasma, with typical

(phase) velocity much higher than thermal velocity of charged particles. Specially this occurs true for transverse waves. For longitudinal waves and low-frequency processes the typical phase velocity is of the order of the particles thermal velocity and therefore their properties can't be described by this model. Specially, this occurs catastrophic for describing purely potential static electric fields in a plasma. For improving the model, firstly, all thermal motion effects of charged particles, such as hydrodynamical effects pressure, viscosity, thermoconductivity and diffusion of particles as well as kinetic velocity effects and energy distributions of particles must be taken into account.

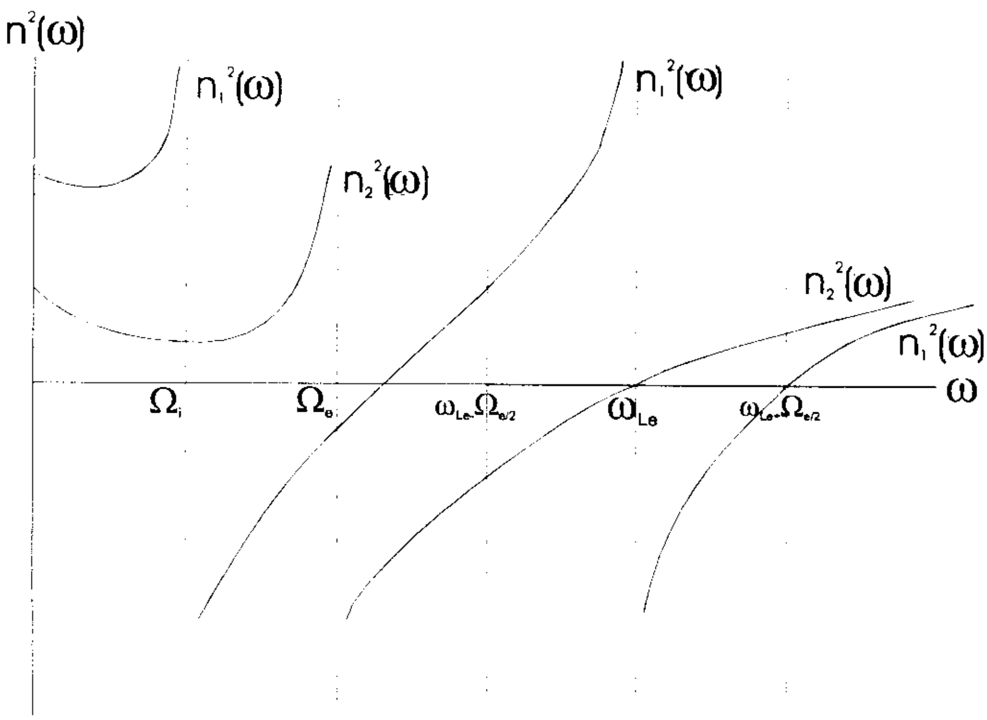

Figure 1 : Dependences $n_{1,2}^2$ (ω) for θ≠ 0,π/2 in the model of independent particles

* 5 Two fluids hydrodynamics

In the previous section it was shown that the model of independent particles occurs incorrect in the low frequency range. For this reason the physicists decided at the end of 30-s that in this frequency range the hydrodynamical descriptions of plasma must be more suitable. At that time two different types of hydrodynamical models were developed. The first one was proposed by I. Langmuir and the second by H. Alfven. Below we will consider only the simplest versions of these models, which is quite sufficient for clarifying the reasons of their success and their failure.

In this section we begin with the I. Langmuir model, known as two fluids hydrodynamics . This model generalizes the independent particles model by taking into consideration the kinetic pressures of electrons and ions. Therefore the equations of motions look as (compare with equations (10)):

$$d\mathbf{v_e}/dt = \partial \mathbf{v_e}/\partial t + (\mathbf{v_e}\nabla)\mathbf{v_e} = -\nabla n_e T_e/mn_e + e/m\{\mathbf{E}+1/c[\mathbf{v_e}\times\mathbf{B}]\} - \nu_e \mathbf{v_e} \tag{36}$$

$$d\mathbf{v_i}/dt = \partial \mathbf{v_i}/\partial t + (\mathbf{v_i}\nabla)\mathbf{v_i} = -\nabla n_i T_i/Mn_i + e_i/M\{\mathbf{E}+1/c[\mathbf{v_i}\times\mathbf{B}]\} - \nu_i \mathbf{v_i}$$

As above these equations must be completed by the Maxwell equations (12) and continuity equations (13). Taking into account the temperatures T_e and T_i, below we will assume that they are constant. Such assumption simplifies the problem in a significant way and at the same time it doesn't influence the validity of the model. To determinate the validity limits of models is our main goal.

The basic equations of two fluids hydrodynamics (36) differ from the equations of motions (10) in the independent particles model by taking in consideration the thermal pressure. For this reason we may hope that low frequency processes for which the independent particles model occurs incorrect will be described quite sufficiently well. To prove this statement let us consider linear electromagnetic properties of spatially homogeneous isotropic plasmas in the model of two fluids hydrodynamics. We will investigate the small perturbations of an equilibrium state in which

$$E_0 = B_0 = 0, \quad v_{0e,i} = 0, \quad n_{0e,i} = \text{const}$$

Then from the linearized equations (35) and (13) one can easily obtain the following expression for the dielectric permittivity of plasma

$$\varepsilon_{i,j}(\omega,\mathbf{k}) = (\delta_{ij} - k_i k_j/k^2)\varepsilon^{tr}(\omega,\mathbf{k}) + k_i k_j/k^2 \varepsilon^{l}(\omega,\mathbf{k}) \tag{37}$$

where

$$\varepsilon^{tr}(\omega,\mathbf{k}) = 1 - \Sigma_\alpha(\omega_{L\alpha}{}^2)/(\omega(\omega+i\nu_\alpha)) \tag{38}$$

$$\varepsilon^{l}(\omega,\mathbf{k}) = 1 - \Sigma_\alpha(\omega_{L\alpha}{}^2)/(\omega(\omega+i\nu_\alpha) - k^2 v_{T\alpha}{}^2)$$

represent the transversal and longitudinal permittivities respectively. Thus we see that they are different. Besides, the transverse dielectric permittivity (38) coincides with (21). Consequently all the difficulties which take place in the independent particles model remain in the considered model of two fluids hydrodynamics. Particularly this model occurs to be incorrect for describing the transverse field penetration into the plasma (skin effect) in low frequency range when $\nu_e \ll \omega \ll v_{Te}/\lambda_{sk}$, where λ_{sk} is given by the relation (26).

At the same time, for the longitudinal dielectric permittivity from (38) it follows

$$\lim_{\omega \to 0} \varepsilon^l(\omega,\mathbf{k}) = 1 + \Sigma_\alpha(\omega_{L\alpha}^2 / k^2 v_{T\alpha}^2) = 1 + 1/k^2 r_D^2 \tag{39}$$

This expression leads to the correct expression for the field potentiel of the static charged particles q located in a plasma at r = 0

$$\Phi(r) = (q/r)e^{-r/r_D} \tag{40}$$

Thus the Debye screening of the potential takes place as it must be in accordance to (8).

Moreover longitudinal dielectric permittivity (38) describes quite correctly the low frequency oscillations, when $\omega \ll \nu_\alpha$, namely the diffusion processes in a plasma, the monopolar diffusions of electrons and ions independently as well as the ambipolar one (In the considered approximation with T_α = const.). Really in this frequency range from (38) it follows

$$\varepsilon^l(\omega,\mathbf{k}) = 1 - \Sigma_\alpha (\omega_{L\alpha}^2/(i\omega\nu_\alpha - k^2 v_{T\alpha}^2)) \tag{41}$$

For the short wave length perturbations, when $k^2 r_{D\alpha}^2 \gg 1$, the solutions of $\varepsilon^l(\omega,\mathbf{k}) = 0$ coincides with the poles of (41)

$$i\omega\nu_\alpha - k^2 v_{T\alpha}^2 = 0 \tag{42}$$

It is easy to understand that this relation corresponds to the diffusion equation for the particles of type α.

$$\partial n_\alpha/\partial t - (v_{T\alpha}2)/\nu_\alpha)\Delta n_\alpha = 0 \tag{43}$$

Thus the coefficient of monopolar particle diffusion in the two fluids hydrodynamics occurs to be equal

$$D_\alpha = v_{T\alpha}^2/\nu_\alpha \tag{44}$$

In the opposite limit of long wave length perturbations when $k^2 r_{D\alpha}^2 \ll 1$, the solution of $\varepsilon l(\omega,\mathbf{k})=0$ can be presented as

$$i\omega\nu_i - k^2(v_s^2 + v_{Ti}^2) = 0, \tag{45}$$

where $v_s = \sqrt{T_e/M}$. As above, this relation corresponds to the equation

(46)

$$\partial n/\partial t - D_a \Delta n = 0$$

which represents the diffusion equation with ambipolar diffusion coefficient

$$D_a = (v_s^2 + v_{Ti}^2)/\nu_i = (T_e + T_i)/M\nu_i \tag{47}$$

The both results concerning monopolar diffusion as well as ambipolar one were confirmed experimentally by I. Langmuir. These facts demonstrate the obvious success of two fluids hydrodynamics.This success firstly was noted by I. Langmuir.

However very quickly appear new difficulties of the model besides remarked above with regard to the low frequency transverse field (skin effect). Namely from the expression (38) for $\varepsilon^l(\omega,\mathbf{k})$ it follows that in a low density collisionless plasma ($\omega >> \nu_\alpha$) when the wavelength of perturbations is sufficiently short, $kr_{D\alpha} >> 1$, there exist the longitudinal oscillations with spectra (for electrons and ions)

$$\omega^2 = k^2 v_{T\alpha}^2 \tag{48}$$

which correspond to the poles of $\varepsilon^l(\omega,\mathbf{k})$. Nobody observed such oscillations and moreover it will be shown below that even in a collisionless plasma they occur very strongly damped. This result was the serious failure of two fluids model.

Finally let us indicate to the very widespread mistake repeated up to day and which follows from two fluids model. We mean the long wave length ($k^2 r_{D\alpha}^2 << 1$) and low-frequency ($\omega^2 << \omega_{Le}^2$) longitudinal oscillations with phase velocity less than electron thermal velocity ($\omega\nu_e << k^2 v_{Te}^2$). Under these conditions from equations $\varepsilon^l(\omega,\mathbf{k})=0$ taking into account the expression (38) we obtain the following dispersion equation

$$\omega^2 - k^2 (v_s^2 + v_{Ti}^2) + i\omega\nu_i = 0 \tag{49}$$

In the limiting case $\omega << \nu_i$, this equation leads to (45) which describes the ambipolar diffusion, confirmed by numerous experiments. However in the opposite limit, when $\omega >> \nu_i$ from (49) we obtain the spectrum of weakly damping oscillations ($\omega \to \omega + i\delta$)

$$\omega = k\sqrt{(T_e + T_i)/M}\,, \quad \delta = -\nu_i/2 \tag{50}$$

I. Langmuir supposed that these oscillations represent the usual accoustic sound oscillations with spectrum $\omega = k\sqrt{\gamma P/P_0}$, $\gamma = 1$ and he called them " ion-accoustic waves". Moreover he really observed such a type of oscillations in a nonisothermal ($T_e >> T_i$) gas discharge plasma. Only one question remained unclear : what is $\gamma = c_p/c_v$ and why $\gamma = 1$ for plasma. From experimental datas for nonisothermal plasma followed that $\gamma = 1$. But why? This question had remained unclear up to 1954 when G. Gardeev clarified it (see below).

Above we restrict ourselves by considering only low frequency processes of isotropic plasma intentionally. Firstly it must be noticed that for high frequency processes with characteristic velocity much higher than the thermal velocities of charged particles the two fluids model corresponds to the independent particles approximation which is quite satisfactory for such processes as it was shown in previous section.

Secondly, just namely this model was proposed for description of low frequency processes by I. Langmuir and namely for them we were convinced that it arised very serious difficulties. For the magnetoactive plasma, when the two fluids model was proposed by I. Langmuir, just at the same time the one fluid magnetohydrodynamic (MHD) was developped by H. Alfven. The MHD represents the generalization of usual hydrodynamics for the conducting liquids and it seems that MHD must be valid only for very high density plasma.

However H.Alfven applied this model for description of ionospheric plasma and it occured very successfully. In the next section we will discuss the one fluid MHD.

* 6 One fluid MHD equations

As it was noticed above, the MHD equations differ from usual hydrodynamics by the additional volumetric force, which affects the conducting media with current **j** by the magnetic field **B**

$$\mathbf{f} = 1/c[\mathbf{j}\times\mathbf{B}] = 1/4\pi[\mathrm{rot}\mathbf{B}\times\mathbf{B}] \tag{51}$$

Taking into consideration this force one can easily make the generalization of usual hydrodynamics on the case of conducting liquid. Supposing the ideal conductivity and neglecting all dissipative processes the MHD equations can be presented as

$$\mathrm{div}\mathbf{B} = 0, \quad \partial\mathbf{B}/\partial t = \mathrm{rot}\,[\mathbf{v}\times\mathbf{B}] \tag{52}$$

$$\partial\mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = -\nabla P/\rho - (1/4\pi\rho)[\mathbf{B}\times\mathrm{rot}\mathbf{B}]$$

Here **v** is the velocity of liquid with density ρ and P is the pressure, which is connected with ρ and temperature T by the state equation

$$P = P(\rho,T) \tag{53}$$

The first achievement of MHD was connected to the analysis of small perturbations of stationary homogeneous equilibrium with

$$\mathbf{v_0} = 0,\ \rho_0 = \mathrm{const},\ P_0 = \mathrm{const},\ \mathbf{B_0} = \mathrm{const}.$$

For the perturbations v_1,ρ_1, and b from (52) we obtain

$$\mathrm{div}\mathbf{b} = 0,\ \mathrm{rot}[\mathbf{v_1}\times\mathbf{B_0}] = \partial\mathbf{b}/\partial t \tag{54}$$

$$\partial\mathbf{v_1}/\partial t = -(v_s^2/\rho_0)\nabla\rho_1 - (1/4\pi\rho_0)[\mathbf{B_0}\times\mathrm{rot}\mathbf{b}]$$

$$\partial\rho_1/\partial t + \mathrm{div}\rho_0\mathbf{v_1} = 0$$

Here v_s is the sound velocity for isoentropic processes, which follows from (53)

$$P_1 \equiv -v_s^2\rho_1 \equiv (\partial P/\partial\rho)_s\rho_1 \tag{55}$$

For the solutions of type $e^{(-i\omega t+ikr)}$ from the linear system (54), one can obtain the dispersion relations

(56)

$$\omega_1{}^2 = k_z{}^2 v_A{}^2,$$

$$\omega_{2,3}{}^2 = k^2/2\left\{\left(v_A{}^2 + v_S{}^2\right) \pm \sqrt{\left(v_A{}^2 + v_S{}^2\right)^2 - 4v_A{}^2 v_S{}^2 \cos^2\theta}\right\}$$

where v_A is the Alfven velocity introduced above and θ is the angle between **k** and $\mathbf{B_0}$.

Thus in the framework of MHD exist 3 branches of small oscillations. The first describes purely transverse waves, **b** and $\mathbf{v_1}$ are perpendicular to $\mathbf{B_0}$ and **k**, is known as Alfven waves. The second and third are called as the fast and slow sound waves in a conducting liquid. It must be noticed that namely 3 types of oscillations were really observed in the ionospheric plasma. Moreover, in the ionospheric plasma the ratio $\beta = (8\pi p_0)/B_0{}^2 \cong v_S{}^2/v_A{}^2 \ll 1$ and therefore the last two branches become separated

(57)

$$\omega_2{}^2 = k^2 v_A{}^2, \quad \omega_3{}^2 = k_z{}^2 v_S{}^2$$

The oscillations with spectrum $\omega_2(k)$ are transverse as well as $\omega_1(k)$ whereas the oscillations with spectrum $\omega_3(k)$ are purely longitudinal and correspond to the isoentropic oscillations of density and pressure in which $\mathbf{v}_1$ is parallel to **k.**

Despite of the observations of mentioned oscillations in ionospheric plasma, some problems have still remained. The first one is quantitative and about some parameters: what are $v_s = \sqrt{(\partial P/\partial \rho)_s} = \sqrt{\gamma T/M}$ and $\gamma = c_p/c_V$?. For ionospheric plasma in accordance to its consistent in the F-layer this quantity must be of the order of γ=5/3, whereas from the experiments it follows that $\gamma \approx 1$ and besides of this the oscillations occur to be very damping. What is the reason of their absorption ? The second problem is more principal: what is the reason of success for MHD in applications to the ionospheric plasma? MHD as usual hydrodynamics must be valid only for dense gaseous, where collisional effects are dominant, whereas ionospheric F-layer plasma seems to be collisionless. Neverthless the predictions of MHD occur in a good agreement with experimental observations.

The first attempt of derivation MHD equations was made starting from the equations of two fluids hydrodynamics (36) at the beginning of 50-s. Really let us suppose the inequalities

$$\omega \ll k v_{Te}, \Omega_i \qquad \nu_e \ll \Omega_e, \quad \nu_i \ll \Omega_i \ll \omega_{Li}$$

Under these conditions the displacement current may be neglected and the Maxwell equation for magnetic field takes the form

(58)

$$\mathrm{rot}\mathbf{B} = (4\pi/c)\,\mathbf{j}$$

Using this equation and taking into consideration that under the above restrictions, plasma can be considered as quasineutral (ne=ni=n), from the equations (36) by summing them, one can obtain:

$$Mn(\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v}) = -\nabla[\,n(T_e+T_i)] - 1/4\pi[\mathbf{B}\times\mathrm{rot}\,\mathbf{B}] - Mn\nu\mathbf{v} \tag{59}$$

where $\mathbf{v}=\mathbf{v_i}$. Besides of this from the first equation of (36) follows

$$\mathbf{E}_\perp = -1/c\,[\mathbf{v}\times\mathbf{B}] \tag{60}$$

As a result the Maxwell equation for electric field takes the form

$$\partial\mathbf{B}/\partial t = -c\,\mathrm{rot}\mathbf{E} = \mathrm{rot}[\mathbf{v}\times\mathbf{B}] \tag{61}$$

Finally completing the above equations by the continuity equation for ions we obtain the following system of one fluid MHD

$$\mathrm{div}\mathbf{B} = 0,\ \mathrm{rot}[\mathbf{v}\times\mathbf{B}] = -\partial\mathbf{B}/\partial t \tag{62}$$

$$\partial\mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = -\nabla P/\rho - 1/4\pi\rho[\mathrm{rot}\mathbf{B}\times\mathbf{B}] - \nu\mathbf{v}$$

$$\partial\rho/\partial t + \mathrm{div}\rho\mathbf{v} = 0$$

where $\rho = Mn$, $P = n(T_e + T_i)$, or $T = T_e + T_i$ = constant. The last relation for P represents the state equation for plasma, which corresponds to (53).

The system (62) coincides practically with the one fluid MHD equations (52). The only difference consists of the existence of the last term in the equation of motion (62). This term takes into account a friction of ions on neutral particles and it is obvious that in the purely one fluid hydrodynamics it does not exist. In this case the system (62) seems more general, it is valid for weakly ionized plasma as well.

Thus the derivation of MHD equations from the two fluids hydrodynamics was a significant success of plasma theory. Nevertheless all the above noted difficulties which are inherent in two fluids hydrodynamics, force the scientist to attempt avoiding them by using the kinetic consideration. More correctly at the end of 30-s the scientists attempted to generalize the Boltzmann's kinetic equation for the case of the systems of charged particles, or in other words, for plasma. We'll speak about this in the next sections.

LECTURE 2

Plasma Kinetic Descriptions

* 7 Boltzmann - Landau kinetic equation

The first attempts of generalization of Boltzmann kinetic equation for a gas of ionized particles were made before the second world war independently by S.Chapman and T.Couling and L.Landau. The basis of kinetic description of systems consisted of a large number of particles is the probability description. Therfore the distribution function of n particles can be introduced as

$$fn(\mathbf{r}_1, \mathbf{p}_1, \mathbf{r}_2, \mathbf{p}_2, \ldots, \mathbf{r}_n, \mathbf{p}_n, t) \tag{63}$$

This function represents the probability that at the moment t the particles with momentums $\mathbf{p}_1, \mathbf{p}_2, \ldots \mathbf{p}_n$ are located at $\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_n$ correspondingly. It is obvious that the distribution function (63) is very general and gives the complete description of the system. However it is very complicated because it depends on too much arguments. As a result it occurs to be practically useless.
Let us remind that the plasma is a gas and in * 1, the corresponding conditions for validity gas approximations were given. Namely these conditions were used by S.Chapman and T.Cooling for a weakly ionized plasma when they attempted to generalize Boltzmann kinetic theory. In * 1, it was shown that the neutral particles can be considered as the hard balls with radius a. Then, a weakly ionized plasma is a gas if (see(4))

$$\eta_0 = a n_o^{1/3} << 1 \tag{64}$$

where n_o is the density of neutrals. In the zero approximation in the condition (64), or in other words, when the particles interaction is completely neglected, then the function fn (63) can be presented as

$$fn(\mathbf{r}_1, \ldots \mathbf{r}_n; \mathbf{p}_1 \ldots \mathbf{p}_n, t) = {}_{i=1}\Pi^n f(\mathbf{r}_i, \mathbf{p}_i, t) \tag{65}$$

Here f(**r**,**p**,t) is a probability that a charged particle with momentum p at the moment t is located at r. It is obvious, that in this approximation this probability is constant and therefore it satisfied the Liouville's equation

$$df(\mathbf{r}, \mathbf{p}, t)/dt = \partial f/\partial t + \mathbf{r}\partial f/\partial \mathbf{r} + \mathbf{F}\partial f/\partial \mathbf{p} = 0 \tag{66}$$

Here **v** is the particles velocity and **F** the force which determines particles motion. For charged particles

$$\mathbf{v} = d\mathbf{r}/dt,\ d\mathbf{P}/dt = \mathbf{F} + e\{\ \mathbf{E} + 1/c[\mathbf{v}\times\mathbf{B}]\} \tag{67}$$

where **E** and **B** are the external electric and magnetic fields, and e the particles charge. Of course the Liouville's equation (66) must be written for each particles of type α.

Moreover, let underline that the Liouville's equation (66) doesn't take into account the particles interaction. In Boltzmann consideration the particles interaction leads to the appearance of the nonzero right side of equation (66). In the lowest approximation it takes into account the pair interactions of particles type α with all particles of type β and therefore

(68)

$$df\alpha/dt = (\partial f\alpha/\partial t)_{st} = \Sigma_\beta(\partial f\alpha/\partial t)_{st}{}^{\alpha\beta} = \Sigma_\beta\Im_{\alpha\beta}(f_\alpha,f_\beta)$$

We will restric ourselves by taking into account only elastic interaction (scattering), as it shown on Figure 2.

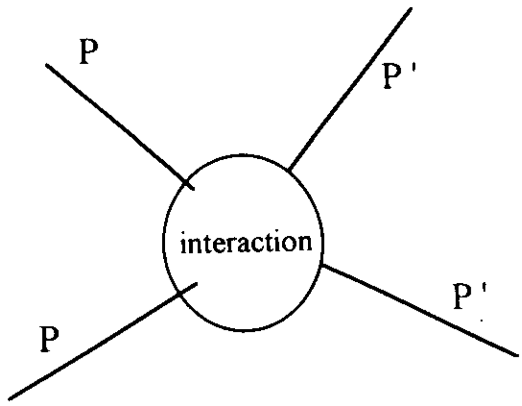

Figure 2

This leads to the following form of collision integral $\Im\alpha\beta\ (f\alpha,f\beta)$

(69)

$$\Im_{\alpha\beta}(f_\alpha,f_\beta) =$$
$$\int d\mathbf{p}_\beta d\mathbf{p}_\beta' d\varepsilon_\beta' v_{\alpha\beta} d\sigma_{\alpha\beta}[f_\alpha(\mathbf{p}_\alpha')f_\beta(\mathbf{p}_\beta')-f_\alpha(\mathbf{p}_\alpha)f_\beta(\mathbf{p}_\beta)]\delta(\mathbf{p}_\alpha+\mathbf{p}_\beta-\mathbf{p}_\alpha'-\mathbf{p}_\beta')\delta(\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\alpha'-\varepsilon_\beta')$$

This expression takes into account the particles momentum $\mathbf{p}$ and energy ε conservations in the scattering processes. Moreover the probabilities of forward and backward scatterings are supposed to be equal. These probabilities are the product of the particles relative velocity $\mathbf{v}_{\alpha\beta} = |\mathbf{v}_\alpha - \mathbf{v}_\beta|$ and scattering crosssection $d\sigma_{\alpha\beta}$. The last quantity depends on

(70)

$$\mathbf{P}_{\alpha,\ \beta} = \pm\ \mu_{\alpha\beta}\mathbf{v}_{\alpha\beta} + (m_{\alpha,,\beta}/(m_\alpha+m_\beta))(\mathbf{p}_\alpha+\mathbf{p}_\beta)$$

$$\mathbf{P}_{\alpha,\ \beta}' = \pm\ \mu_{\alpha\beta}\mathbf{v}_{\alpha\beta}\mathbf{n}+(m_{\alpha,,\beta}/(m_\alpha+m_\beta))(\mathbf{p}'_\alpha+\mathbf{p}_\beta')$$

n is the vector in the direction of particle α velocity in the frame of centrum of inertia (in which $\mathbf{p}_\alpha+\mathbf{p}_\beta = 0$) and $\mu_{\alpha\beta} = m_\alpha \cdot m_\beta /(m_\alpha+m_\beta)$

For the scattering of charged particles on the neutrals (hard balls) we have

(71)

$$d\sigma_{\alpha\beta}\ a^2 d\Omega = a^2 \sin\theta d\theta d\varphi$$

where $d\Omega$ is the solid angle of scattering. The expression (69), when taking in consideration (71), describes the elastic scattering of charged particles on neutrals in a plasma. In this sense Chapman and Couling supposed that the kinetic equation (68) can be applied to the weakly ionized plasma. It must be noticed however that in their interpretation the electric and magnetic fields in the left side of (68) (see (66) and (67)) are external and only external fields. They did not understand the idea of selfconsistent fields, which was clear much earlier for I. Langmuir in his model of independent particles.

The next important progress in developing of plasma kinetic theory was made in 1937 by L. Landau. Starting from the Boltzmann collision integral (69), he derived the kinetic equation for completely ionized plasma. For this aim L. Landau used the Rutherford formula for Coulomb scattering

$$d\sigma_{\alpha\beta}/d\Omega = 4\pi e_\alpha^2 e_\beta^2/(\mu_{\alpha\beta}^2\, v_{\alpha\beta}^4 \sin^4(\theta/2)) \qquad (72)$$

However it is wellknown that this expression leads to the divergence of the total crosssection of scattering. How it can be avoided? At this question the answer was found by L.Landau and this answer was full of genius. He noticed that in a plasma takes place Debye screening of Coulomb potential which is a consequence of the validity of gas approximation

$$\eta_1 = e^2 n^{1/3}/\langle\varepsilon\rangle \sim e^2 n^{1/3}/T \ll 1 \qquad (73)$$

Under this condition the potential energy of charged particles interaction in a plasma looks as

$$U(r) = (e^2/r)e^{-r/r_D} \qquad (74)$$

It must be noticed that the condition (73) is equivalent to the requirement $U(r_{av}) \ll T$, where $r_{av} \cong n^{-1/3}$. At the same time, the expression (74) means that the characteristical radius of charged particles interaction in a plasma is r_D and this radius in accordance to (73) is much larger than the average distances between particles $\sim n^{-1/3}$

$$n^{1/3} r_D \cong r_D / n^{-1/3} \cong \sqrt{T/e^2 n^{1/3}} \cong 1/\eta_1^{\,1/2} \gg 1 \qquad (75)$$

In this sense the above condition is opposite to (64) if instead of a we substitute r_D . Despite this, L.Landau used the Boltzmann collision integral (69) substituting the expression (72) in it. This unsubstantiality was strongly justified by N. Bogolyubov in 1946 when he developed mathematically correct method of derivation of the kinetic equations.

Besides of the inequality (75) L.Landau supposed that for Coulomb scattering an other inequality takes place as well. Namely in this process

$$|\mathbf{p}_{\alpha\beta}' - \mathbf{p}_{\alpha\beta}| \ll p_\alpha,\ p_\beta, \qquad (76)$$

which means that the change of particles momentum is small, or the scattering angle $\theta \ll 1$. This assumption together with Born approximation which is valid when

$$e^2/r_{min} \ll T \qquad (77)$$

allow him to get the convergence collision integral. This integral is known as Landau collision integral and it looks as

(78)

$$\Im_{\alpha\beta}(f_\alpha,f_\beta)=(\partial/\partial P_{\alpha i})\int d\mathbf{P}_\beta(2\pi e_\alpha{}^2 e_\beta{}^2\, L/u^3)(u^2\delta_{ij}-u_iu_j)((\partial f_\alpha/\partial P_{\alpha j})f_\beta-(\partial f_\beta/\partial P_{\beta j})f_\alpha)$$

Here $u=v_\alpha - v_\beta$ is the relative velocity of scattered particles and the quantity L

(79)

$$L = {}^{rD}\int_{r\,min} dr/r = \ln r_D/r_{min} = \ln(T/(e^2n^{1/3})) \sim 10 >> 1$$

is called as Coulomb logarithm.

The kinetic equation (68) with collision integral (78) is known as the equation Boltzmann-Landau. Below we will use it for describing electromagnetic properties of completely ionized plasma. It must be noticed here that L. Landau as S.Chapman and T.Couling was sure that in the left side of his equation electromagnetic fields **E** and **B** are only external. The interaction of particles is completely taken into account in the collision integral (77). But this belief, of course, was an annoying mistake of great scientist.

* 8 Relaxations of momentum and energy

Let us now follow S.Chapman and T.Couling and L.Landau to consider the relaxations of particles momentum and energy in a plasma slightly deviated from thermodynamical equilibrium. In this connection, it must be noticed that the above obtained collision integrals occur to be identically zero for the equilibrium Maxwell distribution

(80)

$$f_{0\alpha} =(n_\alpha/(2\pi m_\alpha T_\alpha)^{3/2})\exp(-m_\alpha v^2/2T_\alpha)$$

Of course, this statement is correct only for stationary and spatially homogeneous distribution (80) and when

$$T_e = T_i = T_0 = T,\ \Sigma_\alpha e_\alpha n_\alpha = 0$$

and only in the case of a plasma without any fields. Let us now consider the small deviations from equilibrium and calculate the time relaxations of nonequilibrium momentum and energy of particles. Suppose that at $t = 0$ the particles (electrons) distribution function differs from Maxwellian by the existence of a small velocity $u_0 << v_{Te}$, or

(81)

$$f_e = (n_e/(2\pi m T_e)^{3/2})e(-m(v-u(t))^2/2T_e)$$

and $u(t=0) = u_0$. The problem is to find the dynamic equation describing time relaxation of u(t). Substituting (81) in the kinetic equation (68) after integration over momentum of electrons one can obtain the following equation

(82)

$$\partial\mathbf{u}/\partial t = -\nu_e\,\mathbf{u}$$

$\nu_{e0} = \pi a^2 v_{Te} n_0$ for weakly ionized plasma

$\nu_e =$

$$\nu_{eff} = 4/3\left(\sqrt{2\pi/m}\, e^2 e_i^2 n_i / T_e^{3/2}\right) L \quad \text{for completely ionized plasma.}$$

It must be noticed that for weakly ionized plasma the equation (82) is exact, whereas for completely ionized plasma the accuracy of this equation is of the order of a factor ~ 1

From (82) the following relation can be obtained

$$\mathbf{u}(t) = \mathbf{u}_0 \exp(-\nu_e t) \tag{83}$$

Thus the momentum relaxation time for electrons in a plasma is equal $\tau \sim 1/\nu_e$

Let us now consider the relaxation of energy. Suppose that at t = 0 the electron temperature T_{e0} differs from the temperature of neutrals Tn (for weakly ionized plasma) or ions T_{i0} (for completely ionized plasma). The problem is to derive the dynamic equation describing relaxation time of electrons temperature Te(t), when

$$f_\alpha = (n_\alpha/(2\pi m_\alpha T_\alpha(t))^{3/2}) e(-m_\alpha v^2/2T_\alpha(t)) \tag{84}$$

Substituting these expressions into the equation (68) and integrating over momentum of particles, as above, after simple calculations we obtain

$$\partial(T_e - T_n)/\partial t = -(\nu_{en} 2m/M_n)(T_e - T_n) \tag{85}$$

$$\partial(T_e - T_i)/\partial t = -(\nu_{eff} 2m/M)(1+|ei/e|)(T_e - T_i)$$

for weakly and completely ionized plasma correspondingly. Here for weakly ionized plasma it was supposed that T_n = const, which follows from obvious inequality $n_o >> n_e$. At the same time, for completely ionized plasma in derivation (85) we take into account, that

$$\partial(T_e + T_i)/\partial t = 0$$

Thus from (85) follows that the energy relaxation time is much larger than that of momentum, or

$$\tau_\varepsilon \sim (M/2m)\, \tau_m >> \tau_m \sim 1/\nu_e \tag{86}$$

In conclusion let us consider the behaviour of the completely ionized plasma in external constant electric field $\mathbf{E_0}$. The solution of this problem shows the above mentioned incorrectness of the calculations of the momentum relaxation time of plasma offered (82). The Boltzmann-Landau equation for this problem looks as

$$(e\mathbf{E_0}/m)\partial f_e/\partial \mathbf{v} = \Sigma_\beta \int d\mathbf{p}_\beta (2\pi e_\alpha^2 e_\beta^2 L/u^3)(u^2\delta_{ij} - u_i u_j)[f_\beta \partial f_e/\partial p_j - f_e \partial f_\beta/\partial p_j] \tag{87}$$

where $\mathbf{u} = \mathbf{v}_\alpha - \mathbf{v}_\beta$ and the summation carried out over β = e,i

If the field $\mathbf{E_0}$ is sufficiently weak we can represent $f_e = f_{0e} + \delta f_e$, where f_{0e} is the Maxwellien distribution (84)(not(81)) and δf_e a small correction. In this case the influence $\mathbf{E_0}$ field on ions is negligible and ion distribution function is identically Maxwellian. For calculating δf_e let us expand it in the series

$$\delta f_e = (\mathbf{vE_0}/E_0)[a_0+a_1(5/2-v^2/2v_{Te}^2)]f_{0e} \tag{88}$$

For the determination of the constant coefficients a_0 and a_1 from (87) the following system can be obtained

$$eE_0/T_e = -\nu_{eff}(a_0+3/2a_1) \tag{89}$$

$$(3/2)a_0 + (13+4\sqrt{2})a_1/4 = 0$$

Here for simplicity we suppose $e_i = -e$.

After solving the system (89) we can calculate the current in a plasma

$$\mathbf{j} = e\int \mathbf{v}f_e d\mathbf{p} = 1{,}96(e^2n_e/m\nu_{eff})\mathbf{E_0} \equiv \sigma\mathbf{E_0} \tag{90}$$

Or for the plasma conductivity we have

$$\sigma=1{,}96(e^2n_e/m\nu_{eff}) \tag{91}$$

The factor 1,96 instead of 1 about which the above remark was done. With increasing the ratio $Z = |e_i/e|$ this factor tends to 1. When $Z > 10$ we can neglect the electron - electron collisions and this factor in (91) becomes equal to 1.

For a weakly ionized plasma quite similar calculation leads to the expression

$$\mathbf{j} = e\int \mathbf{v}f d\mathbf{p} = (e^2n_e/m\nu_{e0})\mathbf{E_0} = \sigma\mathbf{E_0} \tag{92}$$

Thus the plasma conductivity is

$$\sigma = e^2n_e/m\nu_{e0} \tag{93}$$

where $\nu_{e0} = \pi a^2 v_{Te}\, n_0$ is the electron neutral collision frequency. For weakly ionized plasma this expression is exact and therefore the Lorentz approximation taking into account only electron-neutral collisions is correct.

Selfconsistent Field Approximation

* 9 Vlasov-Maxwell equations

Above we emphasized many times on the fact that in the Boltzmann equation for a weakly ionized plasma and in the Boltzmann-Landau equation for a completely ionized one the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields are proposed to be external and only external. As a result of this assumption, all relaxation processes, considered in these equations, are aperodically damping in time and are determined by particles collisions (electrons collisions in considered cases).

The first who draw world scientists attention to the inconsistence of such a treatment of electric and magnetic fields in kinetic theory was A. Vlasov. In his famous work publicated in 1938, A. Vlasov showed that in the lowest approximation of the gaseous parameter η_1, the interaction between the charged particles can be taken into account if in the Liouville's equation electromagnetic fields are considered not only as external, but as full fields satisfying Maxwell equation with induced charges and current densities

$$\rho = \Sigma_\alpha e_\alpha \int f_\alpha \mathbf{dp}, \quad \mathbf{j} = \Sigma_\alpha e_\alpha \int \mathbf{v} f_\alpha \mathbf{dp} \tag{94}$$

Thus the interaction of plasma particles with each fields are to be taken into account because the distribution function f_α itself satisfies the kinetic equation of the lowest approximation

$$\partial f_\alpha/\partial t + \mathbf{v}\partial f_\alpha/\partial \mathbf{r} + e_\alpha \{ \mathbf{E} + (1/c)[\mathbf{v}\times\mathbf{B}]\} \partial f_\alpha/\partial \mathbf{p} = 0 \tag{95}$$

Now we can write the field equations in the form

$$\text{div}\mathbf{E} = 4\pi\Sigma_\alpha e_\alpha \int f_\alpha e\mathbf{dp} + 4\pi\rho_0, \qquad \text{rot}\mathbf{E} = -(1/c)\partial\mathbf{B}/\partial t \tag{96}$$

$$\text{div}\mathbf{B} = 0 \text{ , rot } \mathbf{B} = (1/c)\partial\mathbf{E}/\partial t + (4\pi/c)(\Sigma_\alpha e_\alpha \int \mathbf{v} f_\alpha \mathbf{dp}) + (4\pi/c)\mathbf{j_0}$$

The system (95) - (96) represents the complete system of selfconsistent equations for **E**, **B** and f_α which describe the plasmas in the lowest approximation of the gaseous parameter.

Only in the following, more higher approximation arises the right side of the kinetic equation (95), taking into account the particles scattering (collisions).

In scientific literature the equation (95) is known as Vlasov equation whereas the complete system (94) - (95) is called Vlasov-Maxwell system of equations. Sometimes they are also called equations for collisionless plasmas taking into account particle interactions only via selfconsistent fields.

Here it must be noticed that the basis of Vlasov equation was not sufficiently strict. First of all it was not understood how in the Liouville's equation the particles correlations can be taken into account, although this equation describes a completely noncorrelated particles system. Moreover in this sense the Vlasov equation for many scientists remainded very doubtful. Among them were the great L. Landau, M. Leontovich, V. Fock and others. In 40-s between the scientists arised the wellknown disputes, the results of which led to new dicoveries and new excellent investigations. Let us briefly discuss these disputes.

Following A. Vlasov, let us consider a small perturbation of the equilibrium Maxwell distribution f_{0e}

$$f_e = f_{0e} + \delta f_e \tag{97}$$

The distribution f_{0e} satisfies the equation (95) in the absence of $\mathbf{E_0}$ and $\mathbf{B_0}$. Besides we suppose that $\rho_0 = \mathbf{j_0} \equiv 0$, i.e. $\Sigma_\alpha\, e_\alpha n_{0\alpha} = 0$. Then from (95) can be obtained linear equation for δf_e

$$\partial\delta f_e/\partial t + \mathbf{v}\partial\delta f_e/\partial\mathbf{r} + \mathbf{eE}\partial f_{0e}/\partial\mathbf{p} = 0 \qquad (98)$$

where the field perturbation **E** must be determined from the system of Maxwell equations (96). Below for simplicity we will restrict ourselves by considering the potential field $\mathbf{E} = -\nabla\Phi$ only and therefore

$$\Delta\phi = -4\pi e \int \delta f_e d\mathbf{p} \qquad (99)$$

The system (98) and (99) represents the complete system of linear equations which allows to investigate the time development of initial perturbations $\delta f_e(0,r,p)$. Suppose that

$$\delta f_e(0,\mathbf{r},\mathbf{p}) = \delta f_0(\mathbf{p})e^{i\mathbf{kr}} \qquad (100)$$

It should be noted that an arbitrary perturbation can be represented as a sum of Fourier harmonics such as (100).

Now we can find the solutions of system (98) and (99) as $(\delta f_e, E) \sim e^{(-i\omega t + i\mathbf{k}\cdot\mathbf{r})}$ and from the existence condition of nontrivial solutions determine $\omega(\mathbf{k})$. Namely this quantity gives the time development of initial perturbations of type (100). Really from (98) it follows that

$$\delta f_e(\mathbf{p}) = (-ie\mathbf{E}\partial f_{0e}/\partial\mathbf{p}/(\omega - \mathbf{k}\cdot\mathbf{v})) = (-\mathbf{ek}\partial f_0/\partial\mathbf{p}/(\omega - \mathbf{k}\cdot\mathbf{v}))\phi \qquad (101)$$

After substituting this expression into the equation (99) the following dispersion equation can be obtained

$$1 - (4\pi e^2/k^2)\int (\mathbf{k}\,\partial f_0 e/\partial\mathbf{p}/(\omega - \mathbf{k}\,.\mathbf{v}))d\mathbf{p} = 0 \qquad (102)$$

which represents the condition of existence the nontrivial solutions of the system (98) - (99).

The main disagreement between A. Vlasov and L. Landau is related to the analysis of the equation (102). A.Vlasov supposed that the pole $\omega = \mathbf{k.v}$ in integrand of the equation (102) must be understood in the sense of principal value. Then for the long range perturbations, $k^2 r_{De}^2 << 1$, he obtained nondamping frequency spectrum *

$$\omega^2 = \omega_{Le}^2 + 3\,k^2\,v_{Te}^2 = \omega_{Le}^2\,(1 + 3k^2 r_{De}^2) \qquad (103)$$

From (103) follows that the group velocity of such perturbations are small in comparison with the thermal velocity of electrons

$$\mathbf{v}_g = \partial\omega/\partial\mathbf{k} = 3\,k\,r_{De}\,\mathbf{v}_{Te} << v_{Te} \qquad (104)$$

* It must be noticed that this spectrum differs from that founded by I. Langmuir in the two fluids model by the factor 3 instead of 1 in (48). This indicates the nonaccuracy of hydrodynamical description of plasma oscillations.

However it must be noted that A. Vlasov understood that the oscillations damping really exists and moreover he supposed that it arises in the second approximation of particles interaction, or

as a result of electron - ion collisions. In this sense he thought, that his equation (98) describes "collisionless plasmas"
Quite another sense of this pole gave L.Landau in his famous paper from 1946, in which he criticized A. Vlasov. In agreement with the causality principle he proposed that

$$1/(\omega - \mathbf{k}\cdot\mathbf{v}) = (\wp/(\omega - \mathbf{k}\cdot\mathbf{v})) - i\pi\delta(\omega - \mathbf{k}\cdot\mathbf{v}) \tag{105}$$

The first term corresponds to the A. Vlasov treatment, whereas the second leads to the oscillations damping ($\omega \to \omega + i\delta$)

$$\delta = -\sqrt{\pi/8}\left(\omega_{Le}/k^3 r_{De}{}^3\right)\exp\left(-\left(1/\left(2k^2 r_{De}{}^2\right)\right)^{-3/2}\right) \tag{106}$$

This damping was called as the Landau damping of plasma oscillations with frequency spectrum (103), which was obtained in the Vlasov approximation. Here it must be noted that L. Landau did not notice at that time that this damping contradicts the momentum relaxation time, obtained by him in 1936 and equal ~ 1/ vei. Only in 1946 everything was clarified by N. Bogolyubov in his famous book "Dynamical problems in statistical Physics". In this book the strong derivation Vlasov equation and Landau collision integral were given as an expansion on powers of gas parameter (73).
Thus now we can write the exact kinetic equation for completely ionized plasma (α=e,i)

$$\partial f_\alpha/\partial t + \mathbf{v}\partial f_\alpha/\partial \mathbf{r} + e_\alpha \{\mathbf{E} + (1/c)[\mathbf{v}\times\mathbf{B}]\}\, \partial f_\alpha/\partial \mathbf{p} = \Sigma_\beta\, (\partial f_\alpha/\partial t)^{\alpha\beta} \tag{107}$$

which can be called as Vlasov-Landau equation. If we add to the right side of this equation the integral of charged particles collision with neutrals this equation can be applied to the weakly ionized plasma also.
Physical meaning of Landau damping was clarified by R. Sagdeev in 1956 when he noticed that it is a result of Cherenkov emission and absorption of the plasma oscillations (103) by the plasma electrons at $\omega = \mathbf{k}\cdot\mathbf{v}$. As for Maxwell distribution $\partial f_0/\partial \mathbf{v} < 0$ then the absorption exceeds on emission and we obtain oscillations damping (see Fig.3)
In conclusion let us repeat once more that for the system of charged particles under the condition of gas approximation the principal interaction between the particles is taken into account by the Vlasov kinetic equation, or in other words the principal interaction is the interaction via selfconsistent fields. Only in the second approximation at least for completely ionized plasma the particles collisions must be taken into account. In this sense the Landau - Boltzmann kinetic equation takes into account the effects of higher order than Vlasov's equation.The Vlasov-Landau equation (107) is that which takes into account not only particles interaction via selfconsistent field but also interaction via their direct collisions. The fundamental property of the system of charged particles consists in that the self consistent interaction surpasses the direct collisions of particles. Namely this property represents the beauty of plasma and makes it as a very interesting and important scientific object.

* 10 Bathnagar-Gross-Krook collision integral

The kinetic equation (107) is very complicated because of its right side which represents nonlinear integral operator. It is difficult to make use of this equation. Therefore in scientific literature very often the various phenomenological and approximate collision integrals are

used. Despite phenomenological character of such collision integrals, sometimes it occurs to be not only qualitatively but quantitatively also correct.
Every model of collision integral must take into account the principal conservation laws such as conservation of particles number, their momentum, and anergy. Of course, we mean only elastic collision integrals. Thus the following relations must be satisfied

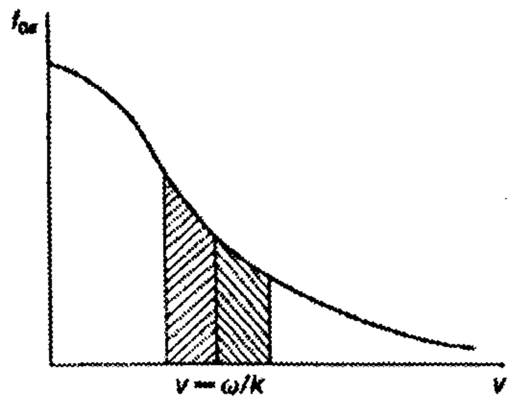

Figure 3

(108)

$$\int d\mathbf{p}_\alpha \, (\partial f_\alpha/\partial t)_{st}{}^{\alpha\beta} = 0,$$

$$\int \mathbf{p}_\alpha \, (\partial f_\alpha/\partial t)_{st}{}^{\alpha\beta} \, d\mathbf{p}_\alpha + \int \mathbf{p}_\beta \, (\partial f_\beta/\partial t)_{st}{}^{\beta\alpha} \, d\mathbf{p}_\beta = 0,$$

$$\int \varepsilon_\alpha \, (\partial f_\alpha/\partial t)_{st}{}^{\alpha\beta} \, d\mathbf{p}_\alpha + \int \varepsilon_\beta \, (\partial f_\beta/\partial t)_{st}{}^{\beta\alpha} \, d\mathbf{p}_\beta = 0,$$

Here ε is energy.
In addition, for the thermodynamically equilibrium (Maxwell) distributions of particles, the collision integrals must be zero. This follows from Boltzmann H-theorem.
Below we will use the most perfect, from our point of view, model of collision integral proposed in 1954 by P. Bathnagar, E. Gross and M. Krook. It looks as

(109)

$$(\partial f_\alpha/\partial t)_{st}{}^{\alpha\beta} = -\nu_{\alpha\beta} \, (f_\alpha - N_\alpha \phi_{\alpha\beta}),$$

where

(110)

$$\phi_{\alpha\beta} = (1/(2\pi m_\alpha T_{\alpha\beta})^{3/2}) \exp[-(m_\alpha (\mathbf{v}-\mathbf{v}_\alpha)^2)/2T_{\alpha\beta}]$$

$$\mathbf{v}_\alpha = 1/N \int d\mathbf{p} \, \mathbf{v} \, f_\alpha \quad , N_\alpha = \int d\mathbf{p} f_\alpha$$

$$T_{\alpha\beta} = (m_\alpha T_\beta + m_\beta T_\alpha)/(m_\alpha + m_\beta),\ T_\alpha = (m_\alpha/2N_\alpha)\int d\mathbf{p}\,(\mathbf{v}-\mathbf{v}_\alpha)^2 f_\alpha$$

For satisfying the relations (108) it is necessary that

$$m_\alpha N_\alpha \nu_{\alpha\beta} = m_\beta N_\beta \nu_{\beta\alpha} \qquad (111)$$

The physical meaning of quantities $\nu_{\alpha\beta}$ is clear from the analysis of relaxation processes which were considered above using Boltzmann and Landau collision integrals. Namely $\nu_{\alpha\beta}^{-1}$ represents the momentum relaxation time of α particles stipulated by their collisions with β particles. So

$$\nu_{e0} = \pi a^2 v_{Te} N_0,\ \nu_{i0} = \pi a^2 v_{Ti} N_0 \qquad (112)$$

$$\nu_{ee} = \left(4/3\sqrt{\pi/m}\right)\left(e^4 N_e L/T_e^{3/2}\right),\ \nu_{ei} = 4/3\sqrt{(2\pi/m)}e^2 e_i^2 N_i L/T_i^{3/2}$$

$$\nu_{ii} = 4/3\sqrt{\pi e_i^4 N_i L/\left(MT_i^{3/2}\right)},\ \nu_{ie} = (m/M)|e_i/e|\nu_{ei}$$

Below these expressions will be used in all estimations.

* 11 About hydrodynamical description of collisionless plasmas

Above it was shown that the Vlasov-Maxwell equations take into account the principal interactions of charged particles and in this sense they can describe all properties of plasmas quite sufficiently. However this system is complicated yet because the distribution function $f(\mathbf{p},\mathbf{r},t)$ is a function of 7 variables. Below we will show that under the definite conditions this system can be simplified and reduced to the the system of equation for hydrodynamical quantities

$$N_\alpha = \int d\mathbf{p} f_\alpha(\mathbf{p},\mathbf{r},t) \qquad (113)$$

$$N_\alpha \mathbf{v}_\alpha = \int d\mathbf{p}\mathbf{v} f_\alpha(\mathbf{p},\mathbf{r},t)$$

$$N_\alpha T_\alpha = \int d\mathbf{p}(m_\alpha v^2/2) f_\alpha(\mathbf{p},\mathbf{r},t)$$

Such simplification is possible in high frequency range, when the characteristic velocity is much larger than the thermal velocities of particles, and in low frequency range, when this velocity exceeds the ion thermal velocity but is much less than the thermal velocity of electrons. In the second case plasma must be nonisothermal with $T_e >> T_i$.

For derivation of hydrodynamical equation we start from Vlasov equation

$$\partial f_\alpha/\partial t + \mathbf{v}\partial f_\alpha/\partial \mathbf{r} + e_\alpha \{\mathbf{E} + 1/c[\mathbf{v}\times\mathbf{B}]\}\partial f_\alpha/\partial \mathbf{p} = 0 \qquad (114)$$

In this equation particles collisions are neglected which means that the characteristical time τ and characteristical scale L_0 of processes must satisfy the inequalities

(115)

$$1/\tau >> \Sigma_\beta \nu_{\alpha\beta}, \; L_0 << v_{T\alpha}/\Sigma_\beta \nu_{\alpha\beta}$$

Multiplying equation (114) on 1 and v and integrating over momentum we obtain

$$\partial N_\alpha/\partial t + \text{div}\, N_\alpha \mathbf{v}_\alpha = 0 \tag{116}$$

$$\partial N_\alpha v_{\alpha i}/\partial t + \partial \Pi_{\alpha ij}/\partial r_j = e_\alpha N_\alpha/m_\alpha \,\{\mathbf{E} + 1/c[\mathbf{v}_\alpha \times \mathbf{B}]\}$$

where

$$\Pi_{\alpha ij} = \int d\mathbf{p}(v_i v_j) f_\alpha(\mathbf{p},\mathbf{r},t) \tag{117}$$

The first equation coupled the first moment N_α to the second one $N_\alpha \mathbf{v}_\alpha$ is the continuity equation and it is closed in hydrodynamical sense. At the same time the second equation which connects the second moment $N_\alpha\, \mathbf{v}_\alpha$ to the third $\Pi_{\alpha ij}$ occurs nonclosed. The problem of deriving hydrodynamical equations consists closing this equation.

In collisionless plasmas exist two possibilities of closing this equation. First concerns the high frequency and fast processes whereas the another concerns the low frequency and slow processes.

In the high frequency range when

$$L_0/\tau \sim \omega/k >> \mathbf{v}_{T\alpha} \tag{118}$$

the thermal motion of particles can be neglected and $f_\alpha \sim \delta\,(\mathbf{v}-\mathbf{v}_\alpha)$. Then from (117) it follows

$$\Pi_{\alpha ij} = N_\alpha v_{\alpha i} v_{\alpha j} \tag{119}$$

Substituting this expression into the equation (116) we obtain the Euler equation

$$\partial \mathbf{v}_\alpha/\partial t + (\mathbf{v}_\alpha.\nabla)\mathbf{v}_\alpha = (e_\alpha/m_\alpha)\{\mathbf{E} + (1/c)[\mathbf{v}_\alpha \times \mathbf{B}]\} \tag{120}$$

The system of equations (116) and (120) together with the definitions of charge and current densities

$$\rho = \Sigma_\alpha e_\alpha N_\alpha, \qquad \mathbf{j} = \Sigma_\alpha e_\alpha N_\alpha \mathbf{v}_\alpha \tag{121}$$

form the complete system of hydrodynamical equations. It is easy to notice that this system coincides with two-fluid hydrodynamical equations if $v_\alpha \to 0$, or in other words with the I. Langmuir hydrodynamics of collisionless plasmas.

The other limit, when the hydrodynamical description of collisionless plasma is valid, is the low frequency limit, when

$$\mathbf{v}_{Ti} << \omega/\mathbf{k} << \mathbf{v}_{Te} \tag{122}$$

The ions in this limit can be considered as a "cold" one, therefore for them the hydrodynamical description is valid ($\mathbf{v}_i = \mathbf{v}$, $N_i = N$)

$$\partial N/\partial t + \mathrm{div}\, N\mathbf{v} = 0 \tag{123}$$

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = (e_i/M)\{\mathbf{E} + (1/c)[\mathbf{v}\times\mathbf{B}]\}$$

As about electrons, in the limit (122), the Vlasov equation

$$\mathbf{v}\partial f_e/\partial \mathbf{r} + (e/m)\{\mathbf{E} + (1/c)[\ \mathbf{v}\times\mathbf{B}]\partial f_e/\partial \mathbf{p} = 0 \tag{124}$$

can be solved exactly.

Let us begin from unmagnetized electrons and purely potential field $\mathbf{E} = -\nabla\phi$. Then the solution of (124) can be presented as

$$f_e = (N_{e0}/(2\pi m T_e)^{3/2}) \exp(-mv^2/2T_e - e_i\phi/T_e) \tag{125}$$

From this follow

$$N_e = N_{e0}\exp(-e\phi/T_e),\ \ \nabla N_e/Ne = e\mathbf{E}/T_e = -e\nabla\phi/T_e \tag{126}$$

It must be noted that in this presentation we supposed T_e = const,which is the consequence of the right inequality (122).

Now we can write the system (123) in a purely hydrodynamical form (remind that B = 0)

$$\partial N/\partial t + \mathrm{div} N\mathbf{v} = 0 \tag{127}$$

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = -|e_i/e|\ \nabla N T_e/NM$$

If one introduces ρ = MN and P = NT_e, where T_e = const (isothermic approximation) then this system coincides with the one fluid hydrodynamics of usual liquid.

Quite analogically can be derived the one fluid MHD equations for nonisothermal , $T_e >> T_i$, and magnetized collisionless plasmas under the conditions

$$\mathbf{k}v_{T\alpha} \sim v_{T\alpha}/L_0 << \Omega_\alpha,\ \ \Omega_i << \omega_{Li} \tag{128}$$

This system of equations coincides with Alfven hydrodynamics of ideal liquid

$$\partial N/\partial t + \mathrm{div}\, N\mathbf{v} = 0 \tag{129}$$

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = -|e_i/e|\nabla N T_e/MN + (1/4\pi NM)[\ \mathbf{B}\times \mathrm{rot}\mathbf{B}]$$

$$\partial \mathbf{B}/\partial t + \mathrm{rot}\ [\mathbf{v}\times\mathbf{B}] = 0,\ \mathrm{div}\ \mathbf{B} = 0$$

Here $\rho = NM$, $P = NT_e$, $T_e >> T_i$ and T_e = const

This derivation of MHD equations was done only in 1956 by V. Silin and Y. Klimontovich. Only after the publication of their paper it becomes clear why the applications of these equations to the low frequency phenomena in the collisionless ionospheric plasma occured so successful. Quite analogically the above given derivation of equations for "cold" two fluids hydrodynamics clarifies the success of their application to the problems of fast radiowaves propagation in the ionospheric plasma.

LECTURE 3

Linear Electrodynamics of Isotropic Plasma

* 12 Linear electrodynamical properties of isotropic collisionless plasma

Below it will be shown that the Vlasov-Landau or Vlasov - Boltzman kinetic equations give the completely adequate descriptions of all properties of gaseous plasma. In this section we will begin from collisionless isotropic plasma and for this reason we will start from Vlasov equation

$$\partial f_\alpha/\partial t + \mathbf{v}\partial f_\alpha/\partial \mathbf{r} + e_\alpha/m_\alpha\{\mathbf{E} + 1/c[\mathbf{v}\times\mathbf{B}]\}\partial f_\alpha/\partial \mathbf{v} = 0 \quad (130)$$

where $\alpha = e,i$. For thermodynamically equilibrium plasma in the absence of external fields

$$f_{0\alpha} = (n_{0\alpha}/(2\pi m_\alpha T_\alpha)^{3/2}) \exp(-m_\alpha v^2/2T_\alpha) \quad (131)$$

Moreover we suppose that plasma is quasineutral

$$\Sigma_\alpha\, e n_{0\alpha} = e n_{0e} + e_i n_{0i} = 0 \quad (132)$$

Let now consider small deviation from $f_{0\alpha}$, or $f\alpha = f_{0\alpha} + \delta f\alpha$. Then for $\delta f\alpha$, we obtain

$$-i(\omega - \mathbf{kv})\, \delta f_\alpha = -e_\alpha\, E\, \partial f_{0\alpha}/\partial \mathbf{p} \quad (133)$$

Here we suppose that in linear approximation $\delta f\alpha \sim \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$. As a result we have

$$\delta f_\alpha = -(i\, e_\alpha \mathbf{E}\, \partial f_{0\alpha}/\partial \mathbf{p})/(\omega - \mathbf{k}\cdot\mathbf{v}) \quad (134)$$

By substituting this expression into the relation

$$j_i = {}_\alpha\Sigma e_\alpha \int v_i \delta f_\alpha\, d\mathbf{p} = \sigma_{ij}(\omega,\mathbf{k})\, E_j \quad (135)$$

we find the conductivity tensor σ_{ij} (ω,k) and then the tensor of dielectric permittivity

$$\varepsilon_{ij}(\omega,\mathbf{k}) = \delta_{ij} + (4\pi i/\omega)\sigma_{ij}(\omega,\mathbf{k}) = \delta_{ij} + \Sigma_\alpha (4\pi e_\alpha^2/\omega)\int d\mathbf{p}(\, v_i\, \partial f_{0\alpha}/\partial p_j)/(\omega - \mathbf{k}\cdot\mathbf{v}) \quad (136)$$

It is obvious that

$$\varepsilon_{ij}(\omega,k) = (\delta_{ij} - (k_i k_j/k^2)\varepsilon^{tr}(\omega,\mathbf{k}) + (k_i k_j/k^2)\varepsilon^{l}(\omega,\mathbf{k}) \quad (137)$$

where

$$\varepsilon^{tr}(\omega,\mathbf{k}) = 1 - \Sigma_\alpha(\omega_{L\alpha}{}^2/\omega^2)\,\Im(\omega/\mathbf{k}\mathbf{v}_{T\alpha}) \tag{138}$$

$$\varepsilon^{l}(\omega,\mathbf{k}) = 1 + \Sigma_\alpha(\omega_{L\alpha}{}^2/k^2 v_{T\alpha}{}^2)[\,1 - \Im(\omega/\mathbf{k}\mathbf{v}_{T\alpha})]$$

When we integrated (136) we took into account the causality principle and the pole $\omega = \mathbf{kv}$ was avoided in the sense of Landau (105). Namely as a result of such consideration it appears in (138) the function $\Im(X)$, which has not only real part for real X, but also imaginary part

$$\Im(X) = X\exp(-X^2/2)\ {}_{i\infty}\!\int^{X} d\tau\,\exp(\tau^2/2) = \begin{cases} 1 + 1/X^2 + \ldots\ldots - i\sqrt{\pi/2}\ e(-X^2/2)X & \text{when } |X| \gg 1 \\ -i\sqrt{(\pi/2)}X + X^2 & \text{when } |X| \ll 1 \end{cases} \tag{139}$$

The imaginary parts for Im $\varepsilon^{l,tr} > 0$ corresponds to the wave dissipation of small oscillations in plasma. This dissipation is stipulated by Cherenkov absorption.

Let us now investigate different limiting cases of ω and $\mathbf{k}$ and clarify the principal meaning of $\varepsilon^{l}(\omega,\mathbf{k})$ and $\varepsilon^{tr}(\omega,\mathbf{k})$. First of all let us consider low frequency (static) limit $\omega \to 0$. Then

$$\varepsilon^{l}(0,\mathbf{k}) = 1 + 1/k^2 r_D{}^2) \tag{140}$$

$$\lim_{\omega\to 0} \varepsilon^{tr}(\omega,\mathbf{k}) = 1 + i\sqrt{(\pi/2)}\omega_{Le}{}^2/\omega\mathbf{k}\mathbf{v}_{Te} = 1 + i(4\pi\sigma^{tr}(0,\mathbf{k})/\omega)$$

The first expression coincides with that obtained in the static limit from the model of two fluids hydrodynamics and corresponds to Debye screening of the field for static point charged particle in a plasma. So we see that the Vlasov equation gives the correct description of electrostatic properties of a collisionless plasmas, or for the fields $\mathbf{E} = -\nabla\Phi$.

More interesting phenomenon is described by the second expression (140). From this expression we see that the collisionless plasma in the static limit has the finite conductivity in connection with the transverse electric field, div $\mathbf{E} = 0$, which is the function of $\mathbf{k}$

$$\sigma^{tr}(0,\mathbf{k}) = (\sqrt{\pi/2})e^2 n_{0e}/m\mathbf{k}\mathbf{v}_{Te} \tag{141}$$

This conductivity is stipulated by the Cherenkov dissipation and leads to the anomalous skin-effect for quasistatic transverse fields in a plasma, the new phenomenon which arises only in collisionless plasmas. To show this let us write the material equation (Ohm's law) corresponding to (141) in the form

$$\text{rot}\mathbf{j} = -(\sqrt{\pi/2})e^2 n_{0e}\mathbf{E}/m\mathbf{v}_{Te} \tag{142}$$

Using this relation from the Maxwell equations can be obtained the following equation for $\mathbf{B}$

$$\text{rotrotrot}\mathbf{B} = (4\pi/c^2)(\sqrt{\pi/2})(e^2 n_{0e}/m\mathbf{v}_{Te})(\partial\mathbf{B}/\partial t) \tag{143}$$

This equation leads to the wellknown formula for anomalous skin-effect- penetration of low *frequency transverse field in collisionless plasmas (for $E \sim exp(i\omega t + i\mathbf{k}\cdot\mathbf{r})$)*

$$k^3 = i(\sqrt{\pi}/2)\omega_{Le}^2\omega/ c^2 v_{Te} \Rightarrow \lambda_{sk} = 1/\mathrm{Im}k \sim (c^2 v_{Te}/\omega\omega_{Le}^2)^{1/3} \tag{144}$$

Namely this formula was firstly obtained by A. Pippard in 1949 who also gives the physical explanation of the phenomenon. But the mathematically correct consideration of the boundary problem of field penetration into the collisionless plasma was done by E. Reuter and E. Sondheimer in 1958; they show that the anomalous skin effect takes place if

$$\omega < \mathbf{k}v_{Te} < \omega_{Le}.$$

Let us consider now the problem of waves propagation in collisionless isotropic plasma, or in other words, find the conditions for existence of nontrivial solutions of type $exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$ in such a plasma in the absence of external field sources. These conditions follow from the field equations which in the space $(\omega,\mathbf{k})$ look as

$$\{k^2\delta_{ij} - k_i k_j - (\omega^2/c^2)\varepsilon_{ij}(\omega,\mathbf{k})\}E_j = 0 \tag{145}$$

For the isotropic media with ε_{ij} as (137) this system separates into the two independent equations

$$E^l \varepsilon^l(\omega,\mathbf{k}) = 0 \tag{146}$$

$$E^{tr}[k^2c^2 - \omega^2\varepsilon^{tr}(\omega,\mathbf{k})]^2 = 0$$

The first describes the longitudinal field with $\mathbf{E} // \mathbf{k}$ and the condition for existence of non trivial solutions is

$$\varepsilon^l(\omega,\mathbf{k}) = 0 \tag{147}$$

This equation is called as dispersion equation for longitudinal waves in an isotropic plasma. Quite analogically from the second equation we obtain the dispersion equation for transverse $(\mathbf{E} \perp \mathbf{k})$ waves

$$k^2c^2 - \omega^2\varepsilon^{tr}(\omega,\mathbf{k}) = 0 \tag{148}$$

In the isotropic plasma this type of waves occurs twice degenerated.
Let us now consider very shortly the spectrum of electromagnetic waves in an isotropic plasma and analyze the solutions of the equations (147) and (148).

<u>1- Longitudinal Waves :</u>
a) Let us begin the analysis of (147) for the high frequency range, $\omega \gg \mathbf{k}\, v_{Te,i}$, when

$$\varepsilon^l(\omega,k) = 1 - (\omega_{Le}^2/\omega^2)(1+3(k^2v_{Te}^2)/\omega^2) + i\sqrt{(\pi/2)}(\omega_{Le}^2\omega/k^3v_{Te}^3)exp(-\omega^2/2k^2v_{Te}^2) \tag{149}$$

Then from (147) it follows ($\omega \to \omega + i\delta$)

$$\omega^2 = \omega_{Le}^2 + 3k^2 v_{Te}^2 \qquad (150)$$

$$\delta = -(\sqrt{\pi}/8)(\omega_{Le}/k^3 r_{De}^3)\exp(-3/2 - 1/2k^2 r_{De}^2)$$

These coincide with (103) and (106) as it should be expected. They describe high frequency plasma oscillations and their absorption due to the Cherenkov mechanism of waves absorption by plasma electrons. Wave damping increases with the increase of k and in the short wave range when $kr_{De} >> 1$, these waves occur aperiodically damping. This spectrum differs from that obtained by I. Langmuir by the factor 3 instead of 1 in $\omega(k)$

b) In the intermediate frequency range, when $v_{Ti} << \omega/k << v_{Te}$, we have

$$\varepsilon^l = 1 - \omega_{Li}^2/\omega^2 + \omega_{Le}^2/k^2 v_{Te}^2 (1 + i\sqrt{(\pi/2)}\,\omega/kv_{Te}) \qquad (151)$$

Substituting this expression into the equation (147) we obtain ($\omega \to \omega + i\delta$)

$$\omega^2 = \omega_{Li}^2/(1 + \omega_{Le}^2/k^2 v_{Te}^2) = \begin{cases} k^2 v_s^2 & \text{if } k^2 r_{De}^2 << 1 \\ \omega_{Li}^2 & \text{if } k^2 r_{De}^2 >> 1 \end{cases} \qquad (152)$$

$$\delta = -\sqrt{(\pi/8)|ei/e|\, m/M}\; \omega^4/k^3 v_s^3$$

Namely the long wave range of these oscillations

$$\omega = kv_s, \quad \delta = -\sqrt{(\pi/8)|ei/e|\, m/M}\; \omega \qquad (153)$$

was investigated by G.Gardeev in 1954 who showed that the frequency spectrum $\omega(k)$ differs from that obtained by I. Langmuir in the model of two fluids hydrodynamics by the dependence only on the electrons temperature T_e and nondependence on T_i. Moreover as $\omega >> kv_{Ti}$ the inequality $T_e >> T_i$ must take place. But under these conditions, the I. Langmuir result occurs to be correct. Besides G.Gardeev showed that these oscillations are damping, the reason of which is the Cherenkov absorption of ion-accoustic (just same as that called by I. Langmuir) oscillations by the plasma electrons. The frequency spectra (150) and (152) are presented on the Figure 4.

c) Finally if $\omega << kv_{Ti}$ then the first expression of (140) is valid and the Debye screening takes place.

2. Transverse Waves :

Let us now consider the transverse waves and analyse the equation (147)

a) In the high frequency range $\omega >> kv_{Te}$, when

$$\varepsilon^{tr}(\omega) = 1 - \omega_{Le}^2/\omega^2 \qquad (154)$$

from (148) follows

$$\omega^2 = \omega_{Le}^2 + k^2c^2 \tag{155}$$

We see that the phase velocity of waves is higher than the light speed. Therefore interaction of such waves with charged particles (emission or absorption) is impossible. As a result in a considered case of collisionless plasma they don't damp and besides the spectrum (155) exactly coincides with spectrum of transverse waves obtained in the model of independent particles in collisionless plasma (lim ve → 0). The spectrum is presented on the Figure 4.

b) Concerning low frequency range when $\omega \ll kv_{Te} = v_{Te}\omega_{Le}/c$ the expression ε^{tr} coincides with (140) corresponding to the anomalous skin effect considered above.

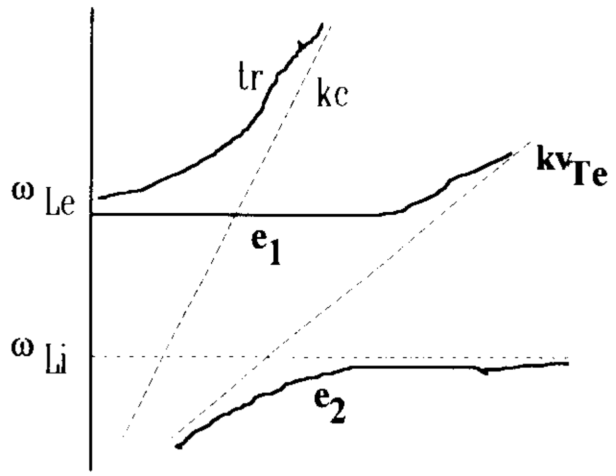

Figure 4

* 13 Collisions influence on the oscillations spectra of isotropic plasma

Below we will restrict ourselves by considering only the qualitative effects caused by the particles collisions in a plasma and BGK collision integral will be considered. Concerning the completely ionized plasma, only the corrections will be given. Thus we will start from the Vlasov-BGK equation

$$\partial f_\alpha/\partial t + \mathbf{v}\partial f_\alpha/\partial \mathbf{r} + e_\alpha\{\mathbf{E}+(1/c)[\mathbf{v}\times\mathbf{B}]\}\partial f_\alpha/\partial \mathbf{p} = -\nu_{\alpha 0}\,(f_\alpha - N_\alpha \phi_{\alpha 0}) \tag{156}$$

Here

$$f_{\alpha 0} = (1/(2m_\alpha T_\alpha)^{3/2})e(-m_\alpha v^2/2T_\alpha) \tag{157}$$

The equilibrium distribution as it is easily seen coincides with Maxwellien $f_{0\alpha} = N_{0\alpha}\phi_{\alpha 0}$. Therefore for a small perturbation of type $\delta f_\alpha \sim \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$ from (156) we obtain

$$-i(\omega - \mathbf{k}\cdot\mathbf{v})\delta f_\alpha + e_\alpha\,\mathbf{E}\partial f_\alpha/\partial \mathbf{p} = -\nu_{\alpha 0}\,(\,\partial f_\alpha - \phi_{\alpha 0} \textstyle\int d\mathbf{p}\,\delta f_\alpha) \tag{158}$$

Here for simplicity the isothermal model of BGK integral was used, that means T_α = const. The equation (158) is the Voltera type integral equation which can be easely solved. We omite the solution and give the final results - the expressions for $\varepsilon^l(\omega,k)$ and $\varepsilon^{tr}(\omega,k)$

(159)

$$\varepsilon^l(\omega,\mathbf{k}) = 1 + {}_\alpha\Sigma(\omega_{Le}{}^2/k^2 v_{T\alpha}{}^2) \times ((1-\Im((\omega+i\nu_{\alpha0})/\mathbf{kv}_{T\alpha}))/(1-(i\nu_{\alpha0}/(\omega+i\nu_{\alpha0})\ \Im((\omega+i\nu_{\alpha0})/\mathbf{kv}_{T\alpha})))$$

$$\varepsilon^{tr}(\omega,\mathbf{k}) = 1 - \Sigma_\alpha\ \omega_{Le}{}^2/(\omega(\omega+i\nu_{\alpha0})\ \Im(\omega + i\nu_{\alpha0})/(\mathbf{kv}_{T\alpha}))$$

As it should be expected for the collisionless plasma when $\nu_{\alpha0} \rightarrow 0$ these expressions coincide with (138).

Let us begin the analysis of the expressions (159) in the static limit when $\omega \rightarrow 0$. It is easy to show that independently of the ratio $\nu_{\alpha0}/kv_{T\alpha}$, we have

(160)

$$\varepsilon^l(0,\mathbf{k}) = 1+1/k^2 r_{De}{}^2$$

Thus in the static limit for collisional plasma as for collisionless one, we have Debye screening of potential fields.

Quite another situation arises for $\varepsilon^{tr}(\omega,k)$. The above result (140) corresponding to the anomalous skin-effect for low frequency transverse field in a plasma is correct for collisional plasma also if

(161)

$$\omega << \nu_e < v_{Te}\omega_{Le}/c$$

Remind that in the collisionless limit, $\nu_e << \omega$, the anomalous skin effect takes place in the frequency range $\omega < v_{Te}\ \omega_{Le}/c$.

In the opposite to (161) limit when $\nu_e > v_{Te}\omega_{Le}/c$ the anomalous skin effect is impossible. Then in the low frequency range, $\omega < \nu_e$, only wellknown normalous skin effect takes place and the formula

(162)

$$\varepsilon^{tr} = 1 + (i\omega_{Le}{}^2/\omega\nu_{e0}) = 1 + 4\pi i\sigma/\omega,\ \sigma = e^2 n_{0e}/m\nu_{e0}$$

obtained in our second lecture is valid*.

Let us now go over high frequency range and consider the particle collisions influence on high frequency phenomena. From the expression (159) for $\varepsilon^l(\omega,\mathbf{k})$ follows that if $\omega \sim \omega_{Le} >> \mathbf{kv}_{Te}, \nu_{e0}$ we find the correction to the expression (149)

(163)

$$\Delta\varepsilon^l = i\ \omega_{Le}{}^2\nu_e/\omega^3$$

Here we take into account also electron-ion collisions and therefore in the formula $\nu_e = \nu_{e0} + \nu_{eff}$. The correction (163) leads to the correction of Landau damping (150) of plasma high frequency oscillations

(164)

$$\Delta\delta = -\nu_e/2$$

Remind that for completely ionized plasma e= 1,96 e^2 n_{oe} / meff

Comparing this correction to (150) we can determine the condition when plasma should be considered as collisional in the high frequency range

(165)

$$\nu_e/\omega_{Le} > (\sqrt{\pi}\exp(-1/2k^2r_{De}^2))/(5{,}5k^3r_{De}^3)$$

In the opposite case when the oscillations are sufficiently short wave length, particles collisions can be neglected. Moreover the quantity (164) allows us to determine when plasma should be considered as completely ionized for high-frequency plasma oscillations

(166)

$$\nu_{eff}/\nu_{e0} = 2\cdot 10^{-5}\,(n_{0e}/n_0)/(ZL/a^2\,T_e^2) \sim Zn_{0e}10^{11}/n_0T_e^2 >> 1 \text{ where } Z = |e_i/e|$$

In the opposite case plasma is weakly ionized. For example if $T_e > 10^4$K then plasma is completely ionized if $n_{0e} > 10^{-3}\,n_0$. At the same time, if $T_e \sim 10^8$K (thermonuclear plasma) then even at $n_0 > 10^{-5}\,n_{0e}$ plasma occurs weakly ionized, the electron-neutral collisions exceeds the collisions between charged particles.
Quantitavely another situation takes place for low-frequency longitudinal oscillations when, $kv_{Ti} << \omega << kv_{Te}$, if in addition to these inequalities $\nu_i << \omega$ and $kv_{Te} > \omega$ are satisfied then the following collisional correction to the (151) arises

* Remind that for completely ionized plasma with e =-e, $\sigma = 1.96\, e^2 n_{0e}/m\nu_{eff}$

(167)

$$\Delta\varepsilon^l = i(\omega_{Li}^2/\omega^3)\cdot \begin{cases} \nu_{i0} & \text{for weakly ionized plasma} \\ 8\nu_{ii}k^2v_{Ti}^2/5\omega^3 & \text{for completely ionized plasma} \end{cases}$$

As a result we obtain the correction to the damping decrement (152)

(168)

$$\Delta\delta = - \begin{cases} \nu_{i0}/2 \\ 4\nu_{ii}k^2v_{Ti}^2/5\omega^2 \end{cases}$$

for weakly and completely ionized plasma correspondingly. Comparison of this expression with the damping decrement (152) leads to the following condition:
if $\nu_i/\omega > \sqrt{Zm/M}$, where

(169)

$$\nu_i = \begin{cases} -\nu_{i0} \\ \end{cases}$$

$- 8/5\ \nu_{ii}k^2v_{Ti}{}^2/\omega^2$

Then the particles collisions determine the low frequency waves absorption and if opposite inequalities take place the Cherenkov mechanism of absorption is dominative.
Besides from the ratio of two expressions (168) we determine the condition when plasma can be considered as weakly ionized for low frequency oscillations and vice versa. Thus in the case of long wave oscillations when the relations (153) are valid we conclude : if

$$n_0/n_{0e} > 10^{11}Z^2/T_eT_i \tag{170}$$

then the plasma should be considered as weakly ionized and as completely ionized in opposite case. For example if $T_e \sim 10^5$K, Ti ~ 10^3K, Z=1 then a plasma only with $n_0/\ n_{0e} > 10^3$ can be considered as weakly ionized.
In conclusion let us analyse the properties of strongly collisional plasma. But before we'll consider the expressions (159) in the limit $|\omega + i\nu_\alpha| >> kv_{T\alpha}$, where the model of two fluids hydrodynamics seems to be valid. From (159) under this condition follows

$$\varepsilon^l(\omega,\mathbf{k}) = 1-\Sigma_\alpha\omega_{L\alpha}{}^2/(\omega+i\nu_{\alpha 0})[\omega-i(k^2v_{Te}{}^2\nu_{\alpha 0})/(\omega+i\nu_{\alpha 0})^2] \tag{171}$$

$$\varepsilon^{tr}(\omega,\mathbf{k}) = 1 - \Sigma_\alpha\omega_{L\alpha}{}^2/(\omega(\omega+i\nu_{\alpha 0}))$$

Comparing these expressions with (38) we conclude that ε^{tr} (ω,k) is identical whereas $\varepsilon^l(\omega,k)$ differs by the factor $\nu_{\alpha n}/(\omega+i\nu_{\alpha n})$ of the term taking into account the thermal motion of particles.
This difference is very principal. Moreover the correctness of the expressions (171) are defined by the condition $|\omega + i\nu_{\alpha,\alpha}| >> kv_{Te}$ and therefore the expressions (38) are correct in two cases: when $\nu_{\alpha 0} >> \omega$ and the mentioned factor becomes equal to unity, or when $\omega >> \nu_{\alpha 0}$, $kv_{T\alpha}$ and the thermal motions of particles is a small correction. In the first case

$$\varepsilon^l(\omega,\mathbf{k}) = 1-\Sigma_\alpha\omega_{Li}{}^2/(i\omega\nu_\alpha-k^2v_{T\alpha}{}^2) \tag{172}$$

This expression coincides with (41) which describes the diffusion processes in a plasma (see lecture 3). In the opposite limit when $\omega >> \nu_{\alpha 0}$, $kv_{T\alpha}$ thermal motions of particles can be neglected and the model of independent particles considered in the lecture 2 is valid.

Linear Electrodynamics of Magnetoactive Plasma

<u>* 14 Linear electromagnetic properties of collisionless magnetoactive plasma</u>

A magnetoactive plasma represents a system with practically infinite number of degrees of freedom. In a magnetized plasma there exist different types and different branches of

oscillations and waves. It is obvious that the investigation of all these oscillations in detail is impossible in one lecture.
Therefore we will restric ourselves by the consideration of only the most specific phenomena which characterized magnetoactive plasma and which every physicist must know.
First of all it must be noted that the charged particles rotate around the magnetic field lines. This rotation can be considered as individual oscillations of particles with frequency equal to the well known Larmor frequency $e_\alpha B_0/m_\alpha c = \Omega_\alpha$. It is easy to understand that if the electromagnetic field frequency is $\omega \sim n\Omega_\alpha$ then the resonance interaction between the field and charged particle must take place. As a result the dielectric permittivity ε_{ij} (ω, k), its hermitian as well as antihermitian parts (which describes energy absorption) have the poles at $\omega \sim n\Omega_\alpha$. In the lecture 2 we showed that such poles arise in the model of independent particles, but only for $n = \pm 1$. Below we will show that the kinetic consideration leads also to the appearence of the poles at $n \neq 1$.
The second obvious phenomenon which arises in a magnetoactive plasma is the magnetic pressure $B_0^2/8\pi$ which follows from the elasticity of magnetic field lines. We have already met this phenomenon in the model of independent particles and namely it is the reason of Alfven type oscillation. Of course this phenomenon exists in two fluids hydrodynamics as well as one fluid (Alfven) MHD, and below it plays a very important role also in a kinetic theory of plasma oscillations.
Finally it is easy to understand that the behaviour of magnetoactive plasma at $\omega = n\Omega_\alpha$ must be somewhat like to the behaviour of isotropic plasma at $\omega \rightarrow 0$, because the Larmor rotation remind the Doppler shift for electromagnetic fields. Below this will be shown by considering the field penetration (skin-effect) into the magnetoactive plasma.
As in the previous section we begin from collisionless plasma described by the Vlasov equation (130). From this equation we obtain the equation for equilibrium distribution function $f_0(\mathbf{p})$ (external magnetic field $\mathbf{B_0}$ is proposed to be parallel to $\mathbf{OZ}$ axis)

$$e[\mathbf{v}\times\mathbf{B_0}]\, \partial f_{0\alpha}/\partial \mathbf{p} = -\Omega_\alpha\, \partial f_{0\alpha}/\partial\varphi = 0 \tag{173}$$

Here φ is the angle in the cylindrical frame : v_z , $v_x = v_\perp \cos\varphi$, $v_y = v_\perp \sin\varphi$. The solution of this equation we have choosed as

$$f_{0\alpha} = (N_{0\alpha}/(2\pi m_\alpha T_\alpha)^{3/2}) \exp(-m_\alpha v^2/2T_\alpha) \tag{174}$$

Besides, it was supposed that the plasma is quasi neutral, or $\sum_\alpha e_\alpha N_{0\alpha} = 0$.
*The detail consideration of linear electromagnetic phenomena in magnetoactive plasma, can be found in many text book on plasma physics.
For a small deviation from $f_{0\alpha}$ which is taken as $\delta f_\alpha \sim e^{-i\omega t + i\mathbf{kr}}$ we obtain

$$-i(\omega - \mathbf{k}\cdot\mathbf{v})\delta f_\alpha - \Omega_\alpha \partial\delta f_\alpha/\partial\varphi = -e_\alpha \mathbf{E}\partial f_{0\alpha}/\partial\mathbf{p} \tag{175}$$

Taking into account the obvious condition of periodicity

$$\delta f_\alpha(\varphi + 2\pi) = \delta f_\alpha(\varphi) \tag{176}$$

the general solution of (175) can be written as

$$\delta f_\alpha = e_\alpha/\Omega_\alpha \ {}_\infty\int^{\varphi} d\varphi' \ \mathbf{E}(\partial f_{0\alpha}/\partial \mathbf{p}) \exp[i/\Omega_\alpha \ {}_\varphi\int^{\varphi'} d\varphi''(\omega - \mathbf{k}\cdot\mathbf{v})_{\varphi''}] \tag{177}$$

Here we suppose that $\delta f_\alpha(\infty) = 0$

Substituting the expression (177) into the formula for induced current (135) we find plasma conductivity and then the dielectric permittivity

$$\varepsilon_{ij}(\omega,\mathbf{k}) = \delta_{ij} + (4\pi i/\omega)\,\sigma_{ij}(\omega,\mathbf{k}) \tag{178}$$

$$= \delta_{ij} + \Sigma_\alpha (4\pi e_\alpha{}^2/\omega\Omega_\alpha) \int d\mathbf{p}\ \partial f_0/\partial\varepsilon_\alpha v_i \ {}_0\int^{\infty} d\varphi' v_j(\varphi') \exp(-i/\Omega_\alpha \ {}_\varphi\int^{\varphi'} d\varphi''(\omega - \mathbf{k}\ \mathbf{v}_{\varphi''}))$$

It can be easily shown that this tensor has 6 independent components: $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy} = -\varepsilon_{yx}$, $\varepsilon_{zy} = -\varepsilon_{yz}$, $\varepsilon_{xz} = \varepsilon_{zx}$ (remind that in the case of isotropic plasma we have only two components ε^{tr} and ε^{l}). However below in general we will consider only two of them. The first quantity

$$\varepsilon_\perp(\omega,\mathbf{k}) = \varepsilon_{xx} \pm i\varepsilon_{xy} = 1 + {}_\alpha\Sigma(2\pi e_\alpha{}^2/\omega)\int (\partial f_{0\alpha}/\partial\varepsilon_\alpha)(v_\perp{}^2/(\omega \pm \Omega_\alpha - k_z v_z))d\mathbf{p} \tag{179}$$

$$= 1 \pm {}_\alpha\Sigma(\omega_{L\alpha}{}^2/\omega(\omega\pm\Omega_\alpha))\Im((\omega\pm\Omega_\alpha)/k_z v_{T\alpha})$$

describes the purely transverse field ($\mathbf{E} \perp \mathbf{k}$) depending only on the parallel coordinates ($k_\perp = 0$, $k_{//} \neq 0$). Whereas the second one

$$\varepsilon(\omega,\mathbf{k}) = k_i k_j \varepsilon_{ij}(\omega,\mathbf{k})/k^2 = 1 - \Sigma_\alpha(4\pi e_\alpha{}^2/k^2)\int d\mathbf{p}\,\partial f_{0\alpha}/\partial\varepsilon_\alpha[1 - \Sigma_n\ \omega \Im_n{}^2(b_\alpha)/(\omega - n\Omega_\alpha - k_z v_z] \tag{180}$$

$$= 1 + \Sigma_\alpha\ \omega_{L\alpha}{}^2/k^2 v_{T\alpha}{}^2[1 - \Sigma_n\ (\omega/(\omega - n\Omega_\alpha))A_n(z_\alpha)\Im((\omega - n\Omega_\alpha)/k_z v_{T\alpha})]$$

describes purely longitudinal field ($\mathbf{E} = -\nabla\phi$, $\mathbf{E}\ //\ \mathbf{k}$) arbitrarily depending on coordinates ($k_\perp \neq 0$, $k_{//} \neq 0$). Here $b_\alpha = k_\perp v_\perp/\Omega_\alpha$, $Z_\alpha = k_\perp{}^2\ v_{T\alpha}{}^2/\Omega_\alpha{}^2$ and $A_n(z) = I_n(z)\exp(-z)$, $I_n(z)$ is the Bessel function.

First of all it must be noted that from the expressions (179) and (180) follows that their poles really correspond to the one particle cyclotron resonances at

$$\omega = n\Omega_\alpha \tag{181}$$

These poles describe one particle oscillations and therefore in the ranges of these frequencies the resonance waves absorption must arise. Indeed from the integrand (179) and (180) we see that under the conditions

$$\omega - n\Omega_\alpha - k_z v_z = 0 \tag{182}$$

the resonance wave absorption takes place. It follows from the Landau prescription that

(183)

$$1/(\omega - k_z v_z - n\Omega_\alpha) = (\wp/(\omega - k_z v_z - n\Omega_\alpha)) - i\pi\delta(\omega - k_z v_z - n\Omega_\alpha)$$

At n=0 the absorption is coming from the Cherenkov mechanism considered in the previous section, whereas the absorption at $n\neq 0$ is known as cyclotron absorption. At the same time, the last one can be treated also as Cherenkov absorption taking into account the Doppler shift $n\Omega_\alpha$ stipulated to the one particle oscillations.

More obvious the Doppler shift is seen from the character of waves propagation described by the dispersion equation

$$| k^2\delta_{ij} - k_i k_j - \omega^2\varepsilon_{ij}(\omega,\mathbf{k})| = 0 \tag{184}$$

For purely longitudinal propagation when $k_\perp = 0$ this equation takes the form

$$k^2c^2 = \omega^2\varepsilon_\perp \tag{185}$$

which corresponds to the purely transverse waves. In the frequency range $\omega\pm\Omega_e << k_z V_{Te}$ from this equation in taking into account (179) we obtain *

$$k^2c^2 = i(\omega_{Le}{}^2\omega/kv_{Te})\sqrt{\pi/2} \tag{186}$$

This equation coincides exactly with (144) and describes the anomalous skin-effect. The penetration depth obviously coincides with (144)

$$\lambda_{sk} = (\Im mk)^{-1} = ((\sqrt{\pi/2})\omega_{Le}{}^2\omega/c^2v_{Te})^{-1/3} \tag{187}$$

At the same time we can rewrite these relations in the language of frequency spectrum, near the cyclotron frequencies when $(\omega\pm\Omega_e) << k_z v_{Te}$ from (185) we obtain

$$\omega = -i(k^3c^2v_{Te}/\omega_{Le}{}^2)\sqrt{2/\pi} \tag{188}$$

We can say that near the cyclotron frequencies, there exist cyclotron waves which intensively are absorbed by the plasma electrons. When the frequency is shifted, this absorption decreases and in the frequency range far from the resonance frequency, when $\omega >> | \omega \pm \Omega_e| >> k_z v_{Te}$ it becomes exponentially weak. Then from (179) we have

$$k^2c^2 = -(\omega_{Le}{}^2\omega/(\omega\pm\Omega_e))[1 - i\sqrt{\pi/2}((\omega\pm\Omega_e)/kv_{Te})e^{-((\omega\pm\Omega_e)^2/2k^2v_{Te}{}^2)} \tag{189}$$

From this equation we find the spectrum of cyclotron waves $(\omega + i\delta)$

$$\omega = \pm\Omega_e - \omega_{Le}{}^2\omega/k^2c^2,\ \delta = -\sqrt{\pi/2}(\omega_{Le}{}^2\omega/k^2c^2)(1/kv_{Te})\exp-((\omega\pm\Omega e)^2/2k^2v_{Te}{}^2) \tag{190}$$

*For simplicity we consider the waves only near electron cyclotron frequencies.

Sometimes these relations are presented in the optical language for the reflection index and absorption coefficient ($k=\omega n/c$, $n \to n+i\chi$). If $|\omega \pm \Omega_e| >> n\omega v_{Te}/c$, then

(191)

$$n^2 = \omega_{Le}^2/\omega(\omega\pm\Omega_e),\ \chi = (\sqrt{\pi}/8)(\omega_{Le}^2 c/\omega^2 v_{Te} n^2)\exp-((\omega\pm\Omega_e)^2 c^2/2n^2\omega^2 v_{Te}^2)$$

In the opposite limit, when $|\omega\pm\Omega_e| << n\omega V_{Te}/c$, this quantity occurs to be essentially complex

(192)

$$n^3 = i(\sqrt{\pi}/2)\omega_{Le}^2 c/\omega^2 v_{Te}$$

On the Figure 5 the dependence of complex reflection index n on the frequency shift $\omega-\Omega_e$ is presented

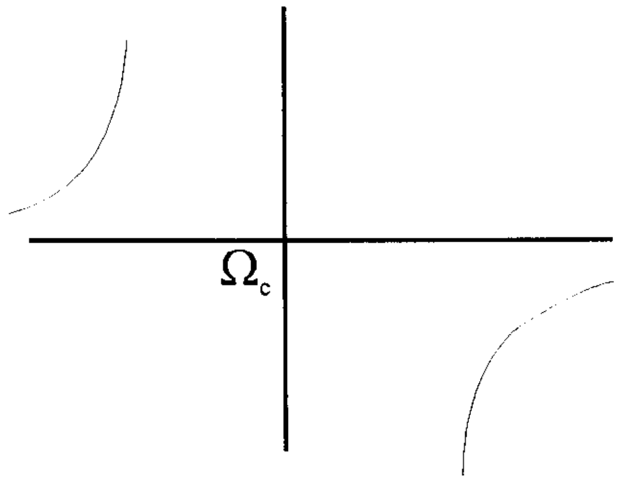

Figure 5

Let us now pass to the quasi longitudinal oscillations of magnetoactive plasma which exist under the conditions of $|\omega - n\Omega_\alpha| << kc$, or they can be considered as slow waves (in taking into account Doppler shift). In the statical case, $\omega \to 0$, from the expression (180) follows

(193)

$$\varepsilon(0,\mathbf{k}) = 1 + 1/(k^2 r_D^2),$$

which means that the electrostatic field of a point charged particle in magnetoactive plasma as well as in isotropic one is screened and the field penetration length is equal to the Debye length.

From the expression (180) it follows that the one particle cyclotron resonances at $\omega = n\Omega_\alpha$ take place for the longitudinal fields also. As a result near the cyclotron frequencies there exist the longitudinal cyclotron waves. Below we will show this for the purely electron plasma and transverse propagation of waves ($k_z = 0$). The dispersion equation for such waves looks as

(194)

$$\varepsilon = 1 + (\omega_{Le}^2/k^2 v_{Te}^2)[1 - {}_n\Sigma(\omega/(\omega - n\Omega_e))A_n(k^2 v_{Te}^2/\Omega_e^2)] =$$

$$1 - 2\ {}_{n=1}\Sigma^\infty(\omega_{Le}^2 n^2 \Omega_e^2/(k^2 v_{Te}^2(\omega^2 - k^2\Omega_e^2)))A_n(k^2 v_{Te}^2/\Omega_e^2) = 0$$

The solutions of this equation are known as Bernstein oscillations in honour of I. Bernstein who theoritically predicted them in 1959. They are presented on the Figure 6.
In conclusion of this section we will discuss very shortly the another branches of electromagnetic waves of magnetoactive plasma which every physicist must know. In the first turn let consider the oscillations of "cold" plasma, or in other words, the limiting case when

$$(\omega - n\Omega_\alpha)/k_z v_{T\alpha} >> 1,\ k_\perp^2 v_{T\alpha}^2/\Omega_\alpha^2 << 1 \qquad (195)$$

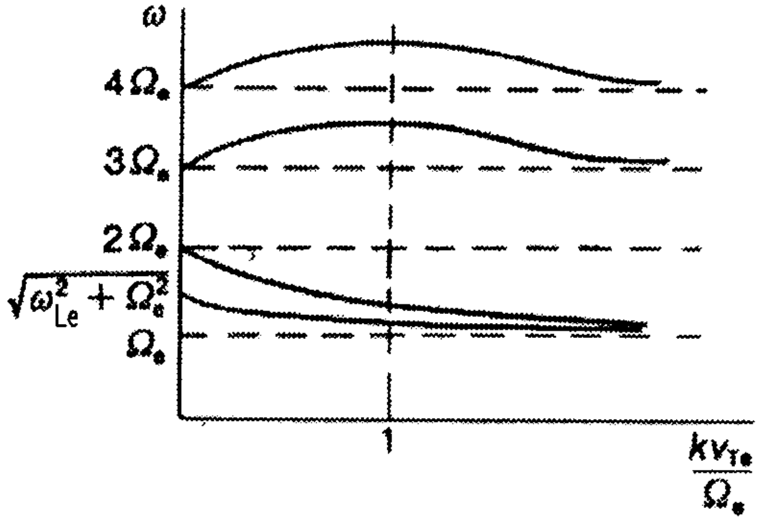

Figure 6

In accordance of these inequalities the phase velocities of waves in taking into account Doppler shift are supposed much larger than the thermal velocities of particles. The dielectric tensor (178) in this limit coincides with the well-known expression considered in the lecture 2 when the model of independent particles was discussed (of course for $v_\alpha \to 0$)

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_\perp & ig & 0 \\ -ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{//} \end{pmatrix} \qquad (196)$$

$$\varepsilon_\perp = 1 - {}_\alpha\Sigma\omega_{L\alpha}^2/(\omega^2 - \Omega_\alpha^2),\ g = \Sigma_\alpha\, \omega_{L\alpha}^2\, \Omega_\alpha/(\omega(\omega^2 - \Omega_\alpha^2)),\ \varepsilon_{//} = 1 - \Sigma\omega_{L\alpha}^2/\omega^2$$

In the lecture 2 the waves described by this tensor were investigated. By this reason here we will restrict only to the statement that inequalities (195) represent the validity of the results of these investigations and more generally the validity of the independent particles model for describing the properties of magnetoactive plasma.
The another simplest model which was considered in the lecture 3 is the Alfven one fluid MHD model for describing nonisothermal $T_e >> T_i$ magnetoactive plasma. It can be easely shown that this model is working under the condition

$$\omega^2 << \Omega_i^2 << \omega_{Li}^2,\ k_\perp^2\, v_{T\alpha}^2/\Omega_\alpha^2 << 1,\ v_{Ti} << \omega/k_z << v_{Te} \qquad (197)$$

Then the tensor (178) looks as

(198)

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & \varepsilon_{yz} \\ 0 & \varepsilon_{vz} & \varepsilon_{zz} \end{pmatrix}$$

$\varepsilon_{yy} = \varepsilon_{xx} = \omega_{Li}{}^2/\Omega i^2$, $\varepsilon_{zy} = -\varepsilon_{yz} = i\omega_{Le}{}^2 k_\perp/k_z\, \omega\Omega_e$, $\varepsilon_{zz} = -\omega_{Li}{}^2/\omega^2 + \omega_{Le}{}^2/k_z{}^2 v_{Te}{}^2$

It must be noted that here we completely neglect the dissipative processes due to the Cherenkov mechanism of wave absorption. Only in this case the expressions (198) provide exact correspondence to the ideal MHD model considered in the Lecture 3.

* 15 Influence of particles collisions on the properties of magnetoactive plasma

Passing to the collisions of particles we wish to notice that as above we will restrict ourselves by considering only a weakly ionized plasma and therefore only the Vlasov equation with BGK collision integral will be solved. This equation for a small perturbation of distribution function looks as

(199)

$$+(\omega - \mathbf{k}\cdot\mathbf{v})\delta f_\alpha + ie_\alpha \mathbf{E}\partial f_{0\alpha}/\partial \mathbf{p} - i\Omega_\alpha \partial\delta f_\alpha/\partial\varphi = -i\nu_\alpha(\delta f_{\alpha 0} - f_{\alpha n}\textstyle\int d\mathbf{p}\delta f_\alpha)$$

$$f_{\alpha 0} = (1/(2\pi m_\alpha T_\alpha)^{3/2})\exp(-m_\alpha v^2/2T_\alpha$$

This Voltera type integral equation can be easely solved. We will present here the result of the solution and calculation of the dielectric tensor. Moreover as above we will write only $\varepsilon_\perp$ and ε. The quantity $\varepsilon_\perp$ is equal

(200)

$$\varepsilon_\perp(\omega,\mathbf{k}) = 1 + (\Sigma_\alpha [(\omega+i\nu_\alpha)/\omega][\varepsilon_{xx}{}^{(\alpha)} - 1 \pm i\varepsilon_{xy}{}^{(\alpha)}] =$$

$$1 - {}_\alpha\Sigma(\omega_{L\alpha}{}^2/\omega\ [(\omega+i\nu_{\alpha 0})\pm\Omega_\alpha]\Im((\omega\pm\Omega_\alpha+i\nu_{\alpha 0})/k_z v_{T\alpha})$$

Here $\varepsilon_{xx}{}^\alpha$ and $\varepsilon_{xy}{}^\alpha$ are the components of dielectric tensor of collisionless plasma with changing $\omega \to \omega + i\nu_{\alpha 0}$.

This quantity describes the purely transverse fields depending only on z (or $k_\perp = 0$, $k_z \neq 0$). For the quantity $\varepsilon(\omega,\mathbf{k})$ which describes purely potential field ($\mathbf{E} = -\nabla\phi$) we have ($k_\perp \neq 0$, $k_z \neq 0$)

(201)

$$\varepsilon(\omega,\mathbf{k}) = 1 + {}_\alpha\Sigma(\omega_{L\alpha}{}^2/k^2 v_{T\alpha}{}^2) \times$$

$$\{1 - {}_n\Sigma(\omega+i\nu_{\alpha 0}/(\omega+i\nu_{\alpha 0}-n\Omega_\alpha))A_n(k_\perp{}^2 v_{T\alpha}{}^2/\Omega_\alpha{}^2)\Im((\omega+i\nu_{\alpha 0}-n\Omega_\alpha)/k_z v_{Te})\} \times$$

$$\{1 - {}_n\Sigma i\nu_{\alpha 0}/(\omega+i\nu_{\alpha 0} - n\Omega_\alpha)\Im((\omega+i\nu_{\alpha 0}-n\Omega_\alpha)/k_z v_{Te})\}^{-1}$$

As in the previous section let us consider the transverse field behaviour near the electron cyclotron frequency, $|\omega - n\Omega_e| << \omega$. Substituting the expression (200) into the equation (185), we obtain that, if $\nu_e << k_z v_{Te} \sim v_{Te}/\lambda_{sk}$ (collisionless plasma), λ_{sk} is given by the relation (187).

However if the opposite inequality takes place (collisional plasma), from (185) we obtain the normalous skin-effect for field penetration

(202)

$$\lambda_{sk} = 1/\Im mk \sim (c^2 \nu_{e0}/\omega\omega_{Le}^2)^{1/2}$$

Quite analogical to (188) we can write this relation in spectral representation

(203)

$$\omega = -ik^2c^2\nu_{e0}/\omega_{Le}^2$$

In conclusion of this section let us consider the behaviour of potential field in a collisional magnetoactive plasma, described by the expression (201). In the static limit, when $\omega \to 0$, independently from the ratio $\nu_{\alpha 0}/kv_{T\alpha}$ we have

(204)

$$\varepsilon(0,\mathbf{k}) = 1+1/k^2r_D^2$$

This means that Debye screening of the potential field of static point charged particles takes place as it was shown for isotropic plasma above.
Analogical to the isotropic plasma can be considered the problem of particles diffusion also. For this aim the expression (201) must be written in the limit ω, $kv_{T\alpha} << \nu_{\alpha 0}$, Ω_α

(205)

$$\varepsilon(\omega,\mathbf{k}) = 1+i_\alpha\Sigma\omega_{L\alpha}^2/k^2v_{T\alpha}^2(((k_\perp^2v_{T\alpha}^2\nu_{\alpha 0})/(\nu_{\alpha 0}^2+\Omega_\alpha^2))+k_z^2v_{T\alpha}^2/\nu_{\alpha 0}) \times$$

$$(\omega + i\nu_{\alpha 0}(k_\perp^2v_{T\alpha}^2/(\Omega_\alpha^2+\nu_{\alpha 0}^2) + k_z^2v_{T\alpha}^2/\nu_{\alpha 0}))^{-1}$$

The poles of this expression describe the monopolar diffusions of electrons and ions in a rare plasma ($\omega_{L\alpha}^2 << k^2v_{T\alpha}^2$)

(206)

$$\partial N_\alpha/\partial t - D_{\perp\alpha}\Delta_\perp N_\alpha - D_{//\alpha}\partial^2N_\alpha/\partial z^2 = 0$$

Here $D_{\perp\alpha} = v_{T\alpha}^2\nu_{\alpha 0}/(\Omega_\alpha^2+\nu_\alpha^2)$, $D_{//\alpha} = v_{T\alpha}^2/\nu_{\alpha 0}$

represent the transverse and longitudinal monopolar diffusion coefficients correspondingly for α = e,i. At the same time in the opposite limit of dense plasma when $k^2v_{T\alpha}^2 << \omega_{L\alpha}^2$ the equation $\varepsilon(\omega,\mathbf{k}) = 0$ describes the ambipolar diffusion of particles in the magnetoactive plasma

(208)

$$\partial N/\partial t - D_{\perp a}\Delta_\perp N - D_{//a}\partial^2N/\partial z^2 = 0$$

Therefore the coefficients of ambipolar diffusion are equal to

(209)

$$D_{\perp a} = (\nu_{e0}(v_{Ti}^2+v_s^2)/((\nu_{e0}\nu_{i0}+(\Omega_e\Omega_i)),\ D_{//a} = (v_{Ti}^2+v_s^2)/\nu_{i0}$$

It can be easily confirmed that there is the well-known Einstein relation between the static partial conductivities of plasma particles and diffusion coefficients (α = e,i)

(210)

$$D_{ij\alpha} = (T_\alpha / e_\alpha{}^2 N_\alpha)\sigma_{ij\alpha}(0)$$

This relation is correct only in static limit and only in thermodynamic equilibrium. Then

$$\sigma_{\perp\alpha}(0) = e_\alpha{}^2 N_\alpha \nu_{\alpha 0}/m_\alpha(\Omega_\alpha{}^2+\nu_{\alpha 0}{}^2),\ \sigma_{//\ \alpha}(0)=e_\alpha{}^2 N_\alpha/m_\alpha \nu_{\alpha 0} \quad (211)$$

represents the transverse and longitudinal partial conductivities of charged particles of type α=e,i.