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RESONANT INTERACTION BETWEEN RELATIVISTIC ELECTRONS AND WHISTLER-MODE WAVES

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These are lecture notes, intended for distribution to participants.

Resonant Interaction Between Relativistic Electrons and Whistler-mode Waves

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① Changes in Resonant Condition at Relativistic Energies

② Modification of Wave Growth.

③ Resonant Diffusion Surfaces

Scattering loss
Stochastic Acceleration

Wave Dispersion Relation for a Cold Plasma:

<u>For field aligned electromagnetic waves</u> ; $\psi = 0$

$$\left(\frac{k^2c^2}{\omega^2}\right)_{\substack{L\\R}} = 1 - \frac{\omega_{pe}^2}{\omega(\omega \pm |\Omega_e|)} - \sum_i \frac{\omega_{pi}^2}{\omega(\omega \mp \Omega_i)}$$

<u>Whistler-Mode</u> : "R" ; $\Omega_i \ll \omega < |\Omega_e|$

$$\boxed{\frac{u_{ph}^2}{c^2} = \frac{\omega^2}{k^2c^2} \approx \frac{\omega|\Omega_e|}{\omega_{pe}^2}\left\{1 - \frac{\omega}{|\Omega_e|}\right\}} \quad ; \quad \psi = 0$$

Normalize $u = \frac{u_{ph}}{c}$; $\hat{\omega} = \frac{\omega}{|\Omega_e|}$; $\boxed{u^2 = \alpha\,\hat{\omega}(1-\hat{\omega})}$

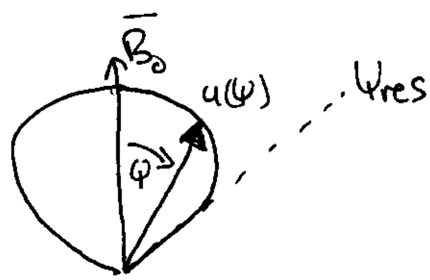
$$\frac{|\Omega_e|^2}{\omega_{pe}^2} \equiv \boxed{\alpha = \frac{B^2/4\pi N}{m_e c^2} \equiv \left(\frac{c_A}{c}\right)^2\left(\frac{m_p}{m_e}\right)}$$ Scaling Parameter

Typical values of $B^2/4\pi N$ ~ few keV -> few MeV.

$$\boxed{10^{-2} \leq \alpha \leq 10}$$ within the Earth's magnetosphere

<u>Oblique Whistlers</u> $\psi \neq 0$

$$\boxed{\hat{u}^2 \approx \alpha\,\hat{\omega}\,(\cos\psi - \hat{\omega})}$$



for $\omega > \omega_{LHR}$

Whistler Dispersion Relation in a Hot Plasma

$$\boxed{K^2 = \frac{\omega^2}{c^2} + \sum_{\text{species}}{}' \frac{\omega\,\omega_{p\sigma}^2}{2c^2}\int \frac{p_\perp \hat{H} f_\sigma(p_\parallel,p_\perp)\,d^3p}{\gamma\omega + \Omega_\sigma - k p_\parallel}}$$

$$\Omega_\sigma = \frac{q_\sigma B}{m_\sigma c} \quad ; \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad ; \quad \hat{H} = \frac{\partial}{\partial p_\perp} + \frac{k}{\gamma\omega}\left(p_\perp \frac{\partial}{\partial p_\parallel} - p_\parallel \frac{\partial}{\partial p_\perp}\right)$$

e.g. Montgomery and Tidman, Plasma Kinetic Theory, 1964

Resonant Energy Transfer between Waves-Particles

$$\boxed{p_\parallel = \frac{\gamma\omega - |\Omega_e|}{k}}$$

for electron-whistler interaction

Alternatively

$$\boxed{p_\parallel = \frac{\gamma\omega + |\Omega_e|}{k}}$$

for electron - Lmode resonance

R-Mode Wave Dispersion Relationship : $U=0$

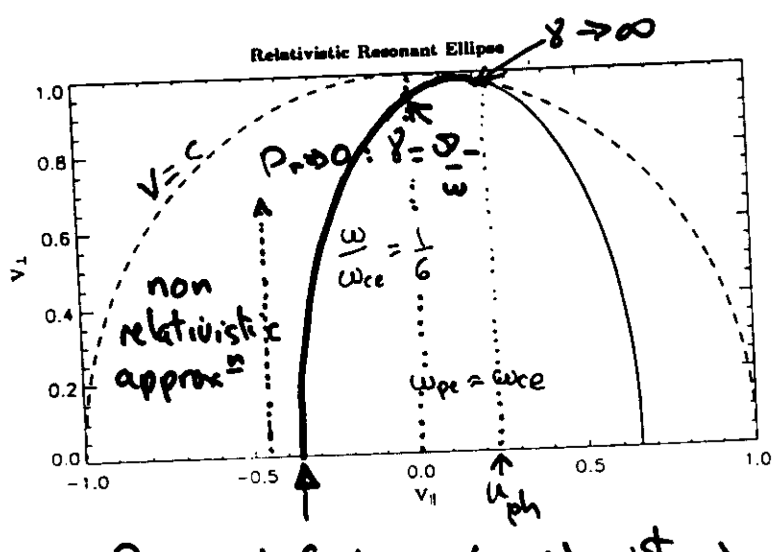
$$c^2k^2 = \omega^2 + 2\pi\omega_{pe}^2 \int_{-\infty}^{\infty} dp_{\parallel} \int_0^{\infty} dp_{\perp} \frac{p_\perp^2 \hat{G} f}{\gamma\omega - \Omega_e - kp_{\parallel}}$$

$$\hat{G} = \omega\frac{\partial}{\partial p_\perp} + \frac{k}{\gamma}(p_\perp \frac{\partial}{\partial p_\parallel} - p_\parallel \frac{\partial}{\partial p_\perp}) = \omega\frac{\partial}{\partial p_\perp} + k\hat{G}_r$$

Resonant Condition : $p_\parallel = \gamma m v_\parallel = \boxed{\frac{\gamma\omega - \Omega_e}{k} = p_r}$

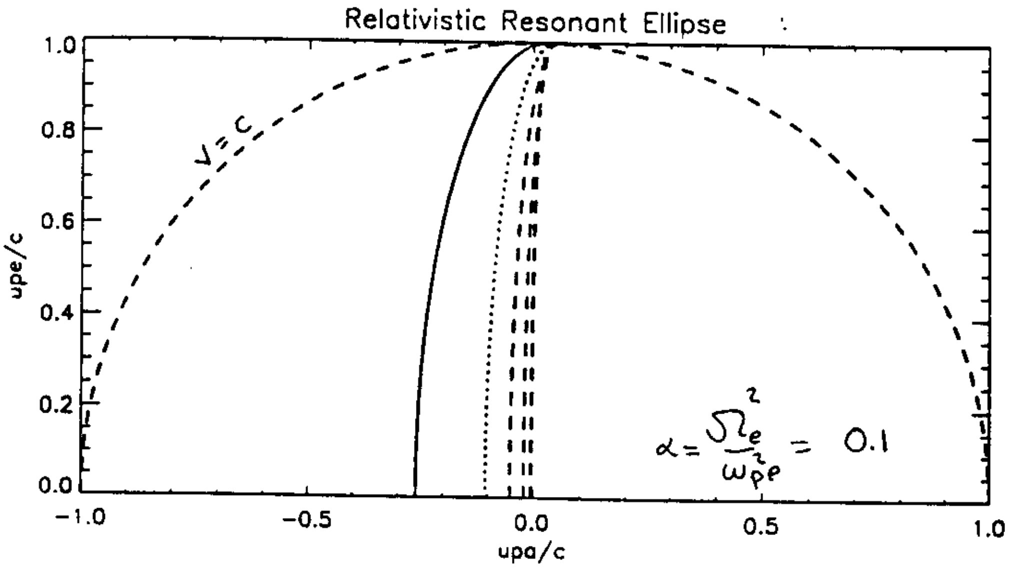
$\gamma = (1 - v^2/c^2)^{-1/2}$

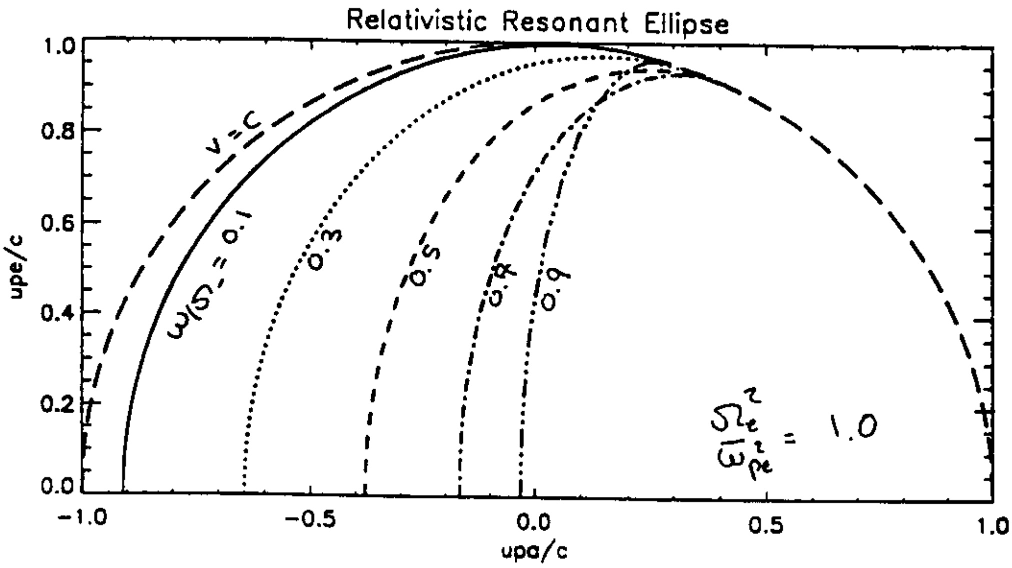
$\frac{v_r}{c} = \left(\frac{\omega}{kc}\right)\left[1 - \frac{\Omega_e}{\omega\gamma}\right]$: An ellipse in velocity space

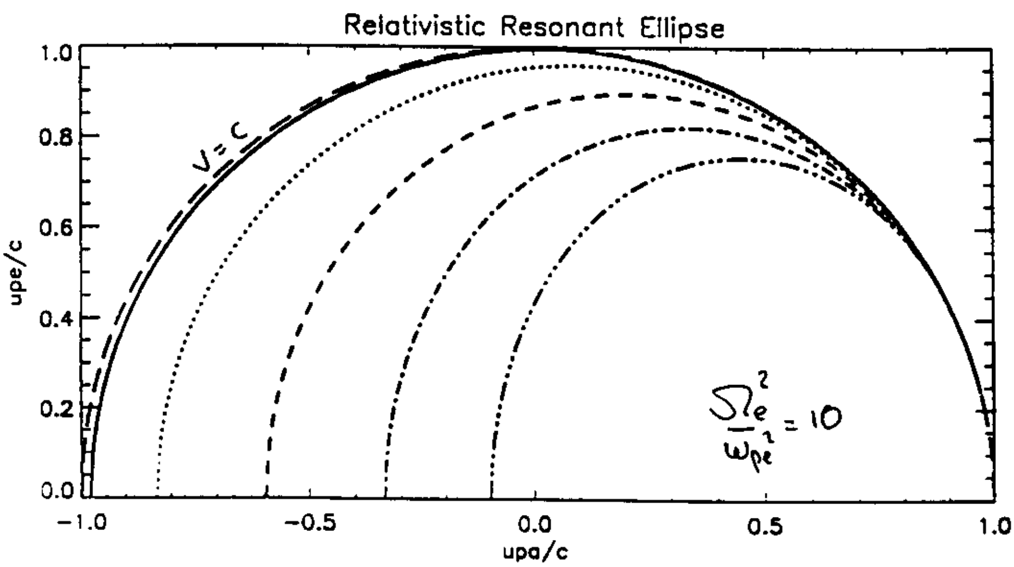


Resonant Contours ($n=1$) 1st order cyclotron

Relativistic effects
(a) Lower V_r near loss cone ($p_\perp \to 0$)
(b) Allow resonace at $p_\parallel = 0$; $\gamma = \frac{\Omega_-}{\omega}$
(c) Allow resonance for $p_\parallel > 0$; $\gamma > \Omega_-/\omega$


Relativistic Resonant Ellipse
upe/c
upa/c
V=c
α = Ω_e^2/ω_pe^2 = 0.1


Relativistic Resonant Ellipse
upe/c
upa/c
V=c
ω/Ω = 0.1
0.3
0.5
0.8
0.9
Ω_e^2/ω_pe^2 = 1.0


Relativistic Resonant Ellipse
upe/c
upa/c
V=c
Ω_e^2/ω_pe^2 = 10

$$\boxed{\text{Relativistic Effects on Wave Growth}}$$

Assume $f = f_0 + \eta f_1$; $\eta \ll 1$

cold background plasma

hot resonant population

$$\boxed{c^2k^2 = \omega^2 - \frac{\omega\omega_{pe}^2}{(\omega - \omega_{ce})} + i\eta 2\pi\omega_{pe}^2 Im \int_{-\infty}^{\infty} dp_{\|} \int_0^{\infty} dp_{\perp} \frac{p_{\perp}^2 \hat{G} f_1}{\gamma\omega - \omega_{ce} - kp_{\|}}}$$

due to cold plasma : f_0 . hot resonant population

Solve this for $\omega = \omega_r + i\omega_i$; $\omega_i \ll \omega_r$

defined by f_0 due to resonant hot electrons

Growth Rate:

$$\boxed{w_i = \frac{\pi\omega_p^2}{\omega + \omega_{ce}\omega_{pe}^2/(\omega - \omega_{ce})^2} N_{res}[A_{eff} - \overbrace{\omega/(\omega_{ce} - \omega)}^{A_c(\omega/\omega_{ce})}]}$$

Critical Anisotropy needed for instability

Effective Pitch-angle Anisotropy

$$A_{eff} = \frac{k\int_0^{\infty} dp_{\perp} \frac{p_{\perp}^2 \hat{G}_r f}{(1 - p_r/(n\gamma))}}{(\omega - \omega_{ce})\int_0^{\infty} dp_{\perp} \frac{p_{\perp}^2 \partial f/\partial p_{\perp}}{[1 - p_r/(n\gamma)]}}$$

Fractional number of resonant electrons

$$N_{res} = \pi \frac{(\omega - \omega_{ce})}{k} \int_0^{\infty} \frac{dp_{\perp} p_{\perp}^2 \partial f/\partial p_{\perp}}{[1 - p_r/(n\gamma)]}$$

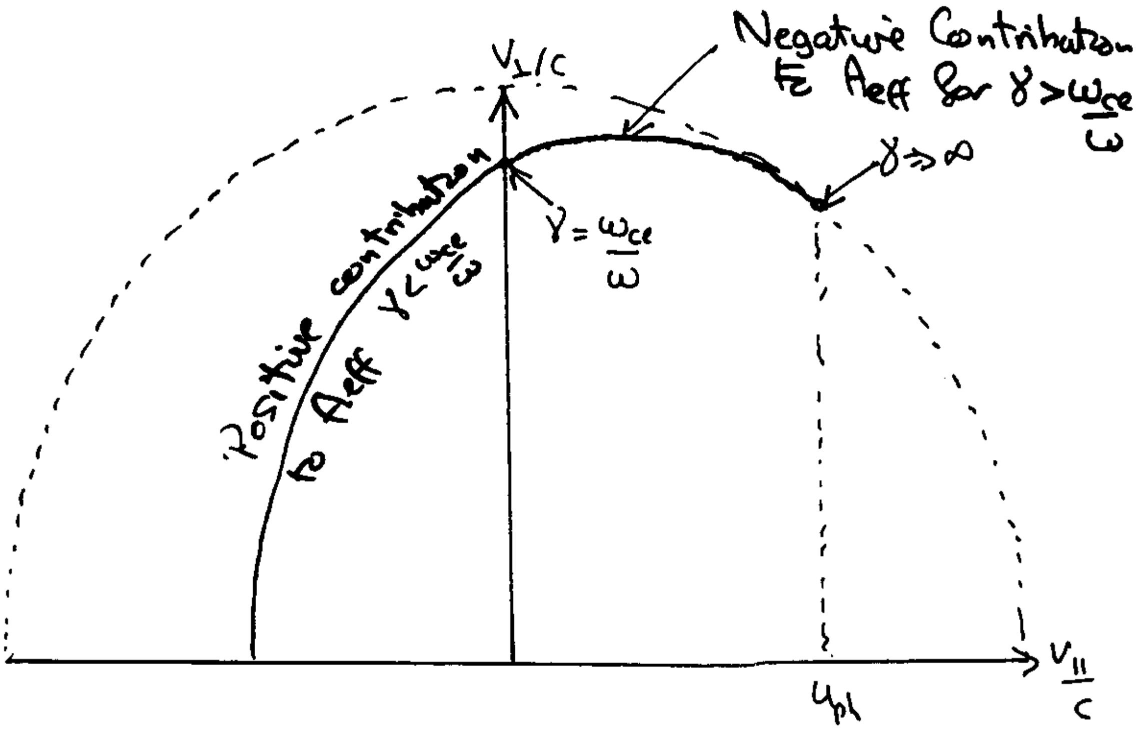
Changes in the Effective Anisotropy due to resonance with Relativistic Electrons

Recall that $\omega_i \sim \left[A_{eff} - A_c\right]$ where $A_c = \dfrac{\omega}{\omega_{ce}-\omega}$

and

$$\boxed{A_{eff} = A\frac{\int_0^\infty dp_\perp \left[\frac{p_\perp^3 (\omega_{ce}/\gamma - \omega) f}{(1-p_r/(n\gamma))}\right.}{(\omega_{ce}-\omega)\int_0^\infty dp_\perp \frac{p_\perp^3 f}{(1-p_r/(n\gamma))}}}$$

Integrate along resonant ellipse.



Since the integrand for A_{eff} becomes negative for $\gamma > \omega_{ce}/\omega$, $A_{eff} < A$ and may become negative for $\omega \to \omega_{ce}$! This will reduce growth at high frequencies

1. Bi-Maxwellian distribution function

$$f(p_{\|}, p_{\perp}) = \frac{1}{\pi^{\frac{1}{2}} a_{\|}} \frac{1}{\pi a_{\perp}^2} exp(-\frac{p_{\|}^2}{a_{\|}^2}) exp(-\frac{p_{\perp}^2}{a_{\perp}^2})$$

$$a_{\|} = (\frac{2T_{\|}}{mc^2})^{\frac{1}{2}}, a_{\perp} = (\frac{2T_{\perp}}{mc^2})^{\frac{1}{2}},$$

Effective Anisotropy

$$\boxed{A_{eff} = A \frac{\int_0^\infty dp_\perp [\frac{p_\perp^3 (\omega_{ce}/\gamma - \omega) f}{(1 - p_r/(n\gamma))}}{(\omega_{ce} - \omega) \int_0^\infty dp_\perp \frac{p_\perp^3 f}{(1 - p_r/(n\gamma))}}}\ ;$$

Thermal Anisotropy

$$\boxed{A = \frac{T_\perp}{T_\|} - 1}$$

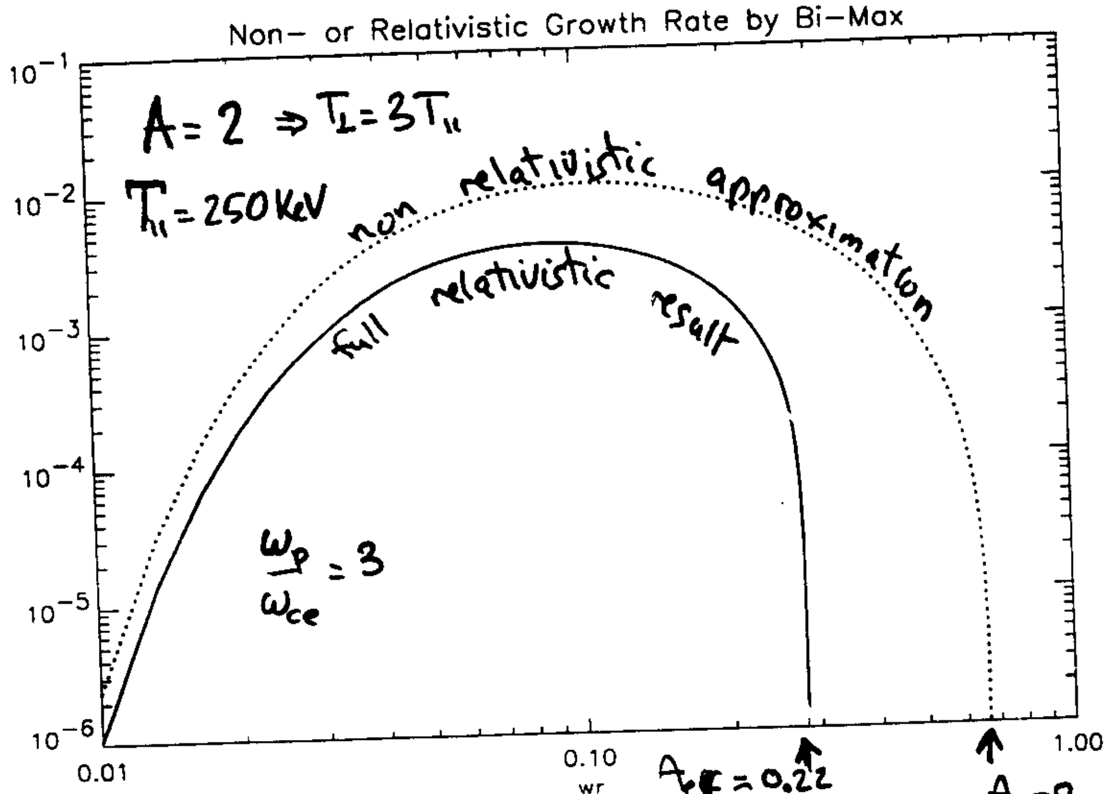
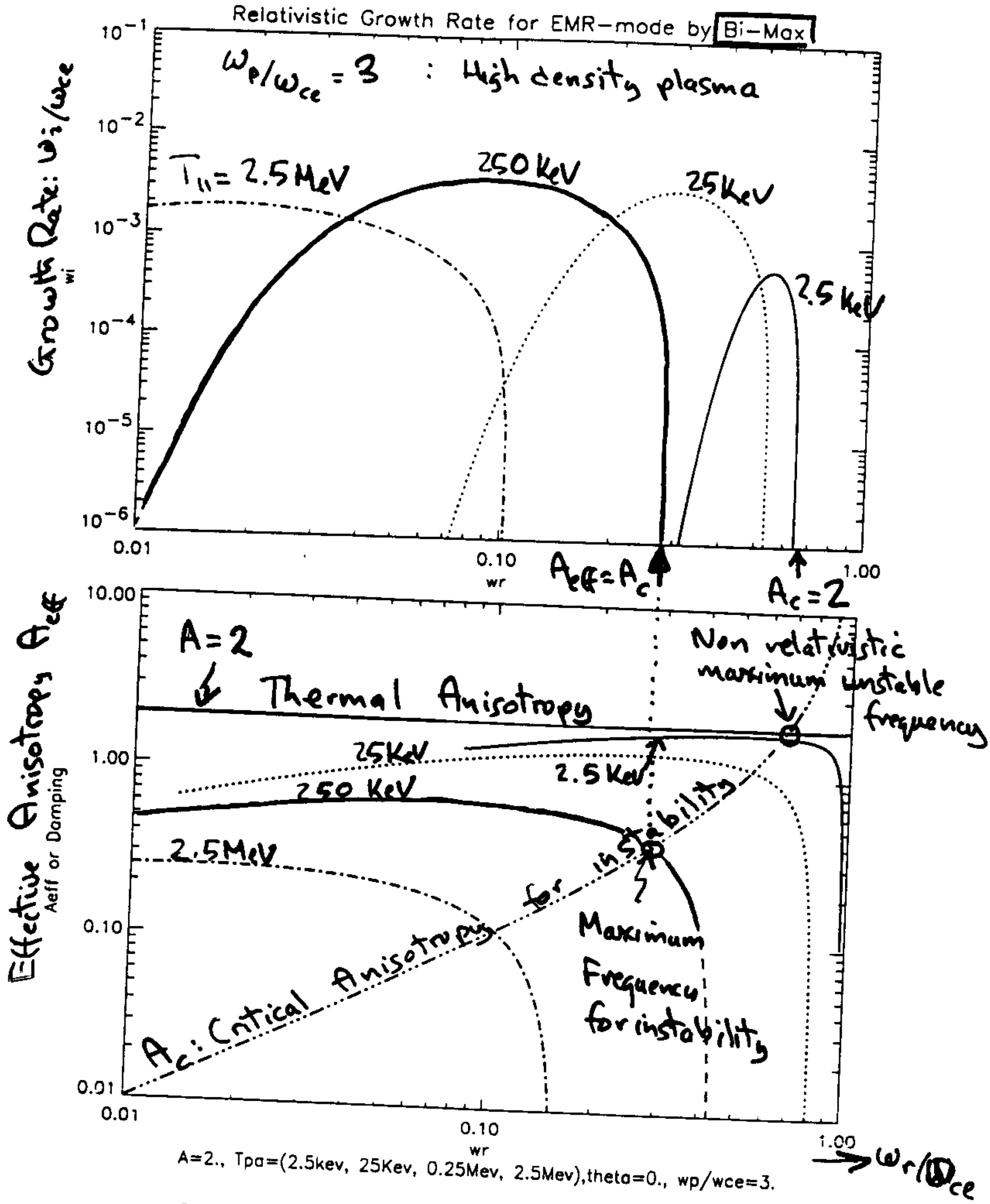


Fig. 2, A=2, Tpa=250kev, theta=0, beta=0.01, wp/wce=3

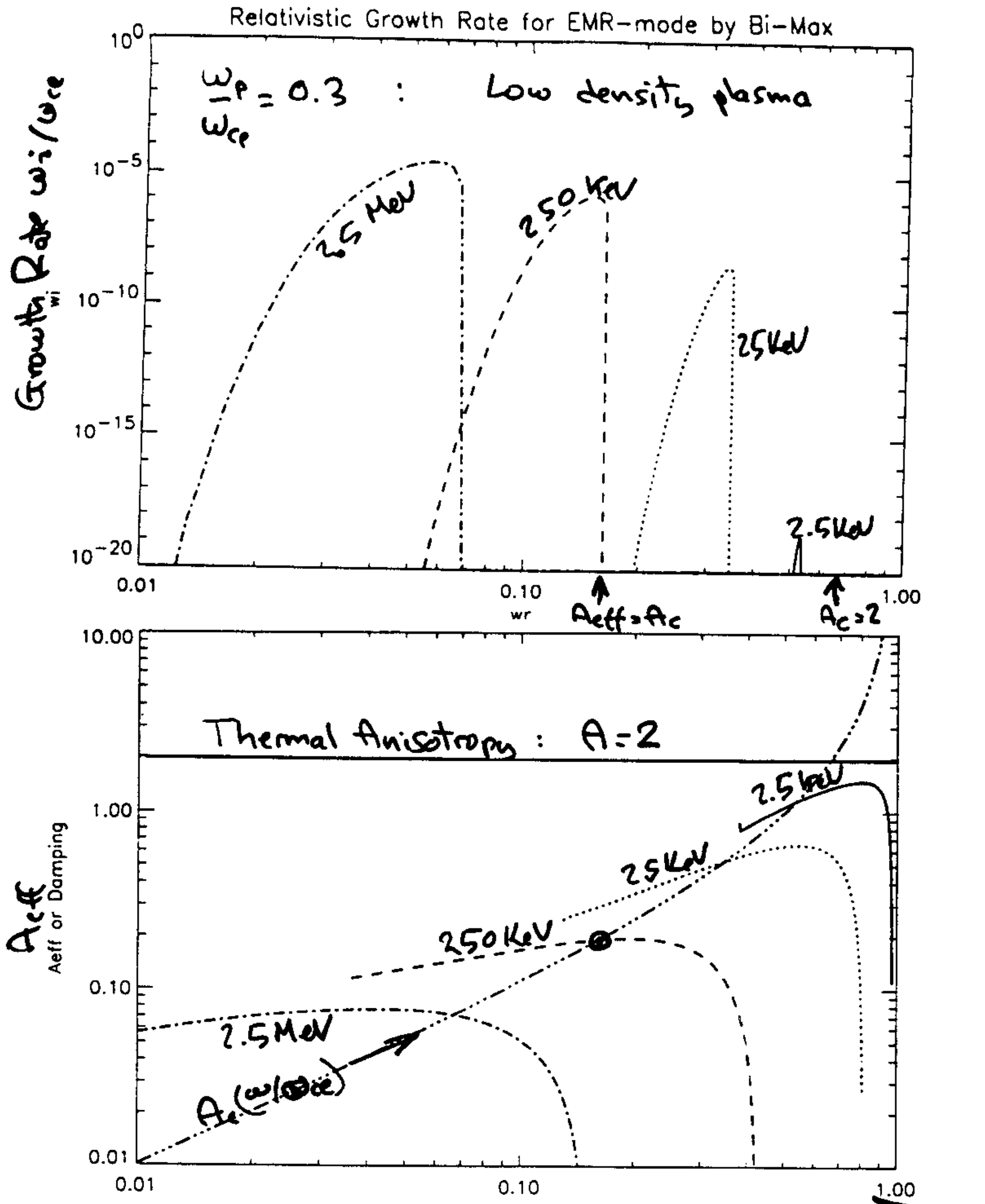
$\Rightarrow \frac{\omega}{\omega_{ce}} = 2/3$



A=2., Tpa=(2.5kev, 25Kev, 0.25Mev, 2.5Mev),theta=0., wp/wce=3.

Since wave growth rate $\omega_i \sim [A_{eff} - A_c]$

There is a dramatic drop in both the magnitude of wave growth and the upper frequency for instability due to relativistic effects of resonant electrons.



A=2., Tpa=(2.5kev, 25Kev, 0.25Mev, 2.5Mev),theta=0., wp/wce=0.3

① Wave Growth Strongly supressed in low density plasma

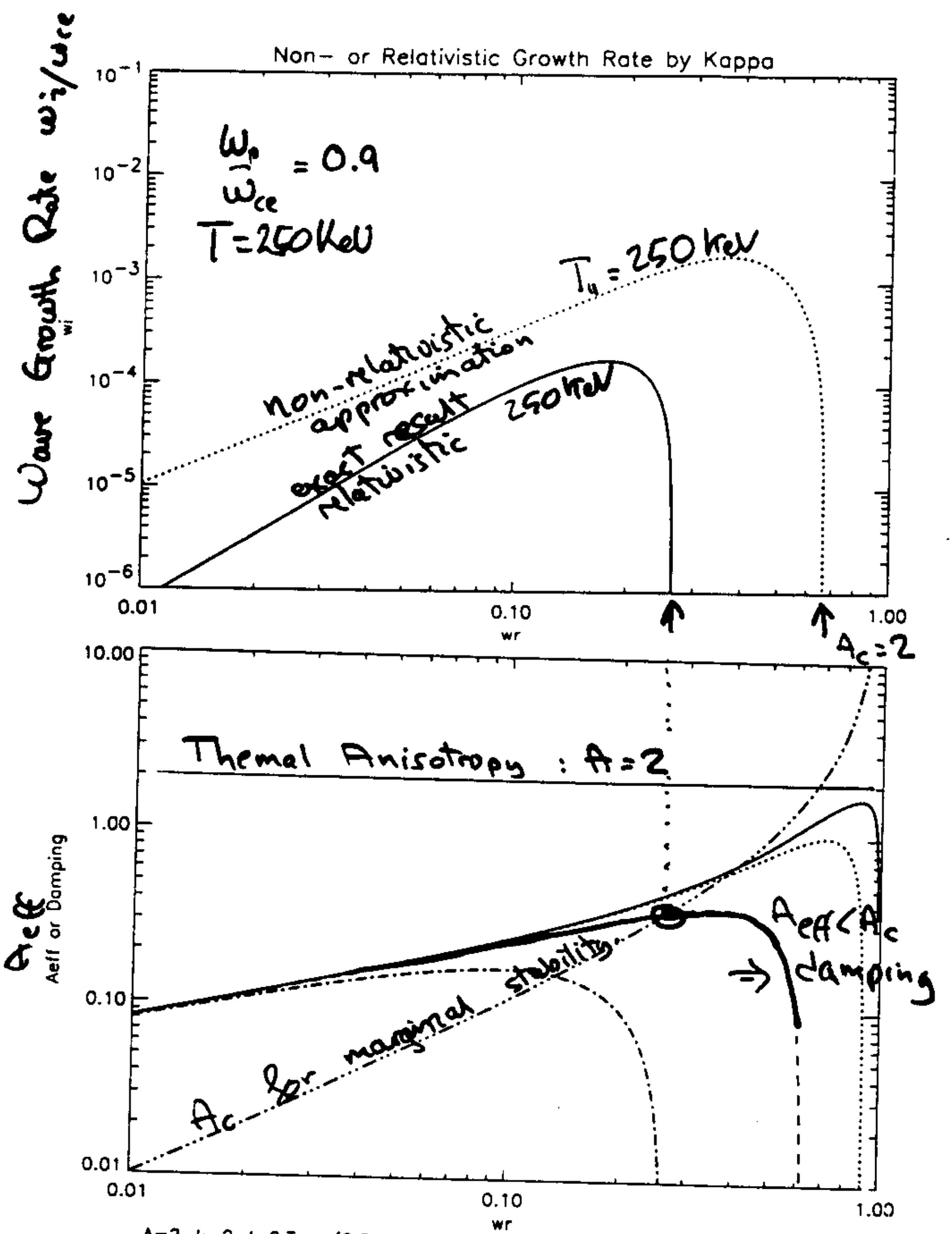
② Reduction in A_{eff} is more effective since $\frac{V_{ph}}{c}$ is increased.

... Growth in a Relativistic Kappa Distribution

$$\boxed{f(p_\parallel, p_\perp) = \frac{N\Gamma(\kappa + l + 1)}{\pi^{\frac{3}{2}}\theta_\perp^2\theta_\parallel\kappa^{l+\frac{3}{2}}\Gamma(l+1)\Gamma(\kappa - 1/2)}\left(\frac{p_\perp}{\theta_\perp}\right)^{2l}\left[1 + \frac{p_\parallel^2}{\kappa\theta_\parallel^2} + \frac{p_\perp^2}{\kappa\theta_\perp^2}\right]^{-(\kappa+l+1)}}$$

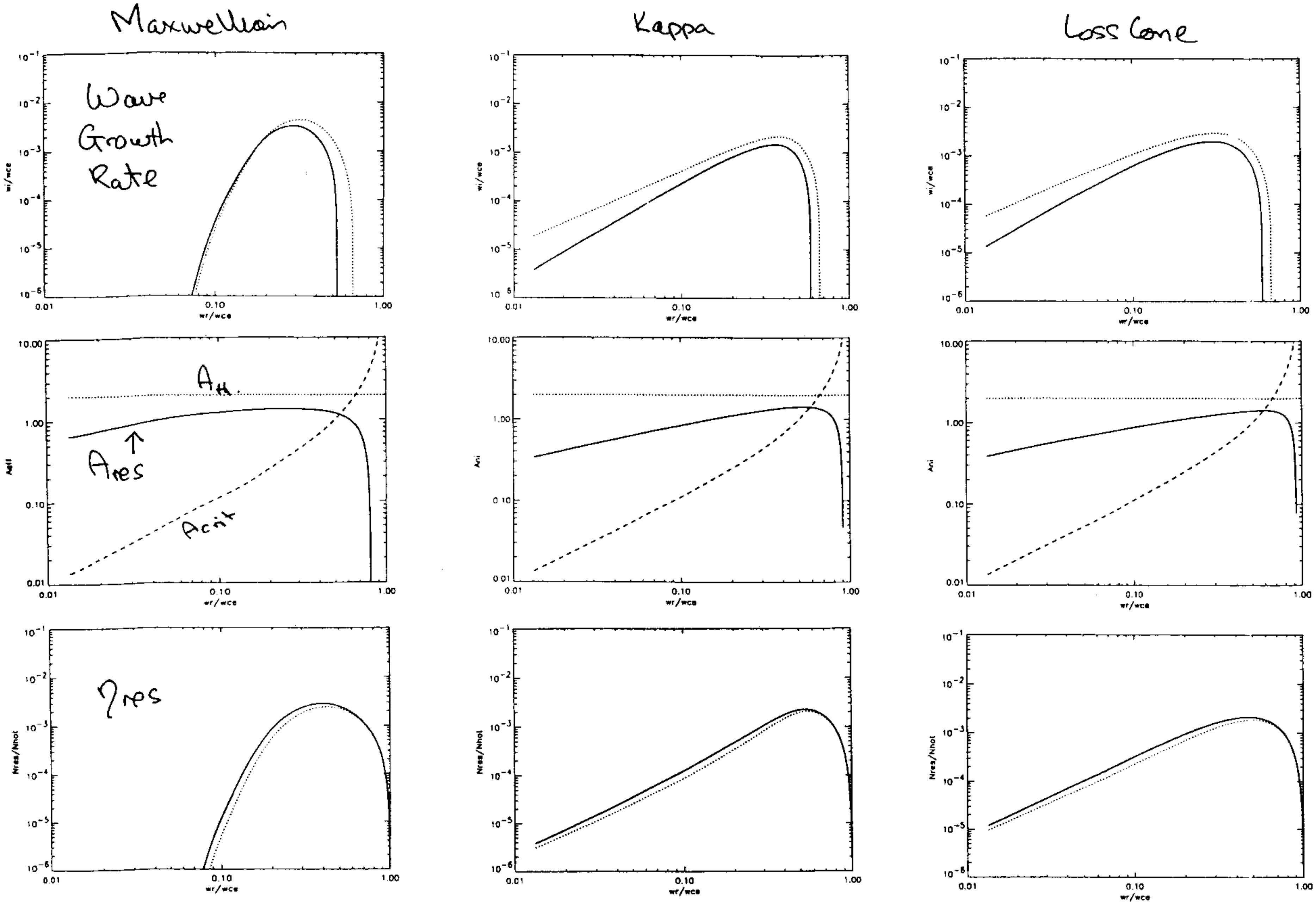
Power law spectrum at high energies.

$$A_{eff} = \frac{\int_0^\infty dp_\perp \frac{p_\perp(\omega_{ce}/\gamma - \omega)f}{(1-p_r/(n\gamma))}\left[l p_\perp + \frac{(\kappa+l+1)p_\perp^3}{f_\kappa \kappa\theta^2}\frac{A-l}{l+1}\right]}{(\omega_{ce} - \omega)\int_0^\infty dp_\perp \frac{p_\perp f}{(1-p_r/(n\gamma))}\left[l - \frac{p_\perp^2(\kappa+l+1)}{f_\kappa\kappa\theta^2}\right]}$$

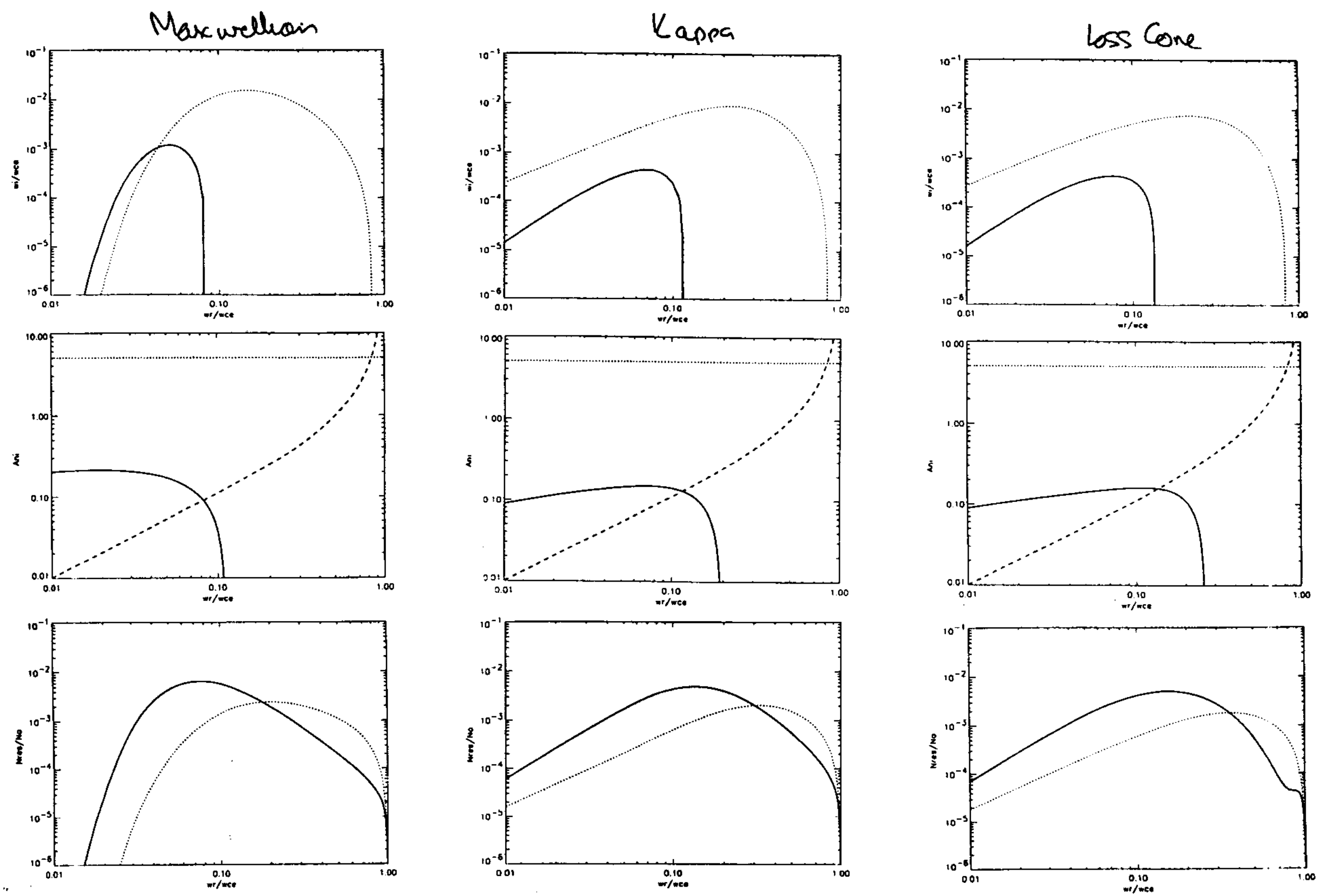


A=2, k=2, l=0,Tpa=(2.5kev, 25Kev, 0.25Mev, 2.5Mev),theta=0, wp/wce=0.9

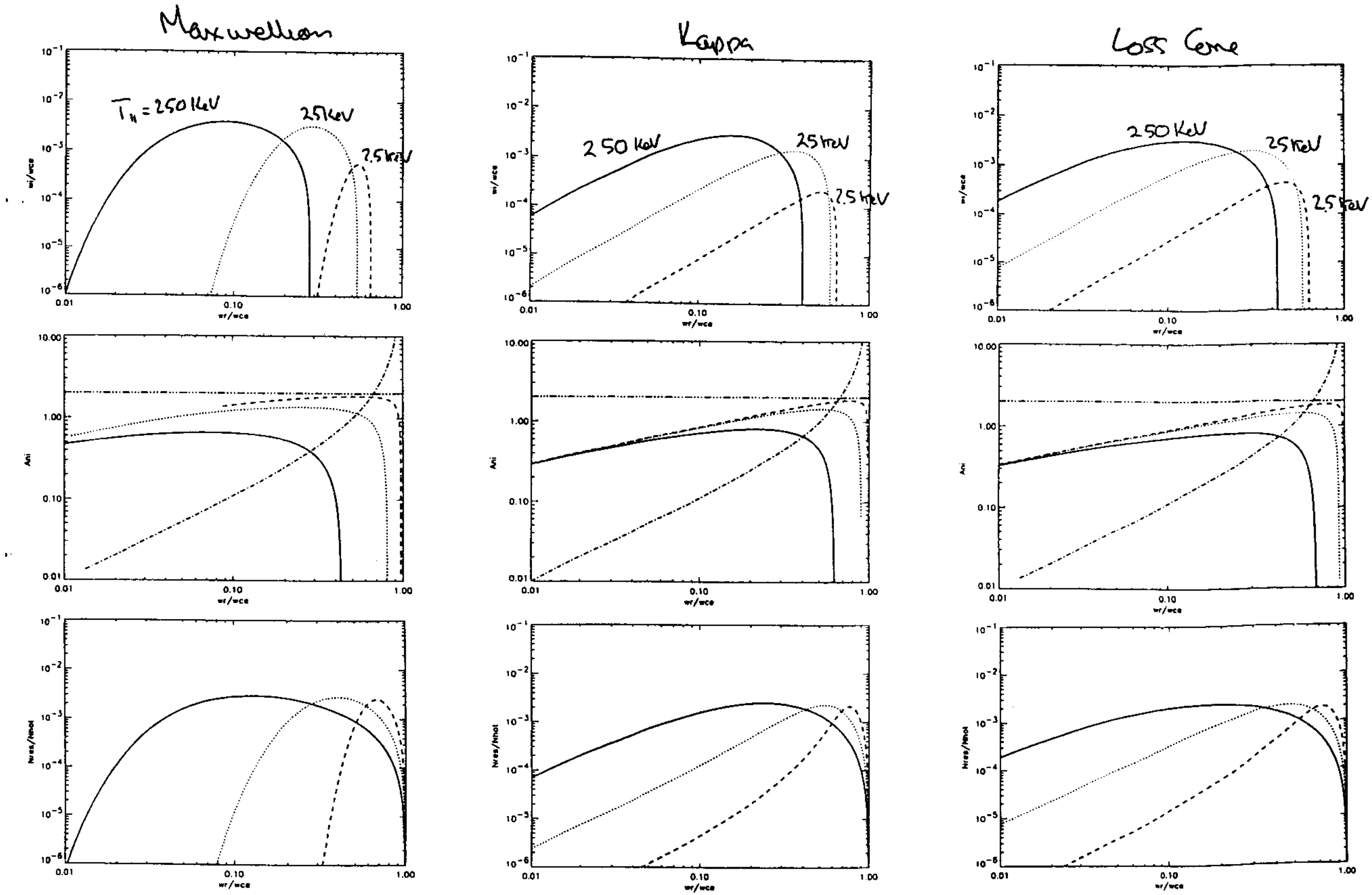
$\omega_p/\Omega_- = 3$; $T_\parallel = 25$ KeV ; $A = T_\perp / T_\parallel = 2$; $\eta_{HOT} = 10^{-2}$; $\theta = 0$



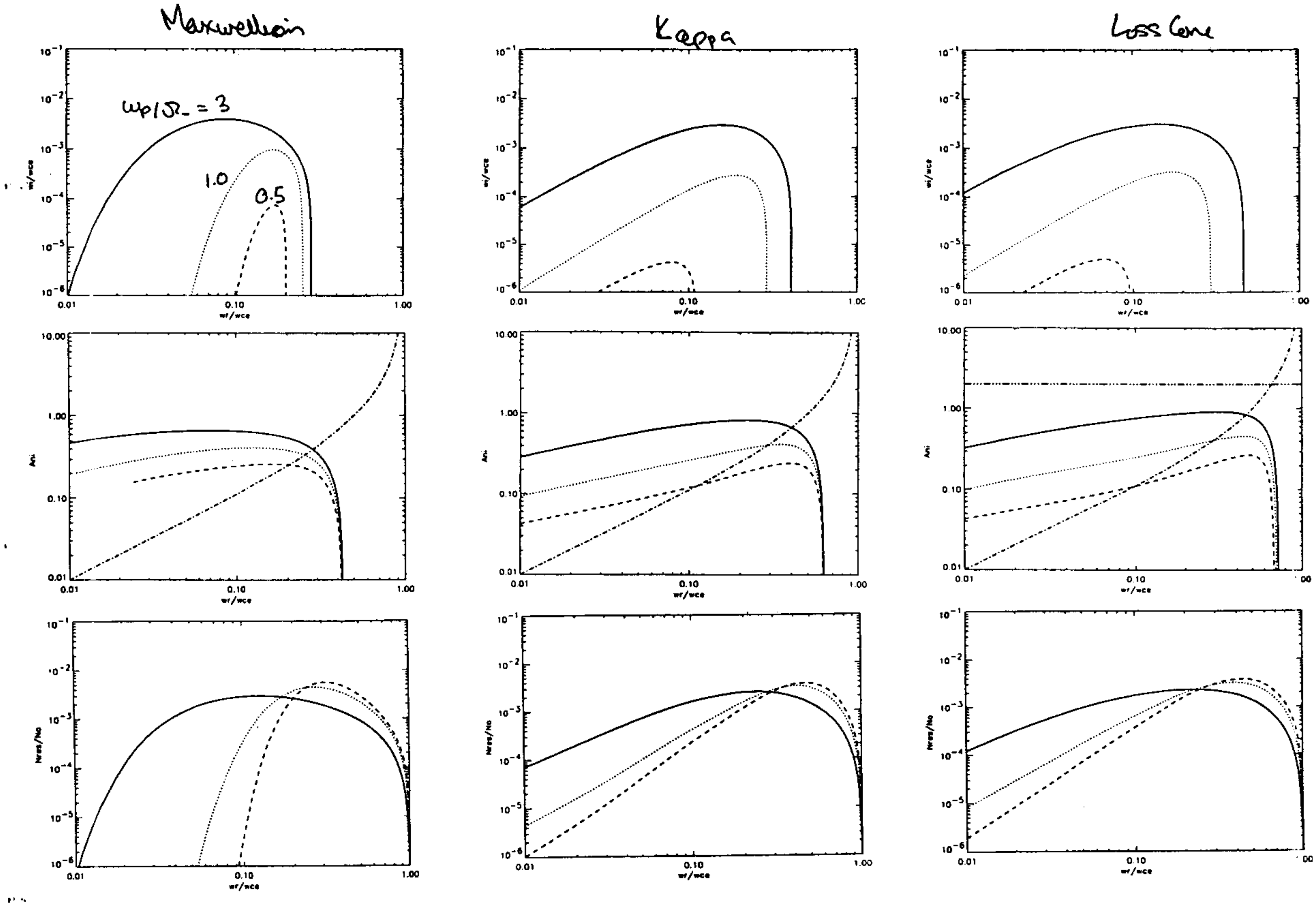
$\frac{\omega_p}{\Omega_-} = 0.6$; $T_{\parallel} = 2.5\,\mathrm{MeV}$; $A = 5$; $\eta_{HOT} = 10^{-2}$; $\theta = 0$



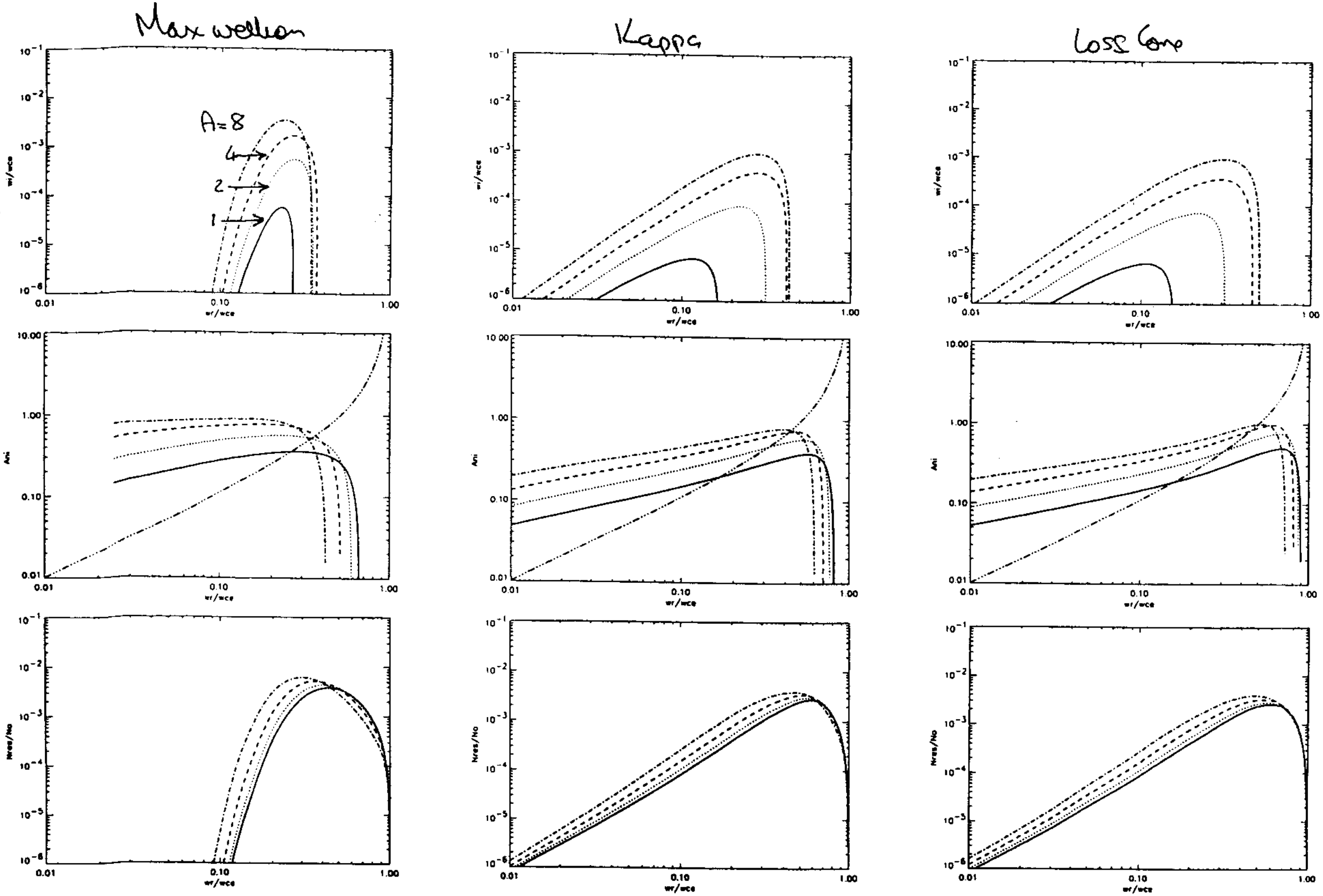
$\omega_p / \Omega_- = 3$; $A = 2$; $\eta_{HOT} = 10^{-2}$; $\theta = 0$



$T_{\parallel} = 250\,\mathrm{keV}$; $A = 2$; $n_{HOT} = 10^{-2}$; $\theta = 0$



$\omega_p/\Omega_- = 0.9$; $T_\parallel = 100$ keV ; $n_{HOT} = 10^{-2}$; $\theta = 0$



Resonant Diffusion Surfaces

At relativistic energies both the resonance condition:

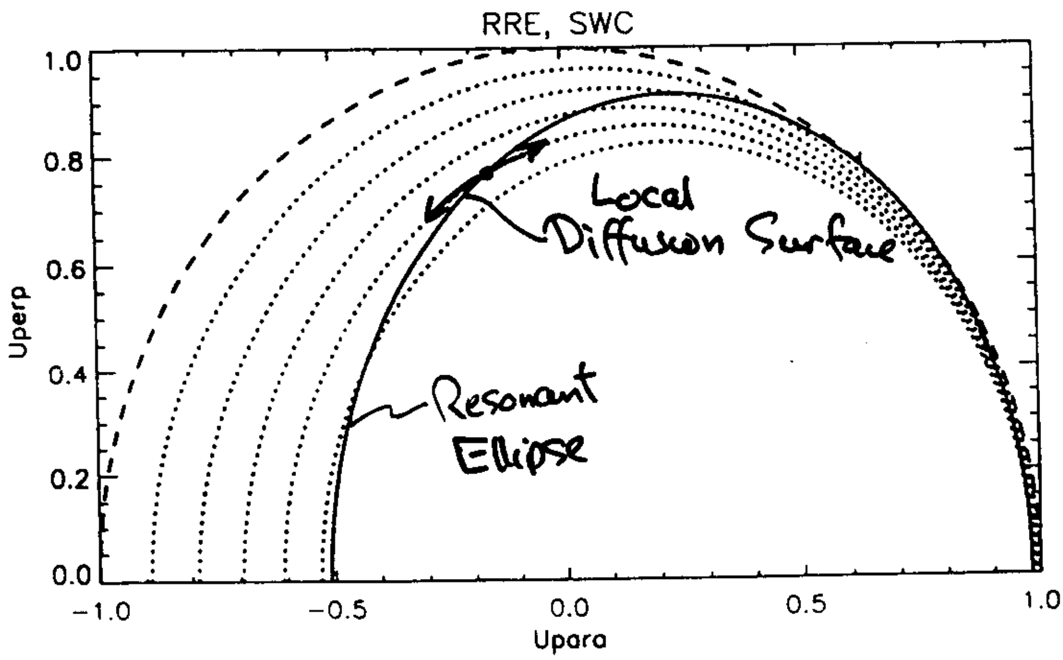
$$\boxed{V_{\parallel} = -u\left[\frac{\omega_{ce}}{\omega}\left(1 - \frac{\omega_{ce}^2}{\omega_p^2}\left(V_{\parallel}^2 - V_{\perp}^2\right)\right)^{1/2} - 1\right]}$$

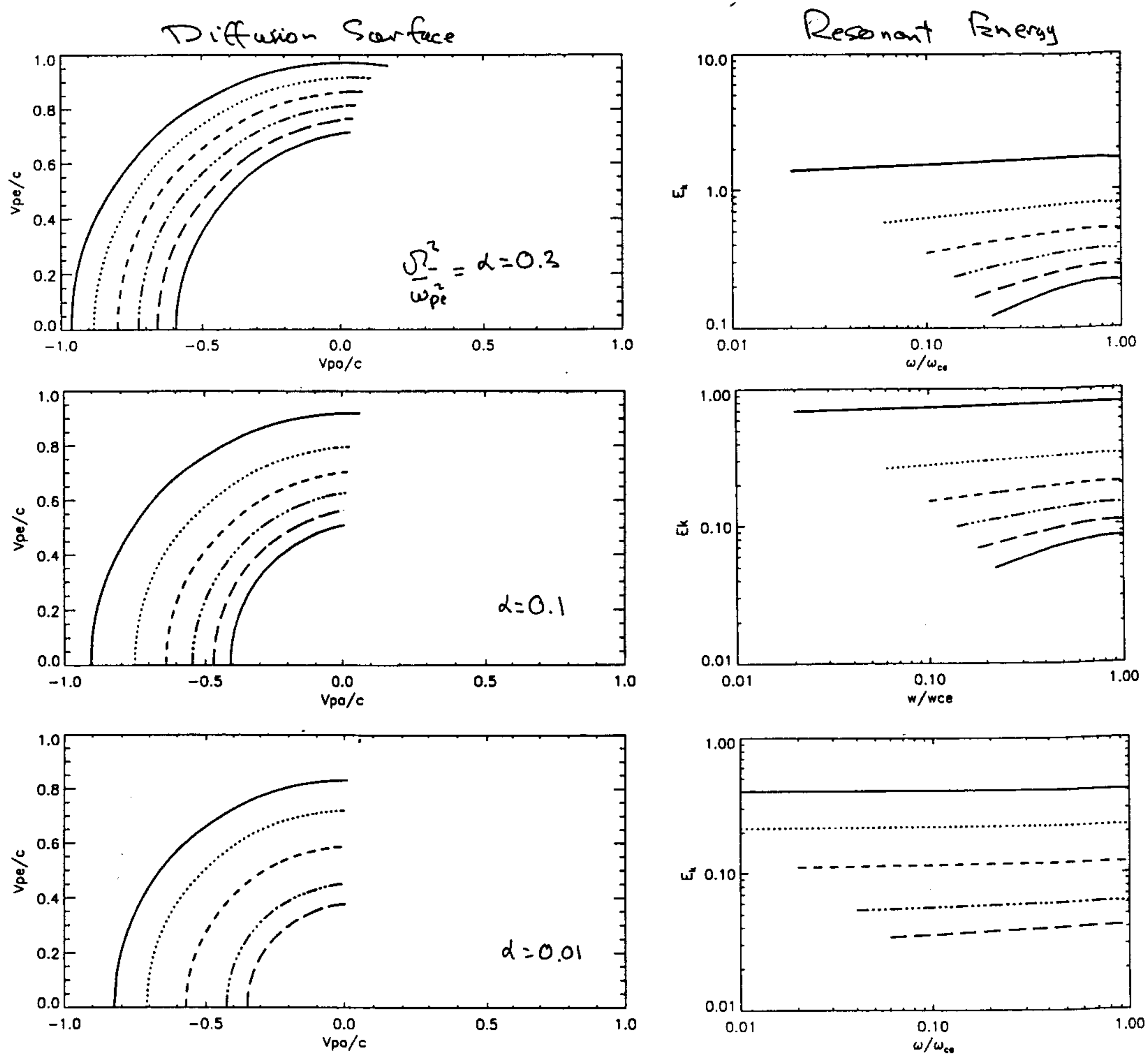
and the resonant diffusion surface:

$$\boxed{v_{\parallel}^2\left(1 - E\alpha^2 u^2\right) - 2uv_{\parallel}(1 - E\alpha) + v_{\perp}^2\left(1 - \alpha u^2\right) = E - u^2}$$

where $\alpha = \left(\frac{\omega_{ce}}{\omega_p}\right)^2$ and E is a constant

become ellipses:

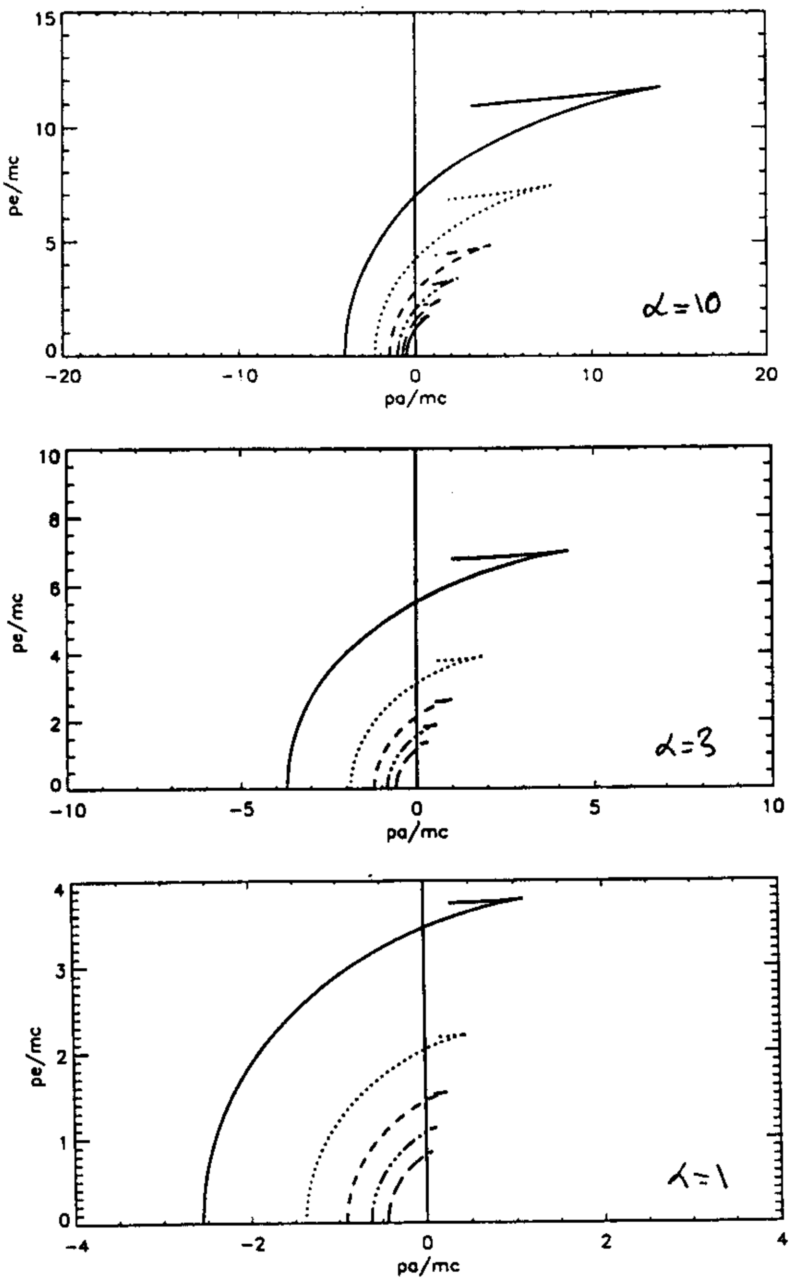




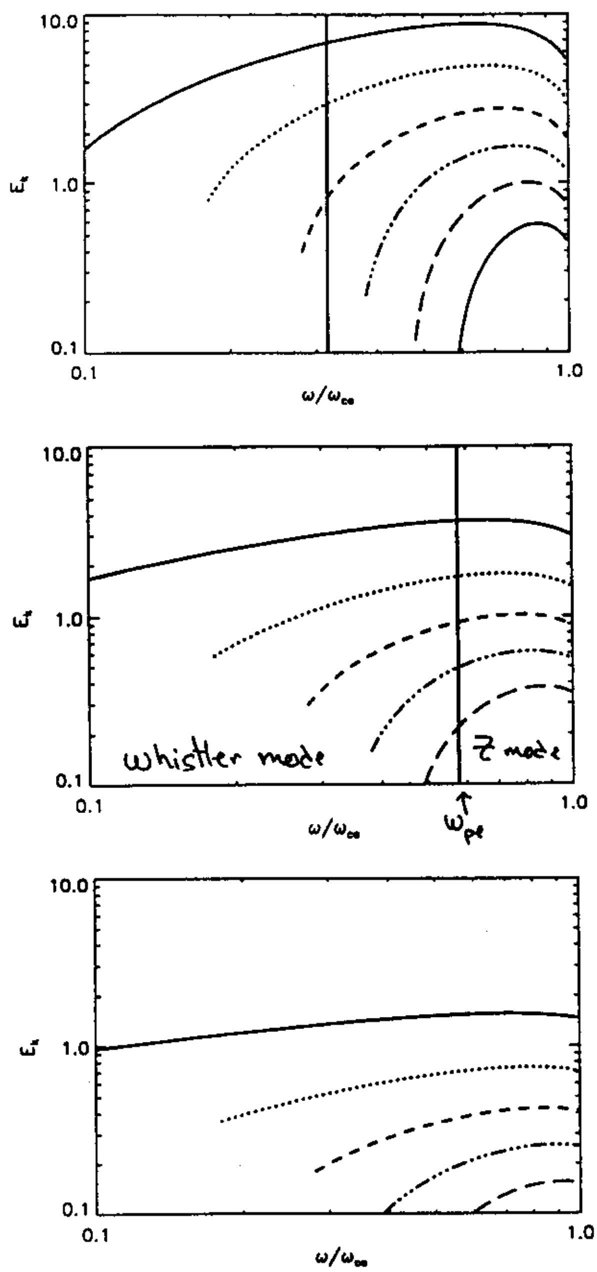
Diffusion Surface
Resonant Energy
$\frac{\Omega_-^2}{\omega_{pe}^2} = \alpha = 0.3$
$\alpha = 0.1$
$\alpha = 0.01$
Vpe/c
Vpa/c
E_k
Ek
ω/ω_{ce}
w/wce

R - Mode Waves
$\alpha \geq 1.0$

Diffusion Surfaces

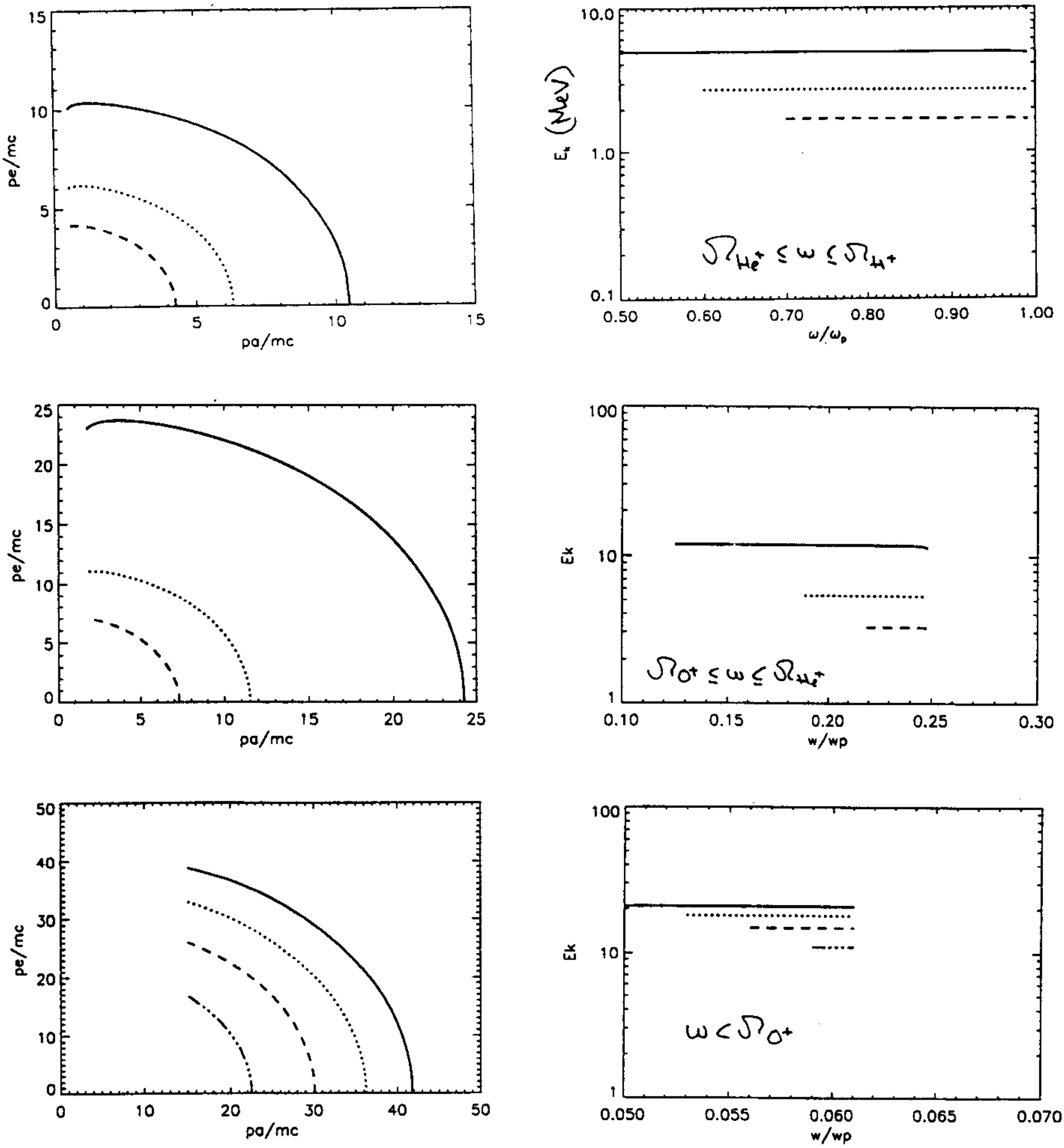


Resonant Energies along the diffusion surface



L - Modo Waves $\alpha = 10^{-2}$

Diffusion Surface | Resonant Energy



$\boxed{\text{L - mode}}$ $\alpha = 1.0$

Diffusion Surface | Resonant Energy