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AUTUMN COLLEGE ON PLASMA PHYSICS

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GENERATION OF SHEAR FLOW IN FLUIDS AND PLASMAS

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These are lecture notes, intended for distribution to participants.

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OUTLINE

- MOTIVATION
- BASIC EQUATIONS
- TWO COUNTER-ROTATING VORTICES
  - LINEAR STABILITY TO SHEAR FLOW INSTABILITY
  - NONLINEAR STUDY
    NUMERICAL
    ANALYTICAL
- FOUR ALTERNATELY COUNTER-ROTATING VORTICES
  - NONLINEAR EVOLUTION
    NUMERICAL
- CONCLUSIONS
- FUTURE WORK

MOTIVATION

TO UNDERSTAND THE ROLE OF GENERATION OF SHEAR FLOW BY VORTICES WHICH CAN LEAD TO "SELF-ORGANIZATION"

EXAMPLES:

1. L-H TRANSITIONS IN TOKAMAK PLASMAS

(R. J. GROEBNER PHYS. FLUIDS B **5**, 2343 (1993))

2. LARGE SCALE FLOW IN RAYLEIGH-BENARD CONVECTION

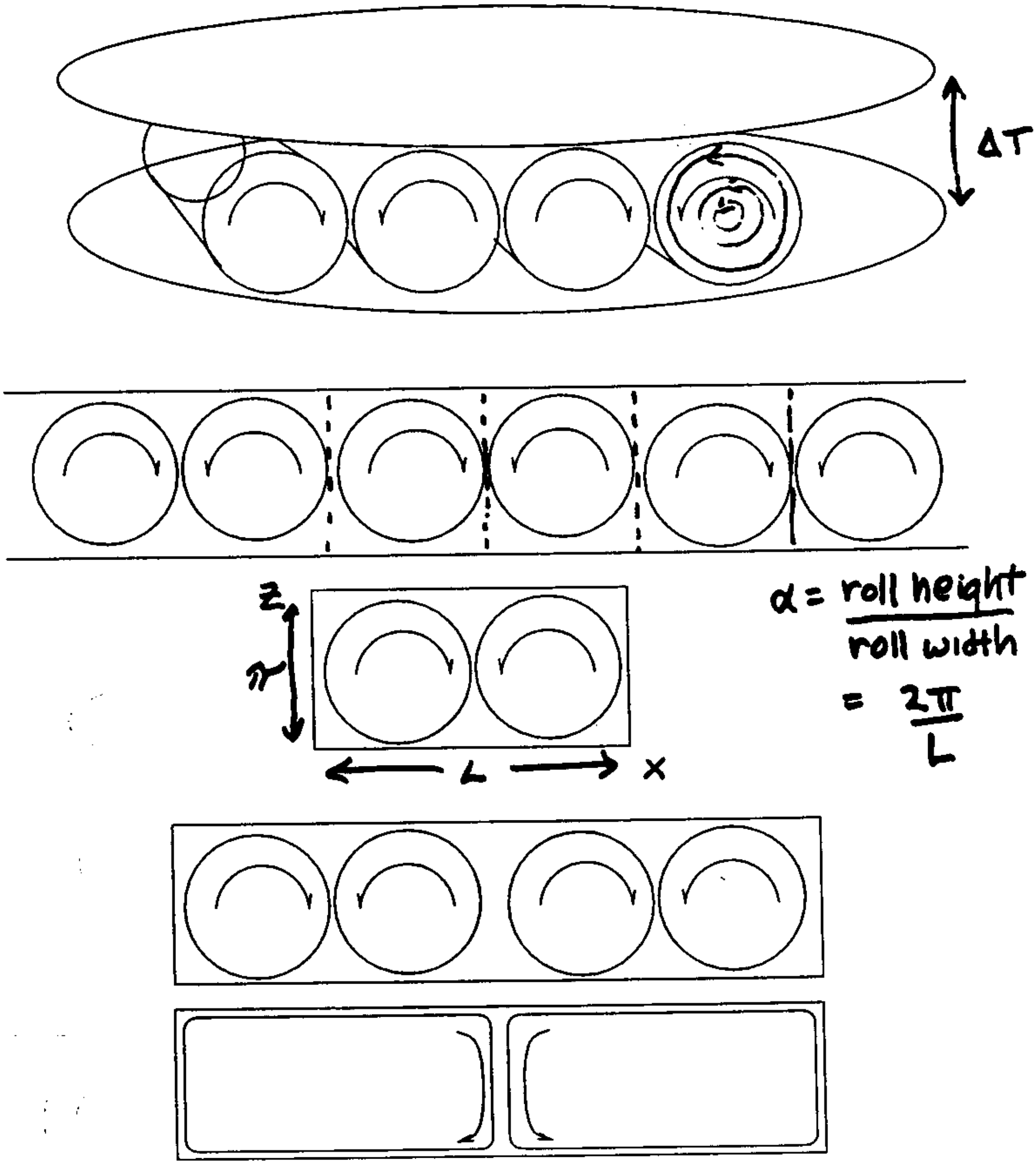
(R. KRISHNAMURTY AND L. N. HOWARD J. FLUID MECH. PROC. NAT. ACAD.SCI. **78**, 1981 (1981)

3. RED SPOT OF JUPITER -- ROSSBY VORTEX IN SHEAR FLOW

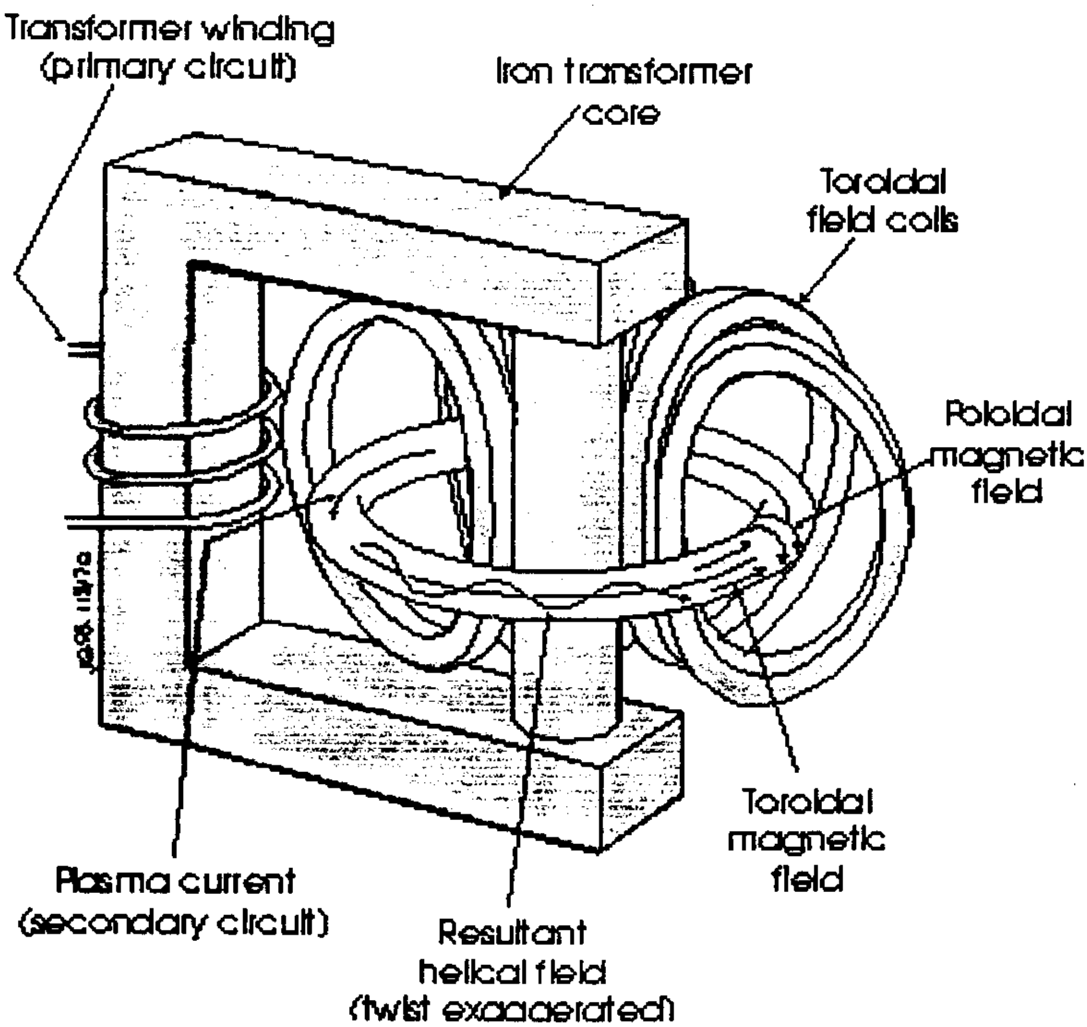
(S. V. ANTIPOV ET AL. NATURE **323**, 238 (1986))

4. STABILITY OF A CHAIN OF DRIVEN VORTICES

(P. TABELING ET AL. J. FLUID MECH. **213**, 511 (1990))

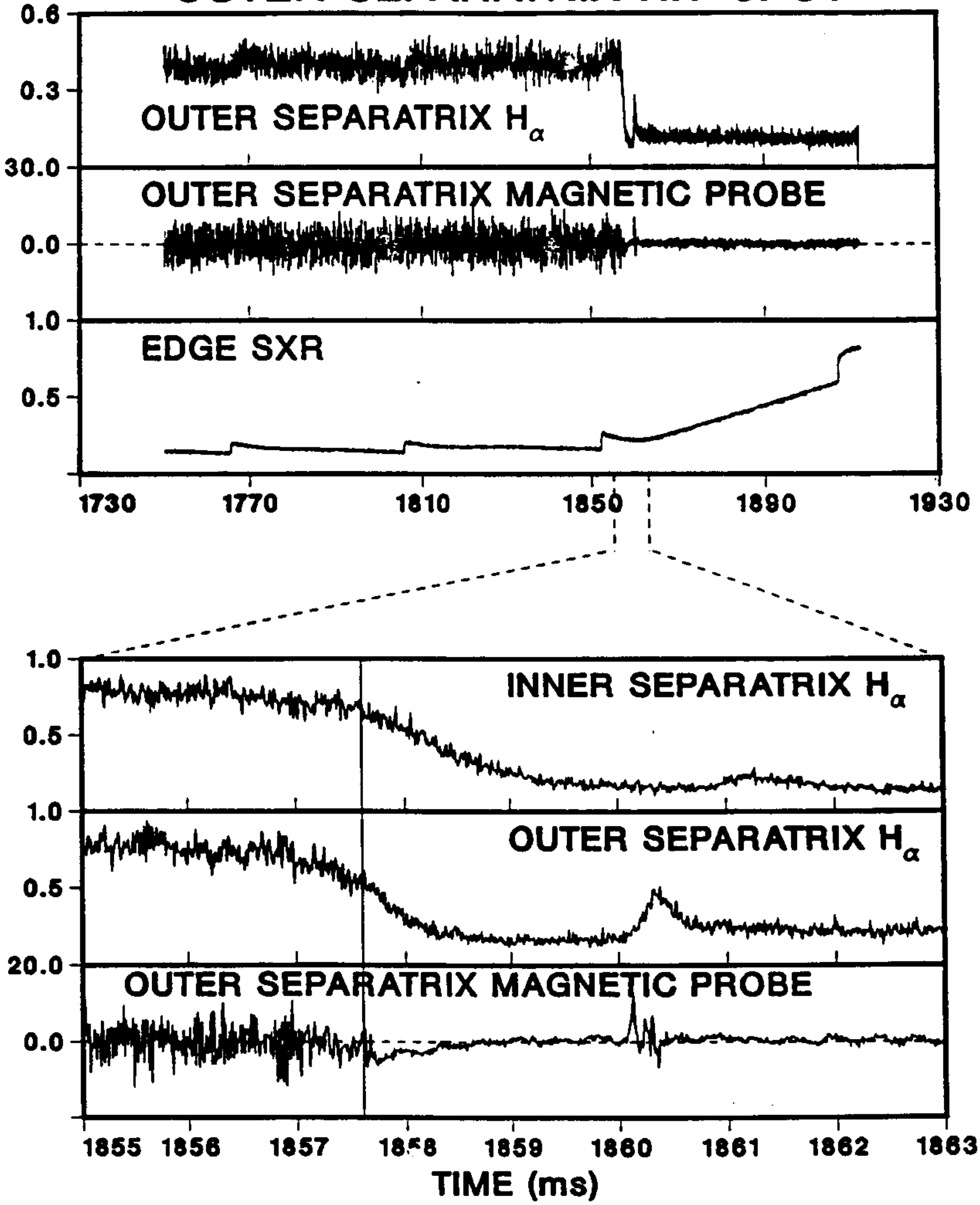

ΔT
z
π
L
x
α = roll height / roll width
= 2π / L

TOKAMAK

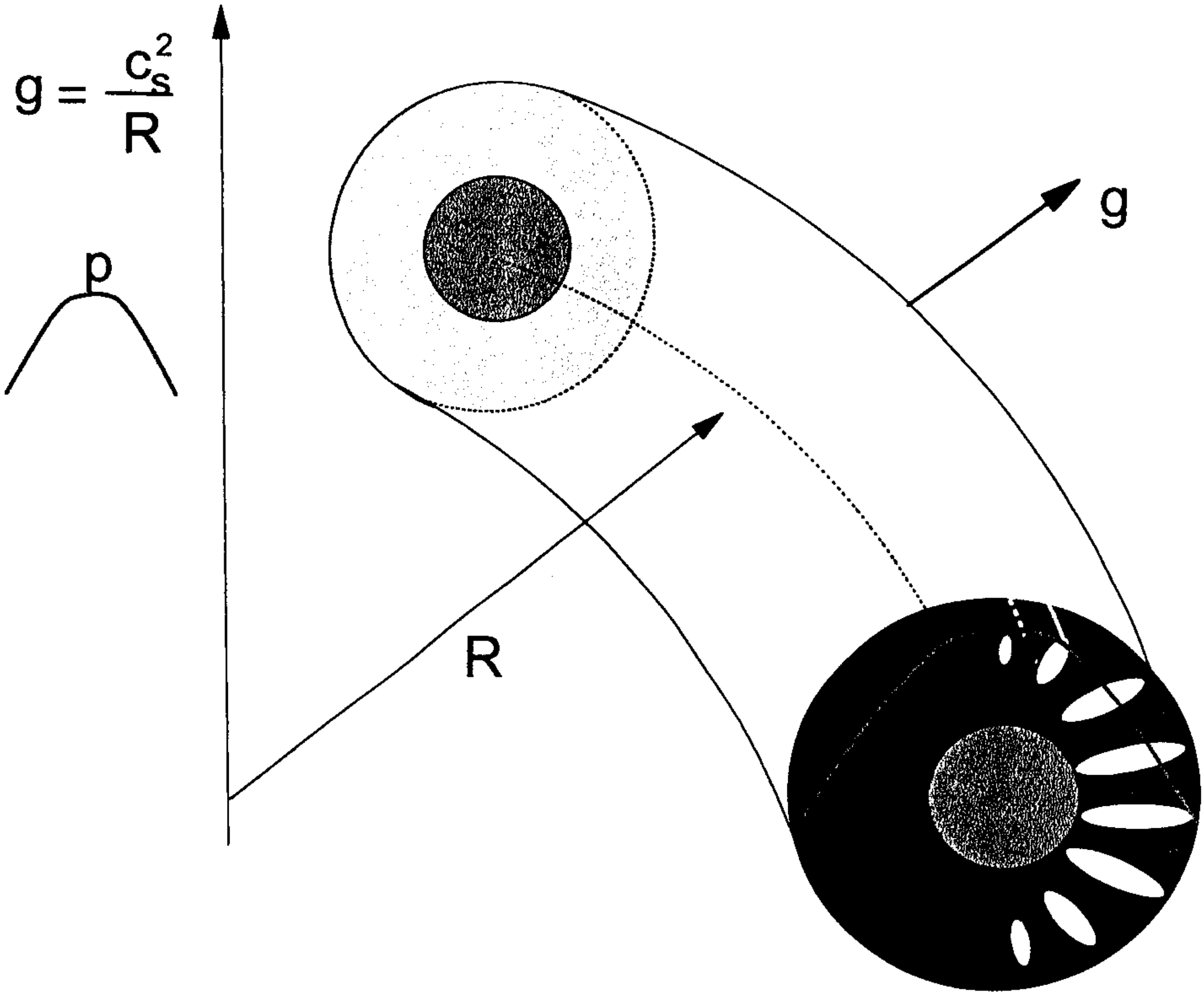


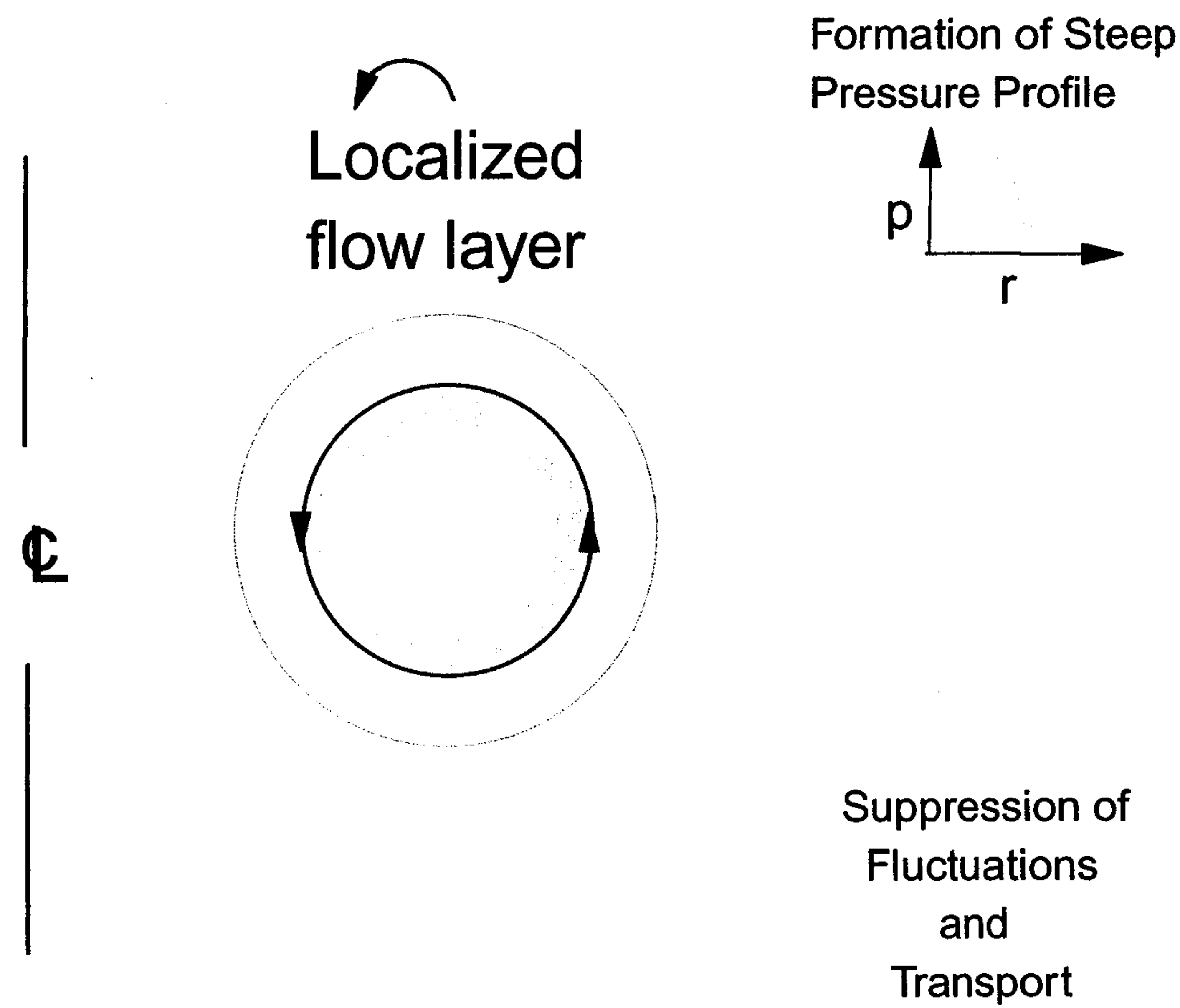
JET HOMEPAGE

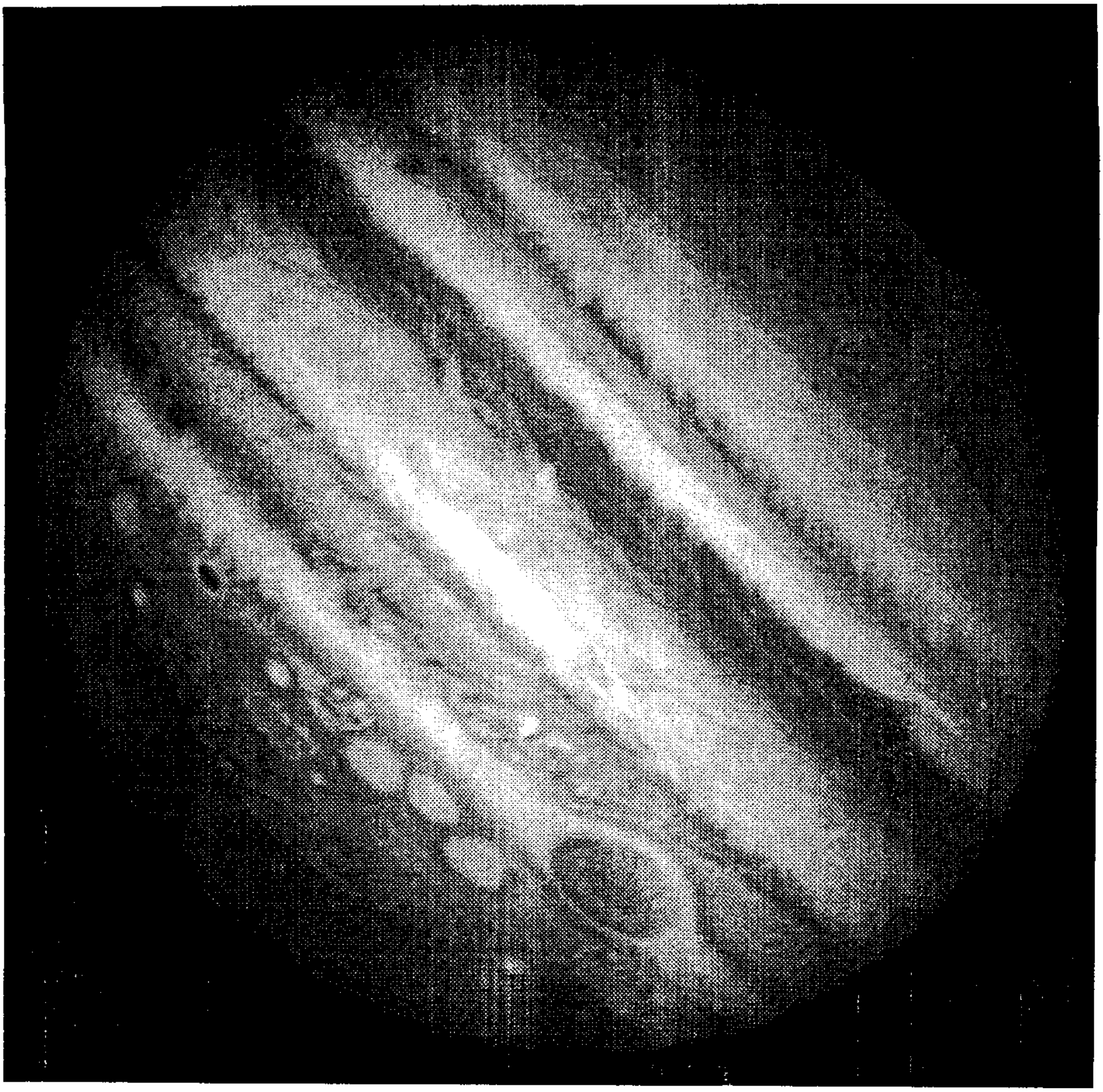
H_α DROP OCCURS FIRST AT OUTER SEPARATRIX HIT SPOT



BALLOONING MODES IN TOKAMAKS




Formation of Steep
Pressure Profile
p
r
Localized
flow layer
℄
Suppression of
Fluctuations
and
Transport



EXPERIMENTAL SET-UP

Chaos in a linear array of vortices

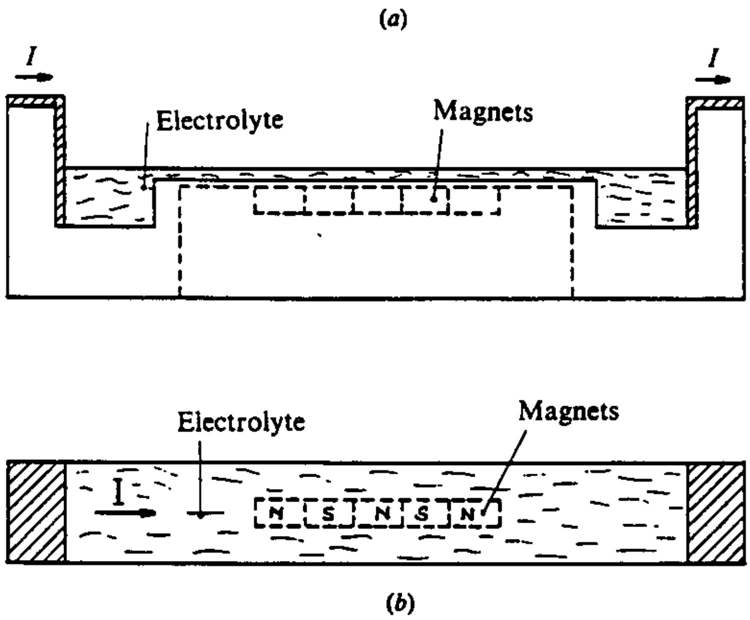


FIGURE 1. The experimental system: (a) cross-sectional view; (b) top view.

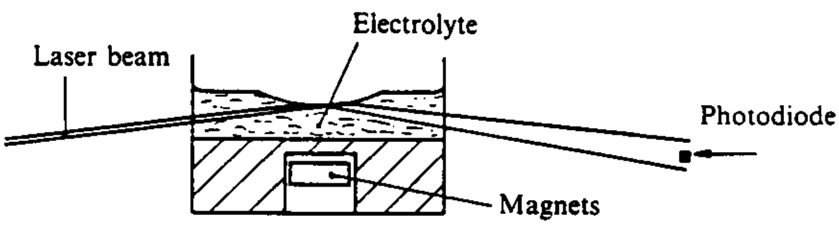
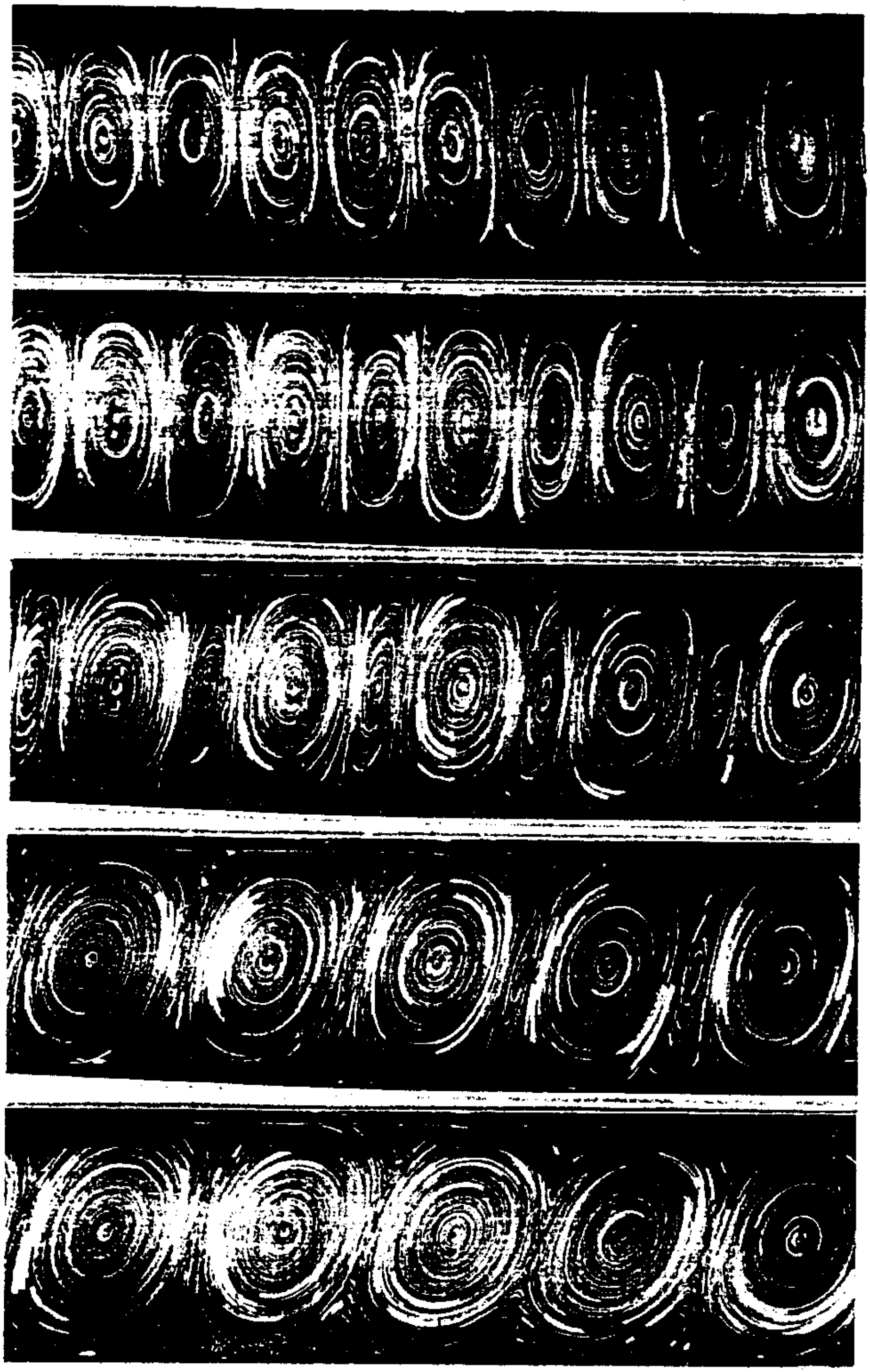


FIGURE 2. Principle of the measurements of the time-dependent evolution of the system.

Electrolyte: sulphuric acid
Length of the cell=35 cms. Width of the cell 3.5 cms.
Thickness of fluid =0.2-0.3 cm to maintain 2D dynamics
Wavelength of forcing =2.24 cms.

Strength of magnets =0.34 T, Currents ~ 10's of mA



a)

b)

c)

d)

e)

Picture 1. – The evolution of the system as I is increased from 0 to 15 mA. In this case, b is 2.5 mm and the line includes 20 magnets with two smaller magnets at the ends. For larger values of I, the system becomes time-dependent. The pictures are restricted to the 10 central vortices of the linear array. *a*) $I = 6.7$ mA, with an exposure time of 3 s. All vortices have the same size. *b*) $I = 7.7$ mA, with an exposure time of 2 s. Half the number of vortices have expanded, at the expense of the others. We are beyond the instability point. *c*) $I = 8.55$ mA, with an exposure time of 1 s. The amplitude of the stationary disturbance has increased. *d*) $I = 10.86$ mA, with an exposure time of 1 s. Same comment as above. *e*) $I = 13.09$ mA, with an exposure time of 1 s. The system is now composed of regular vortices of the same sign.

BASIC EQUATIONS

2D ISOTHERMAL NAVIER-STOKES EQUATIONS

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = -c_s^2 \nabla \rho + \nu \nabla^2 \mathbf{v} + \mathbf{S}$$

$$\mathbf{v} = \left(v_x, v_y\right) \quad c_s^2 = (T/m)^{1/2}$$

$$v/c_s << 1 \Rightarrow \text{incompressible} \Rightarrow \nabla \cdot \mathbf{v} = 0, \quad \rho \approx \rho_0$$

$$\mathbf{v} = -\nabla \phi \times \mathbf{z}, \qquad \omega = \mathbf{z} \cdot \nabla \times \mathbf{v} = \nabla^2 \phi$$

$$\frac{\partial \omega}{\partial t} + \mathbf{v} \cdot \nabla \omega = \mu \nabla^2 \omega + F$$

$$\mu = \nu/\rho, \qquad F = \mathbf{z} \cdot \nabla \times \mathbf{S}/\rho$$

BOUNDARY CONDITIONS

- PERIODIC IN X
- FREE SLIP IN Y ➔ $\partial v_x / \partial y = 0$, HARD WALL➔ $v_y=0$
  ➔ $\omega=0$

Boussinesq Equations

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = \sigma \frac{\partial \Theta}{\partial x} + \sigma \nabla^2 \omega \qquad (1)$$

$$\frac{\partial \Theta}{\partial t} + \mathbf{u} \cdot \nabla \Theta = R \frac{\partial \Psi}{\partial x} + \nabla^2 \Theta \qquad (2)$$

$$\mathbf{u} = -\nabla \times \Psi \hat{\mathbf{y}} \qquad (3)$$

$$\omega = \nabla^2 \Psi \qquad (4)$$

$$\omega(x, z) = \omega(x + L, z) \qquad (5)$$

$$\Theta(x, z) = \Theta(x + L, z) \qquad (6)$$

$$\left.\begin{array}{r} \Theta = 0 \\ \omega = 0 \end{array}\right\} z = 0, \pi \qquad (7)$$

where

$$T = T_0 - \frac{\Delta T z}{\pi} - \Theta \qquad (8)$$

BASIC EQUATIONS

$$\frac{dn}{dt} - 2n\boldsymbol{\kappa}\cdot\mathbf{v}_\perp^0 - \nabla\cdot\left(\frac{nc}{\Omega_i B}\frac{d}{dt}\nabla_\perp\phi\right) + \nabla_{||}(nv_{||}) = D\nabla_\perp^2 n$$

$$\nabla\cdot\left(\frac{nc}{\Omega_i B}\frac{d}{dt}\nabla_\perp\phi\right) - \frac{2cT_e}{eB}\mathbf{b}\times\boldsymbol{\kappa}\cdot\nabla n - \nabla_{||}\frac{j_{||}}{e} = 0$$

$$\frac{dv_{||}}{dt} = -c_s^2\nabla_{||}n$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_\perp^0\cdot\nabla, \quad \mathbf{v}_\perp^0 = -\frac{c}{B}\nabla\phi\times\mathbf{b}, \quad \mathbf{b} = \mathbf{B}/|\mathbf{B}|$$

$$\eta j_{||} = -\nabla_{||}\phi + \frac{T_e}{e}\nabla_{||}\ln n, \qquad \boldsymbol{\kappa} = \mathbf{b}\cdot\nabla\mathbf{b}$$

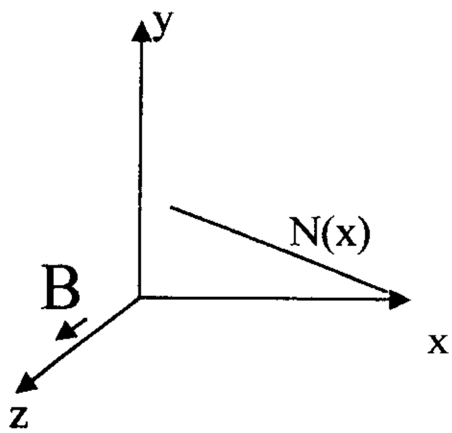
$$\mathbf{B} = \left(\mathbf{e}_\varphi + \frac{\epsilon}{q}\mathbf{e}_\theta\right)(1+\epsilon\cos\theta)^{-1}$$

IF PARALLEL DYNAMICS IGNORED AND CURVATURE “BAD” AND INDEPENDENT OF θ, EQUATIONS ARE VERY SIMILAR TO THAT FOR RAYLEIGH-BENARD CONVECTION.

HASEGAWA-MIMA-CHARNEY EQUATION

$$\left(1-\nabla_\perp^2\right)\frac{\partial\phi}{\partial t}+V\frac{\partial\phi}{\partial y}-\nabla\phi\times z\cdot\nabla\nabla_\perp^2\phi=0$$

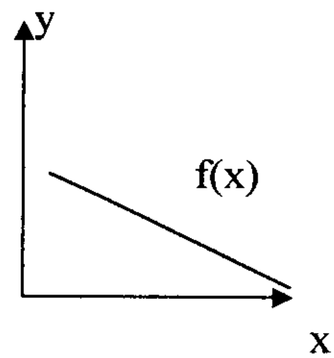
FOR DRIFT WAVES:



$t \Longrightarrow \Omega_i t, \qquad x \Longrightarrow x/\rho_s,$

$\phi \Longrightarrow e\phi/T_e, \qquad V=\rho_s/L_n$

FOR ROSSBY WAVE



f_0 : Coriolis frequency
ξ : Vertical (along y) displacement of the atmosphere with scale H_0
$\rho=\left(gH_0\right)^{1/2}/f_0$: Rossby scale - length

$t \Longrightarrow f_0 t, \qquad x \Longrightarrow x/\rho$

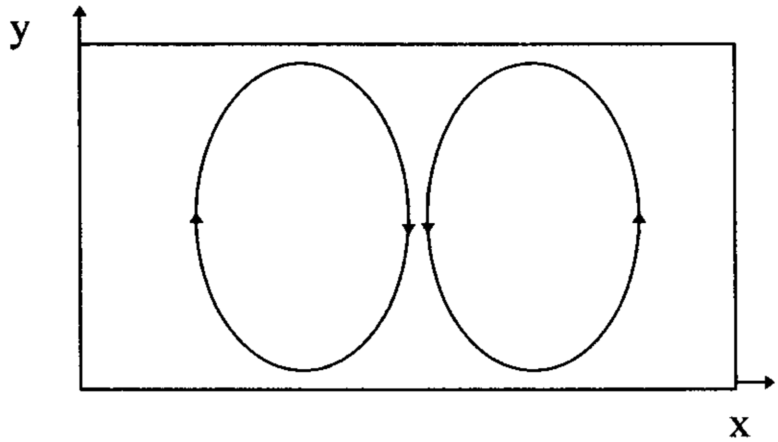
$\phi \Longrightarrow \xi/H_0, \; V=\rho/L_c$

Define potential vorticity $\omega=\phi-\nabla^2\phi-Vx$ then HMC equation is convection equation for potential vorticity.

NORMALIZED EQUATIONS FOR NUMERICAL STUDIES

NORMALIZE $x \rightarrow x/L_y$, $t \rightarrow t\mu / L_y^2$

$$\frac{\partial \omega}{\partial t} + v \cdot \nabla \omega = \nabla^2 \omega + F\mathrm{Sin}(mx / L_x)\mathrm{Sin}(y)$$

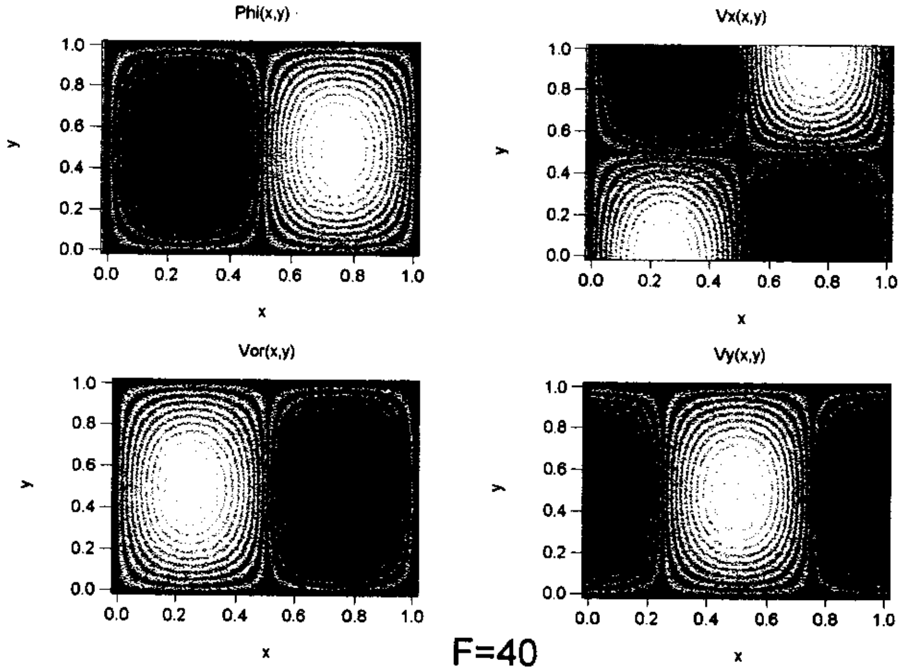


$$0 \le x \le 2\pi \quad 0 \le y \le \pi$$

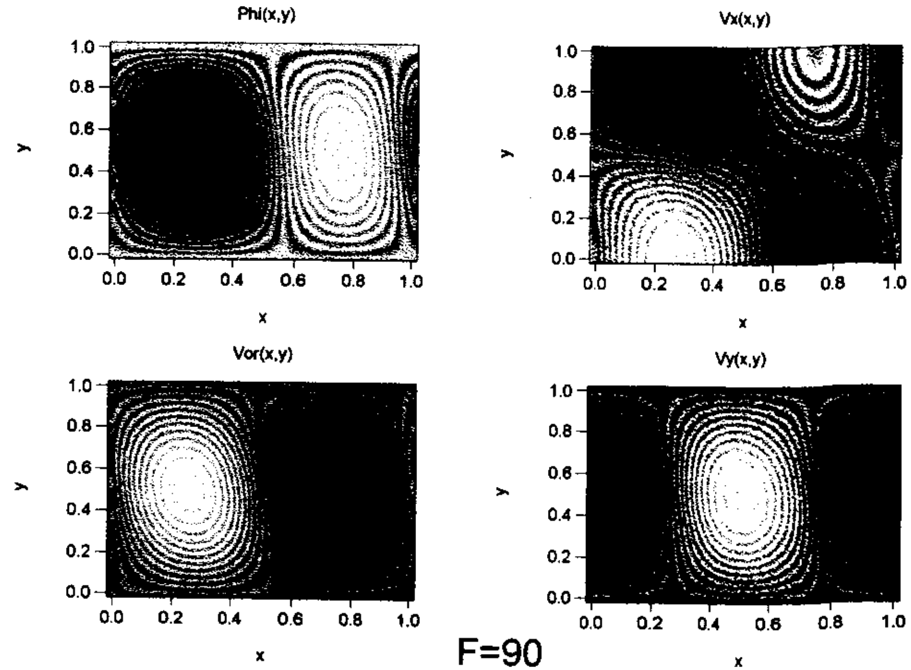
THREE DIMENSIONLESS PARAMETERS: L_x, m and F

- VORTICITY EQUATION SOLVED USING AN EXPLICIT SCHEME

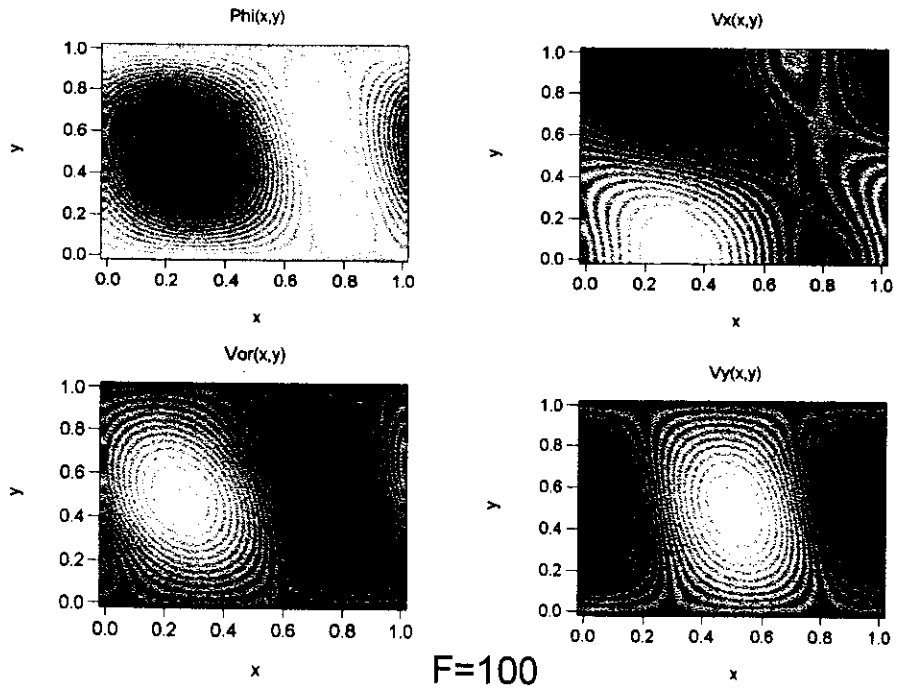
- INVERSION OF POISSON'S EQUATION TO OBTAIN ϕ : FFT IN x AND TRIDIAGONAL INVERSION IN y



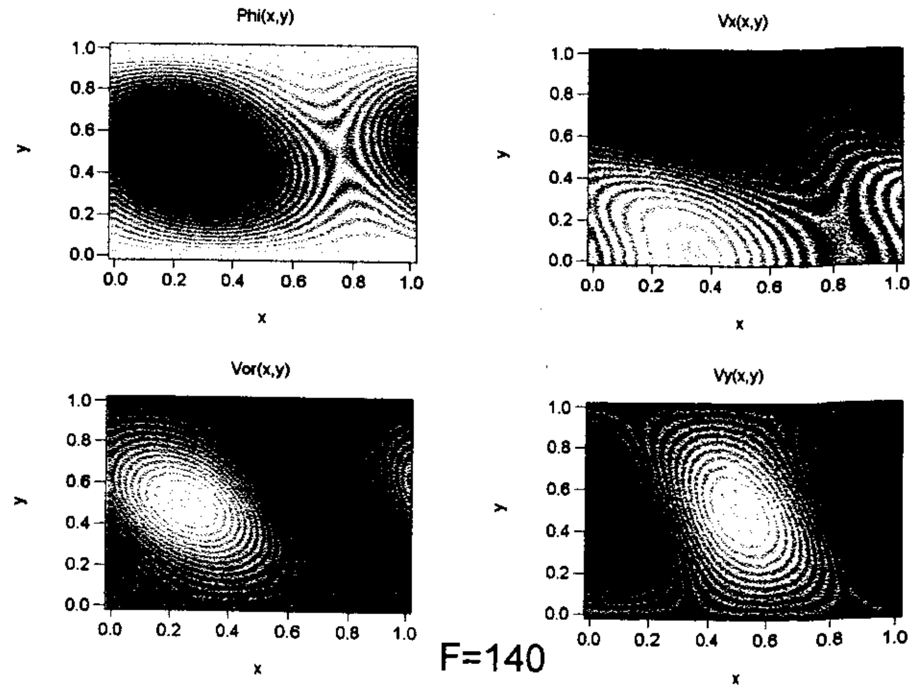
F=40



F=90



F=100



F=140

POINTS TO REMEMBER

1. In all four cases of forcing, the nonlinear state is a fixed point.

2. Tilt of the vortices can go either way.

3. For F=40, steady state has source balancing viscosity damping

4. For F=90, vortices tilt, vortex with vorticity co-directional with the shear flow grows in size.

5. For F=100, there is one vortex with a wavelength twice the size of the original wavelength.

6. For larger forcing F=140, there is a topological change in the stream function. Some streamlines are now open. This is the Cat's Eye Equilibrium.

7. For even larger forcing F > 140, the number of open streamlines increases compared to the number of closed ones.

NUMERICAL CODE

1. LINEAR TIME DEPENDENT CODE

$$\frac{\partial \tilde{\omega}}{\partial t} + \mathbf{v} \cdot \nabla \tilde{\omega} = -\tilde{\mathbf{v}} \cdot \nabla \omega + \mu \nabla^2 \tilde{\omega}$$

NORMALIZATIONS: $x \rightarrow x/L_y$, $t \rightarrow tV_{0y}/L_y$

$\mu \rightarrow \mu/V_{0y}L_y$

EQUILIBRIUM

$$\phi = \phi_0 \mathrm{Sin}(\pi y)\mathrm{Sin}\left(\frac{2\pi x}{L_x}\right)$$

$$\omega_0 = -\lambda^2 \phi_0, \quad \lambda^2 = \pi^2\left(\frac{4}{L_x^2} + 1\right)$$

FOR LINEAR STUDIES WE CHOOSE ϕ_0 SUCH THAT $v_{0y}=1$

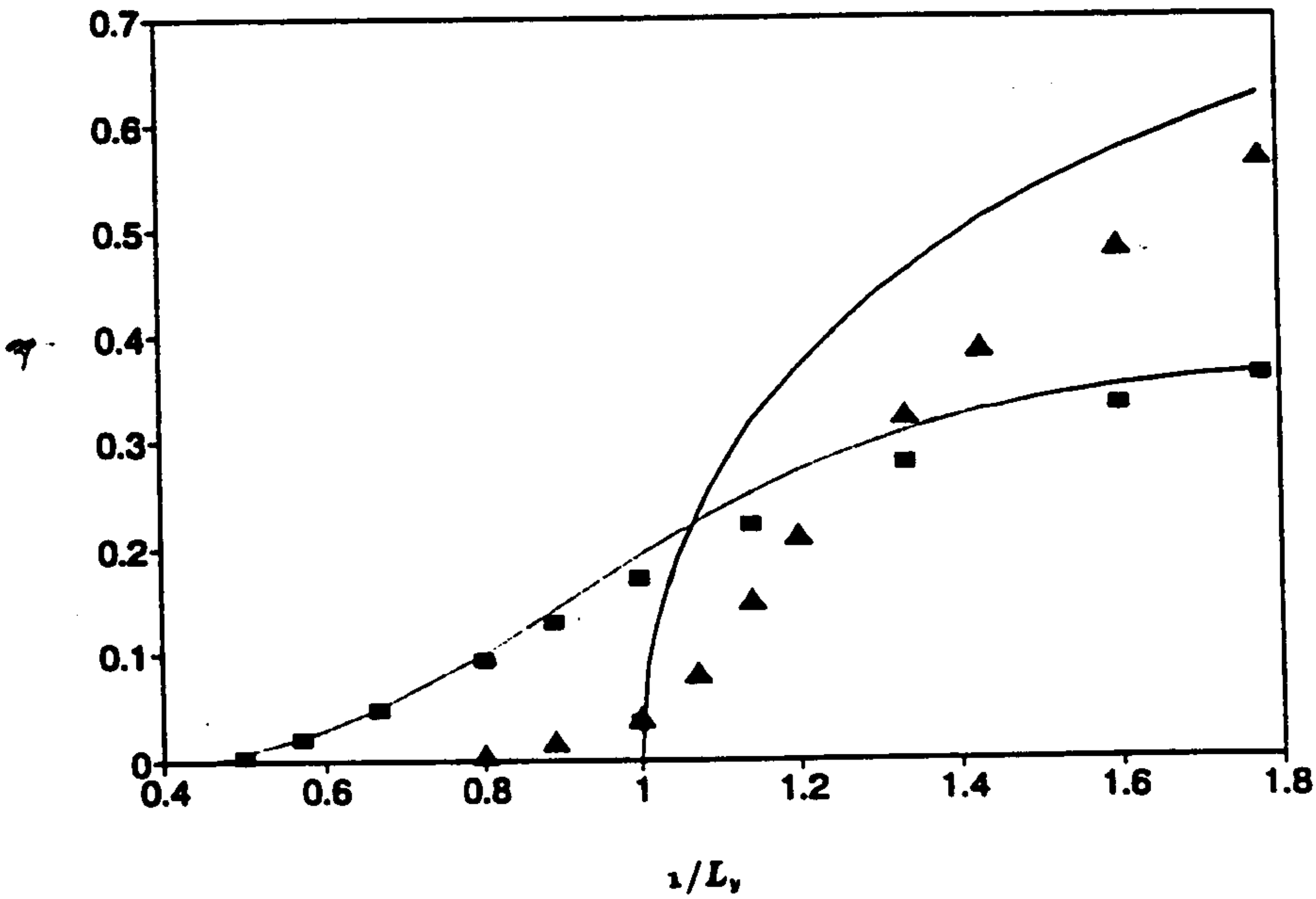
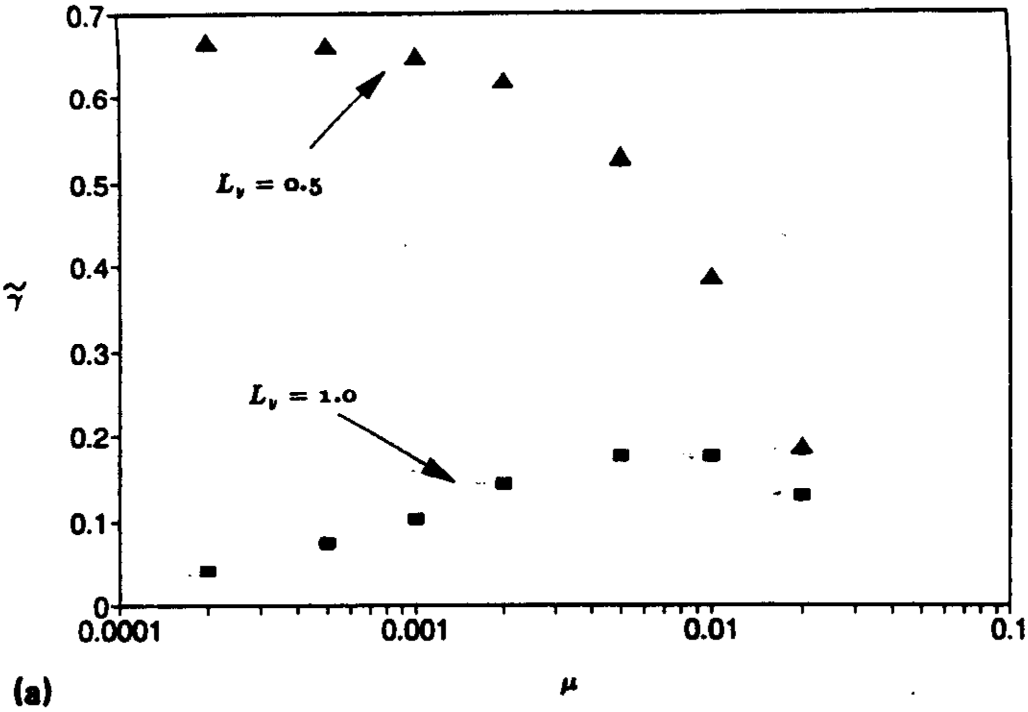
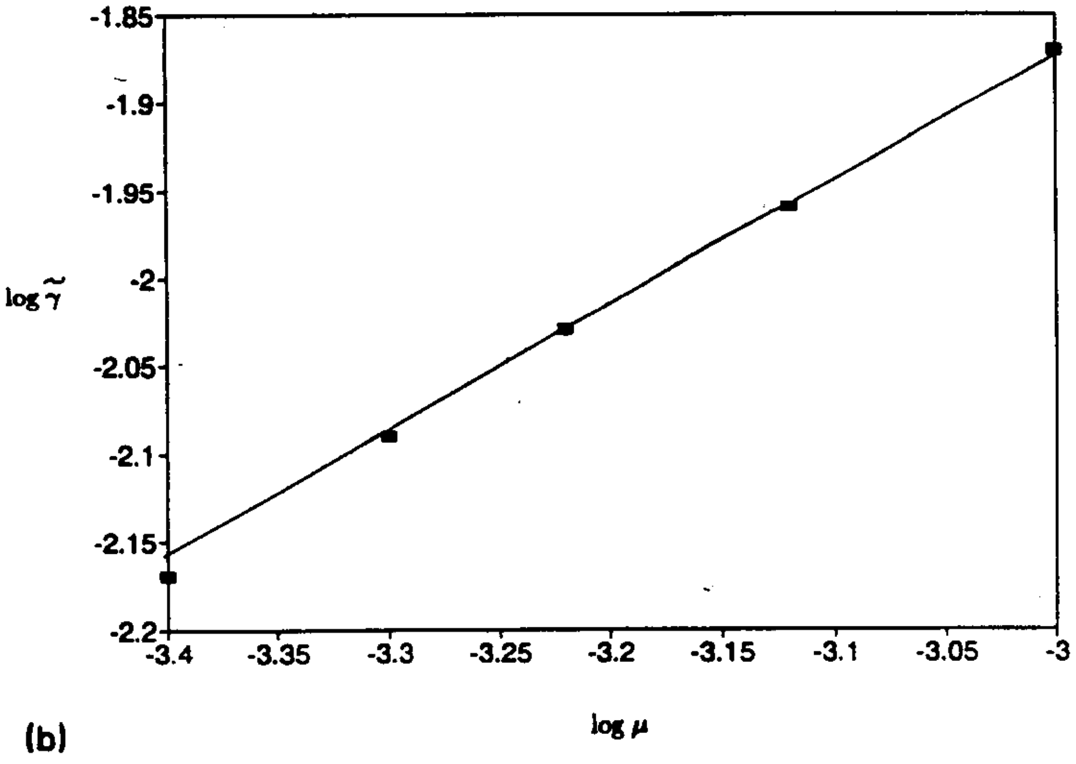


FIG. 4. Normalized growth rate $\tilde{\gamma}$ as a function of L_y for $\mu=10^{-2}$ (squares), $\mu=2\times10^{-4}$ (triangles). Ideal stability from (10b) and viscous stability for $\mu=10^{-2}$ from (11) are also shown.





F

μ

FIG. 3. Normalized growth rate $\tilde{\gamma}$ as a function of viscosity μ (a) for $L_y = 1/2$, $L_y = 1.0$; (b) $\log_{10} \tilde{\gamma}$ as a function of $\log_{10} \mu$ near $\mu = 0$ for $L_y = 1.2$; the solid line is $\tilde{\gamma} \propto \mu^{3/4}$.

NONLINEAR FOUR-WAVE MODEL

$$\phi(x,y,t) = \phi_0(t)\mathrm{Sin}\left(\frac{\pi y}{L_y}\right)\mathrm{Sin}\left(\frac{2\pi x}{L_x}\right) + \underbrace{\phi_1(t)\mathrm{Sin}\left(\frac{\pi y}{L_y}\right)}_{\text{shear flow}}$$

$$+\phi_2(t)\mathrm{Sin}\left(\frac{2\pi y}{L_y}\right)\mathrm{Cos}\left(\frac{2\pi x}{L_x}\right) + \underbrace{\phi_3(t)\mathrm{Sin}\left(\frac{3\pi y}{L_y}\right)}_{\text{preserves average vorticity}}$$

$$\kappa_0^2\dot{\phi}_0 = \frac{\pi^2}{L_x L_y}\left(\kappa_1^2 - \kappa_2^2\right)\phi_1\phi_2 + \frac{3\pi^2}{L_x L_y}\left(\kappa_2^2 - \kappa_3^2\right)\phi_2\phi_3 - \mu\kappa_0^4\phi_0 + S$$

$$\kappa_1^2\dot{\phi}_1 = \frac{\pi^2}{2L_x L_y}\left(\kappa_2^2 - \kappa_0^2\right)\phi_0\phi_2 - \mu\kappa_1^4\phi_1$$

$$\kappa_2^2\dot{\phi}_2 = \frac{\pi^2}{L_x L_y}\left(\kappa_0^2 - \kappa_1^2\right)\phi_0\phi_1 + \frac{3\pi^2}{L_x L_y}\left(\kappa_3^2 - \kappa_0^2\right)\phi_0\phi_3 - \mu\kappa_2^4\phi_2$$

$$\kappa_3^2\dot{\phi}_3 = \frac{3\pi^2}{2L_x L_y}\left(\kappa_0^2 - \kappa_2^2\right)\phi_0\phi_2 - \mu\kappa_3^4\phi_3$$

$$\kappa_0^2 = \left[\frac{4\pi^2}{L_x^2} + \frac{\pi^2}{L_y^2}\right],\ \kappa_1^2 = \frac{\pi^2}{L_y^2},\ \kappa_2^2 = \left[\frac{4\pi^2}{L_x^2} + \frac{4\pi^2}{L_y^2}\right],\ \kappa_3^2 = \frac{9\pi^2}{L_y^2}$$

VORTICITY BALANCE

AVERAGE THE VORTICITY EQUATION

$$\frac{\partial\langle\omega\rangle}{\partial t}=\mu\left\langle\frac{\partial^2\omega}{\partial y^2}\right\rangle$$

WHERE $\langle\bullet\rangle=\int_0^{L_y}dy\int_0^{L_x}dx\,\bullet$

1. INVISCID CASE (μ=0) AND d/dt $\neq$ 0 (LINEAR CASE)

$$\frac{d}{dt}(\phi_1+3\phi_3)=0 \quad\Rightarrow\quad \phi_1=-3\phi_3$$

2. VISCOUS CASE ($\mu \neq 0$) AND d/dt = 0 (NONLINEAR CASE)

$$\mu(\phi_1+27\phi_3)=0 \quad\Rightarrow\quad \phi_1=-27\phi_3$$

THUS THE ϕ_3 TERM CRUCIAL FOR STABILITY ANALYSIS AS WELL AS FOR NONLINEAR STEADY STATE SOLUTIONS

LINEARIZED THREE-WAVE EQUATIONS

$$\phi_1, \phi_2, \phi_3 \sim e^{\gamma t} << \Phi_0(\text{Given})$$

$$\kappa_1^2\left(\gamma+\mu\kappa_1^2\right)\phi_1 = \frac{\pi^2}{2L_xL_y}\left(\kappa_2^2-\kappa_0^2\right)\Phi_0\phi_2$$

$$\kappa_2^2\left(\gamma+\mu\kappa_2^2\right)\phi_2 = \frac{\pi^2}{L_xL_y}\left(\kappa_0^2-\kappa_1^2\right)\Phi_0\phi_1 + \frac{3\pi^2}{L_xL_y}\left(\kappa_0^2-\kappa_3^2\right)\Phi_0\phi_3$$

$$\kappa_3^2\left(\gamma+\mu\kappa_3^2\right)\phi_3 = \frac{\pi^2}{2L_xL_y}\left(\kappa_2^2-\kappa_0^2\right)\Phi_0\phi_2$$

'INVISCID' LIMIT (μ –> 0)

$$\frac{\phi_3}{\phi_1} = \frac{1}{3}$$

$$\gamma\phi_1 = \frac{3\pi^2}{2L_xL_y}\Phi_0\phi_2 \qquad\qquad \gamma\phi_2 = \frac{2\pi^2}{L_xL_y}\frac{\left(L_y^2-L_x^2\right)}{\left(L_x^2+L_y^2\right)}\Phi_0\phi_1$$

$$\gamma = \frac{\pi^2}{L_xL_y}\left[\frac{3\left(L_y^2-L_x^2\right)}{\left(L_x^2+L_y^2\right)}\right]^{1/2}\phi_0$$

INSTABILITY IF $L_y > L_x$. STABILIZATION BY ϕ_3

COMPLETE DISPERSION RELATION

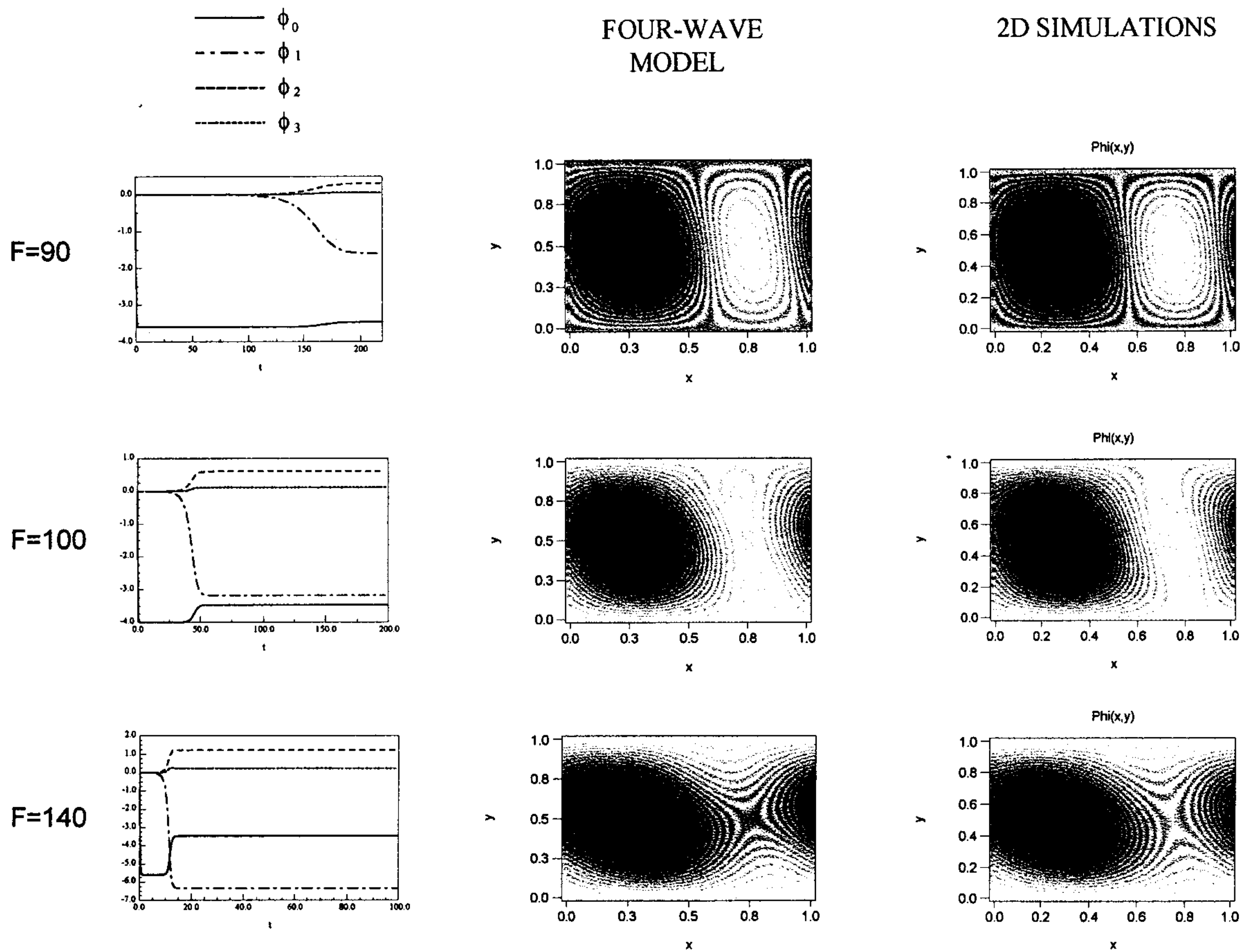
$$\kappa_1^2\kappa_2^2\kappa_3^2\left(\gamma+\mu\kappa_1^2\right)\left(\gamma+\mu\kappa_2^2\right)\left(\gamma+\mu\kappa_3^2\right)=\frac{\Pi_{xy}^2}{2}\left(\kappa_2^2-\kappa_0^2\right)\times$$

$$\left[\kappa_3^2\left(\kappa_0^2-\kappa_1^2\right)+9\kappa_1^2\left(\kappa_0^2-\kappa_3^2\right)\right]\phi_0^2$$

WITH $\Pi_{xy}=\dfrac{\pi^2}{L_x L_y}$

MARGINAL STABILITY (γ=0)

$$\mu=\mu_c=\frac{\phi_0}{2\sqrt{3}}\frac{\left(5-\dfrac{L_x^2}{L_y^2}\right)^{1/2}}{\left(1+\dfrac{L_x^2}{L_y^2}\right)}$$



φ 0
φ 1
φ 2
φ 3
FOUR-WAVE MODEL
2D SIMULATIONS
F=90
F=100
F=140
Phi(x,y)
t
x
y

NONLINEAR STEADY STATE SOLUTIONS OF THE FOUR-WAVE MODEL

SET d/dt=0 IN FOUR ODE'S

$$\phi_0 = \pm\sqrt{12}\,\frac{L_x}{L_y}\,\frac{\left[\dfrac{L_y^2}{L_x^2}+1\right]}{\left[5\dfrac{L_y^2}{L_x^2}-1\right]^{1/2}}\,\mu$$

$$\phi_1 = \pm\left[\frac{27}{4}\,\frac{\left(\phi_0-\phi(0)\right)}{\left(20\dfrac{L_y^2}{L_x^2}+11\right)}\right]^{1/2}\left[4\frac{L_y^2}{L_x^2}+1\right] \qquad \phi(0) = -\frac{F}{\mu\kappa_0^4}$$

$$\phi_2 = \frac{2\mu}{3}\,\frac{L_x}{L_y}\,\frac{\phi_1}{\phi_0}$$

$$\frac{\phi_3}{\phi_1} = -\frac{1}{27}$$

(SAME RESULT FOR AVERAGE VORTICITY CONDITION FOR TIME-INDEPENDENT NONLINEAR SOLUTION)

TWO FIXED POINTS

1. $\phi_0 = -\frac{F}{\mu\kappa_0^4}, \quad \phi_1 = \phi_2 = \phi_3 = 0$

2. $$F > F_c = 2\sqrt{3}\,\frac{\left[\frac{L_x^2}{L_y^2}+1\right]\left[\frac{4\pi^2}{L_x^2}+\frac{\pi^2}{L_y^2}\right]^2}{\left[5-\frac{L_y^2}{L_x^2}\right]^{1/2}}\,\mu^2$$

: STABILITY CONDITION FOR SHEAR FLOW INSTABILITY, THE SECOND EQUILIBRIA GIVEN BY THE ABOVE SET OF EQUATIONS.

FOR NUMERICAL WORK NORMALIZATIONS SUCH THAT $\mu=1$, $L_x = L_y = \pi$

$$F_c = 50\sqrt{3}, \quad \phi_0 = -2\sqrt{3}, \quad \phi_1 = \pm\frac{15\sqrt{3}}{2\sqrt{31}}\left(\frac{F}{25}+2\sqrt{3}\right)^{1/2}$$

$$\phi_2 = \mp\frac{5}{2\sqrt{31}}\left(\frac{F}{25}-2\sqrt{3}\right)^{1/2}, \quad \phi_3 = \mp\frac{5\sqrt{3}}{18\sqrt{31}}\left(\frac{F}{25}-2\sqrt{3}\right)^{1/2}$$

MINIMUM ENSTROPHY STATE

$$\frac{\partial \omega}{\partial t} + \mathbf{v} \cdot \nabla \omega = \mu \nabla^2 \omega$$

DEFINE

$$E = \frac{1}{2}\int_0^{L_y} dy \int_0^{L_x} dx |\nabla \phi|^2 = \frac{1}{2}\int_0^{L_y} dy \int_0^{L_x} dx v^2 \quad \text{ENERGY}$$

$$\Omega = \frac{1}{2}\int_0^{L_y} dy \int_0^{L_x} dx \omega^2 \qquad \text{ENSTROPHY}$$

"HYPOTHESIS" : FIND STATES WITH MINIMUM ENSTROPHY WITH ENERGY CONSTANT

$\delta(\Omega - \lambda E) = 0$ λ LAGRANGE MULTIPLIER

$$\nabla \times \nabla \times \mathbf{v} = -\nabla^2 \mathbf{v} = \lambda \mathbf{v}$$

$$\phi = \phi_0 \text{Sin}\left(\frac{n\pi y}{L_y}\right) \text{Sin}\left(\frac{2m\pi x}{L_x}\right)$$

$$\lambda = \pi^2\left(4m^2 + n^2\right)$$

SMALLEST λ FOR m=0 AND n=1. SINCE $\lambda=\Omega/E$ THIS IS STATE WITH MINUMUM ENSTROPHY.

SUMMARY OF RESULTS FOR TWO VORTEX CASE

1. FOR $F > F_C$ (OR $\mu < \mu_c$) SHEAR FLOW INSTABILITY OCCURS

LINEAR STABILITY STUDY

- FOR INVISCID CASE INSTABILITY IF $L_x/L_y < 1$
- FOR VISCOUS CASE $\gamma \propto \mu^{3/4}$
- FOUR WAVE MODEL DEVELOPED WITH EXTRA TERM TO ACCOUNT FOR CONSERVATION OF MEAN VORTICITY
- GOOD AGREEMENT BETWEEN NUMERICAL RESULTS AND LINEARLIZED FOUR-WAVE MODEL

NONLINEAR STUDY FOR TWO DRIVEN VORTICES

- $F < F_C$ STEADY STATE WITH FORCING BALANCING VISCOUS DRAG
- $F > F_C$ STEADY STATE
  1. TILTED VORTICES
  2. SINGLE VORTEX
  3. SINGLE VORTEX IN SHEAR FLOW (CAT'S EYE EQUILIBRIUM)
- GOOD AGREEMENT BETWEEN 2D NUMERICAL WORK, FOUR-WAVE NONLINEAR MODEL AND OBSERVATIONS OF TABELING ET AL. FOR LOW FORCING

NONLINEAR STUDY WITHOUT FORCING

- FOR $\mu < \mu_c$, INITIAL TWO VORTEX STATE TILTS GENERATES SHEAR FLOW (AS IN FORCED CASE)
- AT LATE TIME, ONLY DECAYING SHEAR FLOW WHICH IS LONGEST WAVELENGTH MODE CONSISTENT WITH PRESENT BOUNDARY CONDITIONS

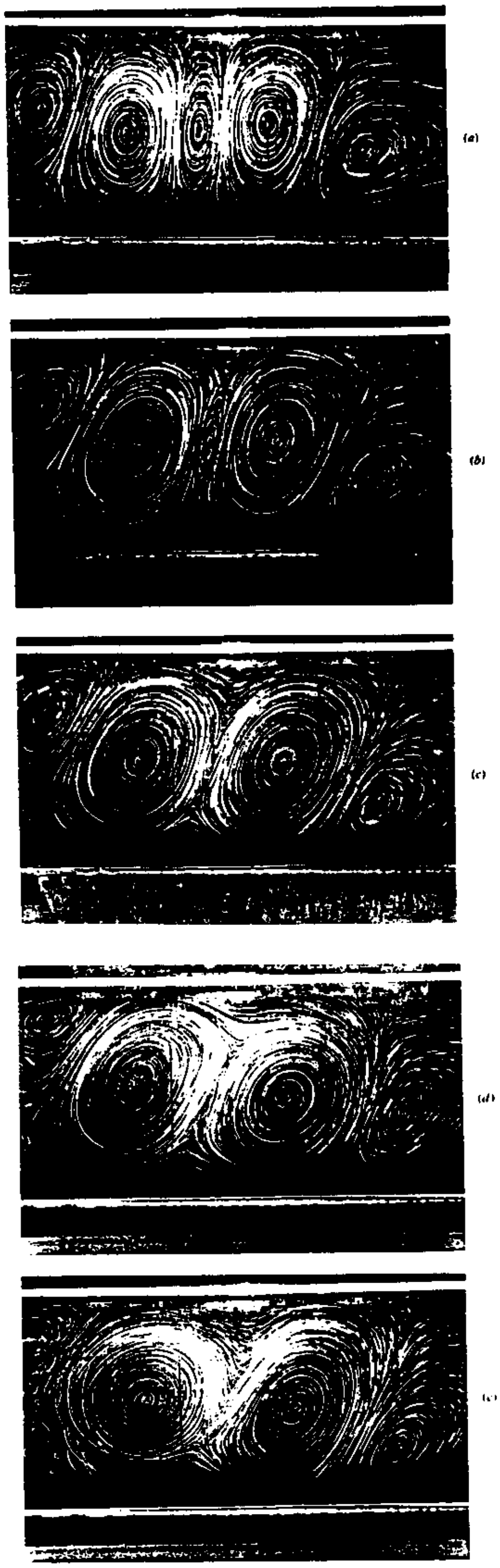


FIGURE 3. Different states of flow for the case of four magnets, for $b = 2.50$ mm and for various values of I: (a) $I = 4.93$ mA; (b) 10.81 mA; (c) 13.09 mA; (d) 20.84 mA; (e) 24.75 mA.

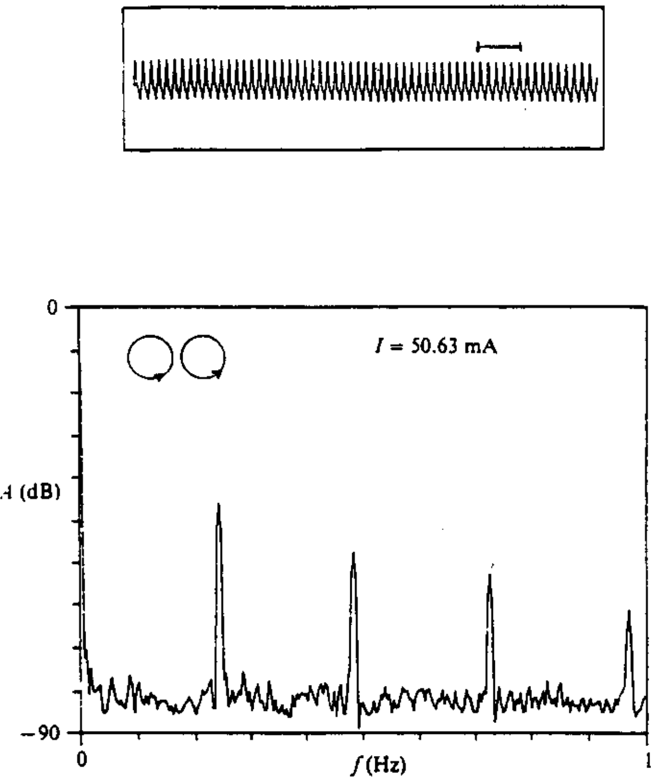


FIGURE 4. Direct time recording and Fourier spectrum of the signal obtained on a detector, for the case of four magnets, for $n = 2.61$ mm.

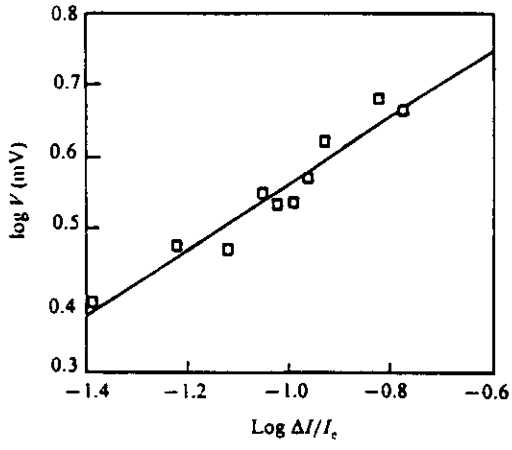


FIGURE 5. Dependence, on a log-log plot, of the amplitude of the time dependent mode on the ratio of ΔI to the critical point I_c, (where $\Delta I = I - I_c$) for the four-magnet case, with $b = 2.11$ mm. In this experiment, I_c is 18.45 mA. The straight line has a slope equal to 0.5.

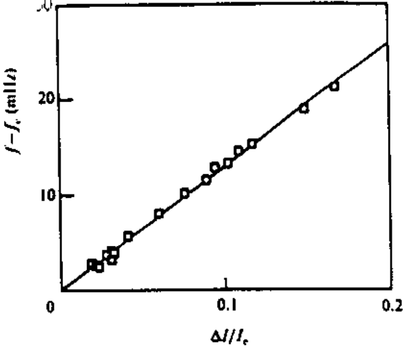


FIGURE 6. Variation of the quantity $f-f_c$ with $\Delta I/I_c = (I-I_c)/I_c$, for the case of four magnets, with $b = 2.11$ mm. In this experiment, $f_c = 148.5$ mHz.

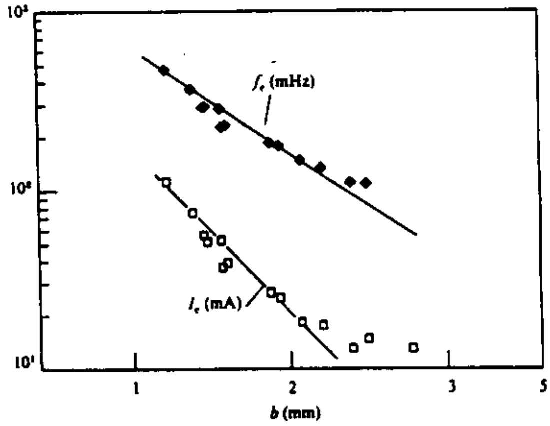


FIGURE 7. Dependence of the critical values of the current and the frequency on the fluid thickness, for the four-magnet case. The straight lines fitting the data for f_c and I_c have slopes -2 and -3 respectively.

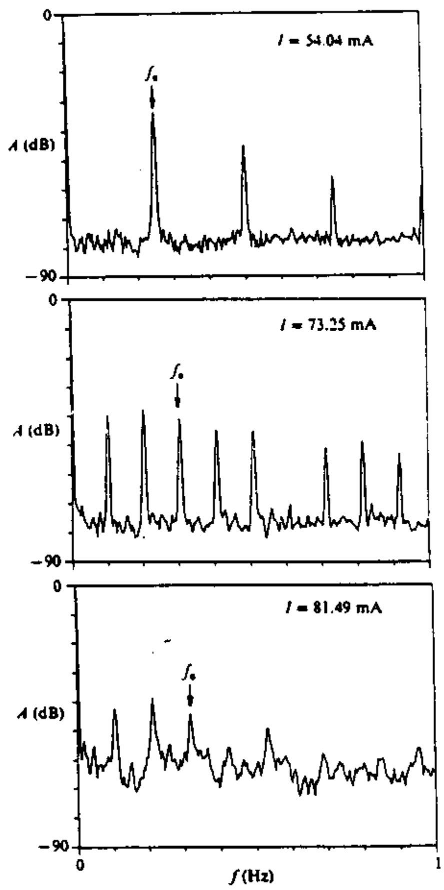


FIGURE 9. Power spectrum analysis of the signal obtained on the photodiodes for different values of I, for the case of four magnets, with $b = 2.50$ mm.

SUMMARY OF OBSERVATIONS FOR THE FOUR VORTEX CASE

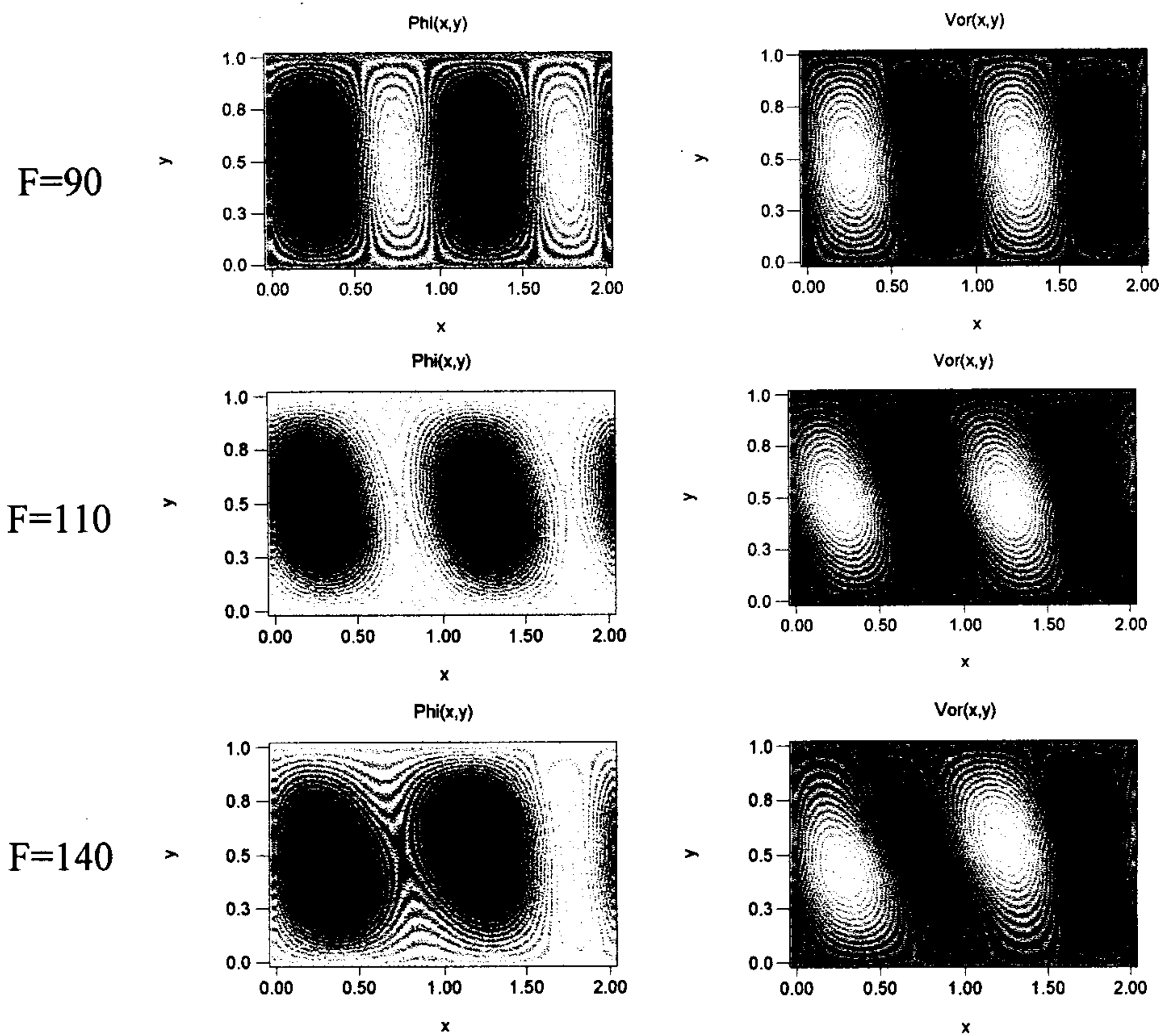
1. FOR LOW CURRENT, GET STEADY STATES :
   - FOUR VORTEX
   - TWO VORTEX WITH SHEAR FLOW

2. FOR $I > I_C$, SHAPE AND THE CENTER OF THE VORTEX FLUCTUATE WITH TIME

   - PERIODIC
   - AMPLITUDE OF OSCILLATION $A \propto \sqrt{(I-I_C)}$
   - $F-F_C \propto (I-I_C)$
   - RANGE OF THIS BEHAVIOR FROM I_C TO $\sim 3\ I_C$

3. TRANSITION TO CHAOS

   - 1/3 SUBHARMONIC (PERIOD THREE) ROUTE TO CHAOS


F=90
F=110
F=140
Phi(x,y)
Vor(x,y)
y
x
1.0
0.8
0.5
0.3
0.0
0.00
0.50
1.00
1.50
2.00

SECONDARY INSTABILITY ASSOCIATED WITH VORTEX-PAIRING OF CO-ROTATING VORTICES

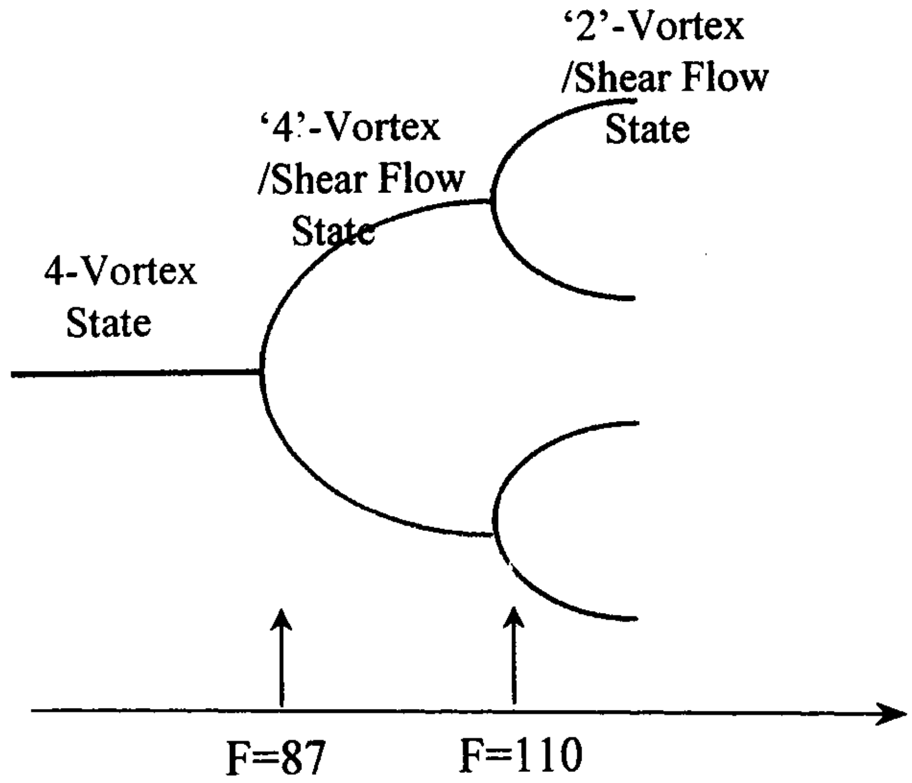
- POINT VORTICES OF EQUAL STRENGTH ROTATE AROUND THE COMMON VORTICITY CENTROID

- FINITE SIZED VORTICES CAN ALSO MERGE IF THE DISTANCE BETWEEN THEIR CENTROIDS IS SMALLER THAN A CRITICAL DISTANCE

- FOR TWO ISOLATED SYMMETRIC VORTICES ONCE MERGING IS INITIATED , THE FINAL STATE IS A SINGLE SYMMETRIC VORTEX

REF: M.V. MELANDER, N. J. ZABUSKY AND J.C. McWILLIAMS J. FLUID MECH. 195,303(1988)

Low Forcing Fixed Points



——— TWO STATES DUE TO POSITIVE AND NEGATIVE SHEAR FLOW VORTICITY

.... TWO STATES FOR EACH SHEAR FLOW STATE DUE TO TRANSLATIONAL SYMMETRY

TIME-DEPENDENT CASES $F > F_C$

PLOTS ARE OF

$\phi(0,1)$: $\mathrm{Sin}(y) \quad 0 < y < \pi$

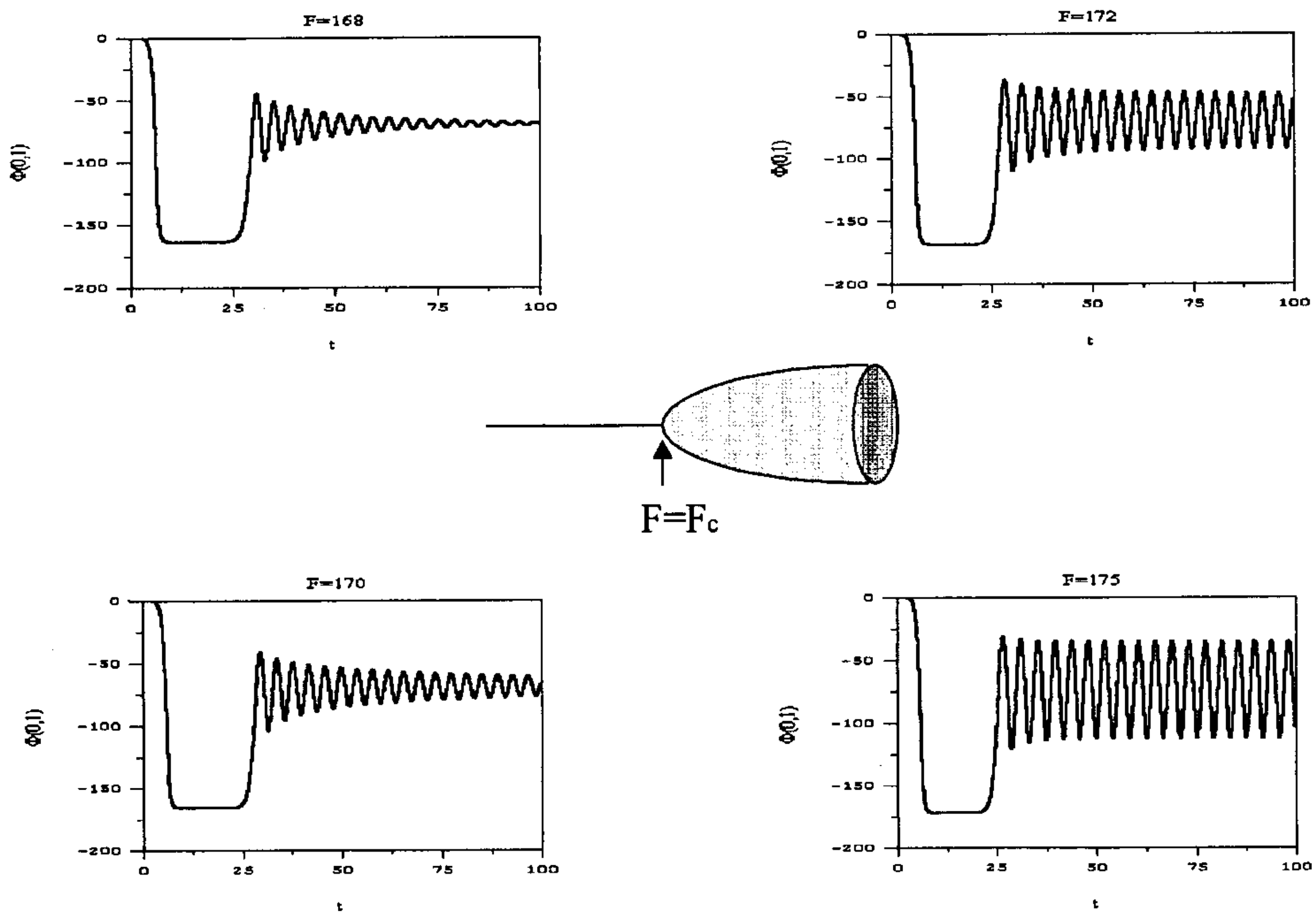
$\phi_s(1,1)$: $\mathrm{Sin}(x)\mathrm{Sin}(y) \quad 0 < y < \pi, 0 < x < 2\pi$

$\phi_c(1,1)$: $\mathrm{Cos}(x)\mathrm{Sin}(y) \quad 0 < y < \pi, 0 < x < 2\pi$

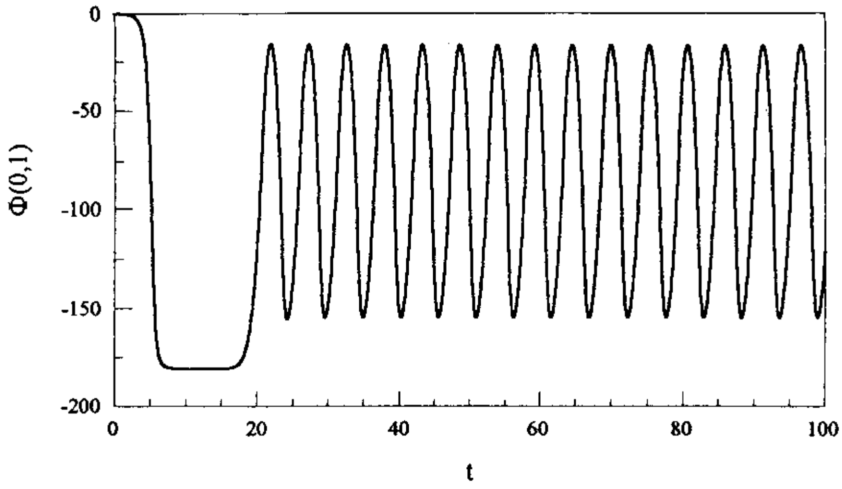
$\phi_s(2,1)$: $\mathrm{Sin}(2x)\mathrm{Sin}(y) \quad 0 < y < \pi, 0 < x < 2\pi$

$\phi_c(2,1)$: $\mathrm{Cos}(2x)\mathrm{Sin}(y) \quad 0 < y < \pi, 0 < x < 2\pi$

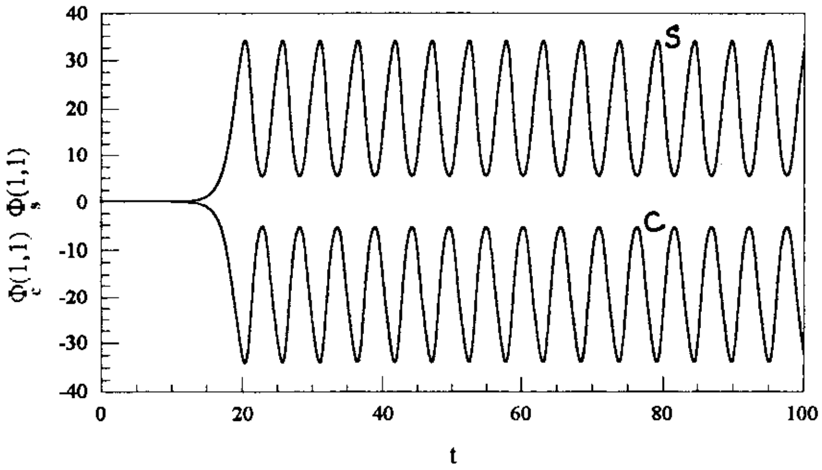
HOPF BIFURCATION



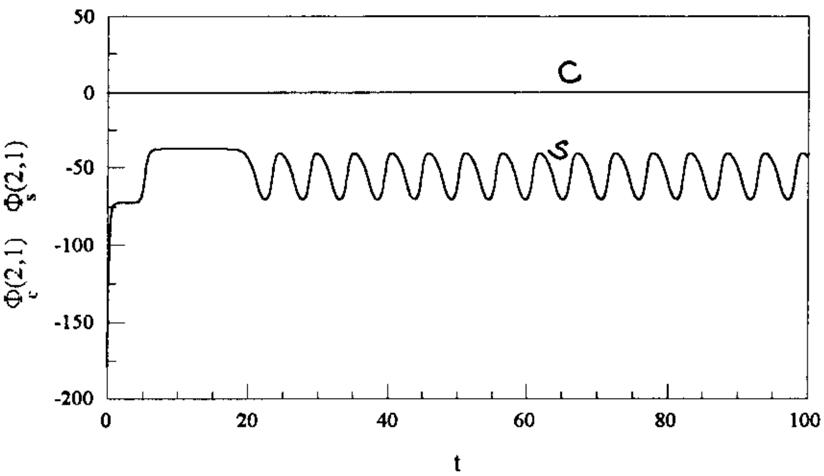
F=182



0
-50
-100
-150
-200
Φ(0,1)
0
20
40
60
80
100
t

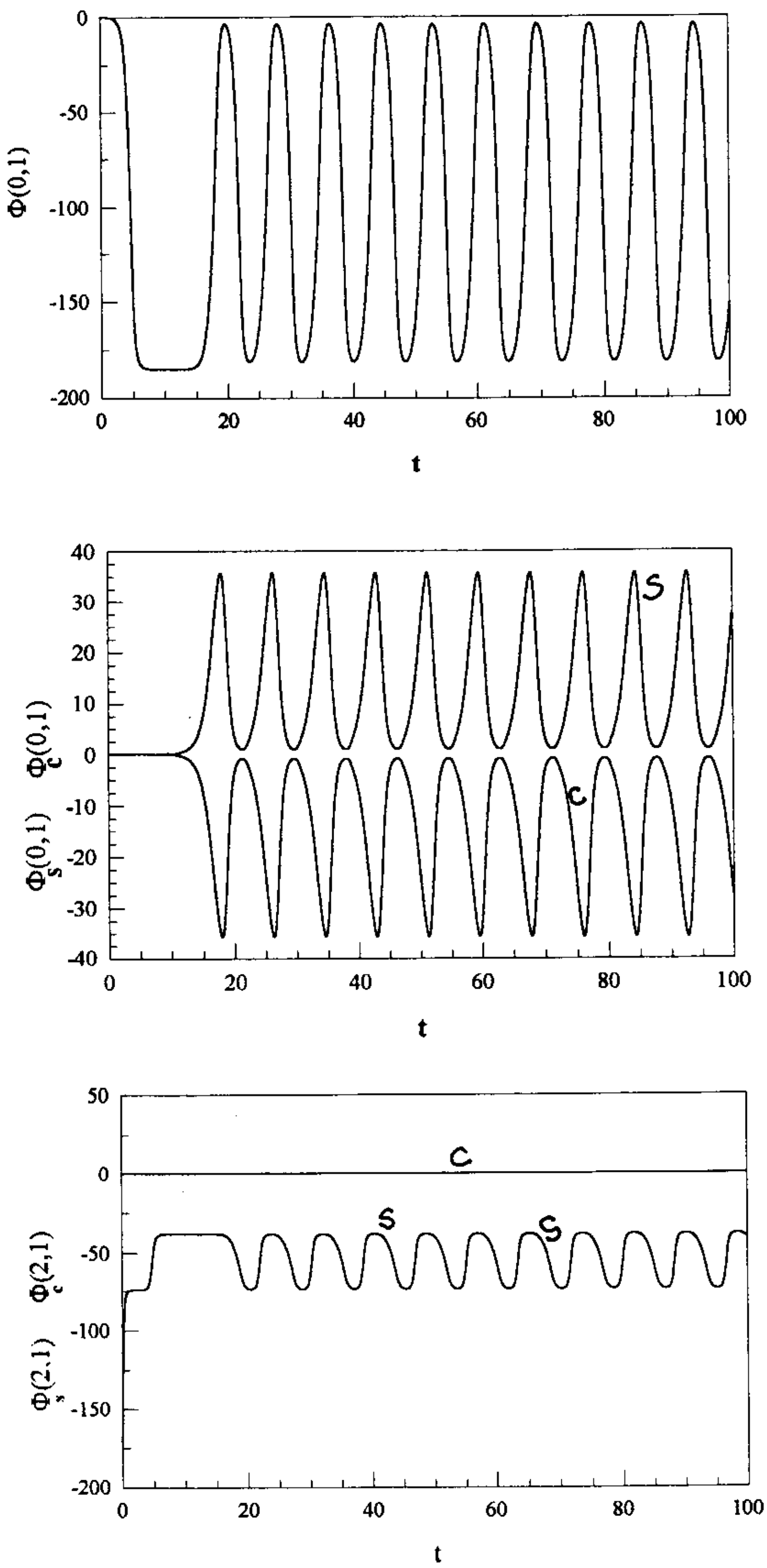


40
30
20
10
0
-10
-20
-30
-40
Φ(1,1) Φ(1,1)
s
c
0
20
40
60
80
100
t

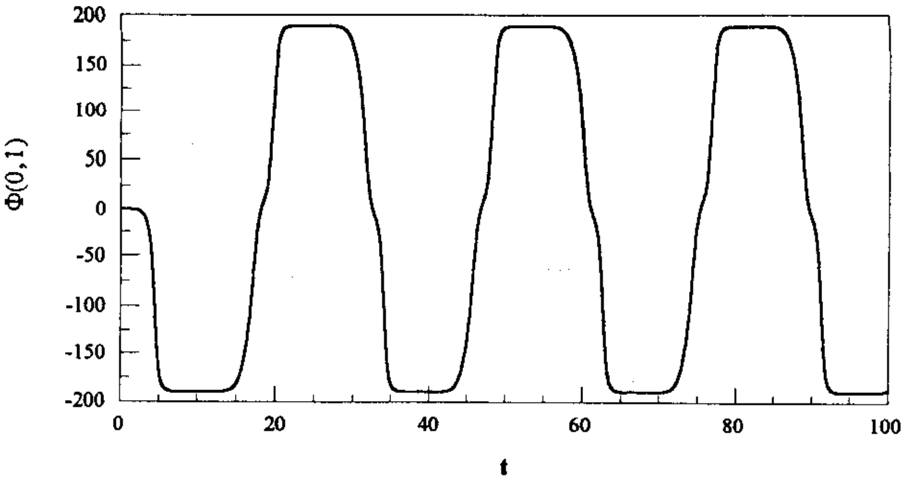


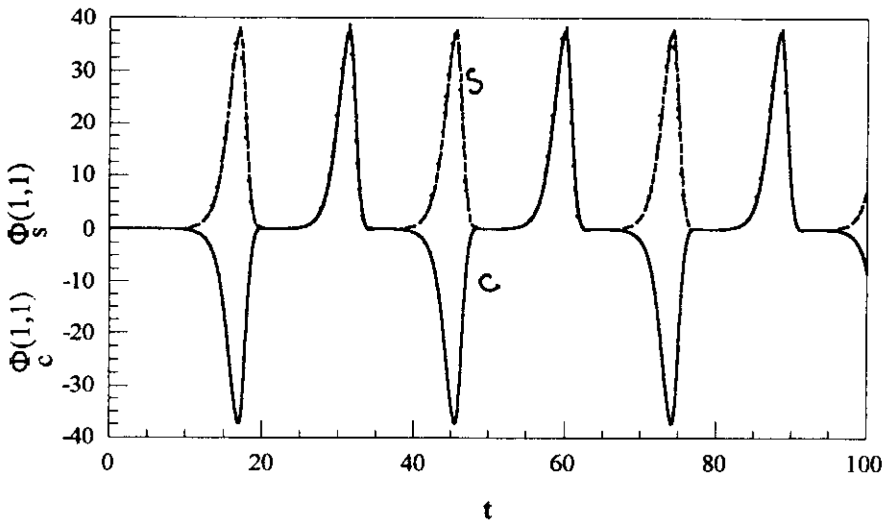
50
0
-50
-100
-150
-200
Φ(2,1) Φ(2,1)
c
s
0
20
40
60
80
100
t

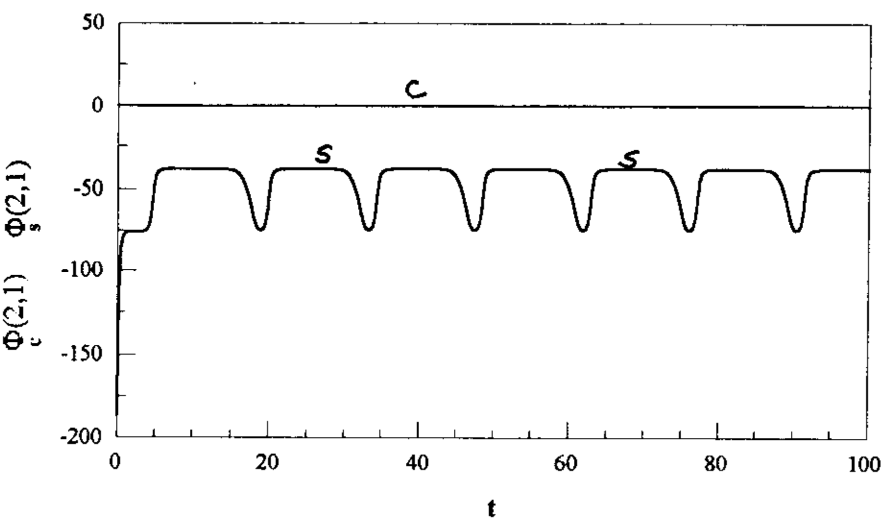
F=186

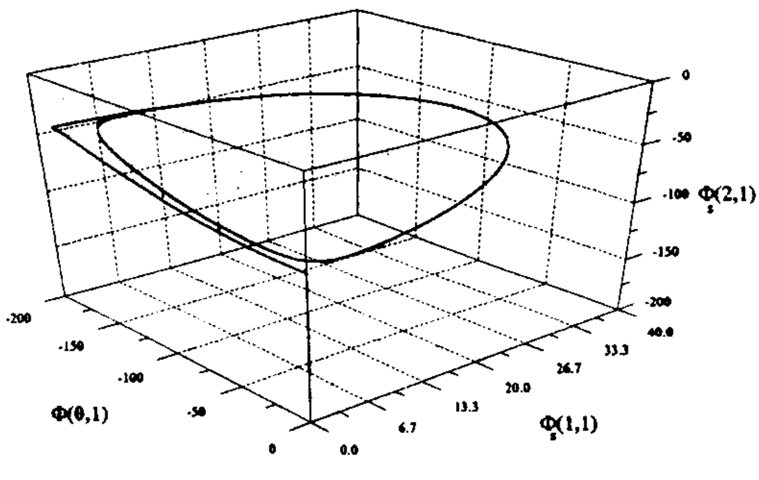


F=190

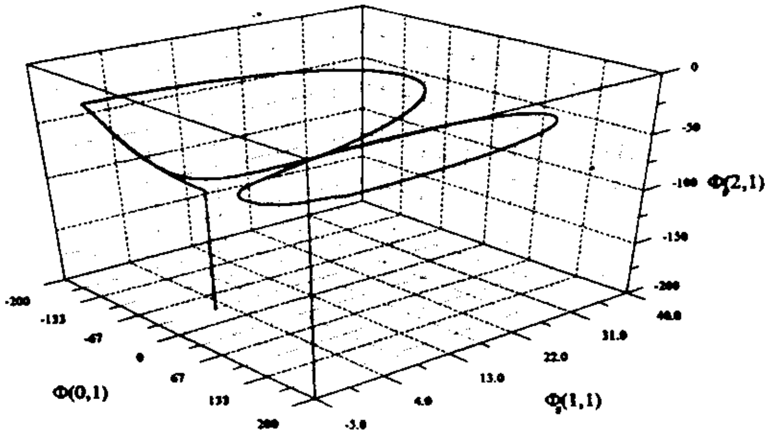






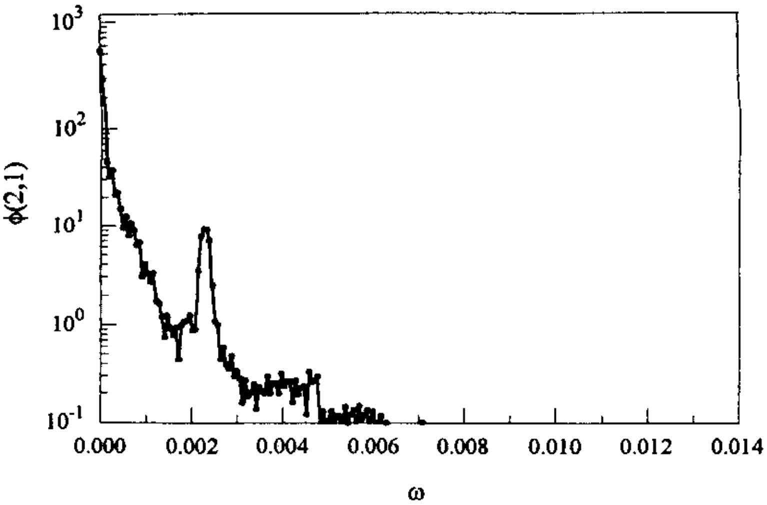


F=186

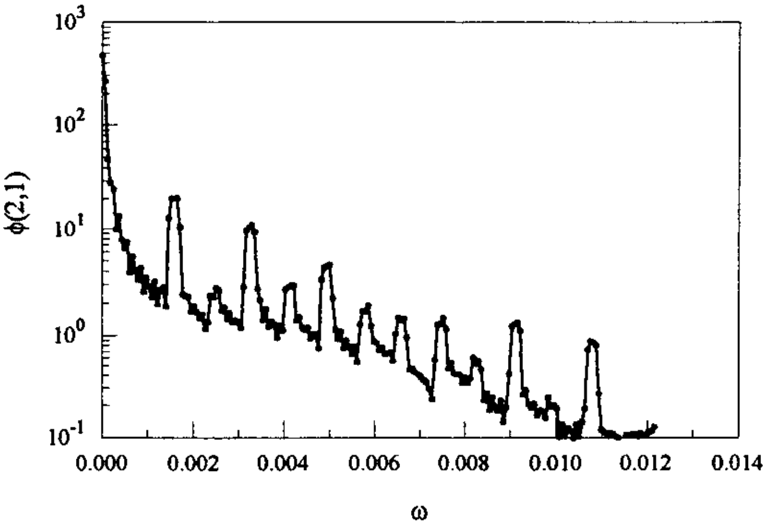


F=190

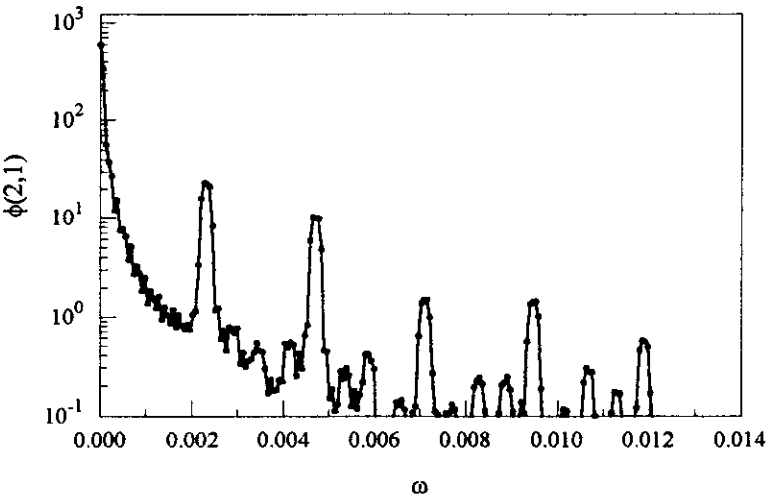
F=175



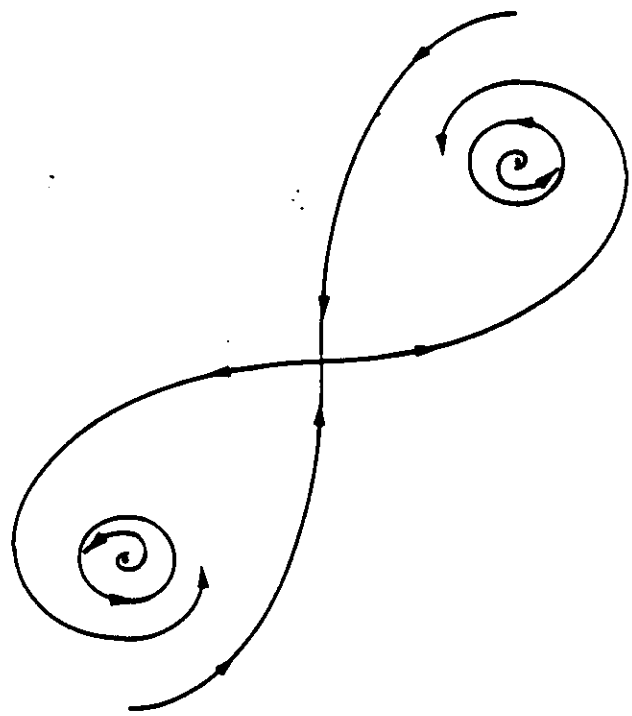
F=300



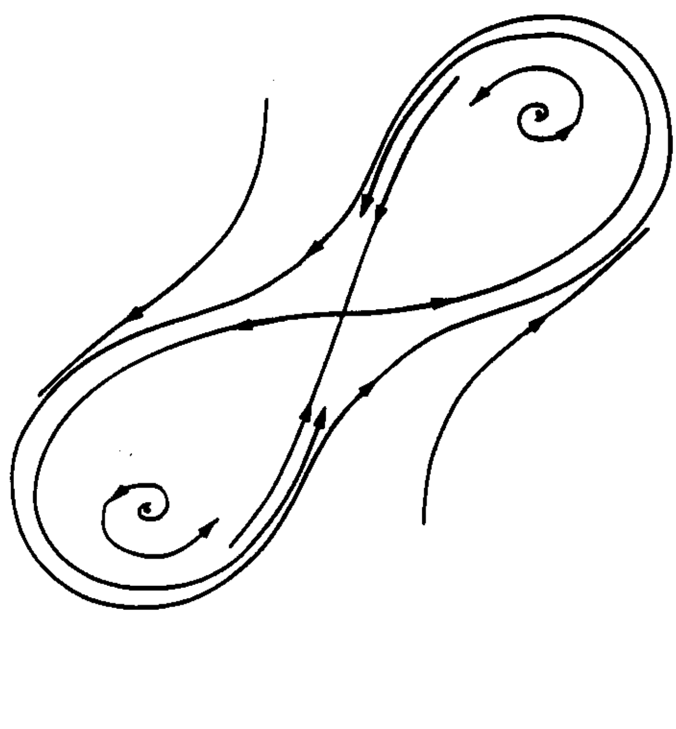
F=400



GLOBAL BIFURCATION

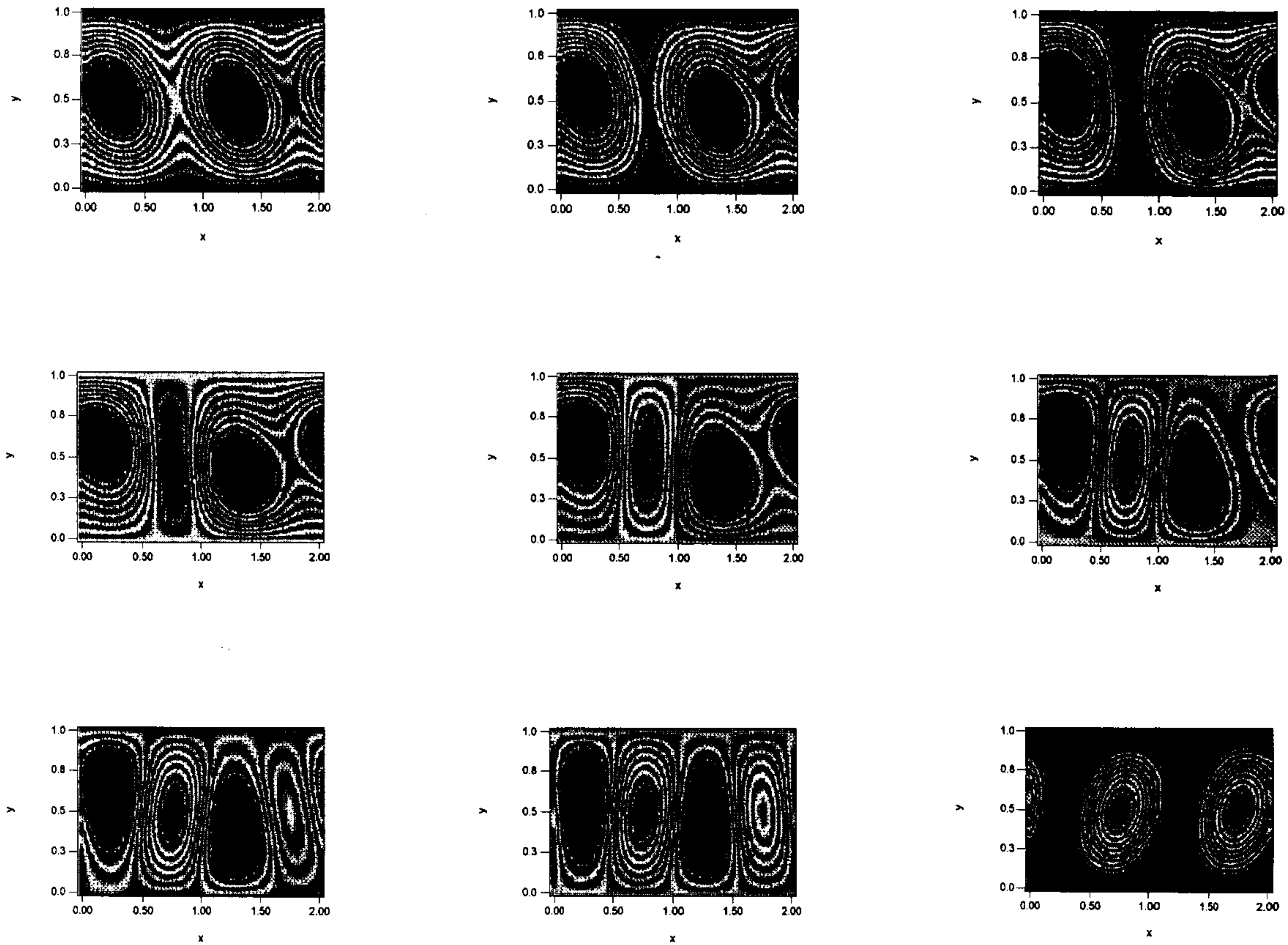


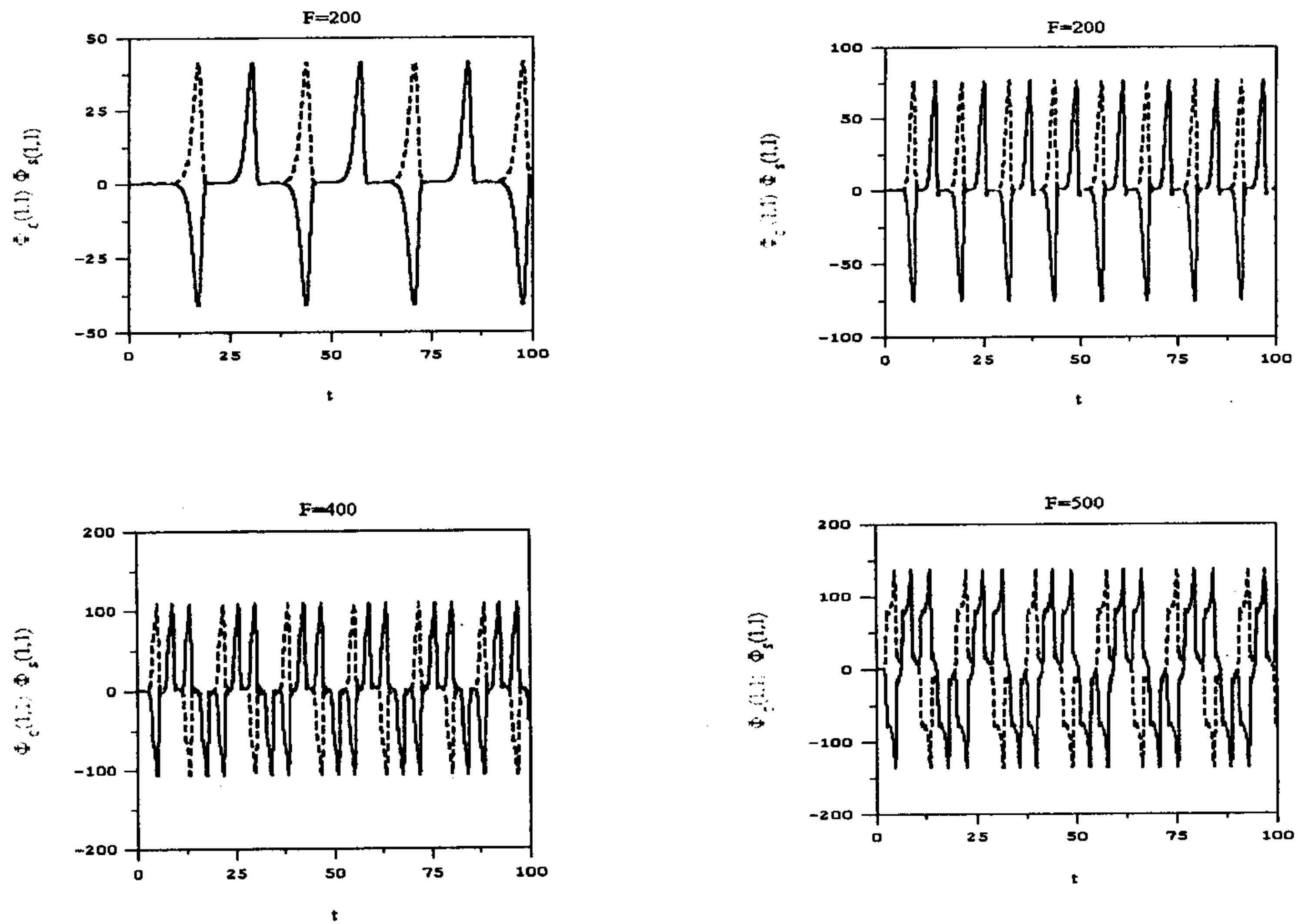
BEFORE



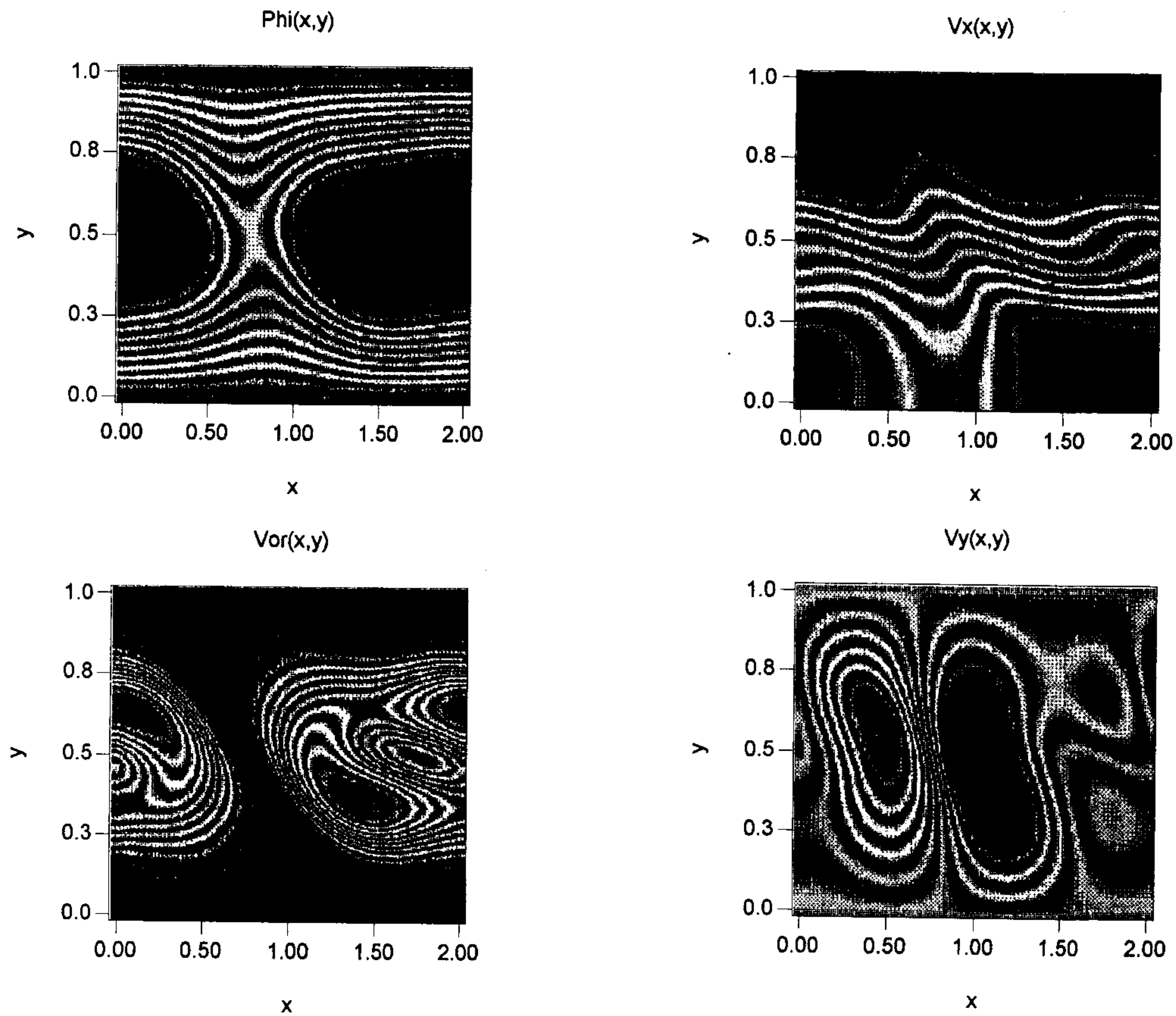
AFTER

$\Phi(x,y)$ F=190




F=200
F=200
F=400
F=500
$\Phi_c(1,1)$ $\Phi_s(1,1)$
t

F=5000



FORCING F	NONLINEAR STATE
$F < 86$	Four vortex state
$86 < F < 110$	Four vortex with shear flow
$110 < F < 170$	Two vortex with shear flow
$F = 170$	Supercritical Hopf Bifurcation
$170 < F < 190$	Periodic state with frequency decreasing with F
$F = 190$	Global bifurcation with self-reversal of shear flow
$190 < F < 400$	Periodic state with amplitude and frequency increasing with F
$400 < F < 700$	Periodic state with frequency decreasing with F
$700 < F < 6000$	Single Vortex with Shear flow steady state
$F > 6000$	Quasi-periodic and chaos

TRANSITION TO CHAOS IN 2D FORCED VORTICES –EIGHT VORTEX CASE

R. BRAUN, F. FEUDEL AND P. GUZDAR

FIGURES

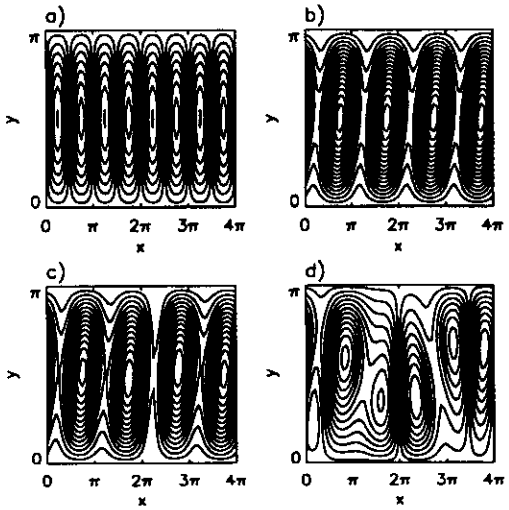


FIG. 1. Streamlines of the steady state branches: a) Steady I (f = 26.7), b) Steady II ($f = 43.3$), c) Steady III ($f = 44.7$), d) Steady IV ($f = 100.0$).

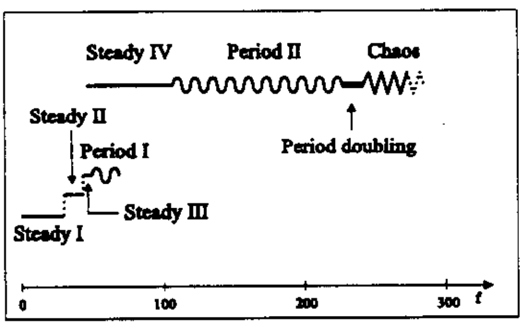


FIG. 2. Schematic bifurcation diagram

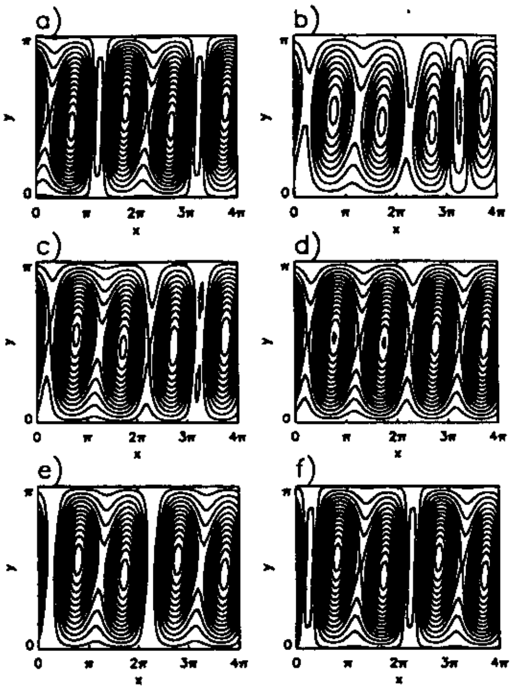


FIG. 3. Snapshots of streamlines belonging to a periodic solution (Period I) at different time points.

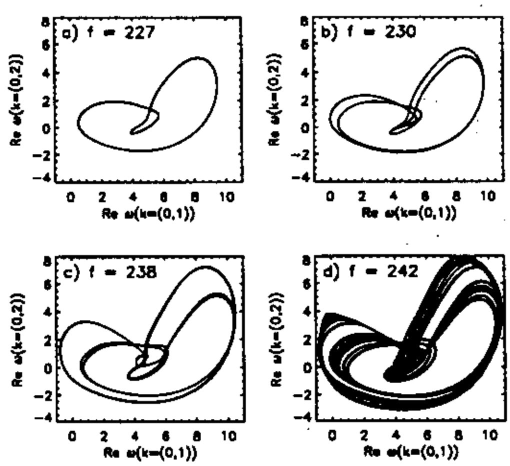


FIG. 4. Projection of the trajectory onto the real parts of the mode $\boldsymbol{k} = (0,1)$ and $\boldsymbol{k} = (0,2)$ in the period doubling cascade on the route to chaos for (a) periodic orbit (Period II), (b) orbit after first period doubling, (c) orbit after second period doubling (d) chaotic attractor (Chaos).

EXTENSION OF PRESENT WORK

1. DEVELOP A LOW-DIMENSIONAL DYNAMICAL MODEL FOR FOUR-VORTEX CASE

2. A SIMILAR STUDY CAN BE UNDERTAKEN FOR DRIVEN VORTICES FOR THE HASEGAWA-MIMA-CHARNEY EQUATION.

   - LINEAR STABILITY OF DRIFT ROSSBY-WAVE TO THE SHEAR-FLOW INSTABILITY INVESTIGATED (GUZDAR PHYS. PLASMAS 1996)
   - NONLINEAR STUDY WITH SOURCE NEEDS TO BE INVESTIGATED
   - RED SPOT OF JUPITER IS ROSSBY-WAVE VORTEX IN A SHEAR FLOW

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