



H4-SMR 1012 - 36

AUTUMN COLLEGE ON PLASMA PHYSICS

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The Electron Beam Plasma Instability: Space Observations, Theory and Simulation

(continued)

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These are lecture notes, intended for distribution to participants.

3. Dynamics of the Wave-Beam Interaction and Quasi-linear Theory

Energy conservation:

$$-\Delta D_b = \Delta W_b + K_e + W_E \quad (1)$$

where $D_b = n_b m u^2 / 2$ beam drift energy density, W_b beam thermal energy, $W_E = \langle E^2 / 8\pi \rangle$ electric field density (averaged over space), $K_e = \Delta W_e + \Delta D_e$ kinetic energy of the bulk electrons associated with wave motion, $K_e \approx \Delta W_e$, as $D_e = 0$ initially $\rightarrow D_e \approx (\Delta D_b)^2 / n m u^2$

$$\omega_e t = 936$$

$$8.8 \times 10^{-5} = (1.1 + 4.2 + 3.5) \times 10^{-5}$$

in units of $n_e m u^2$.

Quasilinear integral:

$$\frac{\partial N_k}{\partial t} = \int dv V_k \left(N_k \frac{k}{m} \frac{\partial}{\partial v} + 1 \right) f(v)$$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v} \int \frac{dk}{2\pi} \frac{k}{m} V_k \left(N_k \frac{k}{m} \frac{\partial}{\partial v} + 1 \right) f(v)$$

where N_k number of plasmons $V_k = M_k \delta(\omega_k - kv)$

$$m \frac{\partial f}{\partial t} = \frac{\partial}{\partial v} \frac{1}{v} \frac{\partial U_v}{\partial t}$$

$$U_v = -mv \Delta N(v, v_u)$$

$$U_k = N_k \omega_k = \omega (\partial \epsilon / \partial \omega) W_k$$

$$U = \sum_k U_k \rightarrow \frac{L}{2\pi} \int dk U$$

$$U_k = \frac{L}{2\pi} \frac{\partial U}{\partial k}$$

4. Interaction with the Bulk Plasma and Mode Coupling

Nonlinear interaction between wave modes:

a.) Nonlinear Landau damping

$$\omega_k \pm \omega_{k'} - (k \pm k')v = 0, \quad \Delta N_k = \pm \Delta N_{k'}$$

$$\frac{\partial N_k}{\partial t} = \sum_{j,k'} \int dv V_{k,\pm k'} [N_k N_{k'} \frac{k \pm k'}{m} \frac{\partial}{\partial v} + N_{k'} \pm N_k] f_j(v)$$

$\epsilon_{po} + e \leftrightarrow \epsilon_{po}$: cancellation between Compton scattering and scattering off shielding cloud $(\epsilon - \nu, \epsilon \nu, -\epsilon^* \nu)$

$\epsilon_{po} + i \leftrightarrow \epsilon_{po}$ $(\epsilon - i - \nu)$

$$\frac{\omega_k - \omega_{k'}}{k + k'}$$

b.) resonant mode coupling $(\epsilon - \nu - i \nu)$

$$\omega_k = \omega_{k'} \pm \omega_{k''} \quad k = k' \pm k''$$

~~$$\Delta N_k = \pm \Delta N_{k'} \pm \Delta N_{k''}$$~~

$$\Delta N_{k''} = \pm \Delta N_{k'}$$

$$\Delta N_k = -\Delta N_{k'}$$

$$\frac{\partial N_k}{\partial t} = \sum_{k',k''} V_{k,k',k''} [N_{k'} N_{k''} \mp N_k N_{k'} - N_k N_{k''}]$$

$\epsilon_{po} \leftrightarrow \epsilon_{po} + iac$

Strong Turbulence (Zakharov 1972)

electron fluid equations : $n_e = n_0 + \delta n_e + \delta n_L$

$$\frac{\partial^2}{\partial t^2} \delta n_e + \omega_e^2 \delta n_e - \alpha v_e^2 \Delta \delta n_e + \nu_e \frac{\partial}{\partial t} \delta n_e = \nabla \cdot \frac{e}{m} \delta n_L \mathbf{E}_H$$

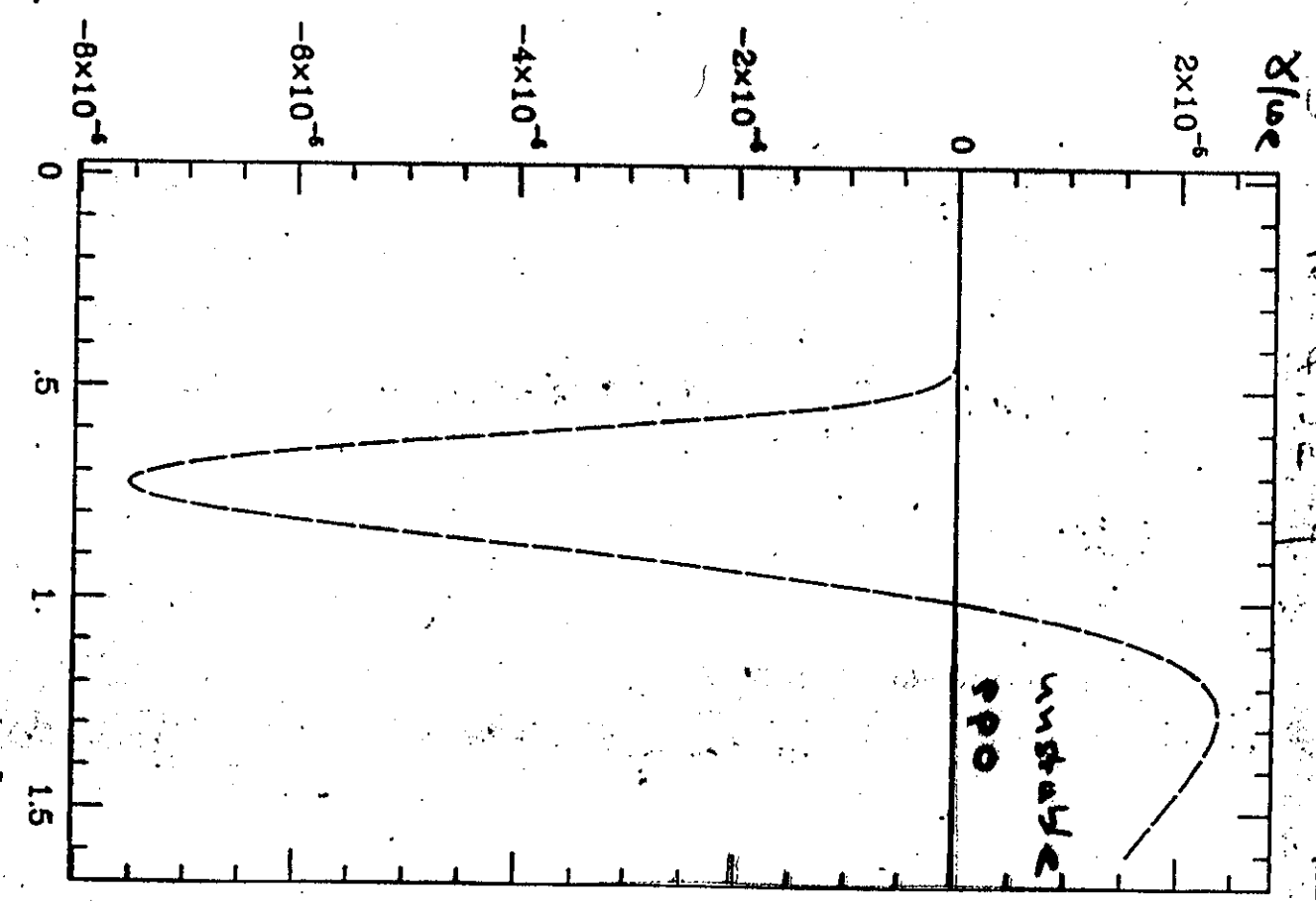
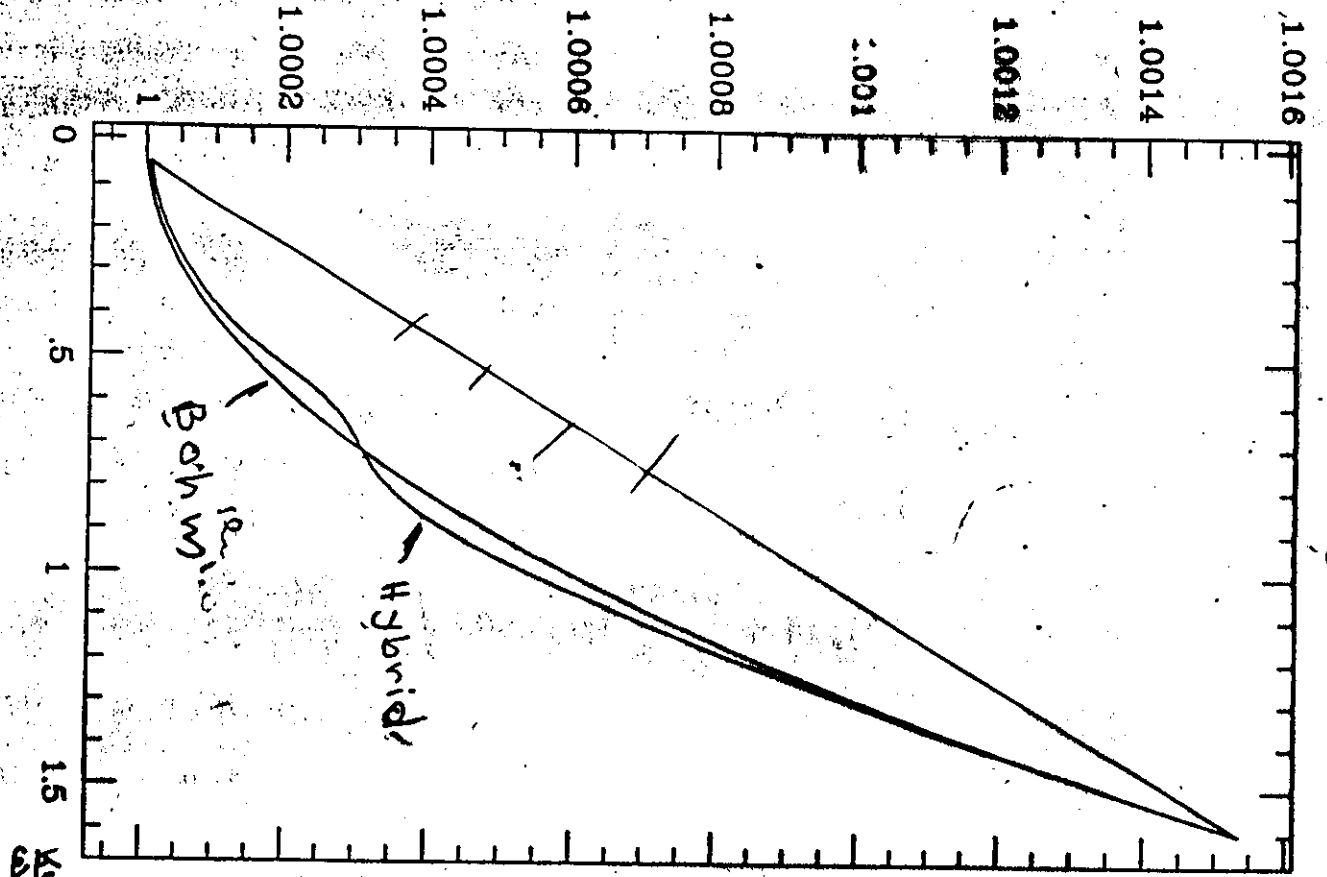
$$\nabla \cdot \mathbf{E}_H = -4\pi e \delta n_e$$

low frequency electron motion: pondermotive force

$$\delta n_L = n_0 \left(\exp \frac{e\Phi - U}{T_e} - 1 \right)$$

$$U = \frac{e^2}{2m\omega_e^2} \langle E_H^2 \rangle$$

ions: quasi-neutrality $\delta n_i = \delta n_L$, hydrodynamic, static or kinetic eqns include modulational instability, decay instability and NLLD by ions as special cases.

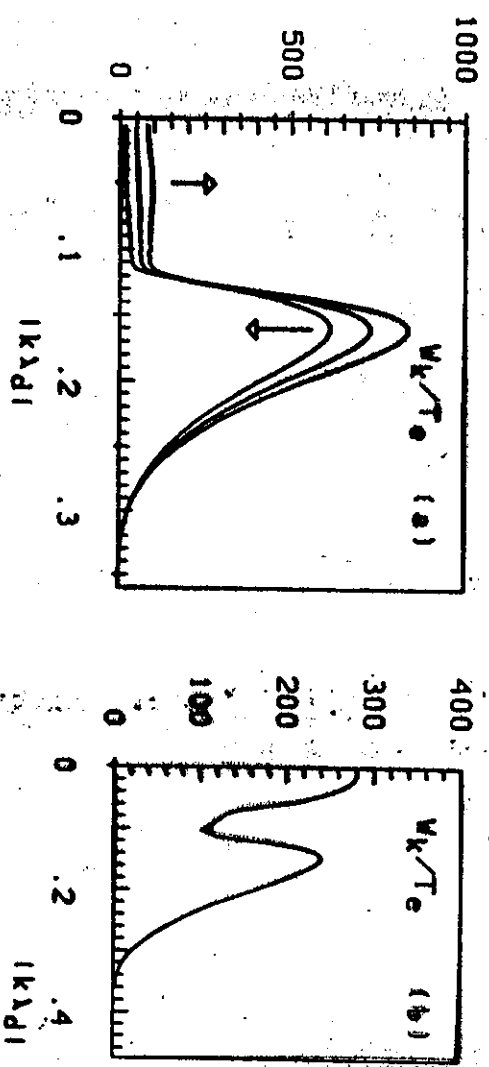


$n_b = 1.0 \text{ E-5}$ $T_b = 0.09$ $T_c = 4.0 \text{ E-4}$ ($n_e = 50$)
 VERY WEAK BEAM

~~3/3~~

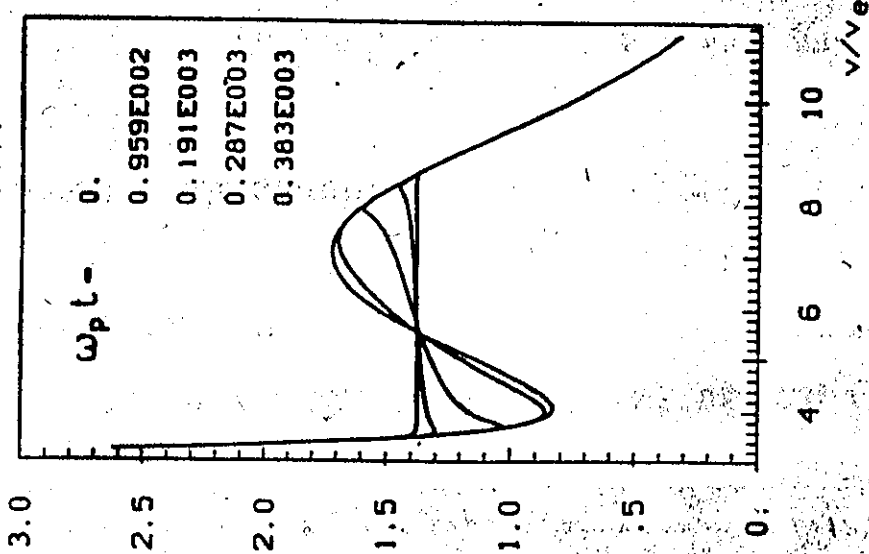
beam ignored in dispersion

$\omega_{et} = 300, 600, 900$ $\omega_{et} = 2700$

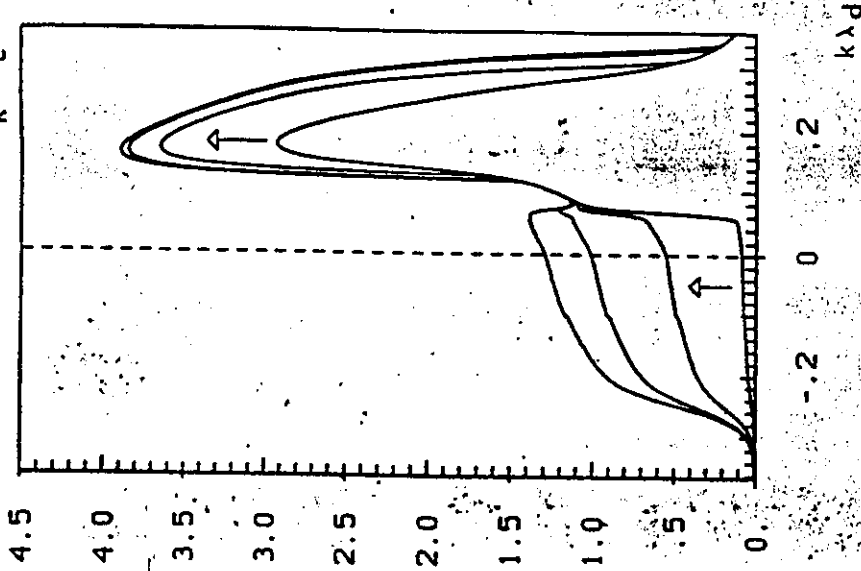


NLID of Langmuir waves:
 build up of condensate at $\omega = 0$
 by scattering off ions
 beam is ignored in dispersion relation
 and coupling coefficient
 actually permissible for extremely weak beams
 d. prev. figure

E-03 ELECTRONS: F(v)



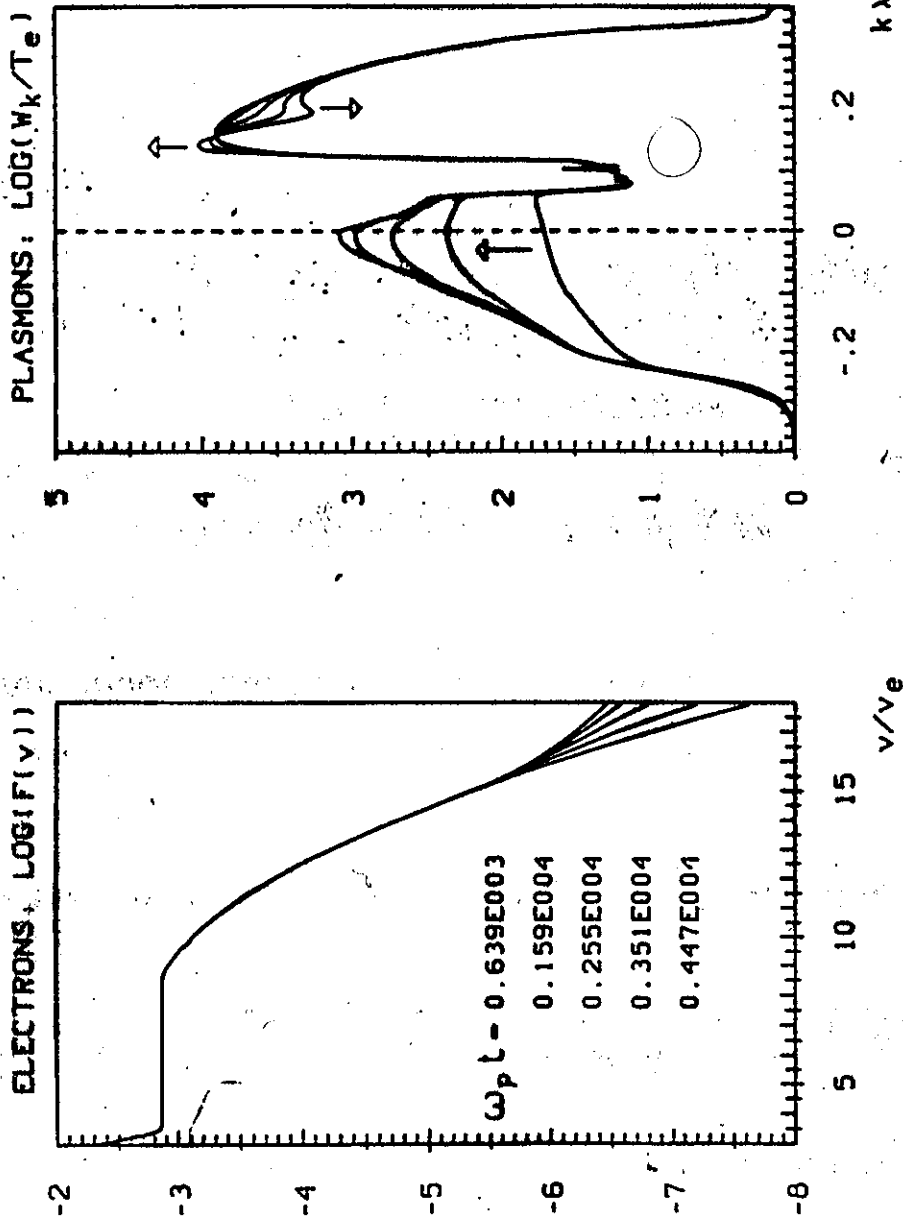
PLASMONS: LOG(W_k/T_e)



beam included
incoupling
coefficient

Fig 5 $\frac{\omega_p}{\omega} = 10^{-2}$ $\frac{\omega_p}{\omega} = 7.07$ $\frac{\omega_p}{\omega} = 0.33$

Strong beam

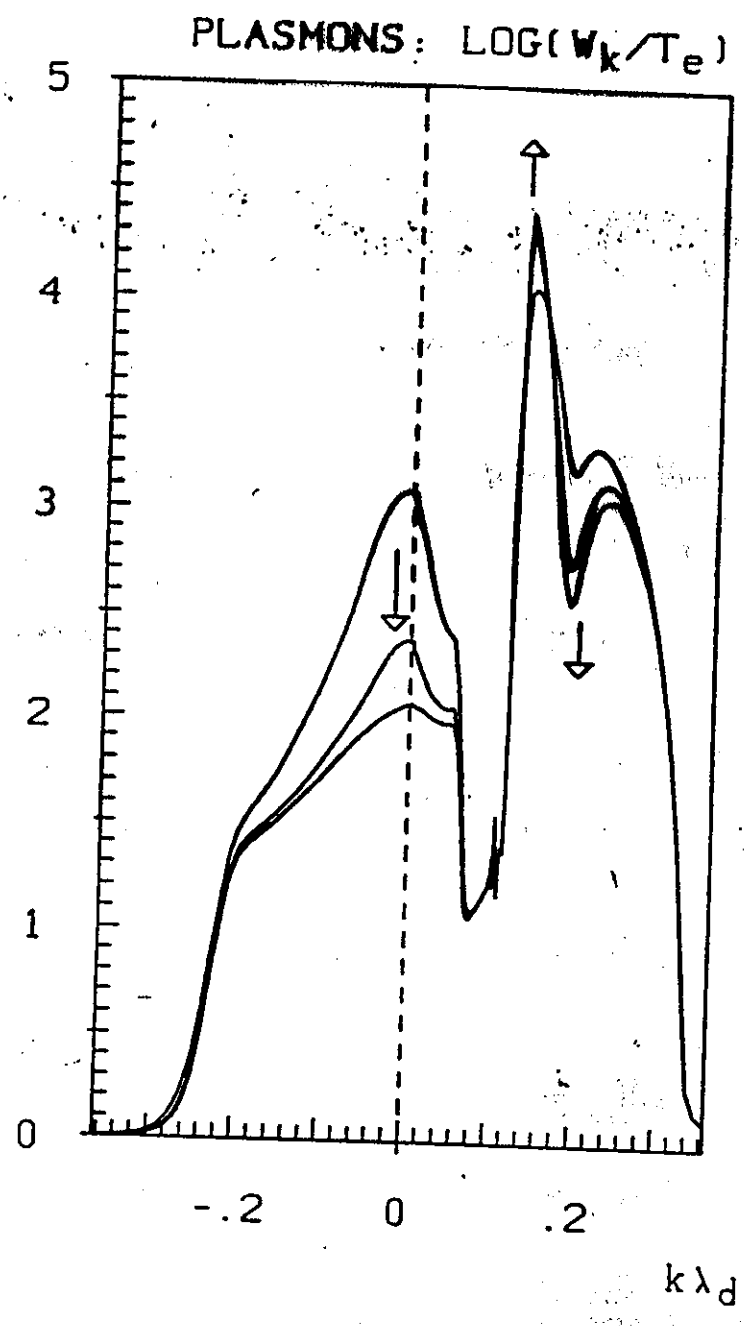


contin. from Fig 5
later times
a secondary condensate of Langmuir
waves is formed, e-tail acc. by fast waves

Fig. 6

$n_b = 0.01$

$\omega_{pe} = 4790$
 8590
 11200



Very late phase
of Fig 5.5
conveyance at $k=0$
is depleted.

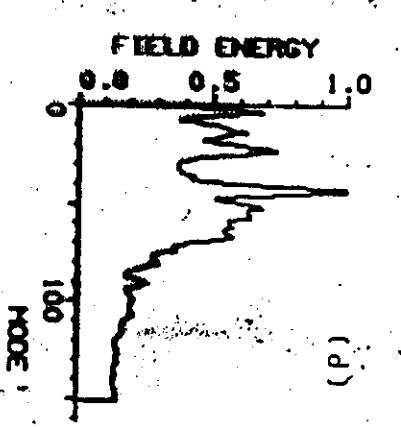
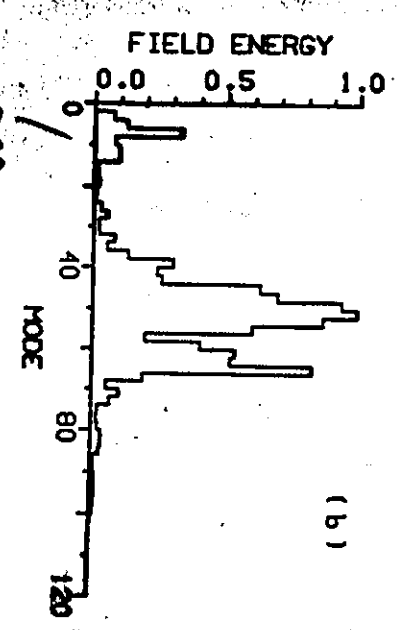
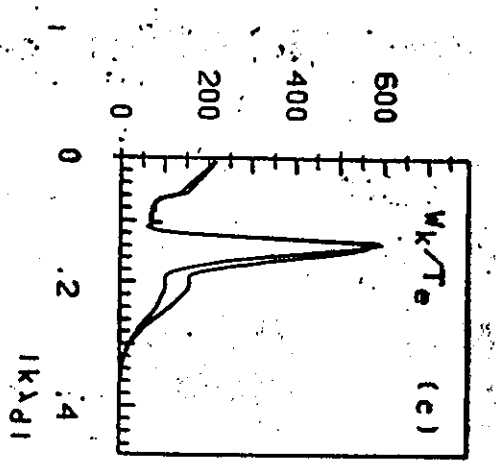
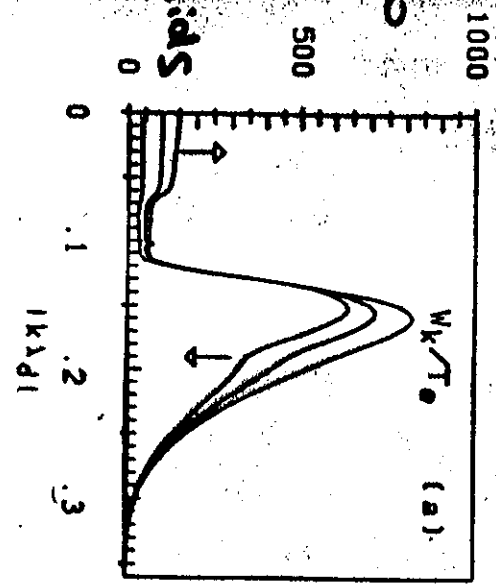
Comparison of QLT+NLCD with Particle Simulation

$\omega_e t = 200, 600, 400$

QLT + NLCD (i)

Muskhelishvili, DuBois 1991

Phys Fluids



Particle Simulation

$\omega_e t = 700$

DuBois 1990 J.G.R.

(Standard propagating)

$\omega_e t = 2600$

$$\frac{E_{\text{QLT}}}{E_{\text{NLCD}}} \approx \frac{W}{nT_e} \left(\frac{u}{u_s} \right)^2$$

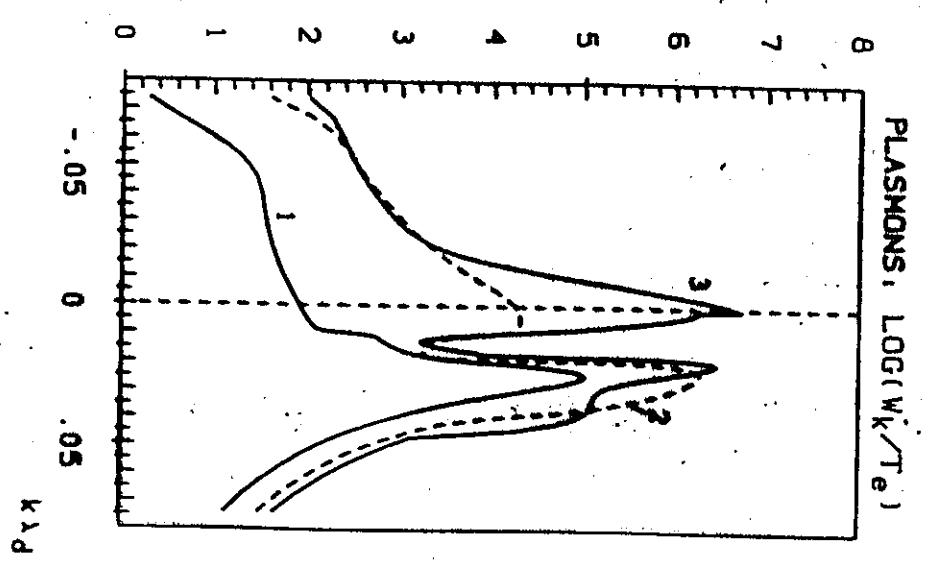
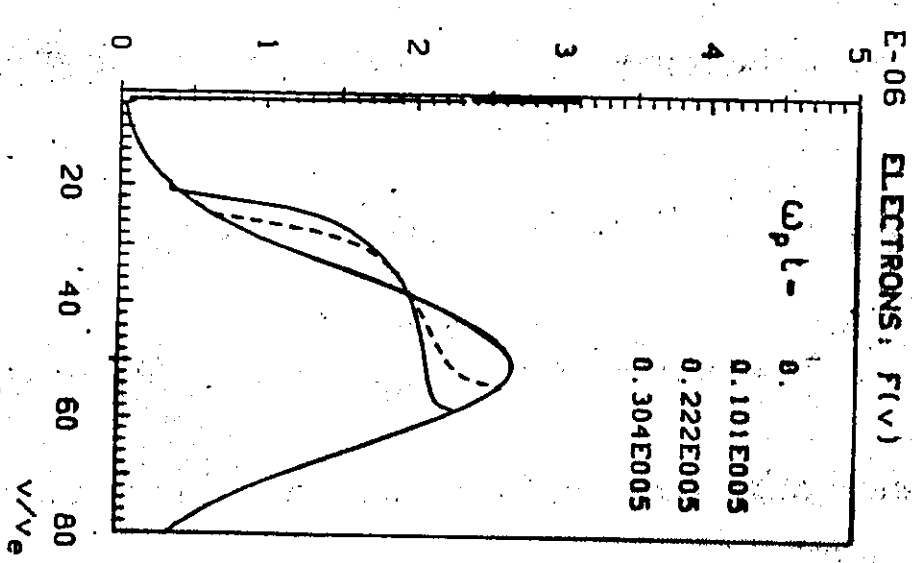
$$\frac{E_{\text{QLT}}}{E_{\text{NLCD}}} \approx 0.3 \frac{W}{nT_e} \left(\frac{u}{u_s} \right)^2$$

$$\frac{W}{n} < 4.2 \left(\frac{u}{u_s} \right)^2 \left(\frac{nT_e}{u_s} \right)^3$$

Papadopoulos, Goldstein, Smith, 1974
 wave in ω_e dispersion velocity

Wash and
Sust beam

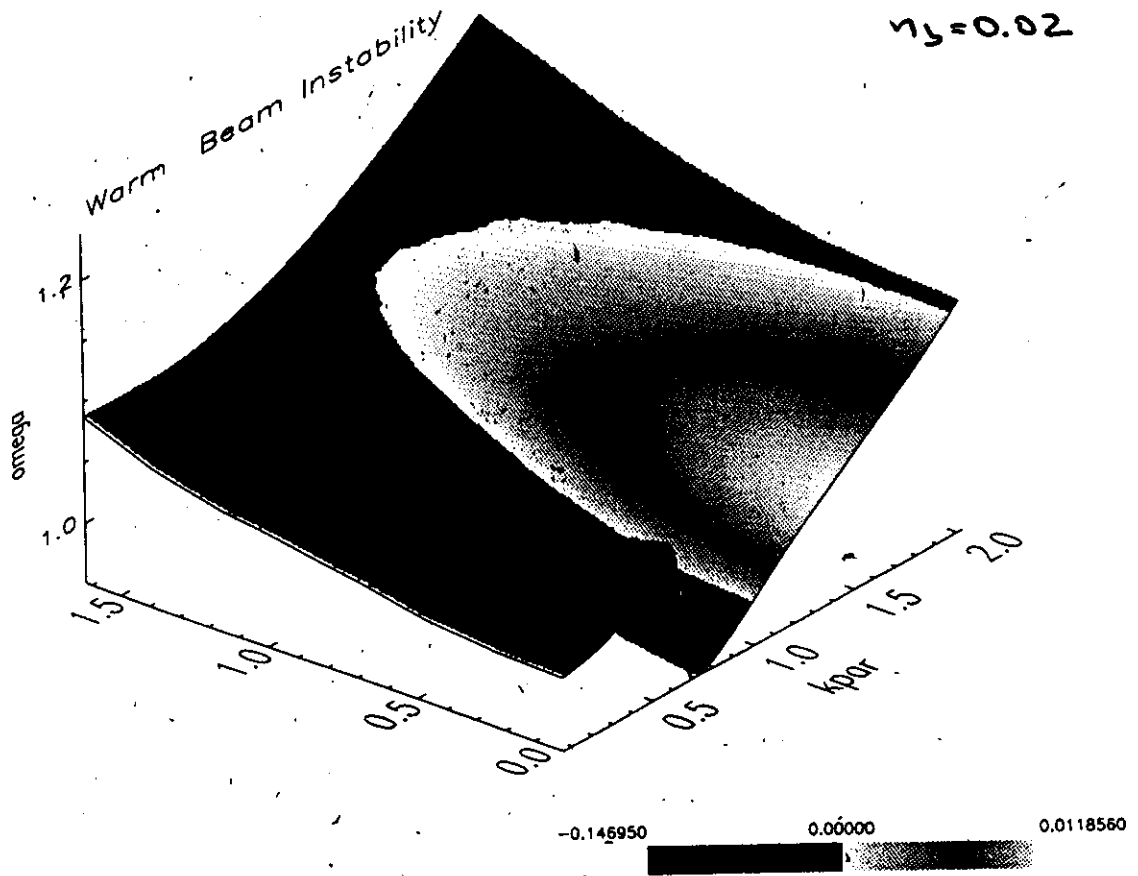
$n_0 = 10^{14}$
 $n/n_0 = 50$
 $v/v_0 = 0.3$



Langmuir condensation forms
at VA D and the process
stabilizes waves resonant with the beam

$1 \approx 1.01 \cdot 10^9 \text{ sec}^{-1}$
 $2 \approx 2.72 \cdot 10^4$

Tb=0.215
 $\eta_3=0.02$



dispersion relation
for a warm beam in unmagnetized
plasma.

beam effect disappears
for more oblique modes,
corresponding to a larger
effective beam temperature

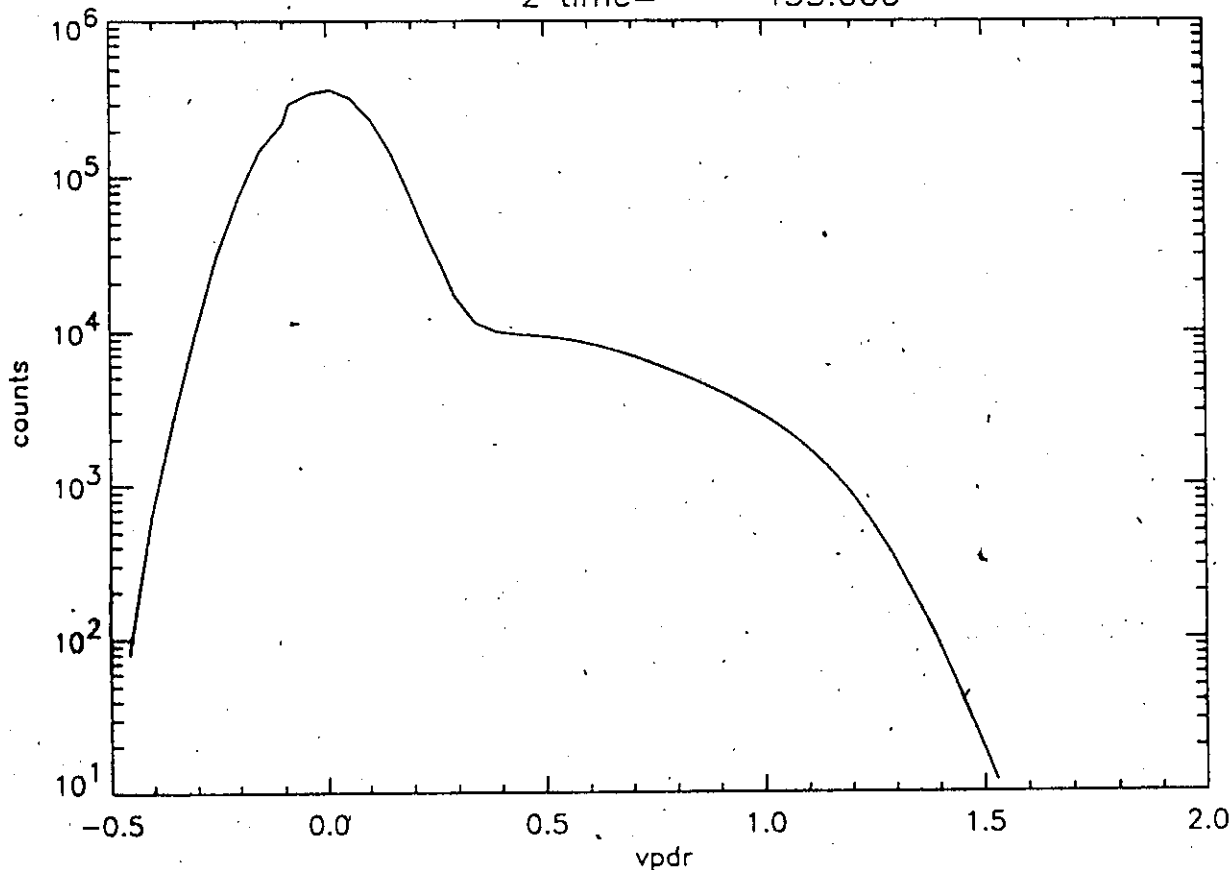
$$T_{\text{eff}} = T_{\parallel} + T_{\perp} \tan^2 \theta$$

2D Kin e.b.i

$f_b(v_u)$

full particle simulation

2 time = 453.000



$f(v_u)$

$\frac{n_b}{n} = 0.03$

$\frac{v_b}{u} = 0.05$

$\frac{\gamma_b}{\gamma} = 0.1$

$R_{ce} = 0.05 \omega_e$

$\frac{\Gamma}{m} = 100$

$\frac{\gamma_b}{\gamma} = 0.001$

($T_e/T_i = 10$)

accel of particles $v \ll u$
 $f(v) \gg f_b(v)$ bulk tail

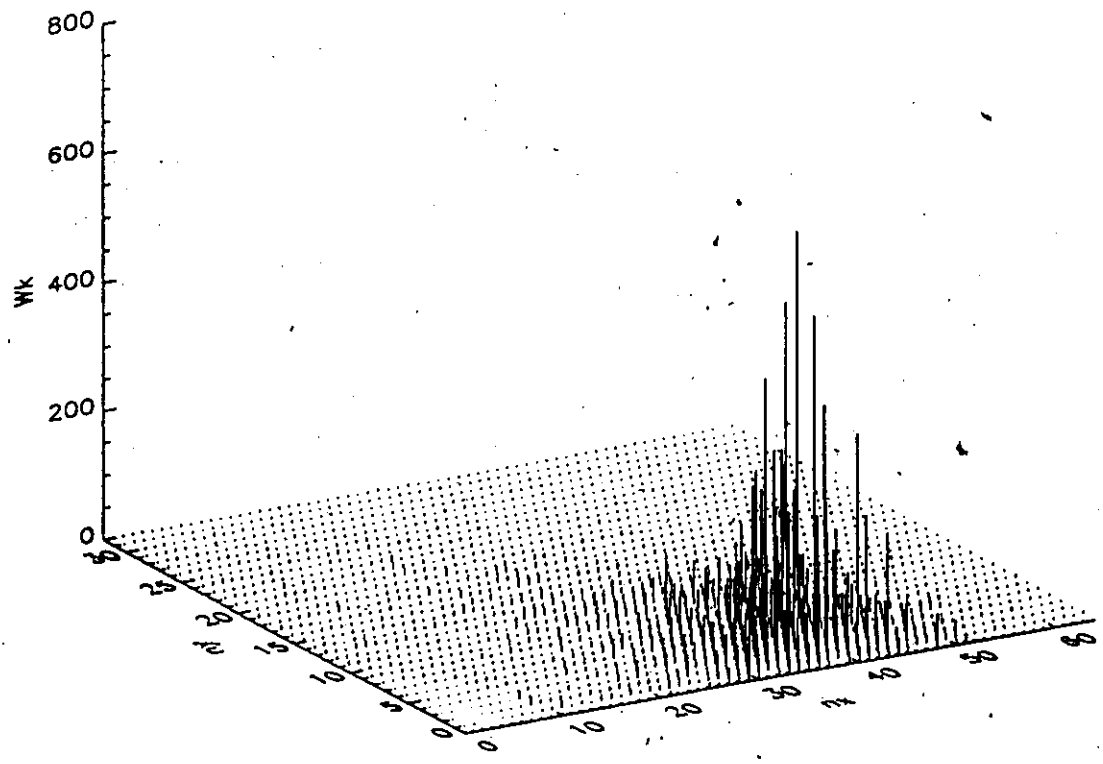
important
 $\rightarrow \frac{\partial f}{\partial v} < 0$

some acc $v < 0$
 $v > u_b$

$\alpha < 0$

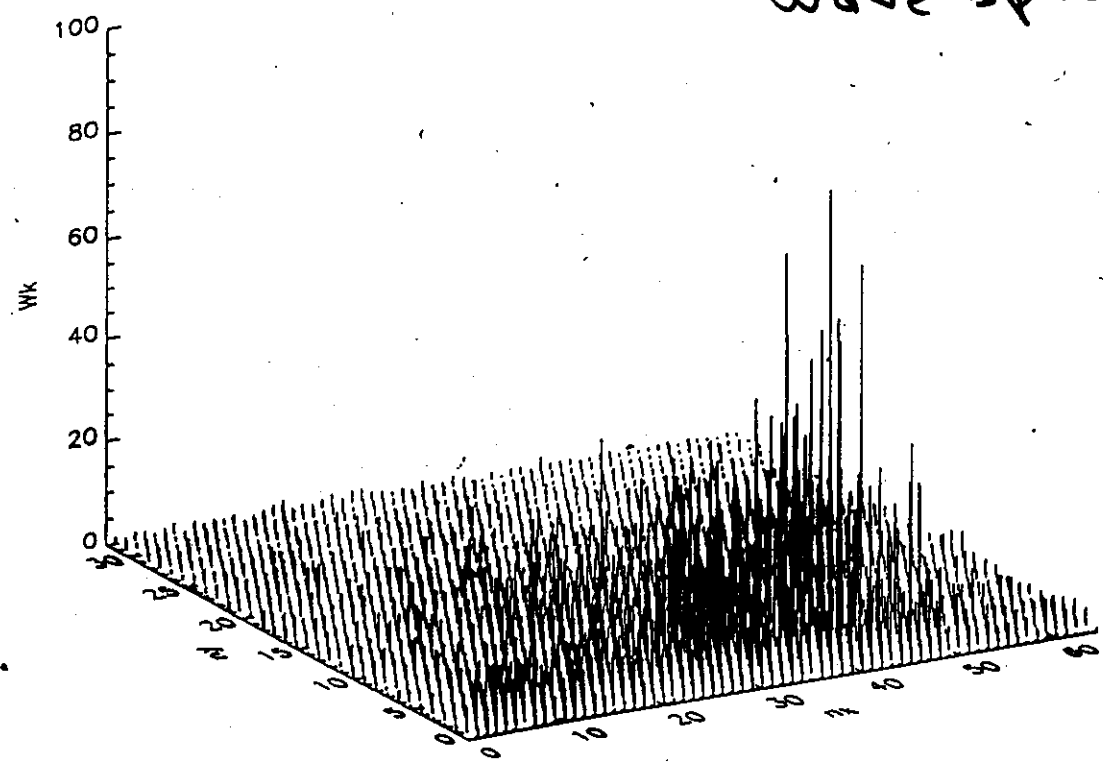
$\frac{S_{15}}{S_{13}} = 0.03$ $\frac{S_{15}}{S_{13}} = 0.05$
 $\frac{S_{15}}{S_{13}} = 0.1$

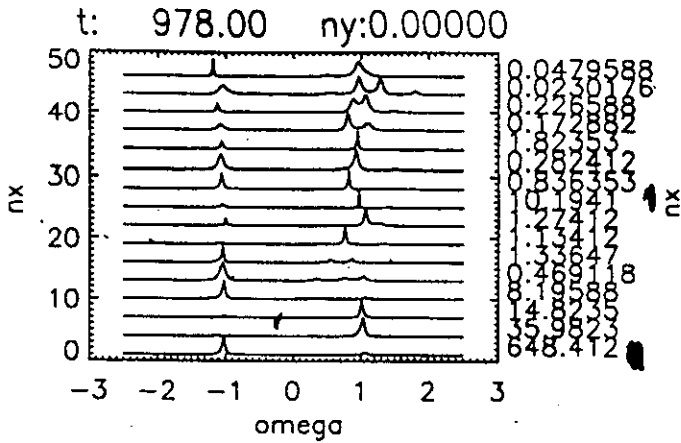
SPECTR 453.0 = $\omega_e t$



SPECTR 603.0

Wave spectrum





Reactive - Kinetic Transition

1. Linear Theory

cut off distribution

cool beam

⇒ reactive instability

excitation of broad band spectrum

$$0 < \omega < \omega_c \approx \omega_e \quad \omega_{opt} < \omega_e$$

2. Observations: Lacombe et al. 1985

3. Nonlinear evolution

a. beam broadening in reactive phase → gentle bump on tail
Kinetic
Shapiro 1963

b. single mode theories
TRAPPING, e folding
suppression of neighboring modes
Drummond et al 1970
Melrose 1988

c. simulation
Kainer, Dawson, Shuman, Coffey 1972
 $\omega_e = 50, n/n_0$

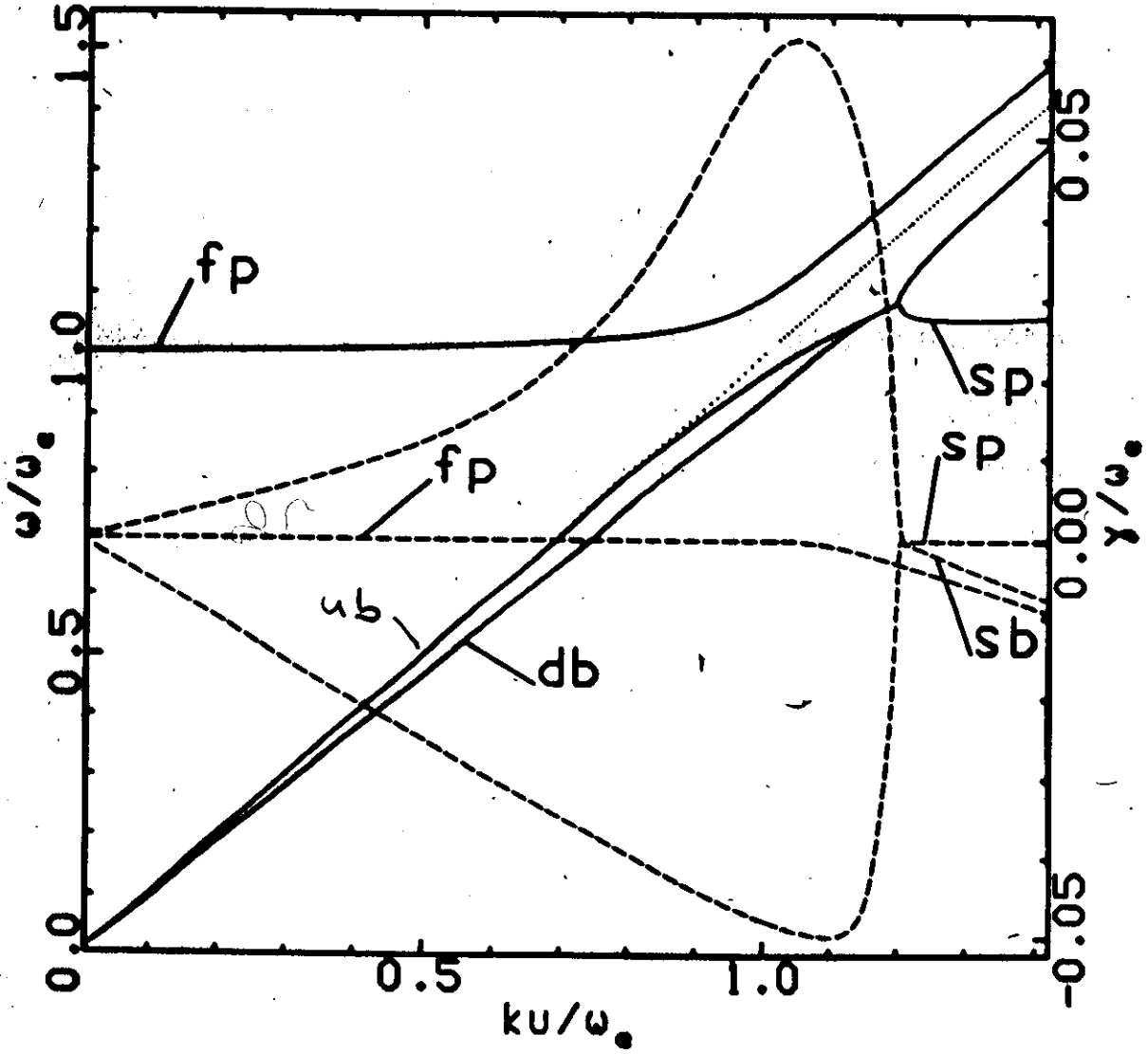
new runs, diagnostics

$n/n_0 = 1 - 5 \cdot 10^{-3}$ volve reactive phase
 $\omega_e = 7$ system length $L \rightarrow$

Conclusion:

1. early reactive phase
Q.L.: $\omega, \delta, \delta b$ short
2. Late reactive phase
N.L.: trapping
still many modes → phase mixing
3. Kinetic phase
Q.L. gentle bump

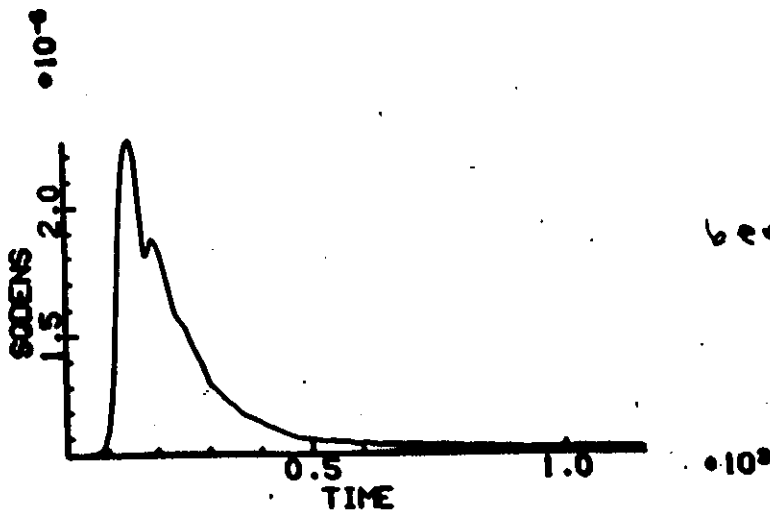
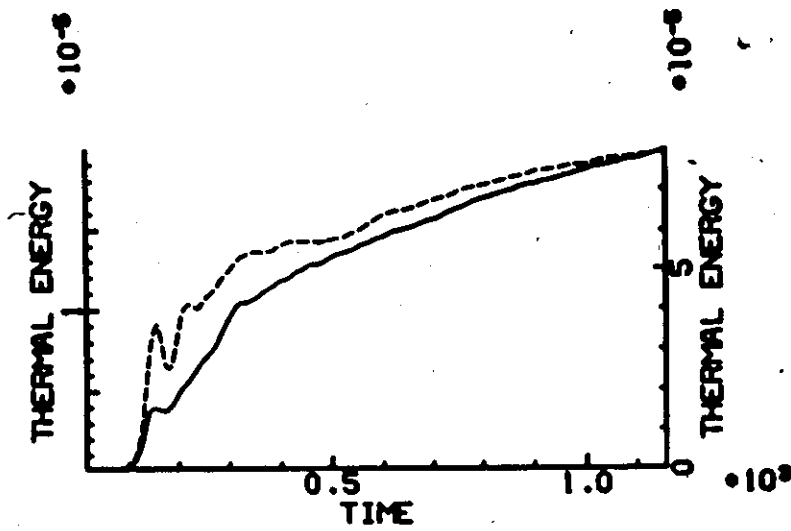
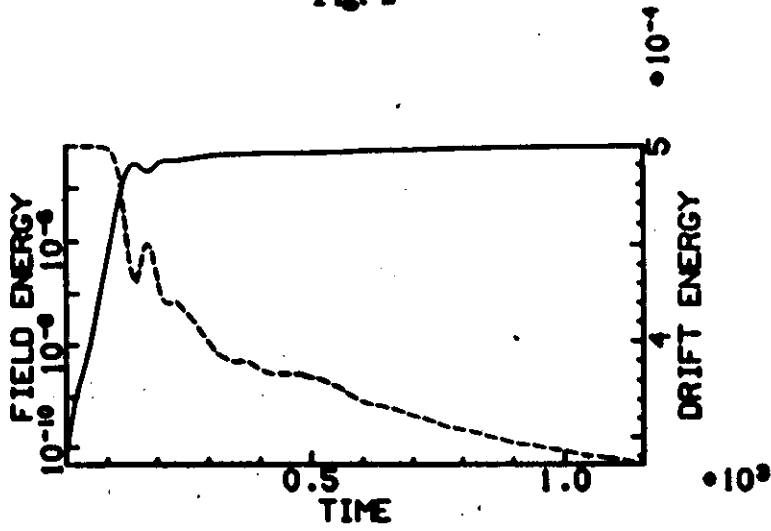
Fig. 1



$\beta_b = 0.001 = \frac{v_b}{v_{te}}$
 $\Gamma_b = T_b / \mu v^2 = 3 \cdot 10^{-4}$
 $\Gamma_e = T_e / \mu v^2 = 0.02$

1
 Reactive instability
 of an electron beam

Fig. 2

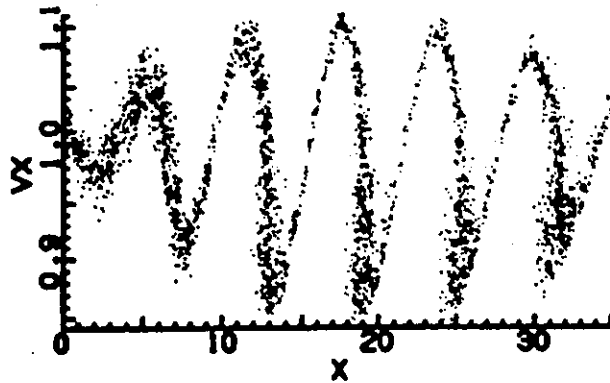


beam modulation
 $\langle n_b(x)^2 \rangle_L$

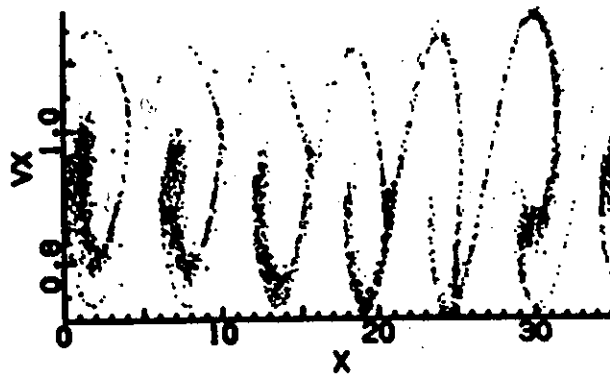
$\hat{n}_b = \frac{n_b}{n} = 0.001$
 $\hat{T}_b = T_b / (mu^2) = 3 \cdot 10^{-4}$
 $\hat{T}_r = T_r / (mu^2) = 0.02$
 Hybrid

C.T. DUM
 JGR 85, 811 (1980)

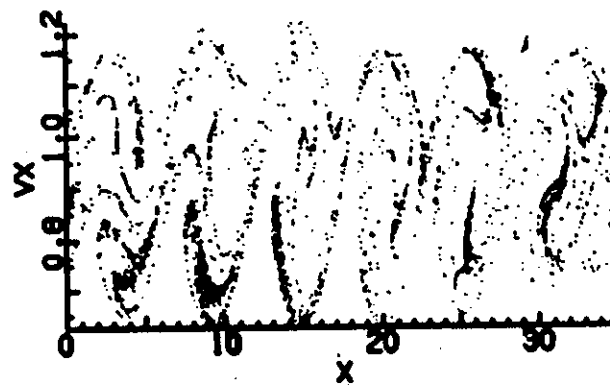
Fig. 3



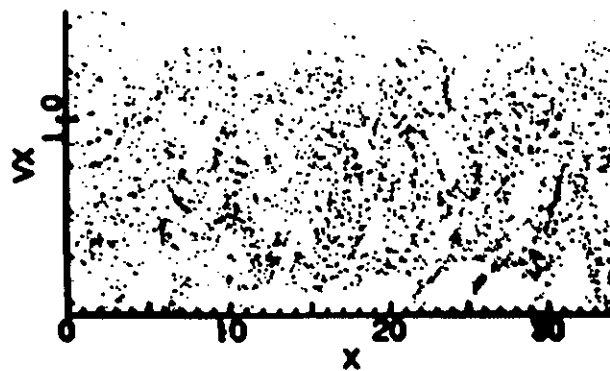
$\omega t = 126.7$



158.3



253.4

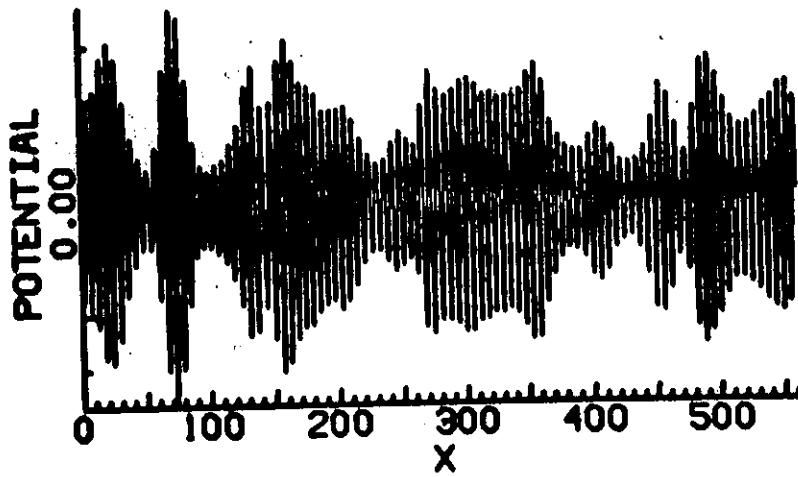


396

3

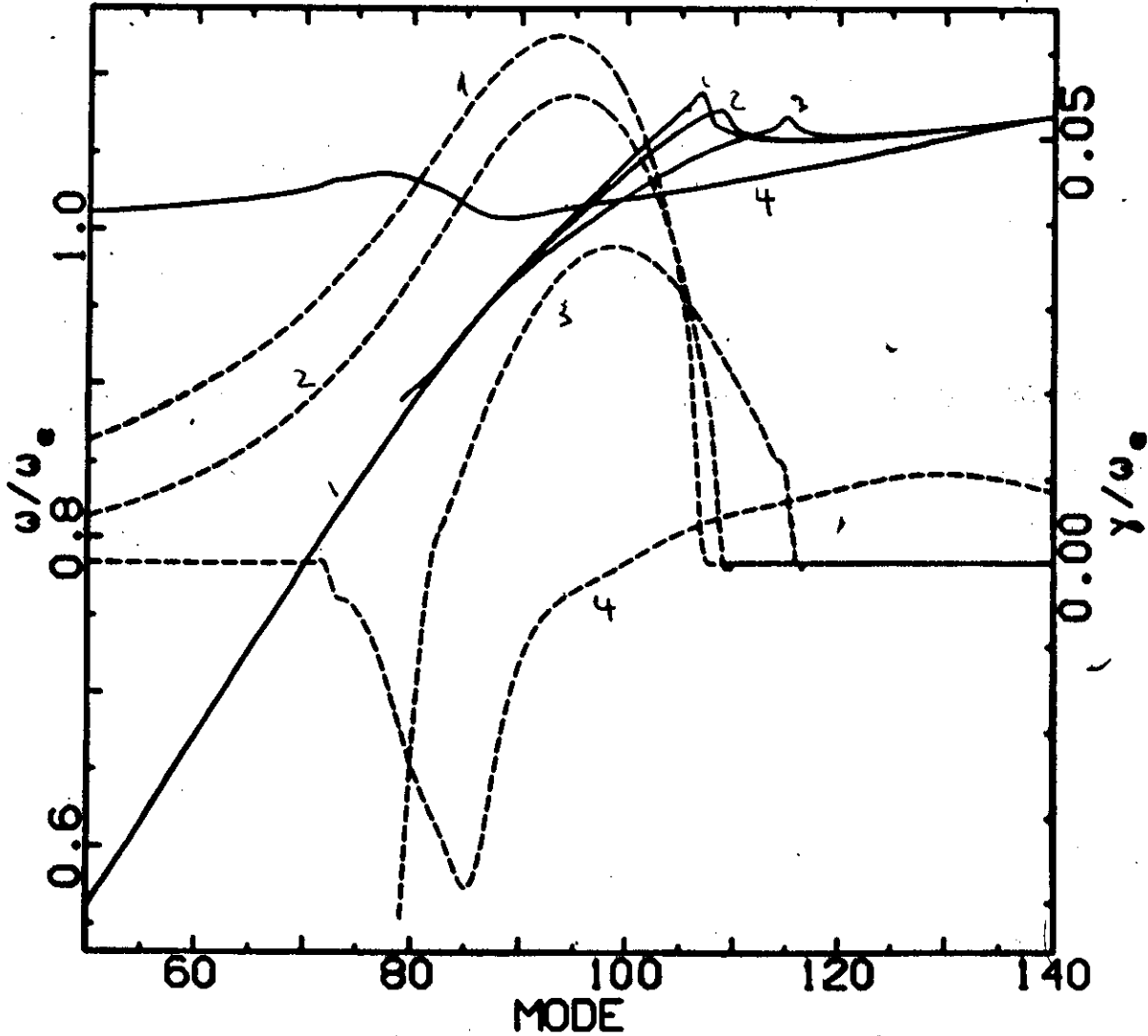
trapping & phase mixing at late times

Fig. 4



$\omega = 158.4$

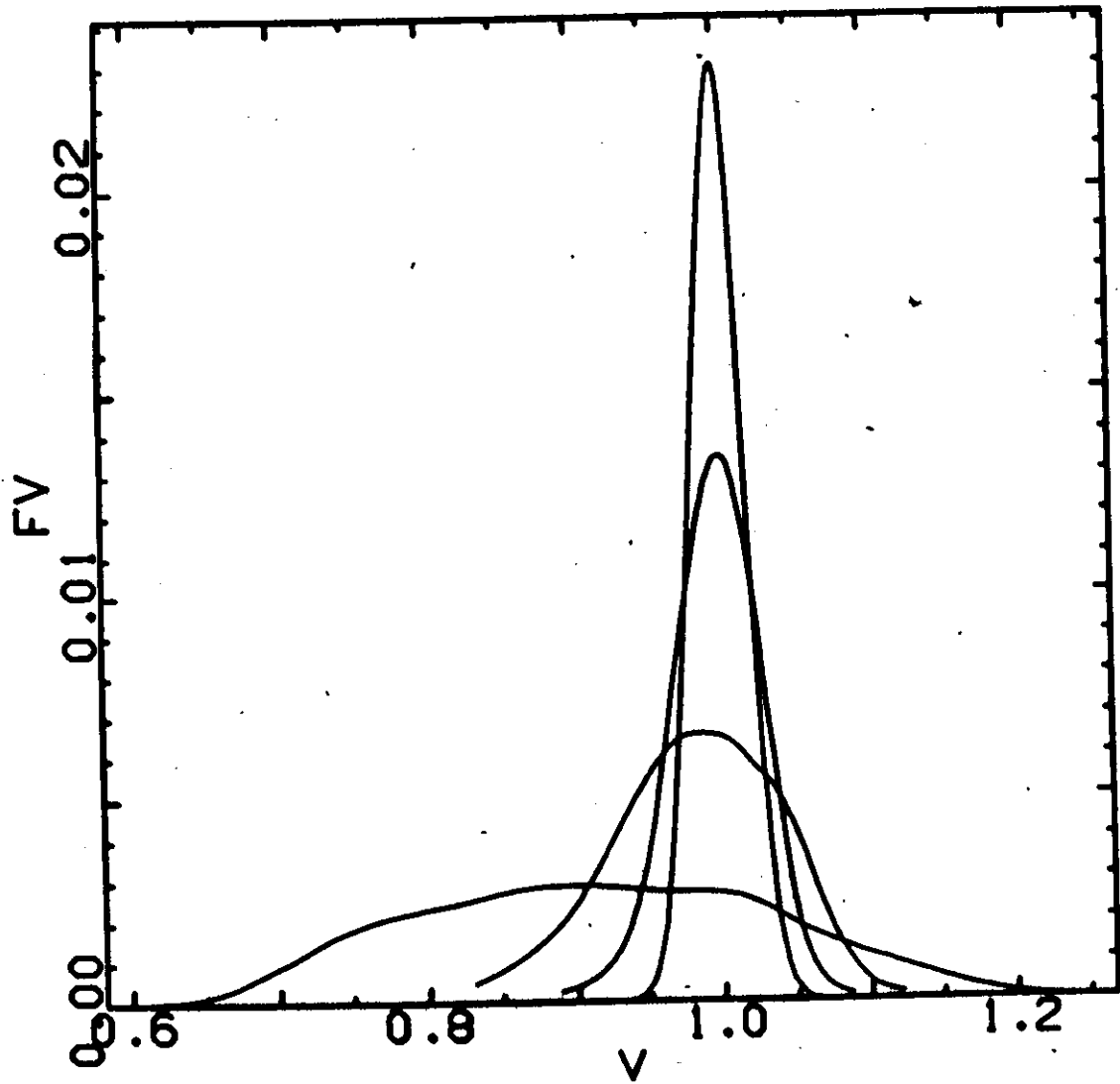
Fig. 6



$\omega_0 = 15.8, 110.3, 126.3, 253.4$
 $\frac{n_b}{n} = 0.001 \quad \frac{T_b}{nm\omega^2} = 2 \cdot 10^{-4} \quad \frac{T_e}{nm\omega^2} = 0.02$

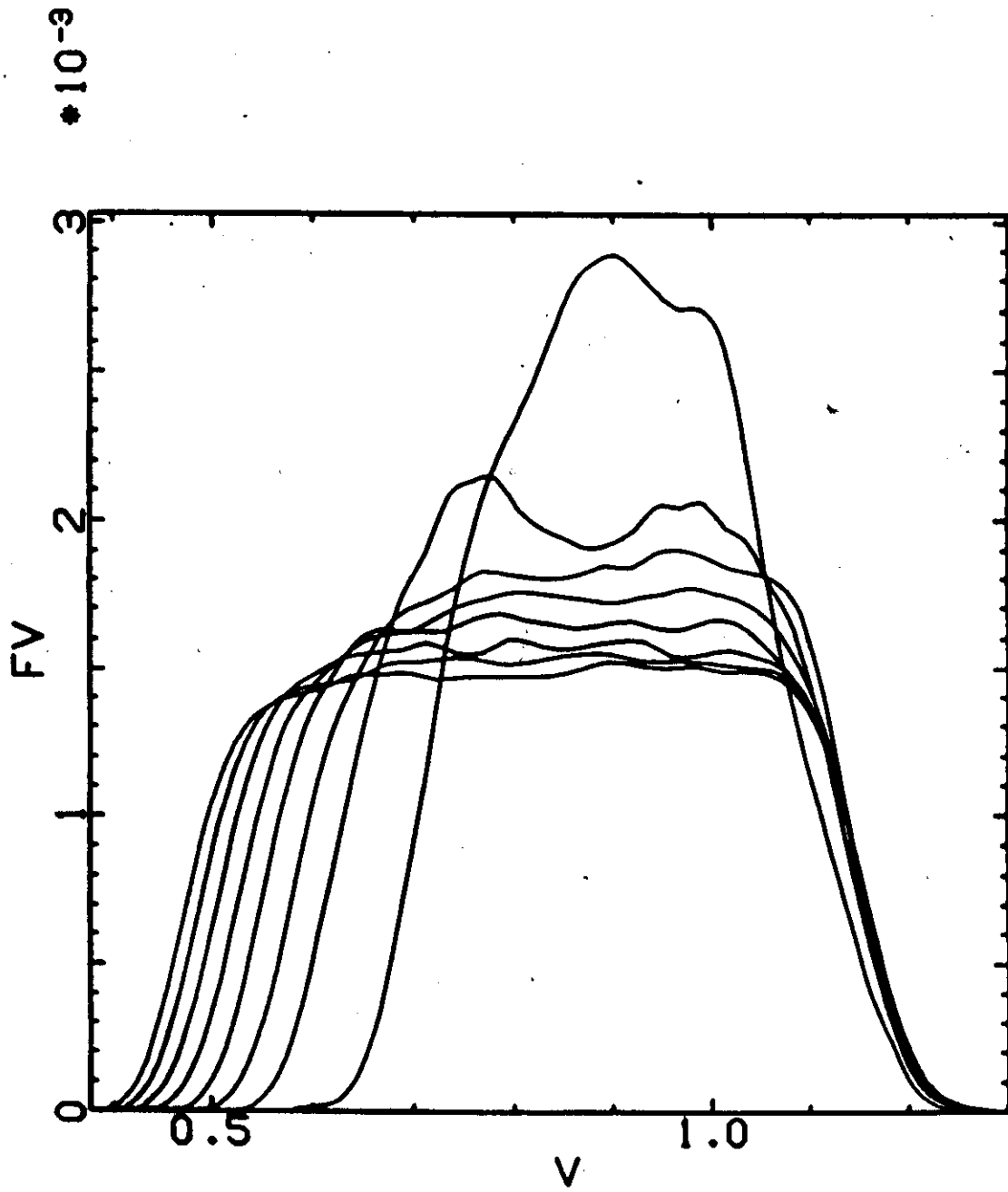
transition from reactive
 beam mode to kinetic
 plasma mode as beam
 is broadened, (cf. Fig. 7)

Fig. 7



$\omega_e = 15.8, 110.9$
 $120.7, 253.4$

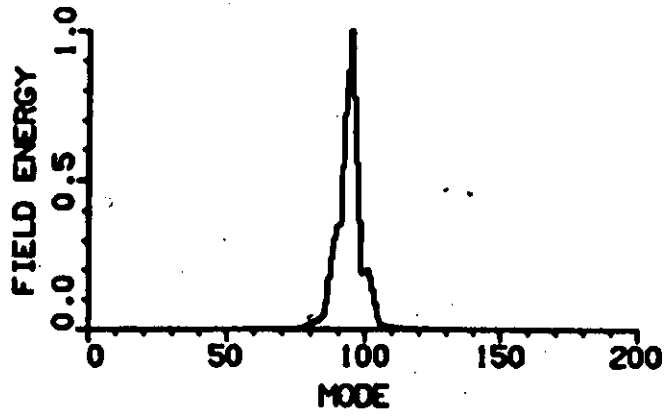
Fig. 8



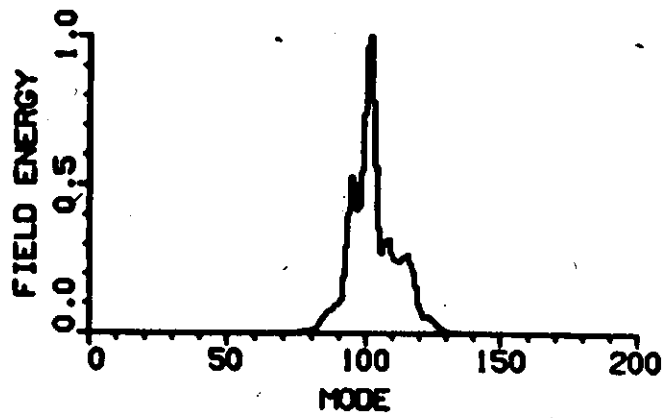
late
kinetic
phase

253.4, 300.1
506.9, 653.6
760.3
1014, 1140

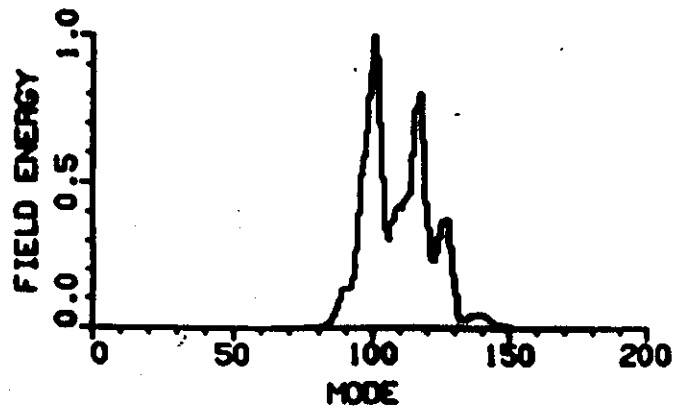
Fig. 9



$\omega = 158.3$



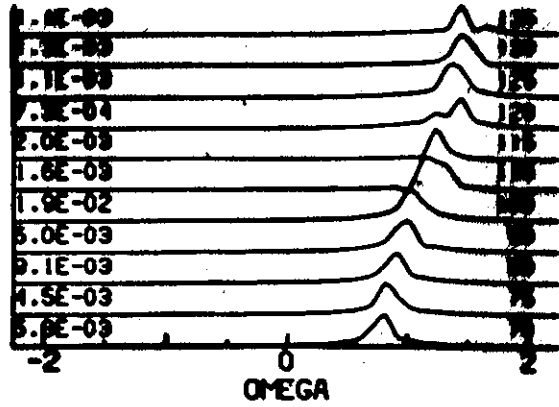
253.4



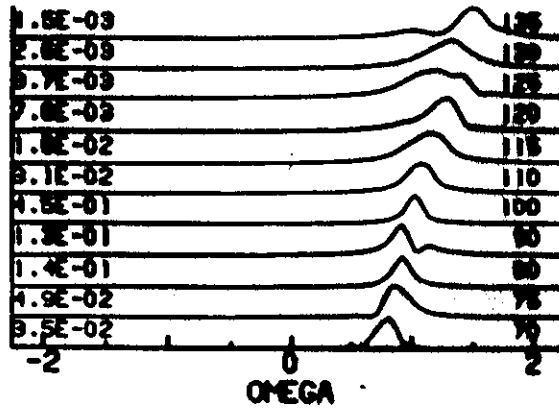
395.9

Fig. 10

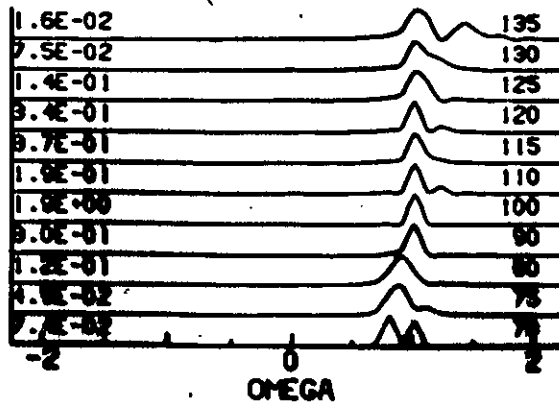
TIME = 17.76 TO 79.20



TIME = 81.12 TO 142.56



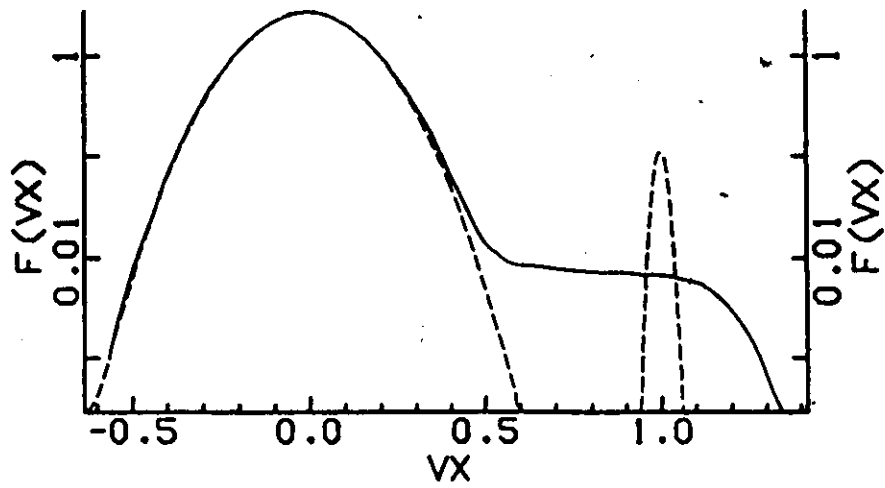
TIME = 207.84 TO 269.28



10
transition from beam mode
w.k.u. to plasma mode wave

Fig. 12

TIME= 205.89



late electron distribution

Down shifted Oscillations

1. Observations

$$v_c = \frac{d}{D_{IFF}} v_{sw}$$

$v/v_e = O(1)$ deep inside foreshock

Fuselier et al. 1985 $\omega < \omega_e$

2. Linear theory

not plasma eigenmode but
negative energy beam mode $\omega \approx k \cdot v$
destabilized by bulk electron
Landau damping

$$0 < \omega < \omega_{max}(\hat{T}_b, \hat{T}_e)$$

3. Nonlinear Evolution: Particle Simul.

bump-in-bulk

large δ
bulk diffusion

spectrum ω, δ
distribution f_n

persist for
long time

Q.L. $\rightarrow \rightarrow$

Overall Conclusions

Quasilinear Evolution

in space: injection $\parallel B$

convection by SW
even B

N.L. effects (mode coupling)

not significant

for foreshock

: cannot prevent
plateau formation

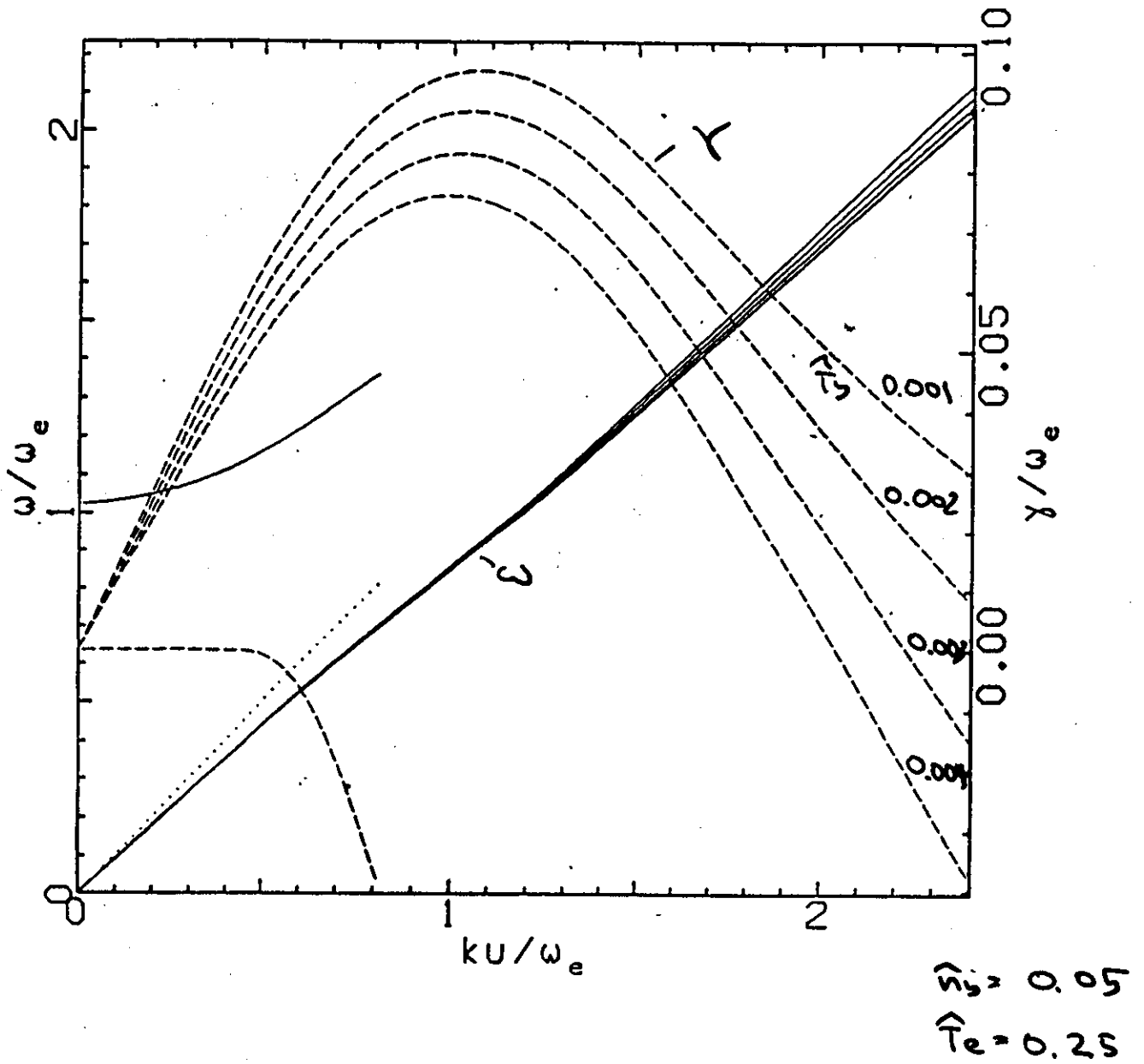
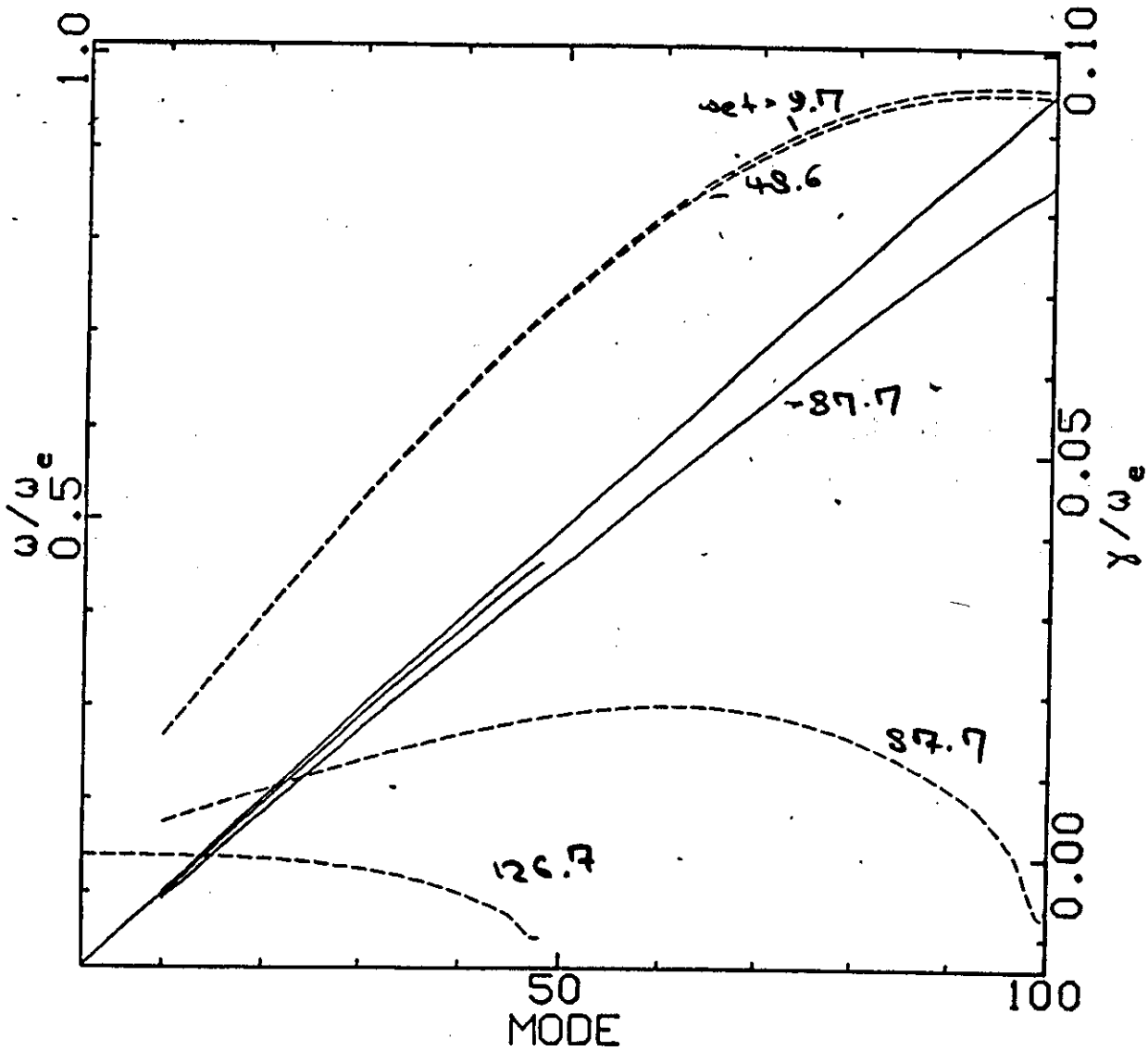


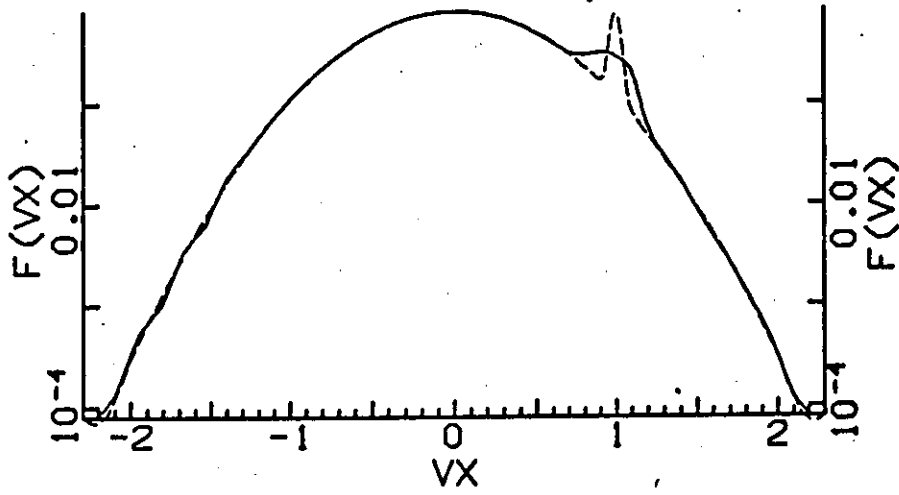
Figure 1
 dispersion relation
 for downshifted oscillations



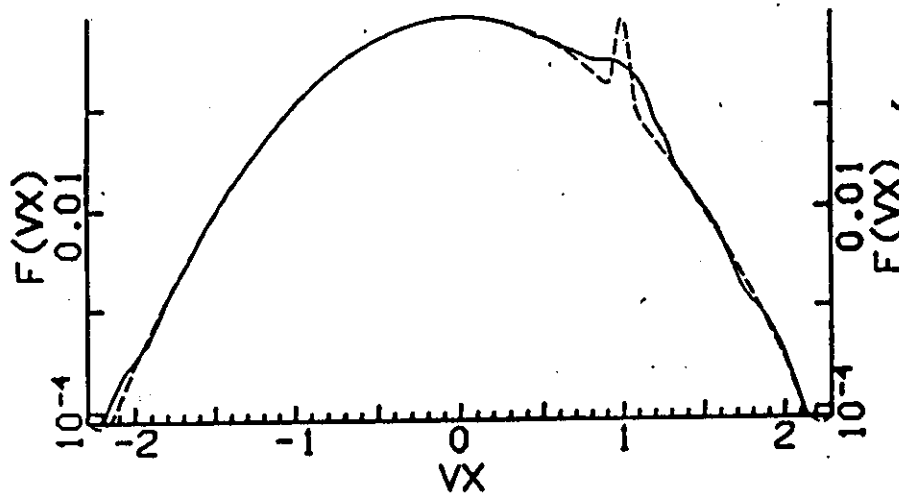
Evolution of dispersion relation

Figure 2

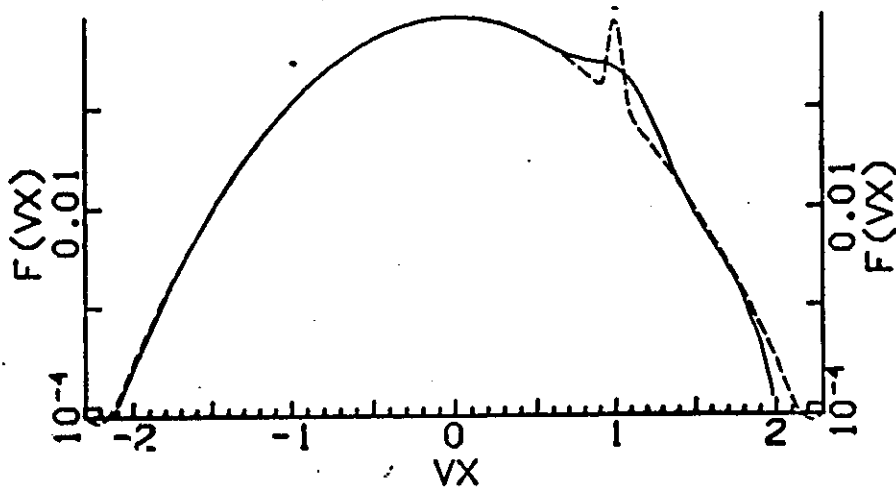
corresponding to evolution
of distribution function (Fig 3)



27.7



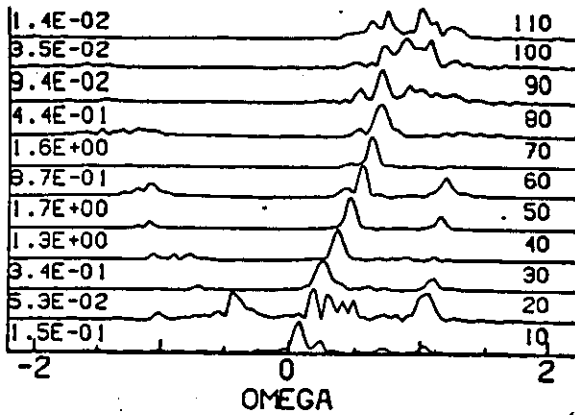
26.6



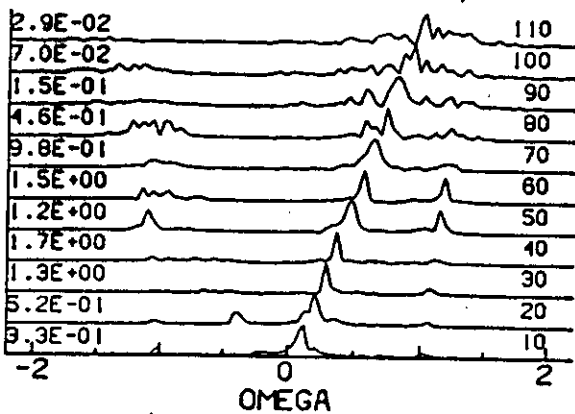
844.9

Figure 3

TIME = 12.15 TO 165.75



TIME = 168.15 TO 321.75



TIME = 480.15 TO 633.75

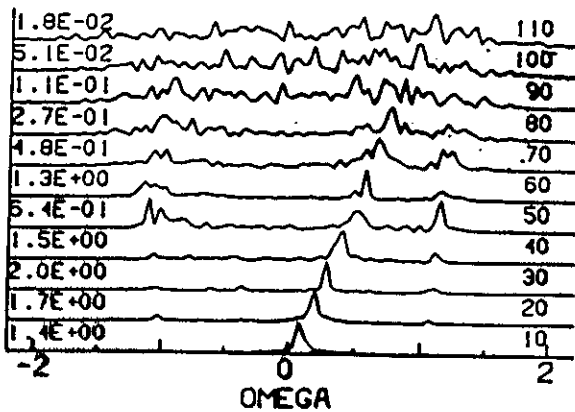
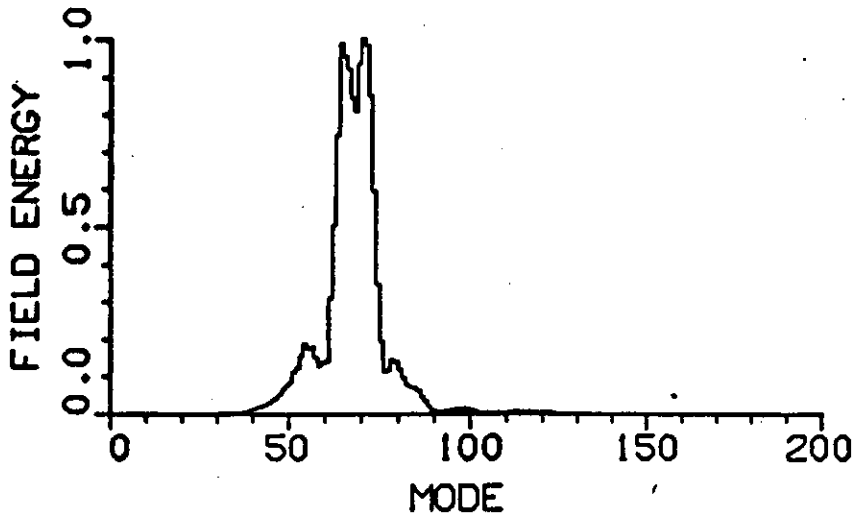
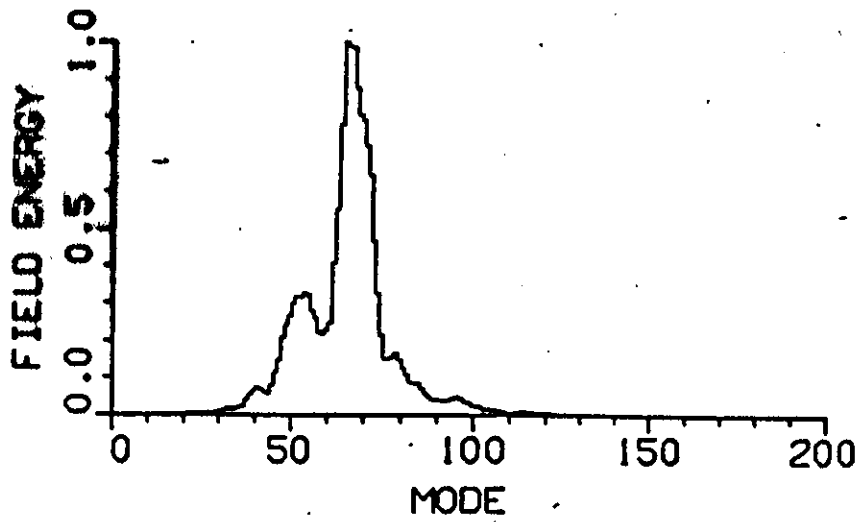


Figure 6

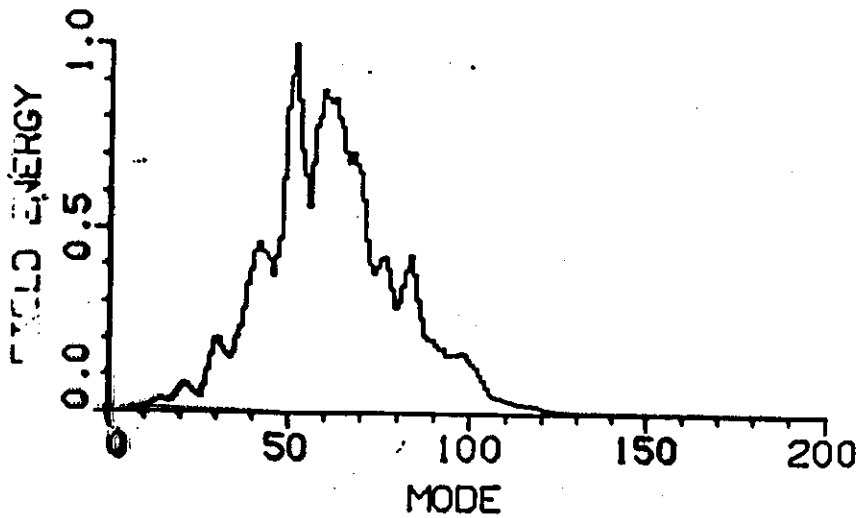
wave spectrum
as a mode number



$\omega_{et} = 87.6$



120.9



594.6

Figure 7

Conclusions

- 1) Textbook approximations have very limited range of applicability
kinetic instability \leftarrow
reactive instability \rightarrow transition
- 2) Q. L. Theory
- 3) Nonlinear Mode coupling
(Scattering off ions) $\omega_k - \omega_{k'} - (k - k')v = 0$
Secondary condensate ($\omega \approx \omega_c$)
backward propagating waves
- 4) New regimes $\omega > \omega_c$
 $\omega < \omega_c$ rel. persistent
- 5) Free energy source
- 6) Simulation
 - a) Linear dispersion relation
Design
Analysis: $f_j(\underline{v}, t)$ non-linear \rightarrow $\omega(k, t)$
 $\gamma(k, t)$ \rightarrow Topology changes possible
modification by strong driving!
($n_b/n_i, U_b/U_i$)
 - b) ω spectra
Number of modes (System size)
 - c) Noise level
 - d) Hybrid simulations
acceleration of ...

