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AUTUMN COLLEGE ON PLASMA PHYSICS

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STRONGLY COUPLED PLASMAS

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These are lecture notes, intended for distribution to participants.

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Strongly coupled plasmas

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Contents:

Coupling parameter

$$\Gamma = \frac{U(l_0)}{T}$$

$$l_0 = n^{-1/3} \quad U(r) = e^2 / r$$

$\Gamma << 1$ - gaseous medium

$\Gamma \approx 1$ - liquid

$\Gamma >> 1$ - glass, crystal

Examples of strongly coupled plasmas:

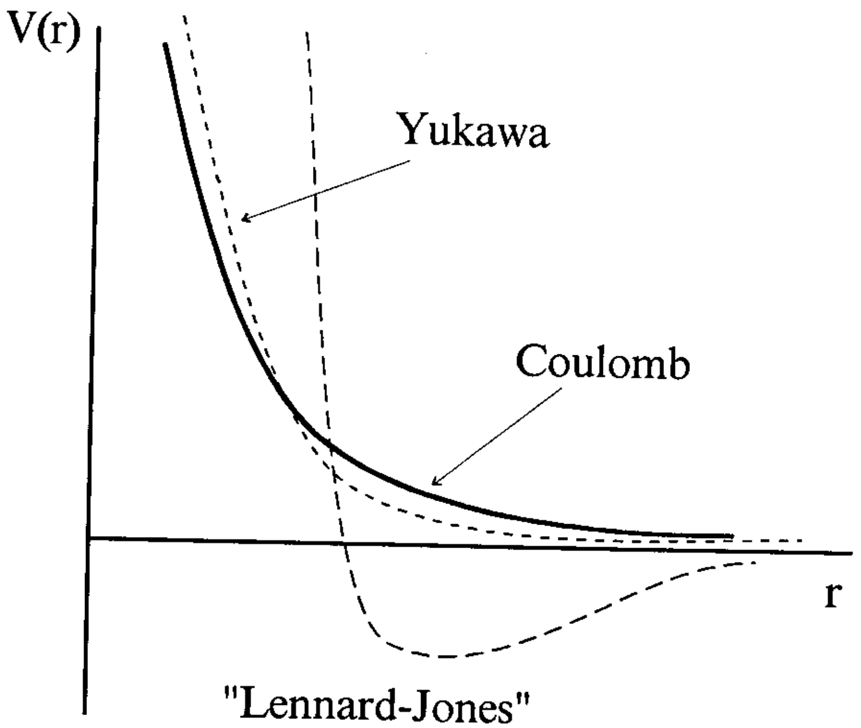
ionic crystals
dusty plasma
pure ion plasma
colloidal plasma

Problems:

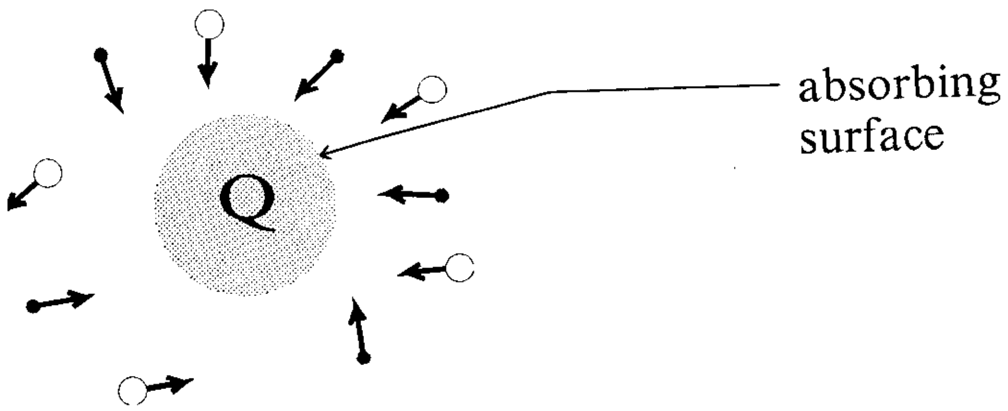
charging,
type of interaction,
confinement,
self-organisation,
phase transitions...

types of interaction

potential	structure	medium
Coulomb $V(r) = \frac{e^2}{r}$	liquid, bcc (Γ=180)	one-component plasma
Yukawa $V(r) = \frac{e^2}{r} \exp(-\kappa r)$	liquid, bcc, fcc	colloidal suspension, dusty plasma?
"Lennard-Jones", e.g. $V(r) = U_0 \left[\left(\frac{a}{r} \right)^2 - \frac{b}{r} \right]$	phase coexistence	dusty plasma?



Dusty plasma
Charging of a single grain



initially: $Q = 0; \quad j_e \approx n_e v_{Te} >> n_i v_{Ti}$

in equilibrium: $Q < 0; \quad j_e = j_i$

OML theory:

$$j_e(r) = \frac{a^2}{r^2} n_0 \sqrt{\frac{T_e}{2\pi m}} \exp\left(-\frac{e\varphi_0}{T_e}\right)$$

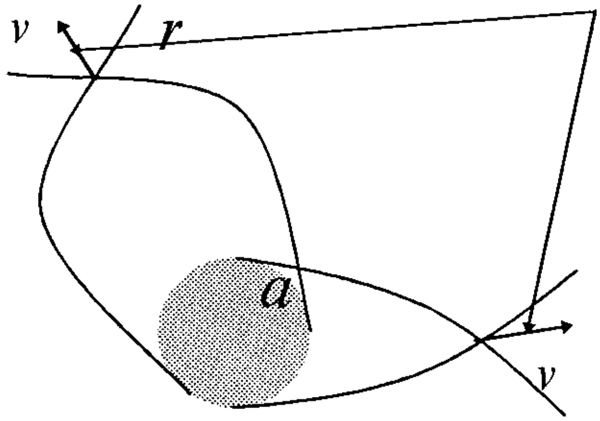
$$j_i(r) = \frac{a^2}{r^2} n_0 \sqrt{\frac{T_i}{2\pi M}} \left(1 - \frac{e\varphi_0}{T_i}\right)$$

equilibrium surface potential:

$\varphi_0 = -\zeta T_e / e; \quad \zeta = 2.5 \ \ (M/m = 1840; \quad T_e = T_i)$

charge: if $a << \lambda_D$, than $Q = a\varphi_0 \leq 10^4 e$

Plasma response:



$$\Omega = \alpha(v)\frac{a^2}{r^2}$$

no particles inside these cones

$$n_i(r) = n_0\left[1 - \frac{e\varphi(r)}{T_i}\right] - \delta n_i^a$$

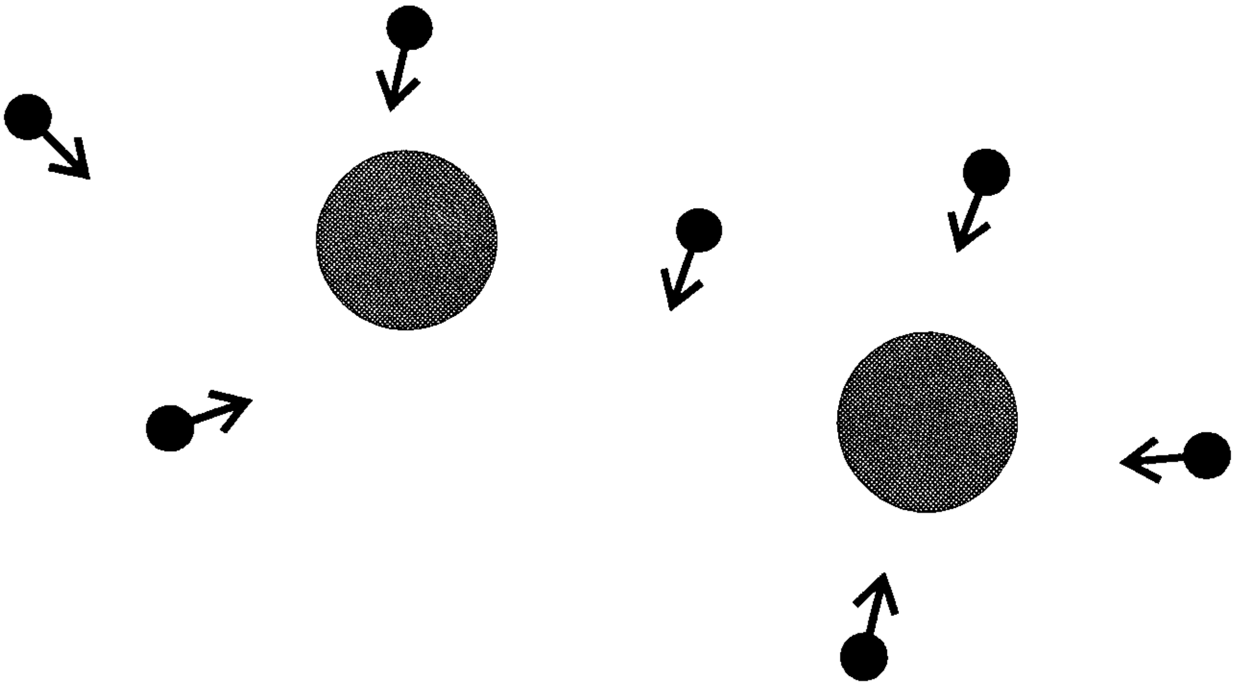
$$\delta n_i^a = n_0\frac{a^2}{4r^2}\left(1 - \frac{2e\varphi_0}{T_i}\right)$$

electrostatic potential of a single grain:

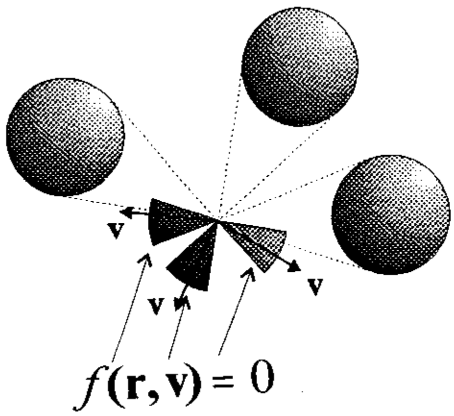
$$\Delta\varphi(r) = 4\pi\sum_{\alpha=e,i}\delta n_\alpha^a$$

$$\varphi(r) \approx \begin{cases} \dfrac{1}{r^2}; & r >> \lambda_D \\ \dfrac{1}{r}\exp\left(-\dfrac{r}{\lambda_D}\right); & r \le \lambda_D \end{cases}$$

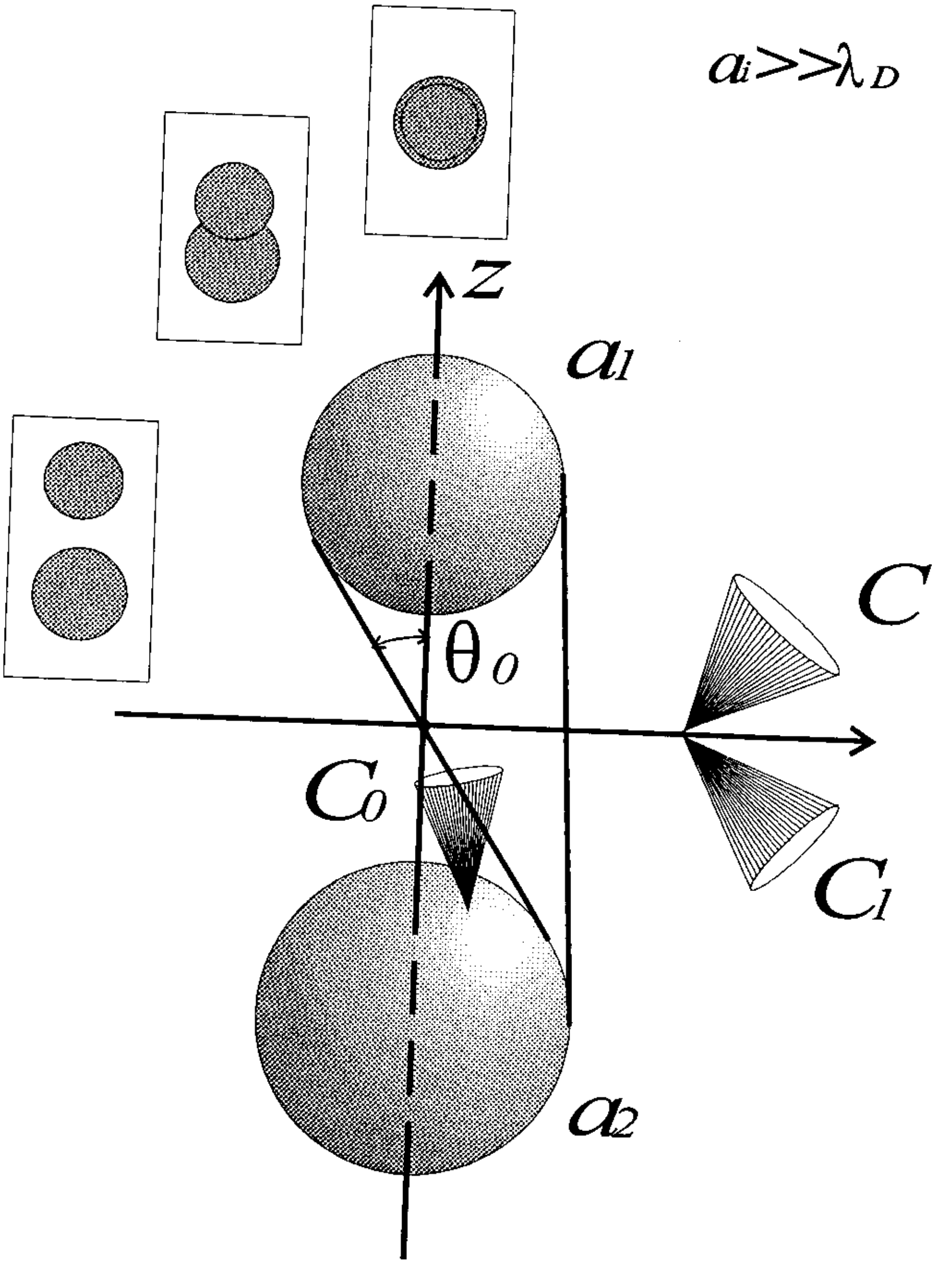
Inelastic collisions - Lesage (1784)



Absorbing grains - Ignatov (1995)

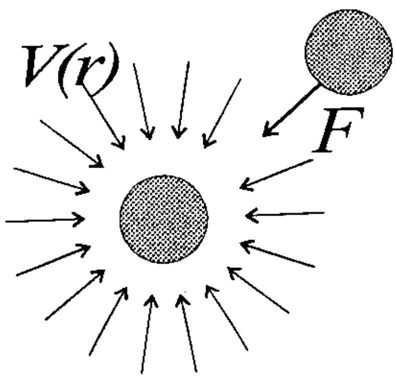


$$f(\mathbf{r},\mathbf{v}) = f_0(\mathbf{v})\prod_i \theta\left(v\sqrt{(\mathbf{r}-\mathbf{R}_i)^2 - a_i^2} - \mathbf{v}(\mathbf{r}-\mathbf{R}_i)\right)$$



$$F = \frac{3\pi}{4} n_0^{(i)} T_i \frac{a_1^2 a_2^2}{R^2}$$

$a_i << \lambda_D$

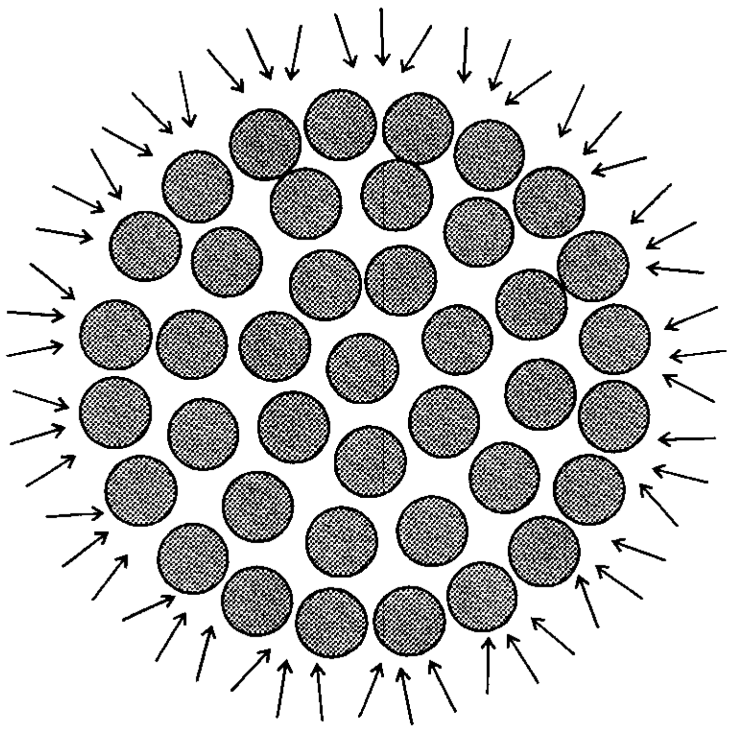


$$J_i \propto a_1^2 n_0^{(i)} v_{Ti}$$

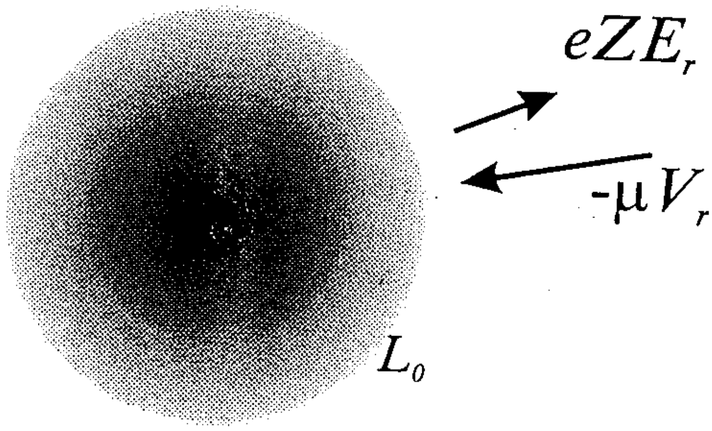
$$V(r) = -\frac{1}{4\pi r^2}\frac{J_i}{n^{(i)}(r)}$$

$$F \approx a_2^2 m_i v_{Ti} n^{(i)}(r) V(r)$$

$$F \approx n_0^{(i)} T_i \frac{a_1^2 a_2^2}{r^2}$$



PLASMA-DUST CLOUD



$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\mathbf{E}\frac{\partial f_\alpha}{\partial \mathbf{v}} = -\int dZ v\sigma_\alpha(v,Z) n_d(r,Z) f_\alpha + I_\alpha^c(v)$$

$$\frac{\partial f_d(Z)}{\partial t} + \mathbf{v}\frac{\partial f_d}{\partial \mathbf{r}} - \frac{eZ}{M}\mathbf{E}\frac{\partial f_d}{\partial \mathbf{v}} = -\frac{1}{M}\frac{\partial}{\partial \mathbf{v}}[\mathbf{A} f_d] - \frac{1}{e}\frac{\partial}{\partial Z}[J(Z) f_d]$$

$$\nabla \mathbf{E} = 4\pi e(n_i - n_e - Z n_d)$$

where

$$\mathbf{A} = \sum_{\alpha=e,i} m_\alpha \int d\mathbf{v}\, \mathbf{v}\left[v\sigma_\alpha(v,Z) + \frac{4\pi e^4 Z^2 L}{m_\alpha v^3}\right] f_\alpha(v)$$

$$J(Z) = \sum_{\alpha=e,i} e_\alpha \int d\mathbf{v}\, v\sigma_\alpha(v,Z) f_\alpha(v)$$

Cold dust:

$$\frac{\partial n_d(t,\mathbf{r},Z)}{\partial t}+\nabla\left(n_d\mathbf{V}_d\right)+\frac{\partial}{\partial Z}(J(Z)n_d)=0$$

$$n_d\left(M\frac{\partial V_d}{\partial t}+M\left(\mathbf{V}_d\nabla\right)\mathbf{V}_d+eZ\mathbf{E}-\mathbf{A}+J(Z)M\frac{\partial \mathbf{V}_d}{\partial Z}\right)=0$$

Steady state: $\mathbf{V}_d=0$

Charge balance:

$$J(Z,\mathbf{r})n_d(Z,\mathbf{r})=0 \quad\Rightarrow\quad n_d(Z,\mathbf{r})=n_d(\mathbf{r})\delta\left(Z-Z_0(\mathbf{r})\right);\ J(Z_0(\mathbf{r}),\mathbf{r})=$$

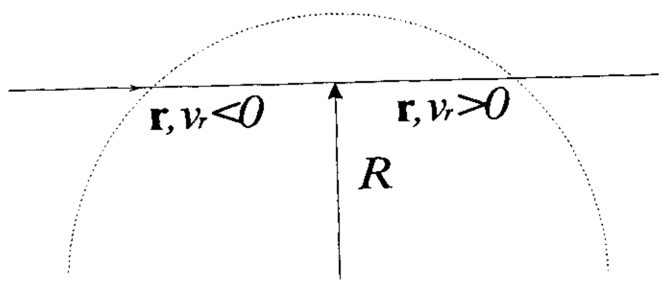
Momentum balance:

$$n_d\left(eZ\mathbf{E}-\mathbf{A}\right)=0$$

Small absorption

$$f_\alpha(\mathbf{r},\mathbf{v})=f_{0\alpha}\left(1-\frac{e_\alpha}{T_\alpha}\phi(\mathbf{r})-\sigma_\alpha(v)\lambda(\mathbf{r},\mathbf{v})\right)$$

$$\lambda(\mathbf{r},\mathbf{v})=\int_R^\infty dr'\,\frac{r'n_d(r')}{\sqrt{r'^2-R^2}}+\mathrm{sgn}(v_r)\int_R^r dr'\,\frac{r'n_d(r')}{\sqrt{r'^2-R^2}};\quad R=\left|\mathbf{r}\times\mathbf{v}\right|/v$$



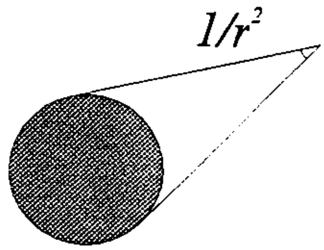
Drag force:

$$A_r = -\beta \frac{1}{r^2}\int_0^r dr' r'^2 n_d(r') \xrightarrow[r\to\infty]{} \frac{1}{r^2} \;;\; \beta \approx n_0 T_i a^3 r_D$$

Density:

$$n_\alpha = n_0\left(1 - \frac{e_\alpha}{T_\alpha}\phi - \langle\sigma_\alpha\rangle\chi(r)\right)$$

$$\chi(r) = \int_0^\infty dr' \frac{r'}{r}\ln\left|\frac{r+r'}{r-r'}\right| n_d(r') \xrightarrow[r\to\infty]{} \frac{1}{r^2}$$



$$n_e + Zn_d - n_i \approx 0 \;\;\Rightarrow\;\; \phi \propto \frac{1}{r^2} \;,\; E_r \propto \frac{1}{r^3}$$

$$eZE < A$$

$$\Downarrow$$

$$n_d(r) \equiv 0 \;\;;\;\; r > L_0$$
$$n_d(r) \neq 0 \;\;;\;\; r < L_0$$

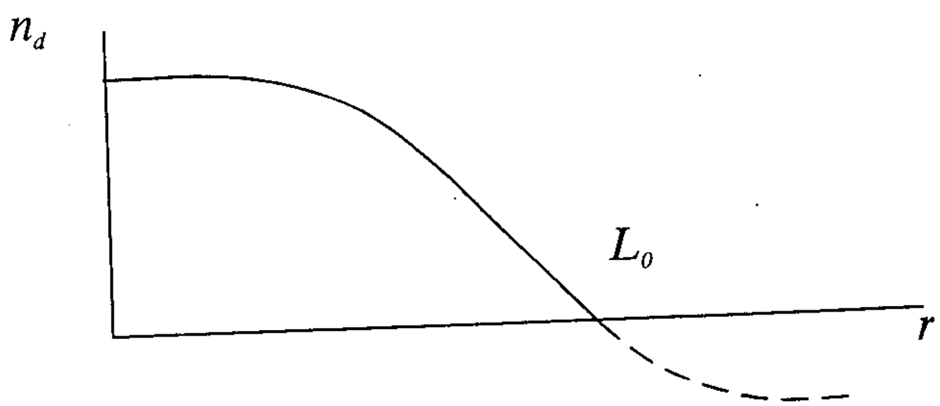
Sharp boundary at L_0

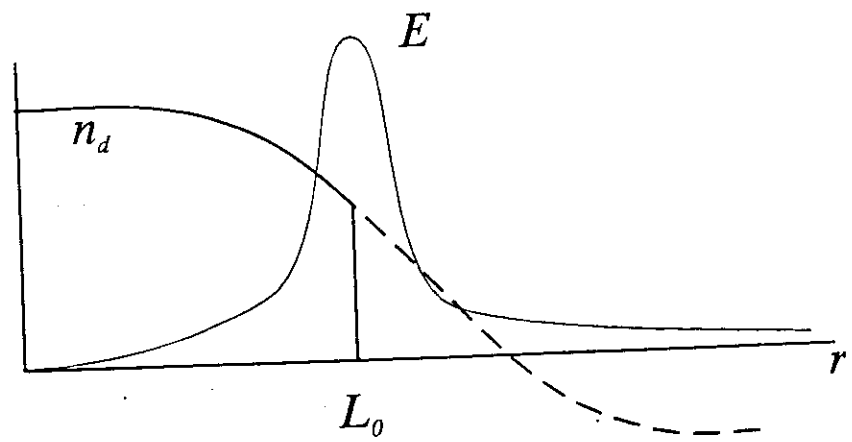
Inside the cloud:

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{dn_d}{dr} = -\frac{1}{l^2}n_d$$

$$l^2 = \frac{4\pi e^2 Z^2 r_D^2}{\beta} \propto \frac{r_D^3}{a}$$

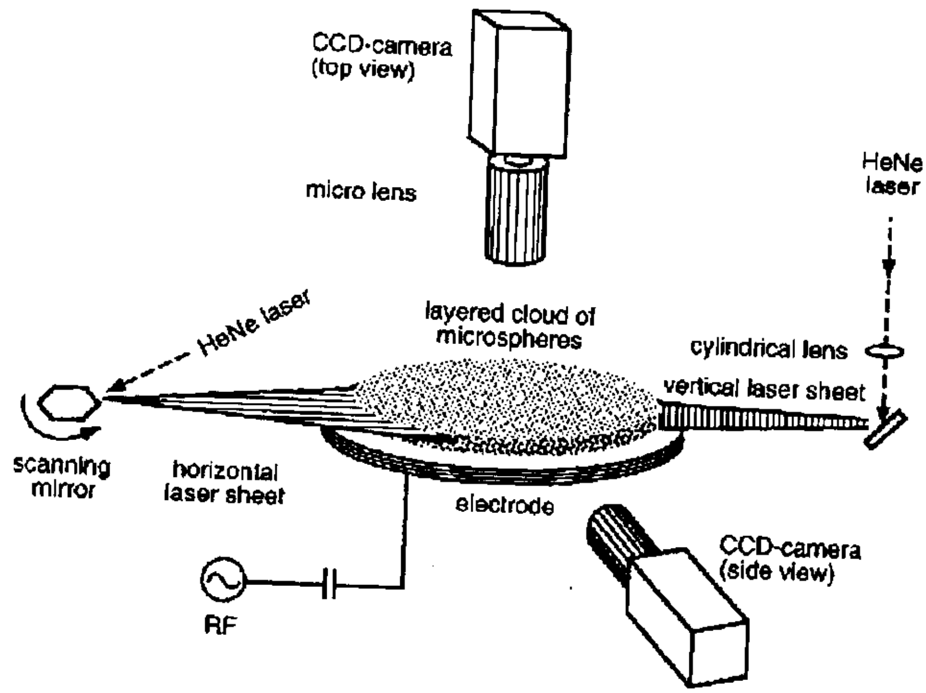
$$n_d(r) = \frac{N}{4\pi l^2}\frac{\sin(r/l)}{r} \quad \Rightarrow \quad L_0 = \pi l$$



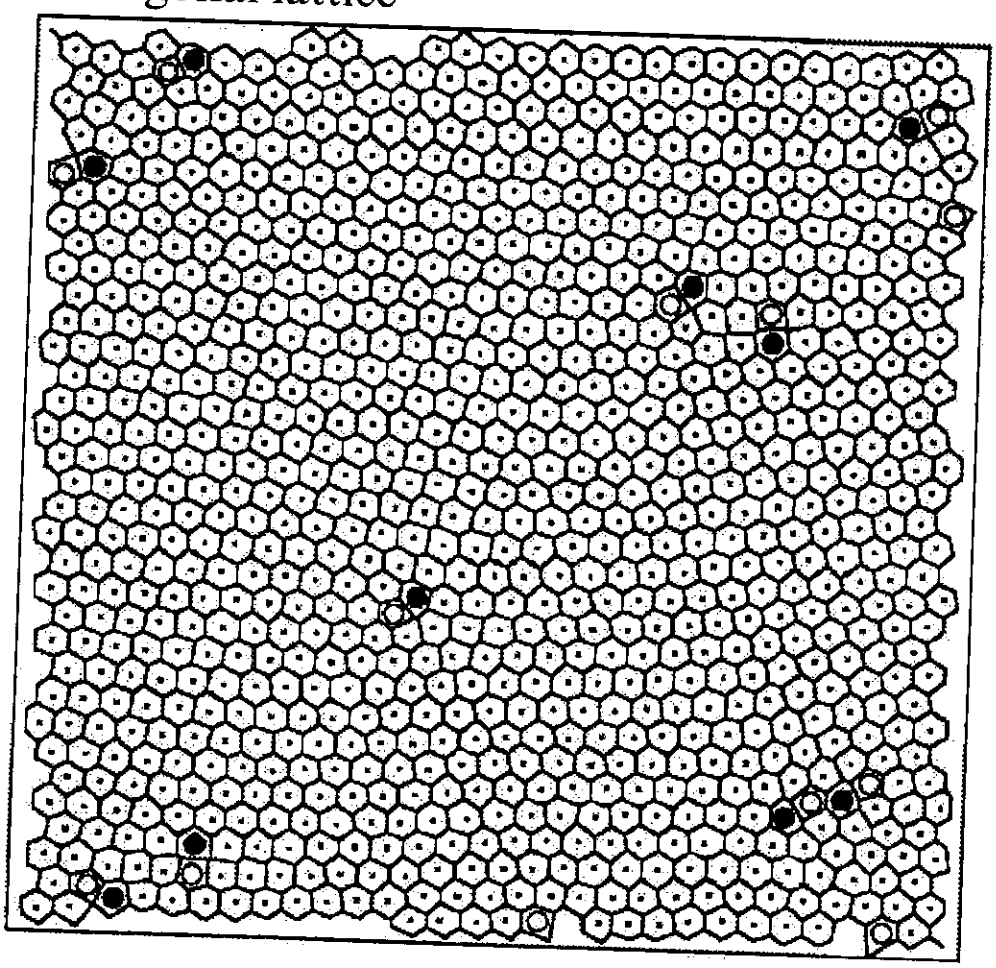


Dust crystal

(Pieper,Goree,Quinn. J. Vac. Sci. Technol. A **14**, 519 (1996))

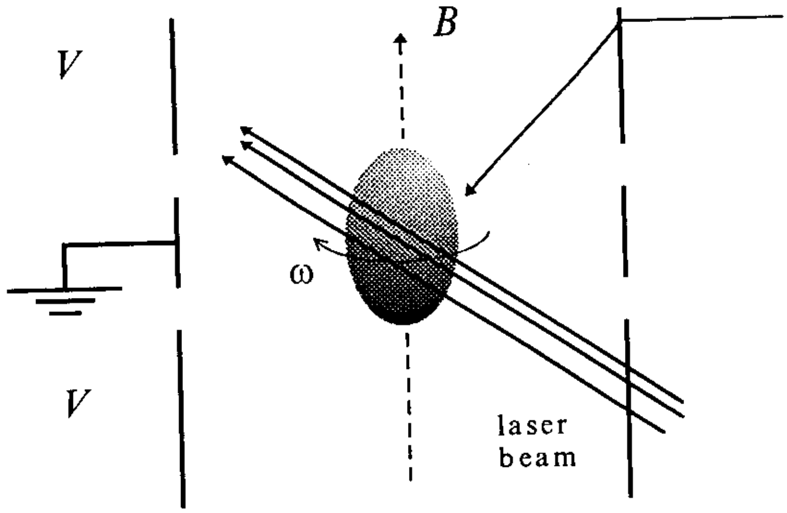


Hexagonal lattice



Pure ion plasmas

Particle traps



$$\phi(\mathbf{r}) = \frac{V_0}{L^2}(2z^2 - \rho^2)$$

Paul trap: ***B=0,*** ***$V=V_0\cos(\omega_0 t)$***

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\cos(\omega_0 t)$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{r}_1\cos(\omega_0 t); \qquad \mathbf{r}_1 = -\frac{e}{m\omega_0^2}\mathbf{E}(\mathbf{r})$$

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1\sin(\omega_0 t); \qquad \mathbf{v}_1 = \frac{e}{m\omega_0}\mathbf{E}(\mathbf{r})$$

$$\frac{d\mathbf{v}_0}{dt} = -\frac{1}{m}\nabla U_p(\mathbf{r}); \qquad U_p(\mathbf{r}) = \frac{e^2}{4m\omega_0^2}\mathbf{E}(\mathbf{r})^2$$

$$U_p(\mathbf{r}) \approx \frac{e^2}{m\omega_0^2}\frac{V_0^2}{L^4}\left(\rho^2 + 4z^2\right)$$

Limitations: $r_1 \ll L$; "Temperature"$\cong U_p$

Penning trap: ***B≠0, V=const***

Single particle:

$$z(t) = z_0 \cos(\omega_\parallel t); \qquad \omega_\parallel{}^2 = \frac{4eV_0}{mL^2}$$

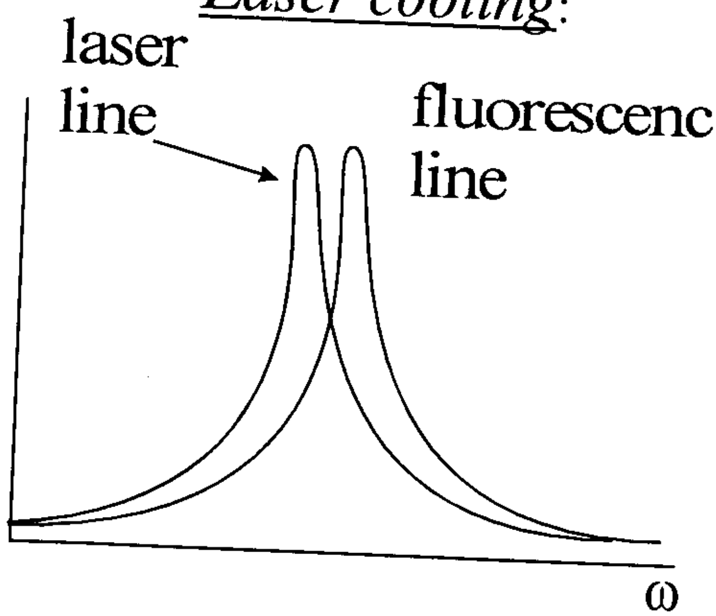
$$x(t) = R\sin(\omega_\perp t);$$

$$x(t) = R\cos(\omega_\perp t); \qquad \omega_\perp{}^2 = \Omega_c^2 \pm \frac{1}{2}\omega_\parallel{}^2$$

trapping condition: $\Omega_c^2 > \frac{1}{2}\omega_\parallel{}^2$

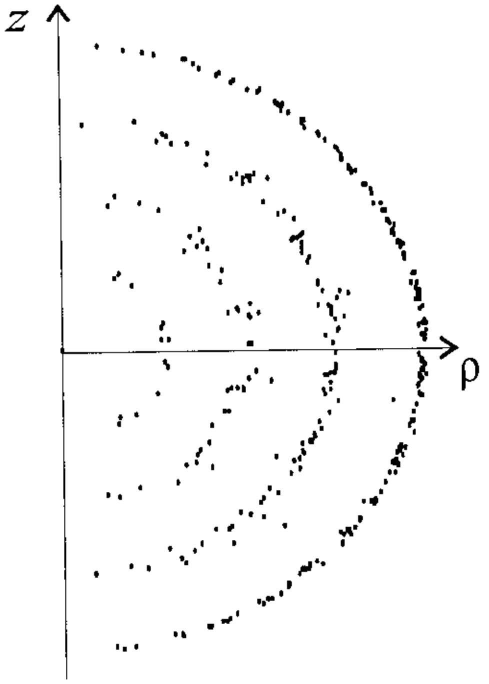
Generally, $V(\mathbf{r}) \propto az^2 + \rho^2$

Laser cooling:

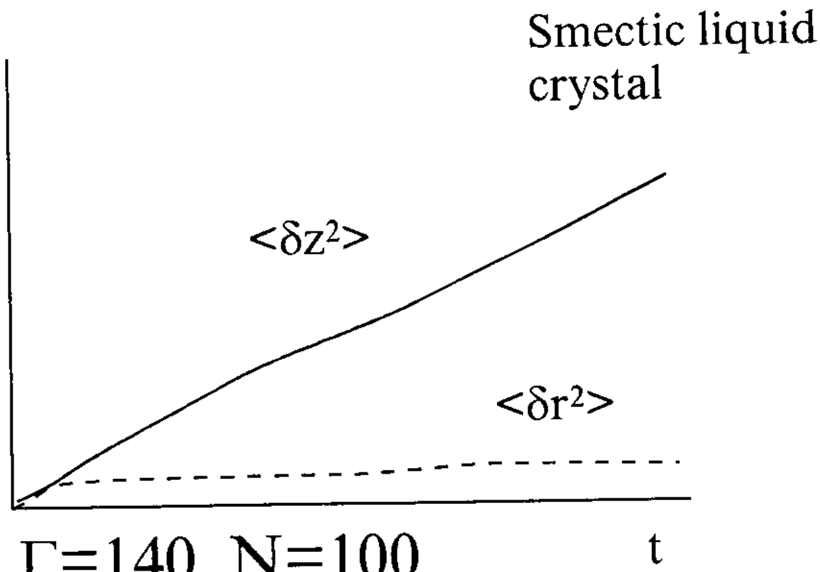


Shell structure of ion cloud

$V(\mathbf{r}) \propto z^2 + \rho^2$



Diffusion



Γ=140, N=100
(Dubin, O'Neil,1988)

Energy estimations

M shells of radii R_i each containing N_i ions

$$W = \sum_{i=1}^{M} \Bigg[\frac{1}{2} m\omega_0^2 R_i^2 N_i \qquad \textit{(External energy)}$$

$$+ \frac{1}{2}\frac{e^2}{R_i} N_i^2 \qquad \textit{(Self energy of the i-th shell)}$$

$$+ \frac{e^2}{R_i} N_i \sum_{j=1}^{i-1} N_j \Bigg] \qquad \textit{(Energy of the i-th shell in the field of all inner shells)}$$

$$\frac{dW}{dR_i} = 0; \quad \frac{dW}{dN_i} = 0; \quad \sum_{i=1}^{M} N_i = N$$

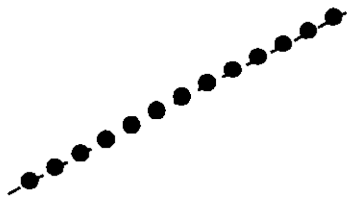
$$\Downarrow$$

$$N_i \approx \frac{9}{5} \frac{i^{4/5}}{M^{9/5}} N$$

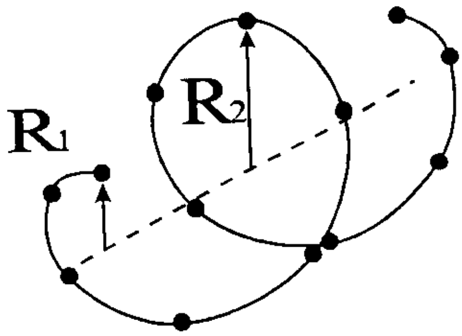
$$R_i \approx \left(\frac{e^2}{m\omega_0^2} \right)^{1/3} \frac{i^{3/5}}{M^{3/5}} N^{1/3}$$

Generally, $V(\mathbf{r}) \propto az^2 + \rho^2$

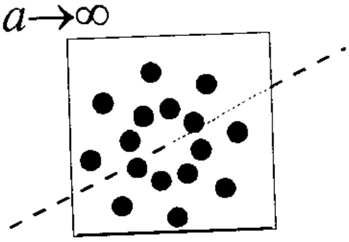
a=0



$a>a_{cr1}$



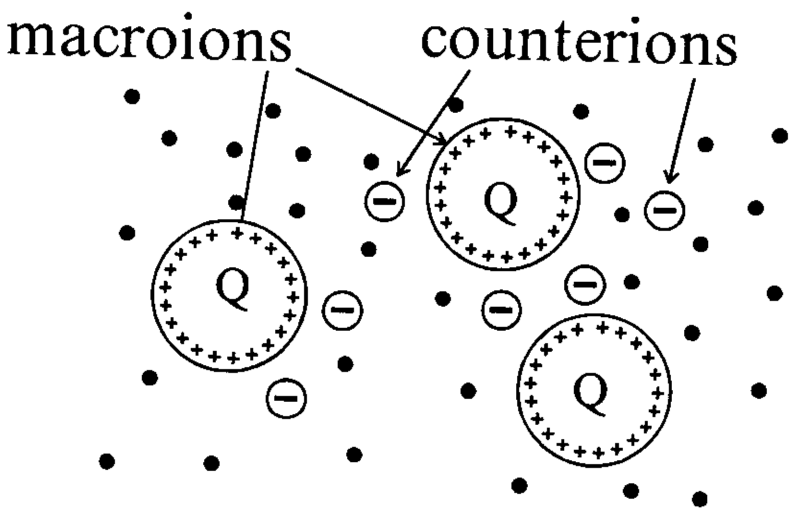
$a\to\infty$



Problems:

- Statistical description: microcanonical ensemble, no thermodynamics, no ergodicity
- Laser cooling
- Classification of structures —no perfect lattices

Colloidal suspensions



size, $a=10^{-2}\div10^{-6}$ cm; charge, $Q=10^2\div10^4\ e$
macroion potential ≈1 V
temperature≈0.03 eV

Direct interaction :

$$V(r) = -\frac{z^{*2}e^2}{\varepsilon r}\exp(-r/\lambda_D); \quad z^* = \frac{e^{a/\lambda_D}}{1+a/\lambda_D}$$

Inderect interaction

(Schram, Trigger, Contrib. Plasma Phys, 37, 251 (1997))

partition integral:

$$Z = \frac{1}{V^{N_c+N_M}} \int dr_1 \ldots dr_{N_c} \int dR_1 \ldots dR_{N_M} \exp[-\beta(U_{CC} + U_{CM} + U_{MM})]$$

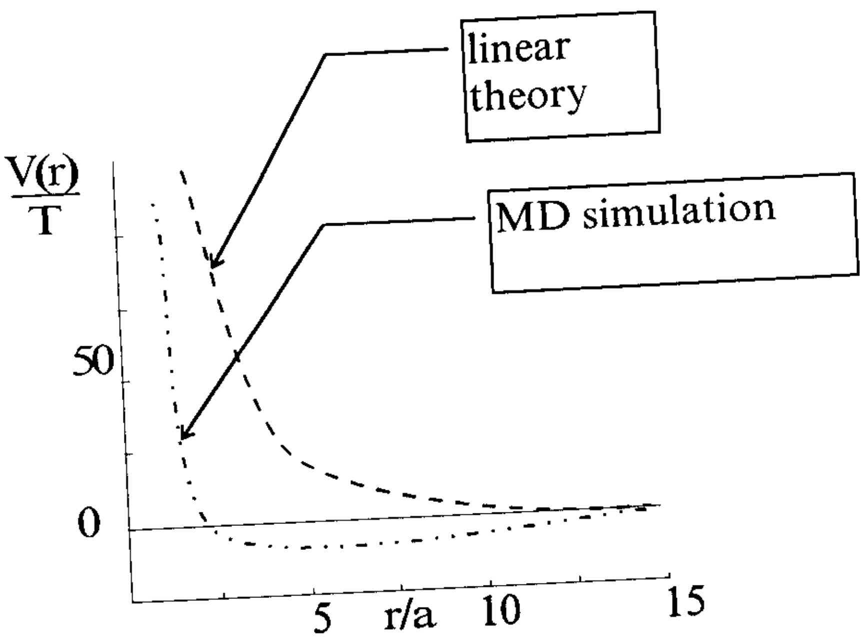
$$\equiv \frac{1}{V^{N_M}} \int dR_1 \ldots dR_{N_M} \exp[-\beta U_{MM}] Z_C, \qquad \text{where}$$

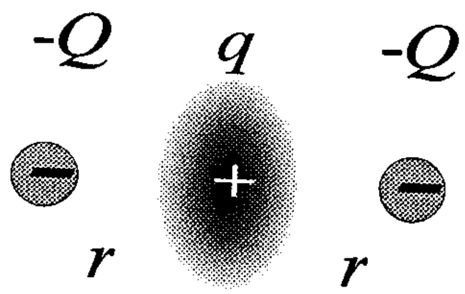
$$Z_C = \frac{1}{V^{N_c}} \int dr_1 \ldots dr_{N_c} \exp[-\beta(U_{CC} + U_{CM})] \equiv \exp[-\beta F_C\{R\}]$$

$$U_{CC} = \sum_{i<j} V_{CC}(r_i - r_j)$$

$$U_{CM} = \sum_{ij} V_{CM}(r_i - R_j)$$

$$U_{MM} = \sum_{i<j} V_{MM}(R_i - R_j)$$





$$U = \frac{Q^2}{2r} - \frac{2Qq}{r} < 0 \quad \Rightarrow \quad q > Q/4$$

Problems:

1. Dependence of the macroion charge on the state of surrounding plasma
2. Can nonlinear screening result in attaction?