INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONES: 224281/2/3/4/5/6
CABLE: CENTRATOM - TELEX 460392-I

SMR/104-18

COLLEGE ON SOIL PHYSICS

19 September - 7 October 1983

MEASURING AND ESTIMATING EFFECTIVE THERMAL CONDUCTIVITY OF INDIAN DESERT SAND

D.R. CHAUDHARY

Department of Physics
University of Rajasthan
Jaipur 302004
India

Measuring and Estimating Effective Thermal Conductivity of Indian Desert Sand

D.R. Chaudhary

Department of Physics, University of Rajasthan, Jaipur-302004, India.

Abstract

A line source in the form of a thermal probe was employed to measure thermal conductivity of desert sand. Hot wire method and acalorimeter were used to determine thermal diffusivity. Specific heat of dry sand was much accurately determined using continuous fall electrical method developed by us. Specific heat, density and diffusivity values enabled to determine thermal conductivity of samples too.

Desert sand being dry, may be considered as a two-phase system. Solid phase (mineral), is uniformly dispersed in fluid phase (dry air). An estimation of the effective thermal conductivity was made considering either phase as thermal resistors of different kind. We considered weighted geometric mean of resistors in series and parallel. This, ohm's law model, predicted bounds on the effective thermal conductivity. These bounds enclose lesser area of the realisable region and suit well to systems like desert sand whose solid to fluid conductivity ratio is large. A rigorous approach made by us considers spheres of solid forming a cubic array in continuous fluid phase. Modification in original flux due to solid spheres enables one to estimating the effective thermal conductivity.

Measured values of the effective thermal conductivity of desert sand were compared with estimated values at porosities and good agreement have been found.

Introduction - Indian desert is situated in the western part of India. The dune sand, is not only quartz, other materials like feldspar, hornblende and secondary minerals form full spectrum of a sand sample from the desert area. It is brown in colour and medium sized sand whose predominant particle size lie near 138 microns and average roundness 0.75. One of the samples from the location (27^o, 76^o) has been described by apparent density 1.52 (g/cm^3), specific gravity 2.6, permeability 5.50×10^4 (cm/s), relative dielectric constant at frequency 9.5×10^9 c/s is 2.8 and pH-value 7.50. It is aeolian soil having sand and little clay; sample is non-alkaline and non-saline; water holding capacity percentage by weight for the aggregate is 30. Thermal conductivity of solid material has been estimated to 8×10^{-3} (cal/cm/s/°C)[1].

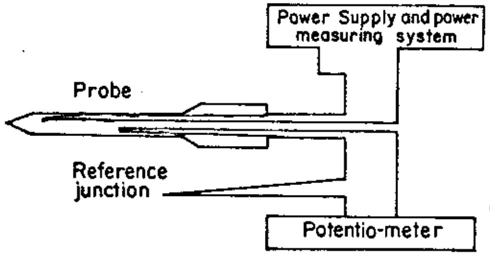
Thermal Parameters Investigation - Thermal conductivity and diffusivity of desert sand have been determined by us using nonsteady state methods while specific heat has been determined using a steady state method.

Probe[2] for the determination of thermal conductivity (λ_e) — When a line heat source is burried in an infinite sample, the change in temperature with time at a point on the source determines thermal conductivity by

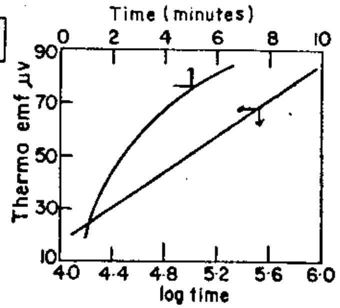
$$\lambda_e = \frac{Q}{4\pi(\theta_2-\theta_1)} \ln \left(\frac{t_2-t_o}{t_1-t_o}\right)$$

θ_2 and θ_1 are two values of temperatures at times t_1 and t_2 ; t_o

is the correction factor in time to be obtained with standard material. Q is the power per unit length supplied to the probe.

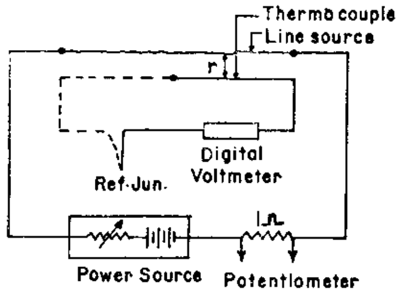


Thermal Probe

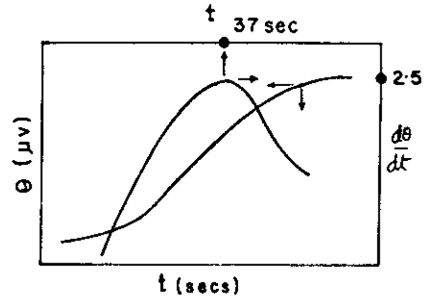


emf generated on Probe

Hot wire[3] for determination of thermal diffusivity (α_e) — When the point of observation in the sample is at a distance r from the line heat source the thermal diffusivity is determined by $\alpha_e = r^2/4t$, where t is the time at the point of inflection of temperature θ ($t = t_{max}$).



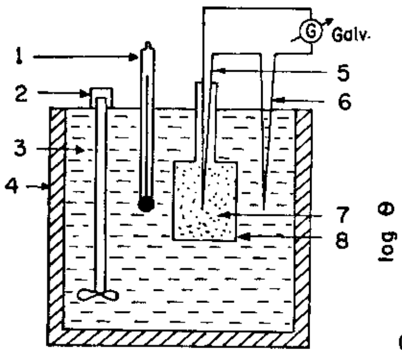
A Hot wire arrangement



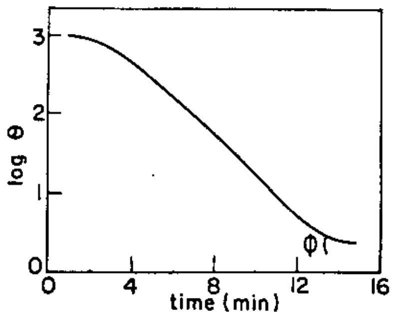
Heating of brick sand

Acalorimeter method[4] for finding diffusivity (α_e) — This method is based on the theory of heating a soil sample encased in a metallic container in the stage of regular thermal regime. Knowing shape factor B of the test body, which in case of cylindrical containers depend on radius r and length l, and by slop of log deflection versus time curve one determines diffusivity of the soil using expression

$$\alpha_e = \frac{\partial}{\partial t} \ln\theta \cdot \frac{1}{\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{l}\right)^2}$$

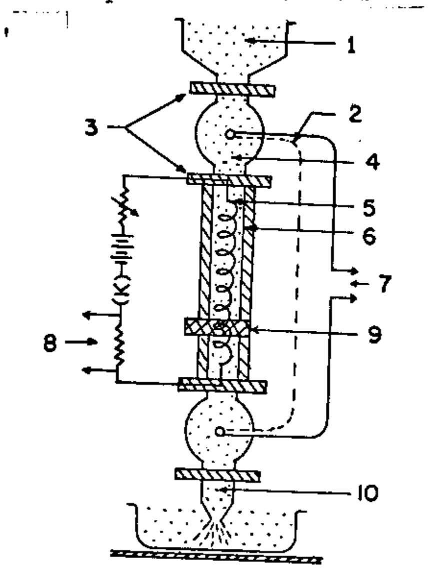


Acalorimeter
(1)Thermometer (2) Stirer (3) Water
(4)Thermostat bath(5) & (6)Thermocouple
(7) Soil (8) Acalorimeter.



Heating curve for dry sand

Continuous fall electrical method[5] for specific heat c determination — Callendar and Barnes' continuous flow method used in finding specific heat of liquids is modified, where the loose sand was allowed to fall through a double walled glass tube. A steady electric current flows through centrally stretched heater. A copper constantan thermocouple was employed to measure the difference in temperatures of incoming and



out going material. To vary rate of fall different diametered nozzles were employed. Specific heat c was calculated from

$$c = (i_1^2 - i_2^2) R \times 10^7 / J(m_1 \Delta\theta_1 - m_2 \Delta\theta_2)$$

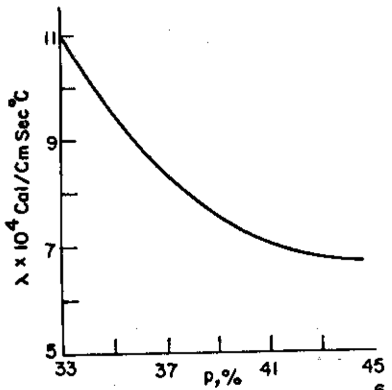
Here i_1 and i_2 are current values when rates of material fall were m_1 and m_2 and corresponding steady state temperatures differences at the ends of glass tube were $\Delta\theta_1$ and $\Delta\theta_2$ respectively. Heat loss from the glass tube were regarded the same in both tests as rise in temperature differed by less than 10 µV.

Fig. Schematic diagram of continuous fall method for the determination of specific heat of loosegranular materials – (1)Reservoir (2)Thermocouple (3)cork (4)sample (5) heater (6)evacuated and silvered double walled glass tube (7)to potentiometer for the measurement of i (9) glass tube coupled to a vibrator and (10) nozzle.

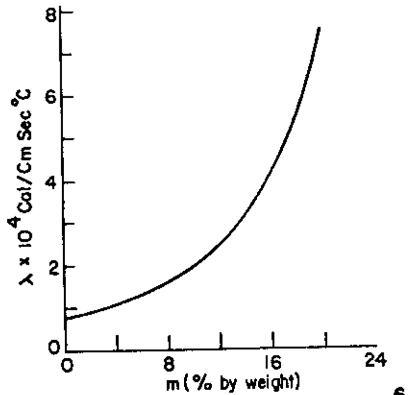
Experimental findings

Sample	Diffusivity ($\alpha \times 10^3$ cm^2/s) Hot wire[3]	Acalorimeter[1]	Specific heat (cal/g/°C) CFM[5]	*Thermal conductivity ($\lambda \times 10^4$ cal/cm/s/°C) Probe[1]	acal.[1]
Dune sand Φ=0.5		1.85±0.02	0.198±0.035	5.88±0.2	5.96±0.10
Brick sand Φ=0.5	1.46			4.66	

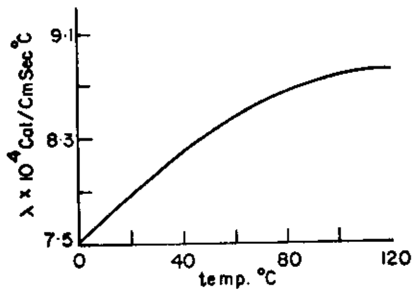
*To convert cal/cm/s/°C into W/m/°K multiply by 418.7.



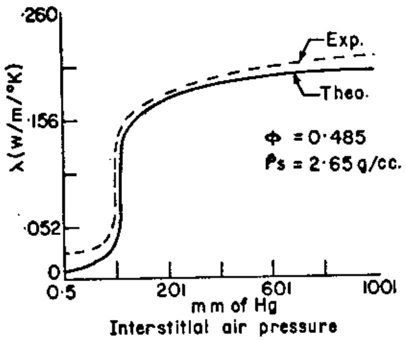
Thermal Conductivity with porosity[6]



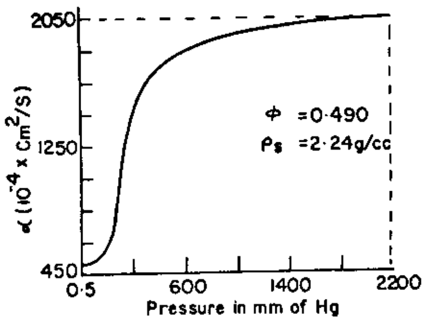
Thermal Conductivity with moisture[6]



Thermal Conductivity with temperature[6]



Thermal Conductivity with Interstitial air pressure (Dune sand)[13]



Thermal diffusisity with Interstitial air pressure (Mud powder)[7]

Predicting λ_e of two phase systems - Dry sand is considered homogeneous in respect of heat flow, which occurs on a molecular scale, and here the process effective is conduction. In such a sample heat flux density H, quantity of heat flowing per unit time per unit of surface area, is proportional to the temperature gradient $\nabla\theta$ ($^{\circ}C/cm$)[7].

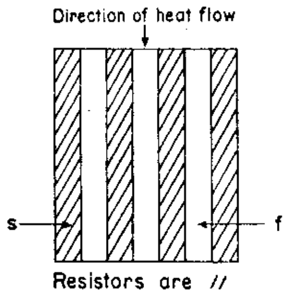
$$H = -\lambda \nabla\theta \qquad (1)$$

The proportionality constant is termed as thermal conductivity. Sand is a composite medium as such an appropriate value of λ is to be introduced in (1). This thermal quantity λ_e characterises the sample from viewpoint of its capacity in transporting heat through it. The effective thermal conductivity λ_e of a soil depends on its mineral composition, texture, phase orientation, amount of air etc. Accounting all these features together an expression of thermal conductivity becomes difficult to obtain. Workable expressions for the estimation of λ_e is to consider a granular material made of two components: continuous medium of conductivity λ_f with volume fraction ϕ ; and dispersed granules of conductivity λ_s with volume fraction $(1-\phi)$. This scheme describes a dry sand or fully saturated sand.

When one assumes that the flux lines are straight i.e. these are not markedly altered through bending toward or away from grains one arrives at semi-empirical relations. Considering the sand as a composite system made of successive flat layers of solid and fluid two plans are visualised[8].

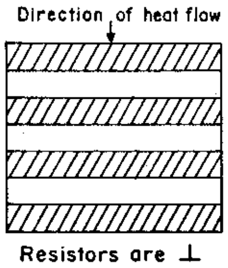
(a) Parallel distribution giving maximum λ_e hence this composite is a poor insulator. Mathematically it is arithmatic mean of constituent conductivities

$$\lambda_e = \phi\lambda_f + (1-\phi)\lambda_s \qquad (2)$$



(b) Series distribution which yields minimum λ_e hence this formation is a better insulator. It is harmonic mean of conductivities

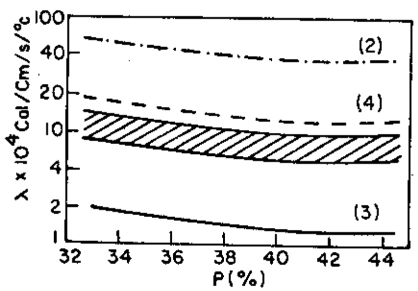
$$\lambda_e = \left[\frac{\phi}{\lambda_f} + \frac{(1-\phi)}{\lambda_s} \right]^{-1} \qquad (3)$$



One then has an intermediate mean which will correspond to randomisation of layers.

$$\lambda_e = \lambda_f^{\phi} \lambda_s^{(1-\phi)} \qquad (4)$$

A plot for extremum (2), (3) and (4) is as shown. Cross hatched area indicates realisable region on sands and soils.



In order to obtain better results an equation analogous to (4) should be taken whose predictions fall within cross-hatched region. We[9] considered weighted geometric mean of maximum thermal conductivity (2) calling

it λ_{11}, and minimum thermal conductivity (3)

Calling it by $\lambda_{\perp}$. We attributed weighting factor to the distribution of phases in a natural sample with regard to the energy flow direction. Let n th fraction of the sample made of alternate zones of solid and fluid be oriented parallel to the direction of heat flow (giving $\lambda_{\parallel}$), then (1-n) th fraction of the sample zones will be oriented perpendicular to the direction of heat flow (giving $\lambda_{\perp}$). The effective thermal conductivity of such a plan will be

$$\lambda_e = \lambda_{11}^{n} \lambda_{\perp}^{1-n}. \qquad (5)$$

'n' in equation (5) is

$$n = \frac{\log\left[\phi \frac{\lambda}{\lambda_f} + (1-\phi)\frac{\lambda}{\lambda_s}\right]}{\log\left[1+\phi(1-\phi)\left\{\frac{\lambda_s}{\lambda_f} + \frac{\lambda_f}{\lambda_s} - 2\right\}\right]} \qquad (6)$$

Using measured values of λ_e on soils and remembering $\lambda_s/\lambda_a \sim 130$ Eq. (6) was reduced to working form

$$n = \frac{0.5\ (1-\log\phi)}{\log\{\lambda_s/\lambda_f\ \phi\ (1-\phi)\}} \qquad (7)$$

Calculated values using (5) and (7) and experimental values of silt loam[1]

ϕ	Experimental ($\lambda_e \times 10^4$) cal/cm/sec/^oC	n calculated using (7)	calculated λ_e ($\lambda_e \times 10^4$) cal/cm/sec/^oC
0.568	4.15	0.421	4.20
0.401	6.88	0.475	7.02
0.332	9.18	0.515	9.26

It gives an idea that in sand like systems about 50 per cent of the sample has set parallel to the direction of heat flow and similar amount perpendicular to the direction of heat flow.

In the expression (6) maximum and minimum values of n are 1 and 0 respectively. Letting $\phi \to 0$ ($\lambda = \lambda_s$ for n = 1) and $\phi \to 1$ ($\lambda = \lambda_f$ for n = 1) one finds that the expression becomes divergent. Using L'Hospital rule we obtain[10]

$$\frac{\lambda_f}{\lambda_s} = 1 + \frac{1}{\lambda}\left(\frac{\partial\lambda}{\partial\phi}\right), \text{ for } n \neq \phi \text{ ; } n = 1,\ 0,\ \lambda = \lambda_s \qquad (8)$$

$$\frac{\lambda_f}{\lambda_s} = 1 - \frac{1}{\lambda}\left(\frac{\partial\lambda}{\partial\phi}\right), \text{ for } n = \phi \text{ ; } n = 0,1,\ \lambda = \lambda_f \qquad (9)$$

For sand like systems whose $\lambda_s > \lambda_a$ we have solution

$$\lambda_H = \lambda_s\, e^{\beta\phi} \text{ and } \lambda_L = \lambda_f\, e^{-\beta(1-\phi)} \qquad (10)$$

where $\beta = (\lambda_f/\lambda_s - 1)$. A linear combination of λ_H and λ_L under $\beta = -\ln(\lambda_f/\lambda_s)$ gives bounds for λ_e. The later condition enters for the reason that logarithmic derivatives of λ_H and λ_L should be equal if (10) is to yield the effective thermal conductivity values at different porosities. Hence,

$$\lambda_e = \lambda_L\phi + \lambda_H(1-\phi) \pm \phi(1-\phi)\sqrt{\lambda_L\lambda_H} \qquad (11)$$

Sample	ϕ	λ_e in $WM^{-1}\,^oC^{-1}$ using (11)	λ_e in $WM^{-1}\,^oC^{-1}$ experimental
Rajasthan Desert Sand	0.385	0.403	0.387
	0.430	0.316	0.312
	0.450	0.285	0.274

One notes the superposition of λ_L and λ_H in eq. (11) is empirical.

Now, let us optimise expression for λ [eq. (5)] with respect to the probability of orientation n. We then have[11],

$$\frac{1}{\lambda}\left(\frac{\partial\lambda}{\partial\phi}\right) = \frac{\lambda_f - \lambda_s}{\phi(\lambda_f-\lambda_s)+\lambda_s} \quad \text{for } n \to 1$$

$$\text{and} \quad \frac{1}{\lambda}\left(\frac{\partial\lambda}{\partial\phi}\right) = \frac{\lambda_f - \lambda_s}{\lambda_f-\phi(\lambda_f-\lambda_s)} \quad \text{for } n \to 0 \qquad (12)$$

The solutions of $(1/\lambda)(\partial\lambda/\partial\phi)$ are to be obtained for $0 < \phi < 1$. To let ϕ neither 0 nor 1 we average boundary conditions of ϕ, on (12), which are (8) and (9). Geometric mean of (8) and (9) is

$$\frac{1}{\lambda}\left(\frac{\partial\lambda}{\partial\phi}\right) = \frac{\lambda_f-\lambda_s}{(\lambda_s\lambda_f)^{1/2}} = a \qquad (13)$$

and their division give,

$$\frac{1}{\lambda}\left(\frac{\partial\lambda}{\partial\phi}\right) = \left(\frac{\lambda_f^2-\lambda_s^2}{\lambda_f^2+\lambda_s^2}\right) = b \qquad (14)$$

Solutions of (13) and (14) under boundary conditions $\lambda \to \lambda_s$ when $\phi \to 0$ and $\lambda \to \lambda_f$ when $\phi \to 1$ are

$$\lambda = \lambda_1 = \frac{1}{(e^a-1)}\left\{(\lambda_f-\lambda_s)\,e^{a\phi} + \lambda_s\,e^a - \lambda_f\right\} \qquad (15)$$

$$\lambda = \lambda_2 = \frac{1}{(e^b-1)}\left\{(\lambda_f-\lambda_s)\,e^{b\phi} + \lambda_s\,e^b - \lambda_f\right\} \qquad (16)$$

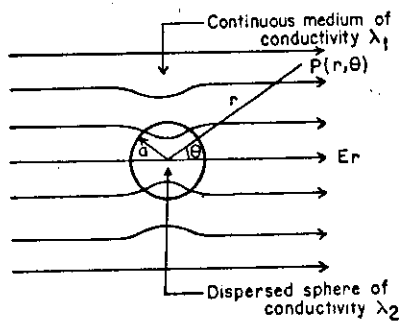
for all the values of a, b and ϕ we have $\lambda_2 > \lambda_1$ and thus

$$\lambda_2 > \lambda_e > \lambda_1$$

We therefore have (15) and (16) the bounds on the effective thermal conductivity of two-phase systems.

System	Porosity	λ_2 ($WM^{-1}K^{-1}$)	λ_1	λ_{exp} ($WM^{-1}K^{-1}$)	Remark
Sand	0.385	1.675	0.067	0.387	The upper bound falls
ZrO_2+air	0.09	1.52	0.924	1.55	close to the expt.
UO_2+air	0.267	5.66	0.089	5.67	values

Exact expression: A rigorous mathematical formulation for conductivity of two-phase system requires conditions of regular shape of grains and orientation of phases. We considered[12] a regular arrangement of spheres of dispersed phase forming a simple cubic lattice. The modifications in the flux because of dispersion, have been derived. A consideration of third order interactions found to predict thermal conductivity closely. For two phase systems these effective interactions yield



$$\lambda_e = \lambda_s\left[1-2x2.309\left(\frac{\lambda_s-\lambda_f}{2\lambda_s+\lambda_f}\right)\phi^{2/3}\right] \qquad (17)$$

In case of dry samples of sand one may use

$$\lambda_e \simeq \lambda_s\,(1-2.309\phi^{2/3}) \qquad (18)$$

Sample	Porosity	λ_e using (18) $WM^{-1}K^{-1}$	$\lambda_{expt.}$ $WM^{-1}K^{-1}$
Desert Sand	0.4297	0.312	0.312
	0.4394	0.269	0.302
	0.4502	0.218	0.289

Effective conductivity of sands at interstitial air pressures[14]:— The nature of λ_e, at different interstitial air pressures, in sand and like systems has also been studied. Our observations show that λ_{ep} at an interstitial air pressure p is predicted through

$$\lambda_{ep} = \lambda_o \left(\frac{p}{p+p_c}\right) \tag{19}$$

Here λ_p is the thermal conductivity of the interstitial air at the pressure p. p_c is termed characteristic value of pressure and is defined through the value of interstitial air pressure at which thermal conductivity of the sample reduces to half of its value at the normal air pressure. It is also found that p_c is inversely proportional to the grain size.

Sample	Porosity	Density gm/cc	Interstitial air Pressure (mm of Hg)	λ_e ($WM^{-1}K^{-1}$) Expt.	Using(19)
Dune sand	0.48	2.659	90	0.165	0.140
			120	0.173	0.154
			180	0.181	0.171

Appendix

(a) Effective thermal diffusivity α_e of sand — In a composite medium effective value of volumetric specific heat C ($cal/cm^3/^oC$) is given by

$$C = \sum_i \rho_i c_i \phi_i \tag{20}$$

where ρ is the density (gm/cm^3) of the sample and c is specific heat in terms of mass ($cal/gm/^oC$). In case of dry sand $\rho_s = 2.58$ gm/cc, $c_s = 0.184$ $cal/gm/^oC$; $\rho_a = 0.0012$ gm/cc, $c_a = 0.24$ $cal/gm/^oC$; $\phi = 0.36$

$C_e = \rho_a c_a \phi + \rho_s c_s (1-\phi) = 0.304$ $cal/cm^3/^oC$

For sand whose $\lambda_e = 6.11 \times 10^{-4}$ $cal/cm/sec/^oC$, α_e which is the ratio of λ_e and C_e will be

$$\alpha_e = \frac{\lambda_e}{C_e} = 2.0 \times 10^{-3}\ cm^2 sec^{-1}.$$

To predict effective thermal diffusivity of a sand like system we assumed a regular dispersion of spheres, in continuous phase, forming a cubic lattice. Accomodating third order interaction[15] the expression for thermal diffusivity is

$$\alpha_e = \alpha'_a \left[1+5.6(1-\phi)^{2/3} \left(\frac{\alpha'_s-\alpha'_a}{\alpha'_s+2\alpha'_a}\right) + 6.298\ (1-\phi) \cdot \frac{(\alpha'_s-\alpha'_a)}{(\alpha_s+2\alpha'_a)}\right] \tag{21}$$

where $\alpha'_a = \lambda_a/C$ and $\alpha'_s = \lambda_s/C$.

For dune sand we have: $\alpha_a' = 2.04 \times 10^{-4}$ cm^{2}sec^{-1}, $\alpha_s' = 26\phi3$ cm^{2}sec^{-1}. Thus (21) predicts $\alpha_e = 1.84 \times 10^{-3}$ cm^{2}sec^{-1}. Experimental value for the sample is 2.13×10^{-3} cm^2 sec^{-1}.

(b) Coefficient of heat storage of sand: Capability of storing the heat or storage quality of a composite sample is characterised by

$$\sqrt{\lambda_e C_e} = \frac{\lambda_e}{\sqrt{\alpha_e}} = b_e \; .$$

For the stated valued dune sand sample heat storage coefficient

$$b_e = 0.014 \text{ cal/cm}^2\text{/sec}^{1/2}\text{/}^{o}\text{C.}$$

References:

1. Chaudhary DR, Some problems of heat transfer in dispersed and porous media - Rajasthan desert sand, Thesis, University of Rajasthan, 1968.
2. Woodside W and Messmer JH, J Appl. Phy 32:9, 1961, 1688.
3. Sharma RG, Pande RN and Chaudhary DR (paper in preparation)
4. Luikov AV, Heat and mass transfer in capillary porous bodies, Pergamon Press, 1966.
5. Chaudhary DR and Veerendra Kumar, A continuous fall electrical method for measuring specific heat of loose granular materials, Am J Phy, 40, 1972, 1176.
6. Chaudhary DR, Agarwal MP and Bhandari RC, An investigation of certain thermophysical characteristics of sands of Rajasthan Desert, Ind J Pure and Appl. Phy, 7, 1969, 252.
7. Van Wijk WR, Physics of plant environment, North Holland Publishing Co., Amsterdam, 1966, 104 and 215.
8. Tye RP, Thermal Conductivity, Vol 1, Academic Press, London, 1969.
9. Chaudhary DR and Bhandari RC, Thermal Conductivity of two-phase porous materials: dry soils, Brit. J. Appl. Phy (J Phys D), 2, 2, 1969, 609.
10. Veerendra Kumar and Chaudhary DR, Prediction of effective thermal conductivity of two-phase porous materials using resistor model, Ind. J Pure and App Phy, 18, 1980, 984.

11. Pande RN, Kumar V and Chaudhary DR, Bounds on the effective thermal conductivity of two-phase systems, Praman, 1983, 26.

12. Pande RN, Kumar V and Chaudhary DR, conduction in homogeneous two-phase systems - Praman (communicated).

13. Pande RN, Saxena NS and Chaudhary DR, Effect of interstitial air pressure and particle size on the effective thermal conductivity of two-phase gas-solid system, Ind. J. pure and App Phys, 1983, (Accepted for Publication).

14. Pande RN, Saxena NS and Chaudhary DR, Measurement of effective thermal conductivity of building construction materials at interstitial air pressures, Ind J Tech, 1983, (Revised manuscript sent).

15. Sharma RG, Pande RN and Chaudhary DR, Estimation of thermal diffusivity in an isotropic homogeneous two-phase system, communicated to Jap J App Phys.