INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONES: 224281/2/3/4/5 6
CABLE: CENTRATOM - TELEX 460392 - I

SMR/104-29

COLLEGE ON SOIL PHYSICS

19 September - 7 October 1983

DRAINAGE OF AGRICULTURAL LANDS

D. GABRIELS

Department of Soil Physics
State University of Ghent
Belgium

DRAINAGE OF AGRICULTURAL LANDS

Dr.ir.D. Gabriels

Department of Soil Physics
State University of Ghent
Belgium.

Because we are interested in the drainage of agricultural land and hence we consider drainage as the removal of excess subsurface water by means of conduits or other conveying devices; it follows that we are concerned with water tables, movement of water through soil, and the relationship that exist between water tables and crops.

In many areas both surface and subsurface drainage may be required.

Surface drainage is accomplished by open ditches and lateral drains.

Subsurface drainage is accomplished by a system of open ditches and buried tube drains into which water seeps by gravity. Water collected in drains is coveyed to a suitable outlet.

Drainage problems differ widely because of the varied nature of physical conditions and crops to be grown. Besides the crops the following factors have to be taken into account : soils, precipitation, topography.

Excessive soil water reduces the exchange of air between soil and atmosphere. Therefore, wet soil conditions are generally accompanied by O_2 deficiency. A considerable amount of O_2 is required in the soil for mineralization of nutrient elements from organic matter by microbiological activity. Deficient aeration reduces this microbiological activity, decreasing the rate at which NH_3^+ and NO_3^- are supplied. Consequently, a tendency towards N deficiency exists in waterlogged soils.

All biological processes are strongly influenced by temperature. Wet soils have a large heat capacity and considerable amounts of heat are required to raise their temperature. Therefore, wet soils are cold and crop growth starts later and is slower than in dryer soils.

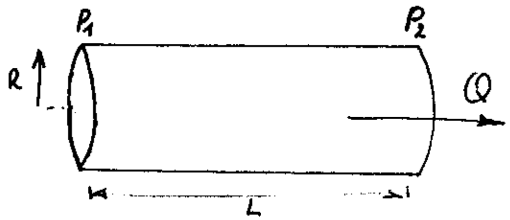
The direct aim of drainage systems in humid regions is to lower the moisture content of the upper layers so air can penetrate more easily to the roots, and transport of CO_2 produced by roots, microorganisms, and chemical reactions is facilitated. Lowering soil moisture content also results in a change in heat budget and higher soil temperatures. This change can be expected to occur in well-drained soils, especially in the spring.

Although the depth of the ground-water table has no direct influence on crop growth, it indirectly determines the prevailing moisture conditions and therefore has an influence on water supply, aeration conditions, and heat properties in soils.

Numerous laboratory and field experiments on the effect of water-table depth on crop yields have been conducted at various locations. The main reason for this possibly is that the water-table depth is easily determined compared to determining other soil properties such as aeration or thermal conductivity.

Flow of water through soils.

The soil may not be regarded as simply a collection of capillary tubes in which Poiseuille's law can be applied.



$$Q = \frac{\pi \, \Delta P \, R^4}{\eta \, L}$$

Soil is a collection of continuous and sometimes discontinuous pore spaces variable in size. It is the effect of a potential gradient (hydraulic gradient) in the soil water which causes the water to move. The earliest recorded investigations to this end were those of Darcy (1856). His law states that the flow of water through porous material is proportional to the hydraulic gradient and to a factor known as the hydraulic conductivity k, which is characteristic of porous media. In mathematical symbols Darcy's law is:

$$Q = k \, i \, A$$

where: Q = volume of water per unit time
- i = hydraulic gradient
- A = cross section of flow area
- k = hydraulic conductivity

$$\frac{Q}{A} = k \, i = v$$

where Q is the volume of water flowing per unit time through a cross-sectional area A.

Q/A has the dimensions of velocity and is called the velocity flux v, which is defined as the flow per unit area.

Measurement of hydraulic head.

Hydraulic heads and gradients can be measured in the field. For saturated soils all that is needed is an open-ended pipe placed in the soil to the proper depth. A measurement of the water level in the pipe will give the hydraulic head at the end of the pipe.

Such a pipe is called a piezometer, which means "pressure meter". The pressure head at the end of the pipe if given by the height of water in the pipe. The gravitational head will be the vertical distance from the end of the pipe to a reference plane. The sum of the pressure head and the gravitational head will equal the hydraulic head.

Hydraulic gradients can be measured by putting several pipes side by side, but at different depths below the soil surface, as shown in figure 6-3. The pressure head at A is h_1, at B is h_2 and at C is h_3. The gravitational heads are respectively z_1, z_2, z_3 and the hydraulic heads are at A, $H_1 = h_1 + z_1$, B, $H_2 = h_2 + z_2$; C, $H_3 = h_3 + z_3$.

The distance from A to B is $z_2 - z_1$, so the hydraulic gradient is:

$$\frac{(h_1 + z_1) - (h_2 + z_2)}{z_2 - z_1} = \frac{h_1 - h_2 + z_1 - z_2}{z_2 - z_1}$$

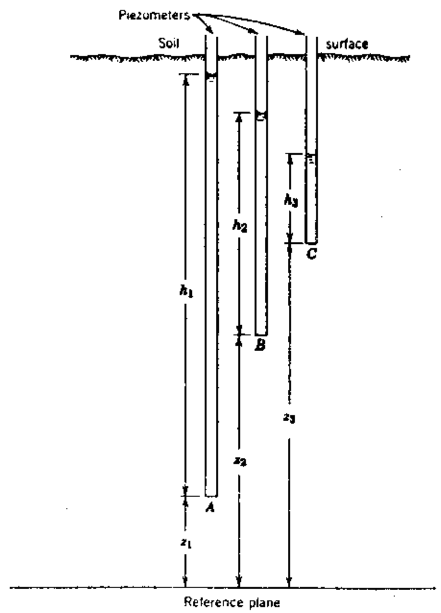


Figure 6-3 Piezometers installed to measure the soil-water pressure and the hydraulic gradients in a vertical direction.

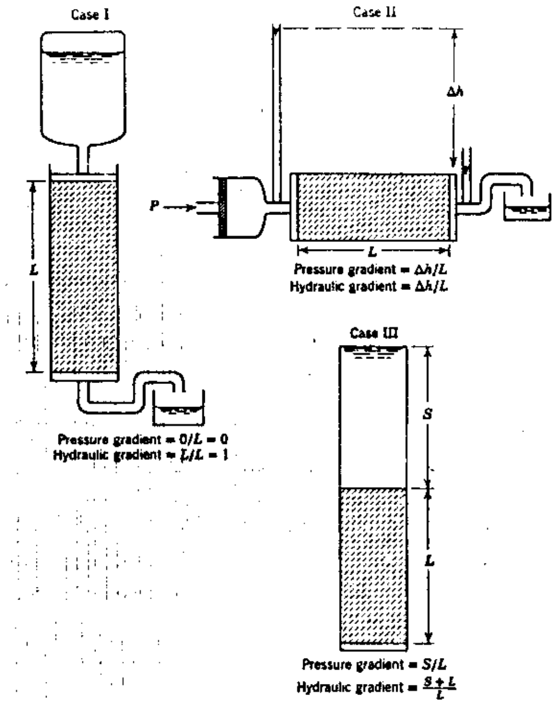


Figure 6-4 Examples of pressure and hydraulic gradients. After Richards.

DEPTH AND SPACING OF DRAINS

One important solution of the drainage problem is a result of the work of Dr. S. B. Hooghoudt of the Netherlands. He considered the water table in equilibrium with the rainfall. The problem solved by Hooghoudt is essentially this: how high will the water table rise for a given rainfall, soil permeability, depth of drain, and spacing of drain? It is also necessary to know the depth to the barrier layer which restricts the downward flow. If drains are installed the water table will rise until the flow into the drains is just equal to the amount of rain or irrigation water infiltrating through the soil surface. At this time the water table is said to be in equilibrium with the rainfall or irrigation water. The problem is to determine the position of the water table at equilibrium. The position of the water table will depend on the following factors:

1. Rate at which water table is replenished by rain water or irrigation water. This is sometimes called the rate of accretion. It is also the drainage coefficient.
2. The soil hydraulic conductivity.
3. The depth and spacing of drains.
4. The depth to an aquiclude or barrier layer.

Other factors such as the rate of plant use of the water, deep seepage, soil stratification, and so forth are usually ignored in the analysis in order to simplify the mathematical treatment.

The above assumptions are then incorporated into a mathematical analysis of the problem that gives as its result the height to which the water table will rise under a specific set of conditions. After determining the height of the water table it is necessary to know whether or not injury will be caused to the plants or to the soil.

A great many field observations and measurements have been made in Holland to determine the important aspect of control of the water table for maximum plant growth. There are two separate factors which are of equal importance in the determination. In the first place it is necessary to keep the water table low enough during the winter months so that the growth of winter crops is not restricted. It is during the winter months that most of the rain falls in Holland, and the drains provide for the lowering of the winter water table. The depth varies with the soil and with the crop that is grown.

During the summer months there is a rainfall deficiency, especially on the lighter soils. During this time it is required that the water table be kept high to supply the plant needs. During the summer months the flow towards the drain is reversed and the flow occurs from the drains out into the field.

HOOGHOUDT'S EQUATION FOR THE WATER TABLE IN EQUILIBRIUM WITH RAINFALL OR IRRIGATION WATER

The problem analyzed by Hooghoudt is presented in Figure 8-1 which shows a homogeneous soil of known permeability with an impermeable stratum lying under it. The soil is assumed to be drained by a series of parallel ditches. It will be shown that the same analysis can be applied to subsurface drains as well.

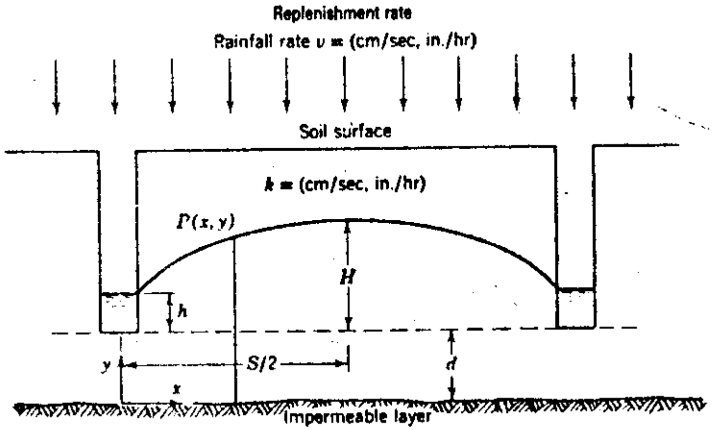


Figure 8-1 Diagram for Hooghoudt's drain-spacing formula. The water table is in equilibrium with the rainfall or irrigation water.

In Hooghoudt's analysis it is assumed that rain is falling at a constant rate on the soil surface. In order to simplify the mathematical analysis, it is assumed that the hydraulic gradient at any point is equal to the slope of the water table above that point. This assumption is known as the Dupuit-Forchheimer (D-F) assumption.

The D-F assumptions imply that water flows horizontally because all the equipotentials are vertical planes. This is, of course, an erroneous picture of the actual flow paths of the water. It is especially incorrect near the drains where the flow paths are quite curved. However, where the slope of the water table is relatively flat the D-F assumptions are nearly valid. The strength of the D-F assumptions lies in the fact that the resulting equations give an accurate value (within 10% of the true value) for the total flow into the drainage facility, even though the individual flow paths are not described accurately.

Hooghoudt's assumptions can be summarized as follows:

... and of hydraulic conductivity k.
... spaced at a distance S apart

3. The hydraulic gradient at any point is equal to the slope of the water table above the point, dy/dx.
4. Darcy's law is valid for flow of water through soils.
5. An aquiclude underlies the drain at a depth d.
6. The rate of replenishment of the water table is v.
7. The origin of coordinates is taken on the aquiclude below the center of one of the drains.

It is evident from an examination of Figure 8-1 that a vertical plane drawn between the center of the two drains is a division plane for the water. All the water entering the soil to the right of this plane flows into the right drain and, similarly, all of the water on the left goes to the left drain.

First consider the flow through a vertical plane drawn from the point P on the water table to the aquiclude. All the water entering the soil to the right of this plane must pass through it on its way to the drain. Since v is the quantity of water entering a unit area of the soil surface then the total quantity of water passing through the plane will be equal to v multiplied by the surface area from the plane to the midpoint between the tile lines. The surface area is equal to $(S/2 - x) \cdot 1$, where 1 stands for a unit distance measured out from the paper. In other words we consider a unit thickness of soil. The quantity of water flowing per unit time through the plane is given by,

$$q_x = \left(\frac{S}{2} - x\right)v \tag{1}$$

We can obtain a second expression for q_x by applying Darcy's law to the flow through the plane. First, remember that the hydraulic gradient at any point is assumed to be equal to the slope of the water table above the point. In other words the hydraulic gradient is equal to dy/dx. Since the distance from the aquiclude to the water table is y, the cross sectional area of flow at the plane is equal to y. Substituting these values in Darcy's law gives

$$q_x = ky\frac{dy}{dx} \tag{2}$$

The right side of equation 1 must equal the right side of equation 2 since the flow in the two instances must be equal. Therefore

$$\left(\frac{S}{2} - x\right)v = ky\frac{dy}{dx}$$

Multiplying through by dx gives

$$\left(\frac{S}{2} - x\right)v\,dx = ky\,dy$$

$$\frac{vS}{2}dx - vx\,dx - [illegible]$$

This is an ordinary differential equation and can be integrated as follows:

$$\int\frac{vS}{2}\,dx - \int vx\,dx = \int ky\,dy$$

On integration

$$\left(\frac{vS}{2}\right)x - \frac{vx^2}{2} = \frac{ky^2}{2}$$

The limits of integration are $x = 0$ then $y = h + d$, and when $x = S/2$ then $y = H + d$. Substituting these limits we have

$$(vS/2)x\Big]_0^{S/2} - \tfrac{1}{2}vx^2\Big]_0^{S/2} = \tfrac{1}{2}ky^2\Big]_{h+d}^{H+d}$$

which results in

$$S^2 = \frac{4k(H^2 - h^2 + 2\,dH - 2\,dh)}{v}$$

which is Hooghoudt's equation for either open ditch drains or subsurface drains such as tile drains.

An important point which will be discussed later is the factor d, the distance from the bottom of the drain to the aquiclude. As it goes to infinity so does S, the drain spacing. This is because the D-F assumptions do not properly account for the radial flow into the bottom of the drain.

For practical purposes the drain is considered to be empty. Hooghoudt's equation then reduces to

$$S^2 = \frac{4kH}{v}(2d + H) \tag{3}$$

HOOGHOUDT'S EQUATION FOR A LAYERED SOIL

In the event that a soil consists of two layers of different hydraulic conductivity then it is possible to use Hooghoudt's procedures to derive a drain spacing formula.

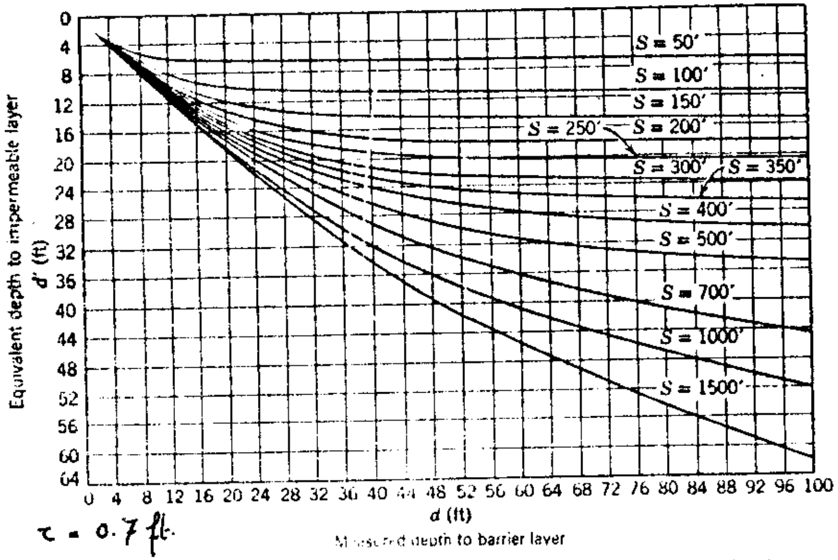
If k_a is the hydraulic conductivity of the layer above the drain line and k_b the hydraulic conductivity below the drain line then Hooghoudt's formula becomes

$$S^2 = \frac{4}{v}(k_aH^2) + \left(\frac{8}{v}k_b\,dH\right)$$

where d is the equivalent depth obtained from Hooghoudt's graphs. A multilayered soil can be treated by taking a weighted-mean of the horizontal conductivities. For example suppose the layer above the drain line consists of three layers of conductivity k_1, k_2, and k_3 having thickness l_1, l_2, l_3. The average will be

$$k_a = \frac{k_1l_1 + k_2l_2 + k_3l_3}{l_1 + l_2 + l_3}$$

As stated above the formula is not valid for large values of d, the depth to the barrier layer. Hooghoudt recognized this difficulty and made a separate analysis for the flow beneath the drain. He assumed that the flow



r = 0.7 ft.

φ = 4 cm

E (m) / D (m)	5	7.5	I0	I2	I5	20	25	30
0,50	0,39	0,42	0,45	0,46	0,46	0,48	0,48	0,48
0,75	0,47	0,53	0,58	0,6I	0,64	0,67	0,69	0,70
I,00	0,52	0,6I	0,68	0,73	0,76	0,8I	0,84	0,87
I,25	0,53	0,66	0,74	0,80	0,87	0,94	I,00	I,05
I,50	"	0,69	0,80	0,87	0,95	I,05	I,I3	I,I8
2,00	"	0,72	0,87	0,95	I,07	I,2I	I,3I	I,40
2,50	"	"	0,90	I,00	I,I4	I,3I	I,46	I,57
3,00	"	"	"	I,0I	I,I9	I,40	I,56	I,7I
4,00	"	"	"	"	I,23	I,49	I,7I	I,90
5,00	"	"	"	"	"	I,54	I,79	2,0I
7,00	"	"	"	"	"	I,55	I,85	2,I2
I0,00	"	"	"	"	"	"	I,86	2,I4
	"	"	"	"	"	"		2,I[illegible]

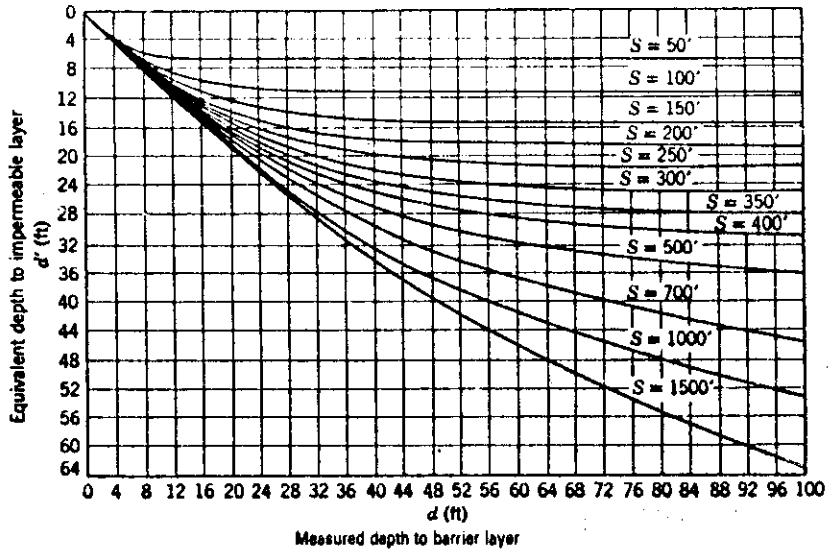


Figure 8-3 Relationship between d and d' where r = 0.8 ft and S is the spacing between the drains. Curves based on Hooghoudt's correction (after Bureau of Reclamation).

is radial in character. He then compared the flow obtained with the radial-flow assumptions to the flow obtained with the horizontal-flow equation and developed a table of "equivalent" depths. Wesseling (1964) indicates that Hooghoudt's table of equivalent depths is correct to about 5%. The values of equivalent depths obtained by Hooghoudt are to be substituted in equation 3 for d.

The identification of the aquiclude layer is often difficult in the field. If the layer has a hydraulic conductivity of one tenth or less than the overlying soil then it can be considered to be an aquiclude. Water can seep down through this aquiclude and this deep seepage may reduce the need for drainage. However, the flow pattern towards the drains is not seriously affected by this deep seepage.

THE DRAINAGE COEFFICIENT. The drainage coefficient is the volume of water that must be drained from a given area in 24 hours. It is expressed as mm/day (in/day). The volume of water is obtained by multiplying the drainage coefficient by the area that is drained.

In humid regions, rainfall is the main source of water that must be drained. However, rainfall rates are seldom constant for a 24 hour period. Also the drainage coefficient is influenced by a number of factors in addition to the rate of rainfall.

APPLICATION OF HOOGHOUDT'S EQUATION IN HUMID AREAS

In Holland the normal criterion states that with a discharge of 0.007 m/day (7mm/day) the water table may not be higher than 50 cm below the soil surface for arable land and 40 cm below the surface for grassland. Under these conditions the drain depth is about 80 to 90 cm.

In France a drainage coefficient of 9 mm/day is used when the water table is 30 cm below the ground surface.

The best way to obtain a value of the drainage coefficient is to measure the outflow from existing drains. Surface runoff must be excluded from these measurements.

One approach used by the Dutch which takes into account the rainfall intensity is as follows. The drainage rate, or v, is taken to be equal to about 5 to 7 mm/day. A surface runoff of about 15 mm/day is assumed. The height to which the water table will rise is then plotted as a function of the rainfall rate. The frequency of the rainfall rate is indicated on the diagram. For example the height of the water table is plotted for rainfalls which are known

to occur in two days, three days, and so forth. Then, from a study of the rainfall distribution through the year, and a consideration of the tolerance of the plant to high water-table conditions, it is decided how serious the drainage problem is. The time of the year is taken into account by considering the temperature, which is held to be a good indicator of the damage that will result to the plant from high water-table conditions. It is thought that the number of day-degrees that the temperature is above zero centigrade is directly related to the damage that will result to the plant. It is recognized that this relationship is not completely true since some plants, such as tomatoes, do not suffer from high water-table conditions until the temperature is about 6 or 7 degrees above zero.

The Drainage Coefficient in Irrigated Areas

Irrigation waters contain substantial quantities of salt; from 0.1 to 4 metric tons/1000 m^3 according to Rhoades (1974). Irrigation water is applied at rates of 10,000 to 15,000 m^3/ha per year and hence between 0.1 to 60 metric tons are added to each hectare annually. Some of the applied salt precipitates in the soil. A small proportion is used by the plants.

The remainder of the salt must be removed from the soil by adding an amount of irrigation water in excess of the crop needs. The excess water leaches the salts out of the soil and into the drain. As much salt must be removed from the soil as is added (less the amount that precipitates). This excess amount of irrigation water is required to maintain the salt balance in the soil.

The output of salt in the drainage water must equal the input of salt in the irrigation water. If we ignore precipitation of salts in the soil, plant uptake of salts we can write

$$\text{salt input} = \text{salt output}$$

The electrical conductivity (EC) is a measure of the amount of salt in the water and

$$EC_{iw}\, D_{iw} = \text{salt input}$$

where EC_{iw} = electrical conductivity of the irrigation water and D_{iw} = volume of irrigation water added. The salt balance equation becomes

$$EC_{iw}\, D_{iw} = EC_{dw}\, D_{dw}$$

where dw refers to the drainage water.

The leaching requirement (LR) is defined as the fraction of irrigation water that must be drained in order to maintain the salt balance or

$$\text{LR} = \frac{D_{dw}}{D_{iw}} = \frac{EC_{iw}}{EC_{dw}}$$

The following assumptions are inherent in the simple formulation of the leaching requirements as given above.

1. The irrigation water is applied uniformly to the soil surface and the hydraulic conductivity of the soil is uniform over the area.
2. No rainfall
3. No removal of salt in the harvested crop.
4. No solution or precipitation of salt in the soil.

The last assumption, "no precipitation of salt in the soil" introduces the greatest error in the calculation.

Appreciable amounts of calcium carbonate, calcium sulphate and to a lesser extent, magnesium carbonate may precipitate in the soil. This precipitation may result in a substantial reduction in the leaching requirement. The amount of precipitation will vary from place to place depending upon the method used to apply the water. At the present time there is no satisfactory theory that enables an accurate prediction of the leaching requirement.

The concept of the leaching requirement is very useful in computing the amounts of drainage water that must be removed from a large area. However, such a computation presupposes that there is adequate control of the applied irrigation water and that the applied irrigation water is just equal to the crop needs plus the leaching requirement.

The maximum concentration of salts, with the exception of surface crusts formed by evaporation, will form at the bottom of the root zone. This concentration will be the same as the concentration of the salts in the drainage water, provided there is no excess leaching and the irrigation water is applied uniformly over the area.

The increase in concentration of salts in the drainage water over the concentration in the irrigation water is a consequence of the consumptive use of water by the crop. The crop will extract the water from the soil but will leave most of the salt behind.

For some field crops an EC_{dw} of 8 mmhos/cm can be tolerated. For irrigation waters with conductivities of 1, 2, and 3 mmhos/cm, respectively, the leaching requirements will be 13, 25, and 38%. These figures are conservative because some of the salt is removed by the crop, and some of the salt may be precipitated in the form of salts such as calcium carbonate or gypsum.

In using the leaching requirement it should be borne in mind that the winter precipitation may well be adequate to leach the soil. All the water that passes through the root zone of the plant must be considered in the use of the equation. The conductivity of the irrigation water should be the weighted average of the conductivities of the rain water EC_{rw}, and the irrigation water, EC_{iw}, as shown in the equation

$$EC_{(rw+iw)} = \frac{D_{rw}EC_{rw} + D_{iw}EC_{iw}}{D_{rw} + D_{iw}}$$

where D_{rw} and D_{iw} are the depths, respectively, of the rainwater and the irrigation water that enters the soil.

In order to use the leaching-requirement concept to analyze the drainage water situation over a large area it is first necessary to know the consumptive use of the crop or crops to be grown. The amount of irrigation water will equal the sum of the consumptive use and the drainage water as given by the

We can eliminate D_{dw} by using equation 4. We then have the depth of irrigation water expressed in terms of the consumptive use and the leaching requirement

$$D_{iw} = \frac{D_{cw}}{(1 - \mathrm{LR})}$$

Rewriting the equation in terms of the conductivity ratio we get

$$D_{iw} = \left(\frac{EC_{dw}}{EC_{dw} - EC_{iw}}\right) D_{cw}$$

It should be remembered that the EC_{dw} represents the salt tolerance of the crop to be grown.

The tolerance of some plants to salts is given in Table 8-1.

The following is an example of the use of the leaching requirement to calculate the rate of replenishment in an irrigated area.

The EC_{iw} is about 1 mmho/cm. The EC_{dw} can be taken as 8 mmho/cm and the consumptive use is 0.35 in./day

$$D_{iw} = \left(\frac{EC_{dw}}{EC_{dw} - EC_{iw}}\right) D_{cw}$$

$$= \left(\frac{8}{8 - 1}\right) 0.35 = 0.40 \text{ in./day}$$

$$v = D_{dw} = \left(\frac{1}{8}\right)(0.40) = 0.050 \text{ in./day.}$$

TABLE 8-1 Relation Between Crop Response and Soil Salinity as Determined by the Saturation Extract Method

Conductivity of Extract, mmhos/cm *at* 25°C	*Crop Response*
0–2	Salinity effects negligible for most crops
2–4	Yields of very sensitive crops may be restricted
4–8	Yields of many crops restricted
8–16	Only tolerant crops yield satisfactorily
above 16	Only a few very tolerant crops yield satisfactorily

$$H^d = (2SR/k)F(2r/2S, h/2S)$$

where H^d = maximum height of the water table above the drains
R = rate of rainfall = v
k = hydraulic conductivity
h = distance from impermeable layer to water table immediately over drains
$2S$ = spacing of drains
r = radius of drain

where

$$F = \frac{1}{\pi}\left\{\ln\frac{2S}{\pi r} + \sum_{m=1}^{\infty}\left[\frac{1}{m}\left(\cos\frac{m\pi r}{S} - \cos m\pi\right)\left(\coth\frac{m\pi h}{S} - 1\right)\right]\right\}$$

Graphs have been prepared by Sadik Toksöz for the solution of the equation.

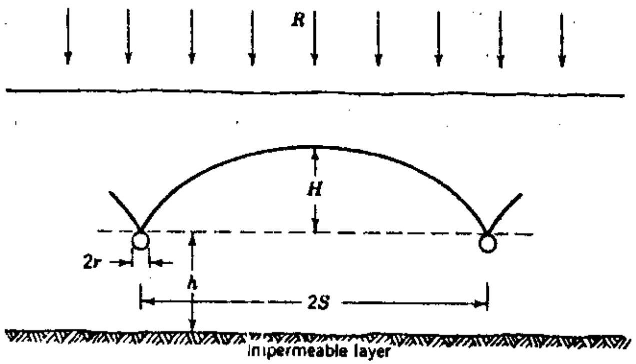


Figure 8-7 Example of the use of the graphs of Sadik Toksöz for the solution of Kirkham's 1958 equation.

H = 0.6 meter
h = 6.0 meters
k = 1.20 meters/day
R = 0.20 liter/sec/hectare = 0.00173 meter/day
$2r$ = 10 centimeters = 0.10 meter

$$L = \frac{H}{h}\left(\frac{k}{R} - 1\right) = \frac{0.6}{6}\left(\frac{1.20}{0.00173} - 1\right) = 69.3$$

$$\frac{h}{2r} = \frac{6}{0.10} = 60$$

From the curve

$$\frac{2S}{h} = \frac{2S}{6} = 19.7$$

$$2S = 118.2$$

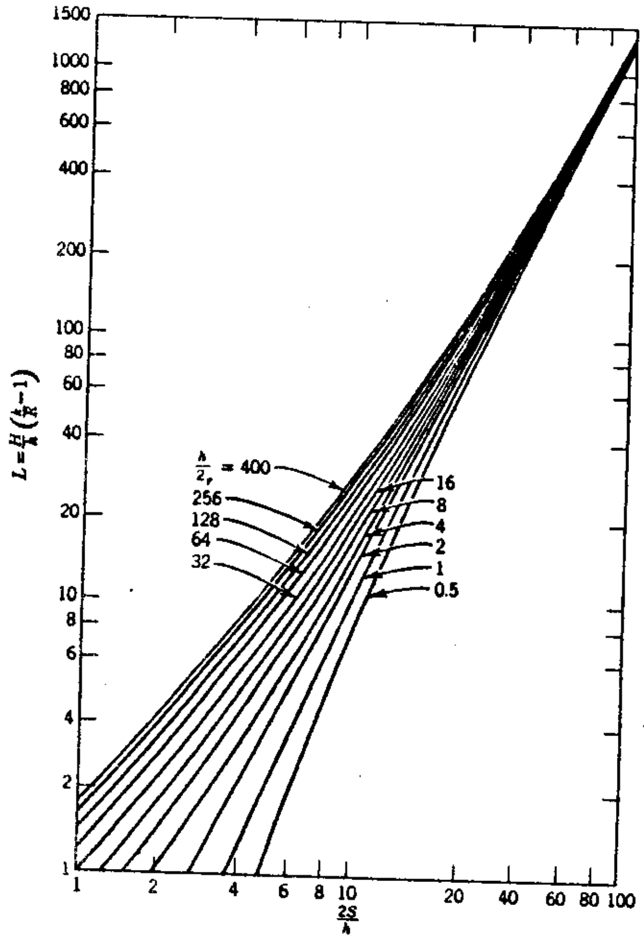


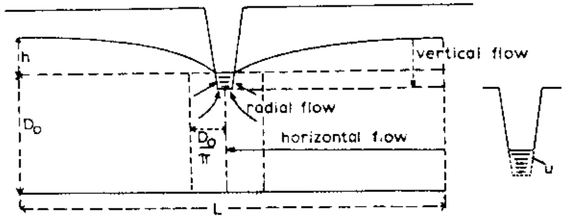
Figure 8-6 Graph for solution of Kirkham's 1958 formula (after Sadik Toksöz).

IMPERMEABLE LAYER AT A RELATIVELY SHALLOW DEPTH ($D < {}^1/_4\, L$)

4.1. Explanation of Ernst's formula

The principle underlying ERNST's formula may be described as follows. The flow is divided into the following three components, a vertical (v), a horizontal (h) and a radial flow (r) (Fig. 5). A given drain spacing being assumed, the hydraulic head required for this flow is calculated for each component with the aid of a given formula. When the sum of the three hydraulic heads required is equal to the total available head ($h = h_v + h_h + h_r$) the result is the drain spacing required. Hence, this formula involves the same kind of trial and error process as does the Hooghoudt formula.

Fig. 5. General principle of the formula of Ernst



ERNST's general formula is as follows:

$$h = q\,\frac{D_v}{K_1} + \frac{qL^2}{8KD} + \frac{qL}{\pi K}\,ln\,\frac{D_o}{u}$$
$$= h_v + h_h + h_r$$

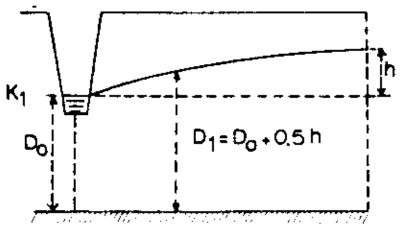


Fig. 6. Homogeneous soil $D_o < {}^1/_4\, L$

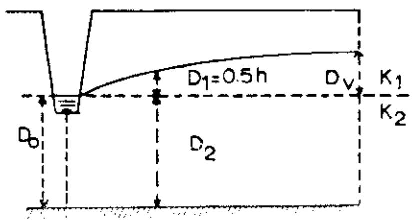
$$h = \frac{qL^2}{8K_1D_1} + \frac{qL}{\pi K_1}\,ln\,\frac{D_o}{u}$$

Fig. 7. Two soil layers. Water level in the drain coincides with boundary K_1/K_2

$$K_1 \gtrless K_2 : h = \frac{qL^2}{8(K_1D_1 + K_2D_2)} + \frac{qL}{\pi K_2}\,ln\,\frac{D_o}{u}$$

$$K_1 << K_2 : h = q\,\frac{D_v}{K_1} + \frac{qL^2}{8K_2D_2} + \frac{qL}{\pi K_2}\,ln\,\frac{D_o}{u}$$

$K_1 >> K_2$: use formula Hooghoudt



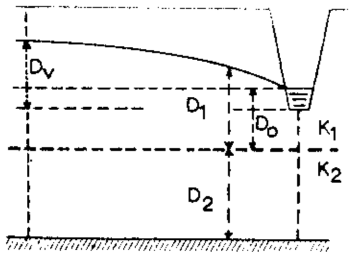


Fig. 8. Two soil layers. Drains entirely in the upper layer

$$h = q\,\frac{D_v}{K_1} + \frac{q\,L^2}{8\,(K_1D_1 + K_2D_2)} + \frac{qL}{\pi K_1}\,ln\,\frac{a\,D_o}{u}$$

The following most frequently occurring situations can be distinguished:

- Homogeneous soils with an impermeable layer at a depth D_o below the level of the drains: $D_o < {}^1/_4\, L$ (Fig. 6).
- The level of the drains coincides with the interface of two layers of varying permeability (Fig. 7).
- The soil below the drains consists of two different layers (Fig. 8).

In connection with the formula the following remarks may be made:

- h = *available hydraulic head* = difference between the level of the water in the drains and midway between the drains. The notation Δh, viz. the difference between two levels, would be more correct in this case and is also employed by ERNST (1954, 1962). For practical reasons, however, the notation h is used here.
- D_v = thickness of the layer for the *vertical component*. The upper limit of this layer invariably coincides with the level of the water midway between the drains, whereas the lower limit varies somewhat according to the position of the drains with respect to layers of different permeability.

  When the drains are located entirely in the uppermost layer, the vertical flow is taken into account up to the bottom of the drains (Fig. 8). If the drain is on the boundary between layers of poor and good permeability, D_v is calculated up to this boundary (Fig. 7).

  In most cases the vertical component is small and may be ignored. For instance, when $q = 0.005$ m, $D_v = 0.6$ m and $K_1 = 0.3$ m, then $h_v = q\,(D_v/K_1) = 0.005 \times 0.6/0.3 = 0.01$ m. But if we have a layer of very poor permeability ($K_1 = 0.02$ m) located on a layer of very good permeability (e.g. heavy basin clay on a sandy river deposit) and for instance $q = 0.010$ and $D_v = 0.60$, then $h_v = 0.010 \times 0.60/0.02 = 0.30$ m, so that h_v should not be neglected.

- KD, viz. without index, denotes the product, or sum of the products of the permeability (K) and thickness (D) of the various layers for the *horizontal component*; this has to be specified in further detail according to the hydrological situation (K_1D_1 or K_2D_2 or $K_1D_1 + K_2D_2$).
- D_2 = thickness of the second layer for the *horizontal component*; this is a given magnitude determined in the field.
  When the level of the drains coincides with the interface of two layers, then $D_2 = D_o$ (Fig. 7).
- D_1 = the average cross section for the *horizontal component* in a layer of permeability K_1. The thickness of this layer cannot be directly inferred from the field data but, where important, will have to be calculated. Depending on the position of the drains with respect to the interface of two layers of different permeability (K_1 and K_2), $D_1 = 0.5\ h$ (Fig. 7) or $D_1 = \frac{D_o + (D_o + h)}{2} = D_o + 0.5\ h$ (Fig. 6 and 8), wherein D_o is the thickness of the layer for the radial component.
- $ln\ (D_o/u)\ /\ \pi K = R_r$ = *radial resistance*. In this case $K = K_1$ or K_2, depending on the position of the drains.
- D_o = thickness of the layer for which the *radial resistance* is calculated. The upper limit of this layer invariably coincides with the level of the water in the drains, whereas the lower limit is formed by an impermeable layer or a layer of differing permeability.
- u = *wetted perimeter* = bottom of the ditch + twice the height of the water in the ditch. Theoretically it would be necessary to take into account the gradient of the slope of the ditch, but such a degree of accuracy has little practical value. For comparison with other formulas, $u = \pi r$.

4.2. ERNST'S AND HOOGHOUDT'S FORMULAS COMPARED

If we now compare ERNST's formula with HOOGHOUDT's, the following differences and correspondences may be observed:

- The component for the vertical flow is disregarded by HOOGHOUDT. In most cases, however, this component is small and may be neglected.
- The horizontal flow or resistance is calculated by HOOGHOUDT over the length L–1.4 D (Fig. 4), whereas ERNST calculates it over the entire drain distance L (Fig. 5).
  As a result the horizontal resistance calculated by ERNST is greater than that calculated by HOOGHOUDT.
- The value calculated by ERNST for the radial resistance is, however, smaller than that calculated by HOOGHOUDT (ERNST: $\frac{1}{\pi K}\ ln\ \frac{D_o}{u} = \frac{1}{\pi K}\ ln\ \frac{D_o}{\pi r} = \frac{1}{\pi K}\ ln\ \frac{0.32 D_o}{r}$ and HOOGHOUDT: $\frac{1}{\pi K}\ ln\ \frac{0.70 D_o}{r}$).

In most cases the sum of the horizontal and radial resistances calculated according to ERNST's formula is substantially the same as the result of HOOGHOUDT's formula. Consequently both formulas usually give practically the same drain spacings. It is only when K_1 is much greater than K_2 ($K_1 >> K_2$) that, unlike HOOGHOUDT's formula, ERNST's formula gives a slightly smaller drain spacing. According to ERNST (1963, p. 35) no acceptable formula has been found for the radial resistance for this special case. ERNST's formula may be employed for this case but without further investigation it is impossible to say what degree of accuracy will be obtained. This depends, for instance, on the position of the drains with respect to the interface of the various layers.