E X A M P L E S

S O I L   W A T E R   P O T E N T I A L S

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" A P P L I E D   S O I L   P H Y S I C S "

by R.J. Hanks and G.L. Ashcroft

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34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONES: 224281/2/3/4/5 6
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COLLEGE ON SOIL PHYSICS

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EXAMPLES: SOIL WATER POTENTIALS

R. HARTMANN

Department of Soil Physics
State University of Ghent
Ghent
Belgium

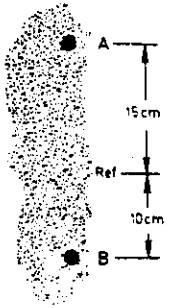
Example 2.1

Given: Two points in a soil. Each point is located a specified vertical distance from a reference elevation. (See the figure at the right.)
Find: The difference in gravitational potential, $\Delta\psi_z$, between the two points.
Solution:

$\psi_{zA} = 15$ cm,

$\psi_{zB} = -10$ cm,

$\Delta\psi_z = \psi_{zA} - \psi_{zB} = 15 \text{ cm} - (-10 \text{ cm}) = 25 \text{ cm}$.



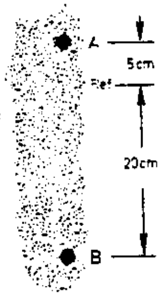
Example 2.2

Given: The same two soil points specified in Example 2.1 but with the reference elevation relocated as shown in the figure at the right.
Find: The difference in gravitational potential, ψ_z, between the two points.
Solution:

$\psi_{zA} = 5$ cm,

$\psi_{zB} = -20$ cm,

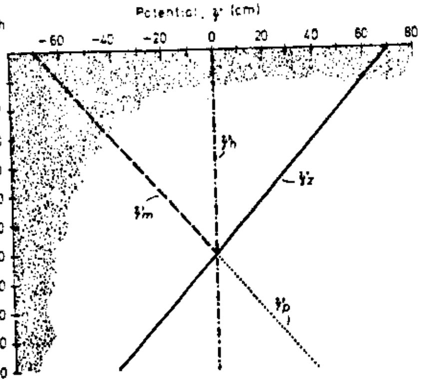
$\Delta\psi_z = \psi_{zA} - \psi_{zB} = 5 \text{ cm} - (-20 \text{ cm}) = 25 \text{ cm}$.



Example 2.3.

Given: A soil in which the liquid water is in equilibrium with a water table at −70 cm and the reference level is chosen as −70 cm.
Find: The values of ψ_p, ψ_m, ψ_z, and ψ_h throughout the soil profile to −110 cm.
Solution:

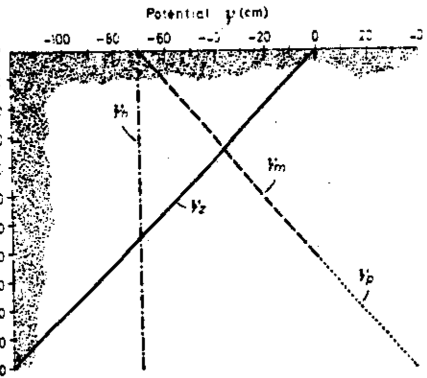
ψ_p	ψ_m	ψ_z	ψ_h	Depth (cm)
0	-70	70	0	0
[illegible]	-60	60	0	-10
[illegible]	-50	50	0	-20
0	-40	40	0	-30
0	-30	30	0	-40
[illegible]	[illegible]	20	0	-50
[illegible]	[illegible]	0	0	-60
[illegible]	[illegible]	0	0	-70
0	0	-10	0	-80
20	0	-20	0	-90
30	0	-30	0	-100
40	0	-40	0	-110



Example 2.4.

Given: The conditions of Example 2.3 except the reference level is the soil surface.
Find: The values of ψ_h, ψ_p, ψ_m, and ψ_z throughout the soil profile to −110 cm.
Solution:

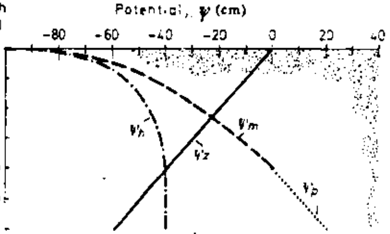
ψ_p	ψ_m	ψ_z	ψ_h	Depth (cm)
0	-70	0	-70	0
0	-60	-10	-70	-10
0	-50	-20	-70	-20
0	-40	-30	-70	-30
0	-30	-40	-70	-40
0	-20	-50	-70	-50
0	-10	-60	-70	-60
0	0	-70	-70	-70
10	0	-80	-70	-80
20	0	-90	-70	-90
30	0	-100	-70	-100
40	0	-110	-70	-110



Example 2.5.

Given: Water is evaporating at the soil surface and there is a water table at −40 cm. The reference level is at the soil surface.
Find: Values of ψ_z, ψ_p, and ψ_h throughout the soil profile to −60 cm. In order to find ψ_h, measured or estimated values of ψ_m must be available. Make estimates of ψ_m for the conditions specified.
Solution:

ψ_p	ψ_m est.	ψ_z	ψ_h	Depth (cm)
0	−100	0	−100	0
0	−45	−10	−55	−10
0	−26	−20	−46	−20
0	−12	−30	−42	−30
0	0	−40	−40	−40
10	0	−50	−40	−50
20	0	−60	−40	−60

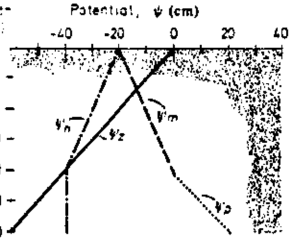


Note: For upward flow, the hydraulic potential at −40 cm must be greater than at −30 cm, etc. Thus, $\psi_{h(-40)} > \psi_{h(-30)} > \psi_{h(-20)} > \psi_{h(-10)} > \psi_{h(0)}$. We may have variations in the gradient $\Delta\psi_h/\Delta z$ with depth; but for upward flow, the sign must always be negative.

Example 2.6.

Given: A soil in which water is flowing into a drain at −40 cm. The reference level is the soil surface.
Find: Values of ψ_p, ψ_z, and ψ_h for the entire soil profile to −60 cm. Estimates must first be made of ψ_m.
Solution:

ψ_z	ψ_p	ψ_m est.	ψ_h	Depth (cm)
0	0	−20	−20	0
−10	0	−15	−25	−10
−20	0	−10	−30	−20
−30	0	−5	−35	−30
−40	0	0	−40	−40
−50	10	0	−40	−50
−60	20	0	−40	−60

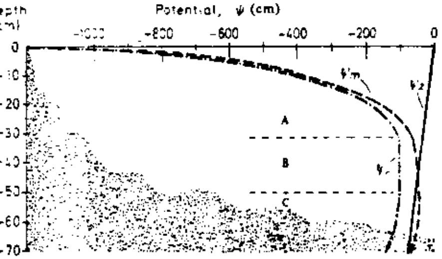


Note: Flow is downward, thus $\Delta\psi_h/\Delta z$ must be positive.

Example 2.7.

Given: A soil was initially quite dry. It received rain that wet the soil from the surface downward to part way through the profile. After a few days, the surface had dried and water moved upward in response to the evaporation. At lower depths, however, the soil water moved downward into the soil that had not been wetted by the rain.
Find: Values of ψ_z, ψ_p, and ψ_h throughout the profile to −60 cm using your estimates for ψ_m
Solution:

ψ_z	ψ_p	Estimated ψ_m est.	ψ_h	Depth (cm)
0	0	−1000	−1000	0
−10	0	−390	−400	−10
−20	0	−180	−200	−20
−30	0	−70	−100	−30
−40	0	−60	−100	−40
−50	0	−50	−100	−50
−60	0	−50	−110	−60
−70	0	−60	−130	−70



Note: In zone A, $\Delta\psi_h/\Delta z$ is negative, thus flow is upward. In zone B, $\Delta\psi_h/\Delta z$ is zero, thus there is no flow. In zone C, $\Delta\psi_h/\Delta z$ is positive, and as a consequence, flow is downward.

WATER BALANCE STUDIES

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Dr. ir. R. Hartmann

Department of Soil Physics

State University of Ghent

B E L G I U M

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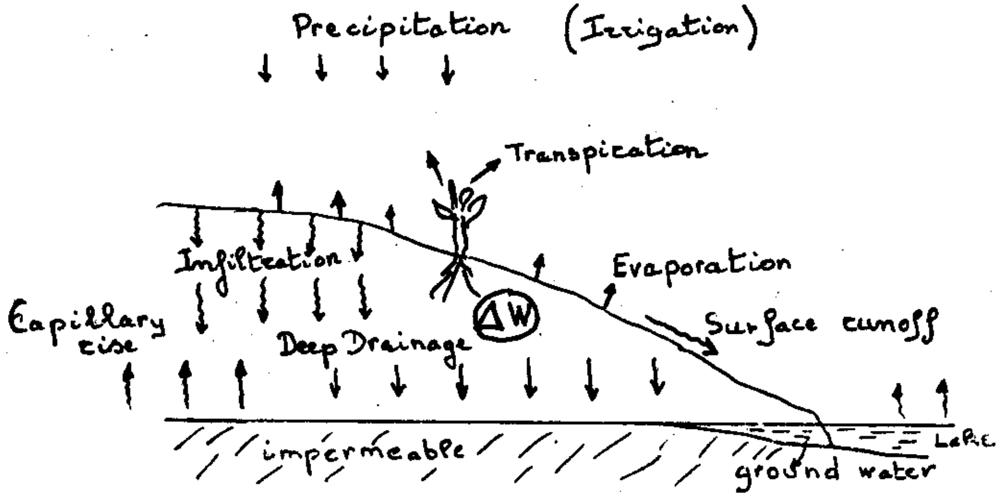
Hydrologic Cycle



ΔW: change in water content during time Δt

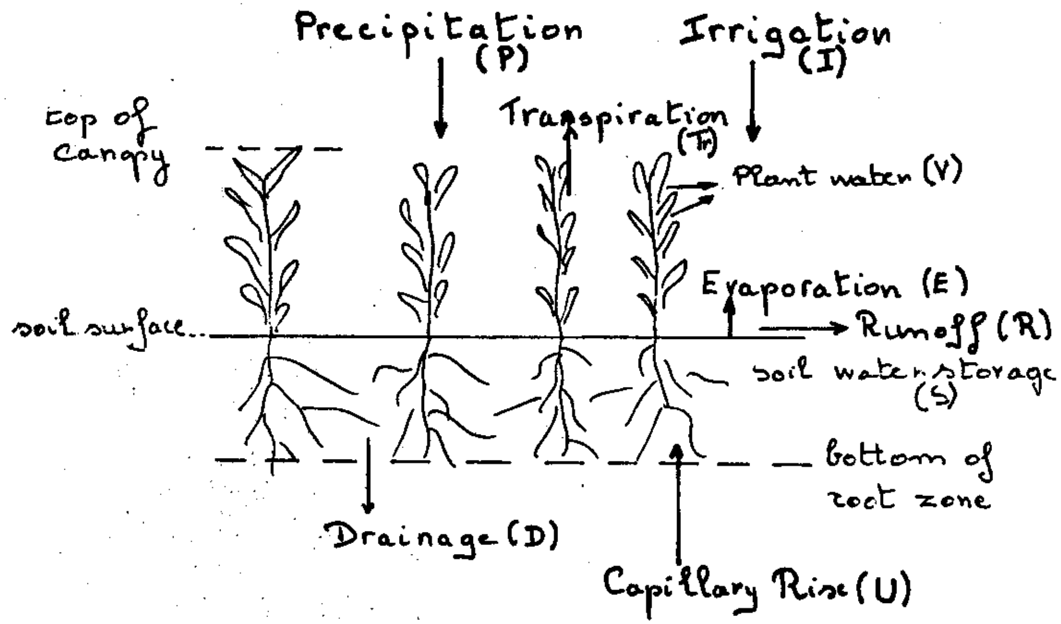
$$\underline{\overline{\Delta W = \text{Water}_{in} - \text{Water}_{out}}}$$

$$W_{in} = P + I_r$$

$$W_{out} = R + D + E + Tr$$

$$\underline{\overline{\Delta W = P + I_r - R - D - [E + T]}}$$

Water Balance



Root zone water balance assuming vertical water flow only:

$$\underbrace{\Delta S + \Delta V}_{\text{changes}} = \underbrace{P + I + U}_{\text{gains}} - \underbrace{(R + D + E + T_r)}_{\text{losses}}$$

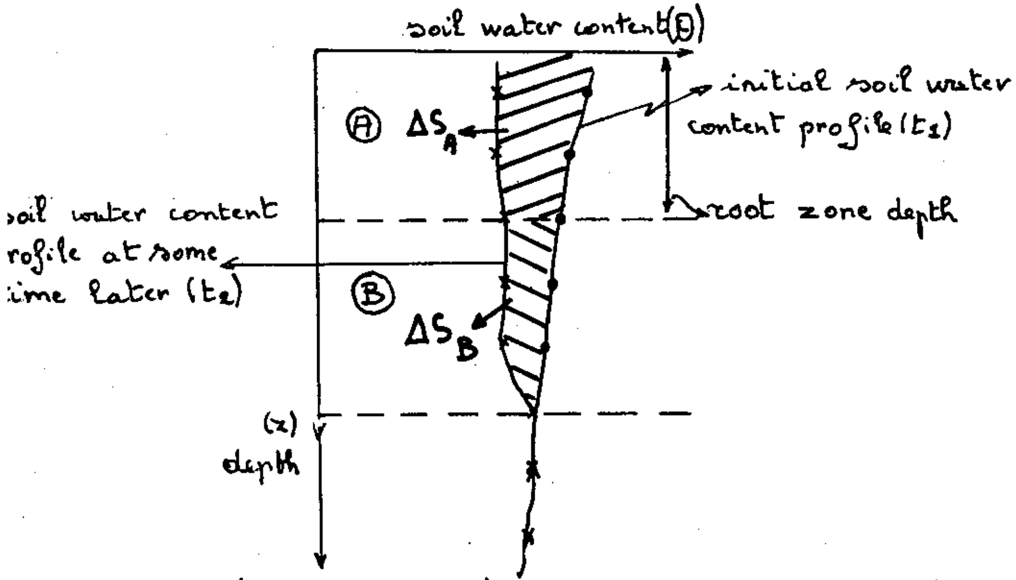
All quantities in terms of volume of water per unit area (equivalent depth units) during the period considered (Δt)

$$\boxed{\Delta S = P + I - R - D - E - T_r}$$

$$\Delta S = P + I - R - D - \underbrace{E - T_r}_{ET_{cr}}$$

$\downarrow$ during dry spells

$$\Delta S = -D - ET_{cr}$$



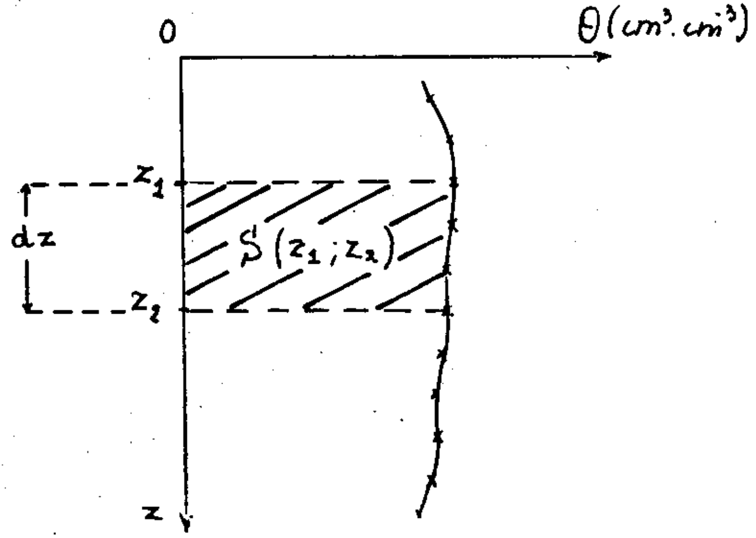
ΔS can be estimated using <u>the neutron probe</u>

but ΔS_A: due to evaporation and drainage!

ΔS_B: due to evaporation and/or drainage!

Direction of water flow is known using <u>tensiometers</u>.

Soil Water Content Profile – Soil Water Storage



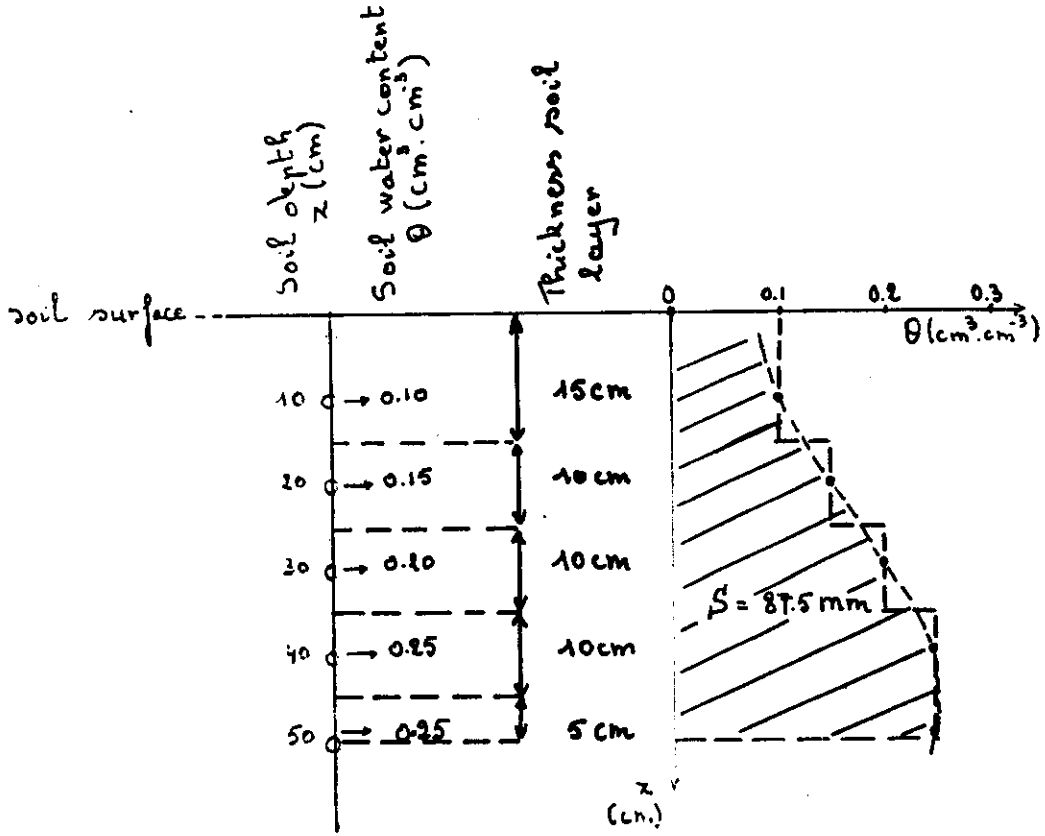
Volume of water in the vertical soil column (dz) with cross-section $d\sigma$

$$\int_{z_2}^{z_1} \theta \cdot dV = d\sigma \int_{z_2}^{z_1} \theta \cdot dz$$

or per unit area

$$\int_{z_2}^{z_1} \theta \cdot dz = \frac{S(z_1;z_2)}{(mm)!}$$

Calculation of the soil water storage



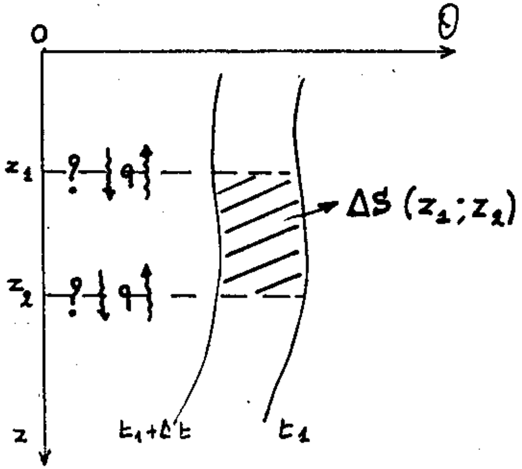
$$S_{50cm} = 100\left[1{,}5\ \theta_{10cm} + 1{,}0\left(\theta_{20cm} + \theta_{30cm} + \theta_{40cm}\right) + 0.5\ \theta_{50cm}\right]$$

or $100\left[1{,}5 \times 0.10 + 1\left(0{,}15 + 0.20 + 0.25\right) + 0{,}5 \times 0{,}25\right] = 87{,}5mm$

In general

$$S_{z_j} = \left[1{,}5\ \theta_{10} + \theta_{20} + \theta_{30} + \cdots + 0{,}5\ \theta_j\right] 100$$

The change of the soil water storage (ΔS) within a soil layer



is due to the transfer of water and is conducted by following two equations:

1. Darcy's law: (for vertical water movement)

2. Mass Conservation law: (expressed in the equation of continuity)

Darcy's law

$$q = -K(\theta)\,\frac{dH}{dz}$$

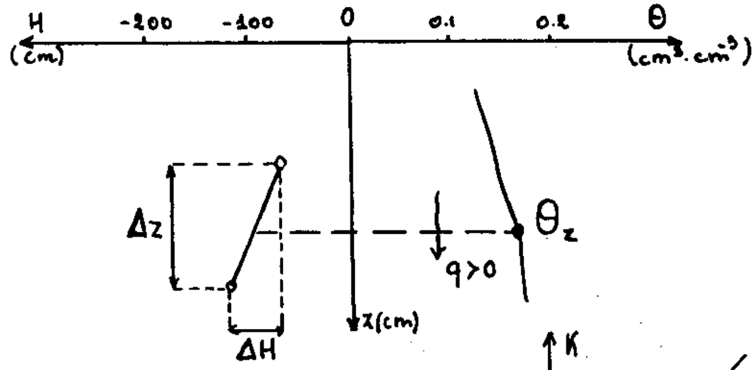
where . q = soil water flux: the amount of water (cm^3) crossing an unit area per unit time.
($\frac{cm^3}{cm^2} \cdot day^{-1} = cm \cdot day^{-1}$)

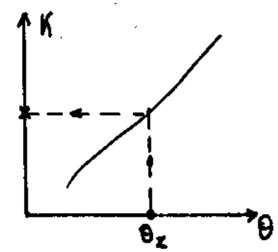
$K(\theta)$: hydraulic conductivity of the soil ($cm.\,day^{-1}$)

H = hydraulic head = h + z (cm)
h: soil water pressure head
z: gravitational head

z = depth (cm)

$\frac{dH}{dz}$ = hydraulic gradient ($cm.\,cm^{-1}$): rate of change of hydraulic head with depth = the driving force that makes water move





Mass conservation law (eq. of continuity)

① Bare Soil

For one-dimensional flow with q_z, being the flux in the direction x the equation in general can be written as follows:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z}$$

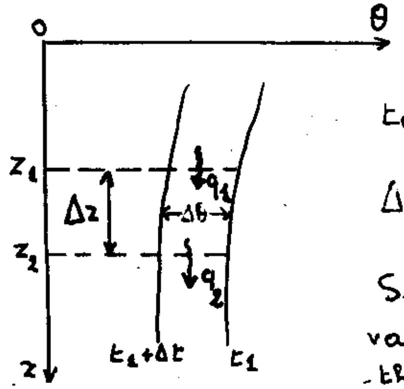
This eq. applied to a soil element as given in the figure below in a case of downward flux gives:

$$\rho_w (q_2 - q_1)\,\Delta t = -\rho_w\ \Delta\theta . \Delta z . 1$$

↳ density of water

or

$$q_2 - q_1 = -\frac{\Delta\theta\Delta z}{\Delta t} = -\frac{\Delta S}{\Delta t}$$



to obtain q_1, the values of q_2, ΔS and Δt have to be known!

Single measurement of the variation of water storage in the element of volume taken into account is not sufficient to obtain q_1

The problem can be solved as follows:

① using Darcy's Law

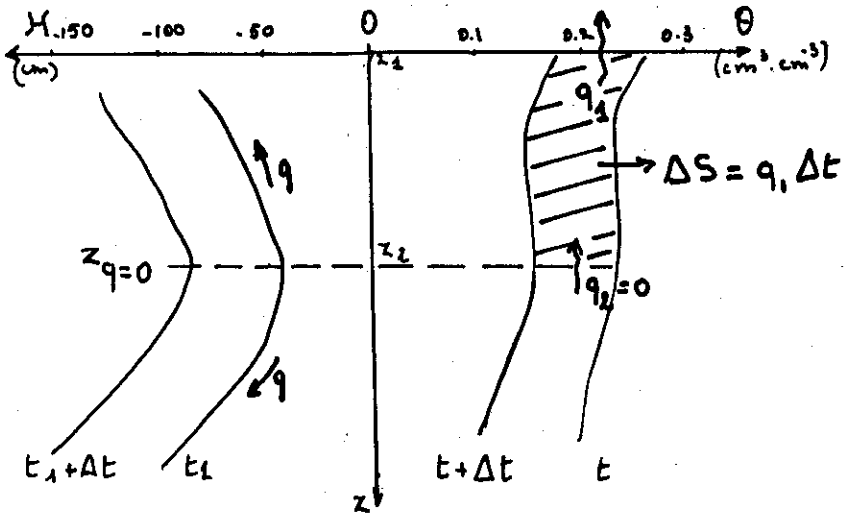
$$q = -K(\theta)\frac{dH}{dz}$$

if $K(\theta)$ and $\frac{dH}{dz}$ is known in the section z_1

② fixing the level of z_1 at a depth where the flux is known, normally z where $q = 0$ viz. where $\frac{dH}{dz} = 0$!

In such case $q_1 = \frac{\Delta S}{\Delta t}$

Such a situation is presented in the figure below and often occurs in the field.



② Cropped Soil

If vegetation is present the continuity eq. is to be written as follows:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z} - J(z,t)$$

water extraction by roots per unit volume of soil and per unit time.

Evaporation (E) or Evapotranspiration (ET_{cr}) for period Δt can be estimated using the water balance equation:

$$\Delta S\Big|_z^0 = P + I - R - \underbrace{D}_{\to\, q_z \cdot \Delta t,\;\; q_z = -K(\theta)\frac{dH}{dz}\big|_z} - \overbrace{E - Tr}^{ET_{cr}}$$

or

$$E + Tr = P + I - R - \Delta S\Big|_z^0 + K(t)\frac{dH}{dz}\Big|_z$$

$\frac{dH}{dz}\big|_z \to >0 \quad q\uparrow$

$\frac{dH}{dz}\big|_z \to <0 \quad q\downarrow$

Evaluation of each component

- $P \to$ raingauge
- $I \to$ through irrigation controll
- $R \to \simeq 0$ or through field erosion experiments
- $\Delta S \to$ neutron probe measurements
- $K(\theta) \to$ from field experiments
- $\frac{dH}{dz} \to$ tensiometer readings

Diagrams illustrating the calculation of E, ET_{cr} and fluxes under different situations

Ⓐ BARE SOIL

Integrating equation $\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z}$ between the soil surface (z=0) and a soil depth (z) gives

$$-\left[q(z,t) - q(0,t)\right] = \frac{\partial}{\partial t}\int_0^z \theta(z,t)\, dz$$

$$q(0,t) = q(z,t) + \frac{\partial}{\partial t}\int_0^z \theta(z,t)\, dz \quad (1)$$

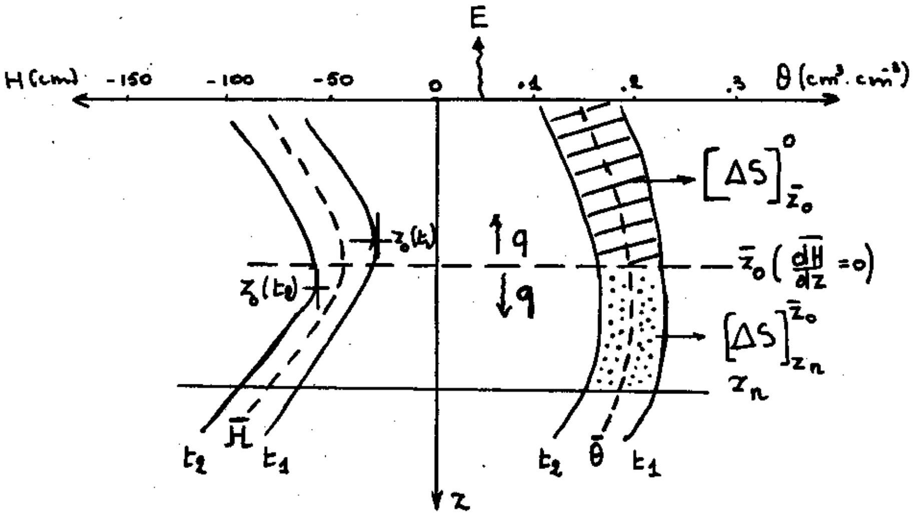
where

- $\int_0^z \theta(z,t)\, dz$ = soil water storage at time t from z=0 to z=z
- $q(0,t)$ = evaporation flux at time t
- $q(z,t)$ = flux at depth z and time t

or (1) $\to$

$$q_1 = q_2 + \frac{\Delta S}{\Delta t}$$

(A 1) Calculation of the fluxes when a plane of zero flux (z where $q=0$: z_0) exists



Evaporation

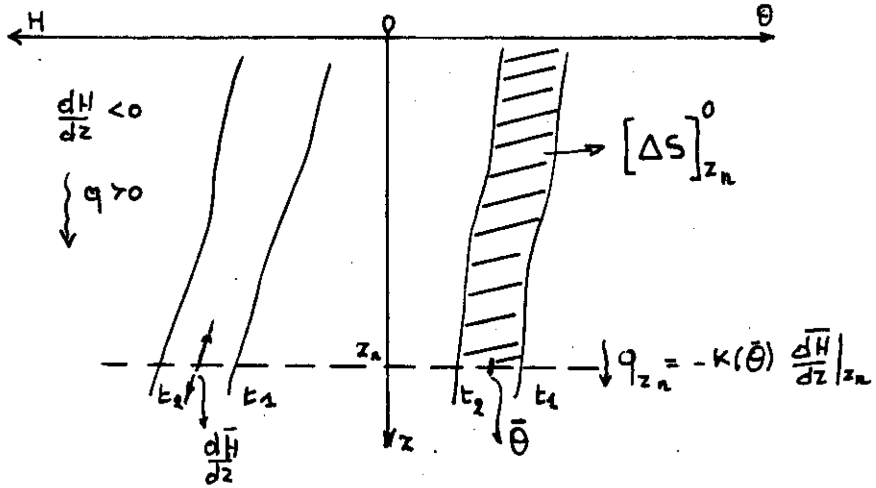
in dry period : $q_1 = \frac{\Delta S}{\Delta t}$ or $\Delta S\Big|_{\bar z_0}^{0} = q_1 \Delta t = E$

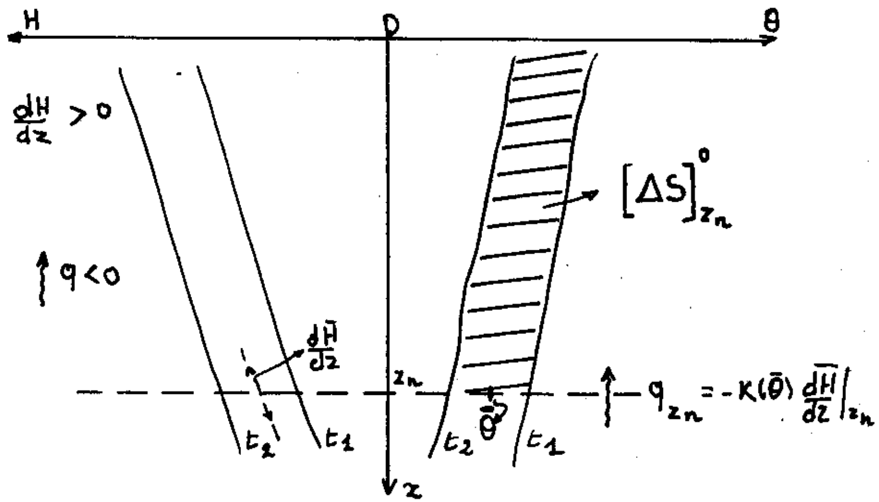
in general : $E = P + I - R - \Delta S\Big|_{\bar z_0}^{0}$

Drainage at depth $z_n > \bar z_0$

$$D = \Delta S\Big|_{z_n}^{\bar z_0} = q_{z_n} \cdot \Delta t = -K(\bar\theta)\,\frac{d\bar H}{dz}\Big|_{z_n} \cdot \Delta t$$

(A 2) Calculation of the fluxes under continuous drainage and upward flow conditions (absence of plane of zero flux).





$$E = P + I - R - [\Delta S]_{z_n}^{0} - \underbrace{\left[-K(\bar\theta)\,\frac{d\bar H}{dz}\Big|_{z_n}\right]}_{0 < q < 0} \Delta t$$

(B) CROPPED SOIL

In this case there is:

- evaporation through the soil surface : $q(0,t)$
- transpiration $T_R(t) = -\int_0^{z_R} J(z,t)\,dz$

with z_R = max. root zone depth.

Integrating equation $\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z} - J(z,t)$ between the soil surface $(z=0)$ and a soil depth (z) gives

$$q(0,t) = q(z,t) + \frac{\partial}{\partial t}\int_0^z \theta(z,t)\,dz + \int_0^z J(z,t)\,dz$$

$$\downarrow$$

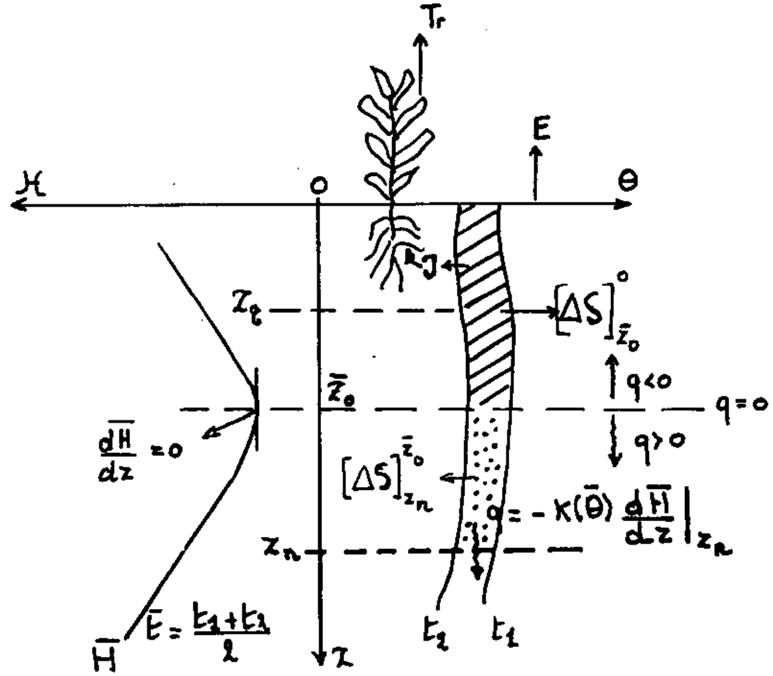
$$q(0,t) = q(z,t) + \frac{\partial}{\partial t}\int_0^z \theta(z,t)\,dz + \int_0^{z_R} J(z,t)\,dz + \int_{z_R}^z J(z,t)\,dz$$

$$\downarrow$$

$$q(0,t) + T_R(t) = q(z,t) + \frac{\partial}{\partial t}\int_0^z \theta(z,t)\,dz + \int_{z_R}^z J(z,t)\,dz$$

$$q_1 + T_R = q_2 + \frac{\Delta S}{\Delta t} + \int_{z_R}^z J(z,t)\,dz$$

(B1a) Calculation of the crop evapotranspiration (ET_{cr}) when the plane of zero flux is located under the maximal root depth ($\bar{z}_0 > z_R$)



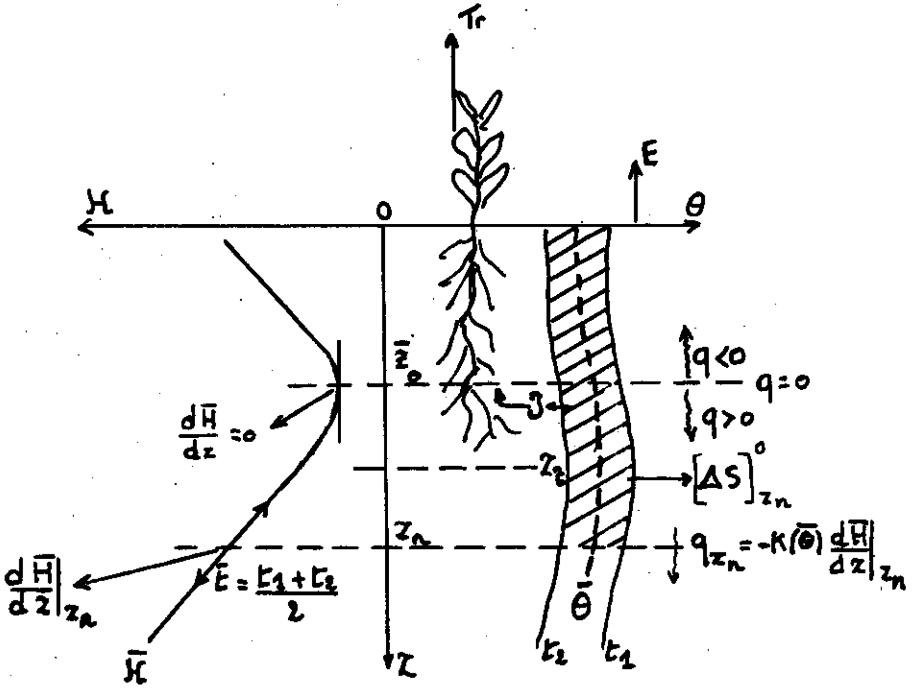
Crop Evapotranspiration

$$ET_{cp} = P + I - R - [\Delta S]_{\bar{z}_0}^{0}$$

Drainage at z_R

$$q_{z_R}\cdot\Delta t = -K(\bar{\theta})\,\frac{d\bar{H}}{dz}\Big|_{z_R}\cdot\Delta t = [\Delta S]_{z_R}^{\bar{z}_0}$$

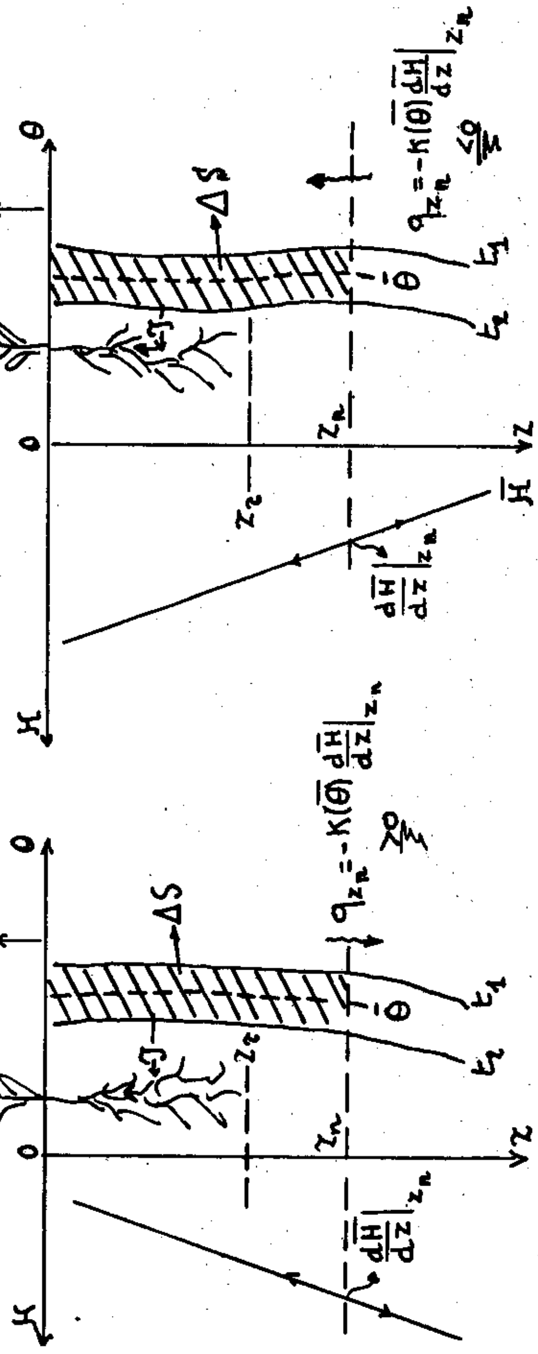
(B1b) Calculation of the crop evapotranspiration (ET_{cr}) when the plane of zero flux is located within the root zone ($\bar{z}_0 < z_R$).



Crop Evapotranspiration

$$ET_{cr} = P + I - R - [\Delta S]^0_{z_n} - \overbrace{\left[-K(\bar{\theta})\,\frac{dH}{dz}\Big|_{z_n}\right]}^{q_{z_n} > 0} \cdot \Delta t$$

(B2) Illustration of the calculation of ET_{cr} in the absence of a plane of zero flux ($z_{q=0}$).



Crop Evapotranspiration

$$ET_{cr} = P + I - R - [\Delta S]^0_{z_n} - \underbrace{\left[-K(\bar{\theta})\,\frac{d\bar{H}}{dz}\Big|_{z_n}\right]}_{0 < q < 0} \Delta t$$