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**FOURTH WORKSHOP ON THREE DIMENSIONAL MODELLING
OF SEISMIC WAVES GENERATION,
PROPAGATION AND THEIR INVERSION**

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***Moment Tensor & Source Depth
Inversion Program
(MomTens)***

User's Guide

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These are preliminary lecture notes, intended only for distribution to participants.

Moment Tensor & Source Depth Inversion Program (MomTens)

User's Guide

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The program "MomTens" retrieves seismic source parameters from long period surface wave spectra and P wave polarity data. The theoretical basis of the method we use is provided in the Appendix. This program runs in Unix environment. It is a windows based application where the user interface is organized through dialogs, menus and toolbars.

Here below are provided the first hand information on the menu and related icons. Further details can be found in the next sections.

File:

New.....
Open.....
Save
Save as...
Exit

Project:

Records.....
Structure models.....
Spectral Discretization.....
Parametric Space.....

Build:

Polarity.....
Moment tensor.....

View:

Current Project.....
DBase WfDisc.....
Results.....



New Project File - Create new Project File .



Open Project File.



Save Project file.



Selection of records to be used for inversion.

This icon opens a window-based dialog allowing removal and add of records.



Stations and related structural models.

This icon opens a window based dialog that relate a station with it's structural model.



Spectra Range Discretization.



Definition of grid in parametric space.



Selection and Rarefaction of Original Polarities.



Moment Tensor, Source Depth Calculation.



View of all project lists and parameters.



View of DataBase table of all wfdisc records.

It is possible to look at waveforms.



View of results.

From SEED Files to Database.

FTAN and Moment Tensor & Source Depth Inversion programs read seismic records from Database. We will illustrate the procedure of going from SEED to Database by the following example:

1. Let "/export/home/smith/DATASEED" is the name of directory containing the SEED file "1998222_Event_1230".

1a. Execute the shell script "mseed2db" by the following command line:

mseed2db A B C 1998 1998222_Event_1230 222 12 30 15 222 14 30 15 Y Y

A is /export/home/smith/DATASEED - directory containing SEED file;

B is /export/home/smith/SEISMOGRAM_DATA_BASES/work - working directory;

C is /export/home/smith/SEISMOGRAM_DATA_BASES/DBE - place directory;

1998 - year, **1998222_Event_1230** - data file name ;

beginning of the records: **222** - julian day, **12** - hour, **30** - min, **15** - sec;

end of the records: **222** - julian day, **14** - hour, **30** - min, **15** - sec;

Y - procflag, **Y(N)** - delete (or not) the working directory.

1b. To go from two SEED files into one Database, use the shell script "mmseed2db", the procedure is as 1a, remember to add the second SEED file name in the command line just right after the first SEED file.

2. Generation of origin Dbase table

Sometimes SEED files do not contain the event information and only in this case we have to generate the origin Dbase table.

You must extract the event information from one of the "pde.origin" Dbase tables or from the "cmt_end_123196.origin" table. The relevant files are included in the Dbase DATASCOPE package.

The two-step extraction procedure is the following:

(a). Execute Dbase program "h2e" for conversion human times to epoch times.

h2e [-y year] [-m month] [-d day] [-H hour] [-M min] [-S sec]

The result is [epoch time], which is the number of seconds since 00:00 hrs, January 1, 1970.

(b). Execute "more" command:

more ../3.2/pde/pde_1995.origin | grep "epoch time" > Event.origin.

Here is an example: **more ../3.2/pde/pde_1995.origin | grep "806366644.50"**

pde_1995.origin refers to the pde catalogue for the 1995 recordings.

or

more ../3.2/cmt/cmt_end_123196.origin | grep "epoch time" > Event.origin

3. The directory containing Database will be named

/export/home/smith/SEISMOGRAM_DATA_BASES/DBE/1998/1998222_Event_1230.

It will contain the following files and directories:

222.wfdisc 1998222_Event_1230.sensor.old 1998222_Event_1230_v.wfedit

1998222_Event_1230.affiliation	1998222_Event_1230.site	
1998222_Event_1230.b051	1998222_Event_1230.sitechan	
1998222_Event_1230.instrument	1998222_Event_1230.sitechan.old	response/
1998222_Event_1230.instrument.old	1998222_Event_1230_l.lastid	
1998222_Event_1230.lastid	1998222_Event_1230_l.wfdisc	wf_l/
1998222_Event_1230.network	1998222_Event_1230_l.wfedit	wf_v/
1998222_Event_1230.origin	1998222_Event_1230_v.lastid	
1998222_Event_1230.sensor	1998222_Event_1230_v.wfdisc	

Command lines for FTAN program execution

Let "/export/home/smith/SEISMOGRAM_DATA_BASES/DBE/1998/1998222_Event_1230" - be the Database directory. It contains the following files and directories:

222.wfdisc	1998222_Event_1230.sensor.old	1998222_Event_1230_v.wfedit
1998222_Event_1230.affiliation	1998222_Event_1230.site	
1998222_Event_1230.b051	1998222_Event_1230.sitechan	
1998222_Event_1230.instrument	1998222_Event_1230.sitechan.old	response/
1998222_Event_1230.instrument.old	1998222_Event_1230_l.lastid	
1998222_Event_1230.lastid	1998222_Event_1230_l.wfdisc	wf_l/
1998222_Event_1230.network	1998222_Event_1230_l.wfedit	wf_v/
1998222_Event_1230.origin	1998222_Event_1230_v.lastid	
1998222_Event_1230.sensor	1998222_Event_1230_v.wfdisc	

To run FTAN program you have to type:

```
gftan 1998222_Event_1230_l Flt 1998222_Event_1230 1998222_Event_1230
```

where:

gftan	- alias name (*),
1998222_Event_1230_l	- wfdisc table name,
Flt	- output Root Dbase name,
1998222_Event_1230	- Root Db tables name for responses,
1998222_Event_1230	- origin Db tables name for event information.

Filtered seismograms will be written in directory "wfout". FTAN program will create a table "Flt.wfdisc" containing the description of directory "wfout".

Note.

(*) alias gftan 'ftanfeo | tee stdin.log | ftan_sol \!*',
where "ftanfeo" and "ftan_sol" are executable files.

Input observed data and output results.

I. Input data.

1. The program reads filtered records and all the information concerning stations and event from Database generated from SEED files. In the example considered in the section "From SEED to Database" all these data are collected in the directory "export/home/smith/SEISMOGRAM_DATA_BASES/DBE/1998/1998222_Event_1230".

1a. Starting a new project (an inversion of a given set of observations) use the icon



or item "New" in the File menu. The following window will appear:

The users have to browse to fill the empty boxes related to the different items reported on the window. The new project name cannot be introduced in the related box shown in the window here up. It has to be browsed and an extension .spr is automatically attributed.

Note.

If you have filtered any seismograms twice or more and then saved the results, you will have two or more filtered records in the wdisc Data base table, which have the same station and channel names. Before executing the MomTens program it is necessary to remove "doubled" records, using Data Base facilities.

1b. It is possible to start a session using a Project File saved earlier.

Push "Open" button  or "Open" menu item. Using "Browse" button and the directory-file navigator find the Project File you would like to work with.

2. Before starting a new Project you have to create a directory structure containing structural models, first arrival polarity data, and surface wave average phase velocity along the paths. The name of the directories should be as follow: MODELS/ POLAR/ PH_VEL/, and should reside in the directory from where the program MomTens is executed.

2a. MODELS - this directory contains the structural models (ascii files) of the source and all the station vicinities used.

Every row represents a layer in the model. The format of the ascii file is the following:

```

N
f1 f2 f3 f4
.....
.....

f1 f2 f3 f4
.....
.....

f2 f3 f4

```

where N is the number of layers within the model (number of rows starting from the 2-nd), f1 is the layer thickness in km, f2 is P wave velocity in km/s, f3 is Swave velocity in km/s, and f4 is density in g/sm^3 . The last row represents the characteristics of the half-space and is without layer thickness.

The files of the structural models must have the following names:

"name.mod", where extension "mod" is fixed, and "name" is arbitrary for station models. But the name of the source model is fixed: this model should be named "000.mod".

2b. POLAR this directory contains ascii file "polarity.ori" with first arrival polarity data.

If polarity data are not available this subdirectory has not to be created.

The format of "polarity.ori" file is the following:

```

f1 f2 f3
.....
.....

f1 f2 f3

```

Every row of this file contains polarity data for one station. f1 is the azimuth of the direction from the source to the station, f2 is the take-off angle of the first arrival, f3=-1 for dilatation, and f3=1 for compression.

2c. PH_VEL this directory contains for every surface wave record to be used in the inversion an ascii file with averaged along path the phase velocity as a function of period. The names of these files have to correspond to the station names. Such a file containing velocities of Love wave recorded at the station "XXX" has to be named "XXX.l"; file with Rayleigh velocities has to be named "XXX.r".

If phase velocity data are not available this subdirectory has not to be created. In this case PREM velocity model for all traces will be used by default.

The format of "XXX.l" and "XXX.r" files is the following:

```

f1 f2
.....
.....

```

f1 f2

where:

f1 is the period in seconds, and f2 is the correspondent surface wave phase velocity.

II. Output results.

If "TestEvent.spr" is the name of the Project, then the program will create in the directory "Event_1998" a subdirectory "TestEvent.SPR". The results of the inversion will be placed in its sub-directory named "RESULTS". The structure of sub-directory "RESULTS" will be organised as follows:

DEPTH.PS/ MAPS.PS/ MECHA.PS/ OBS_CALC.PS/ POLARITY.PS/

All these directories contain output graphics in postscript format. This output is described in the section "View".

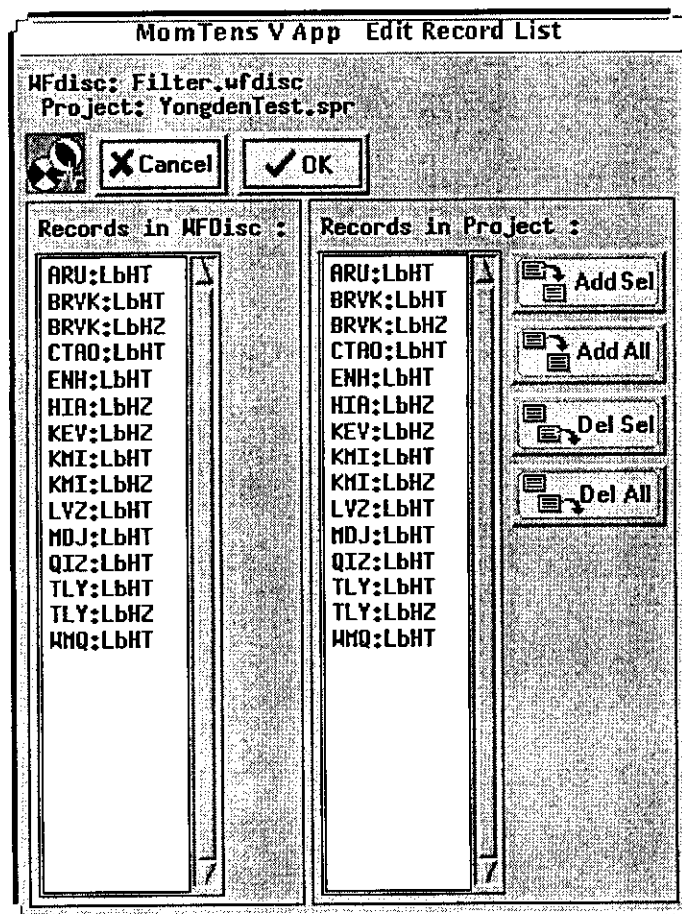
Preparing data for inversion

Before we start the inversion procedure, the following steps have to be followed



1. Selection of records to be used for inversion

The window-based dialog as shown here below selects the records to be used for inversion. The left column are the available records, while the right one corresponds that will be inverted. By default the content of the two columns is the same.



2. Structural models and related stations

This window below shows the lists of Models and stations. Clicking on any model from the left

column (files from directory MODELS) and any station from the right column will relate the station to its right model.

/export/home/mostinsk/YONGDEN/YongdenTest.spr

The Equivalence Models and Stations Dialog.

Models	Models Stations	Stations
000	BRVK	BRU
BRVK	BRVK	BRVK
CTAO	HIA	CTAO
ENH	CTAO	ENH
KEY	ENH	HIA
KIP	KEY	KEY
KHI	KEY	KHI
LVZ	MDJ	LVZ
QIZ	KHI	MDJ
SCZ	LVZ	QIZ
SPA	QIZ	TLY
STG	TLY	WMO
TAM	WMO	
TLY		
VND		
WMO		



3. Spectral Range Discretization

3a. Appropriate Spectral Range for every recorded wave

During preliminary filtering by FTAN program, a spectral domain, defined by maximum and minimum periods, is to be determined for every wave where the signal is of a good quality. These period values go in the text boxes "T min" and "T max" in the uppermost left part of the window. To associate this pair of values to any record, the user has to click on the right record name.

3b. Spectral grid

After the spectral boundaries for every record are fixed, the spectral range for inversion has to be chosen.

Minimum and maximum periods has to be typed in "T min" and "T max" text boxes in the central part of the window. A maximum number of spectral samples (≤ 18) has to be typed in the text box Nw. The number of samples for the Fast Fourier Transform program calculating the spectra

is the last parameter to be defined. Push the arrow in the combo box, choose the number, and push "Get" button.

Actual spectral boundaries for every record will appear in the central window. If you will mark by cursor any record in this window, spectral samples related to this record will appear in the right window.

3c. Minimum period for phase spectra inversion

By default the observed phase spectra will be used for the inversion in the full spectral domain. Model PREM will be used in this case to calculate the surface wave phase velocities.

If you will click on the check box "PhVel" and will type in the text box T min (in the lowermost left part of the window) any value of the period, then the phase spectra will be used for the inversion only for the periods larger than this value.

If the check box "PhVel" is highlighted and the directory PH_VEL containing the files with surface wave average phase velocities is already existing, then these velocities interpolated on the grid defined above will be used for computations.

/export/home/mostinsk/YONGDEN/YongdenTest.spr

The Spectral Range Discretization.

Spectral Range for every wave

T min(sec):

T max(sec):

Seis	T_min	T_max
ARU:LbHT	30.00	100.00
BRVK:LbHT	50.00	90.00
BRVK:LbHZ	30.00	100.00
CTAO:LbHT	50.00	80.00
ENH:LbHT	30.00	100.00
HIA:LbHZ	35.00	75.00
KEY:LbHZ	30.00	100.00
KMI:LbHT	40.00	90.00
KMI:LbHZ	40.00	90.00
LVZ:LbHT	30.00	100.00
MDJ:LbHT	30.00	100.00
QIZ:LbHT	50.00	90.00
TLY:LbHT	30.00	100.00
TLY:LbHZ	30.00	100.00
WMQ:LbHT	60.00	90.00

☒ PhVel T min:

Spectral Range

T min(sec):

T max(sec):

Nu <= 18 : Total:

Seis	T_min	N	T_max	N	W	T
ARU:LbHT	31.752	12	99.902	1	0.19788	31.7519
BRVK:LbHT	50.568	5	83.592	2	0.18561	33.8512
BRVK:LbHZ	31.752	12	99.902	3	0.17334	36.2478
CTAO:LbHT	50.568	4	71.860	4	0.16107	39.0095
ENH:LbHT	31.752	12	99.902	5	0.14880	42.2268
HIA:LbHZ	36.248	8	71.860	6	0.13652	46.0225
KEY:LbHZ	31.752	12	99.902	7	0.12425	50.5679
KMI:LbHT	42.227	7	83.592	8	0.11198	56.1096
KMI:LbHZ	42.227	7	83.592	9	0.09971	63.0154
LVZ:LbHT	31.752	12	99.902	10	0.08744	71.8596
MDJ:LbHT	31.752	12	99.902	11	0.07517	83.5918
QIZ:LbHT	50.568	5	83.592	12	0.06289	99.9024
TLY:LbHT	31.752	12	99.902			
TLY:LbHZ	31.752	12	99.902			
WMQ:LbHT	63.015	3	83.592			



4. Parametric range definition

This window allows to define the grid for the source parametric model. This model is defined by double couple depth (in km) and 3 angles (in degrees): dip, strike and slip. For each of these parameters you have to define the start value, the step, and the number of samples (N Step) using correspondent text box.

Note:

It will be impossible to close up this window and a warning message appears, in the case where the parameters do not satisfy the following constrains:

Dip N Step <= 6;

Strike N Step <= 12;

Slip N Step <= 12;

<start of dip> + <step of dip> * (<N Step of dip> - 1) <= 90;

<start of strike> + <step of strike> * (<N Step of strike> - 1) <= 360;

<start of slip> + <step of slip> * (<N Step of slip> - 1) <= 360.

For a complete exploration of the focal sphere, it is enough to assign in the following window half

of the full interval for strike and slip angles. The symmetric mechanisms are also investigated during the inversion. The window shown here below contain the default value for each parameter. As it investigates the full focal sphere.

/export/home/mostinsk/YONGDEN/YongdenTest.spr

The Depth Range Input
 Start(kn) Step(kn) N Step

The Dip Range Input
 Start Step N Step

The Strike Range Input
 Start Step N Step

The Slip Range Input
 Start Step N Step



5. Selection and rarefication of polarity data

Push the arrow in the combo box, choose the angle of polarity data smoothing and confirm by pushing OK. Stereographic projections of focal sphere with original and selected polarities will appear in a GHOSTVIEW window. To go back to the main window, select File and then Quit.

Smoothing of Original Polarities



Calculations

Pushing this button when all input data and parameters are prepared, you will start the procedure of inversion by exploration of the source parametric space.

View of results and input data



View of all project lists and parameters



View of DataBase Wfdisc table

Attention:

Don't Edit wfdisc using this option!

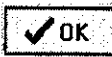


View of results

Pushed of this button the following window will be opened

/export/home/mostinsk/YONGDEN/YongdenTest.spr

View of Results

View inversion	maximum visible - minimum residual difference (from 0 to 1.)
☒ amplitude spectra inversion	0.5
☐ amplitude spectra + P polarities inversion	number of best solutions to be shown
☐ amplitude + phase spectra inversion	10

The left window frame allow you to choose one of three following items correspondent to the three types of inversion considered in the Appendix.

(a) Amplitude spectra inversion

After you have chosen this item, type in the upper right text box the value defining the difference between the maximum and minimum residual. Only these possible orientations of main double couple stress axis which give the residual between observations and synthetic within this interval will be shown at the maps of the quality of different possible axis orientation.

Push button OK, and following results will appear in different windows:

- Map of the quality of different possible T and P axes orientation. Every point at the map represents the stereographic projection of the axis orientation on the lower hemisphere. The color shows the quality of this orientation. In the case under consideration the maps for T and P axes are similar because of the symmetry considered in Appendix, This is why only one map is presented.
- Four equivalent best focal mechanisms and seismic moment.
- Residual as function of source depth (see Appendix).
- Observed and synthetic surface wave amplitude spectra calculated for the best values of varying parameters.

(b) Amplitude spectra + P polarities inversion

After you have chosen this item, type in the upper right text box the value defining the difference between the maximum and minimum residual which you would like to be shown at the maps.

Type in the lower right text box the number of best solutions to be shown by the nodal planes stereographic projections. Push button OK, and following results will appear in different windows:

- Maps of the quality of different possible T and P axes orientation.
- The best focal mechanism, selected polarities and seismic moment.
- A set of first N (N is the number typed in the lower right text box) best focal mechanisms and seismic moments.
- Residual as function of source depth.
- Observed and synthetic surface wave amplitude spectra calculated for the best values of varying parameters.

(c) Amplitude + phase spectra inversion

After you have chosen this item, follow the instructions given in the previous paragraph (b).

Appendix

Description of technique

Instant point source can be described by the moment tensor - a symmetric 3x3 matrix $\mathbf{M}$. Seismic moment M_0 is defined by equation

$M_0 = \sqrt{\frac{1}{2} \text{tr}(\mathbf{M}^T \mathbf{M})}$, where $\mathbf{M}^T$ is transposed moment tensor $\mathbf{M}$, and

$\text{tr}(\mathbf{M}^T \mathbf{M}) = \sum_{i,j=1}^3 M_{ij}^2$. Moment tensor of any event can be presented in the

form $\mathbf{M} = M_0 \mathbf{m}$, where matrix $\mathbf{m}$ is normalised by condition $\text{tr}(\mathbf{m}^T \mathbf{m}) = 2$.

We are considering a double couple instant point source (a pure tangential dislocation) at a depth h . Such a source can be given by 5 parameters: double couple depth, its focal mechanism which is characterising by three angles: strike, dip and slip or by two unit vectors (direction of principal tension $\mathbf{T}$ and direction of principal compression $\mathbf{P}$) and seismic moment M_0 . Four of these parameters we determine by a systematic exploration of the four dimensional parametric space, and the 5-th parameter M_0 - solving the problem of minimisation of the misfit between observed and calculated surface wave amplitude spectra for every current combination of all other parameters.

Under assumptions mentioned above the relation between the spectrum of the displacements $u_i(\mathbf{x}, \omega)$ in any surface wave and the total moment tensor $\mathbf{M}$ can be expressed by following formula

$$u_i(\mathbf{x}, \omega) = \frac{1}{i\omega} [M_{jt} \frac{\partial}{\partial y_t} G_{ij}(\mathbf{x}, \mathbf{y}, \omega)] \quad (1)$$

$i, j = 1, 2, 3$ and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x}, \mathbf{y}, \omega)$ in equation (1) is the spectrum of Green function for the chosen model of medium and wave type (see Levshin, 1985; Bukchin, 1990), $\mathbf{y}$ - source location. We assume that the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model (Woodhouse, 1974; Babich *et al.*, 1976). The surface wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. The amplitude spectrum $|u_i(\mathbf{x}, \omega)|$ defined by formula (1) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum (Bukchin, 1990). The average phase velocities of surface waves are usually not well known. For this reason as a rule we use only amplitude spectra of surface waves for determining source parameters under consideration. We use observed surface wave phase spectra only for very long periods.

Surface wave amplitude spectra inversion

If all characteristics of the medium are known the representation (1) gives us a system of equations for parameters defined above. Let us consider now a grid in the space of these 4 parameters. Let the models of the media

be given. Using formula (1) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω . Let $u^{(i)}(\mathbf{x}, \omega)$ be any observed value of the spectrum, $i = 1, \dots, N$; $\varepsilon_{\text{amp}}^{(i)}$ - corresponding residual of $|u^{(i)}(\mathbf{x}, \omega)|$. We define the normalised amplitude residual by formula

$$\varepsilon_{\text{amp}}(h, \varphi, \mathbf{T}, \mathbf{P}) = \left[\left(\sum_{i=1}^N \varepsilon_{\text{amp}}^{(i)2} \right) / \left(\sum_{i=1}^N |u^{(i)}(\mathbf{x}, \omega)|^2 \right) \right]^{1/2} \quad (2)$$

The optimal values of the parameters that minimize ε_{amp} we consider as estimates of these parameters. We search them by a systematic exploration of the five dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions. Fixing the value of one of varying parameters we put in correspondence to it a minimal value of the residual ε_{amp} on the set of all possible values of the other parameters. In this way we define one residual function on scalar argument and two residual functions on vector argument corresponding to the scalar and two vector varying parameters: $\varepsilon_h(h)$, $\varepsilon_T(\mathbf{T})$ and $\varepsilon_P(\mathbf{P})$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters. From geometrical point of view these functions describe the lower boundaries of projections of the 4-D surface of functional ε on the coordinate planes.

It is well known that the focal mechanism can not be uniquely determined from surface wave amplitude spectra. There are four different focal mechanisms which will radiate this same surface wave amplitude spectra. These four equivalent solutions represent two pairs of mechanisms symmetric with respect to the vertical axis, and within the pair differ from each other by the opposite direction of slip.

To get a unique solution for the focal mechanism we have to use in the inversion additional observations. For these purpose we use very long period phase spectra of surface waves or polarities of P wave first arrivals.

Joint inversion of surface wave amplitude and phase spectra

Using formula (1) we can calculate for chosen frequency range the phase spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed phase spectra give us a residual $\varepsilon_{\text{ph}}^{(i)}$ for every point of observation, every wave and every frequency ω . We define the normalised amplitude residual by formula

$$\varepsilon_{\text{ph}}(h, \varphi, \mathbf{T}, \mathbf{P}) = \frac{1}{\pi} \left[\left(\sum_{i=1}^N \varepsilon_{\text{ph}}^{(i)2} \right) / N \right]^{1/2} \quad (3)$$

We determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{ph})(1 - \varepsilon_{amp}) . \quad (4)$$

Joint inversion of surface wave amplitude spectra and P wave polarities

Calculating radiation pattern of P waves for every current combination of parameters we compare it with observed polarities. The misfit obtained from this comparison we use to calculate a joint residual of surface wave amplitude spectra and polarities of P wave first arrivals. Let ε_{amp} be the residual of surface wave amplitude spectra, ε_p - the residual of P wave first arrival polarities (the number of wrong polarities divided by the full number of observed polarities), then we determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_p)(1 - \varepsilon_{amp}) . \quad (5)$$

Before inversion we apply to observed polarities a smoothing procedure which we will describe here briefly.

Let us consider a group of observed polarities (+1 for compression and -1 for dilatation) radiated in directions deviating from any medium one by a small angle. This group is presented in the inversion procedure by one polarity prescribing to this medium direction. If the number of one of two types of polarities from this group is significantly larger then the number of opposite polarities, then we prescribe this polarity to this medium direction. If no one of two polarity types can be considered as preferable, then all these polarities will not be used in the inversion. To make a decision for any group of n observed polarities we calculate the sum $m = n_+ - n_-$, where n_+ is the number of compressions and $n_- = n - n_+$ is the number of dilatations. We consider one of polarity types as preferable if $|m|$ is larger then its standard deviation in the case when +1 and -1 appear randomly with this same probability 0.5. In this case n_+ is a random value distributed following the binomial law. For its average we have $M(n_+) = 0.5n$, and for dispersion $D(n_+) = 0.25n$. Random value m is a linear function of n_+ such that $m = 2n_+ - n$. So following equations are valid for the average, for the dispersion, and for the standard deviation σ of value m

$$M(m) = 2M(n_+) - n = n - n = 0, \quad D(m) = 4D(n_+) = n, \quad \text{and} \quad \sigma(m) = \sqrt{n}.$$

As a result, if the inequality $|m| \geq \sqrt{n}$ is valid then we prescribe +1 to the medium direction if $m > 0$, and -1 if $m < 0$.

References

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