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**FOURTH WORKSHOP ON THREE DIMENSIONAL MODELLING
OF SEISMIC WAVES GENERATION.
PROPAGATION AND THEIR INVERSION**

28 September - 9 October 1998

Complex Heterogeneous Faulting Models

presented by:

Raul MADARIAGA

Laboratoire de Géologie URA CNRS 1316

Ecole Normale Supérieure

24 rue Lhomond

75231 Paris Cedex 05

France

These are preliminary lecture notes, intended only for distribution to participants.

Complex Heterogeneous Faulting Models

Notes for the 4th Workshop of Seismic Wave Generation, Propagation and Inversion

ICTP, Trieste 28 September - 9 October 1998

by

Raúl Madariaga
Laboratoire de Géologie URA CNRS 1316,
Ecole Normale Supérieure,
24 rue Lhomond, 75231 Paris Cedex 05, France,

with the essential help of

Kim Bak Olsen
Institute for Crustal Studies
UCSB, Santa Barbara, CA 93016-1100, USA

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These notes are dedicated to Boris Kostrov who sadly left us not long ago. He was one of the finest scientist I had the chance to meet; our rare but intense conversations, spiced with foul smelling cigarettes and superb cups of coffee are one of my fondest memories.

1 Introduction

Earthquake are probably one of the most obvious manifestation of complexity in earth sciences; and are taken by many earth and physical scientists as a paradigm for the spontaneous development of complexity in a natural system driven by an external source of energy (the external source being plate tectonics). Papers in physical journals containing the word earthquake are almost as frequent as those in earth science.

Yet this complexity is not obvious to many earth scientists: A large segment of our research community still looks for regularity, repeatability, characteristic earthquakes, etc in seismic records and in seismotectonics. The most obvious example of this approach to earthquake research is the Parkfield Project where a limited time series was projected into the future in order to predict the next event in the series. Many evidences that the previous Parkfield earthquakes were actually not very regular nor very similar were "filtered" out. It is clear now that this naive naturalistic approach has little chances of succes. Or does it?

On the other extreme of the research spectrum, a group of earth scientists recently declared in papers and notes published in the best science journals that earthquakes are an example of a conceptual model called self-organized criticality (SOC) and that they are intrinsically unpredictable. Never mind that SOC is only "observed" in numerical experiments done in computers, the only system where no length or time scales interfere with SOC. Although SOC is an interesting and provocative idea, its elevation to paradigm of earthquake occurrence requires a major research effort that the authors did not really do.

Where do seismologists stand? It seems to me that most earthquake seismologists have taken a distance with these arguments and continue to work in their usual wave business, oblivious to the controversies. This leaves the field open to all sort of speculations, naive computer models and serious conceptual mistakes regarding the origin of seismic energy. Is this the right attitude to take? I believe that both extreme approaches, the dogged search for regularity and the imposition of the SOC dogma have serious pitfalls. I also firmly believe that seismologists must confront the challenge of understanding earthquakes in their full *multidisciplinary* spectrum.

In this notes I will discuss the evidence drawing from different sources that I do not have the time to refer properly. I hope authors will forgive this "légèreté".

These notes contain three parts:

- **Evidences for Earthquake Complexity**

- Time and space variability of seismicity. Is Gutenberg Richter a universal feature of earthquakes or does it fail for isolated faults? This is without doubt the most important argument by partisans of the SOC model of seismicity.
- Are small and large earthquakes different or is this just a problem of catalog completeness? Recent observations by Wesnousky and others of Gutenberg-Richter distribution for isolated faults seem to indicate that large earthquakes may be different from smaller ones.
- Slip and stress fields inverted from near and far-field observations as well as geodetic motions are very heterogenous at all scales.

- **From the simplest toy-models to realistic 2D fault models**

- A study of the Burridge Knopoff (BK) toy-model and the current status of knowledge about this system. New notes and a review of work by Carlson, Langer, Shaw, Schmittbuhl and Vilotte.
- A study of a simple 2D crack model and its agreement with the BK model. Reprint of paper in the Proceedings of the National Academy of Sciences by Cochard and Madariaga

- **Towards realistic earthquake models. Seismic rupture propagation in a 3 dimensional earthquake model**

- A study of rupture propagation in a simplified model of the Landers earthquake in the mojave desert that occurred on July 28, 1992. Starting from results obtained by Wald and Heaton (1994) we made a fully dynamic model of this event. Introductory Notes plus a Reprint of paper by Olsen, Madariaga and Archuleta published in Science.

2 Evidences for Earthquake Complexity

This part is presented in the form of copies of transparencies annotated to draw attention to certain points. I apologize for not doing this properly but scanners are not my strength.

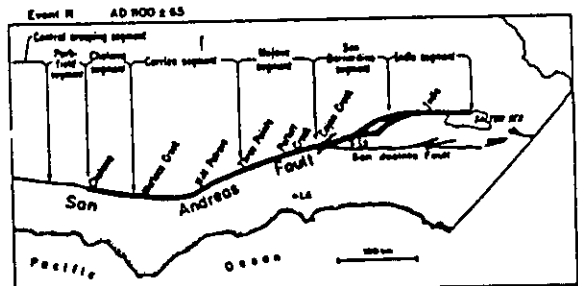
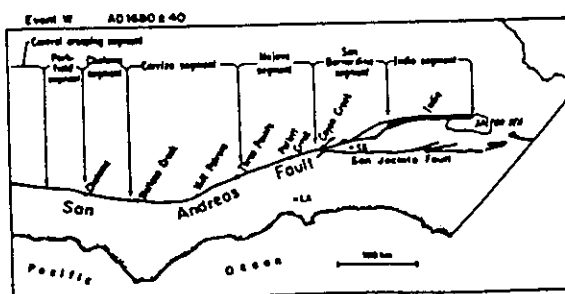
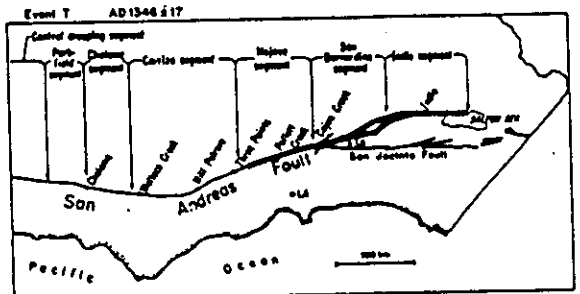
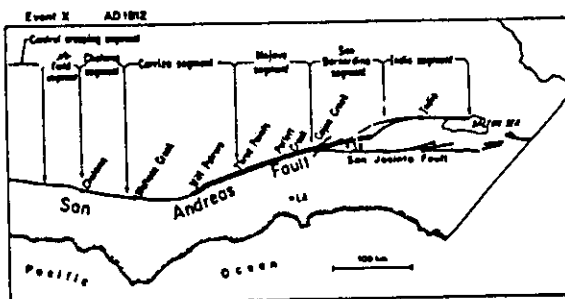
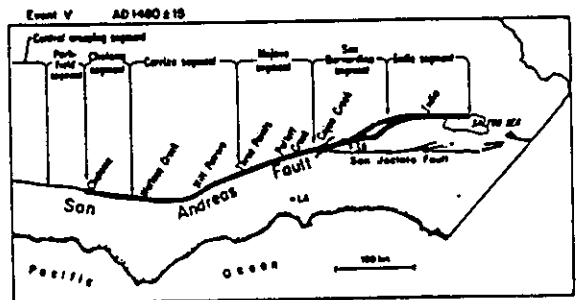
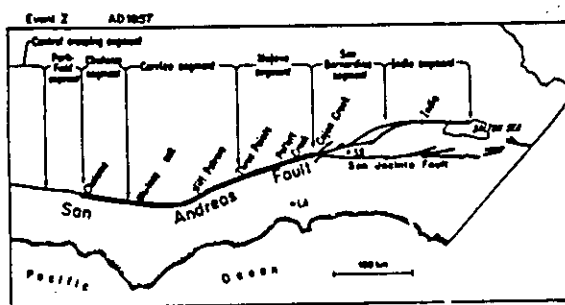
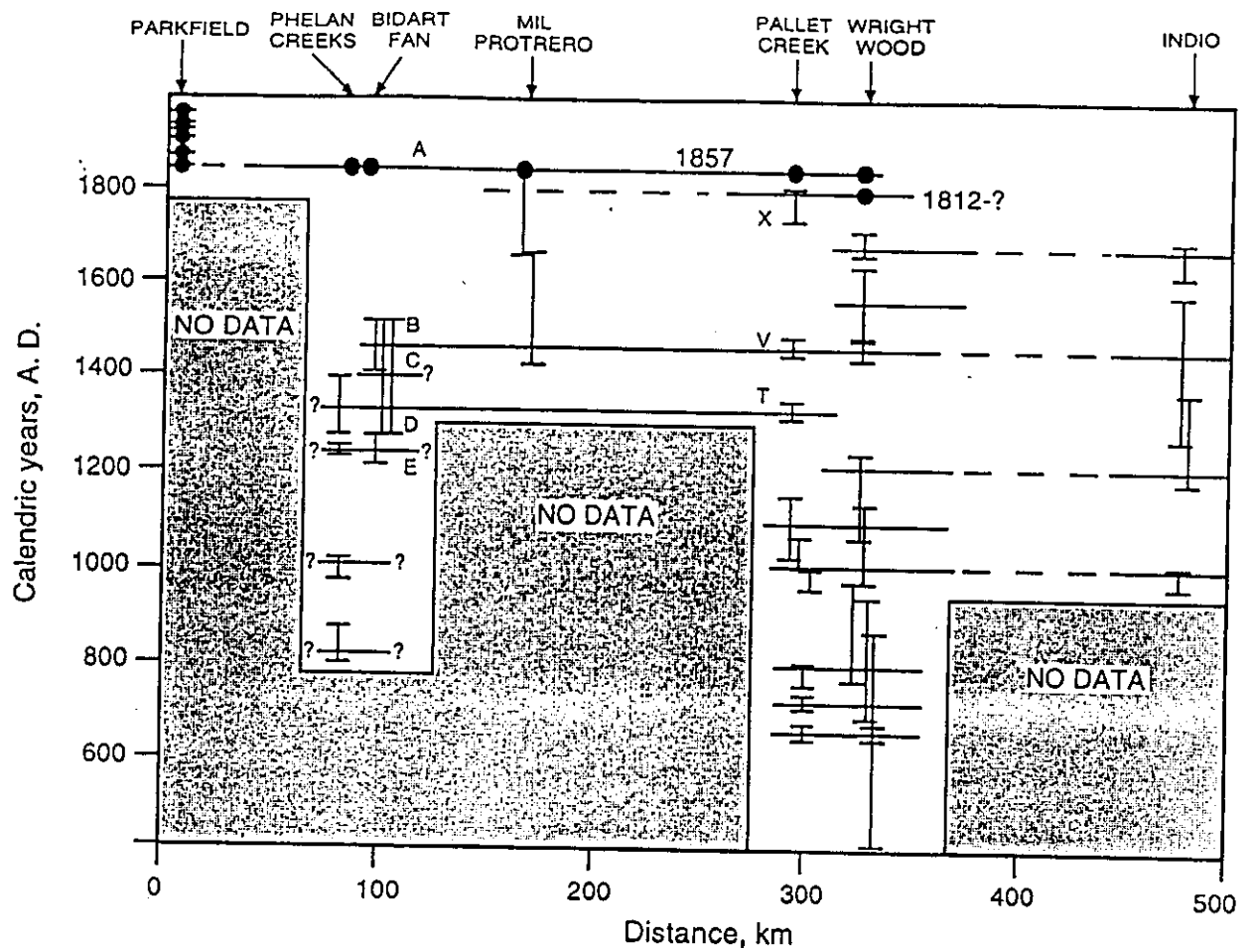
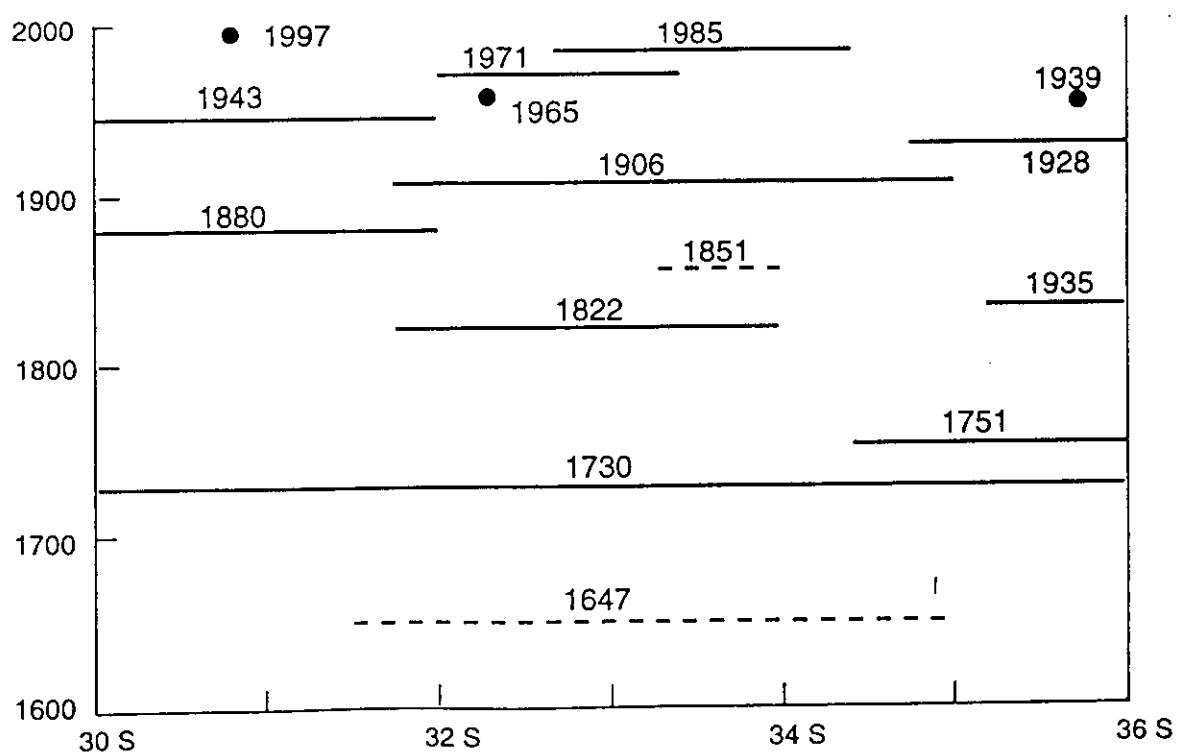
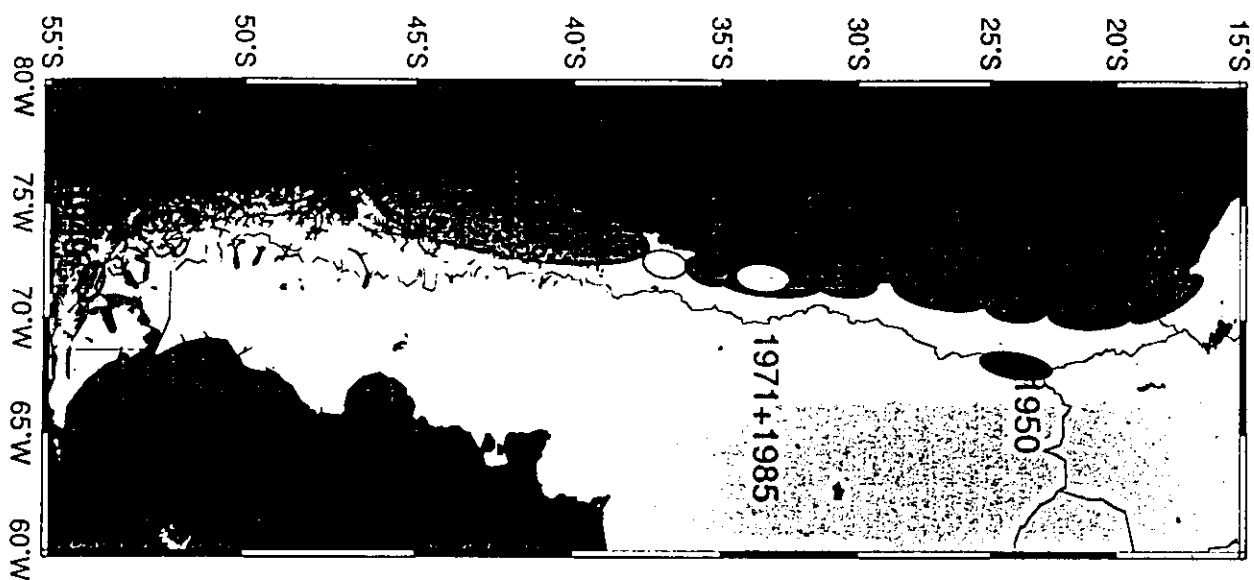
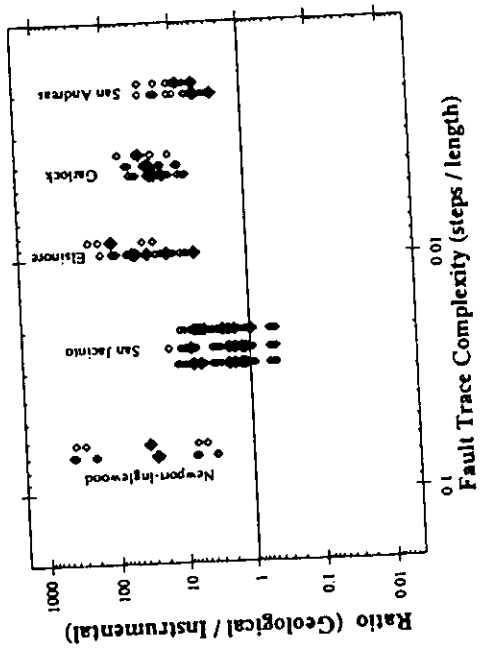
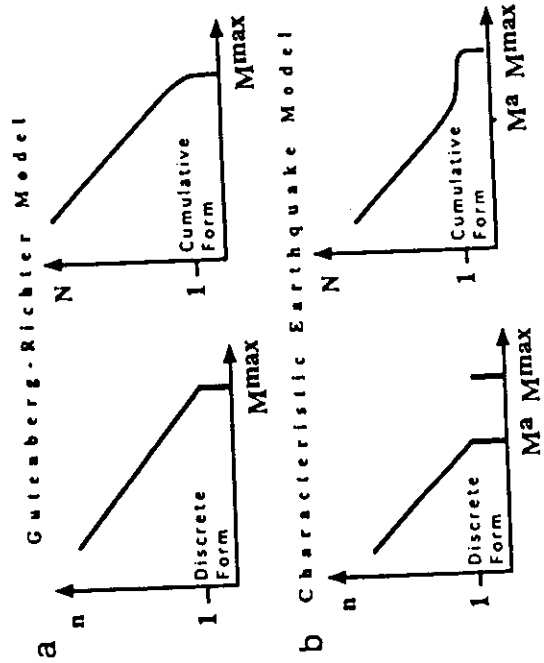
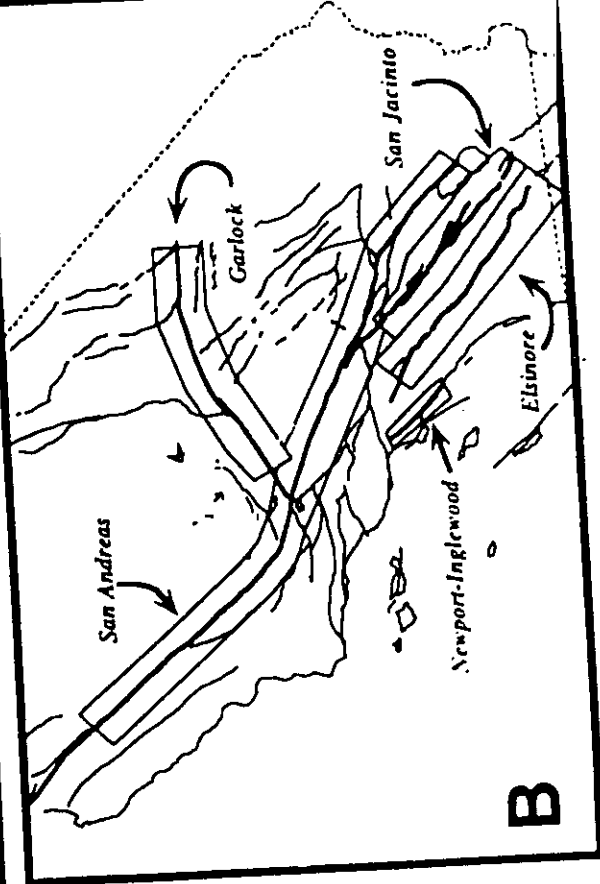
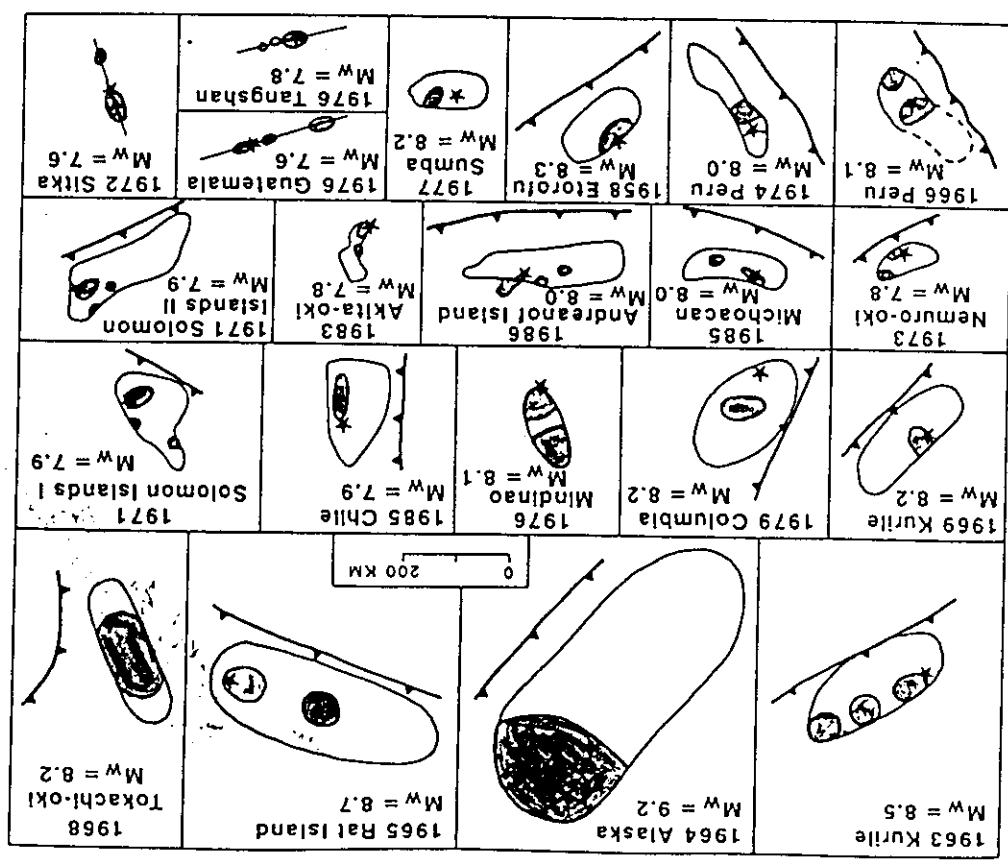
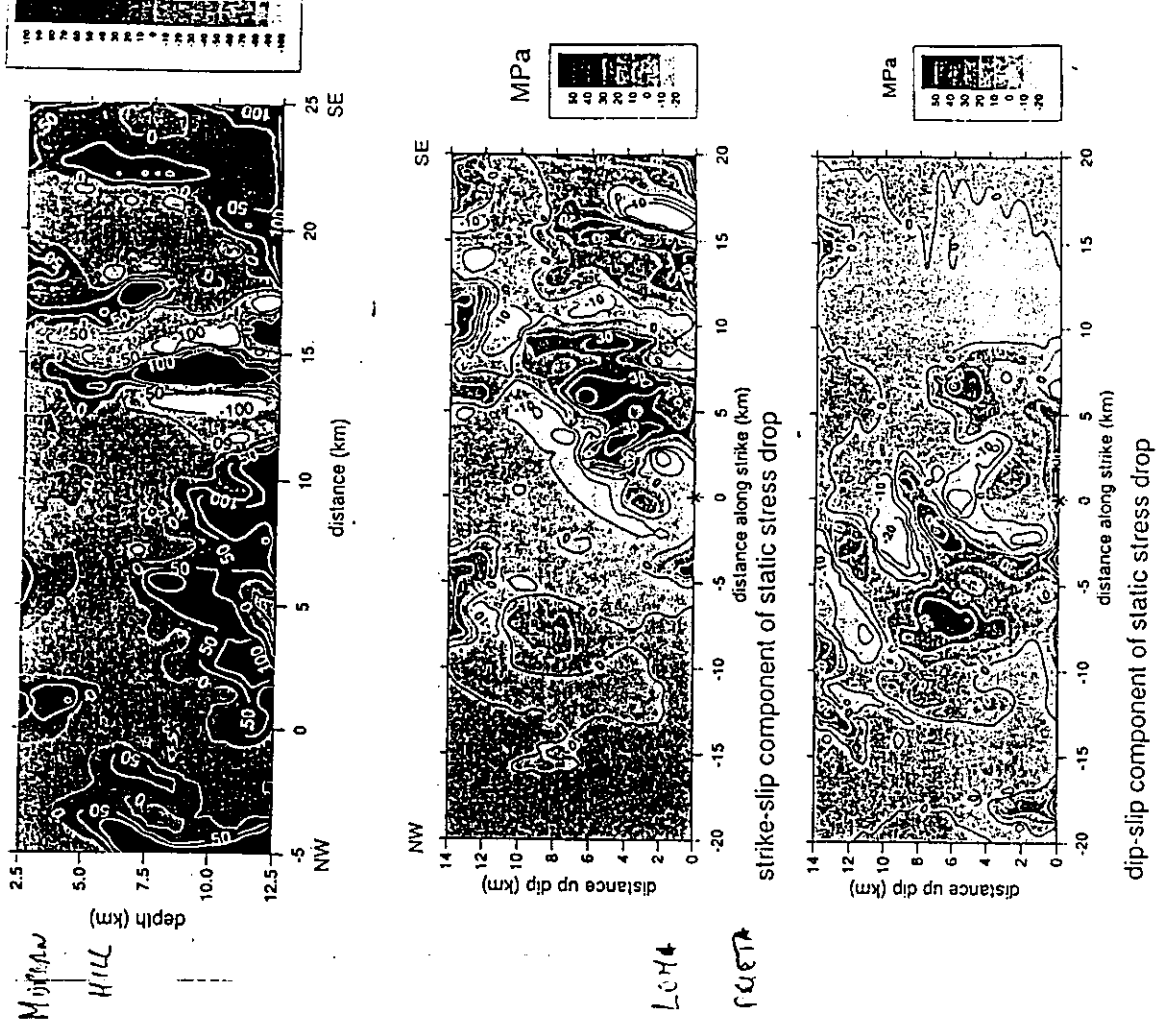


Fig. 13. Plausible earthquake history for the southern half of the San Andreas fault for the past millennium. See text for discussion of data and assumptions.





ADP System



3 From the simplest toy-models to realistic 2D fault models

The simplest slider-and-spring toy-models

The simplest model of an earthquake-like phenomenon that one can imagine is the block and spring model used by Brace, Byerlee, Burridge and Knopoff to illustrate stick slip, the essential non-linear property of friction that is believed to be at the base of the earthquake instability. Two essential hypotheses:

First, why friction? Although not every body agrees with this point of view, I strongly believe that all tectonic earthquakes are due to slip on preexisting faults. Almost no new fractures are produced by earthquakes simply because the energy required to make fresh fractures is much higher than what it takes to produce slip on a fault. In this notes we will not discuss alternative earthquake models based on statistical mechanics of phase transformations because I do not understand their foundation in observations. The failure of preexisting faults is entirely controlled by friction and geometry. In the continental interiors faults are short and discontinuous (some say immature), they produce only small earthquakes. Here geometry controls the earthquakes. On plate boundaries or major intracontinental tectonic features like the Altyn Tagh fault in Western China or the Gulf of Corinth, faults form a relatively flat network of closely connected subfaults that work as more or less continuous system. In this case friction, understood in the general sense as resistance to slip, is the dominating feature. Of course on large plate boundary faults earthquakes are also segmented and the role of geometry is important; but it seems to me that in order to understand large earthquakes we have first to put the accent on friction.

Second, what instability? Earthquakes occur on a time scale that is extremely short compared to all relevant geological time scales, even those related to tidal loads. Aki and Brune showed 30 years ago that all earthquakes are similar and a single time scale, size divided by wave speed, characterizes all natural earthquakes. This time scale is not that of a fundamental physical property of friction but that of wave propagation. Thus we must have at the heart of the earthquake process a loss of traction across the fault that the elastodynamic forces can only restore on the time scale of wave propagation around the fault.

The instability of friction can be illustrated very easily with the simple model of a block of mass M driven by a spring of stiffness k . The driver moves at a constant speed v_0 so that the spring exerts a linearly increasing force $f = kv_0t$ on the block. Displacement u of the block is resisted by friction $\sigma(u)$. In general σ must be a complex function of displacement, speed of the block and some additional "internal" variables like temperature, time of contact, etc.

For simplicity let us consider the simplest case of a classical Coulombian friction that says

$$\begin{aligned} u &= 0 & \text{for } f < \sigma_s \\ \sigma &= \sigma_d & \text{for } u > 0 \\ \dot{u} &= 0 & \text{asoonafter } \dot{u} < 0 \end{aligned} \tag{1}$$

As long as the force exerted by the spring is less than σ_s , the block does not move, $u = 0$ and f increases. Once f overcomes the threshold σ_s (the "static" friction) slip

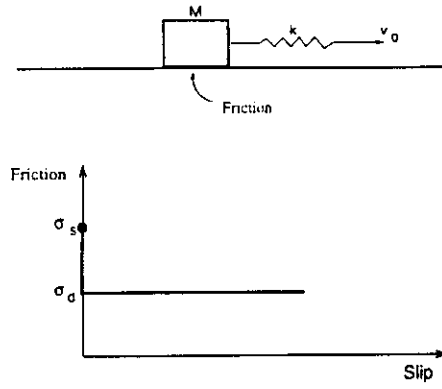


Figure 1: Simple one degree of freedom oscillator, probably the simplest conceivable model of an earthquake like system. At the bottom the Coulomb friction law

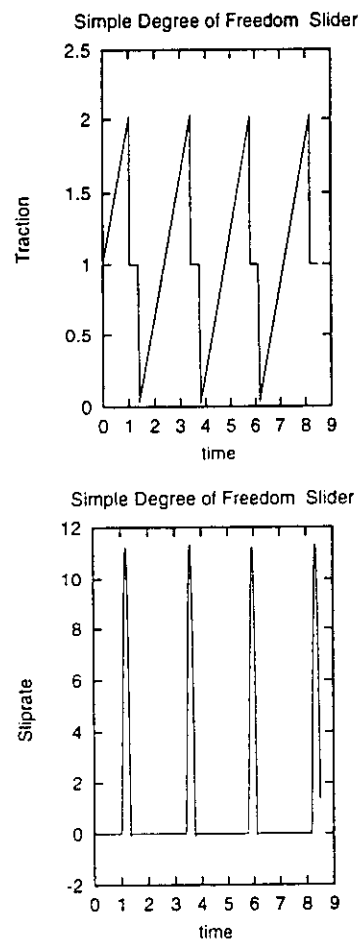


Figure 2: Simple one degree of freedom oscillator. Stress (top) and slip rate (bottom)

starts and friction drop **instantly** to σ_d , the kinematic friction. From that moment on the movement of the slider is controlled by Newton's equation of motion

$$M\ddot{u} = -ku + \Delta\sigma H(t) \quad (2)$$

with initial conditions $u = 0, \dot{u} = 0$. $\Delta\sigma = \sigma_s - \sigma_d$ is the "stress drop" that acts as the driving force for the system. Equation 2 is a harmonic oscillator of natural frequency $\omega_0 = \sqrt{k/M}$, driven by the stress drop $\Delta\sigma$. The oscillations are arrested (the interface heals) *instantaneously* when the velocity $\dot{u}$ crosses zero. The solution is very simple: For $t < 2\pi/\omega_0$

$$\begin{aligned} u(t) &= \frac{\Delta\sigma}{k}[1 - \cos(\omega_0 t)] \\ \dot{u}(t) &= \frac{\Delta\sigma}{k}\omega_0 \sin(\omega_0 t), \end{aligned} \quad (3)$$

while for $t > 2\pi/\omega_0$

$$\begin{aligned} u(t) &= 2\frac{\Delta\sigma}{k} \\ \dot{u}(t) &= 0 \end{aligned} \quad (4)$$

Let us remark the factor of 2 in front of the final slip. This is the overshoot a characteristic of block-and-spring slider. The overshoot occurs because the slider model does not dissipate energy, so that at the end of the slip process the spring is in a state of tension. In crack models overshoot is small because the radiation of elastic waves damps the movement on the fault.

The solution is easier to understand looking at figures 5 and 3. After slip arrest the process starts all over again, the continuous motion of the loader produces an increasing force f on the block that yields again once the force overcomes the static friction, and so on. We observe that the block motion is episodic. The name stick-slip comes from here: stick when the block is not moving, slip when it rapidly jumps ahead driven by stress drop.

Now that we have finished this section, why did I call this is a toy-model? Although phenomenologically this model acts like somewhat like earthquakes, the similarity is only vague:

1. Earthquakes radiate seismic waves and our toy model does not! If a sudden jump in the earth were not to produce elastic waves we would never notice it and nobody would call this an earthquake.
2. More subtle, slip events in the block model are driven by an external force, while in the earth there is no external driver. The driving force is slow creeping motion below the seismogenic zone as was proposed by Reid in his elastic rebound model of the 1906 San Francisco earthquake.
3. Finally, at the origin of recent controversies, earthquake have sizes determined by the length scale of the seismogenic zone or the the fault size. Our toy model is a zeroth-dimensional object. It has NO LENGTH SCALE, therefore no moment.

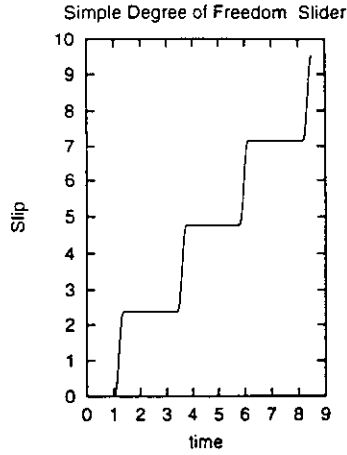


Figure 3: Slip as a function of time for the simple one degree of freedom oscillator.

We will try to improve on this models, making them increasingly realistic. In the process we will loose the simplicity of the harmonic oscillator and we will be forced to do numerical simulations. What we learn from these simple models will guide our way through these more agitated waters.

Improvements on the single slider model

One of the main problems with the single slider is that the simple two-state Coulomb “friction law” is very unphysical. Numerous improvements have appeared in the litterature but I can not review all of them here. I will discuss briefly two:

Slip weakening friction

The Coulombian friction law with a sudden jump from static to dynamic friction is unacceptable because no energy is spent in reducing friction. No matter what friction law we use, energy dissipation requires the introduction of a finite length scale that I will call the slip weakening distance (D_0), a minimum slip distance for friction to drop from σ_s to σ_d . This process of weakening of the contact between the block and the substratum dissipates a finite amount of energy that we call G which is of the order of the stress drop by the slip weakening distance, i.e. $G \simeq \Delta\sigma d_0$. Although the details of the slip weakening process are very important, it is the introduction of the intrinsic length scale that is the most essential change in the physics.

Understanding the need for an intrinsic length scale in the simple oscillator is a very fundamental physical notion: energy being the product of a force by a distance, this distance must be found either in the oscillator or in the friction law. As we saw earlier there is no intrinsic length scale in the simple oscillator, so that the distance for finite energy dissipation must come from the friction law.

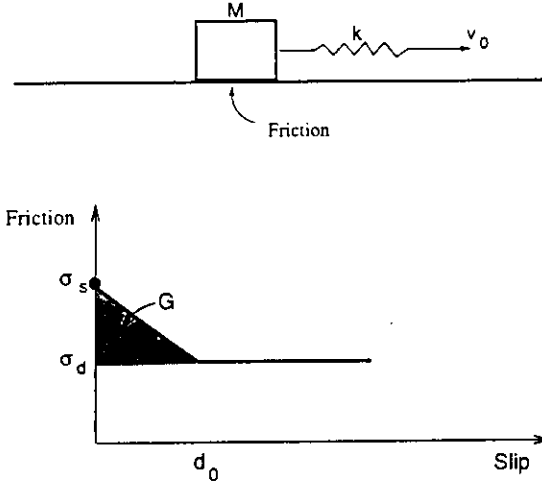


Figure 4: Simple one degree of freedom oscillator, probably the simplest conceivable model of an earthquake like system. At the bottom the Slip weakening friction law

For the sake of illustration we have introduced a simple linear slip weakening model that was first studied by Ida and Aki (1972):

$$\begin{aligned}
 u &= 0 & \text{for } f < \sigma_s \\
 \sigma &= \sigma_s - \Delta\sigma \frac{u}{d_0} & \text{for } d_0 > u > 0 \\
 \sigma &= \sigma_D & \text{for } u > d_0 \\
 u &= 0 & \text{for } \dot{u} > 0
 \end{aligned} \tag{5}$$

The solution to this problem can be found numerically (analytical solutions probably exist but I did not find them).

It is clear that the slip weakening process only acts at the beginning of the slip episodes, smoothing the relaxation of stress. The essential features of the simple slider model remain the same and I will not discuss them any further. The slip weakening process is actually very important in order to regularize the friction law in the Burridge and Knopoff model when several harmonic oscillators are coupled together by coil springs.

Laboratory based Sliprate and State dependent friction

A major effort to understand friction was started by Dieterich and coworkers in the late 70s. These studies form the basis of current knowledge about fault creep, healing, aging, contact cementation, etc. Although I will attract the ire of many colleagues, I do not think that these models change the basic phenomenology of the **dynamic oscillations** in the stick slip process. This is logical: the friction laws of Dieterich-Ruina and Ruina-Dieterich are low speed quasi-static friction laws; when the slider is moving fast, state remains frozen, inertial forces dominate the slider so that it acts again as a harmonic oscillator driven by stress drop. The friction law is important because it determines the

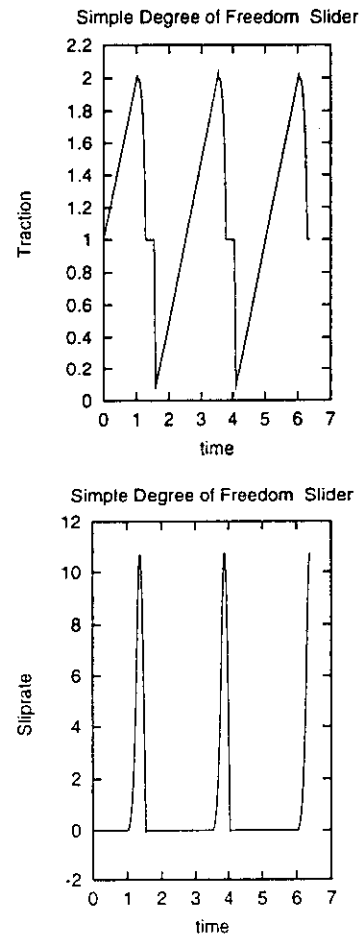


Figure 5: Simple one degree of freedom oscillator. Stress (top) and slip rate (bottom)

stress drop. These results are already present in a seldom cited paper by Gu and Wong (1992) and are briefly improved here and in a forthcoming paper by Lavallée and myself.

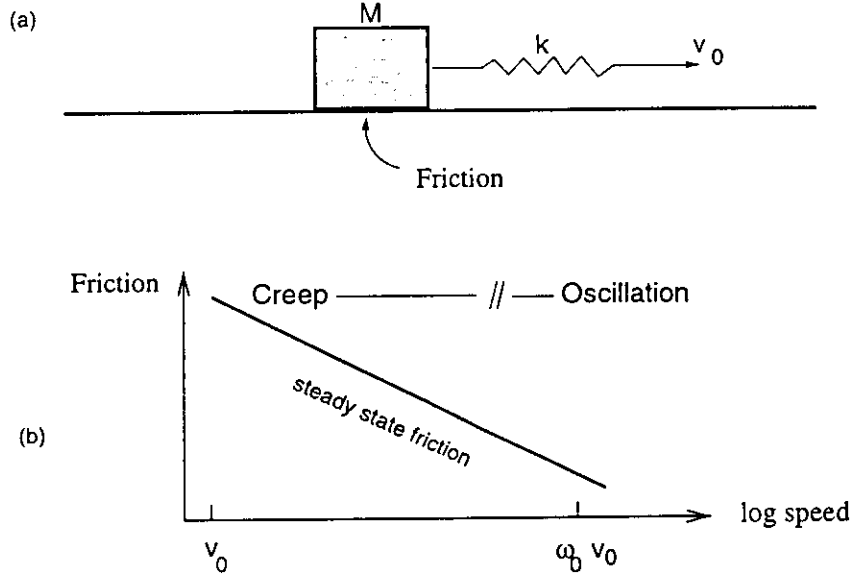


Figure 6: Simple one degree of freedom oscillator. The top (a) shows the oscillator. The bottom shows the dependence of steady state friction on the velocity of the block. In the velocity domain we distinguish two domains: a slow or creep zone when velocities are low with respect to non-dimensional frequency ω_0 and a fast oscillatory domain in which v is of the order of ω_0 .

For simplicity I will consider only one of Dieterich Ruina law. The equation of motion for the block-and-spring model is

$$\begin{aligned}\dot{\theta} &= -v/L(\theta + B \log v/v_0) \\ M\ddot{u} &= -ku + \Delta\sigma(\theta, v)\end{aligned}\tag{6}$$

where θ , u , v are the state variable, the displacement (relative to the motion of the pulling spring) and the velocity of the slider, respectively. t is time and stress drop is given by

$$\Delta\sigma(\theta, v) = \sigma_0 - \sigma = -\theta - A \log v/v_0\tag{7}$$

where σ_0 is the reference traction when the oscillator is moving at constant speed v_0 . As usual σ_0 does not enter in the equation of motion.

Finally, when the slider moves at a constant velocity v , steady state stress and state are given by

$$\begin{aligned}\theta_{ss}(v) &= -B \log v/v_0 \\ \tau_{ss}(v) &= -(B - A) \log v/v_0\end{aligned}\tag{8}$$

The steady state friction is shown schematically in Figure 6b.

The parameters of the Ruina-Dieterich friction law are

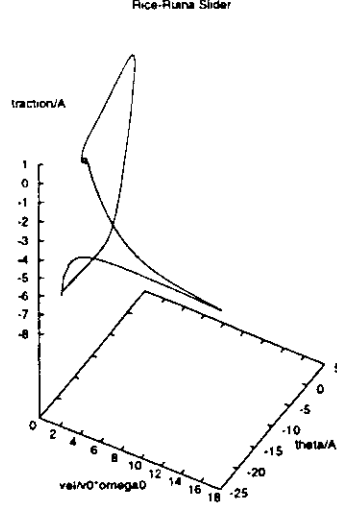


Figure 7: Stress vs. log velocity and state (theta)

- A, B The two coefficients that measure stress sensitivity to $\log v$
- L The relaxation length of the state variable.
- v_0 The reference creep velocity at which the slider moves when unperturbed.

Ruina, Rice, Gu etc considered solutions of equations 6 around the steady state solution. Tse and Rice and Gu and Wong have gone well beyond the perturbative solutions.

The most important feature of equations 6 is that they contain two widely different time scales. A *slow time scale* L/v_0 that controls slow creeping motion of the block, and a *fast time scale*, the period of the oscillator $T = 2\pi/\omega_0$, where $\omega_0 = \sqrt{k/M}$ is its natural frequency.

A numerical example

As a typical example of the motion of the Ruina-Dieterich oscillator I solved the system 6 following set of parameters:

$$\begin{aligned} (B - A)/A &= 0.25 \\ kL/A &= 0.20 \\ \omega_0 &= 10^{12} v_0/L \end{aligned} \tag{9}$$

For this example the ratio between the fast and slow time scales is 10^{12} a very large number indeed.

The result is shown in Figure 7 as trajectories of the oscillator in the $(\log v, \theta, \tau)$ phase space.

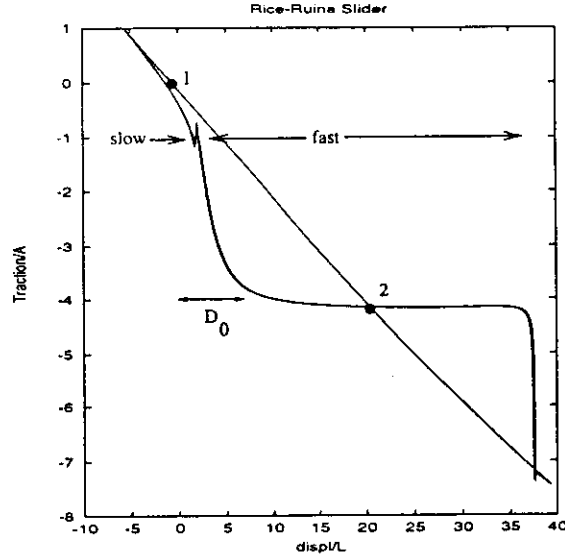


Figure 8: Variation of Stress with displacement in the BK model

Slip weakening in the Ruina-Dieterich model

Although frequently the Ruina-Dieterich friction law is presented as being fundamentally different from traditional slip-weakening models of rupture resistance as proposed by Ida(1972), it actually is a combination of a creep law at low speeds and a slip weakening model at high speeds. This is clearly shown in Figure 10 where we plot stress as a function of slip. Two regimes are clearly seen. The most important for earthquake dynamics is the high speed regime which corresponds to the flat stress zone at the center of the figure. To a very good approximation, stress drop measured as in Figure 10 is

$$\Delta\sigma_d = (B - A) \log(\omega_0 L / v_0)$$

The straight diagonal line seen across the figure is the stress vs. spring force relation

$$\sigma = k(u - v_0 t)$$

Point 2 is the intersection of the dynamic friction with this relaxation line and defines a slip of

$$u_2 = \frac{B - A}{k} \log(\omega_0 L / v_0)$$

which characterizes the maximum slip of the oscillator.

After a period of slip-rate weakening due to creep, the fast regime starts near slip equal 0. At this point indicated by 1 in Figure 10, the slope of the stress-slip curve is very close to $-\xi$. Slip weakening occurs over a distance of roughly $d_0 = 5L$

To sum up, at high speeds the Ruina-Dieterich friction model behaves just like the slip-weakening model (5) in which the slip weakening distance and stress drop are

$$d_0 = 5L$$

$$\Delta\sigma = (B - A) \log(\omega_0 L / v_0) \quad (10)$$

As expected, the Ruina-Dieterich oscillator behaves differently from the slip-weakening oscillator only at slow speeds, when creep effects are important.

A two slider model

In a second effort we demonstrate that slider models spontaneously produce complexity. The model we study was proposed by Huang and Turcotte (1992) and has not been thoroughly studied for general friction laws. I did not have time to do it so that I will present their results qualitatively. The model consists in two identical masses connected to a driver by springs of constant k and interacting through longitudinal coil of stiffness κ . This is a two-block model of the famous Burridge and Knopoff model discussed in next section. Obviously if the two blocks are identical and the friction law is the same these two blocks act like a single block of mass $2M$ and stiffness $2k$. The coil κ is always relaxed.

Two-slider Toy Model

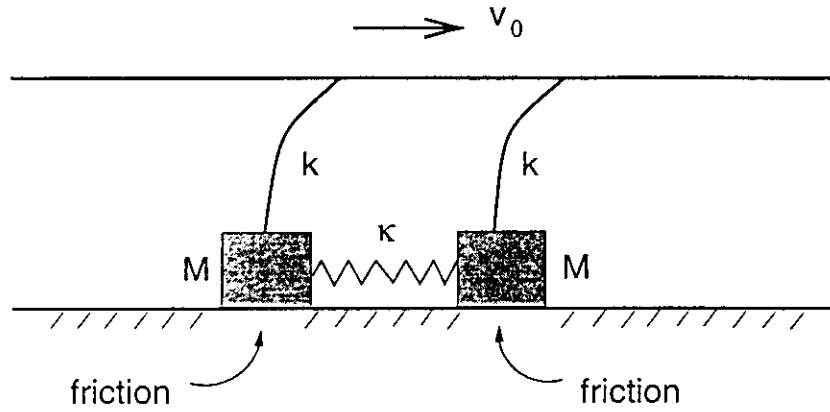


Figure 9: The two coupled harmonic oscillators studied by Huang and Turcotte (1992)

Without entering into the details of the solution largely explained by Turcotte in his book on Fractals, we find that when $\kappa > k$, the system becomes chaotic. The reason is simple, when $\kappa < k$, the oscillator can store energy in the coil spring. Thus between two slip episodes in one or the other of the springs, some energy is stored in the coil springs. This energy is used during the next slip episode to maintain the desynchronization of the system.

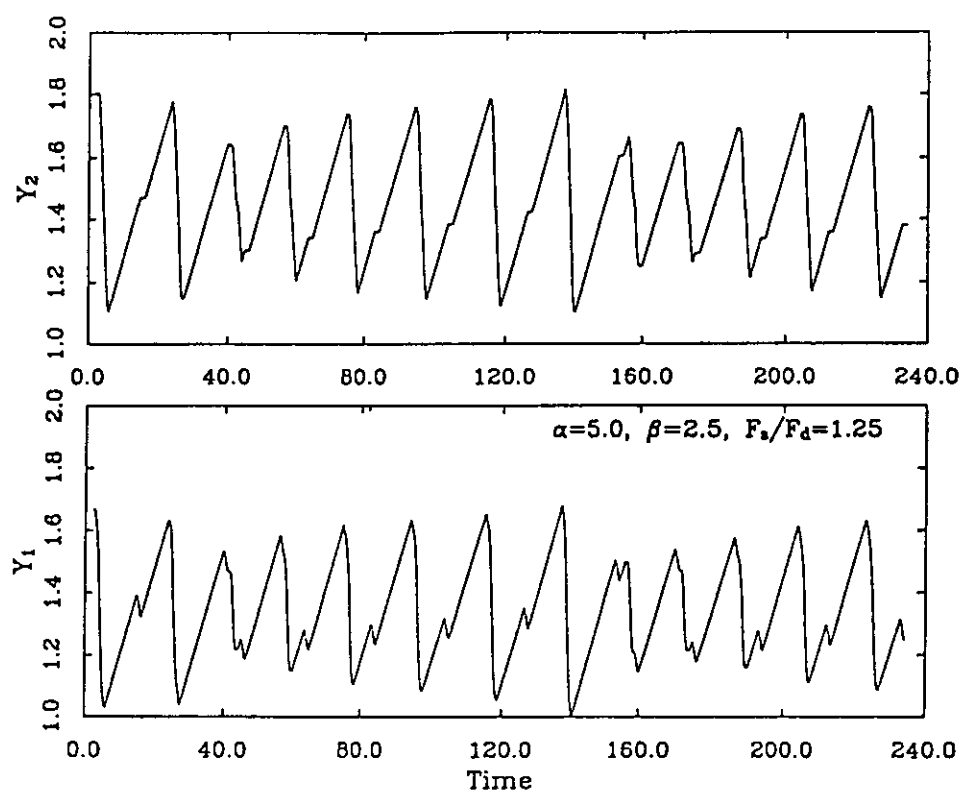


Fig. 4. Time dependence of the displacements Y_1 and Y_2 for the example illustrated in Figure 3.

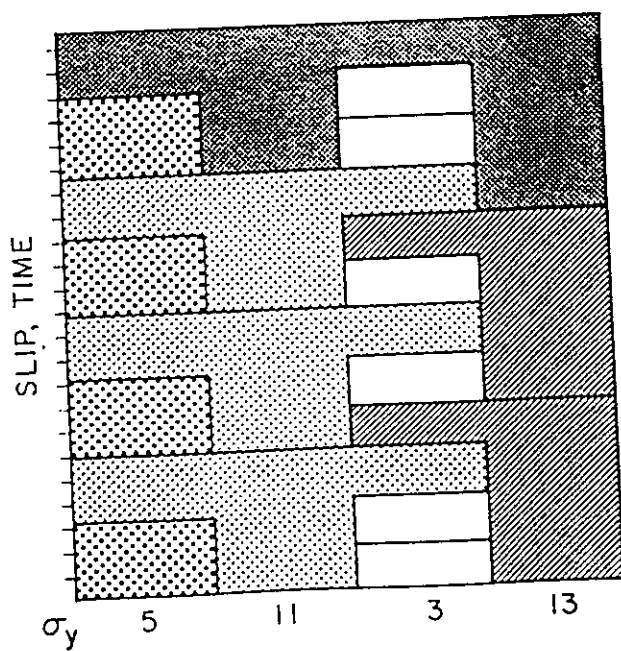


Fig. 5.37 Results of a complete cycle of a simple four-block coupled model.

The Burridge and Knopoff model

Probably the best known of all toy models of seismicity is the mass and spring model proposed by Knopoff and Burridge to study the seismicity of an extended fault. This model is essentially a large set of *identical* blok-and-spring oscillators of mass M driven by leaf springs of stiffness k and coupled by weaker coil springs of stiffness κ . Huang and Turcotte's model referred to in the previous section is a minimal example of this model. Assuming that friction can be written in the formal form $\sigma(u, \dot{u})$, Burridge and Knopoff model satisfies the following system of equations:

$$M\ddot{u}_i = -ku_i - \kappa(u_{i-1} - 2u_i + u_{i+1}) + \sigma(u, \dot{u}) \quad (11)$$

where $u_i(t)$ is the slip of mass element i . For a set of N oscillators this leads to $N - 1$ second order equation plus boundary conditions at the end of the system.

The Burridge-Knopoff Model

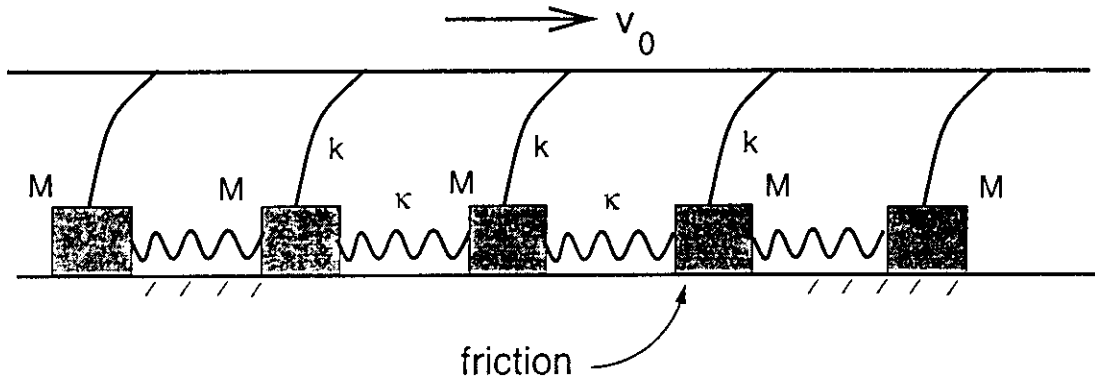


Figure 10: The Burridge and Knopoff earthquake model

Studies of the system 11 have been subject to a major controversy since Cao and Aki (1976) and Carlson and Langer (1989) discovered that slip in this system may become disorganized for certain friction laws. The main criticism was spelled by Rice (1993) who found that for non-regularized friction laws like the Coulomb friction, or the rate-weakening friction used by Carlson and Langer (1973) equations 11 lack a well defined continuum limit. This is a very subtle argument that basically means that the BK model is not the discrete version of a continuum mechanics model. The question is then would a properly posed problem have the same behavior as Carlson and Langer found? Rice studied a fault model for using the Ruina-Dieterich friction law described in the previous section and found perfectly regular oscillations.

The question we have to answer is, was Rice studying the same problem as Carlson and Langer (1989) or is his model intrinsically different because the Ruina-Dieterich friction law has very little rate dependence. The answer to this question is in fact contained in our study of the single block. If we regularize the friction law by taking a finite slip

weakening length scale d_0 , the model has a well defined continuum limit and still behaves like Carlson and Langer proposed for length scales longer than a certain minimum fault length. Thus it seems that the difference in the results from Carlson and Langer and Rice comes from the different nature of the friction laws used by either authors. While Carlson and Langer's law has a strong slip rate dependence, Rice's friction law is almost insensitive to rate dependence. I am convinced that the difference between Rice's results and Carlson and Langer's comes not from the lack of small scale cut-off but rather from a fundamental difference in the behavior of friction at high slip rates. In order to get a physical feeling for this we have to study the regularization of Carlson and Langer's friction.

We can regularize Carlson and Langer model by the use of simple friction law that is both slip and rate-weakening. As we saw earlier, slip-weakening introduces a characteristic length scale d_0 the slip weakening distance. Events for which slip does not reach the slip-weakening limit do not develop into inertial oscillations or BK earthquakes. These events represent creep in the system. When slip at some of the sliders overcomes the slip-weakening limit, a freely slipping event (an "earthquake") occurs. This event may or may not grow into a large event depending on the details of the state of stress before the event. A fundamental change of behavior occurs when the event becomes larger than a certain length scale initially identified by Carlson and Langer and studied more precisely by Schmittbuhl and Vilotte. It turns out that for regular friction laws with finite fracture energy dissipation, the BK model has not just one but TWO length scales: d_0 the small scale cut-off, and a second, longer length scale $L = kd_0/\kappa$. The latter separates two regimes of rupture: ruptures that are smaller than L , represent small events that tend to smooth the slip distribution and occur as foreshock or aftershocks of the main events which have rupture lengths longer than L . These macro-events tend to create complexity in the system because they leave the sliders slightly out of their equilibrium position. As shown by Shaw, Langer, Carlson, Vilotte and Schmittbuhl the macro-events load the coil springs favoring heterogeneity, while the small events mainly act to reduce the load in the coils. There are thus two fundamentally different regimes in this model due to the presence of the two independent length scales. For the large events to generate heterogeneity it is necessary that the friction law has a mechanism to arrest slip spontaneously before the stress is completely relaxed. For this reason it is now known that complexity in perfectly homogeneous BK models does only develop for certain friction laws with rate weakening. It is unfortunate that the understanding of this problem took such a long time.

It must be stressed that this discussion was extremely technical, because a perfectly homogeneous BK model is such a particular object. Had we allowed for lateral variations of rupture resistance or other properties of the springs or masses, the system would have become completely chaotic as already seen for the very simple two-slider model of Huang and Turcotte.

The figures show solutions and examples of complexity.

3.0.1 Conclusion from spring-and-slider models

In conclusion, it appears to me that it is obvious that BK-type earthquake models generate complex earthquake series for a large class of friction laws. The onset of complexity is favored by early healing of slip, variable friction properties along the fault. If certain models

lack complexity, they are probably inadequate. As shown by Shaw, Langer, Schmittbuhl and Vilotte for the BK model and Cochard and Madariaga for a shear fault, rate weakening produces spontaneous heterogeneity. The mechanism is simple: rate weakening promotes premature healing of the fault by an instability clearly identified by Cochard and Madariaga. Rupture propagate in the form of narrow pulses called nowadays Heaton pulses but which were actually introduced in seismology by Haskell.

Strictly slip weakening friction models or weakly rate dependent ones like the Ruina Dieterich friction law, do not become unstable at slow slip rate, therefore they do not produce premature healing and they favor complete unloading of faults. If a fault is completely relaxed after an earthquake there is no way to produce complexity because the load is applied homogeneously to all the sliders. In this case complexity may be easily generated by differences in frictional properties along the fault.

This discussion is very fastidious: it is obvious to any seismologist familiar with slip distributions, seismic radiation and seismicity that earthquakes are very complex. So why all this fuss? I think that the reason is that we are at the dawn of the first realistic models of rupture in the earth and the challenge is right there in front of us. We have the computer power to study these models, the main hurdle is the friction law. Is a simple friction law enough (I believe so and so do Shaw, Carlson, Langer and others) or do we have to include all the gory details of actual friction as insists Rice? Do we have to run fault models that combine creep and fast oscillations as in the single slider toy-models, or can we separate the fast and slow regimes in the numerical simulations? I am sure that these questions will not remain unanswered very long.

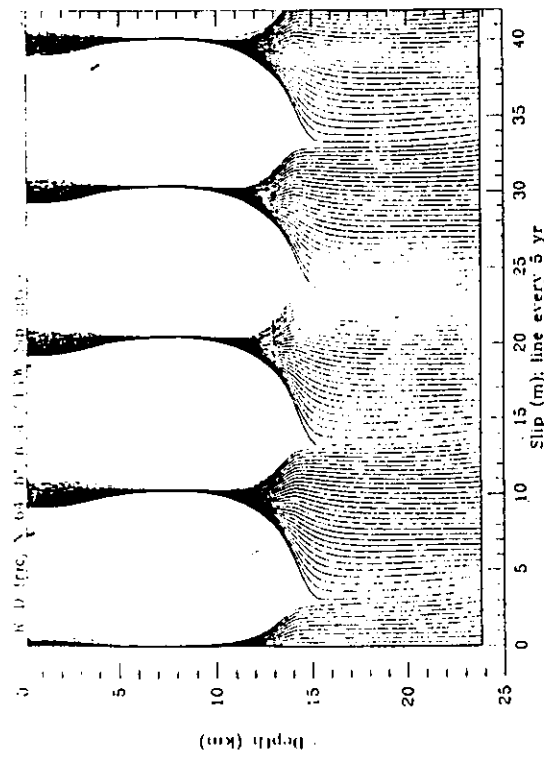


Figure 4. Slip versus depth at 5-year interval in dynamic simulation of long history with the model realization of Figures 2 and 3. The response consists of quasi-periodic large events.

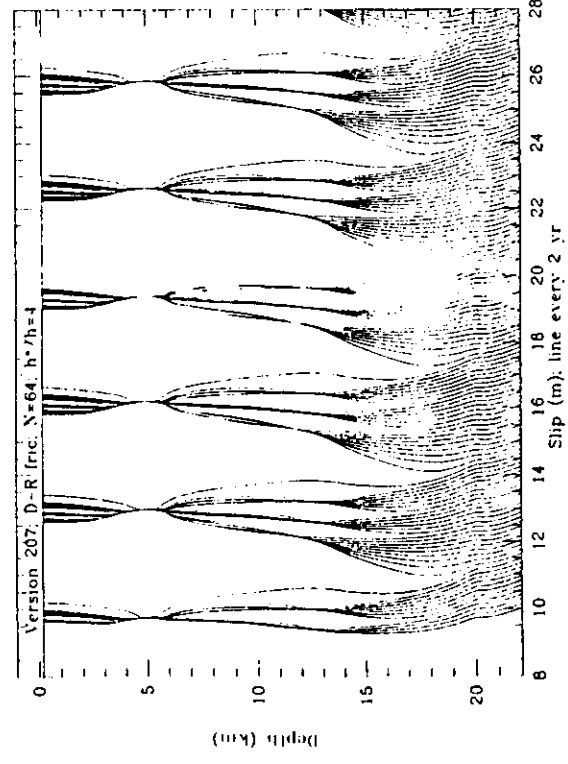
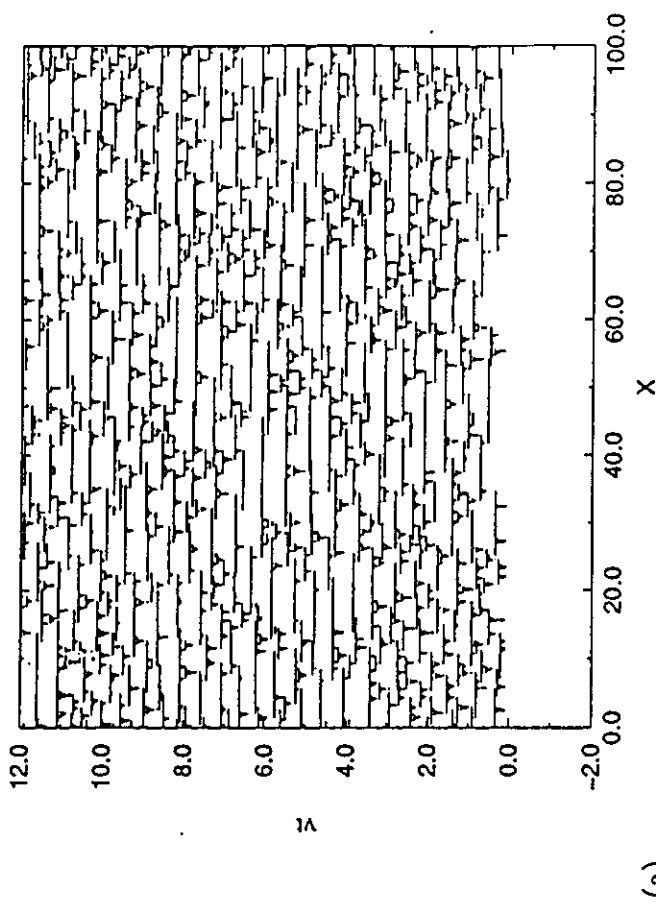
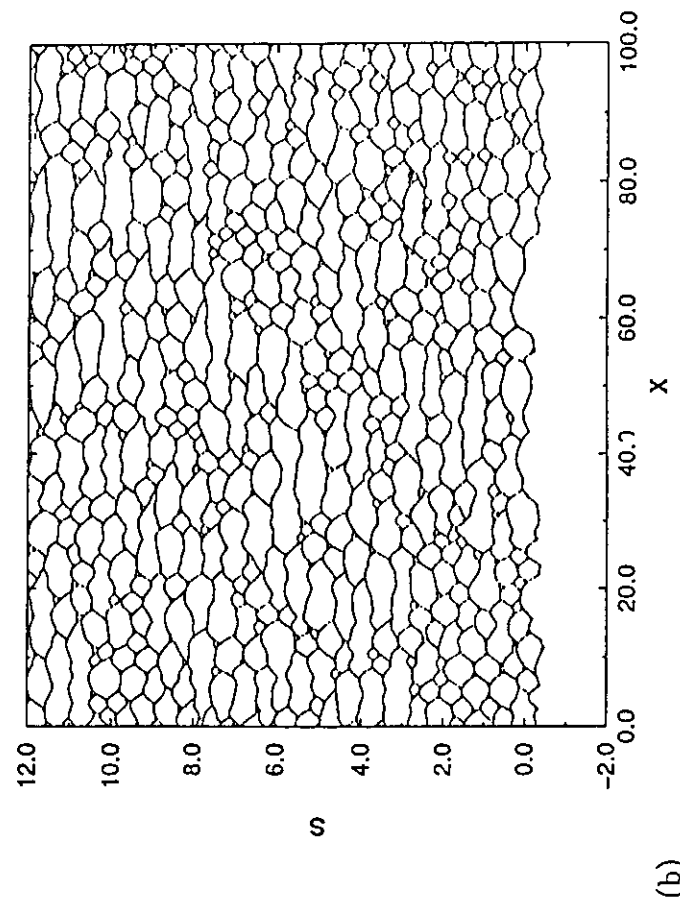


Figure 5. Slip versus depth at 2-year interval in dynamic simulation of long history with the "aging/slowness" version of the rate- and state-dependent friction. The creep process is active, p has step to 10 MPa offset from σ_n at 6 km and lithostatic gradient to 10 km, and $h = h^*/4$. The response shows slip complexities associated with the strong heterogeneities of p .



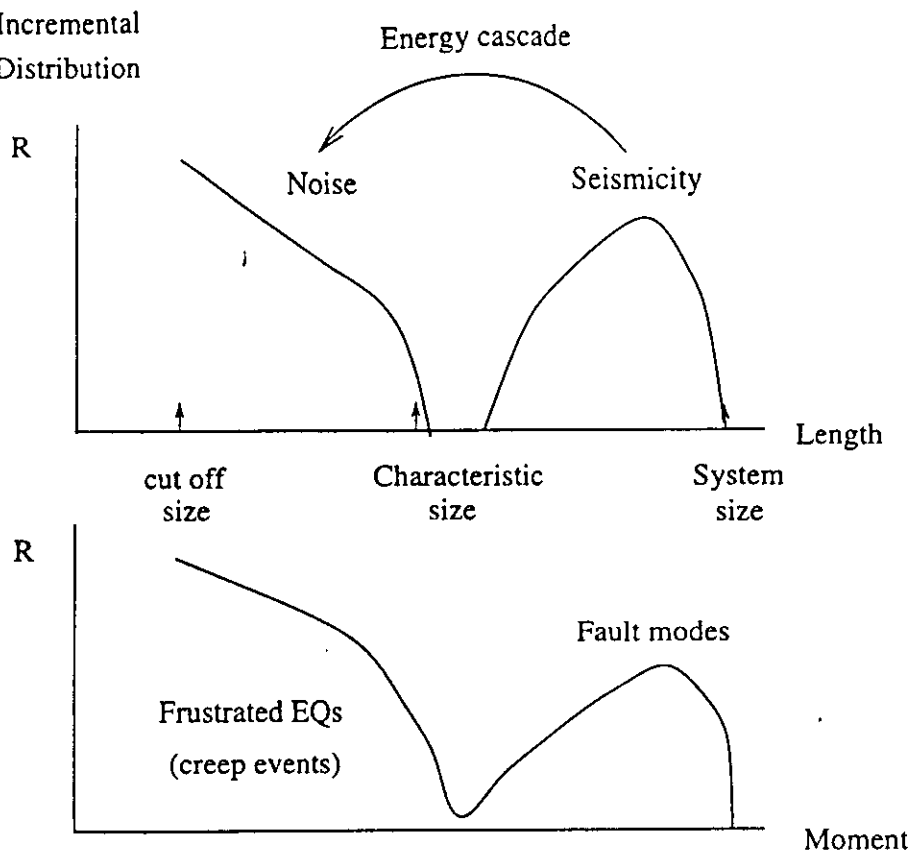
(a)



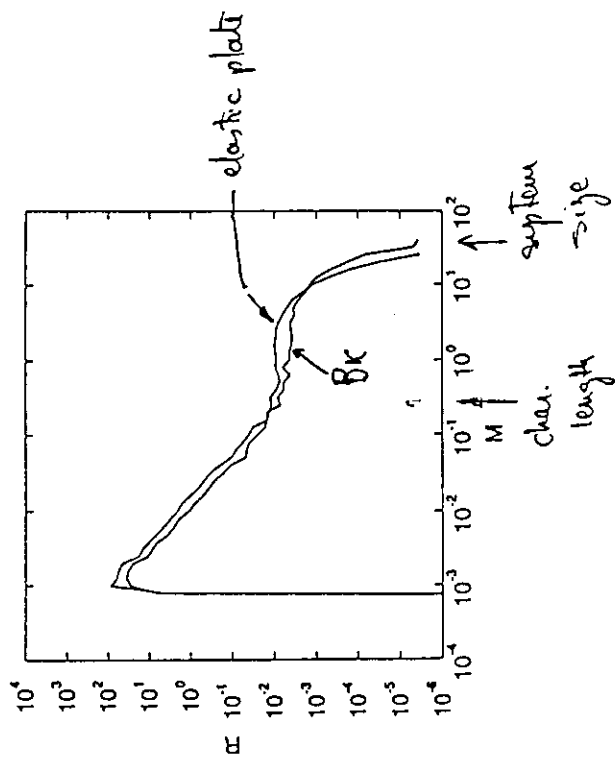
(b)

ADP System

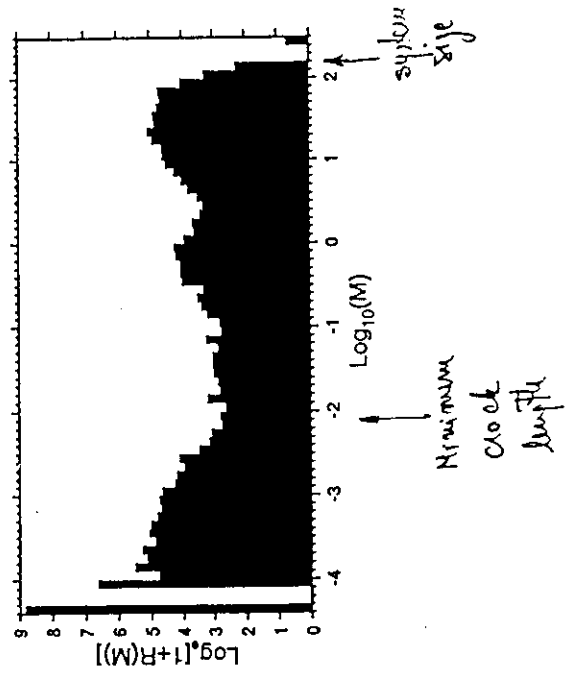
Incremental
Distribution

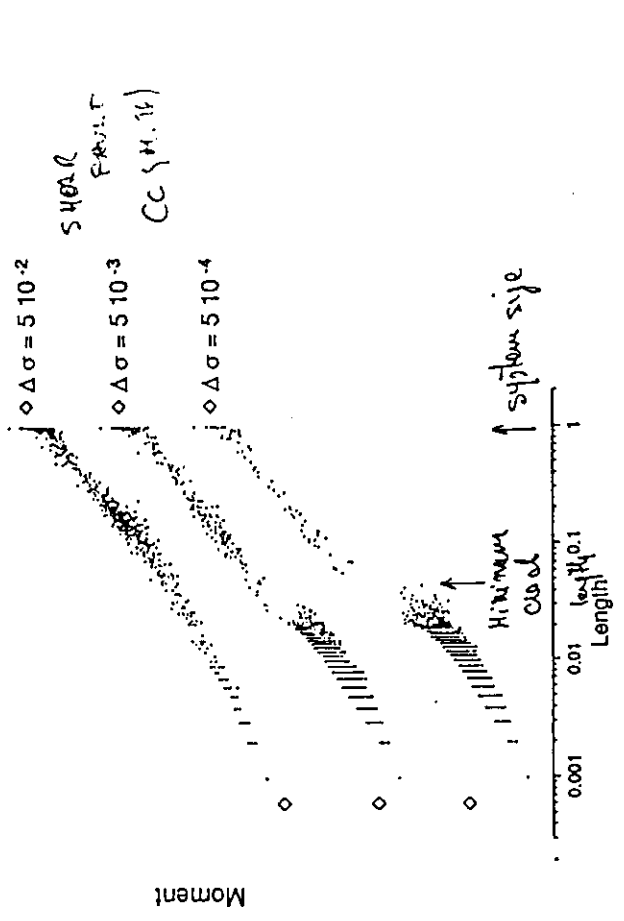
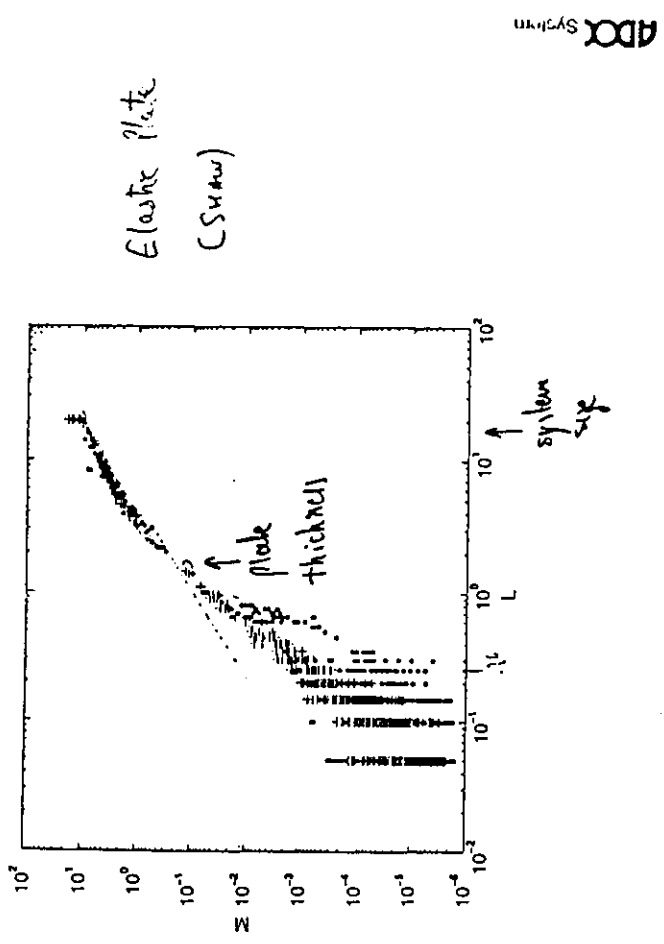
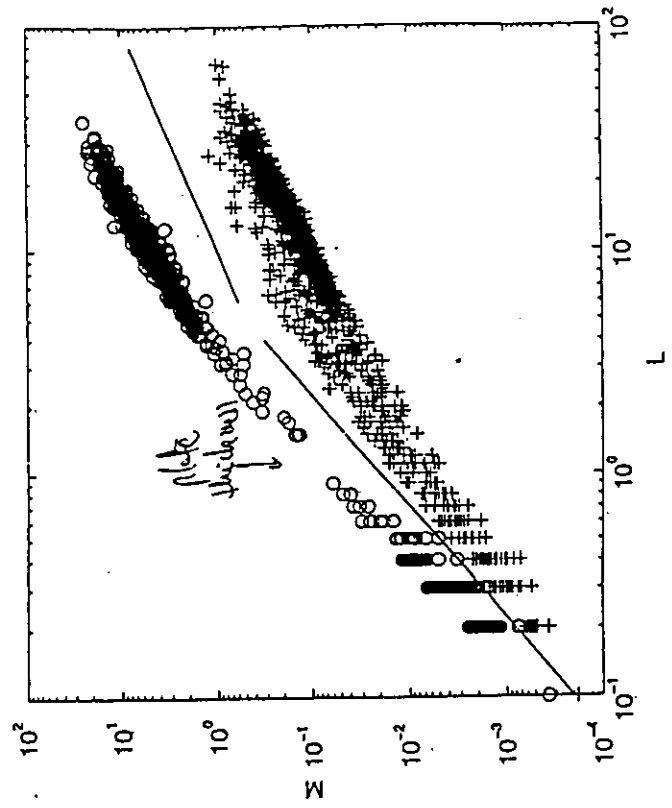
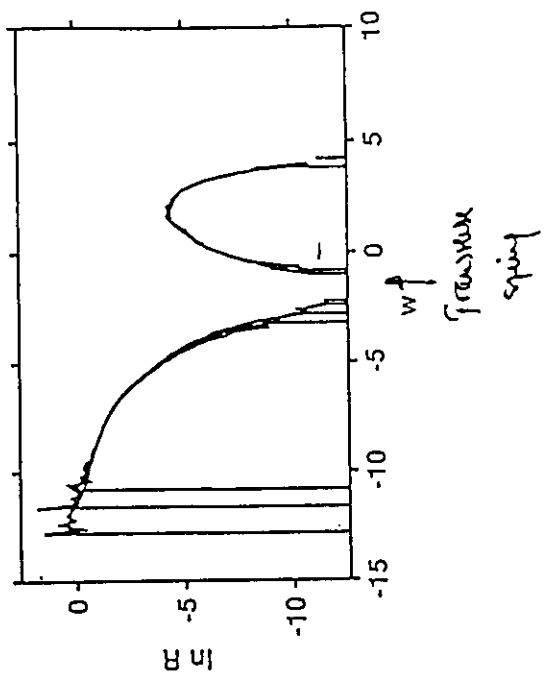


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ADX System

4 Towards realistic earthquake models

In this section we study in some detail the modeling of rupture in a very simple infinite linear elastic medium that contains a localized mature fault. Although this is not necessary, we assume for simplicity that the fault is a single planar surface completely embedded inside the elastic medium. This problem has no exact solutions except for the so-called self-similar circular fault studied by Kostrov in his classical papers of 1964-1973.

We study numerical solutions of the 3D elastic wave equation:

$$\rho \frac{\partial^2}{\partial t^2} \mathbf{u} = \nabla \cdot \sigma \quad (12)$$

where $\mathbf{u}(\mathbf{x}, t)$ is the displacement vector field, a function of both position $\mathbf{x}$ and time t , and $\rho(\mathbf{x})$ is the density of the elastic medium. Associated with the displacement field $\mathbf{u}$ we have the stress tensor $\sigma(\mathbf{x}, t)$ defined by

$$\sigma = \lambda \nabla \cdot \mathbf{u} \mathbf{I} + \mu [(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T], \quad (13)$$

where $\lambda(\mathbf{x})$ and $\mu(\mathbf{x})$ are Lamé's elastic constants. We can transform this system into a more symmetric velocity-stress formulation (Madariaga, 1976; Virieux and Madariaga, 1982; Virieux, 1986):

$$\begin{aligned} \rho \frac{\partial}{\partial t} \mathbf{v} &= \nabla \cdot \sigma + \mathbf{f} \\ \frac{\partial}{\partial t} \sigma &= \lambda \nabla \cdot \mathbf{v} \mathbf{I} + \mu [(\nabla \mathbf{v}) + (\nabla \mathbf{v})^T] + \dot{\mathbf{m}}, \end{aligned} \quad (14)$$

where $\mathbf{v}(\mathbf{x}, t)$ is the particle velocity vector; and $\mathbf{f}(\mathbf{x})$ and $\mathbf{m}(\mathbf{x})$ are the force and moment source distributions, respectively.

Slip Boundary Conditions on the Fault

For simplicity and due to present limitations of the numerical method, we assume that the fault is a flat horizontal plane perpendicular to the z -axis in the elastic medium. Due to frictional instability a rupture zone can spread along the fault; let $\Gamma(t)$ be this rupture zone at time t . The kinematic inversion studies cited in the introduction show that $\Gamma(t)$ will usually be a collection of one or more rupture zones propagating along the fault.

The main feature of a seismic rupture is that, at any point $\mathbf{x}$ inside the rupture zone $\Gamma(t)$, displacement and particle velocities are discontinuous. Let

$$\mathbf{D}(\mathbf{x}, t) = \mathbf{u}^+(\mathbf{x}^+, t) - \mathbf{u}^-(\mathbf{x}^-, t) \quad (15)$$

be the slip vector across the fault, i.e. the jump in displacement between the positive and the negative side of the fault. The notation $\mathbf{x}^\pm$ indicates a point immediately above or below the fault, and $\mathbf{u}^\pm$ are the corresponding displacements. In the following we assume that shear faults do not open up because of the strong normal stress component, i.e., slip

has only two components different from zero (i.e. $\mathbf{D} = [D_x, D_y, 0]$), but this restriction could be easily removed.

Slip is associated through the solution of the wave equation (14) to a change in the traction $\mathbf{T} = \boldsymbol{\sigma} \cdot \mathbf{e}_z = [\sigma_{zx}, \sigma_{yz}, \sigma_{zz}]$ across the fault:

$$\Delta \mathbf{T}(\mathbf{x}, t) = \Delta \boldsymbol{\Sigma}[\mathbf{D}] \quad \text{for} \quad \mathbf{x} \in \Gamma(t), \quad (16)$$

where $\Delta \boldsymbol{\Sigma}[\mathbf{D}]$ is a complex singular functional of $\mathbf{D}$ and its time and space derivatives. The operator $\Delta \boldsymbol{\Sigma}$ has to be computed explicitly in the BIE methods (see, e.g., Geubelle and Rice, 1995; Fukuyama and Madariaga, 1995, 1997), but not in FD since in this case $\Delta \mathbf{T}$ is obtained numerically. In fact, all we need is a numerical procedure that computes the stress change $\Delta \mathbf{T}$ given the slip $\mathbf{D}$ distribution at earlier times. The boundary condition in (16) imposes no *a priori* conditions on the signs of neither $\Delta \mathbf{T}$ nor $\mathbf{D}$, except that there should be no interpenetration of matter.

Friction Laws

The main assumption in seismic source dynamics is that traction across the fault is related to slip at the same point through a friction law that for our purposes can be expressed in the functional form

$$\mathbf{T}(\mathbf{D}, \dot{\mathbf{D}}, \theta_i) = \mathbf{T}_{\text{total}} \quad \text{for} \quad \mathbf{x} \in \Gamma(t). \quad (17)$$

Friction $\mathbf{T}$ is a function of at least slip, but an increasing amount of experimental evidence shows that it is also a function of slip rate $\dot{\mathbf{D}}$ and several state variables denoted by $\theta_i, i = 1 \dots N$. For more details see Dieterich (1978, 1979), Rice and Ruina (1983), but see also Ohnaka (1996) for an alternative point of view.

The traction that appears in friction laws is the total traction $\mathbf{T}_{\text{total}}$ on the fault which can be expressed as the sum of a preexisting field stress $\mathbf{T}^0(\mathbf{x})$ and the stress change $\Delta \mathbf{T}$ due to slip on the fault obtained from (16). The pre-stress is caused by tectonic load of the fault and will usually be a combination of purely tectonic loads due to internal plate deformation, plate motion etc, and the residual stress field remaining from previous seismic events on the fault and its vicinity. In the rest of this paper we assume that the preexisting stress is known.

Using (16) we can now explicitly formulate the friction law on the fault (17):

$$\mathbf{T}(\mathbf{D}, \dot{\mathbf{D}}, \theta_i) = \mathbf{T}^0(\mathbf{x}) + \Delta \mathbf{T}(\mathbf{x}, t) \quad \text{for} \quad \mathbf{x} \in \Gamma(t). \quad (18)$$

Friction as defined by (18) is clearly a vector. For the appropriate study of a shear fault we need to write (18) as a system of two equations. In spite of the importance of friction for earthquake dynamics it has rarely been studied for a fault that extends in two dimensions inside a 3D elastic medium. Here, following earlier work by Archuleta and Day (1990), Day (1982) and Spudich (1992) we choose to use a very simple approach that will certainly have to be refined in the future, assuming that slip rate and traction are anti-parallel, i.e.,

$$\mathbf{T}(\mathbf{D}, \dot{\mathbf{D}}, \theta_i) = -T(D, \dot{D}, \theta_i) \mathbf{e}_v, \quad (19)$$

where $\mathbf{e}_V = \dot{\mathbf{D}}/\|\dot{\mathbf{D}}\|$ is a unit vector in the direction of instantaneous slip rate. With this assumption, the boundary condition reduces (18) to the special form

$$-T(D, \dot{D}, \theta_i)\mathbf{e}_V = \mathbf{T}^0(\mathbf{x}) + \Delta\mathbf{T}(\mathbf{x}, t) \quad \text{for} \quad \mathbf{x} \in \Gamma(t). \quad (20)$$

Figure ?? (left) shows the vector diagram implied by this equation. The only fixed vector in this diagram is the prestress that is known. Friction and slip rate are collinear but anti-parallel. Stress change $\Delta\mathbf{T}$ is in general neither collinear with pre-stress nor with friction. We know of no other condition that can be applied to this diagram. Dissipation is insured by the anti-parallelism of friction and slip rate. In order to compare with previous results we also implemented a simpler version of friction that we call the “scalar boundary condition”. We assume that slip is only allowed in the direction of the initial stress which is everywhere parallel to the x -axis, i.e., $\mathbf{T}^0(\mathbf{x}) = [T_x^0(\mathbf{x}), 0]$ and $\mathbf{D}(\mathbf{x}, t) = [D_x(\mathbf{x}, t), 0]$,

$$\Delta T_x(\mathbf{x}, t)n = T(D, \dot{D}, \theta_i) - T_x^0(\mathbf{x}) \quad \text{for} \quad \mathbf{x} \in \Gamma(t) \quad (21)$$

This boundary condition, that may be graphically described as if the fault had “rails” aligned in the x direction, has been applied in most 3D source models starting with Madariaga (1976).

Both boundary conditions (20) and (21) require a friction law that relates scalar traction T to slip, its derivatives and possible state variables. Although any realistic friction law can be implemented with our method, in our study of the Landers earthquake of June 28, 1992 we used what we consider to be the simplest realistic friction law: the slip weakening model that we used earlier with the block-nd-spring slider (see, eq 5). In this friction law, slip is zero until the total stress reaches a peak value (yield stress) that we denote with T_u . Once this stress has been reached, slip D starts to increase from zero and $T(D)$ decreases linearly to zero as slip increases:

$$\begin{aligned} T(D) &= T_u \left(1 - \frac{D}{D_0}\right) & \text{for} \quad D < D_0 \\ T(D) &= 0 & \text{for} \quad D > D_0, \end{aligned} \quad (22)$$

where D_0 is a characteristic slip distance. The notation in this example is somewhat different from that in (5). But the analogy would be clearer if we noticed that $\Delta\sigma = T_u$ and that $\sigma_d = 0$ in (22). This friction law has been used in numerical simulations of rupture by Andrews (1976), Day (1982b), Harris and Day (1993) and many others. Slip weakening at small slip is *absolutely necessary for the friction law to be realizable*, otherwise stress σ_z becomes infinite at the rupture front, so that seismic ruptures would spread at either S, Rayleigh or P wave velocities until they stop. Of course, in numerical implementations stress is never infinite so that rupture velocity is numerically limited. This is the so-called Irwin criterion described by Das and Aki (1977) and used by many authors since their publication. In our simulations we want to avoid numerical control of the rupture front by all means, so that we have studied in detail the values of D_0 that render the numerical method independent of grid size.

It is well known that the slip-weakening friction model is an oversimplification of actual friction laws. As shown by Dieterich (1978, 1979), Rice and Ruina (1983) and Ruina (1983) a realistic friction law should incorporate slip rate sensitivity and aging as a

minimum. A friction law that has all these characteristics is the Ruina-Dieterich friction law already discussed with the single degree of freedom oscillator. Although it is possible to use their friction laws in our numerical computations, this would require the integration of additional equations for the state variables and the cost would increase enormously. It is my firm belief (not necessarily shared by other authors) that

Dimensional Analysis

In all our simulations we used non-dimensional variables. This has the advantage of clearly showing how different variables scale with stresses and distances. We choose the following dimensional variables:

- Distances along the fault are measured in units of Δx , the grid interval.
- Wave velocities are measured in units of β , the shear wave velocity.
- Stress is measured in units of T_u , the peak frictional resistance (yield stress) in the the friction laws described in (22) and (??).

All other dimensions are determined by the previous three. In particular

- Time is measured in units of $\Delta t = H\Delta x/\beta$ where β is the shear wave velocity. H is the so-called CFL (Courant-Friedrich-Lewy) parameter that controls stability of the numerical method. In our simulations it was usually taken as 0.25 in order to insure stability and good accuracy.
- Displacement is measured in units of $T_u/\mu \times \Delta x$.
- Particle velocities are measured in units of $T_u/\mu \times \beta$.

Slip and slip rate are normalized by $2T_u/\mu \times \Delta x$ and $2T_u/\mu \times \beta$. The factor of 2 is not really necessary, but we decided to follow a tradition in seismological publications. We also assume in the numerical computations that the P-wave velocity α is equal to $\sqrt{3}\beta$. Finally, D_0 , the slip weakening distance in (22) is measured in units of slip (i.e. $2T_u/\mu \times \Delta x$), and V_0 , the rate weakening parameter, is measured in units of slip rate (i.e., $2T_u/\mu \times \beta$).

It is important to remark that we used the more traditional choice Δx , the grid interval, as unit of length. We preferred Δx to a physical length such as the slip-weakening distance D_0 as unit of displacement since, as discussed below, the relation between D_0 and distances along the fault is dependent on load and rupture history.

Scaling of Rupture with the Slip-Weakening Distance D_0

An essential requirement for an accurate numerical method is that the numerical solution becomes independent of grid size beyond the use of a certain number of grid points per wavelength. In our problem the shortest physical distances are the radius of the asperity R and the width of the rupture front. The latter depends on the slip-weakening distance D_0 as shown by Ida (1972) and Andrews (1976). For 2D faults and for the slip weakening law (22) this width, L_c , is

$$L_c = \frac{4\mu}{3\pi} \frac{T_u}{T_{ext}^2} D_0 \quad (23)$$

We have assumed as in (22) that the residual friction at high slip rates is $T_f = 0$. This expression is valid for a constant stress level T_{ext} outside the asperity. For a stress distribution that scales only with the size of the grid, the fault scales with D_0 . However, it would be convenient to scale our numerical computations by L_c instead of D_0 , but unfortunately (23) depends on the load through the initial stress field T_{ext} . If this external stress varies spatially, the denominator has to be replaced by a complex expression that contains the stress intensity. Furthermore, (23) is only valid asymptotically, when the crack tip is very sharp. For these reasons we decided to use Δx and not D_0 as length scale in our numerical simulations.

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Dynamic friction and the origin of the complexity of earthquake sources

RAÚL MADARIAGA* AND ALAIN COCHARD

Département de Sismologie, Centre National de la Recherche Scientifique Unité Associée 195, Institut de Physique du Globe de Paris et Université Paris 7, case 89, 4 Place Jussieu, 75252 Paris Cedex 05, France

ABSTRACT We study a simple antiplane fault of finite length embedded in a homogeneous isotropic elastic solid to understand the origin of seismic source heterogeneity in the presence of nonlinear rate- and state-dependent friction. All the mechanical properties of the medium and friction are assumed homogeneous. Friction includes a characteristic length that is longer than the grid size so that our models have a well-defined continuum limit. Starting from a heterogeneous initial stress distribution, we apply a slowly increasing uniform stress load far from the fault and we simulate the seismicity for a few 1000 events. The style of seismicity produced by this model is determined by a control parameter associated with the degree of rate dependence of friction. For classical friction models with rate-independent friction, no complexity appears and seismicity is perfectly periodic. For weakly rate-dependent friction, large ruptures are still periodic, but small seismicity becomes increasingly nonstationary. When friction is highly rate-dependent, seismicity becomes nonperiodic and ruptures of all sizes occur inside the fault. Highly rate-dependent friction destabilizes the healing process producing premature healing of slip and partial stress drop. Partial stress drop produces large variations in the state of stress that in turn produce earthquakes of different sizes. Similar results have been found by other authors using the Burridge and Knopoff model. We conjecture that all models in which static stress drop is only a fraction of the dynamic stress drop produce stress heterogeneity.

A fundamental problem for understanding seismicity and the nature of large earthquakes is the origin of stress and slip complexity on seismic faults. Although seismicity has been carefully studied, the dynamics of the observed complexity of seismic sources is poorly understood although it is generally considered to be mainly due to fault segmentation. Initial work on seismic source dynamics (1–3) was derived from models of mode I fracture. In mode I, residual stress after opening a crack is obviously zero. This assumption had the virtue of simplifying the solution of the crack problems and allowed us to calculate radiation (4) and compare it with empirical models of radiation (5). At the end of the 1970s, the first models of complexity (asperities and barriers) were proposed but their relation to seismicity was not explored (6–8). A fundamental question that was left unanswered in these models was how do asperities arise on a fault loaded by very slow tectonic forces? According to current models, asperities are due to preseismic slip and foreshocks that concentrate stresses on locked areas of the fault; so that it is not enough to consider individual events, but the complete seismicity of the fault before a large event has

to be studied in detail. To do this we have to specify the state of stress before and after an earthquake. In most of the previous work on earthquake dynamics as reviewed by Kostrov and Das (9), it was invariably assumed that residual stress after any event was uniform and equal to zero. This is very unlikely as we have attempted to show in our recent work (10, 11).

The question of the origin of complexity was posed from a completely different point of view by research on self-organized criticality (12), and on the artificial seismicity of the box and spring model of Burridge and Knopoff (13–18). These studies suggested that stress and slip would become spontaneously heterogeneous because of nonlinear instabilities in the mechanical model of a frictional fault. These results led to an interesting controversy because quasi-static models studied by Rice using the experimentally derived friction laws by Dieterich (19, 20) and Ruina (21) did not give rise to any significant heterogeneity. Rice and coworkers (22, 23) suggested that heterogeneity may be due to the lack of a continuum limit of some of these models.

Without waiting for the clarification of nature of heterogeneity of problems for the Burridge and Knopoff models, we started a study of a simple antiplane fault model (10, 11) where the only change with respect to Kostrov's original formulation (3) was that the friction law was assumed to be rate dependent, as suggested by Carlson and Langer (14). In the present paper, we present results for a simple fault with rate-dependent friction that has been regularized at small scales by using a slip-weakening zone (24). We will demonstrate that depending on the value of a single parameter that measures the rate dependence of friction, we either produce complexity or suppress it. We are of course quite aware that our fault models are very crude and lack many complexities of actual faults; yet some of them have a large number of the features observed in actual seismicity including slip and stress complexity, events of many different sizes, variable recurrence times, etc.

Definition of the Model

Let us consider, as in Fig. 1, a two-dimensional antiplane crack of length $2L$ embedded in a linear elastic medium of elastic constant μ and mass density ρ . The elastic medium is subject to a slowly increasing uniform shear stress $\sigma_{yz}(x, y, t) = T_c(t)$, due to slow plate motion or intraplate deformation. As the external stress, $T_c(t)$, increases with time, the frictional resistance to slip between the walls of the fault is eventually overcome, and slip occurs. The amount of slip and the history of slip on the fault depends on the nonlinear friction law that relates shear stress transmitted across the fault $T(x, 0, t)$, to slip $D(x, 0, t)$, slip velocity $V(x, 0, t) = \dot{D}(x, 0, t)$, and possibly to certain state variables $\theta_i(x, 0, t)$. The exact nature of this

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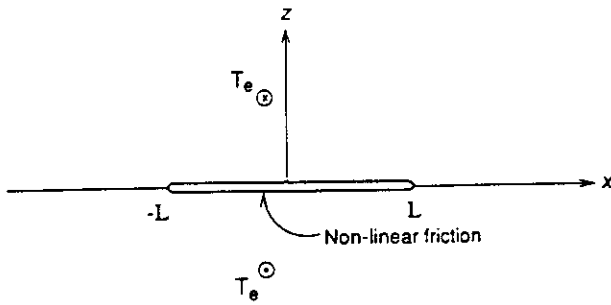


FIG. 1. Simple model of an antiplane fault in a homogeneous linear elastic medium. The fault extends from $-L$ to L and is clamped at the ends by unbreakable barriers. The fault is driven by a slowly increasing homogeneous shear stress field. The only possible source of heterogeneity in this problem is the nonlinear frictional interaction between the walls of the fault.

friction law will be discussed after we set up the elastodynamical problem.

We have assumed antiplane slip, so that the only component of displacement that we consider is $u = u_y$ and the relevant traction component across the fault is $T = -\sigma_{yz}$. Thus, u satisfies the wave equation

$$\frac{1}{\beta^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}, \quad [1]$$

where β is the shear wave speed. By using the symmetry about the fault plane $z = 0$, we find the appropriate boundary conditions for the solution of the problem in the upper half-space ($z > 0$):

$$u(x, 0^+, t) = 0 \quad \text{on } -\infty < x < -\ell_1(t) \text{ and } \ell_2(t) < x < \infty \quad [2]$$

outside the slipping part of the fault delimited by $\ell_1(t)$ and $\ell_2(t)$,

$$T(x, 0^+, t) = T_c(t) + T(D, V, \theta) \quad \text{on } -\ell_1(t) < x < \ell_2(t) \quad [3]$$

inside the fault. $T(D, V, \theta)$ is the friction law. This problem is then a typical crack problem with mixed boundary conditions, which consists in finding the slip $D(x, 0, t) = 2u(x, 0^+, t)$ on the fault.

In spite of the apparent simplicity of this crack problem, in most cases it is intractable by analytical methods because of the mixed boundary conditions (Eqs. 2 and 3). As discussed (25), finite differences are too inaccurate because of high frequency dispersion, so that we adopted a boundary integral equation method that we believe is the most appropriate for solving this kind of crack problem (26, 27). The appropriate boundary integral equation (10) is

$$T(D, V, \theta) = T_c(x) - \frac{\mu}{2\beta} V + S, \quad [4]$$

where S is the space-time convolution integral

$$S(x, t) = \frac{\mu}{2\pi} \int_0^t \int_{-\beta\tau}^{\beta\tau} K(\xi, \tau) \frac{\partial}{\partial \xi} \Delta u(x - \xi, t - \tau) d\xi d\tau, \quad [5]$$

with $K(x, t) = \sqrt{t^2 - x^2/\beta^2}/(xt)H(t - \|x\|/\beta)$. The convolution over space has a Cauchy-like singularity at $\xi = x$. The convolution over time, on the other hand, is regular because $K(x, t) \rightarrow 0$ when $x \rightarrow 0$. The details of the numerical method used to solve this problem and its implementation in a CM5 parallel computer have been discussed (10). Thus it is sufficient to say that Eq. 4 is a nonlinear algebraic equation for the simultaneous solution of D , V , T , and θ .

Let us remark that the fault is assumed to be clamped at the ends by unbreakable barriers. This boundary condition is different from the periodic conditions used by Carlson and Langer (14). To apply periodic boundary conditions, we would have to change the Green functions used to derive Eq. 4.

Friction Law

The nature of the solution of Eq. 4 is completely determined by the friction law $T(D, V, \theta)$. Extensive work on rock friction has been discussed by Dieterich (28) and does not need to be repeated here. Experimental evidence (20, 21, 29) shows that friction laws at low slip rates should at least include three elements: (i) direct stress change for rapid increases in slip velocity, (ii) an intrinsic time constant for the response to abrupt changes in velocity, and (iii) velocity weakening at steady-state slip.

At present we are unable to introduce such laws in our numerical method because they were proposed for very low values of slip velocity, so that the solution of the boundary integral equation problem would have to take into account slip rates varying over more than six orders of magnitude (i.e., from $\mu\text{m/s}$ for fault creep up to m/s for seismic slip). Since the time step for the solution of Eq. 4 is controlled by the faster time scales present in it, the slow evolution of the fault in the interseismic period would have to be computed at the time steps appropriate for the dynamic regime, which is several orders of magnitude less than those that are actually needed. Equations of this type are called stiff and require special techniques for their solution as shown by Tse and Rice (30) for a spring-loaded massive slider. A possible method to increase the time steps during the interseismic period would be to modify Eq. 4 at low speeds by using its quasi-static approximation as proposed by Perrin *et al.* (31), but we have not implemented this option yet.

Because of the difficulties discussed in the previous paragraph, we have used a simplified friction law. The most sweeping assumption is that the fault is perfectly locked during the inter-seismic period so that successive events can be treated independently. In the Dieterich-Ruina rate- and state-dependent laws, slip occurs at all times no matter how small the total stress. In the period when the fault is locked, we assume that there is no slip on the fault, so that stress increases steadily during the continuous load $T_c(t)$. When the total stress on a certain point of the fault reaches a threshold T_u , slip begins and we immediately switch to the dynamic equation (Eq. 4).

The particular friction law assumed in our computations is

$$\begin{aligned} T(D, V, \theta) &= T_u(1 - D/U_0) \quad \text{for } D < U_0 \\ T(D, V, \theta) &= 0 \quad \text{for } D > U_0 \text{ and } \theta > \theta_0 \\ T(D, V, \theta) &= T_{sp}(1 - \theta/\theta_0) \quad \text{for } D > U_0 \text{ and } \theta < \theta_0, \end{aligned} \quad [6]$$

to which we add the following evolution equation for θ

$$\frac{d\theta}{dt} = -\frac{1}{\tau_0}(\theta - V), \quad [7]$$

where D is slip measured from the beginning of the current slip episode, θ is a state variable, and V is instantaneous slip velocity. At steady state, θ is equal to the slip rate, so that θ differs from slip rate V only during fast rate changes. The first-order evolution equation for θ introduces a relaxation time scale τ_0 and a corresponding relaxation length $D_c = \beta\tau_0$. These are the small scale parameters needed to regularize the fault model at small scales and avoid the intrinsic discreteness of our previous models (10). The dependence of friction on slip and steady-state slip rate (θ) is better appreciated in Fig. 2. As slip begins, there is a slip-weakening process that simulates

energy dissipation near the rupture front. U_0 controls the minimum size of a fault patch before it becomes unstable. As shown by Dieterich (32) and Ohnaka (33), this is an observable feature of realistic friction laws. In the state- and rate-dependent laws, slip-weakening is produced by the direct change of stress due to rapid increases in slip rate. Dynamic simulations of simple crack models by Okubo (34) showed that rate- and state-dependent laws behave in a very similar way to slip-weakening models near the rupture front.

The other main feature of the friction law described in Fig. 2 is slip rate weakening at low slip rates. This is a more controversial feature that has been included in the rate- and state-dependent friction laws in the form of a logarithmic decrease of friction with increasing velocity. In the present study, we assume a linearly decreasing dynamic friction as a function of increasing steady-state slip rate to simulate the simpler frictional models used by Carlson and Langer (14). They assumed that friction was a simple function of slip rate only:

$$T(D, V, \theta) = T_{sp} \frac{V_0}{V_0 + V} \quad \text{for all } V, \quad [8]$$

so that stress increased as the slip rate decreased. Rate dependence may destabilize healing depending on the value of T_{sp} (11). For small T_{sp} , the friction law is not very different from the usual rate-independent friction. For large values of T_{sp} , the destabilizing effect is more pronounced and spontaneous heterogeneity develops for large values of this parameter.

That friction decreases with slip rate seems to be a well documented feature of many frictions including those of Dieterich (19, 20) and Ruina (21). In these friction laws, steady-state frictional stress is proportional to a logarithm of steady-state slip rate of the form $T_{ss}(V_{ss}) = -a \log(V_{ss})$, so that frictional stress decreases more mildly as a function of slip rate than in the model of Carlson and Langer (14).

Fig. 2 also shows the trajectory of a point on the fault in the slip-steady-state-slip-rate space. As slip begins, stress slowly decreases transferring load to neighboring points on the fault. Simultaneously slip velocity increases rapidly because any stress relaxation is initially compensated by increases in slip velocity. The time scale of this rapid velocity increase is controlled by the relaxation time τ_0 and the slip-weakening time U_0/v_R , where v_R is the rupture front speed. In our computations, these two time scales are of the same order as they are also in the slip- and rate-dependent models. After this brief time, the fault slips freely at high slip rates. Slip rate becomes determinant for the fate of the fault only when the slip rate is less than the threshold V_0 (which corresponds to θ_0). At that point, stress increases as slip rate decreases, further driving the local point on the fault toward locking. This locking process is a highly nonlinear feature of the friction law that is entirely due to local conditions on the fault and is completely

independent of the propagation of stopping phases or other signals along the fault.

Simulation of Seismicity on a Finite Fault

Let us now study the seismicity on the fault model that we have just described. In the following simulations, all parameters will be rendered nondimensional by the following choices: stress is scaled by μ , the rigidity. We choose $T_u/\mu = 1$ as the peak traction resistance on the fault. We scale length by the unit step Δx used to discretize the fault. The total fault length $L/\Delta x = 511$ in most of the simulations presented here. Time is scaled as $t \times \beta/\Delta x$, slip scales are scaled as $D \times \mu/(2T_u \times \Delta x)$, and slip rate is scaled as $V \times \mu/(\beta \times 2T_u)$. The shortest nondimensional time scale admissible in our computations was 0.5 so that the Courant parameter of our numerical solutions is also $h = 0.5$. We could have used the maximum stable value $h = 1$, but we prefer to stay on the safer side. All the other parameters appear in the friction law and were adopted as follows: $U_0 = 10$, $\tau_0 = 3$, $\theta_0 = 1$, and T_{sp} will be varied as explained later. Let us remark that when $T_{sp} = 0$, we get the classical Coulomb friction law with initial slip-weakening. Computations using this law were extensively studied by Andrews (26) and produce perfectly repeatable earthquake sequences in our computations. For $T_{sp} = 1$, the final stress is equal to the peak stress on the fault as assumed in the original work by Carlson and Langer (14). In this case, events can be artificially triggered by relaxing a point on the fault. We consider these two values as extreme models of slip rate sensitivity of friction. As we will show later, a transition in the behavior of the fault occurs around $T_{sp} = 0.6$.

Let us emphasize that in all our calculations the material parameters μ , β , T_u , τ_0 , θ_0 , and U_0 are considered to be uniform along the fault. The presence or lack of heterogeneity of stress and slip will be entirely due to the nonlinear features of the friction law (Eq. 6).

The slip-weakening parameter U_0 determines the shortest scale in our problem. Let us define the total length of an event ℓ as the total number of fault elements involved in the rupture. In our simulations, events occur at scales ℓ shorter and longer than U_0 . Only the larger events are considered as seismic events; small events, such that $\ell < U_0$, never develop the fast rates typical of seismic events and friction never drops to 0 as allowed by the friction law of Fig. 2. These events would disappear if we could include fault creep in our model.

Simulation of a Relatively Smooth Fault ($T_{sp} = 0.4$). Let us consider first a model that is just below the critical value of partial stress drop. In Fig. 3, we show the velocity and stress field for a typical large event in this model. Because of limited resolution, we have decimated the output so that Fig. 3 (and also Fig. 4) presents only 1 line in 10. Although the slip velocity field does not resemble the classical self-similar crack models of Burridge (1) or Madariaga (4) much, slip continues for a long time after the passage of the rupture front. A weak stopping phase is clearly observed traveling across the fault, so that that slip duration is controlled by the total length of the slip zone. An interesting feature of the slip velocity field is the variation of velocity intensity near the rupture front. After a period of growth, the velocity peak decreases temporarily near the center of the fault. This is quite different from models with constant stress drop in which velocity intensity grows like the square root of distance from the nucleation point. This is a clear manifestation of incipient complexity. The stress field shows a clear decrease after the passage of the rupture front although several slip events leave a trace inside the fault. At the end of the rupture process, stress drops to a value close to 0.2 and is rather uniform. After reloading of the fault by the slow external stress field, the next event in the series will occur under a mildly complex stress distribution that is not enough

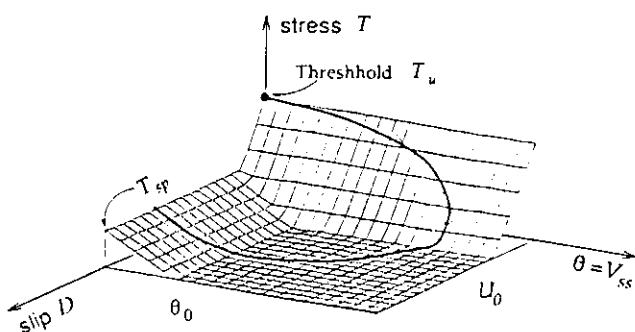


FIG. 2. Main features of the friction law assumed in this paper. Friction is represented as a function of slip (D) and state variable (θ). Transient slip rate may be different from θ in the course of rapid changes in slip rate.

Fig. 7 by the broken lines. Let us add to this distribution some heterogeneity and look at the slip and stress distribution, as shown in Fig. 7 with the solid lines. We observe that a modest change of the slip distribution produces large variations in the residual stress pattern. Since stress can only change inside a limited range ($0 < T < T_u$), these lateral variations put some fault elements closer to rupture than others. As a consequence, the future seismicity of the fault is completely determined by these heterogeneities in the residual stress field. Thus the mechanism that generates complexity in our model is clearly identified, although we do not know what is the bifurcation value for the control parameter T_{sp} . We propose that the key to the creation of heterogeneity in our models is partial stress drop. This is not at all a new concept in seismology; Brune (5) proposed from observational arguments that most earthquakes presented partial stress drop and suggested that the dynamic stress drop (T_d in our case) was much larger than the static stress drop. The main difference between our results and the suggestion by Brune (5) is that he considered a model where partial stress drop was uniform along the fault, while in our models a partial stress drop triggers strong lateral variations of the static stress drop.

Conclusions

We have demonstrated that, for certain highly rate-dependent friction laws, a simple antiplane fault embedded in a homogeneous medium can spontaneously become complex. This complexity has several interesting features:

(i) Premature locking of the fault, so that slip duration at any point of the fault is independent of the total size of the fault. Premature healing produces partial stress drop, so that stress heterogeneity may be simply due to the extreme sensitivity of fault stress to very small changes in the slip distribution.

(ii) Self-healing slip pulses are spontaneously generated for large values of T_{sp} . These pulses were proposed by Heaton (35) and they were explained in our previous work (10).

(iii) Stress heterogeneity and partial stress drop are manifestations of the same underlying instability. Partial stress drop occurs for all friction models that have a strong rate-dependent friction. Partial stress drop disorganizes the fault for the simple reason that stress drop of neighboring points will be highly variable.

(iv) Slip gradient (dislocation density) and stress heterogeneity appear when small scale modes of slip on the fault can express themselves. For full stress drop models like rate-independent friction laws, these small scale modes are suppressed by the requirement that the stress drop be fixed, uniform, and determined only by constitutive parameters. In that case, only material heterogeneity can produce complexity.

(v) Seismic events (i.e., events whose length is greater than the length of the slip-weakening zone) follow an L^2 scaling law in which seismic moment scales like the product of partial stress drop and the square of the length of the zone that actually slipped during the event. Thus, the regularization length that is included in our slip-weakening model has no influence on the properties of large seismic events. We find that the size of the slip pulse that traverses the fault determines the actual final size of the rupture. If the slip pulse is large, it simply reaches further along the fault.

(vi) Seismic-like events in our simulations are both periodic and simple for small values of the control parameter T_{sp} . For values larger than a critical value situated around $T_{sp} = 0.6$, we observe events of all sizes and a much higher rate of seismicity.

In conclusion, we have shown that a rate-dependent friction can spontaneously produce heterogeneity for large values of a control parameter. This limit corresponds to that of the friction law used by Carlson and Langer (14) in their study of the Burridge and Knopoff model. At the other extreme, the rate-independent friction suppresses these instabilities for the very simple reason that partial stress drop is eliminated from the outset. It is very likely that both material heterogeneities and dynamically generated complexity play a role in determining the observed complexity of faulting and seismic events. Given the little knowledge that we currently have about the friction laws at high slip rates, we firmly believe that heterogeneity should be explored without preconceived assumptions about which of material and dynamically generated heterogeneity dominates in the earth.

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of the high-temperature structural modifications of iron between 30 and 60 GPa. Funamori *et al.* (17) found γ iron to be stable up to 32 GPa and 1500 K. Saxena *et al.* (4, 12) suggested that iron underwent a phase transformation to β phase with δ hcp structure at 40 to 60 GPa. They recorded x-ray patterns on quenched samples previously heated with intentionally strong temperature gradients [see figure 2 in (12)]. In such an experimental procedure, samples were most likely subjected to strong stress fields that might bias structural determinations. Dubrovinsky *et al.* (18) recently provided experimental data supporting a δ hcp structure. The major evidence was the presence of 004 and 103 Bragg lines, which however disappeared with prolonged heating; thus raising the possibility that the reflections were from an unstable intermediate iron structure. Yoo *et al.* (5) recognized a metastable new polymorph at lower pressures and located it within the previously defined γ iron stability field. They did not observe a phase transformation when laser heating ϵ iron between 40 and 60 GPa. The absence of a phase transformation in this range disagrees with the other reports (2–4) and our study.

All of our spectra recorded at high-temperatures from 30 to 60 GPa and at 100 GPa correspond to the described orthorhombic lattice. The stability field of this polymorph thus appears similar to that proposed by Saxena *et al.* (4) for the β phase. We believe that the main difference between their study and our work lies in the interpretation of the diffraction patterns, and thus propose that the structure of the β

phase is the orthorhombic lattice. From our results, it appears that the stability field of γ iron should not extend much above 32 GPa, while that of β iron extends up to at least 100 GPa (Fig. 6).

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Three-Dimensional Dynamic Simulation of the 1992 Landers Earthquake

K. B. Olsen,* R. Madariaga, R. J. Archuleta

The 1992 Landers, California, earthquake (magnitude 7.3) was modeled as the propagation of a spontaneous rupture controlled by a realistic prestress distribution with the use of a three-dimensional finite-difference method. The dynamic rupture reproduces the general slip pattern used to compute the initial stress distribution and generates near-fault ground motions at the surface similar to observations. The simulated rupture propagates on the fault along a complex path with highly variable speed and rise time, changing the magnitude and pattern of the stress significantly. The method provides the framework to estimate earthquake rupture parameters from recorded seismic and geodetic data.

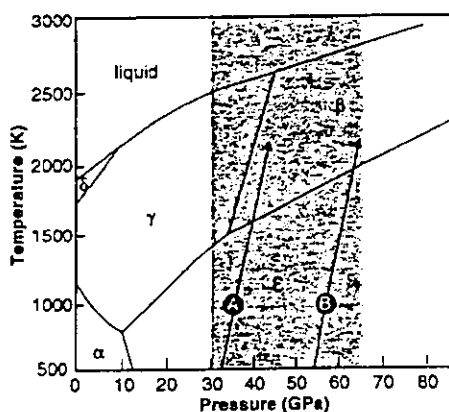


Fig. 6. Revised pressure-temperature phase diagram of iron [derived from (12)]. The A and B arrows are an estimate of the pressure-temperature path of iron during laser heating under pressure. We maintain β iron as a name for the newly determined orthorhombic structure. Melting curve was determined by Boehler *et al.* and Shen *et al.* (3, 19). Persistence of the γ phase at 32 GPa and 1500 K was indicated by in situ x-ray experiments (17).

The rupture of earthquakes and the resulting ground shaking have usually been studied from two complementary but different perspectives. The rupture is typically analyzed by seismologists interested in the initiation, propagation, and healing of earthquake ruptures using complicated models (1, 2), whereas the resulting ground shaking is generally computed in specific areas on the basis of a simple kinematic definition of

the source (3). Here, we combine the two efforts into a single integrated approach, including the computational efficiency of recent kinematic methods (4). We assume rupture on a planar, vertical fault, although it would be possible in principle to simulate more complex ruptures.

The main assumption in seismic source dynamics is that traction and slip, the relative displacement of one side of the fault with respect to the other, are related by a friction law across the fault zone. This friction law is a function of local rupture parameters including slip (D), slip rate ($\dot{D}$), and certain state variables (5) that describe the aging of fault contacts, fluid pressure, temperature, and several other slowly vary-

K. B. Olsen and R. J. Archuleta, Institute for Crustal Studies, University of California, Santa Barbara, CA 93106–1100, USA.
R. Madariaga, Laboratoire de Géologie, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05, France.

*To whom correspondence should be addressed.

ing properties of the fault. The total traction T , and therefore friction, on the fault can be expressed as the sum of a preexisting stress field T^0 and the stress change ΔT due to slip on the fault

$$T(D, \dot{D}, \theta) = T^0 + \Delta T \quad (1)$$

where θ refers to possible state variables. Prestress T^0 is generated by a combination of tectonic loads caused by plate motion and internal deformation and the residual stresses from earlier seismic events on the fault and its vicinity. We modeled a single event, assuming that the prestress is known. The change in traction ΔT is related through the solution of the wave equation to the history of slip on the fault.

We used a fourth-order staggered-grid, finite-difference solution to the three-dimensional elastic wave equation (6) to compute the stress change for a given slip history on the fault. Our scheme includes a free-surface boundary condition at the top and sponge layers (7) in addition to absorbing boundary conditions (8) at the remaining grid boundaries. The use of fourth-order, spatially accurate derivatives in the finite-difference scheme ensures that all stresses and velocities have the appropriate symmetries about the fault plane (9). This property reduces many of the numerical instabilities that have prevented accurate simulations of fault dynamics by the use of finite-difference methods (10).

We used a friction law for which slip is zero until the total stress reaches a critical yield value (T_0). Once this stress has been attained, slip rate (and therefore slip) increases rapidly from zero while traction across the fault decreases to zero (11),

$$T(D) = T_0 \left[1 - \frac{|D|}{D_0} \right] \text{ for } |D| < D_0$$

$$T(D) = 0 \quad \text{for } |D| > D_0 \quad (2)$$

Here D_0 is the so-called slip-weakening distance. The decrease of friction T with increasing slip $|D|$ is a universal property that is necessary for the friction law to be dissipative (12). If slip weakening is omitted, stress becomes infinite at the rupture front so that, after a numerical transient, seismic ruptures would continue propagating forever at exactly the terminal rupture velocity. The most important feature introduced by this friction law is a characteristic length D_0 that defines a minimum size for a stable rupture zone (13).

Although friction is a vector property, we assumed that slip occurs only in the direction parallel to the applied shear stress. In this way, slip becomes a scalar ($D = |D|$) in the simulations and reduces the number of rupture parameters needed in the model (14). This assumption is consistent with

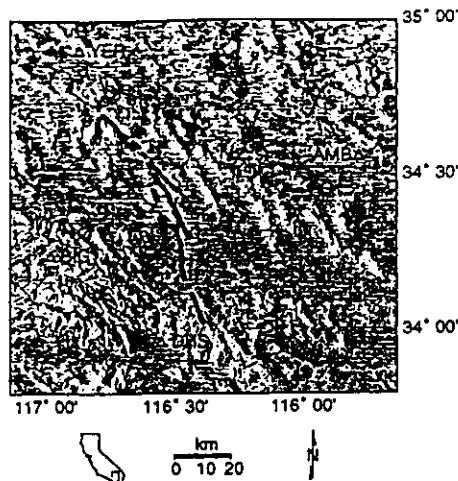


Fig. 1. Topographic map showing the surface rupture from the 1992 Landers earthquake. The fault trace is depicted by the black lines, and the triangles depict the location of stations at which we compared our modeling result with recorded seismograms.

kinematic source inversion results (15) and recent numerical experiments (16) suggesting that slip is predominantly aligned with the shear prestress, along the long dimension of a shallow strike-slip fault (simple shear).

The 28 June 1992 Landers earthquake (magnitude 7.3) occurred in a remote area of the Mojave Desert in southern California (Fig. 1), but the rupture process has been extensively studied (17) because of its large size, proximity to the southern California metropolitan areas, and wide coverage by seismic instruments. Several studies inverted for the rupture history of this event from a combination of seismograms, geodetic data, and geologic data (18–20). The overall kinematics of the seismic rupture are thought to be understood (18–20), making the Landers earthquake an appropriate test case for our dynamic modeling method.

The fault that ruptured during the Landers earthquake can be divided into three segments: the Landers–Johnson Valley (LJV) segment to the southeast, where the hypocenter was located; the Homestead Valley (HV) segment in the central part of the fault; and the Camp Rock–Emerson (CRE) segment to the northwest. For the numerical simulation, we replaced the three segments of the fault by a single 78-km-long vertical fault plane extending from the surface down to a depth of 15 km.

The most important parameter required for dynamic modeling is the initial stress on the fault before rupture starts; all other observable characteristics of the seismic rupture, including the motion of the rupture front, are determined by the friction law. To estimate the initial stress field, we used

Table 1. Three-dimensional modeling parameters.

Spatial discretization (m)	200
Temporal discretization (s)	0.013
Yield stress (bars)	120
Fault length (km)	78
Fault width (km)	15
Slip-weakening parameter (m)	0.8

Table 2. One-dimensional earth structure and parameters; V_P , P-wave velocity; V_S , S-wave velocity.

V_P (km/s)	V_S (km/s)	Density (g/cm ³)	Thick- ness (km)	Depth (km)
3.80	1.98	2.30	1.50	1.50
5.50	3.15	2.60	2.50	4.00
6.20	3.52	2.70	22.0	26.0
6.80	3.83	2.87	6.00	32.0

the slip distribution for the Landers earthquake proposed by Wald and Heaton (18). From this slip distribution, we computed the change in static traction along the longitudinal direction of the fault using our finite-difference method. We then reversed the sign of this stress change and added a constant "tectonic stress" of 50 bars in order to produce the initial stress field T^0 on the Landers fault (21, 22). Thus, the initial stress field (T^0 in Eq. 1) is the sum of internal stresses on the fault due to preslip and previous earthquakes, plus some external spatially uniform tectonic load.

Now that we have defined the initial stress, we need to specify the friction law. The values of the yield stress determining the initiation of slip and the slip weakening distance D_0 , which must be a fraction of the slip on the fault (21), are more or less unconstrained and may vary on the fault. The general effect of an increase in yield stress and the slip-weakening parameter is to increase rupture resistance. For the moment, D_0 remains unconstrained, although a recent study of the 1995 Kobe earthquake in Japan has proposed rather large values for D_0 (23). Because of the lack of constraints, we used a constant value of D_0 even though any distribution of the parameter is allowed by our method. We found that a constant yield stress level of 120 bars and D_0 equal to 80 cm produce a total rupture time and final slip distribution in agreement with kinematic inversion results. Before the simulation, we constrained the initial stress T^0 on the fault to values just below the specified yield level (120 bars) to prevent rupture from several locations. We adopted the same regional one-dimensional (1D) model of velocities and densities as used in (18) in our numerical simulations (Tables 1 and 2).

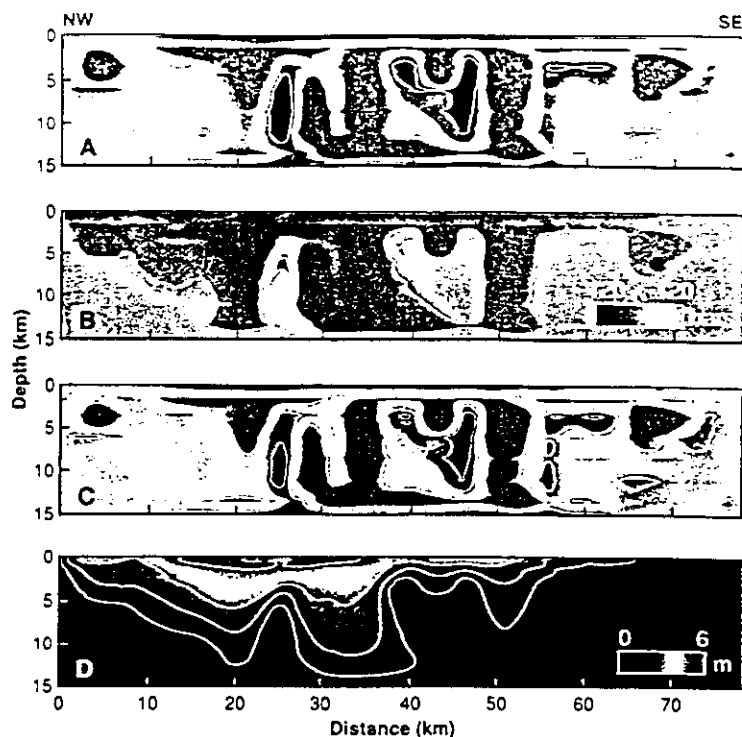
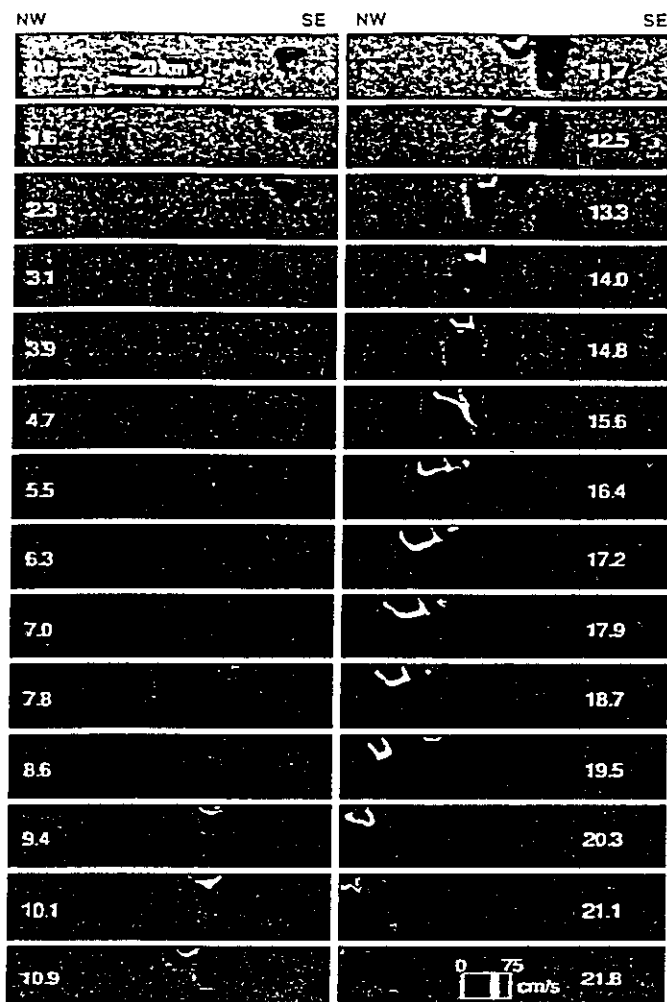


Fig. 2 (left). Snapshots of our dynamic rupture simulation of the Landers earthquake on the fault plane. The snapshots depict the horizontal slip rate. **Fig. 3 (right).** (A) Initial stress, (B) final stress, (C) stress drop, and (D) final slip distribution from the dynamic rupture simulation. Contours shown for (A) through (C) are -120 , 0 , and $+120$ bars; the contour interval for (D) is 1 m, with the first contour at 1 m.

The initiation of rupture was forced by a lowering of the yield stress in a small patch of radius 1 km inside a high-stress region near the hypocenter toward the southern end of the LJV fault strand, as inferred from the kinematic results (Fig. 2). Soon after initiation, the rupture slows down as a result of the relatively low stress surrounding the initial asperity and shrinks to a small patch of slip near the surface (see snapshots between 3.1 and 7 s, Fig. 2). Then the rupture enters a high-stress area (7.8 s) about 50 km from the northwestern edge. At this point, the rupture becomes more complex. Part of the rupture propagates vertically down and stops (7.8 to 13.3 s, Fig. 2), while another part moves laterally close to the surface. The main part of the rupture moves ahead in the general rupture direction where its strength increases, and it enters a high-stress area about 35 km from the northwestern edge of the fault, representing the HV fault segment. After 14.8 s, the rupture spreads to the full width of the fault with the highest intensity near the surface. Here, the rupture pulse resembles the fast, almost instantaneous self-healing phase with a fi-

nite slip duration proposed by Heaton (24) for large earthquakes. At 15.6 s, the rupture enters the CRE segment of the fault, where the rupture extends to greater depths as it propagates with a "tail" of slowly healing fault material left behind. Toward the northwestern edge of the fault, the rupture leaps ahead for the last time (18.7 s) onto a smaller patch of higher stressed material near the surface. After about 22 s, the rupture loses energy and terminates near the upper northwestern corner of the fault.

The earthquake has effectively released the stress on all the most heavily loaded areas of the fault (Fig. 3, A through C). The stress has increased within and particularly near the edges of the areas that showed little or no slip (for example, about 25 to 27 km and 37 to 48 km from the northwestern edge of the fault at a depth of 3 to 12 km) to values similar to or larger than the yield stress specified for the earthquake that just occurred. This result has, to our knowledge, not been reported in earlier studies and suggests that these areas could be the locations of nucleation for a future earthquake on the fault. The final slip distribution

(maximum 6.1 m) represents a smooth version of the one (18) used to compute the initial stress (Fig. 3D).

There are important similarities and differences between the kinematic inversion results (18–20) and our dynamic reconstruction of the Landers event. Both methods find that a confined rupture pulse generally propagates unilaterally along a continuous path on the fault. A more detailed comparison of the rupture pulse is complicated by the relatively lower resolution of the kinematic inversion. The rupture velocity shows a strong variation during the earthquake for the dynamic simulation (Fig. 4A). On average, the rupture here propagates with velocities close to the local S-wave speed and terminates about 22 s after initiation, in agreement with the kinematic results (18–20). However, the rupture velocity varies from subsonic to supersonic during the simulation. The subsonic rupture velocities generally occur within and near the low-stress areas on the fault, whereas the supersonic ones dominate within highly stressed patches of the fault where the rupture resistance is relatively low. Supersonic

Fig. 4. (A) Image and contour plot of the slip rate averaged over a 2-km-wide horizontal band intersecting the hypocenter. Lines depicting a range of rupture velocities are superimposed for comparison. The contour interval is 25 cm for slip between 25 and 175 cm. (B) Image and contour plot of the rise times; contours shown are 2, 4, and 6 s.

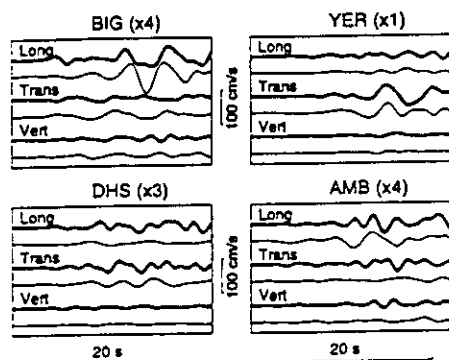
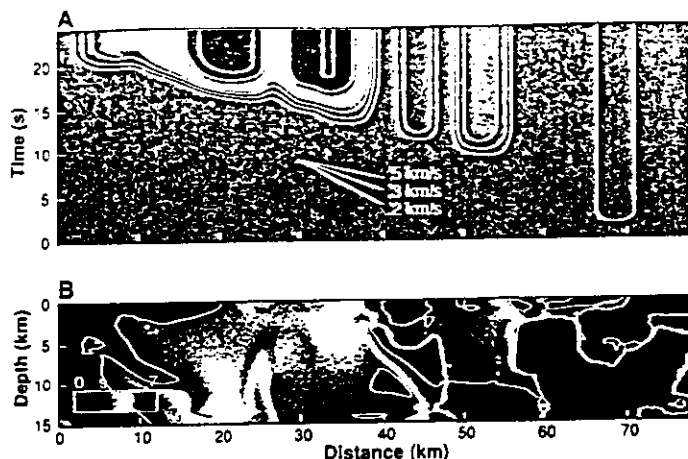


Fig. 5. Comparison of simulated (thin lines) to observed (thick lines) seismograms at four stations (see locations in Fig. 1). Long, Trans, and Vert depict longitudinal, transversal, and vertical components, respectively. Amplitude scaling factors listed at the station abbreviation have been applied to each pair of simulated and observed seismograms. Both simulations and observations are shown within a bandwidth of 0.1 to 0.5 Hz.

velocities are particularly dominating near the surface, allowing the shallow fault to lead the rupture. This result is not expected from the kinematic inversions where the rupture generally propagates faster at depth (15). These shallow supersonic rupture velocities are likely caused by the free surface, which promotes the generation of S to P converted head waves (S^*) (25).

The existence of supersonic rupture velocities has been documented in earlier rupture models (26), but evidence from seismic data (27) is sparse, perhaps because of the limited resolution of the inversion methods. The nearly horizontal time-distance contours in certain areas on the fault represent "jumps" into areas of higher stress (for example, at 4 to 10 km in Fig. 4A and snapshots at 18.7 to 19.5 s in Fig. 2). In addition to promoting supersonic velocities, the free surface enhances the slip and slip rate near the surface, in agreement with earlier dynamic earthquake

modeling results (28). Rise times, calculated as the time required to accumulate from 10% to 90% of the total slip, are larger in areas of relatively low stress, where the rupture slows down as a result of increased rupture resistance. The pattern and magnitude of the rise times (Fig. 4B) agree with kinematic results (19). The resolution of the rise times varies, with the lowest values in the areas where the least amount of slip occurred.

Our model of the rupture process can be compared with observations to determine how the model fits the more complex real event. We have compared the synthetic seismograms generated by our model with the observed velocity seismograms at four stations located in the forward [Yermo (YER)] and backward [Desert Hor Springs (DHS)] rupture directions, as well as southwest [Big Bear Lake (BIG)] and northeast [Amboy (AMB)] of the fault (Figs. 1 and 5). Both synthetic and observed seismograms are bandpass-filtered to frequencies between 0.1 and 0.5 Hz, and the horizontal ground motion is rotated to the approximate longitudinal (154°) and transverse (64°) directions of the simulation. The main features of the low-frequency ground motion for both amplitude and wave form are reproduced by the synthetic seismograms at the four stations. The best fit is obtained for the relatively stronger ground motion recorded in the forward rupture direction (YER). The fit at the other three stations is worse; this is due in part to the weaker ground motion in the back-rupture direction where the relative influence of the crustal structure and fault geometry is larger. Effects of deviations of the simple 1D model are furthermore increased by the large distance from the fault to AMB (65 km). Unfortunately, AMB is the only station available at northeastern azimuths from the fault.

Besides the initial distribution of stress, the only constraint of our rupture model is

given by the friction parameters that are not measured. For our Landers simulation, we used only two such parameters, the slip-weakening distance D_0 and the yield stress level. The values of these parameters were selected from a few sensitivity simulations. For instance, the rupture speed and healing of the fault are critically determined by the level of the yield stress and D_0 . If D_0 is chosen to be less than about 60 to 80 cm, the rupture duration and therefore the rise times are much shorter than those obtained from kinematic inversion (18–20); larger values of D_0 together with a yield stress higher than about 120 bars introduce rupture resistance at a level that prevents the rupture from propagating at all. Although the rupture duration and rise times are strongly related to D_0 , the final slip distribution remains practically unchanged for constant values of D_0 that allow rupture propagation. It is presently unclear whether a spatially varying slip-weakening parameter or a slip-rate-dependent friction law (or both) is needed. Thus, we realize that the friction parameters are somewhat unconstrained. However, we believe that our numerical method provides a framework to constrain these parameters by inversion of observed seismic and geodetic data.

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5. The nature of the friction that applies to an active fault is a major subject of research [J. Dieterich, *J. Geophys. Res.* **84**, 2161 (1979); *Am. Geophys. Union Geophys. Monogr. Ser.* **24**, 103 (1981); J. Rice and A. Ruina, *Trans. ASME J. Appl. Mech.* **50**, 343 (1983); A. Ruina, *J. Geophys. Res.* **88**, 10359 (1983)]. In this work, we have used a friction law with slip weakening but no rate dependence.
6. The staggered-grid, finite-difference method was first introduced by R. Madanaga [*Bull. Seismol. Soc. Am.* **66**, 639 (1976)] and later analyzed by, among others, J. Virieux [*Geophys. J.* **93**, 1933 (1984); *ibid.* **51**, 389 (1986)], A. Leander [*ibid.* **53**, 1425 (1986)], and K. Olsen [thesis, University of Utah (1994)].
7. C. Ceran, D. Kosloff, R. Kosloff, and M. Resner [*Geophysics* **50**, 725 (1985)] showed that the postulation of a zone (sponge) of artificial attenuation near the grid edges helps eliminate numerical artifacts. In our simulation, we found that a 20-point-wide sponge zone satisfactorily eliminated the artifacts.
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9. The actual numerical implementation in a fourth-order scheme is more complex, making use of the

- symmetries in the finite-difference grid. For example, the fault-parallel components of the velocity as well as the normal and σ_{xx} shear stresses are antisymmetric, whereas the fault-perpendicular component of the velocity and the σ_{yy} and σ_{zz} stresses are symmetric across a vertical fault in the xz plane. By careful examination of the numerical implementation, it can be shown that, if the numerical values of the velocity and stress have the aforementioned symmetries about the fault plane at the N th time step, then they do at upgraded $(N + 1)$ th time step as well. Details of the numerical application of the boundary conditions are described in (16).
10. J. Vrioux and R. Madariaga, *Bull. Seismol. Soc. Am.* **72**, 345 (1982).
 11. Slip-weakening friction laws similar to Eq. 2 have been extensively studied by rock mechanicians in Japan; see, for example, M. Ohnaka, Y. Kuwahara, and K. Yamamoto [*Tectonophysics* **144**, 109 (1987)], M. Ohnaka [*Proc. Natl. Acad. Sci. U.S.A.* **93**, 3795 (1996)], and M. Matsu'ura, H. Kataoka, and B. Shibasaki [*Tectonophysics* **211**, 135 (1992)].
 12. Actually, the only physical requirement is that friction dissipates energy, such that T integrated over the fault will be less than zero. To our knowledge, the consequences of this less strict requirement have not been reported in the literature.
 13. The study of slip weakening and the conditions for spontaneous rupture were introduced in seismology by Y. Ida [*J. Geophys. Res.* **77**, 3796 (1972)], although the conditions had been used in dynamic fracture mechanics [B. Kostrov, L. V. Nikitin, L. M. Flintman, *Mech. Solids* **4**, 112 (1969)]. The friction law used in our simulation is described by A. Palmer and J. Rice [*Proc. R. Soc. London Ser. A* **332**, 527 (1973)] and in the excellent review by J. Rice [*In Physics of the Earth Interior*, A. Dziewonski and E. Boschi, Eds. (North-Holland, Amsterdam, 1980), pp. 555–649]. For a complete review of the relation between seismic faulting and rupture mechanics, see (2).
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 21. This method for computing the static stress drop during an earthquake has been used by, among others, T. Miyatake [*Geophys. Res. Lett.* **19**, 349 (1992)], G. Beroza and T. Mikumo [*J. Geophys. Res.* **101**, 22449 (1996)], and T. Mikumo and T. Miyatake [*Bull. Seismol. Soc. Am.* **85**, 178 (1995)].
 22. Several investigators have used quasi-dynamic models of recent large earthquakes (21). In those pioneering computations, the boundary conditions on the fault were different from ours. In particular, they used the stress excess or Irwin fracture criterion [G. R. Irwin, in *Fracturing of Metals*, F. Jonasen, W. Roop, R. Bayless, Eds. (American Society for Metals, Cleveland, OH, 1947), pp. 147–166] instead of a friction law with an internal length scale (D_0). Irwin's criterion lacks a continuum limit in the sense of J. R. Rice [*J. Geophys. Res.* **98**, 9885 (1993)]. Their numerical resolution was almost two orders of magnitude less than ours, which prevented the inclusion of a length scale in the friction law.
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 24. T. Heaton, *Phys. Earth Planet. Inter.* **64**, 1 (1990).

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26. S. Day, *Bull. Seismol. Soc. Am.* **72**, 1881 (1982).
27. The first report of locally supersonic rupture velocities is due to R. Archuleta [*J. Geophys. Res.* **89**, 4559 (1984)], who inverted by trial-and-error accelerograms from the Imperial Valley earthquake, California, 1979. In the temporal reconstruction of the 1992 Landers rupture (magnitude 7.3) (18), the rupture velocity varies significantly from start to end. Between 15 and 17.5 s, the rupture velocity is estimated at 4 km/s, which is faster than the fastest S-wave velocity of 3.52 km/s in the fault zone used for the source inversion.
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on the SGI Origin 2000 at the Materials Research Laboratory, University of California, Santa Barbara (NSF grant CDA95-01954), and in part on the Sparc20 server at the Institute for Crustal Studies, University of California, Santa Barbara (NSF grant EAR 96-28682), and the Southern California Earthquake Center (SCEC) (USC 572726 through NSF cooperative agreement EAR-8920136). R.M.'s work was supported by the Environment Program of the European Community under Project Strong Ground Motion from Earthquakes. Movies of the Landers earthquake simulation can be found on the World Wide Web at URL <http://quake.crustal.ucsb.edu/~kbolsen>. This is Institute for Crustal Studies contribution 0272-64EQ and SCEC contribution 383.

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Mesoporous Platinum Films from Lyotropic Liquid Crystalline Phases

George S. Attard,* Philip N. Bartlett, Nicholas R. B. Coleman, Joanne M. Elliott, John R. Owen, Jin Hai Wang

The lyotropic liquid crystalline phases of surfactants exhibit a rich polymorphism of structures that have long-range periodicities and whose characteristic repeat distances range from 2 to 15 nanometers. The electrochemical reduction of platinum salts confined to the aqueous environments of these phases leads to the deposition of platinum films that have a well-defined long-ranged porous nanostructure and high specific surface areas. These results suggest that the use of liquid crystalline plating solutions could be a versatile way to create mesoporous electrodes for batteries, fuel cells, electrochemical capacitors, and sensors.

The use of micellar solutions of surfactants in the synthesis of oxide ceramics is currently the subject of considerable research, because the resulting materials, which combine high specific surface areas and larger pore sizes than are achievable in conventional zeolites, are of interest for applications in catalysis and size-selective chemical separations (1–9). We have shown that the synthesis of nanostructured oxides can also be achieved by using high surfactant concentrations (>30 weight %) (10), so that the reaction mixture is in a homogeneous liquid crystalline phase. The advantage of conducting the synthesis in a single phase system is that it is possible to exploit the rich lyotropic polymorphism exhibited by surfactants to produce oxides whose nanostructures are in effect casts of the structures of the liquid crystalline phases in which they were formed. The diversity of periodic nanostructures that is accessible is illustrated by the following examples. Materials synthesized in normal topology hexagonal (H_1) phases have a system of aligned pores disposed on a hexagonal lattice, whereas materials prepared in normal topology cubic

($Ia3d$) phases have a system of interconnected cylindrical pores that form a gyroid lattice. Similarly, the use of fluid lamellar (L_a) phases produces materials with lamellar nanostructures.

Metals with nanostructures analogous to those of mesoporous oxide ceramics are of considerable interest for applications in catalysis, batteries, fuel cells, capacitors, and sensors (11–14). Recently we reported that nanostructured platinum can be produced by the chemical reduction of metal salts dissolved in the aqueous domains of a hexagonal lyotropic liquid crystalline phase (15). This nanostructured material (which is denoted by H_1 -Pt to indicate the phase used in its synthesis) is obtained as a coarse granular material with a specific surface area between 17 and 23 m² g⁻¹, which is comparable with that of platinum black (20 to 26 m² g⁻¹). Such granular nanostructured metals can be used directly in catalysis, but they are of limited relevance for electrochemical or sensor devices in which the metal is required as a thin film. For those applications, the ability to predetermine the nature of the film's nanostructure through the use of different liquid crystalline phases, together with the control that can be achieved over the characteristic dimensions of the channels, provides new methods for obtaining optimized nanostructures. Here

Department of Chemistry, University of Southampton, Southampton SO17 1BJ, UK.

* To whom correspondence should be addressed. E-mail: gza@soton.ac.uk

