

SMR: 1059/29

**FOURTH WORKSHOP ON THREE DIMENSIONAL MODELLING
OF SEISMIC WAVES GENERATION.
PROPAGATION AND THEIR INVERSION**

28 September - 9 October 1998

*Theory and Application of Mode in
Laterally Varying Media*

presented by:

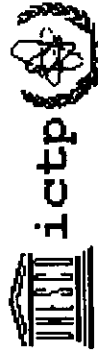
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These are preliminary lecture notes, intended only for distribution to participants.



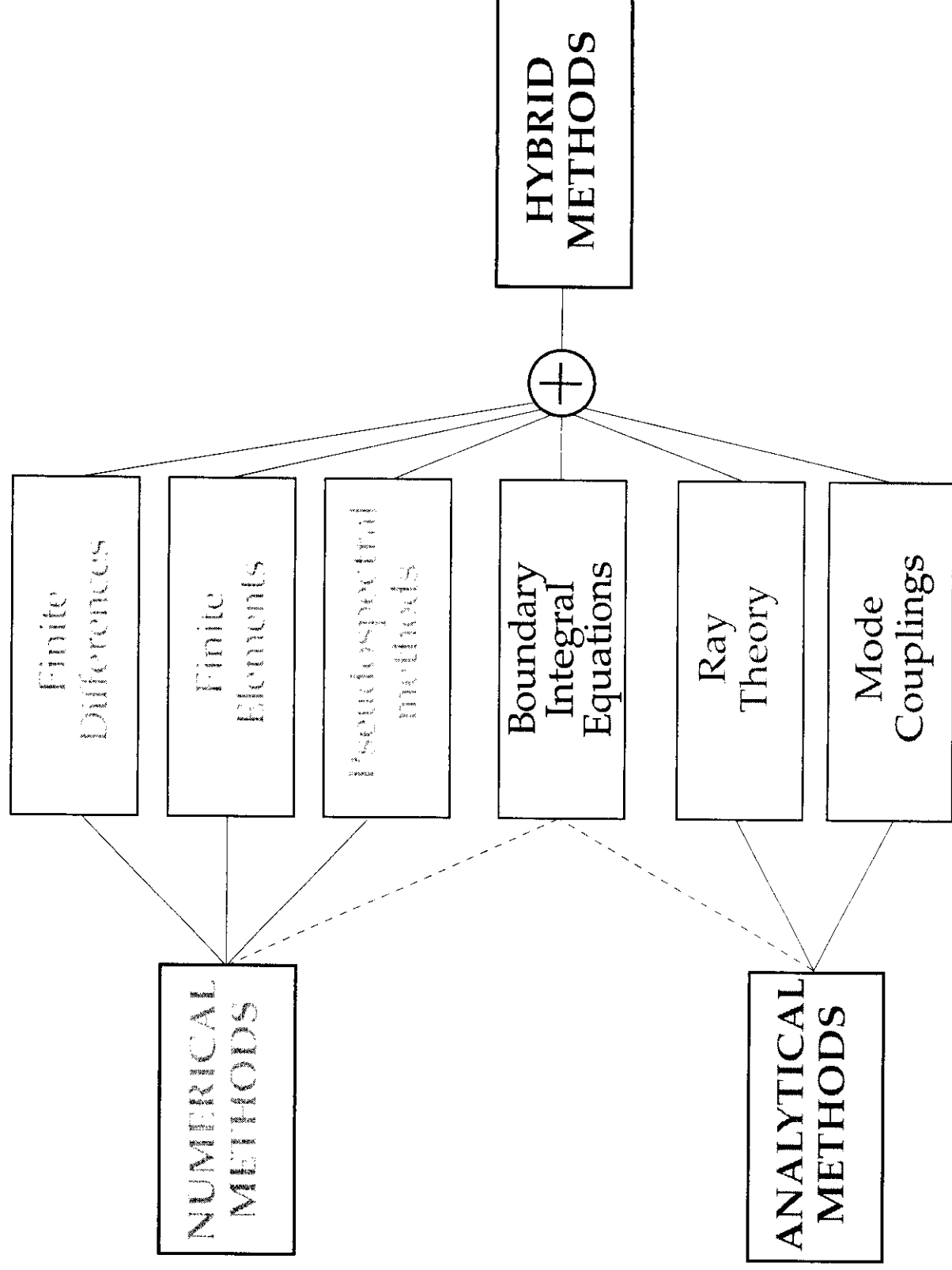
**FOURTH WORKSHOP ON THREE-DIMENSIONAL
MODELLING OF SEISMIC WAVES GENERATION,
PROPAGATION AND THEIR INVERSION**

**Theory and application of mode
propagation in laterally varying media:**

- a) Spectral theory of Love and Rayleigh operators**
- b) Considerations on site response estimation**

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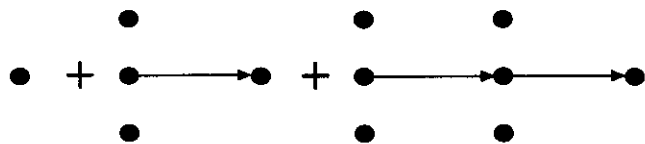
Wave propagation in LHM



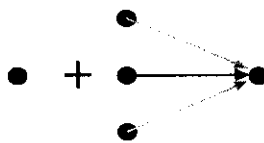
Mode coupling techniques

MODE COUPLINGS

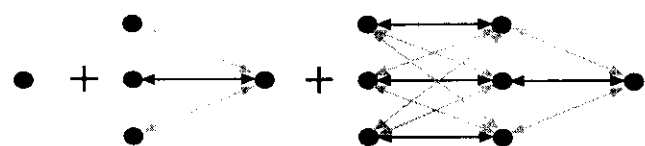
WKBJ



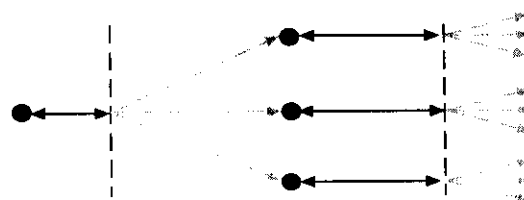
Born approximation



IIT

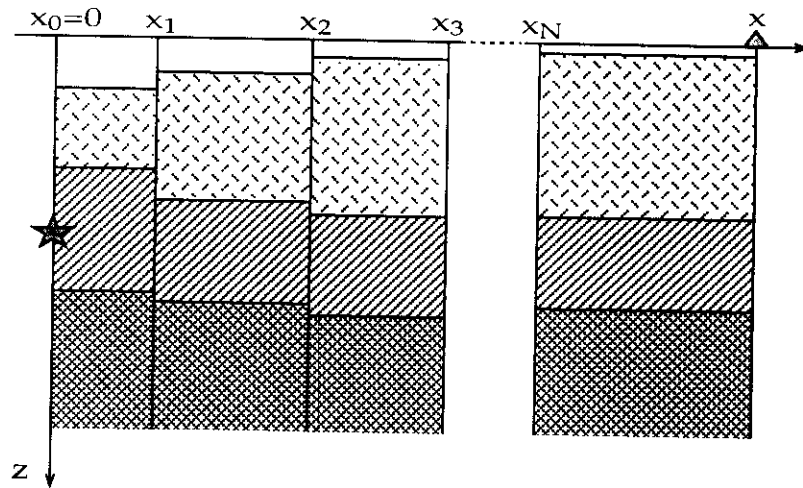


Alsop method



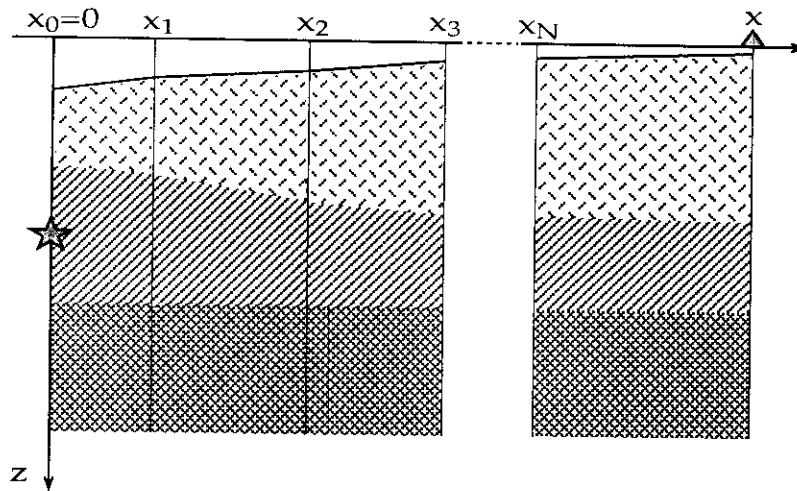
Laterally heterogeneous models

a)



$$u_i^{mL}(\omega) = \frac{e^{-i\pi/4}}{\sqrt{8\pi\omega}} e^{-i\omega \sum_{j=1}^N \frac{x_j}{c_j}} \frac{\chi_L^m(h_s, \varphi)}{\sqrt{c_m v_m I_m}} \bigg|_S \sum_{j=1}^N \gamma_{mm}^{jL} \frac{F_y^{(i)m}(z)}{\sqrt{v_m I_m}}$$

b)



$$u_i^{mL}(\omega) = \frac{e^{-i\pi/4}}{\sqrt{8\pi\omega}} e^{-i\omega \int_S^R \frac{dx}{c}} \frac{\chi_L^m(h_s, \varphi)}{\sqrt{c_m v_m I_m}} \bigg|_S \frac{F_y^{(i)m}(z)}{\sqrt{v_m I_m}}$$

Alsop approach

- The 2-D model is built up by a set of layered quarterspaces in welded contact
- The set of eigenfunctions, at a **given frequency**, form a **complete** set
- One mode of the first structure is normally incident at the vertical interface

$$\phi_m^I + \sum_{n=1}^{\infty} R_{mn}^{I\ II} \phi_n^I = \sum_{k=1}^{\infty} T_{mk}^{I\ II} \phi_k^{\ II}$$

$$k_m^I \phi_m^I - \sum_{n=1}^{\infty} k_n^I R_{mn}^{I\ II} \phi_n^I = \frac{\mu_{II}}{\mu_I} \sum_{k=1}^{\infty} k_k^{\ II} T_{mk}^{I\ II} \phi_k^{\ II}$$

Limits:

The completeness assumption is wrong, as the Sturm-Liouville problem is **singular**

S-L regular problem:

$$\frac{d}{dx} \left(p(x) \frac{df}{dx} \right) + (\lambda q(x) - w(x)) f = 0$$

$$\alpha f(a) + \alpha' \left. \frac{df}{dx} \right|_a = 0$$

$$\beta f(b) + \beta' \left. \frac{df}{dx} \right|_b = 0$$

- eigenvalues are **real** and **discrete**
- eigenfunctions are orthogonal (q)
- eigenfunctions are a **complete** set

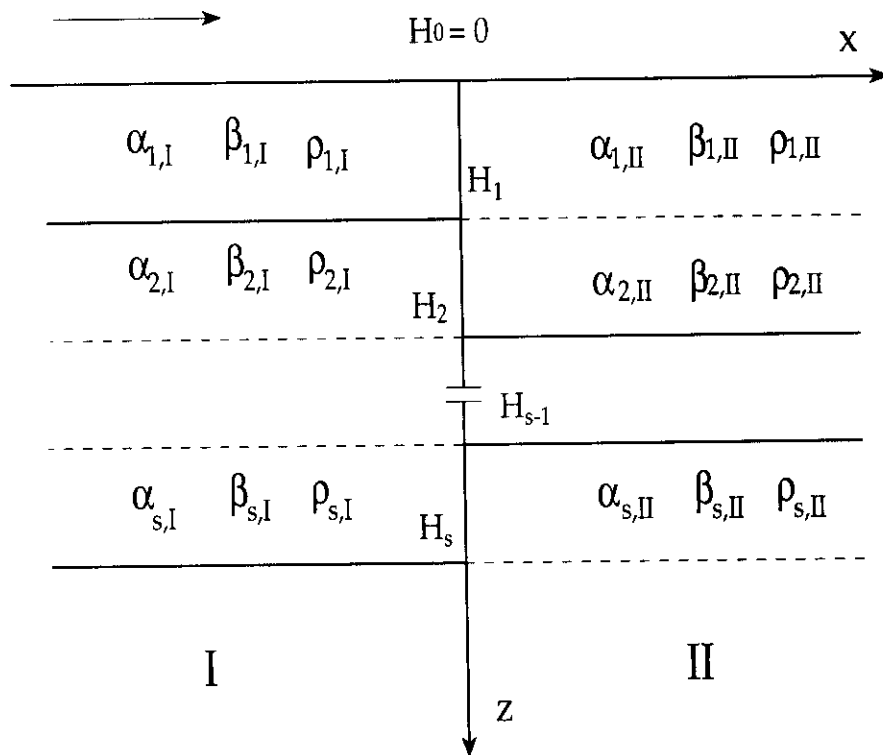
$$\frac{d}{dz} \left(\mu_f(z) \frac{d_f \phi}{dz} \right) + (\omega^2 \rho_f(z) - k^2 \mu_f(z)) f \phi = 0$$

$$\mu(0) \left(\frac{d\phi}{dz} \right) \Big|_0 = 0 \quad \mu(z_{\max}) \left(\frac{d\phi}{dz} \right) \Big|_{z_{\max}} = 0$$

otherwise the problem is **singular**

2-D basic model

- two quarter-spaces in welded contact



- **Stress-Displacement** vectors for a given Love mode, at a given frequency

$$\mathbf{A}_I = (0, v_I, 0, 0, \sigma_{xyI}, 0) \quad \mathbf{A}_{II} = (0, v_{II}, 0, 0, \sigma_{xyII}, 0)$$

- **Scalar Product**

$$\langle \mathbf{A}_I, \mathbf{A}_{II} \rangle = \frac{1}{2i} \int_0^\infty [v_I \bar{\sigma}_{xyII} - \bar{v}_{II} \sigma_{xyI}] dz$$

Coupling coefficients

$$\gamma_T^{(m,m')} = \frac{\langle \mathbf{A}_T^{(m)}, \mathbf{A}_{II}^{(m')} \rangle}{\langle \mathbf{A}_I^{(m)}, \mathbf{A}_I^{(m)} \rangle^{1/2} \langle \mathbf{A}_{II}^{(m')}, \mathbf{A}_{II}^{(m')} \rangle^{1/2}}$$

$$\Gamma_T^{(m,m')} = \frac{\langle \mathbf{A}_T^{(m)}, \mathbf{A}_{II}^{(m')} \rangle}{\langle \mathbf{A}_{II}^{(m')}, \mathbf{A}_{II}^{(m')} \rangle} = \gamma_T^{(m,m')} \frac{\langle \mathbf{A}_I^{(m)}, \mathbf{A}_I^{(m)} \rangle^{1/2}}{\langle \mathbf{A}_{II}^{(m')}, \mathbf{A}_{II}^{(m')} \rangle^{1/2}}$$

$$\gamma_R^{(m,m')} = \frac{\langle \mathbf{A}_R^{(m)}, \mathbf{A}_I^{(m')} \rangle}{\langle \mathbf{A}_I^{(m)}, \mathbf{A}_I^{(m)} \rangle^{1/2} \langle \mathbf{A}_I^{(m')}, \mathbf{A}_I^{(m')} \rangle^{1/2}}$$

$$\Gamma_R^{(m,m')} = \frac{\langle \mathbf{A}_R^{(m)}, \mathbf{A}_I^{(m')} \rangle}{\langle \mathbf{A}_I^{(m')}, \mathbf{A}_I^{(m')} \rangle} = \gamma_R^{(m,m')} \frac{\langle \mathbf{A}_I^{(m)}, \mathbf{A}_I^{(m)} \rangle^{1/2}}{\langle \mathbf{A}_I^{(m')}, \mathbf{A}_I^{(m')} \rangle^{1/2}}$$

$$\begin{aligned} \langle \mathbf{A}_I^{(m)}, \mathbf{A}_I^{(m')} \rangle &= \frac{(k_I^{(m)} + k_I^{(m')})}{2} \int_0^\infty \mu_I(v_I^{(m)} v_I^{(m')}) dz = \\ &= \delta_{mm'} k_I^{(m)} \int_0^\infty \mu_I(v_I^{(m)})^2 dz = \delta_{mm'} \omega U_I^{(m)} I_I^{(m)} \end{aligned}$$

$$\gamma_T^{(m,m')} = \frac{(k_T^{(m)} + k_{II}^{(m')})}{2} \frac{\int_0^\infty \mu_{II}(v_T^{(m)} v_{II}^{(m')}) dz}{\sqrt{\omega U_I^{(m)} I_I^{(m)}} \sqrt{\omega U_{II}^{(m')} I_{II}^{(m')}}}$$

Groundshaking site effects

“... the unique behaviour of a site that persists given all the potential sources of earthquake ground motion ...” Field, 1996

1) Surface topography effects (convexity)

sensitivity to:

- a) type of wavefield
- b) angle of incidence
- c) shape and sharpness

2) Soft surface layering

a) 1-D: trapping of waves for impedance contrast (**vertical resonances**)

$$f_n = (2n+1)\beta/4H$$

$$A \approx (\rho_2 v_2)/(\rho_1 v_1)$$

b) 2-D 3-D: complex energy focusing for diffraction effects
(**basin edge waves**)

Empirical techniques for SR estimation

$$O_{ij}(\omega) = E_i(\omega) \cdot P_{ij}(\omega) \cdot S_j(\omega)$$

$$\ln O_{ij}(\omega) = \ln E_i(\omega) + \ln P_{ij}(\omega) + \ln S_j(\omega)$$

1) Weak (and strong) motion

- a) **S/B spectral ratio**
(Borcherdt, 1970)
- b) **generalized inversion scheme**
(Andrews, 1986)
- c) **coda waves analysis**
(Margheriti et al., 1994)
- d) **parametrized source and path inversion**
(Boatwright et al., 1991)
- e) **H/V spectral ratio (receiver function)**
(Lermo et al., 1993)

2) Microtremors

- a) **peak frequencies examination**
- b) **S/B spectral ratio**
- c) **H/V spectral ratio**
(Nagoshi, 1971; Nakamura, 1989)
- d) **array analysis**
(Malagnini et al., 1993)

Nakamura's technique

1) Usual transfer function

$$T_F = \frac{S_{HS}}{S_{HB}}$$

2) Effect of Rayleigh waves

$$R = \frac{S_{VS}}{S_{VB}}$$

$$T_{FF} = (S_{HS}/S_{HB}) \cdot (S_{HS}/S_{HB})$$

3) Nakamura's hypothesis

$$S_{HB}/S_{VB} = 1$$

4) New transfer function

$$T_{FF} = \frac{S_{HS}}{S_{VS}}$$

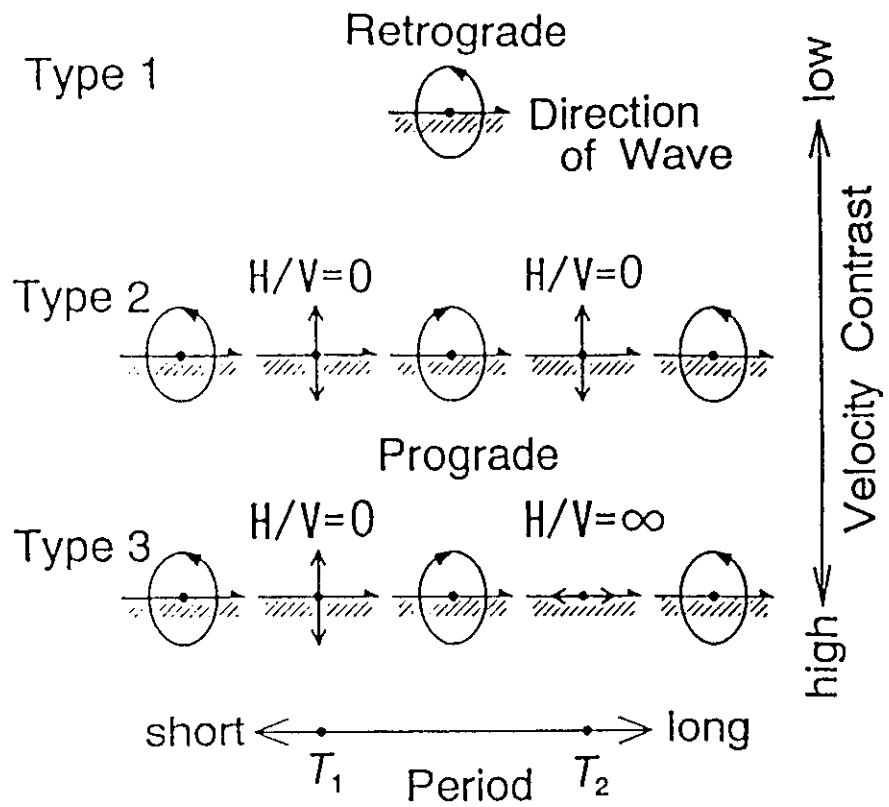


Figure 2. Particle orbit of three types of fundamental-mode Rayleigh waves for the models in Figure 1.

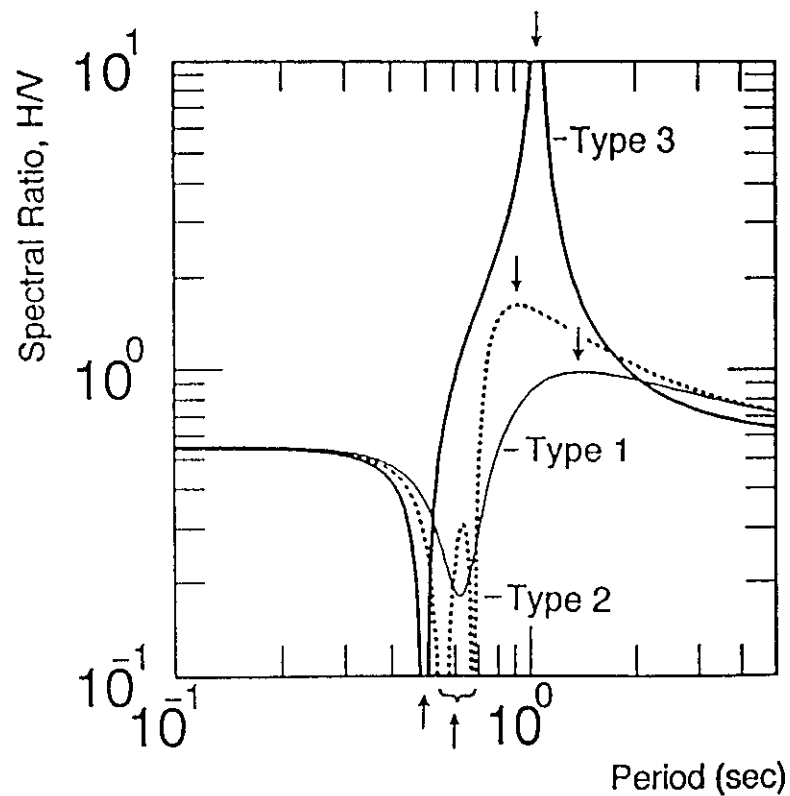


Figure 3. Three types of H/V ratio of fundamental-mode Rayleigh waves. Downward arrows show peak periods, and upward arrows show trough periods: type 1, $V_{S1} = 250$ m/sec; type 2, $V_{S1} = 200$ m/sec; type 3, $V_{S1} = 50$ m/sec.

Koyu
and
Ohwachi,
1998

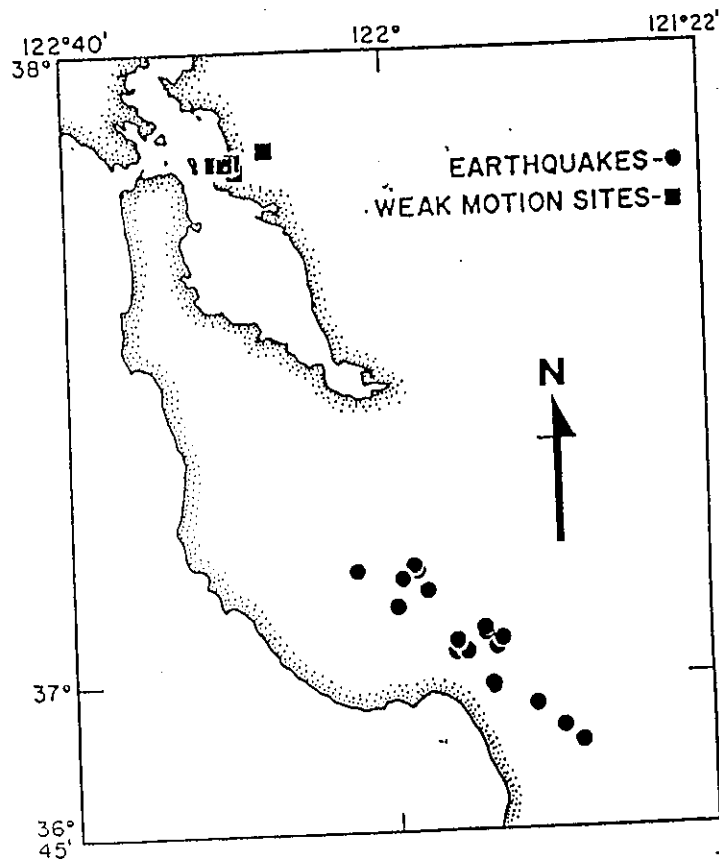
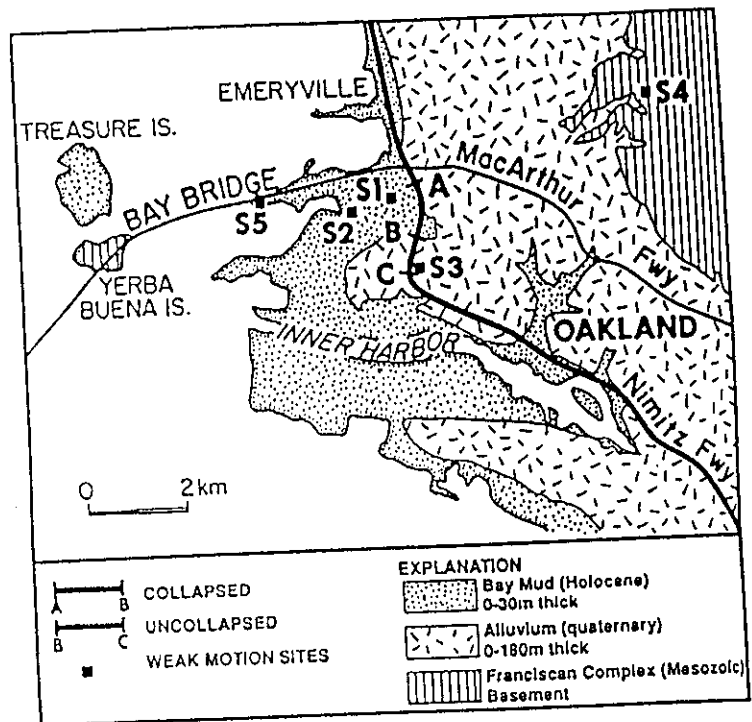


Figure 1. (a) Map of Oakland, California, region showing the surface geology and the locations of the five sites (S1, S2, S3, S4, and S5) occupied during the aftershock deployment. (b) Regional map showing the epicenter locations (circles) and recording sites (squares).

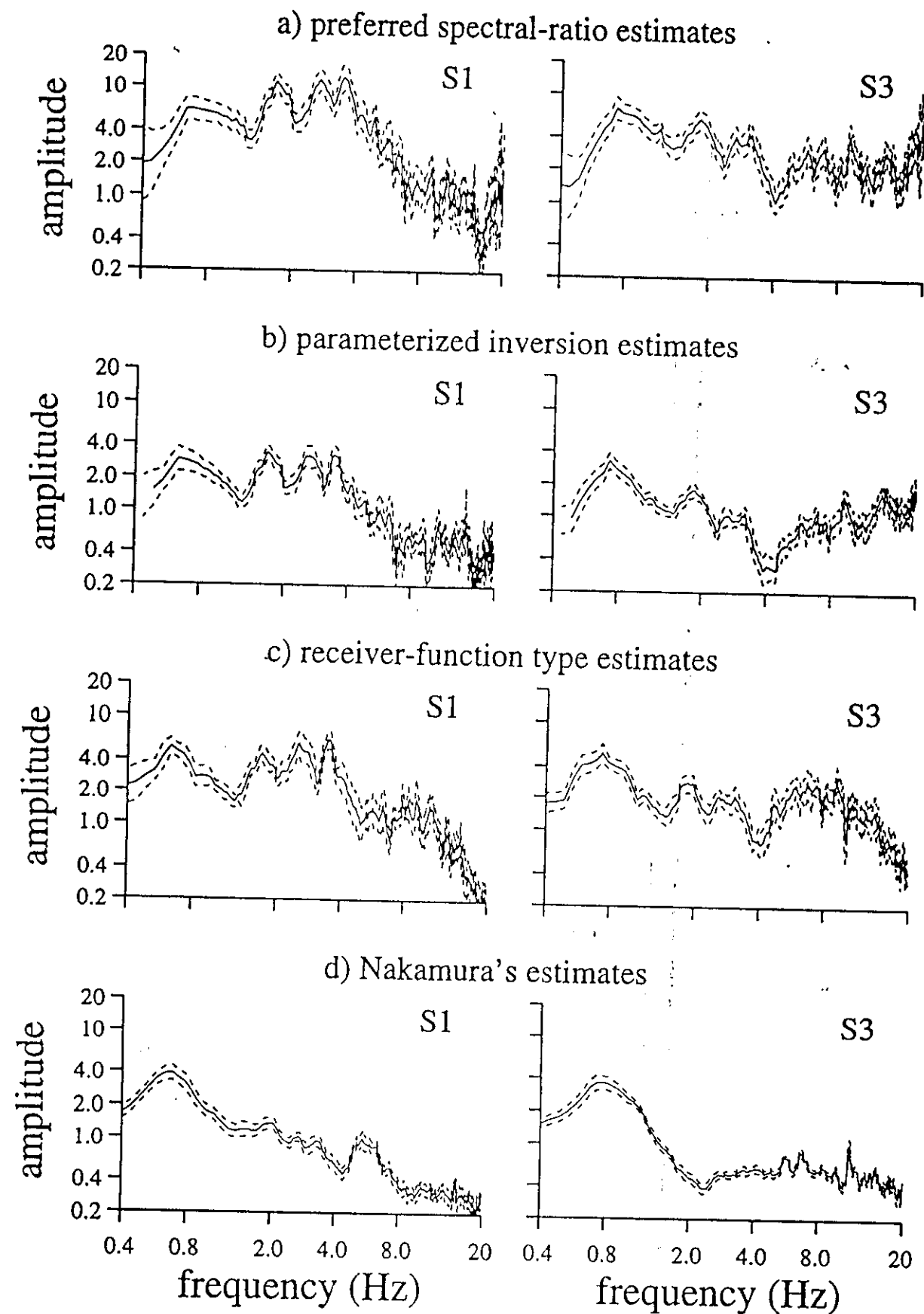


Figure 4. Comparison of the site-response estimates obtained from the non-reference-site-dependent techniques (for S1 and S3). The dashed lines represent 95% confidence limits of the mean. (a) The preferred spectral-ratio estimates (reproduced from Fig. 2d). (b) The site-response estimates obtained from the parameterized inversion (where all data except those for the reference site were included). (c) Receiver-function-type estimates (average horizontal- to vertical-component spectral ratios of aftershock shear waves). (d) Nakamura's estimate (average horizontal- to vertical-component spectral ratios of ambient noise). These noise data were collected using Kinematics (SH-

Field
and
Jacob,
1995

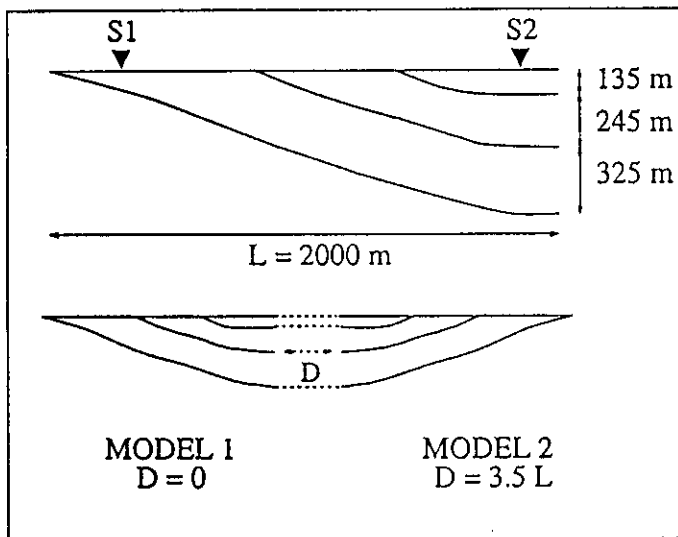


Figure 11. Generic model of a symmetric basin. The geotechnical data are the same as for the Brisbane model (Table 2). Also plotted are the two study sites: S1 and S2.

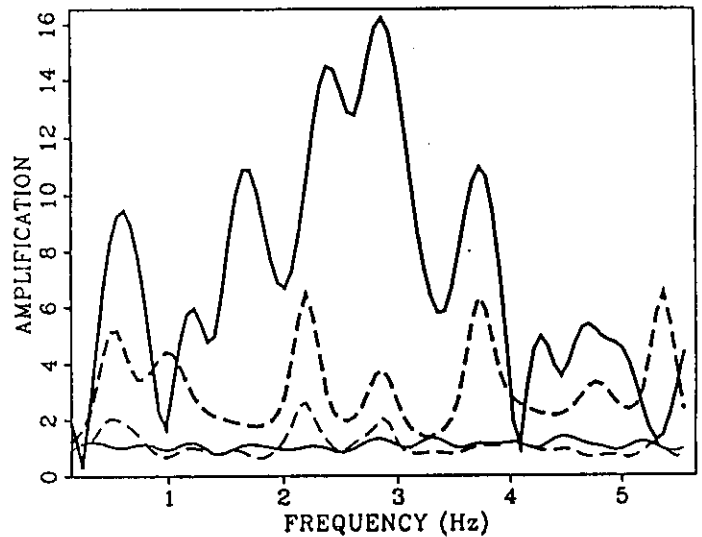


Figure 12. 2D site response, for vertical incidence, computed at site S2 for model 1 (Fig. 11) using SBSR (continuous thick line) and HVNR (thin dashed line). Also plotted, the 1D response of the same site (thick dashed line) and SBSR (continuous thin line) computed at a site situated near the edge of the basin (site S1).

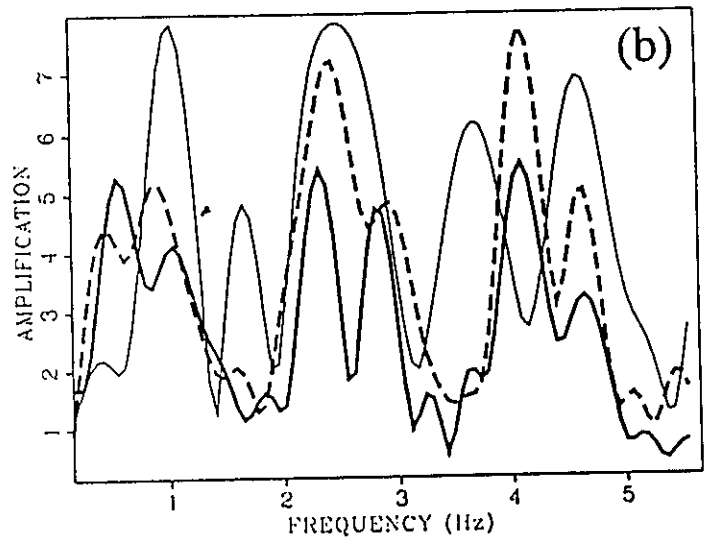
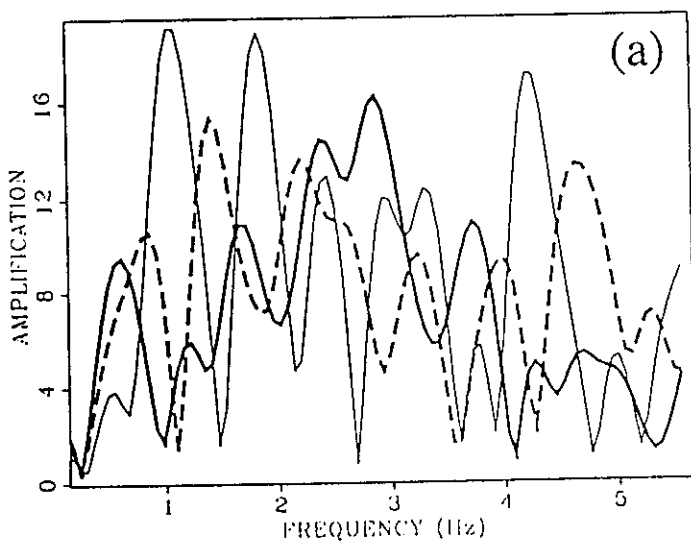
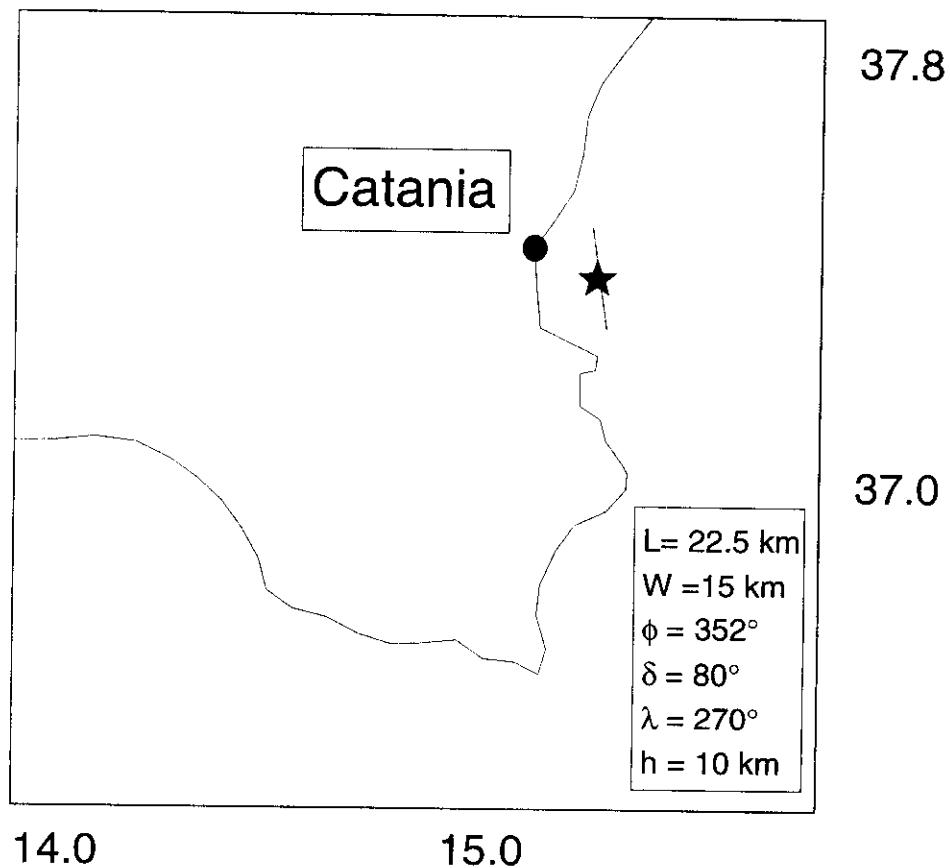
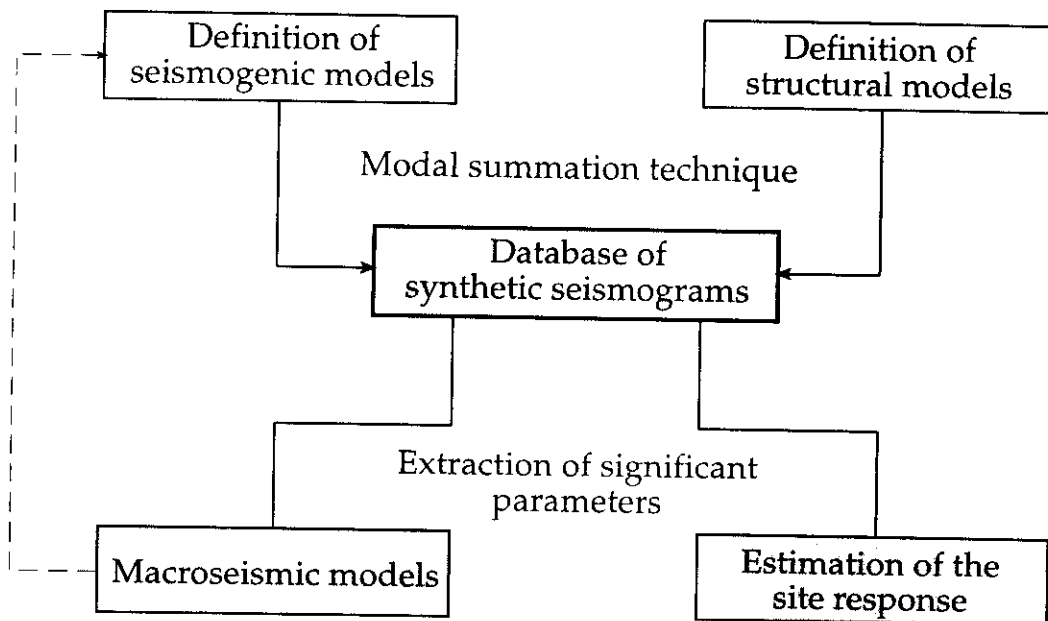


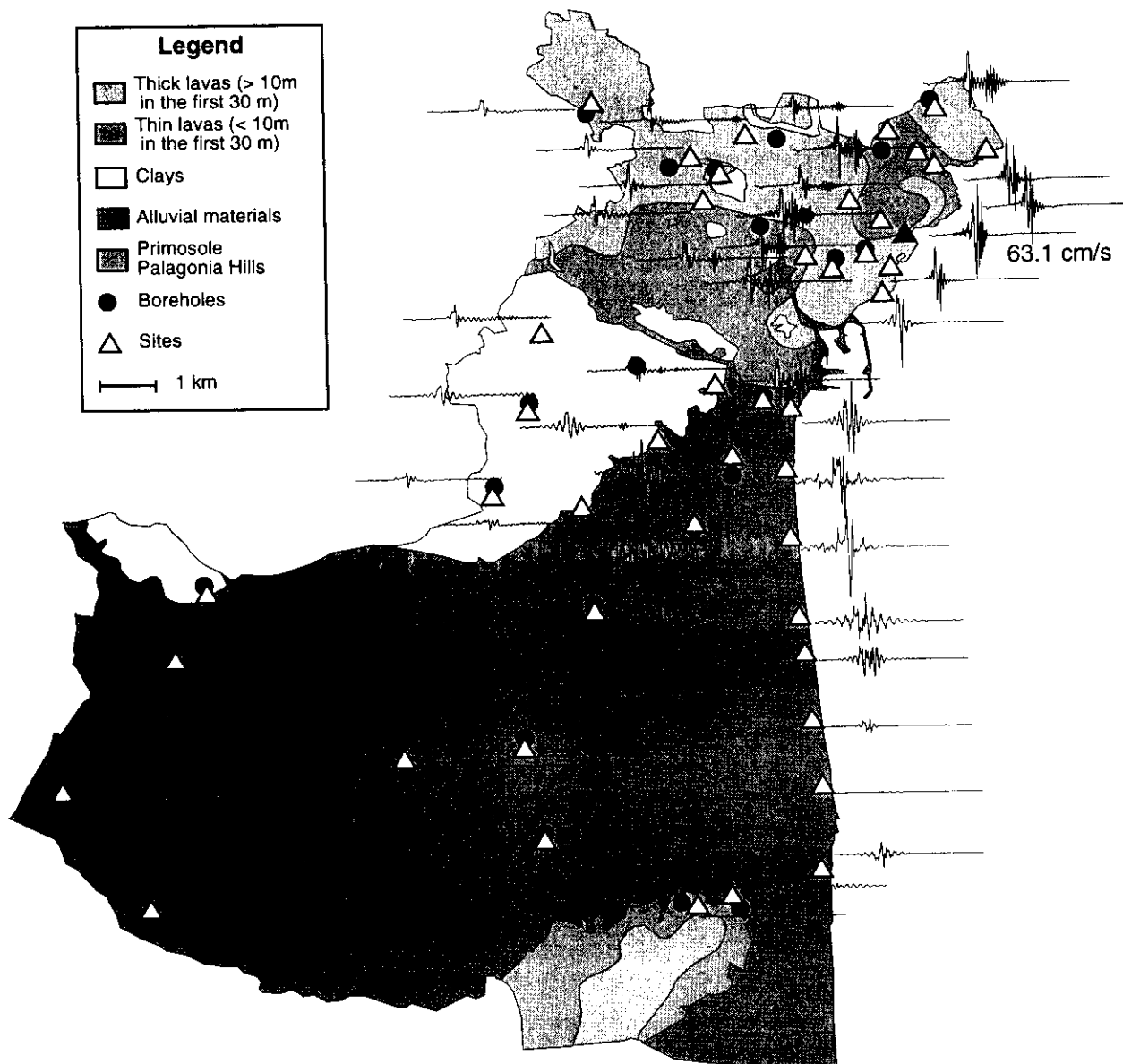
Figure 13. Site response based on SBSR computed at site S2 (Fig. 11) of model 1 (left) and model 2 (right) for three different angles of incidence: 0° (continuous thick line), 30° (thick dashed line), and 60° (continuous thin line).

REALISTIC DEFINITION OF THE SEISMIC INPUT: CATANIA AREA (SICILY)



Ground motion scenario in the Catania area for a magnitude 7 earthquake on the Hyblean fault

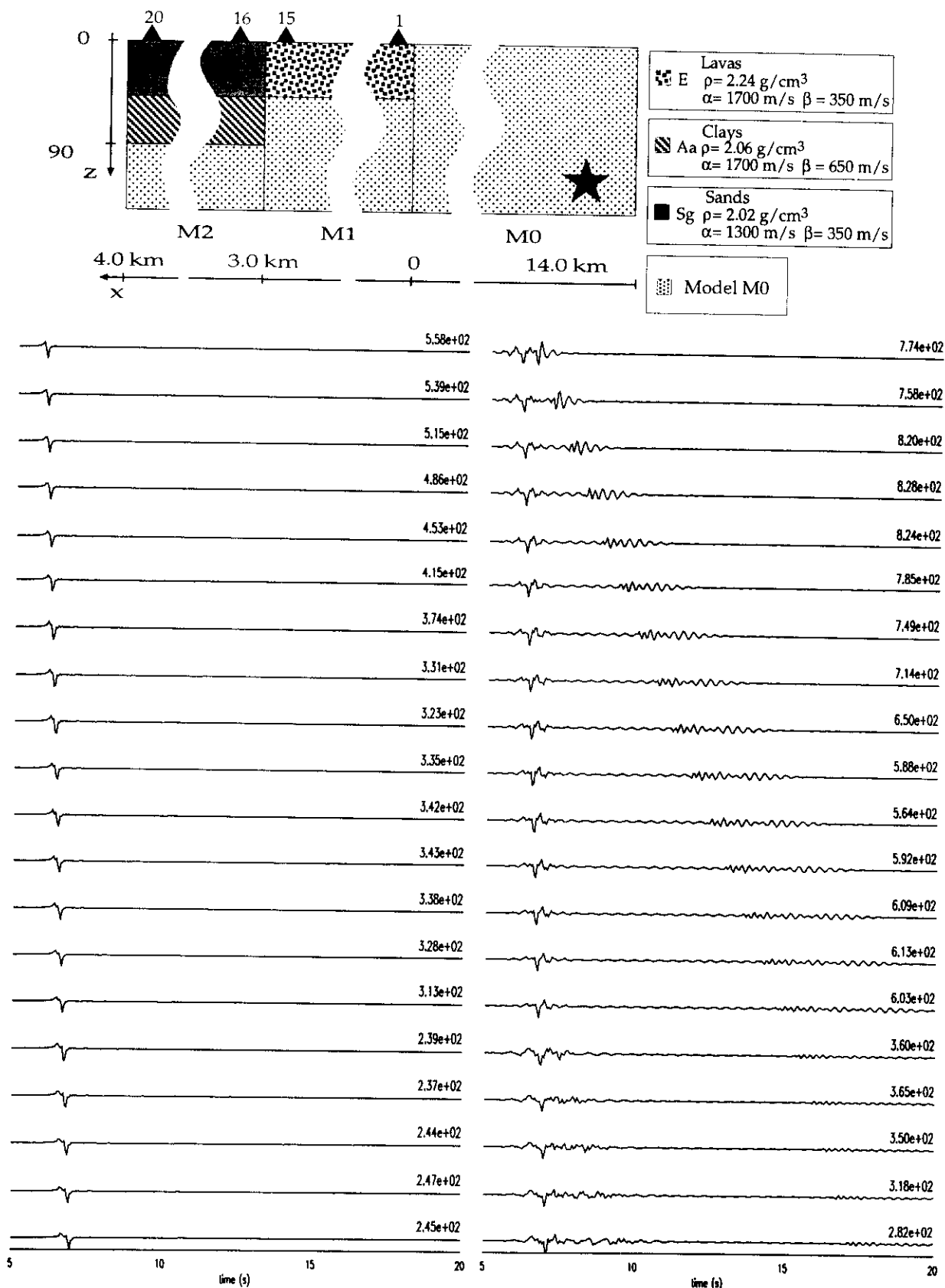
Velocity time series



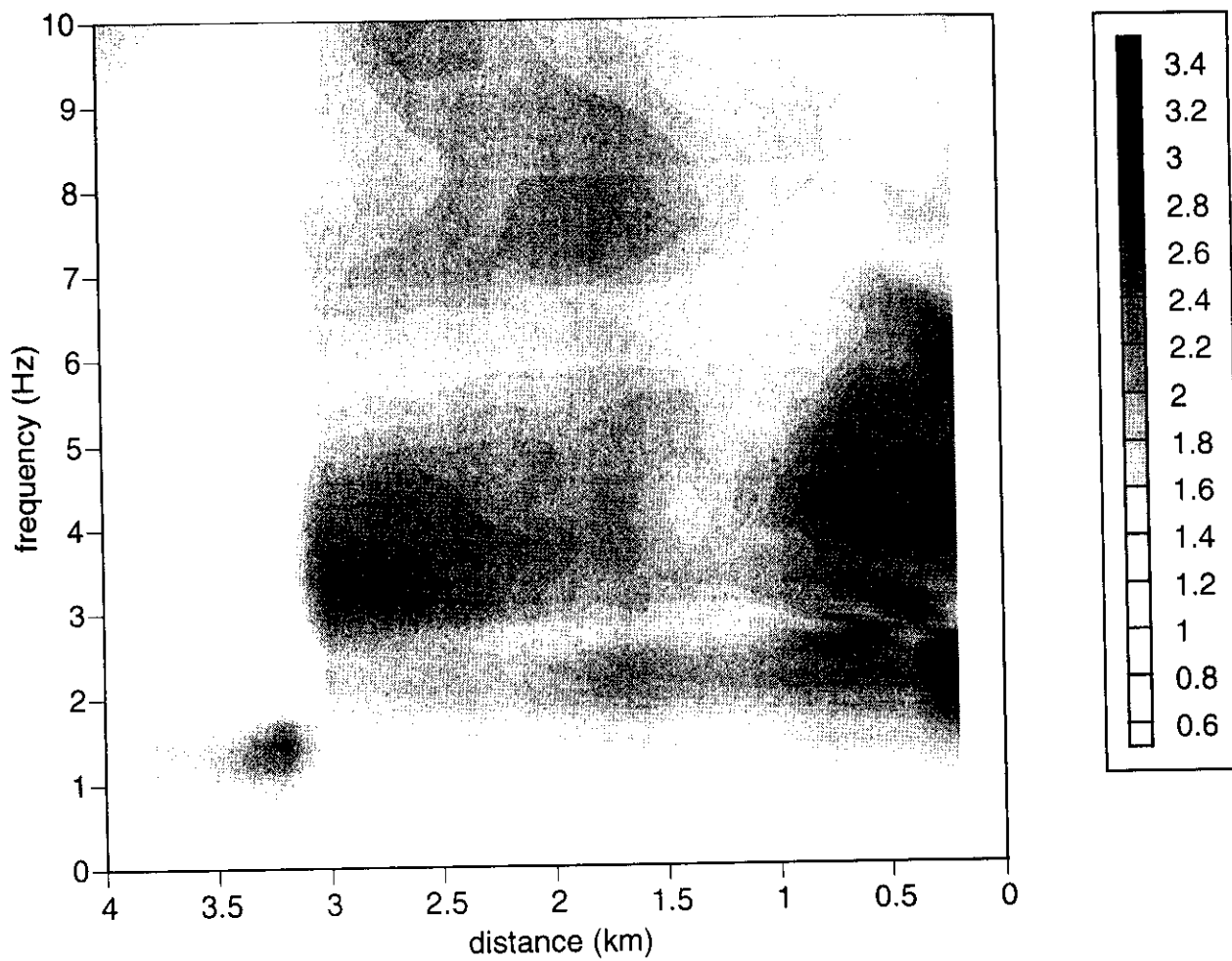
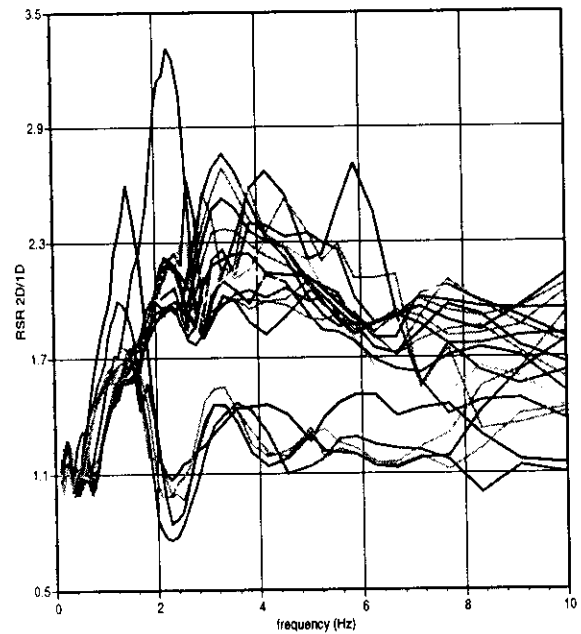
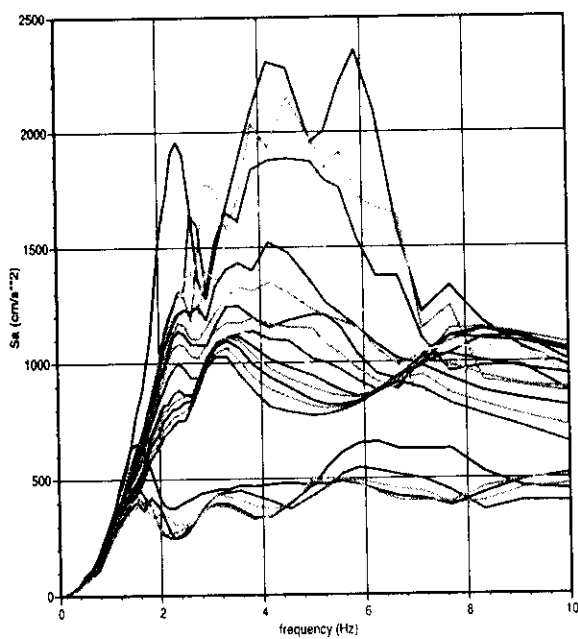
DST



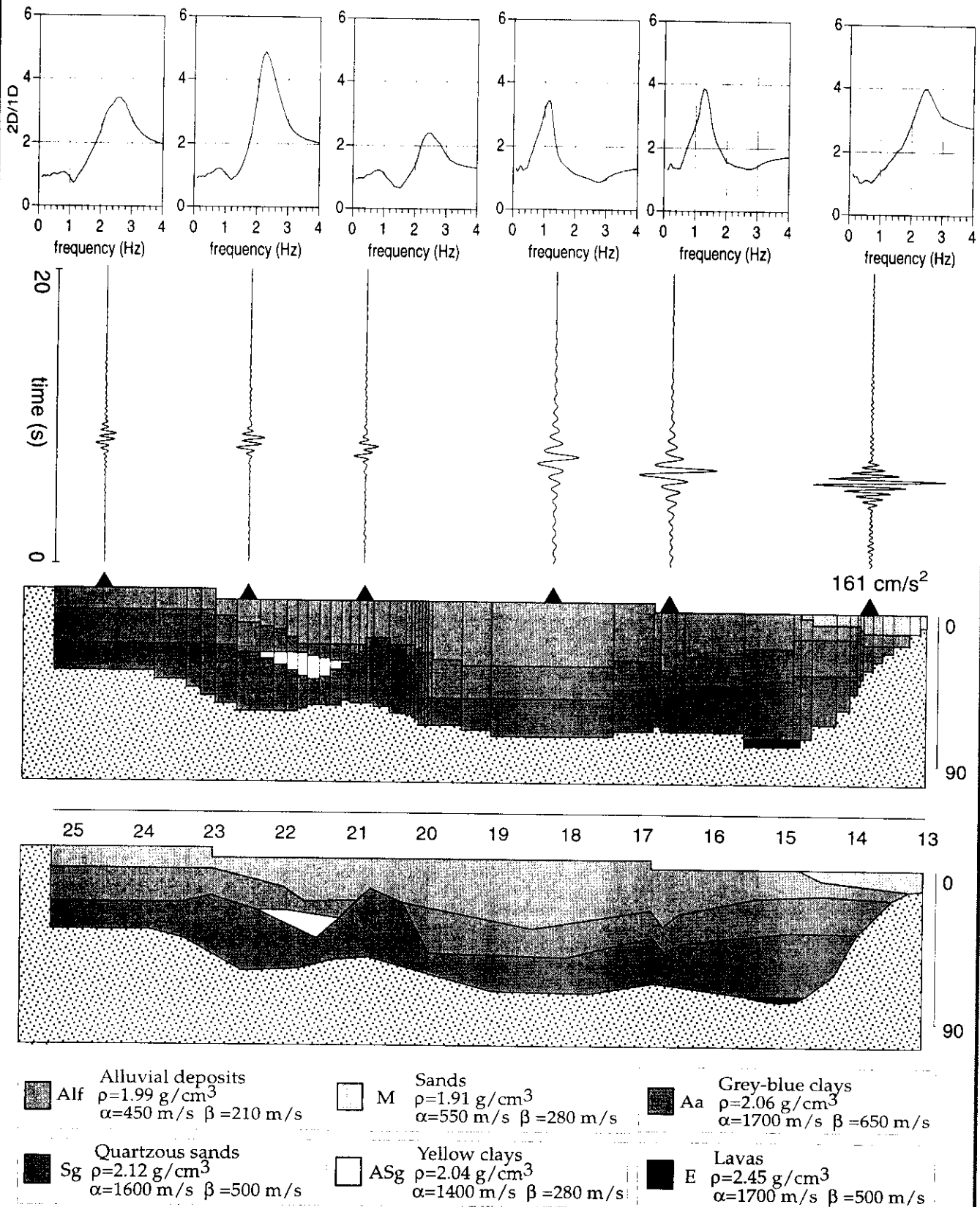
Simplified section S4: time signals



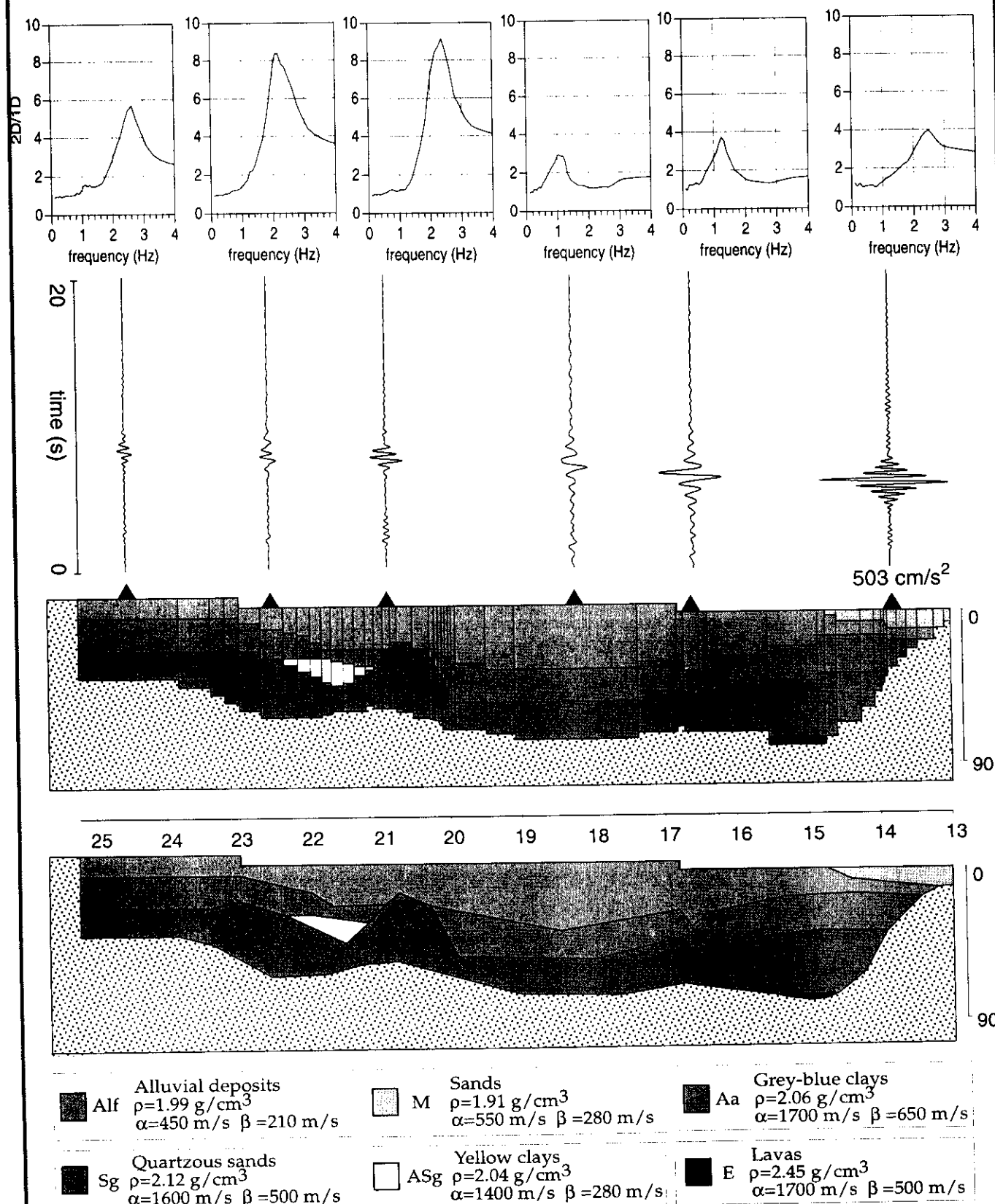
Simplified section S4: RSR



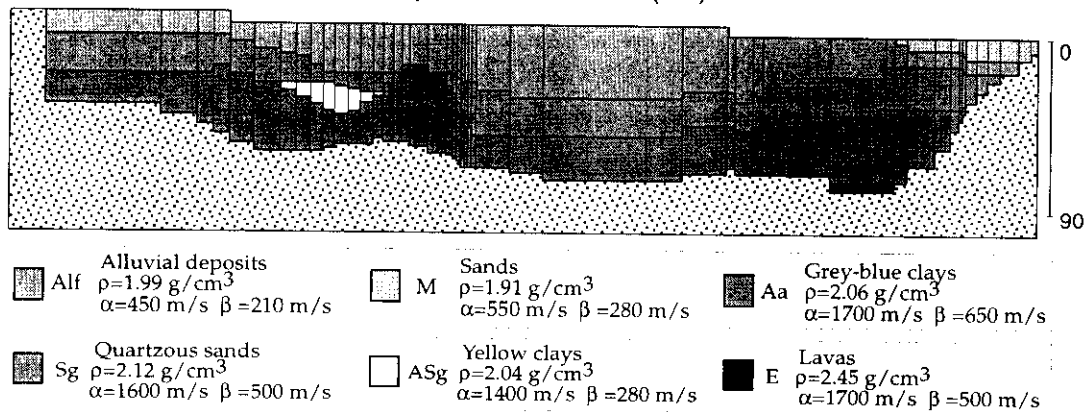
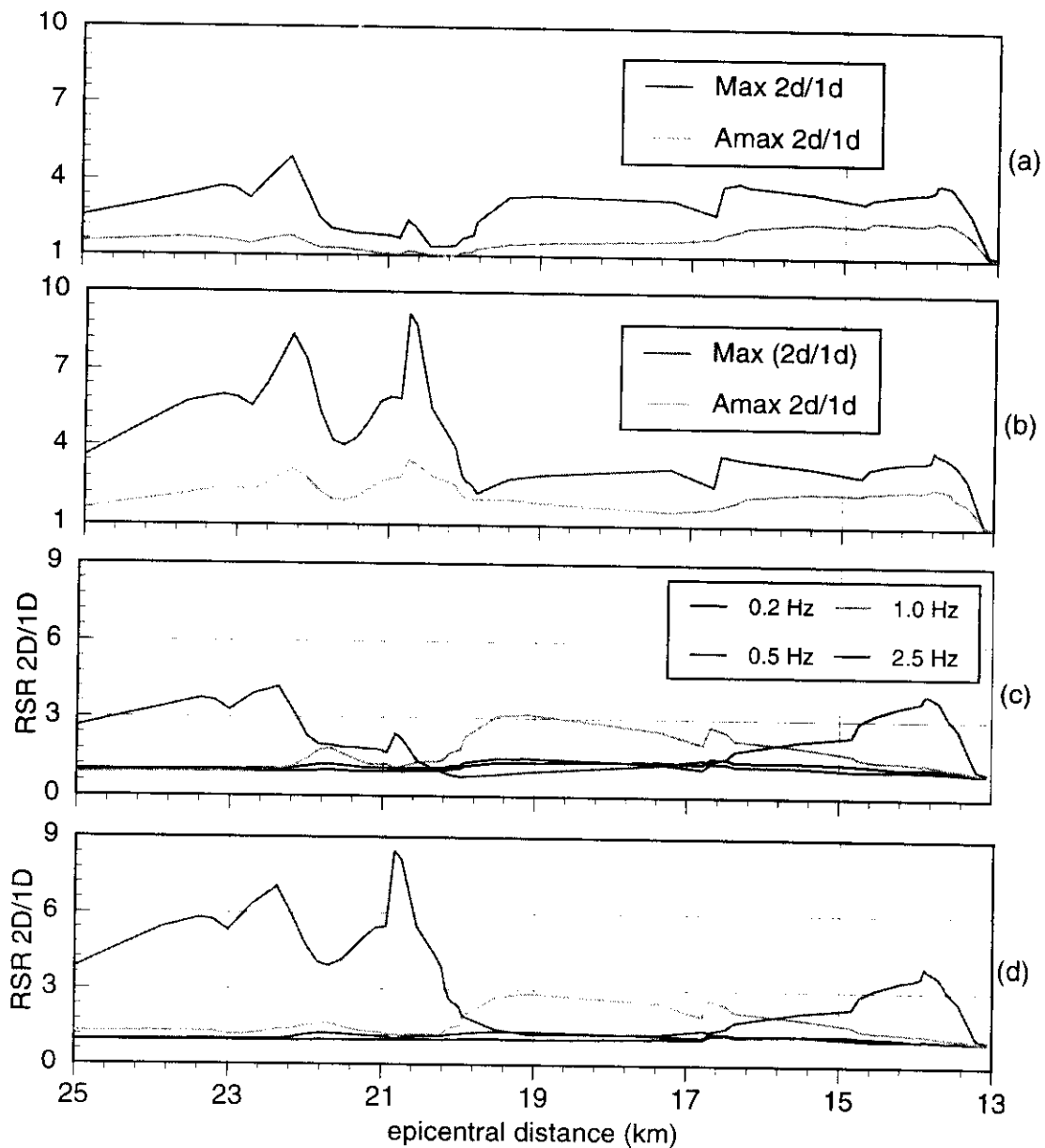
Selected signals along S10 for the scenario earthquake



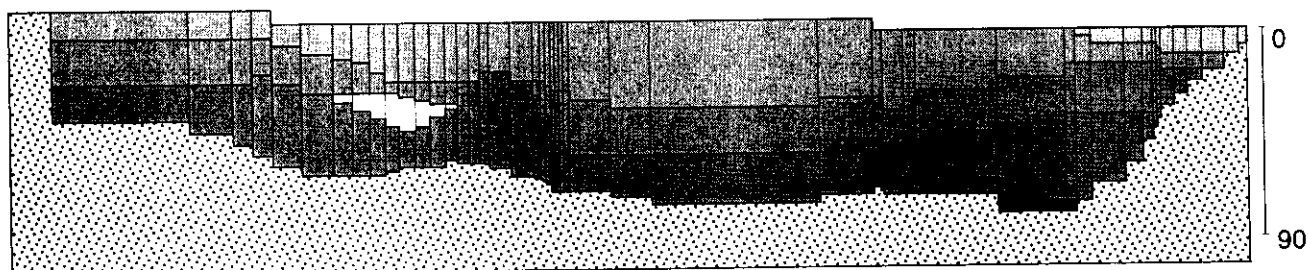
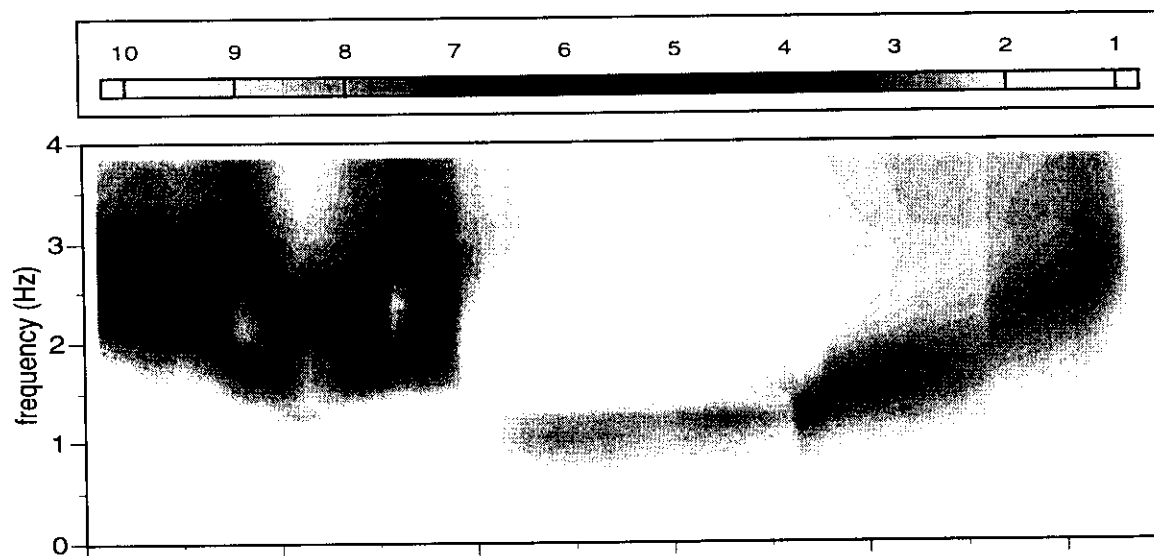
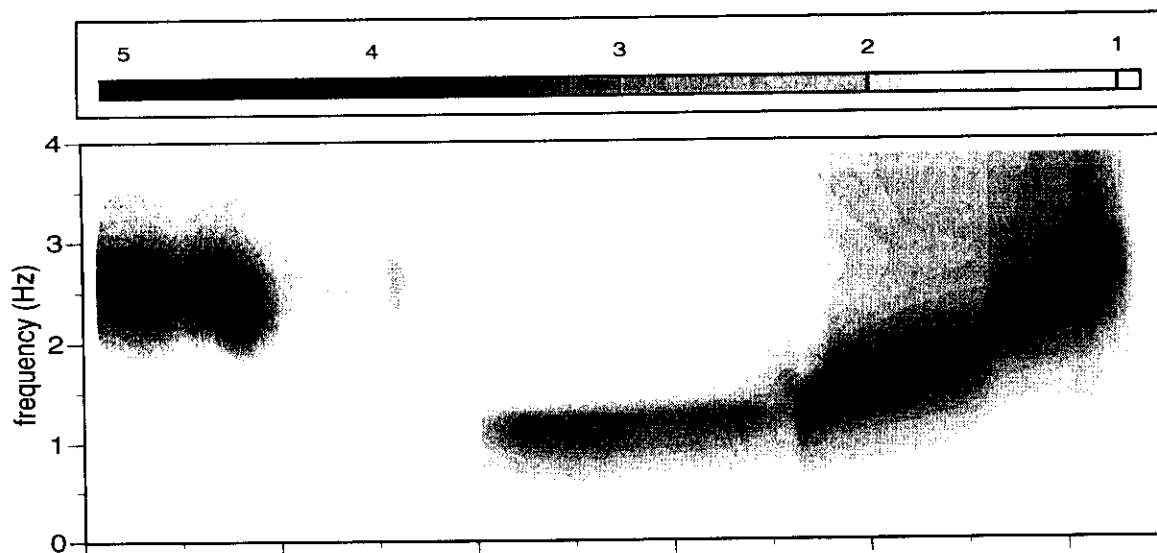
Selected signals along S10 for the scenario earthquake (strike=180°)



Amplification factors RSR 2D/1D versus epicentral distance



Response spectra ratios 2D/1D versus frequency and epicentral distance



25	24	23	22	21	20	19	18	17	16	15	14	13
Alf	Alluvial deposits $\rho=1.99 \text{ g/cm}^3$ $\alpha=450 \text{ m/s}$ $\beta=210 \text{ m/s}$			M	Sands $\rho=1.91 \text{ g/cm}^3$ $\alpha=550 \text{ m/s}$ $\beta=280 \text{ m/s}$			Aa	Grey-blue clays $\rho=2.06 \text{ g/cm}^3$ $\alpha=1700 \text{ m/s}$ $\beta=650 \text{ m/s}$			
Sg	Quartzous sands $\rho=2.12 \text{ g/cm}^3$ $\alpha=1600 \text{ m/s}$ $\beta=500 \text{ m/s}$			ASg	Yellow clays $\rho=2.04 \text{ g/cm}^3$ $\alpha=1400 \text{ m/s}$ $\beta=280 \text{ m/s}$			E	Lavas $\rho=2.45 \text{ g/cm}^3$ $\alpha=1700 \text{ m/s}$ $\beta=500 \text{ m/s}$			

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