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OF SEISMIC WAVES GENERATION,
PROPAGATION AND THEIR INVERSION**

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*On the inverse problem for
earthquake rupture:
The Haskell-type source model*

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These are preliminary lecture notes, intended only for distribution to participants.

On the inverse problem for earthquake rupture: The Haskell-type source model

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Abstract. In order to gain insight into how to invert seismograms correctly to estimate the details of the earthquake rupturing process, we perform numerical experiments using artificial data, generated for an idealized faulting model with a very simple rupture and moment release history, and solve the inverse problem using standard widely used inversion methods. We construct synthetic accelerograms in the vicinity of an earthquake for a discrete analog of the Haskell-type rupture model with a prescribed rupture velocity in a layered medium. A constant level of moment is released as the rupture front passes by. We show that using physically based constraints, such as not permitting back slip on the fault, allow us to reproduce many aspects of the solution correctly, whereas the minimum norm solution or the solution with the smallest first differences of moment rates in space and time do not reproduce many aspects for the cases studied here. With the positivity of moment rate constraint, as long as the rupturing area is allowed to be larger than that in the forward problem, it is correctly found for the simple faulting model considered in this paper, provided that the rupture velocity and the Earth structure are known. If, however, the rupture front is constrained either to propagate more slowly or the rupturing area is taken smaller than that in the forward problem, we find that we are unable even to fit the accelerograms well. Use of incorrect crustal structure in the source region also leads to poor fitting of the data. In this case, the proper rupture front is not obtained, but instead a "ghost front" is found behind the correct rupture front and demonstrates how the incorrect crustal structure is transformed into an artifact in the solution. The positions of the centroids of the moment release in time and space are generally correctly obtained.

Introduction

With the deployment of high dynamic range, broad band digitally recording seismometers, and the availability of supercomputers, it has become feasible to consider the problem of inverting seismograms to obtain the details of the moment release time history and distribution on earthquake faults. The solution of this problem is important for the following reasons. Since the moment release on faults is generally expected to be nonuniform, one can identify regions of high and low moment release, or slip deficit. The slip or moment distribution obtained from such inversions can be used to infer the stress drop distribution due to the earthquake [e.g., Miyatake, 1992] that in turn can be used to estimate stress accumulation on faults. The slip deficit as well as the stress accumulation history on the fault can then lead to inferences about the times of future earthquakes on the fault. An example

where a portion of a fault with slip deficit in one earthquake ruptured relatively soon afterwards in another earthquake is the 1986 Andreanof Islands earthquake [Das, 1990]. The rupture zone of the Andreanof Islands earthquake was contained entirely within that of the 1957 Aleutian earthquake ($M_w = 8.6$), yet the 1986 earthquake of $M_w = 8.0$ occurred only 29 years later on a plate boundary that is believed to have a much larger characteristic repeat time. Noting that the region of major moment release in the 1986 earthquake coincided with the region of the 1957 event that had essentially no aftershocks, Das [1990] identified the 1986 earthquake to be due to the slip deficit left after the 1957 event. Another situation in which the inverse problem solution is useful is when one can relate the variations in moment release on the fault to the morphology, say jogs or bends, or cross-cutting physical features on faults, and so on. Such understanding can lead in the long term to successful prediction of the expected ground motion at particular sites of special interest, say, the locations of critical structures such as power plants, dams, bridges, etc., in regions where large earthquakes are expected. Finally, once the motion on the fault is reconstructed, the entire displacement field can be found by solving the appropriate

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forward problem. This makes it possible to estimate the motion at some site where there is damage but where there was no seismometer [e.g., *Suhadolc et al.*, 1990], thus enabling the cause of the damage, for example, focusing of waves on the site, to be investigated.

The inverse problem for the earthquake source was first formulated by *Kostrov* [1970], and discussed by *Kostrov* [1975] and by *Kostrov and Das* [1988]. Of the numerous studies that estimate the rupture and moment release history during earthquakes, we mention here the papers that develop a new method, or extend an existing method of inversion for both the spatial and temporal moment release pattern on the fault. These include *Olson and Apsel* [1982], *Kikuchi and Kanamori* [1982], *Hartzell and Heaton* [1983], *Kikuchi and Fukao* [1985], *Beroza and Spudich* [1988], *Mendoza and Hartzell* [1988a, 1988b, 1989], *Olson and Anderson* [1988], *Das and Kostrov*, [1990, 1994], *Hartzell et al.* [1991], and *Hartzell and Liu* [1995].

The limitations of such inversions have, however, not yet been studied sufficiently. For example, how close is the solution of this problem, which is well known to be unstable, to the actual moment distribution? How does poor knowledge of crustal structure in the source region affect the estimate of the rupture front location and speed? Since such inversions are not unique, what methods can one use to choose the "correct" solution from among the multiplicity of solutions? The last question cannot, in fact, be answered when working with real data, since the actual moment release at the depths where earthquakes occur is unknowable. In their studies of the great 1986 Andreanof Islands earthquake and the great 1989 Macquarie Ridge earthquake, *Das and Kostrov* [1990, 1994] attempted, using teleseismic data, to choose solutions from among the many possible ones. They demonstrated that more than one slip distribution can fit the data equally well. For the Macquarie Ridge earthquake, alternative slip distributions that could be interpreted as due to a propagating crack or to isolated asperities rupturing fit the data. The different rupture models would clearly lead to different stress accumulation patterns and histories on the fault [*Ruff*, 1983].

In geophysical inverse problems, the solution is often stabilized by using nonphysical prior bounds, such as finding the minimum norm solution or the smoothest solution. *Hartzell and Heaton* [1983] and *Das and Kostrov* [1990, 1994] investigated physically based constraints that can be used to stabilize the solutions. We shall show that physical constraints, such as not allowing back slip on the fault, produce the proper results, whereas the minimum norm solution and the solution with the smallest first differences do not, for the cases studied in this paper. *Hartzell and Heaton* [1983], *Mendoza and Hartzell* [1988a, 1988b, 1989], *Hartzell et al.* [1991], and *Hartzell and Liu* [1995] have used various stabilizing bounds in their inverse problem solutions and *Hartzell and Liu* [1995] summarize many features of such prior bounds.

In this paper we shall address only some specific aspects of the questions raised above by using artificial data where we do know what the correct solution is. We shall use the ver-

tical components of synthetic accelerograms constructed using the "far-field" approximation in the vicinity of the earthquake epicenter for source receiver distances in the 15 to 35 km range, generated by a very idealized model of earthquake rupture. We shall take the fault rupture model to be a Haskell-type dislocation [*Haskell*, 1964, 1966, 1969] propagating at a constant rupture velocity. This model has the great advantage of being very simple and has been widely used in seismic source studies, both for the forward problem and for the inverse problem. *Madariaga* [1978] proved that it is an appropriate model for simulating radiation with wavelengths longer than the fault width, as in this paper. Some of the studies of the inverse problem referred to above use essentially this model. We therefore use this simple model to obtain insight into the inverse problem. In this study we confine ourselves to using accurate data and seismograms close to the earthquake source. We do not discuss the teleseismic problem, although some of the results could be adapted to that case by scaling of the fault size and wave periods used here. Neither do we discuss the effects of noise in the data. Rather, we aim to gain insight into the basic problem of solving such unstable inverse problems by studying a very simple and idealized situation. Most importantly, working with synthetic data provides the possibility of identifying artifacts of the solution and their causes.

We first set out briefly the method used to generate the synthetic ground motion data. We next describe the inverse solution method, generate many sets of synthetic data for different faulting models, invert them, and present the results. Finally, we examine the limitations of the inverse problem for earthquake faulting, identifying, for example, which source properties we might be able to infer reliably, which ones depend strongly on knowledge of proper crustal structure, and so on.

Description of the Mathematical Problem

The formulation of the problem in terms of the slip rate or slip on the fault is well known and is stated only briefly here. Using the representation theorem (e.g., equation (3.2) of *Aki and Richards* [1980]; equation (3.2.18) of *Kostrov and Das* [1988]) and neglecting body forces, the displacement record at a station located at point $\mathbf{x}_1$ on the Earth's surface can be expressed in terms of the slip distribution over a fault surface, Σ , as an integral equation of the first kind [*Das and Kostrov*, 1990]

$$u_k(\mathbf{x}_1, t_1) = \int_0^{t_1} dt \iint_{\Sigma} K_{ik}(\mathbf{x}_1, \mathbf{y}_1, t_1, t) a_i(\mathbf{y}_1, t) dS, \quad (1)$$

where $i, k = 1, 2, 3$, $u_k(\mathbf{x}_1, t_1)$ are the components of the displacement vector, $a_i(\mathbf{y}_1, t)$ are the components of the slip, and $K_{ik}(\mathbf{x}_1, \mathbf{y}_1, t_1, t)$ are the components of the impulse response of the medium at $(\mathbf{x}_1, t_1)$, due to a dislocation point source at $(\mathbf{y}_1, t)$. By moving the time derivative that exists in the kernel K to the slip term in (1), we obtain an equivalent representation in terms of the slip-rate distribution over the fault, with the corresponding kernel G . In short,

$$\mathbf{u} = \mathbf{K} * \mathbf{a} = \mathbf{G} * \dot{\mathbf{a}}, \text{ where } \mathbf{K} = \dot{\mathbf{G}} \quad (2)$$

where $\mathbf{u}$ is the displacement vector, $\mathbf{a}$ and $\dot{\mathbf{a}}$ are the slip and slip rate vectors, respectively, asterisk denotes convolution over the fault area and source duration, and dot denotes time derivative. We shall assume that the fault is planar and that the slip direction is constant over the fault. We can use either form of (2) to find slip or slip rate, if we know the kernels $\mathbf{K}$ and $\mathbf{G}$ and the displacement seismograms. Slip rate is easily converted into moment rate given the modulus of rigidity.

Discretizing the problem, by dividing the fault into square cells and the source time function into steps, reduces (2) to the system of equations

$$\mathbf{A}\mathbf{x} \approx \mathbf{b}, \quad (3)$$

where $\mathbf{A}$ is the matrix obtained by integrating $\mathbf{G}$ or $\mathbf{K}$ over fault cells and time steps, each column of $\mathbf{A}$ being the appropriate discretized set of the kernel for all stations corresponding to different cells and time instants, ordered in the same way as the observations $\mathbf{b}$, and $\mathbf{x}$ are the unknown slips or slip rates. In this paper, we use the formulation in terms of the slip rates and the kernel $\mathbf{G}$.

The Green functions are determined using the multimodal summation method for a layered, anelastic medium [Panza, 1985; Panza and Suhadolc, 1987; Florsch et al., 1991]. In this paper we shall only use the vertical component of motion and Rayleigh wave modal summation to determine the Green function kernel. We have chosen to work initially with the vertical component only to make the matrix $\mathbf{A}$ smaller, for a given number of stations. The effect of including the horizontal component on the inversion will be investigated in the future.

The medium consists of homogeneous layers separated by first-order discontinuities. The modal summation method allows exact and complete solution of the full wave equation for a laterally homogeneous medium in a preassigned (ω, c) interval, where ω is the angular frequency, and c is the phase velocity [e.g. Aki and Richards, 1980]. The seismic source

is included in the computations using the formulation due to Harkrider [1964] and Ben-Menahem and Harkrider [1964]. The extended fault is modeled as a grid of point sources, and the synthetic seismogram at each station due to the moment release on the fault is computed by summing the contributions from each point source with appropriate delays and weights [Panza and Suhadolc, 1987].

In this paper the Green functions are computed for a maximum frequency of 1 Hz. The frequency domain is sampled with 200 points in the range DC to 1 Hz, which gives a frequency step of 0.005 Hz, yielding good frequency resolution between 0.1 Hz and 1 Hz. The upper frequency of 1 Hz implies minimum wavelengths on the order of 1 km for the velocity models considered in this paper. The achievable spatial resolution on the fault at a given instant of time is on the order of 0.5 km. The size of the time step used in constructing the Green functions is taken as about 0.1 s (more precisely as $200/2048 = 0.09765$ s, where 200 is the number of points in the frequency domain and 2048 is the number of points in the discrete Fourier transform).

Construction of the Synthetic Data

The synthetic accelerograms to be used in the inversion are constructed by performing the convolution in (2) for the particular faulting models considered. In this study the forward problem is a discretized form of the Haskell dislocation propagating unilaterally over a rectangular fault at a constant rupture velocity, $v_r = 0.7v_s$, where v_s is the shear wave speed in the medium. Figure 1 shows the fault geometry together with a schematic diagram of the propagating rupture. The fault is taken to be of pure dip-slip type with a 30° dip and with the top of the fault located at a depth of 1 km below the Earth's surface.

The discrete approximation (3), of equation (1), must be fine enough to be a good approximation to the integral in (1), must be representative for the wave lengths involved in the data, and yet be feasible to solve. The spatial cell size Δx

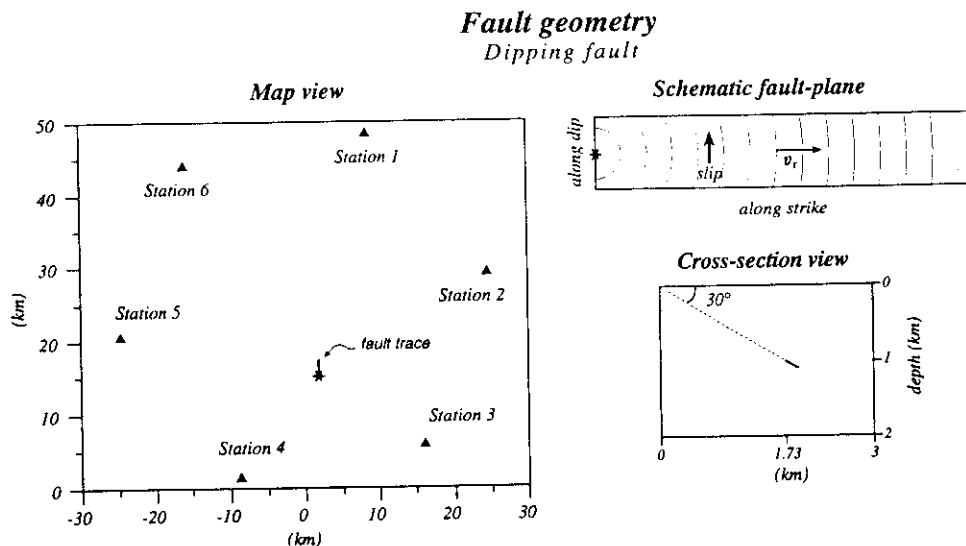


Figure 1. Fault and station geometry.

Table 1. Summary of Inversion Method, Parameters, and Results

Case	Source Medium (inv)	Faulting Parameters						Constraints Used	Inversion Method	Centroid ^a in Time (inv)	Centroid ^b Along Strike (inv)	Remarks
		Top of Fault (inv, km)	nh (fwd)	nh (inv)	nt (fwd)	nt (inv)	Centroid ^a in time (fwd)					
1a	M1	1	1	1	21	21	10.5	R1(0.7v _S),MTO	SVD	10.6	26.0	Only total moment correctly reproduced
(2)	M1	1	1	1	21	21	10.5	R1(0.7v _S),MTO,P	LP	10.5	25.5	Entire solution correctly reproduced
1b	M1	1	1	1	21	21	10.5	R2,MTO	SVD	10.6	26.0	Same as case 1a
(2)	M1	1	1	1	21	21	10.5	R2,MTO,P	LP	10.5	25.5	Entire solution reproduced
1c	M1	1	1	1	21	21	10.5	R1(0.5v _S),MTO,P	LP	9.8	17.3	Poor solution
1d	M1	1	1	1	21	21	10.5	R1(0.5v _S),MTO,P,M	LP	9.9	17.3	"
2a	M1	1	1	5	21	21	10.5	R1(0.7v _S),P	LP	10.6	26.0	Some aspects of solution correct
(2)	M1	1	1	5	21	21	10.5	R1(0.7v _S),P,MTO	LP	10.4	25.5	Entire solution correctly reproduced
2b	M1	1	5	1	21	21	10.5	R1(0.7v _S)	LP	10.9	26.2	Some aspects of solution correct
(2)	M1	1	5	1	21	21	10.5	R2	LP	10.3	23.9	Rupture front correctly found
3a	M2	2.05	1	1	21	38	10.5	R2,MTO,P	LP	20.3	24.3	Poor solution
3b	M2	2.05	1	5	21	38	10.5	R2,MTO,P	LP	11.1	20.3	"

$nx = 51$; $\Delta x = 50m$; $\Delta t = 0.097...s$ in all forward (fwd) and inverse (inv) cases.

FORWARD PROBLEM: Centroid along strike, measured from hypocenter/ $\Delta x = 25.5$; source in medium M1.

^aTime centroid is $/\Delta t$. ^bCentroid along strike is $/\Delta x$.

MTO is "more than once".

For meaning of other constraints, see text.

on the fault is taken as 50 m and the time step size Δt in the source time function as approximately 0.1 s, both for the forward and inverse problems. The rupture front is discretized using the same spatial and temporal gridding, with the moment being released at the center of each cell and in the middle of each time step as the rupture front crosses any portion on the cell within the time step interval. The moment is released only once in each cell, at the time the cell ruptures. The level of moment released at each cell is taken as constant and equal to 1×10^{11} N m. The problem using finer discretization in the forward than in the inverse problem is the subject of another paper [Das et al., 1995].

The size of the fault varies in the different cases considered here. Table 1 summarizes the length and width of the rupture area for each case. Figure 2 shows velocity profiles with depth for the Earth structures *M1* and *M2* that we shall use in this study. *M1* has a low velocity layer that is absent in *M2*. The *Q* values in the two models range from a value of 20 in the surficial layers to 100 in the deep sediments and 400 beneath them. The six stations, distributed equally in azimuth around the fault (Figure 1), are chosen so as not to involve nodal directions. The hypocenter is marked by a star and the source receiver distances lie in the 15 to 35 km range. All the synthetic accelerograms are sampled at the same time interval as the time step size used to construct the Green function, about 0.1 s, and the entire accelerogram is used in the inversion.

Solution of the Inverse Problem

Since the integral equation (1) is unstable, we need to stabilize it by the use of additional constraints. Olson and Apsel

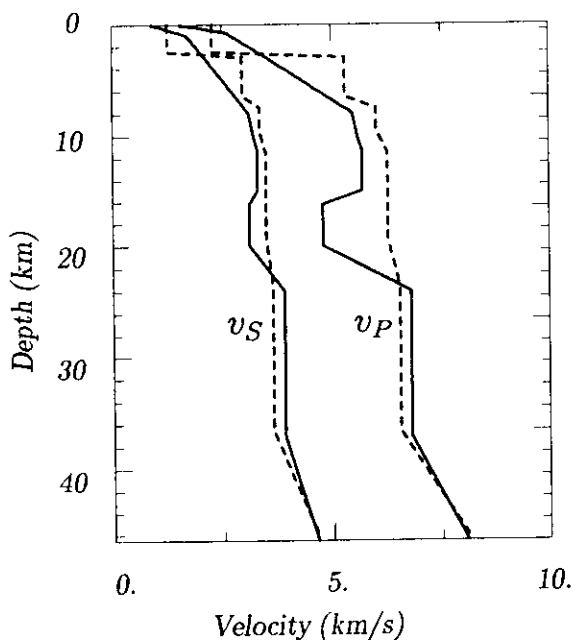


Figure 2. The P and S wave velocity profiles with depth in the crust for medium *M1* (solid line) and medium *M2* (dashed line).

[1982], Hartzell and Heaton [1983], Das and Kostrov [1990, 1994], and Hartzell and Liu [1995], among others, have identified constraints that can be used for this purpose. The physically based constraints we shall use in this paper are as follows:

1. Slip rate $\dot{x} \geq 0$ for all points on the fault for all time ("positivity" constraint *P*).
2. The final moment must equal some preassigned value ("moment" constraint *M*).
3. The rupture is constrained to move at, or more slowly than, some preassigned speed ("strong causality" constraint *R1*). It is important to point out that there is no rupture criterion involved in the discrete inverse problem, and by "rupture speed" we mean the rupture speed of some (unspecified) rupture triggering signal.

4. The moment rate is zero in any cell and time step that would produce a signal before the first arrival at any station from the hypocentral cell ("weak causality" constraint *R2*). In the case when there is insufficient station coverage, if this constraint is used without the constraint *R1* also being enforced, the inversion may permit super P "rupture speeds".

5. In the forward problem, moment is released only once as the cell breaks. In the inverse problem, a cell may be allowed to release moment more than once. The phrase 'more than once' (MTO) will be used to denote this case.

We select constraints for the different cases studied (Table 1) in order to gain insight into the effects on the solution of the constraints. Constraints of this type have been called "hard prior bounds" by Jackson [1979] and Backus [1988].

In addition, it is possible to improve the stability of the problem by the use of "soft prior bounds", such as finding the solution with the minimum norm or the smoothest solution in some sense, say, the solution with the smallest first differences. This is done by adding a term to the penalty functional $\|b - Ax\|_p$, where $p = 1, 2, \dots$, as desired. For example, to obtain the minimum norm solution, one then minimizes $\|b - Ax\|_p + \eta \|x\|_p$, where η is some weighting factor. To find the solution with the smallest first difference (in space and/or time), one minimizes $\|b - Ax\|_p + \eta$ times the chosen norm of the appropriate first differences of x , and so on. Such stabilizations have been used in the papers on inverse problem solution referred to earlier.

To solve the constrained linear system (3), we shall use two standard methods, discussed by Press et al. [1986], Tarantola [1987], and Parker [1994]. First, we solve (3) using the method of singular value decomposition (SVD), in which we minimize $\|b - Ax\|_2$. If the results are not satisfactory, for example, if the moment obtained is not the right one, then we again use SVD but constrain the moment to a preassigned value (constraint *M*). If the results are still not satisfactory, for example, if there are large negative moment rates on the fault, we remove small singular values and examine the solution. Finally, in some instances we shall find the solution with the smallest first differences in space and time and compare it with the correct solution. In the second approach, we solve (3) using various combinations of the physical constraints discussed above and the method of linear programming, in which the 1-norm of the penalty functional

is minimized. In some instances when using this second approach, we shall find solutions with the smallest second differences, following the formulation developed and applied to the earthquake faulting problem by *Das and Kostrov* [1990, 1994]. For some cases we use only the second approach to solve the inverse problem. The inversion method used together with the constraints and the results obtained for each case are summarized in Table 1.

Results

More than 40 inversions were performed. Selected cases are discussed in detail below.

Case 1

The synthesized vertical accelerograms in this case correspond to a unilateral rupture that spreads out over a 2.55 km-long and 50 m-wide fault in 21 time steps with a rupture velocity of $v_r = 0.7v_s$ in *M1*. The number of cells (nx) along fault strike is 51 and the number (nh) along the fault dip is 1. The number of time steps will be denoted by nt . The shortest wavelength (about 1.2 km for this case) is much larger than the fault width. Figure 3 shows the rupture model, the moment rate history and distribution and the final moment over the fault for this forward problem. (The corresponding source time function is shown later in Figure 5a.) The six synthesized accelerograms are displayed in Figure 4. Simply in order to check our programs, we solve this problem with the same rupture model and the same Earth structure as in the forward problem. The system of equations has 51 unknowns and about 1500 equations (sum of all the samples in all the accelerograms) and is an exact one. A simple unconstrained SVD solution is found to agree with the forward problem to machine accuracy.

Case 1a: Rupture front constrained to actual front.

We next constrain the rupture velocity v_r to be the same as in the forward problem ($0.7v_s$), but without restricting the number of times each cell behind the rupture front releases moment (MTO). The number of unknowns is now 554. The unconstrained SVD solution is close to the moment rate distribution of Figure 3 but with some negative values behind the rupture front, the magnitude of these moment rates being about 1% of the constant moment rate level of the forward problem. The solution fits the accelerograms to several significant figures. The solution source time function is compared to the true one in Figure 5a. For the first few time steps, when the area of slip on the fault is not large, the agreement is good, but at later times the source time function obtained oscillates about the correct solution. Though the total moment is correctly reproduced without being constrained, the final moment distribution on the fault, plotted in Figure 5b, also oscillates around the actual solution. Thus the negative moment rates, though small, when summed in space to produce the source time function or in time to obtain the final moment, have a nonnegligible contribution. Excluding small singular values did not improve the situation significantly. The centroids of the moment distribution in time and along strike are

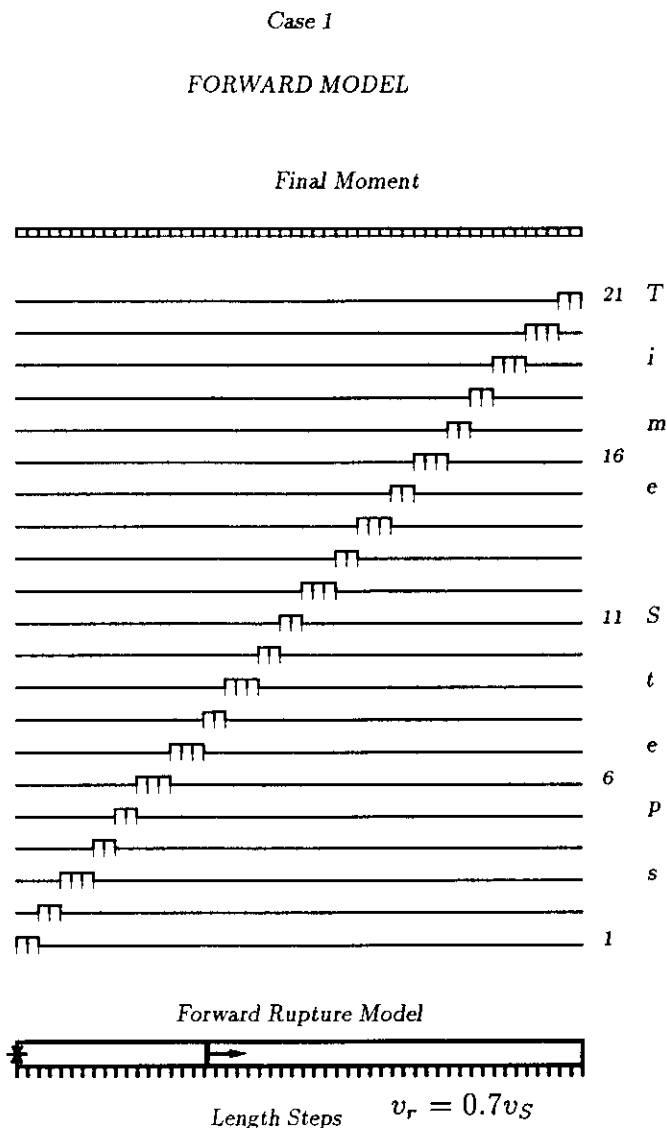


Figure 3. The forward model for case 1. The rupture model is displayed at the bottom of the figure, the moment rate time history along the fault length above it and the final moment along the fault is illustrated at the top. The fault length is taken along the abscissa and time is plotted along the ordinate. The numbers at the right indicate the time step. The level of moment release at each cell is 1×10^{11} N m. The cells that are slipping at each time step are indicated by tick marks below the trace. The scales used for plots similar to this will be kept the same throughout the paper to facilitate comparison, though the shift between time steps might vary in later figures in the interest of clarity. The solutions of the inverse problems for case 1, case 2a, and case 3 should be compared to this figure.

shown in Table 1. Both centroids are found to be close to the correct ones.

We next find the solution with the smallest first differences in space and time, but are still unable to remove the large number of small negative moment rates from the solution. We then solve the problem using linear programming and

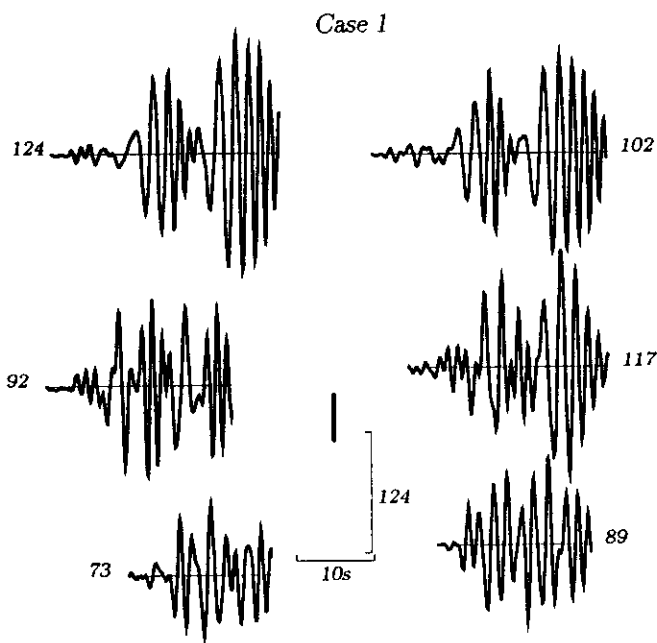


Figure 4. The synthetic accelerograms for the fault and station geometry of Figure 1, for the forward model of case 1. The thick line is the fault strike and the numbers next to each accelerogram give the maximum ground acceleration, multiplied by 100, in centimeters per second squared.

enforcing the moment rate to be positive (Case 1a(2)). All aspects of the forward model are now reproduced exactly. Even though cells have the freedom to release moment during more than one time step, it is found that each cell releases moment only once. Thus for the simple rupture model used here, we can reproduce the moment rate distribution and history on the fault if we know the rupture front, the focal mechanism and fault geometry, and Earth structure, by constraining the moment rate to be positive but without constraining the total moment. Since the predicted accelerograms cannot be distinguished from the original accelerograms (Figure 4), they are not plotted.

Case 1b: Weak causality applied to rupture front. We next apply the weak causality constraint (R2), with cells behind the causal front being allowed to release moment as often as necessary (MTO). We solve the inverse problem using both SVD and linear programming. The number of unknowns is now 987. The conclusions are the same as in case 1a. Thus as long as the positivity constraint is enforced, we can reproduce the rupture process using the weak causality constraint, knowing the fault mechanism, the fault geometry, and the Earth structure.

Case 1c: Rupture front constrained to propagate more slowly than in the forward problem. We next consider the same problem as in case 1a, but constrain the rupture front to a velocity of $0.5v_S$. We use only the linear programming approach in this case. The moment is not constrained and cells are allowed to release moment as often as necessary, but the moment rate is constrained to be positive. The number of unknowns is now 398. Owing to the low rupture speed con-

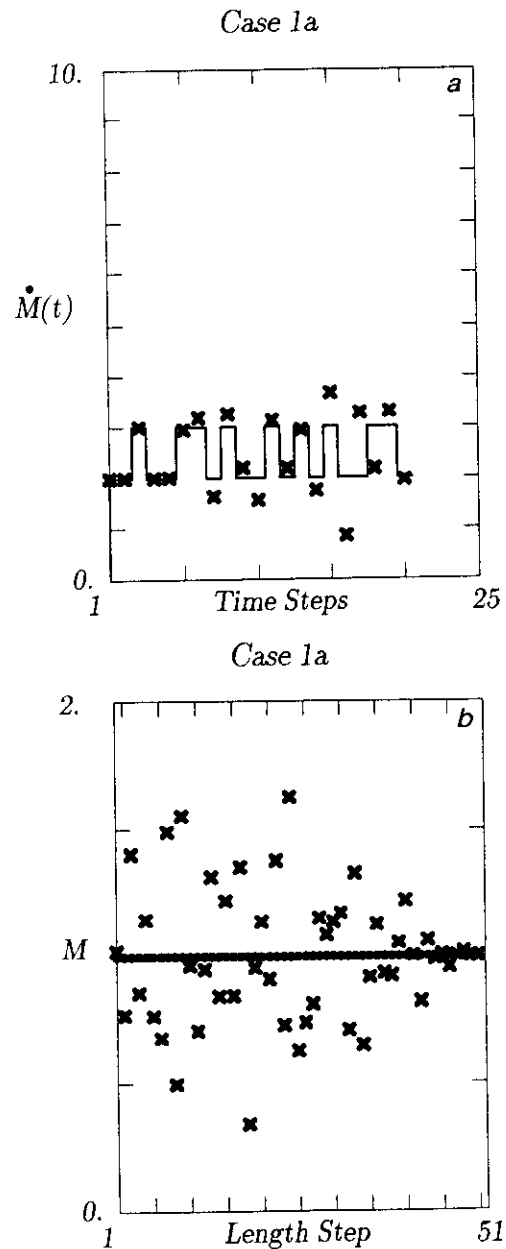


Figure 5. (a) Moment release per time step, multiplied by 1×10^{11} N m, on the fault for the forward problem (solid line) and the solution (crosses) for case 1a. In the forward problem either two or three cells break at each time step. The number of cells allowed to break at each time step is the same in the forward and in the inverse problem in this case, but the figure demonstrates that the amount of moment released at each time step in the inversion does not agree with that in the forward problem. (b) Final moment distribution, multiplied by 1×10^{11} N m, on the fault for the forward problem (dots) and the solution (crosses) for case 1a.

straint used in the inversion, only part of the fault can rupture in the total rupturing time, which is determined by the durations of the synthetic accelerograms. Figure 6 shows the forward and inverse rupture models and the moment rate history obtained. Figure 7a shows the fit to the data; the fit is not per-

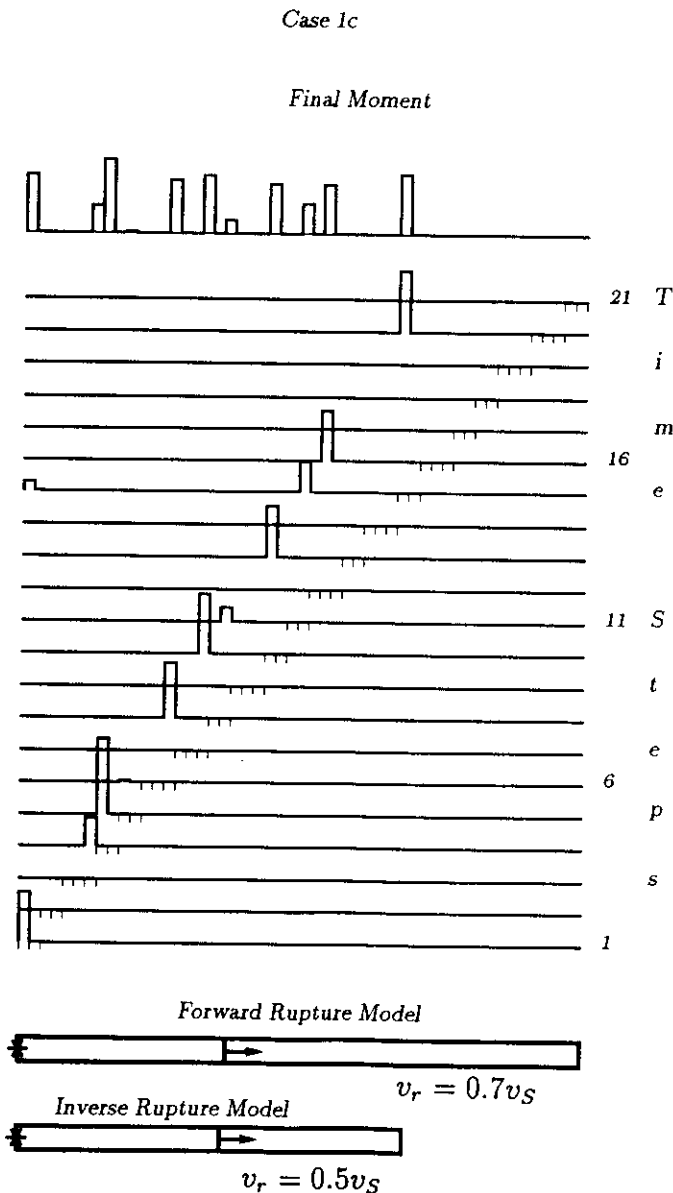


Figure 6. Same as Figure 3 but for the inversion in case 1c. Compare with Figure 3.

fect, the l_1 -misfit as defined by Das and Kostrov [1990] being 47%. However, the fit shown in Figure 7a would be considered very good for real data. Neither the moment rate distribution and history, the final moment, nor the source time function (Figure 7b) is reproduced well. The centroid in time is not far from the correct one but, as expected, the centroid along strike is not correctly obtained (see Table 1). Note also a cell rupturing well behind the rupture front. The moment was 20% larger than in the forward problem.

Case 1d: Rupture front constrained to propagate more slowly than that in forward problem, moment constrained. The solution obtained for this case using linear programming is not significantly better than for case 1c. Thus constraining the rupture front to propagate at too low a speed produced a poor fit to the data, which provides a clue that our inversion model is incorrect. We next attempt to

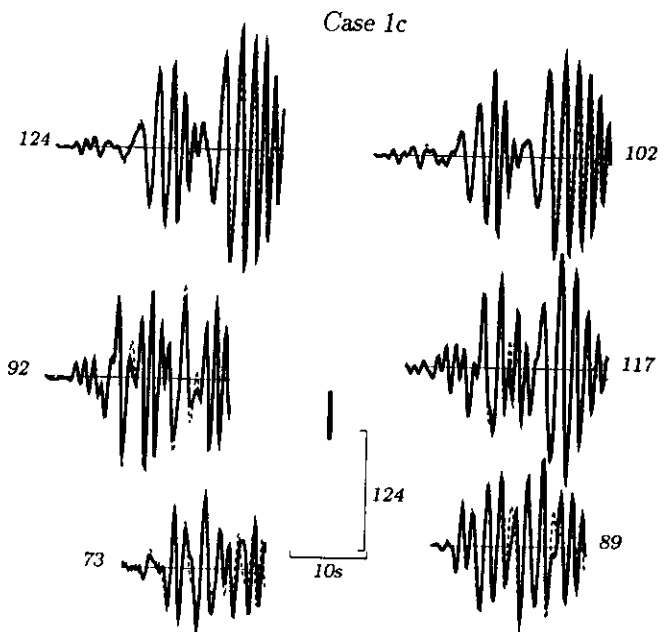


Figure 7a. Same as Figure 4 but for case 1c.

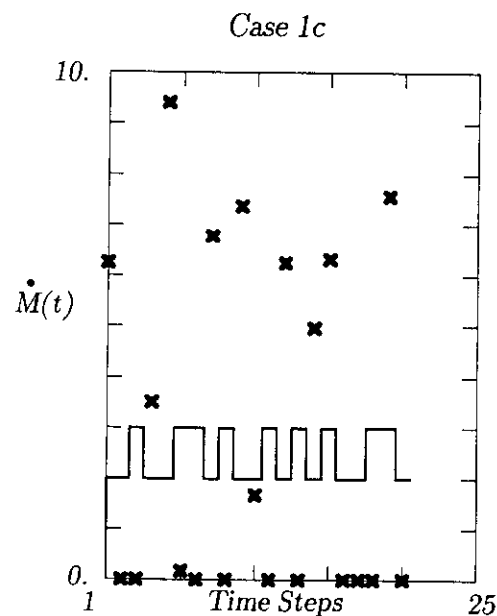


Figure 7b. Comparison of the data (solid lines) with synthetics (dashed lines) for case 1c. Notations and symbols are the same as in Figure 4.

smear out the moment distribution behind the rupture front by minimizing the maximum moment rate, as described by Das and Kostrov [1994], with 10% additional misfit to the data being permitted. It is found that the moment does spread out more evenly behind the rupture front but is still far from the correct solution. The results of a smoothed solution in which the sum of the moduli of the second differences of the moment rates were minimized, as formulated by Das and Kostrov [1994], with 10% additional misfit to the data being permitted, is very similar.

Case 2a

Case 2a

Final Moment

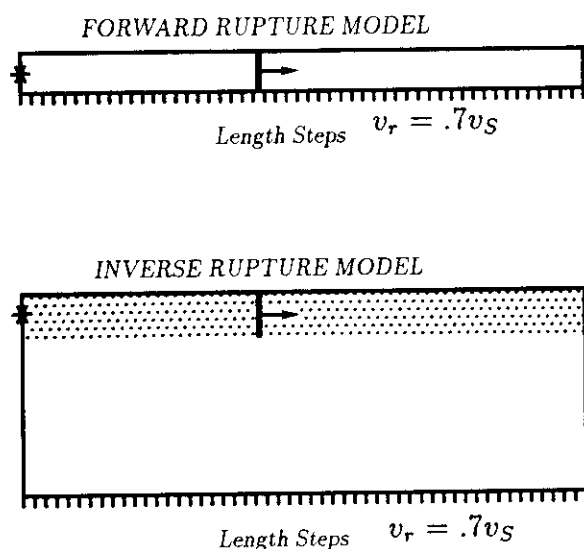


Figure 8. The forward and inverse rupture models used in case 2a. The top of the fault is at the same depth, 1 km, below the Earth's surface and the rupture nucleation points are marked by the asterisks. The shaded area in the inverse model is the part of the fault that slipped in the forward problem.

Case 2

We next consider a set of cases to obtain insight into the effect of inverting seismograms using a narrower or wider fault region than in the forward problem. We use the linear programming approach to solve the inverse problem in all cases in this section.

Case 2a: Inversion for wider fault than in forward problem. The forward and inverse rupture models for this case are illustrated in Figure 8. The forward problem is the same as in case 1, the data being shown in Figure 4, but the inverse model is taken as a 2.55 km x 250 m fault, with the top of the fault being at the same depth as in the forward case (1 km) and embedded in the same structure, *M1*. The rupture in the inversion model nucleates at the same point and propagates at the same speed ($0.7v_s$) as in the forward model, with each cell releasing moment only once as the rupture passes. The number of unknowns in this problem is 255. The only other constraint used is the positivity of moment rate. In the solution, the fit to the data is exact to three decimal places and the total moment is correctly reproduced. Figure 9a shows the moment rate history at the hypocentral depth level (stippled region in Figure 8), where the moment was released in the forward problem. Comparison with the correct solution (Figure 3) reveals that the level of moment release at the rupturing cells is not correctly obtained. Often a large moment is released at a certain cell without any moment being released at adjacent cells. No moment was released at the deeper levels even though this was allowed in the inversion. The final

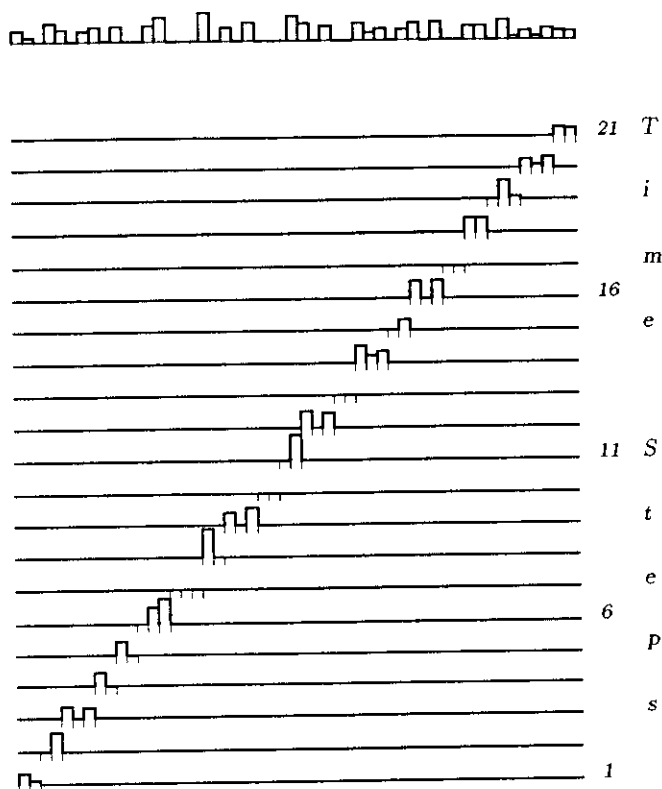


Figure 9a. Same as Figure 3 but for the inverse problem case 2a, plotted at the hypocentral cell level. Compare with Figure 3. No moment was released at the deeper parts of the fault, though this freedom was allowed in the inversion.

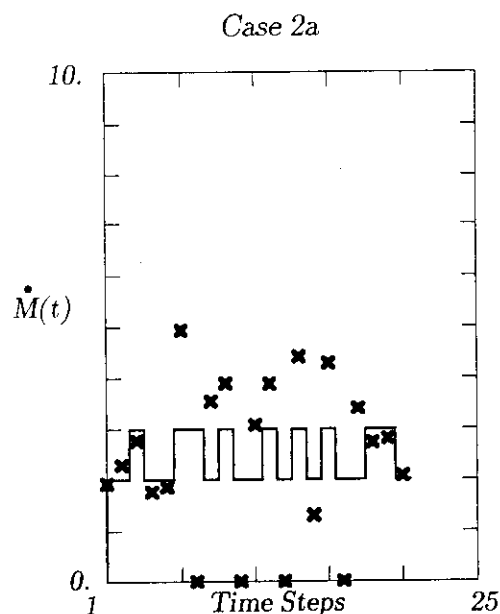


Figure 9b. Same as Figure 5a but for case 2a.

moment distribution, plotted at the top of Figure 9a, shows that the constant moment release over the fault is not reproduced. The source time function (Figure 9b) is also not reproduced correctly, but Table 1 shows that the centroids obtained are close to the correct ones. Next, the inverse problem is solved without any restriction on how often cells behind the rupture front are allowed to release moment (case 2a(2)). The number of unknowns is now 3731. The fit to the data is excellent and the total moment is correctly reproduced. The moment rate history at the hypocentral level and the final moment distribution on the fault are similar to that in case 2a. Thus when the positivity of moment rate is enforced, the width of the rupturing area and the moment centroids are correctly found, provided the rupture velocity and the Earth structure are known, even though the fault width in the inversion is larger than that used in creating the synthetic accelerograms. The moment release history, the final moment distribution, and the source time function are not, however, reproduced correctly.

Case 2b: Inversion for narrower fault area than in forward problem. The forward and inverse faulting models used in this case are illustrated in Figure 10. We construct synthetic accelerograms for the case of a 2.55 km x 250 m fault, with the rupture propagating at a constant speed of $0.7v_s$. Each cell is allowed to release moment only once as the rupture front passes. This is very similar to the classical "Haskell model" but with a curved rupture front. We perform the inversion using a 2.55 km x 50 m fault, with the top of the fault being at the same depth (1 km) as in the forward case and embedded in the same structure, *M1*. The rupture nucleation point and the rupture speed are the same in the inverse and forward models. The moment is not constrained and each cell is allowed to release moment only once as the rupture front passes. The number of unknowns in this case is 51. The l_1 misfit of the solution to the data is 18%, this

difference being essentially undetectable by eye. The rupture process in time is plotted in Figure 11a. The total moment is found to be 95% of the correct value but the uniform moment release at the rupture front is not reproduced correctly; very large moment is released at certain fault cells but none at adjacent ones. The source time function, shown in Figure 11b, is not correctly reproduced. The spatial and temporal centroids of the moment distribution are found to be very close to the actual solution (Table 1). We then solve this same problem using the weak causality constraint (case 2b(2)) in order to allow more freedom in the inversion, but permit cells to release moment only once. The number of unknowns increases to 1038. The l_1 misfit is 12% and the moment is larger by about 10%. Figure 12 shows that although the rupture front position is not preassigned, the moment release is confined primarily to the vicinity of the true rupture

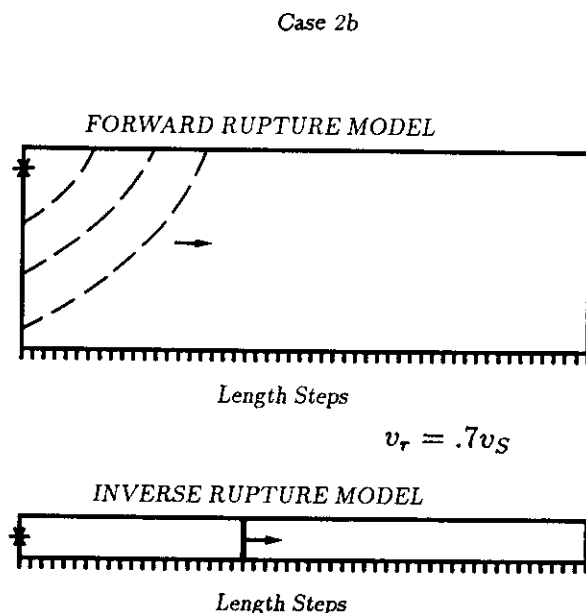


Figure 10. Same as Figure 8 but for case 2b.

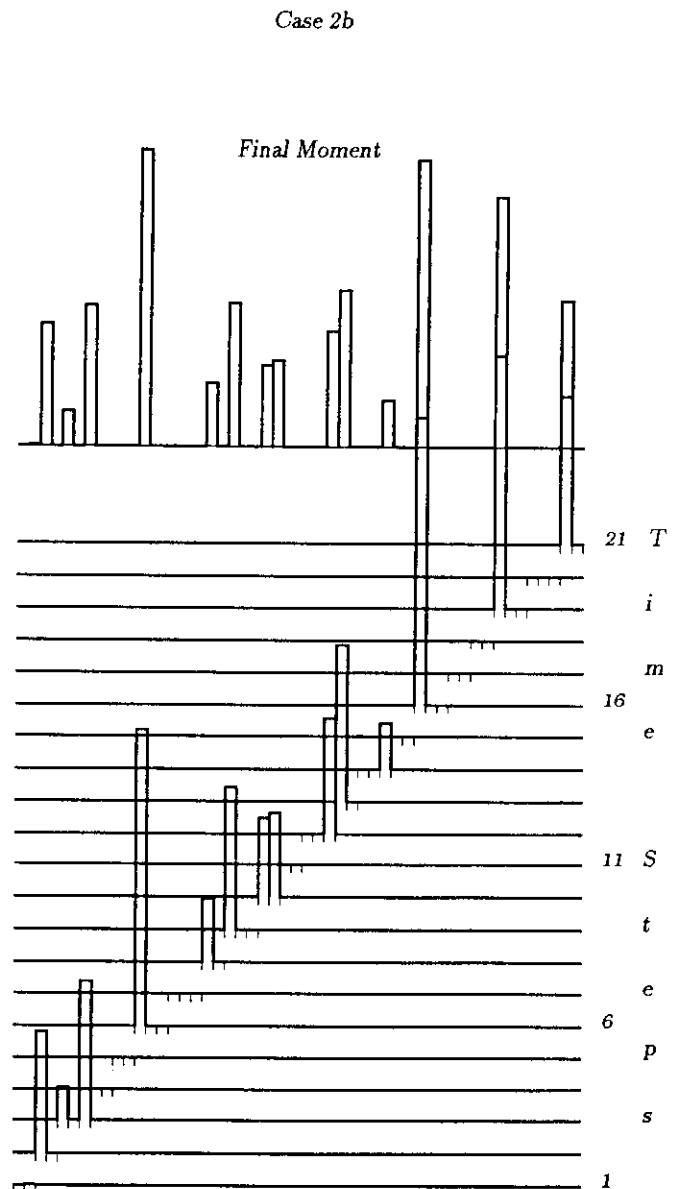


Figure 11a. Same as Figure 3 but for the inverse problem case 2b.

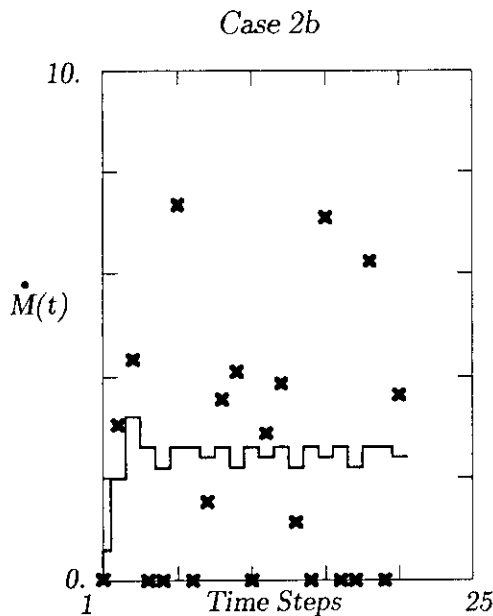


Figure 11b. Same as Figure 5a but for case 2b.

front, with some moment release both ahead and well behind it. The centroid of the moment release in both time and along fault strike is found correctly, but the source time function is not reproduced.

Case 3

We next consider a set of cases to find the effect of using incorrect Earth velocity models. Clearly, there is potential for such errors in Earth velocity to be aliased into artifacts in the solution. Here we demonstrate what some of these artifacts can be.

Case 3a: Effect of incorrect Earth structure. We construct synthetics for the 2.55 km x 50 m fault in medium $M1$, with the top of the fault located at a depth of 1 km below the Earth's surface (the forward problem of case 1). The rupture speed v_r is taken as $0.7v_s$ of medium $M1$. We solve the inverse problem using the same fault geometry but in medium $M2$. The rupture speed in the inversion is 70% of the shear wave speed of medium $M2$. Performing an SVD inversion, we obtain a very poor fit to the data with many negative moment rates. Adding constraints clearly will not improve the fit to the data. We then solve the problem using the linear programming approach and the positivity of moment rate constraint, but we are still unable to fit the data. Since the two media are different in the source region (Figure 2), we next determine the hypocentral depth in $M2$ for which the travel times of the first arrivals to the six stations are closest to those for the original source depth in $M1$, and place the fault at this depth (2.05 km) for the inversion. The forward and inverse faulting models are illustrated in Figure 13. We do not pre-assign the rupture speed but use only the weak causality constraint and allow grids to release moment more than once. Owing to the weak causality constraint, regions of the fault farthest from the nucleation region are found to rupture only

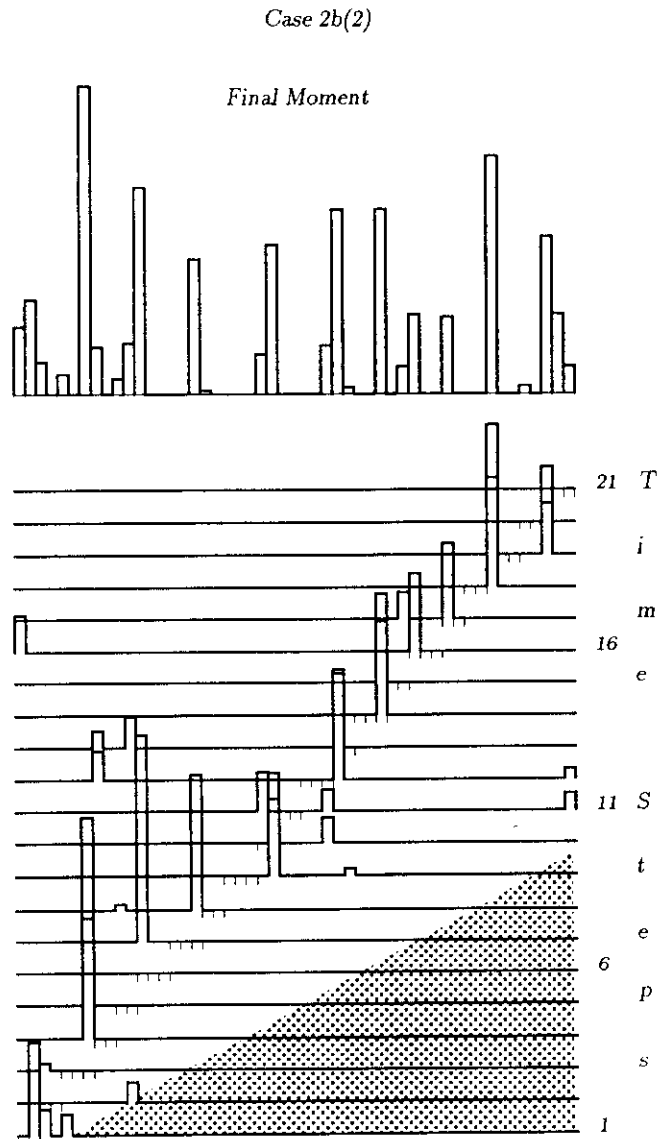


Figure 12. Same as Figure 3 but for the inverse problem case 2b(2). The tick marks shown at each time step are those for the forward problem. The region in space and time excluded by the weak causality constraint is indicated by stippling.

five time steps after nucleation, implying an apparent super P wave rupture speed. The duration of the entire source process is determined by the length of the synthetic accelerograms which is found to be 38 time steps for this inversion, the time step size being the same as in the forward problem, that is, approximately 0.1 s. The difference in the rupture durations of the forward and inverse cases is due to the different durations of the Green functions in the two media. The number of unknown moment rates is now 1818; the total moment is not constrained. Figure 14 shows the fit of the solution to the accelerograms. The fit is far from good, the l_1 norm of the misfit being 84%. Figure 15 displays the moment rate history and the final moment obtained. The first notable result is that the rupture front position is not correctly obtained and

Case 3a

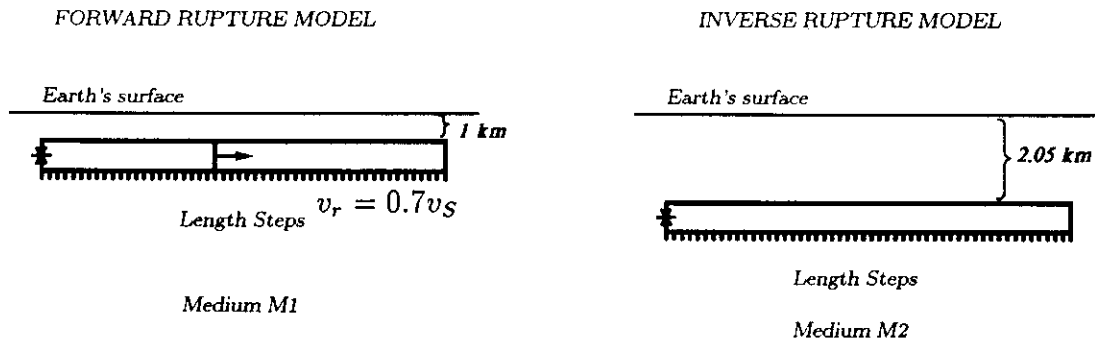


Figure 13. The forward and inverse rupture models used in case 3a. The top of the fault is at depth 1 km below the Earth's surface in the forward case and at depth 2.05 km in the inverse case. The rupture velocity in the forward problem is 70% of the shear wave speed of medium M1. The constraint R2 is used for the inversion in this case so that the rupture speed is not preassigned. The rupture nucleation points are marked by the asterisks.

the moment release appears to be somewhat randomly distributed on many parts of the fault, though an incoherent front can be identified, as indicated by the dotted line on Figure 15. The average speed of this front is about 60% of the shear wave speed of medium M2 at the level where the fault is located. The final uniform moment distribution of the forward problem and the source time function are not correctly reproduced. The moment obtained is 55% larger than the actual moment. The position of the centroid along strike is close to the correct one. The centroid in time, however, is far from correct which is not surprising since the duration of the process in the inversion is much longer than the correct one. But the most remarkable result here is the moment that is released

at later times on the fault. Figure 15 shows an additional coherent moment release in space and time appearing from $nt = 20$ on, defining a second moment release front. We call this a "ghost front" and it illustrates how the incorrect structure manifests itself as an artifact of the solution, and would lead in the real case to being interpreted physically as a secondary rupture front. The moment release ahead of the rupture front could potentially be interpreted in the real case as evidence for super shear rupture speed and the random nature of the moment release at some other places as evidence of "asperities" rupturing. The poor fit of the synthetics to the data is the clue that our input model is incorrect.

Case 3b: Effect of incorrect Earth structure with a larger fault size. Finally, we use a larger fault, 2.55 km x 250 m in the inverse model, with the top of the fault located at a depth of 2.05 km, to see if this additional freedom improves the fit to the data. We find that the fit does not improve significantly. Constraining the moment only worsens the fit, as expected. Hence with the incorrect structure we are simply unable to fit the data.

Discussion and Conclusions

Using synthetic data, we solve the inverse problem for a very simple faulting model in order to gain insight into solutions of such unstable problems. We demonstrate that the constraint of positivity of moment rate on the fault is essential to reproducing all facets of the solution, namely the moment release history and distribution, the source time function, and the final moment distribution. With this constraint, we find that even if we do not preassign the rupture front position it is identifiable in the inversion, for all practical purposes, when the medium properties are known and for the simple rupture model considered here. The centroids of the moment release in space and time are generally found to be

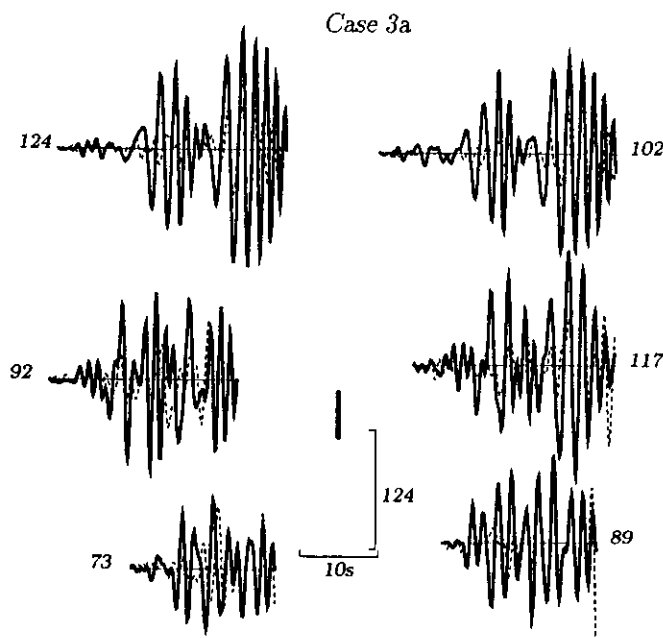


Figure 14. Same as Figure 4 but for case 3a.

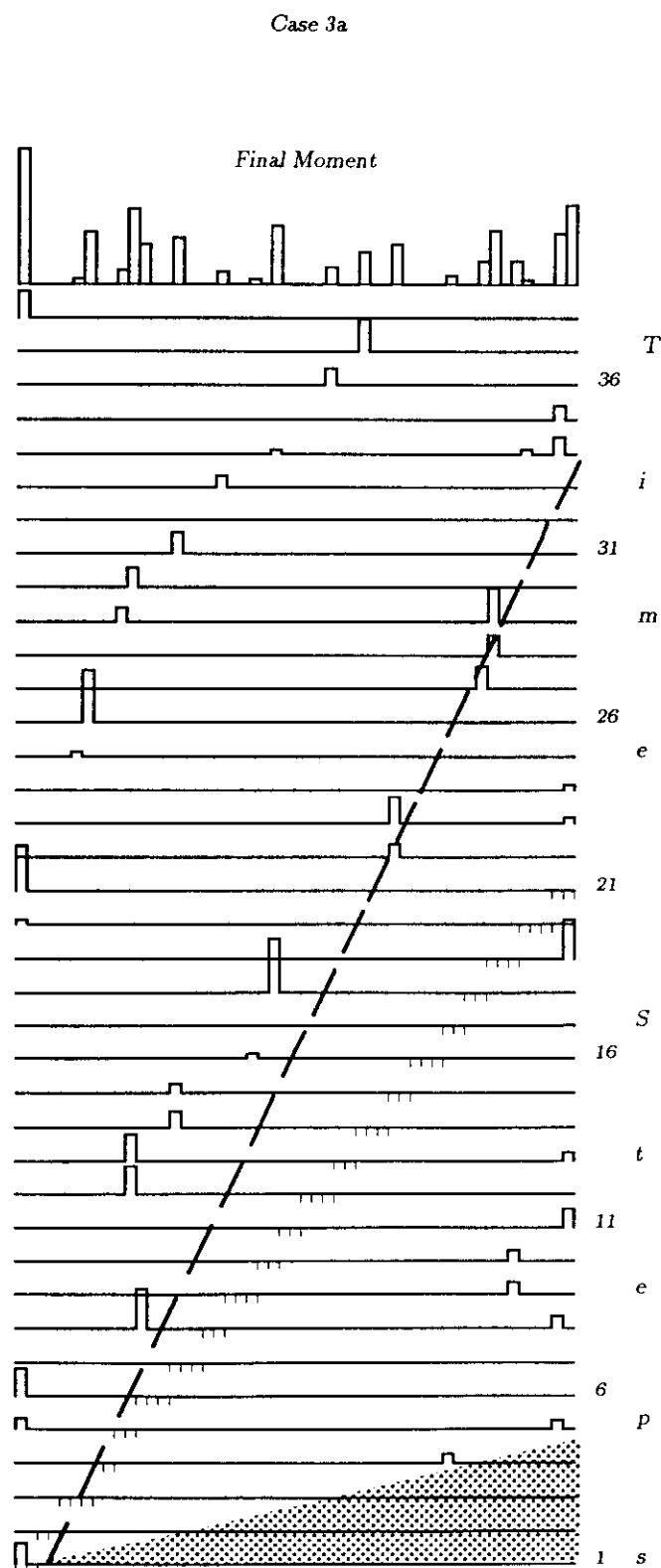


Figure 15. Same as Figure 12 but for the inverse problem case 3a. The tick marks shown at each time step are those for the forward problem in medium $M1$. The dashed line identifies the incoherent rupture front, propagating at an average speed of about 60% of the shear wave speed of the medium $M2$ at the depth where the fault is located.

close to the correct ones, even in cases where the fit to the data is poor, except that when the rupture front is constrained to propagate more slowly than in the forward problem (cases 1c and 1d), the spatial centroid is not correctly obtained. However, the constant level of moment release and the uniform final moment distribution of the forward problem generally are not correctly reproduced. In inversions of real data, such artifacts might be interpreted as evidence for heterogeneous faulting. We are unable to fit the data adequately if the rupture velocity is constrained to be lower than that in the true velocity or if the fault is constrained to be narrower than its true width. Use of incorrect crustal structure also has this effect. In the latter case, the position of the main rupture front is not obtained correctly. Instead, an additional coherent "ghost front" is obtained behind the rupture front, illustrating how poor knowledge of crustal structure can be manifest as an artifact in the solution.

The problem remains that in many cases the fit to the data is very good even when the faulting process is poorly reproduced, so that in the real case it would be difficult to know when one has obtained the correct solution. Then one must follow the suggestion of *Das and Kostrov* [1990, 1994] and consider many possible solutions, seeking physical characteristics that persist in many solutions. For example, if solutions resulting from differing constraints all show that the main moment release was at a particular region of the fault or give nearly the same average rupture velocity, then we may have some confidence in these features of the solution. Using data from the 1989 Macquarie Ridge earthquake, *Das and Kostrov* [1994] showed how to perform further optimizations to see if a particular common feature persists. If it does, then can one have some confidence that it is truly representative of the actual faulting process. This study, using artificial, noise-free data, also shows that small variations in the quantities obtained, such as rupture velocity, moment release over the fault, and so on, may not be reliable. The complications in using real, noisy data, deconvolution of instrument responses, and so on, will only make the situation even more difficult.

The results presented here suggest that it is essential to carry out a study such as this before inverting real data in order to have some idea of the limitations of the inversion for the particular case under investigation.

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