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**FOURTH WORKSHOP ON THREE DIMENSIONAL MODELLING
OF SEISMIC WAVES GENERATION.
PROPAGATION AND THEIR INVERSION**

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***Synthetic Seismograms from Multimode
Summation - Theory and Computational Aspects***

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These are preliminary lecture notes, intended only for distribution to participants.

SYNTHETIC SEISMOGRAMS FROM MULTIMODE SUMMATION — THEORY AND COMPUTATIONAL ASPECTS

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From the latest developments of algorithms for the computation of eigenvalues and eigenfunctions of Love (SH) and Rayleigh (P-SV) waves for flat anelastic layered media, it is possible to construct, with highly satisfactory efficiency and accuracy, complete broad-band synthetic seismograms. Therefore, computer programs based on the addition of modes are quite versatile and can be used to model, in a realistic way, seismological and seismic signals, also for laterally heterogeneous media.

Keywords: Friuli Imperial Valley; Love-wave; Raleigh-wave; seismic waves; synthetic seismogram

1. Introduction

In spite of the very considerable efforts made by seismologists and theoreticians, a satisfactory theory is still missing which describes accurately wave propagation in three-dimensional models of the Earth. If the extremely time consuming numerical procedures, based on finite differences or finite element methods, are excluded, all the existing analytical methods involve significant approximations. The modal summation method (Panza 1985, Florsch et al. 1989) is practically free from approximation in the one dimensional case and can be efficiently extended, introducing approximations of variable and to some extent quantifiable size, to two- and three-dimensional cases (Vaccari et al. 1989). The method allows us to construct very realistic signals, also in the relatively simple one-dimensional case, and can be very easily applied for a quantitative and realistic earthquake hazard assesement.

2. Waves in multilayered media

The medium is assumed to consist of homogeneous layers, separated by first-order discontinuities. If a medium is continuously inhomogeneous, it is replaced by a number of homogeneous layers; in smooth gradient zones it is usually sufficient to choose roughly half the dominant wavelength as layer thickness, whereas in transition zones with larger velocity gradients the layer thickness should be further reduced. The advantage of the homogeneous-layer approximation is that inside each layer the equation of motion takes a relatively simple form and can be solved exactly. Its disadvantage is that boundary conditions have to be fulfilled at many interfaces. Analytical methods for inhomogeneous layers — in contrast to numerical, e.g. finite-difference methods — are not yet developed to a point where they re-

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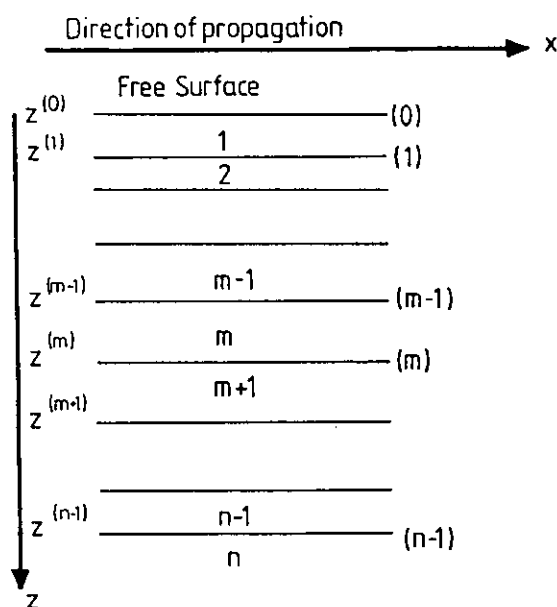


Fig. 1. Layered model and reference system

ally can complete with the methods for homogeneous layers. At present, within the frame of IGG-ICTP activities, a large project is being developed for the formulation of the theory and related computer code for the construction of complete synthetic seismograms for three-dimensional anelastic media, based on modal summation.

The equation of motion for a homogeneous, isotropic elastic medium is

$$\rho \mathbf{u}_{tt} = (\lambda + 2\mu) \text{grad div } \mathbf{u} - \mu \text{rot rot } \mathbf{u} \quad (1)$$

where $\mathbf{u}$ is the displacement vector, $\mathbf{u}_{tt}$ its second order time derivative, ρ is the density and λ and μ are the Lamé parameters. Body forces due to gravity and seismic sources are not included in Eq. (1): it is assumed that gravity has no other effect than to determine, via self compression, the constant values of ρ , λ and μ , and sources of seismic waves are included through their known contribution to $\mathbf{u}$ (Harkrider 1964). In order to simplify the discussion as far as possible, we shall consider solutions of the elastic equations of motion in the form of plane waves rather than attempt to treat the more complex case of waves diverging from a point-source. This does not involve a loss of generality in the computation of the dispersion function since the point-source solution may be developed by integration of plane-wave solutions (Harkrider 1964), with a preassigned precision depending upon source-receiver distance and wavelength (Panza et al. 1973).

Let us consider plane waves of angular frequency ω and horizontal phase velocity c propagating in a semi-infinite medium made up of n parallel, homogeneous, isotropic layers. The x axis is taken parallel to the layers with the positive sense in the direction of propagation. The positive z axis is taken as directed into the

medium. The various layers and interfaces are numbered away from the free surface, as shown in Fig. 1. We treat first waves of the Rayleigh type (P-SV motion), i.e. there is no displacement in the y direction and the amplitude diminishes exponentially in the positive z direction in the semi-infinite layer. The case of waves of Love type (SH motion) is treated in Section 4.

3. P-SV waves

For the m -th layer let ρ_m = density, d_m = thickness, λ_m and $\mu_m = \rho_m \beta_m^2$ = Lamé elastic constants, α_m = velocity of propagation of dilatational waves, β_m = velocity of propagation of rotational waves, $k = \omega/c$ = horizontal wave number, $\gamma_m = 2(\beta_m/c)^2$, u_m = displacement component in the x direction, w_m = displacement component in the z direction, σ_m = normal stress, τ_m = tangential stress.

For $m < n$ $r_{\alpha_m} = [(c/\alpha_m)^2 - 1]^{1/2}$ if $c > \alpha_m$ and $r_{\alpha_m} = -i[1 - (c/\alpha_m)^2]^{1/2}$, if $c < \alpha_m$; furthermore $r_{\beta_m} = [(c/\beta_m)^2 - 1]^{1/2}$, if $c > \beta_m$ and $r_{\beta_m} = -i[1 - (c/\beta_m)^2]^{1/2}$, if $c < \beta_m$. Finally, if $m = n$, $r_{\alpha_m} = -i[1 - (c/\alpha_m)^2]^{1/2}$ and $r_{\beta_m} = -i[1 - (c/\beta_m)^2]^{1/2}$.

Then periodic solutions of the elastic equation of motion for the m -th layer may be found by combining dilatational wave solutions,

$$\begin{aligned} \Delta_m &= (\partial u_m / \partial x) + (\partial w_m / \partial z) = \\ &= \exp[i(pt - kx)] [\Delta'_m \exp(-ikr_{\alpha_m} z) + \Delta''_m \exp(ikr_{\alpha_m} z)] \end{aligned} \quad (2)$$

with rotational wave solutions

$$\begin{aligned} \delta_m &= (1/2)[(\partial u_m / \partial z) - (\partial w_m / \partial x)] = \\ &= \exp[i(pt - kx)] [\delta'_m \exp(-ikr_{\beta_m} z) + \delta''_m \exp(ikr_{\beta_m} z)] \end{aligned} \quad (3)$$

where Δ'_m , Δ''_m , δ'_m and δ''_m are constants.

With the sign conventions defined above, the term in Δ'_m represents a plane wave whose direction of propagation makes an angle $\cot^{-1} r_{\alpha_m}$ with the $+z$ direction when r_{α_m} is real, and a wave propagated in the $+x$ direction with amplitude diminishing exponentially in the $+z$ direction when r_{α_m} is imaginary. Similarly, the term in Δ''_m represents a plane wave making the same angle with the $-z$ direction when r_{α_m} is real and a wave propagated in the $+x$ direction with amplitude increasing exponentially in the $+z$ direction when r_{α_m} is imaginary. The same applies to the terms in δ'_m and δ''_m with r_{β_m} substituted for r_{α_m} (see Figs 2a, 2b, 2c, 2d).

Dropping the term $\exp[i(\omega t - kx)]$ the displacements and the pertinent stress components corresponding to the dilatation and rotation, given by Eqs (2) and (3), can be written:

$$u_m = -(\alpha_m/\omega)^2 (\partial \Delta_m / \partial x) - 2(\beta_m/\omega)^2 (\partial \delta_m / \partial z) \quad (4)$$

$$w_m = -(\alpha_m/\omega)^2 (\partial \Delta_m / \partial z) + 2(\beta_m/\omega)^2 (\partial \delta_m / \partial x) \quad (5)$$

$$\begin{aligned} \sigma_m &= \rho_m \{ \alpha_m^2 \Delta_m + 2\beta_m^2 [(\alpha_m/\omega)^2 (\partial^2 \Delta_m / \partial x^2) + \\ &\quad + 2(\beta_m/\omega)^2 (\partial^2 \delta_m / \partial x \partial z)] \} \end{aligned} \quad (6)$$

$$\begin{aligned} \tau_m &= 2\rho_m \beta_m^2 \{ -(\alpha_m/\omega)^2 (\partial^2 \Delta_m / \partial x \partial z) + \\ &\quad + (\beta_m/\omega)^2 [(\partial^2 \delta_m / \partial x^2) - (\partial^2 \delta_m / \partial z^2)] \}. \end{aligned} \quad (7)$$

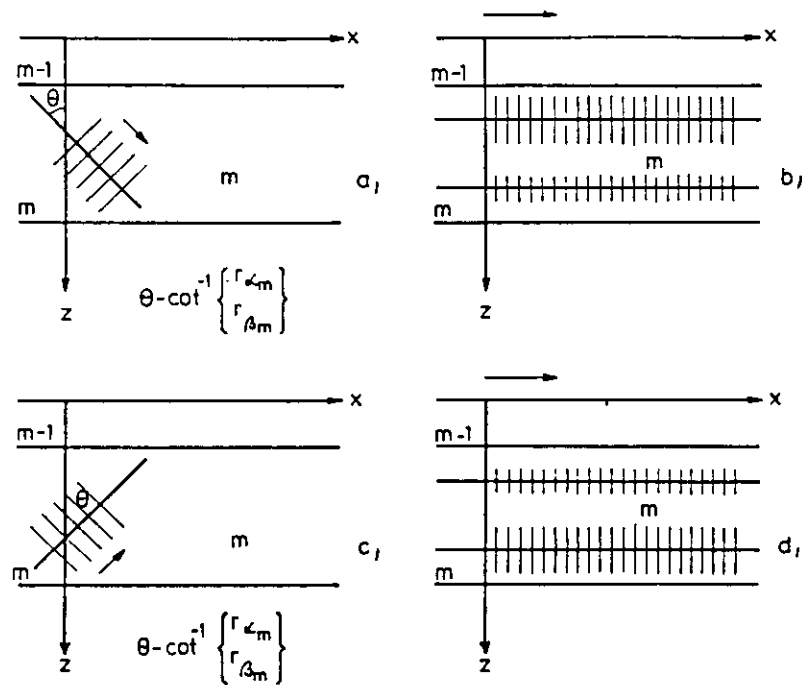


Fig. 2. With the sign conventions defined in Fig. 1, the term in Δ'_m of Eq. (2) represents a plane wave whose direction of propagation makes an angle $\cot^{-1} r_{\alpha_m}$ with the $+z$ direction when r_{α_m} is real, and a wave propagated in the $+z$ direction with amplitude diminishing exponentially in the $+z$ direction when r_{α_m} is imaginary. Similarly, the term in Δ''_m Eq. (2) represents a plane wave making the same angle with the $-z$ direction when r_{α_m} is real and a wave propagated in the $+z$ direction with amplitude increasing exponentially in the $+z$ direction when r_{α_m} is imaginary. The same applies to the terms in δ'_m and δ''_m Eq. (3) with r_{β_m} substituted for r_{α_m} (see Figs 2a, 2b, 2c, 2d)

The boundary conditions at an interface between two layers require that these four quantities should be continuous. Continuity of the displacements is assured if the corresponding velocity components $\dot{u}_m$ and $\dot{w}_m$ are made continuous and, since c is the same in all layers, we may take the dimensionless quantities $\dot{u}_m/c$ and $\dot{w}_m/c$ to be continuous. Substituting the expressions (2) and (3) in Eqs (4) to (7) and expressing the exponential functions of ikr_z in trigonometric form, we find

$$c\dot{u}_m = A_m \cos p_m - iB_m \sin p_m + r_{\beta_m} C_m \cos q_m - ir_{\beta_m} D_m \sin q_m \quad (8)$$

$$c\dot{w}_m = -ir_{\alpha_m} A_m \sin p_m + r_{\alpha_m} B_m \cos p_m + iC_m \sin q_m - D_m \cos q_m \quad (9)$$

$$\sigma_m = \varrho_m(\gamma_m - 1)A_m \cos p_m - i\varrho_m(\gamma_m - 1)B_m \sin p_m + \varrho_m\gamma_m r_{\beta_m} C_m \cos q_m - i\varrho_m\gamma_m r_{\beta_m} D_m \sin q_m \quad (10)$$

$$\tau_m = i\varrho_m\gamma_m r_{\alpha_m} A_m \sin p_m - \varrho_m\gamma_m r_{\alpha_m} B_m \cos p_m - i\varrho_m(\gamma_m - 1)C_m \sin q_m + \varrho_m(\gamma_m - 1)D_m \cos q_m \quad (11)$$

where

$A_m = -\alpha_m^2(\Delta'_m + \Delta''_m)$, $B_m = -\alpha_m^2(\Delta'_m - \Delta''_m)$, $C_m = -2\beta_m^2(\delta'_m - \delta''_m)$, $D_m = -2\beta_m^2(\delta'_m + \delta''_m)$, $p_m = kr_{\alpha_m}[z - z^{(m-1)}]$, $q_m = kr_{\beta_m}[z - z^{(m-1)}]$, $z^{(m-1)}$ is the depth of the upper interface of the m -th layer and $\Delta'_m, \Delta''_m, \delta'_m, \delta''_m$ are the constants defined in (6) appearing in the depth-dependent part of the dilatational and rotational wave solutions:

$$\Delta'_m \exp(-ikr_{\alpha_m} z) + \Delta''_m \exp(ikr_{\alpha_m} z) \quad (12)$$

$$\delta'_m \exp(-ikr_{\beta_m} z) + \delta''_m \exp(ikr_{\beta_m} z). \quad (13)$$

3.1 Evaluation of eigenvalues and eigenfunctions

For a continental model, the vanishing of the two components of stress at the free surface yields:

$$-\varrho_1(\gamma_1 - 1)A_1 - \varrho_1\gamma_1 r_{\beta_1} C_1 = 0 \quad (14)$$

$$\varrho_1\gamma_1 r_{\alpha_1} B_1 - \varrho_1(\gamma_1 - 1)A_1 = 0. \quad (15)$$

Thus the submatrix $\Lambda^{(0)}$ defined in Knopoff (1964) and Schwab (1970) can be written in the form

$$\Lambda^{(0)} = \begin{vmatrix} -\varrho_1(\gamma_1 - 1) & 0 & \varrho_1\gamma_1 & 0 \\ 0 & \varrho_1\gamma_1 & 0 & -\varrho_1(\gamma_1 - 1) \end{vmatrix} \quad (16)$$

At the m -th interface, the continuity of displacement and stress yields

$$A_m \cos P_m - iB_m \sin P_m + r_{\beta_m} C_m \cos Q_m - \\ - ir_{\beta_m} D_m \sin Q_m = A_{m+1} + r_{\beta_{m+1}} C_{m+1} \quad (17)$$

$$-ir_{\alpha_m} A_m \sin P_m + r_{\alpha_m} B_m \cos P_m + \\ + iC_m \sin Q_m - D_m \cos Q_m = r_{\alpha_{m+1}} B_{m+1} - D_{m+1} \quad (18)$$

$$\varrho_m(\gamma_m - 1)A_m \cos P_m - i\varrho_m(\gamma_m - 1)B_m \sin P_m + \\ + \varrho_m\gamma_m r_{\beta_m} C_m \cos Q_m - i\varrho_m\gamma_m r_{\beta_m} D_m \sin Q_m = \\ = \varrho_{m+1}(\gamma_{m+1} - 1)A_{m+1} + \varrho_{m+1}\gamma_{m+1} r_{\beta_{m+1}} C_{m+1} \quad (19)$$

$$i\varrho_m\gamma_m r_{\alpha_m} A_m \sin P_m - \varrho_m\gamma_m r_{\alpha_m} B_m \cos P_m - \\ - i\varrho_m(\gamma_m - 1)C_m \sin Q_m + \varrho_m(\gamma_m - 1)D_m \cos Q_m = \\ = \varrho_{m+1}\gamma_{m+1} r_{\alpha_{m+1}} B_{m+1} + \varrho_{m+1}(\gamma_{m+1} - 1)D_{m+1} \quad (20)$$

where $P_m = kr_{\alpha_m} d_m$, $Q_m = kr_{\beta_m} d_m$ and d_m is the layer thickness. Thus the interface submatrices defined by Schwab (1970) have the form

$$\Lambda^{(m)} = \begin{vmatrix} \cos P_m & -i \sin P_m / r_{\alpha_m} \\ -ir_{\alpha_m} \sin P_m & \cos P_m \\ \varrho_m(\gamma_m - 1) \cos P_m & -i\varrho_m(\gamma_m - 1) \sin P_m / r_{\alpha_m} \\ i\varrho_m\gamma_m r_{\alpha_m} \sin P_m & -\varrho_m\gamma_m \cos P_m \end{vmatrix}$$

$$\begin{array}{cccc}
 \cos Q_m & & -ir_{\beta_m} \sin Q_m & \\
 i \sin Q_m / r_{\beta_m} & & -\cos Q_m & \\
 \varrho_m \gamma_m \cos Q_m & & -i \varrho_m \gamma_m r_{\beta_m} \sin Q_m & \\
 -i \varrho_m (\gamma_m - 1) \sin Q_m / r_{\beta_m} & & \varrho_m (\gamma_m - 1) \cos Q_m & \\
 \hline
 -1 & 0 & -1 & 0 \\
 0 & -1 & 0 & 1 \\
 -\varrho_{m+1}(\gamma_{m+1} - 1) & 0 & -\varrho_{m+1} \gamma_{m+1} & 0 \\
 0 & \varrho_{m+1} \gamma_{m+1} & 0 & -\varrho_{m+1}(\gamma_{m+1} - 1)
 \end{array} \quad (21)$$

and, noting that, when imposing surface waves conditions, in the half space $\Delta_n'' = \delta_n'' = 0$, $A_n = B_n = -\alpha_n^2 \Delta_n'$ and $C_n = D_n = -2\beta_n \omega_n'$, the submatrix representing the $(n-1)$ th interface has the form

$$\Lambda^{(n-1)} = \begin{vmatrix} \dots & -1 & -r_{\beta_n} \\ \dots & -r_{\alpha_n} & 1 \\ \dots & -\varrho_n(\gamma_n - 1) & -\varrho_n \gamma_n r_{\beta_n} \\ \dots & \varrho_n \gamma_n r_{\alpha_n} & -\varrho_n(\gamma_n - 1) \end{vmatrix} \quad (22)$$

where the first four columns are the same as those of $\Lambda^{(m)}$ with $m = n-1$. For each layer, $\Lambda^{(i)}$ ($i = 1, n$) submatrices represent the denominators of Cramer's system solutions when the boundary conditions are applied. In more compact notation it can be written

$$\Delta_R = \begin{vmatrix} |\Lambda^{(0)}| & & & \\ & |\Lambda^{(1)}| & & \\ & & \ddots & \\ & & & |\Lambda^{(n-2)}| \\ & & & & |\Lambda^{(n-1)}| \end{vmatrix} \quad (23)$$

where the non zero elements are only written. A condition for surface waves to exist is $\Delta_R = 0$, which defines the dispersion function for Rayleigh waves:

$$F_R(\omega, c) = \Delta_R = 0. \quad (24)$$

The discrete solutions (ω, c) of Eq. (24) describe, in each of the layers, body waves or surface waves depending upon the real or imaginary nature of r_{α_m} and r_{β_m} . More precisely real values of r_{α_m} and r_{β_m} correspond to P- and S-waves while imaginary values of r_{α_m} and r_{β_m} correspond to surface waves. Therefore the modal summation method allows to solve in an exact and complete way the full wave equation in a preassigned (ω, c) interval. In other words it is possible to describe all the rays propagating with phase velocity less than a preassigned maximum value. It is easy to prove that using the modal representation the upper limit for the phase velocity is represented by the S-wave velocity value assigned to the half space used to terminate the structure at depth.

Once the eigenvalue problem is solved it is possible to determine eigenfunctions, i.e. displacements and stresses.

[illegible]

4. SH waves

$$u_m = w_m = \sigma_m = \tau_m = 0 \quad (26)$$

$$v_m = \exp i(\omega t - kx)[v'_m \exp(-ikr_{\beta_m} z) + v''_m \exp(ikr_{\beta_m} z)] \quad (27)$$

$$v_m = \mu_m(\partial v_m / \partial z) =$$

$$= ik\mu_m r_{\beta_m} \exp i(\omega t - kx)[v_m'' \exp(ikr_{\beta_m} z) - v_m' \exp(-ikr_{\beta_m} z)]. \quad (28)$$

$$\dot{v}_m/c = (\dot{v}_{m-1}/c) \cos Q_m + i v_{m-1} (\mu_m r_{\beta_m})^{-1} \sin Q_m \quad (29)$$

$$v_m = i(v_{m-1}/c)\mu_m r_{\beta_m} \sin Q_m + v_{m-1} \cos Q_m. \quad (30)$$

4.1 Evaluation of eigenvalues and eigenfunctions

From Eqs (29) and (30) the layer matrix can be defined:

$$a_m = \begin{bmatrix} \cos Q_m & \frac{i \sin Q_m}{\mu_m \cdot r_{\beta_m}} \\ i\mu_m \cdot r_{\beta_m} \cdot \sin Q_m & \cos Q_m \end{bmatrix}. \quad (31)$$

For the multimode surface-wave eigenvalue computations, using notation of Schwab and Knopoff (1972), the dispersion function can be written as the modified product for layer-matrices:

$$F_L(\omega, c) = b_n \cdot b_{n-1} \cdot b_{n-2} \cdot \dots \cdot b_1 \quad (32)$$

where n is the number of layers, including the lower halfspace. In Eq. (32) b_n is given by:

$$\begin{aligned} b_n &= (s, -1) \text{ if the halfspace is solid} \\ b_n &= (0, -1) \text{ if the halfspace is liquid} \\ b_n &= (1, 0) \text{ if the halfspace is rigid} \end{aligned} \quad (33)$$

where

$$s = -\mu_n \cdot \left(1 - \left(\frac{c}{\beta_n}\right)^2\right)^{\frac{1}{2}} \quad (34)$$

b_m ($0 < m < n$) is given by:

$$b_m = \begin{bmatrix} \cos Q_m & \frac{\sin Q_m}{\mu_m \cdot r_{\beta_m}} \\ \mu_m \cdot r_{\beta_m} \cdot \sin Q_m & \cos Q_m \end{bmatrix} \quad \text{if } c > \beta_m \quad (35)$$

$$b_m = \begin{bmatrix} \cosh Q_m^* & \frac{\sinh Q_m^*}{\mu_m \cdot r_{\beta_m}^*} \\ -\mu_m \cdot r_{\beta_m}^* \cdot \sinh Q_m^* & \cosh Q_m^* \end{bmatrix} \quad \text{if } c < \beta_m \quad (36)$$

$$b_m = \begin{bmatrix} 1 & \frac{p \cdot d_m}{\mu_m \cdot c} \\ 0 & 1 \end{bmatrix} \quad \text{if } c = \beta_m \quad (37)$$

where we have introduced the real part of imaginary quantities

$$\left. \begin{aligned} r_{\beta_m}^* &= -\left(1 - \left(\frac{c}{\beta_m}\right)^2\right)^{\frac{1}{2}} \\ Q_m^* &= \frac{p \cdot r_{\beta_m}^* \cdot d_m}{c} = k \cdot r_{\beta_m}^* \cdot d_m \end{aligned} \right\} \quad \text{if } c < \beta_m. \quad (38)$$

The modified matrix product of b_m and b_{m-1} is defined as follows:

$$[b_m \cdot b_{m-1}]_{jk} = \begin{cases} (b_m)_{j\ell} \cdot (b_{m-1})_{\ell k} & \text{if } (j+k) \text{ is even} \\ (-1)^{j+1} \cdot (b_m)_{j\ell} \cdot (b_{m-1})_{\ell k} & \text{if } (j+k) \text{ is odd} \end{cases} \quad (39)$$

The mathematical solution of the surface wave propagation allows two types of waves in the solid halfspace, exponentially increasing and decreasing with depth. To avoid infinite values of the solution, the coefficient of exponentially increasing wave in the halfspace must vanish (surface waves condition). If the halfspace is supposed to be liquid, the deepest interface is at the analogy of the mantle-core boundary. In analogy with the case of P-SV waves, imaginary values of r_{β_m} correspond to surface waves, while real values of r_{β_m} correspond to S-waves. More precisely real values of r_{β_m} correspond to S-waves while imaginary values of r_{β_m} correspond to surface waves. Therefore also for SH-waves, the modal summation method gives a solution in an exact and complete way for the full wave equation in a preassigned (ω, c) interval. In other words it is possible to describe all the rays propagating with phase velocity less than a preassigned maximum value. It is easy to prove that using the modal representation, the upper limit for the phase velocity is represented by the S-wave velocity value assigned to the half space used to terminate the structure at depth.

With the geometry shown in Fig. 1, the computation of the eigenfunctions at the layer interfaces can be performed as follows (Schwab 1970):

$$\begin{bmatrix} v_m \\ v_m \end{bmatrix} = \begin{bmatrix} \cos Q_m & \frac{\sin Q_m}{k \cdot \mu_m \cdot r_{\beta_m}} \\ -k \cdot \mu_m \cdot r_{\beta_m} \cdot \sin Q_m & \cos Q_m \end{bmatrix} \cdot \begin{bmatrix} v_{m-1} \\ v_{m-1} \end{bmatrix} \quad \text{if } c > \beta_m \quad (40)$$

$$\begin{bmatrix} v_m \\ v_m \end{bmatrix} = \begin{bmatrix} \cosh Q_m^* & \frac{\sinh Q_m^*}{k \cdot \mu_m \cdot r_{\beta_m}^*} \\ k \cdot \mu_m \cdot r_{\beta_m}^* \cdot \sinh Q_m^* & \cosh Q_m^* \end{bmatrix} \cdot \begin{bmatrix} v_{m-1} \\ v_{m-1} \end{bmatrix} \quad \text{if } c < \beta_m \quad (41)$$

$$\begin{bmatrix} v_m \\ v_m \end{bmatrix} = \begin{bmatrix} 1 & \frac{d_m}{\mu_m} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_{m-1} \\ v_{m-1} \end{bmatrix} \quad \text{if } c = \beta_m \quad (42)$$

where v_m is the displacement and v_m the stress at the interface m . Notice that:

$$\dot{v}_m = i\omega v_m \quad (43)$$

For the last interface, supposing a solid terminating halfspace, we shall use:

$$\begin{cases} v_{n-2} \cdot \cos Q_{n-1} + \frac{v_{n-2} \cdot \sin Q_{n-1}}{k \cdot r_{\beta_{n-1}} \cdot \mu_{n-1}} & \text{if } c > \beta_{n-1} \\ v_{n-2} \cdot \cos Q_{n-1}^* + \frac{v_{n-2} \cdot \sin Q_{n-1}^*}{k \cdot r_{\beta_{n-1}}^* \cdot \mu_{n-1}} & \text{if } c < \beta_{n-1} \\ v_{n-2} + v_{n-2} \cdot \frac{d_{n-1}}{\mu_{n-1}} & \text{if } c = \beta_{n-1} \end{cases} \quad (44)$$

These computations are performed using the initial values $(v_0, v_0) = (1, 0)$ at the free surface.

5. Computation of group velocities

Following (Schwab and Knopoff 1972) the group velocity, u , is obtained from:

$$u = \frac{c}{1 - (\omega/c)(\partial c/\partial \omega)} \quad (45)$$

where standard implicit function theory is applied to the dispersion function F to obtain

$$dc/d\omega = -(\partial F/\partial \omega)_c / (\partial F/\partial c)_\omega. \quad (46)$$

Equation (46) is obviously valid both when F indicates Love as well as Rayleigh dispersion function. Details about the analytic computation of (46) are given for Love waves in Appendix A.

6. Energy integral

Along with eigenvalues and eigenfunctions, the integrals:

$$I_{1R} = \int_0^\infty \varrho(z)[y_1^2(z) + y_3^2(z)]dz \quad (47)$$

for Rayleigh waves, and

$$I_{1L} = \int_0^\infty \varrho(z) \left(\frac{v(z)}{v(0)} \right)^2 dz \quad (48)$$

for Love waves, where $y_1 = w(z)/w(0)$ and $y_3 = u(z)/w(0)$, are required in multimode synthesis of theoretical seismograms. For a sequence of homogeneous solid layers, these integrals can be written as

$$I_{1R} = c^2[r_{\alpha_1}B_1 - D_1]^{-2} \sum I_{(m)} \quad \text{with } m = 1, 2, \dots, n.$$

The integrals $I_{(m)}$ are given by Eqs (51) and (53) of Schwab et al. (1984).

For SH-waves we have:

$$I_{1L} = \begin{cases} \left(\frac{c}{v_0} \right)^2 \cdot \sum_{m=1}^n I_{(m)} & \text{for the (S-L) case} \\ \left(\frac{c}{v_0} \right)^2 \cdot \left(\sum_{m=1}^n I_{(m)} \right) + I_{(S-S)} & \text{for the (S-S) case} \end{cases} \quad (49)$$

with

$$\begin{aligned} I_{(m)} &= \int_{z_{m-1}}^{z_m} \varrho_m \cdot \left(\frac{\dot{v}(z)}{c} \right)^2 dz \\ I_{(S-S)} &= \int_{z_{n-1}}^{\infty} \varrho_n \cdot \left(\frac{\dot{v}(z)}{c} \right)^2 dz. \end{aligned} \quad (50)$$

The integrals $I_{(m)}$ can be computed analytically, both for Rayleigh and Love waves (Schwab et al. 1984, Florsch et al. 1989). For details about the analytic computation of (49) see Appendix B.

7. Mode follower and structure minimization

Since all the problems connected with the loss of precision at high frequencies have been solved (Schwab et al. 1984) the summation of higher modes of surface waves gives a generation of complete synthetic seismograms even at high frequencies. The key point in the use of multimode summation, both for Love and Rayleigh modes, is an efficient computation of the phase velocity for the different modes at sufficiently small frequency intervals Δf with sufficient precision. To be efficient it is not advisable to determine at each frequency and for each mode the zeros of the dispersion function using the standard root-bracketing and root-refining procedure (Schwab and Knopoff 1972). This must be only used when strictly necessary, as for instance at the beginning of each mode. For all other points i of each mode, the phase velocity can be estimated by cubic extrapolation, using the values of the phase slowness $s = 1/c$ and df/ds already determined at frequencies f_{i-2} and f_{i-1} . However, the precision that can be reached in this way is not satisfactory, thus the phase velocity value must be refined. This can be done by an iterative cubic fit in the $F - c$ plane.

Once the problem of an efficient determination of phase velocities is overcome, two other main problems must be solved at each frequency: (a) to correctly follow a mode and (b) to determine the minimum number of layers to be used. The problem of correctly following a mode arises in the high-frequency domain ($f > 0.1$ Hz), where several higher modes are very close to each other. The determination of the minimum number of layers to be used — structure minimization — is critical in order to reach a high precision in phase velocity determination spending the minimum possible computer time. In order to ensure high efficiency in the computation of synthetic seismograms, it is necessary to compute the phase velocity, phase attenuation, group velocity, ellipticity, energy integral and eigenfunctions and their maximum depth of penetration at constant frequency intervals. To reach a maximum frequency of 10 Hz, a satisfactory step is 0.05 Hz. To determine the total number of modes present in the frequency interval considered, we fix $c = c_0$ a value close to β_n , where β_n is the S-wave velocity in the half-space, and we increment f to find its values corresponding to zeros of the dispersion function $F(f, c_0)$ (Schwab and Knopoff 1972). Obviously, starting from $f = 0$, the first zero in $F(f, c_0)$ corresponds to the fundamental mode, the second to the first higher mode, and so on. The values of f for which $F(f, c_0) = 0$ are used as starting frequencies (the lowest frequencies) for the computation of the different modes. Once the starting frequency for each mode is defined, it is possible to compute, beginning from the fundamental mode, all dispersion relations. This is accomplished by keeping f fixed and varying c , the procedure being applied at all of the equally spaced frequency points of the chosen frequency interval.

More details about the mode follower and the complete description of the procedure for structure minimization are given by Panza and Suhadolc (1987).

8. Attenuation due to anelasticity

The treatment of anelasticity requires, for causality reasons, the introduction of body wave dispersion (BWD) (Futterman 1962). In a medium with constant Q , the P- and S-wave phase-velocity can be expressed:

$$A_1(\omega) = \frac{A_1(\omega_0)}{1 + \frac{2}{\pi} \cdot A_1(\omega_0) \cdot A_2(\omega_0) \cdot \ln\left(\frac{\omega_0}{\omega}\right)} \quad (51)$$

$$B_1(\omega) = \frac{B_1(\omega_0)}{1 + \frac{2}{\pi} \cdot B_1(\omega_0) \cdot B_2(\omega_0) \cdot \ln\left(\frac{\omega_0}{\omega}\right)} \quad (52)$$

The layer index m is omitted in Eqs (51) and (52). $A_1(\omega_0)$ and $A_2(\omega_0)$ are the P-wave velocity and the P-wave phase attenuation, while $B_1(\omega_0)$ and $B_2(\omega_0)$ are the S-wave velocity and the S-wave phase attenuation at the reference angular frequency ω_0 (see also Panza and Suhadolc 1987). The quantities A_1 and A_2 and B_1 and B_2 are related to the complex body-wave velocity α and β (Schwab and Knopoff 1972):

$$\frac{1}{\alpha} = \frac{1}{A_1} - i \cdot A_2 \quad (53)$$

$$\frac{1}{\beta} = \frac{1}{B_1} - i \cdot B_2 \quad (54)$$

In the computation we have chosen the reference angular frequency $\omega_0 = 2\pi$ radians. In anelastic media the surface wave phase velocity c must be expressed as a complex quantity:

$$\frac{1}{c} = \frac{1}{C_1} - i \cdot C_2 \quad (55)$$

C_1 is the attenuated phase velocity and C_2 is the phase attenuation, which is necessary for the computation of seismograms. C_2 can be estimated by using the variational technique (Takeuchi and Saito 1972, Aki and Richards 1980). The phase attenuation C_2 is given by Panza (1985), Florsch et al. (1989), Panza and Suhadolc (1987). For Rayleigh waves:

$$C_2 = (2\omega I_3 \bar{k})^{-1} / \text{Im}(I_4)$$

where $\bar{k}$ is the wave number in the perfectly elastic case and:

$$I_3 = \int_0^\infty \left\{ \left[(\lambda + 2\mu) - \frac{\lambda^2}{(\lambda + 2\mu)} \right] y_3^2 + \frac{1}{k} \cdot \left(y_1 y_4 - \frac{\lambda}{(\lambda + 2\mu)} y_2 y_3 \right) \right\} dz$$

$$I_4 = \int_0^\infty \left\{ \delta(\lambda + 2\mu) \left[\frac{1}{(\lambda + 2\mu)^2} (y_2^2 + 2k\lambda y_2 y_3) + k^2 \left(1 + \frac{\lambda^2}{(\lambda + 2\mu)^2} \right) y_3^2 \right] + \right.$$

$$+ \delta\mu \frac{1}{\mu^2} y_4^2 - \delta\lambda \left[\frac{2k}{(\lambda + 2\mu)} (y_2 y_3 + k\lambda y_3^2) \right] \Big\} dz$$

and $y_2 = \sigma(z)/w(0)$ and $iy_4 = \tau(z)/w(0)$,

$$\begin{aligned} \delta\mu &= \varrho(\beta_1^2 - \beta_2^2 - \bar{\beta}^2) + 2i\varrho\beta_1\beta_2 \\ \delta\lambda &= \varrho \left[(\alpha_1^2 - \alpha_2^2 - \bar{\alpha}^2) - 2(\beta_1^2 - \beta_2^2 - \bar{\beta}^2) \right] + i\varrho 2(\alpha_1\alpha_2 - 2\beta_1\beta_2) \\ \delta(\lambda + 2\mu) &= \varrho(\alpha_1^2 - \alpha_2^2 - \bar{\alpha}^2) + i2\varrho\alpha_1\alpha_2. \end{aligned}$$

In these expressions α and β are the compressional and shear-wave velocities in the perfectly elastic case.

For Love waves (Florsch et al. 1989, Panza and Suhadolc 1987):

$$C_2 = \frac{\int_0^\infty \mu \cdot B_1 \cdot B_2 \cdot \left(\frac{\sigma_z^2}{\mu^2 \cdot k^2} + v^2 \right) dz}{c \int_0^\infty \mu \cdot v^2 dz}. \quad (56)$$

All these integrals can be calculated analytically, since simple analytic expressions are known for the eigenfunctions. The details of the computations of Eq. (56) are given in Appendix B.

The most important effect of the attenuation is the modification of the wave velocities and the decay of amplitude in the final computations of seismograms. As the variational technique is only an approximated method, the C_2 values can be in error by as much as 20 per cent in comparison with the exact method. This error arises mainly from the use of the elastic and therefore real eigenfunctions to compute the phase attenuation.

Recently Day et al. (1989) showed the limits of the variational technique in the locked mode approximation, which can be obtained by limiting the model with a rigid or liquid halfspace. They showed that an error in amplitudes up to 100 per cent can occur when dealing with low Q -values. The error increases when the Q -values undergo large variations with depth. Introducing a solid halfspace in the model and using the structure minimization procedure prevents this kind of error.

9. Response to buried sources

To include the seismic source in the computations, the formulation due to Harkrider (1970) is used. A detailed description of the fault model of an earthquake used in the following computations is given by Panza et al. (1973). Accordingly, in the reference system of Fig. 1, for the double couple point source, the asymptotic expression of the Fourier time transform of the j -th Love (U_L)- or Rayleigh (U_R^R, U_R^V)-mode displacement at the free surface at a distance r from the source can be written as:

$$U_L = R(\omega) \cdot e^{i\Phi_0} \cdot e^{-\frac{i3\pi}{4}} \cdot k_L^{1/2} \cdot \chi_L(\Theta, h) \cdot A_L \cdot \frac{e^{-ik_L r}}{\sqrt{2\pi r}} \cdot e^{-\omega r C_{2L}} \quad (57)$$

$$U_R^R = R(\omega) \cdot e^{i\Phi_0} \cdot |n| \cdot e^{-\frac{13\pi}{4}} \cdot k_R^{1/2} \cdot \varepsilon_0 \cdot \chi_R(\Theta, h) \cdot A_R \cdot \frac{e^{-ik_R r}}{\sqrt{2\pi r}} \cdot e^{-\omega r C_{2R}} \quad (58)$$

$$U_R^V = e^{-\frac{13\pi}{4}} \cdot \varepsilon_0^{-1} \cdot U_R \quad (59)$$

where $R(\omega)$ is the Fourier transform of the equivalent point-force time function, n is the unit vector perpendicular to the fault and has units of length, $\Phi_0 = \arg R(\omega)$ is the initial phase and $\varepsilon_0 = -u^*(0)/w(0)$ is the ellipticity. The factors A_R and A_L are given by:

$$A_L = \frac{1}{2 \cdot c \cdot u \cdot I_{1L}} \quad (60)$$

and

$$A_R = \frac{1}{2 \cdot c \cdot u \cdot I_{1L}} \quad (61)$$

where c and u are the phase and group velocities for Love and Rayleigh waves respectively.

The effect of anelasticity is expressed by the term:

$$e^{-\omega r C_2} \quad (62)$$

where C_2 , which indicates the phase attenuation either for Love or Rayleigh waves, can be determined as shown in Section 8.

$\chi(\theta, h)$ is the azimuthal dependence given by:

$$\chi_R(\Theta, h) = d_0 + i \cdot (d_{1R} \sin \Theta + d_{2R} \cos \Theta) + d_{3R} \sin 2\Theta + d_{4R} \cos 2\Theta$$

for P-SV waves, and by

$$\chi_L(\Theta, h) = i \cdot (d_{1L} \sin \Theta + d_{2L} \cos \Theta) + d_{3L} \sin 2\Theta + d_{4L} \cos 2\Theta$$

for SH waves.

$$\begin{aligned} d_0 &= \frac{1}{2} \cdot B(h) \cdot \sin \lambda \cdot \sin 2\delta \\ d_{1R} &= -C(h) \cdot \sin \lambda \cdot \cos 2\delta \\ d_{2R} &= -C(h) \cdot \cos \lambda \cdot \cos \delta \\ d_{3R} &= A(h) \cdot \cos \lambda \cdot \sin \delta \\ d_{4R} &= -\frac{1}{2} \cdot A(h) \cdot \sin \lambda \cdot \sin 2\delta \\ d_{1L} &= G(h) \cdot \cos \lambda \cdot \cos \delta \\ d_{2L} &= -G(h) \cdot \sin \lambda \cdot \cos 2\delta \\ d_{3L} &= \frac{1}{2} \cdot V(h) \cdot \sin \lambda \cdot \sin 2\delta \\ d_{4L} &= V(h) \cdot \cos \lambda \cdot \sin \delta \end{aligned}$$

Θ is the angle between the strike of the fault and the epicenter-station direction, λ is the rake angle, δ is the dip angle and h is the source depth. The source geometry and the coordinate system associated with the free surface is given in Fig. 3. $A(h)$, $B(h)$, $C(h)$, $G(h)$ and $V(h)$ depend on the values of the eigenfunctions at

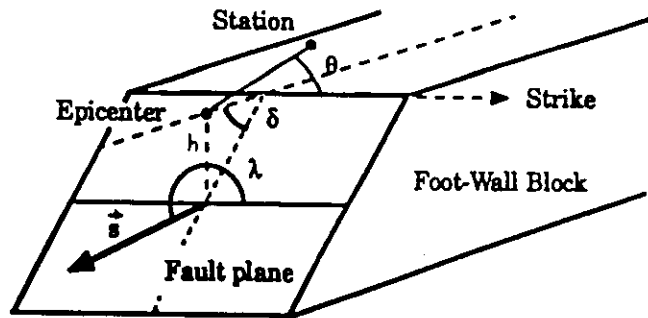


Fig. 3. Source geometry and coordinate system associated with the free surface. Θ is the angle between the strike of the fault and the epicenter-station direction, δ is the dip, λ is the rake and h is the source depth

the hypocenter:

$$\begin{aligned}
 A(h) &= -\frac{u^*(h)}{w_0} \\
 B(h) &= -\left(3 - 4\frac{\beta^2(h)}{\alpha^2(h)}\right) \frac{u^*(h)}{w_0} - \frac{2}{\rho(h)\alpha^2(h)} \frac{\sigma^*(h)}{\dot{w}_0/c} \\
 C(h) &= -\frac{1}{\mu(h)} \frac{\tau(h)}{\dot{w}_0/c} \\
 G(h) &= -\frac{1}{\mu(h)} \left(\frac{v^*(h)}{\frac{v_0}{c}}\right) = \frac{1}{k \cdot \mu(h)} \cdot \frac{v(h)}{v_0} \\
 V(h) &= \frac{\dot{v}_s(h)}{\dot{v}_0} = \frac{v_s(h)}{v_0}
 \end{aligned}$$

The asymptotic expressions (57), (58) and (59) allow the computation of synthetic seismograms with at least 3 significant figures as long as $kr > 10$ (Panza et al. 1973) and is equivalent to the expression in terms of the seismic moment (e.g. see equations (7.148), (7.149) and (7.150) by Aki and Richards 1980). The seismogram related to a given mode is obtained by the inverse Fourier transform of (57), (58) and (59).

The extension of these results to the available formalism for sources with finite dimensions and durations is quite straightforward; the necessary details can be found in Panza and Suhadolc (1987).

10. Two dimensional models

The expressions (57), (58) and (59), describing the displacement due to surface-wave modes, have been generalized to the case of two stratified quarterspaces in welded contact (Levshin 1985, Vaccari et al. 1989). For example the radial compo-

ment of displacement spectrum can be written:

$$U = R(\omega) \cdot e^{-i(3\pi/4)} \cdot e^{-i\omega(\ell/c + \ell'/c')} \cdot e^{-(\ell C_2 + \ell' C'_2)} \cdot \frac{1}{\sqrt{J}} \cdot \sqrt{\frac{\omega}{2\pi}} \cdot \frac{\epsilon'}{\sqrt{2cuI_{1R}}} \cdot \Gamma_{jj'} \cdot \sqrt{\frac{\cos \varphi}{\cos \varphi'}} \cdot \frac{\chi_R(\Theta, h)}{\sqrt{2c^2 u I_{1R}}} \quad (63)$$

It is assumed that the source and the receiver are situated far from the sharp vertical discontinuity in comparison to the biggest wavelength of interest. Equation (63) represents the radial displacement carried by the n -th Rayleigh mode generated by a joint source at $M(r, \ell)$ of a medium j , then transmitted through the boundary between j and j' and recorded at point $M'(r', \ell')$ on the surface of medium j' as mode n' . As it is evident from Fig. 4 R and R' are respectively the distances of source and receiver from the vertical interface, ℓ and ℓ' are the paths travelled by incident and generated waves, ϕ and ϕ' are the angles of incidence and refraction. In Eq. (63) the primed quantities refer to the medium where the receiver is placed while the unprimed quantities refer to the medium containing the source. $R(\omega)$ is the Fourier transform of the time function relative to the source, c and c' are the phase velocities, u and u' the group velocities, I and I' are energy integrals ϵ' is the Rayleigh mode ellipticity of the receiver's medium, $\chi_R(\Theta, h)$ is the radiation pattern evaluated for the medium containing the source, $J = (\cos \phi \cos \phi' / c)[(rc / \cos^3 \phi) + (r'c' / \cos^3 \phi')]$ describes the geometrical spreading of the surface wave energy. The effect of anelasticity is expressed by $e^{-\omega(\ell C_2 + \ell' C'_2)}$ where C_2 and C'_2 are the phase attenuations in the two media (for more details see Panza 1985).

The coupling coefficients $\Gamma_{jj'}(\omega, \phi, \phi', n, n')$, necessary to describe reflection and transmission phenomena at the boundary, will be discussed in Section 10.1.

10.1 Coupling coefficients

The problem of reflection and transmission of surface waves through lateral discontinuities existing inside the earth cannot be solved in an analytical way. Several methods based on different approximations have been proposed to define and estimate surface waves reflection and transmission coefficients. Here only the transmission problem will be discussed, but the same procedure can be adopted to describe the reflection problem.

The approximations suggested by Gregersen and Alsop (1974) were chosen to evaluate a set of coupling coefficients that gives a picture of how the energy carried by the normal modes, characteristic of the medium with the source, and transmitted through the discontinuity, is redistributed among the normal modes, of the medium with the receiver.

The starting point is the stress-displacement system of the incoming surface wave mode. Decomposing the incident wave into the P-component and the SV-component, we turn to a solvable problem of reflection and transmission of P-SV waves. Referring to Fig. 5, the problem will be solved for every single section on the vertical interface as if it would be infinite, using well known formulae based on Snell's law and continuity conditions of displacement and traction at the boundary.

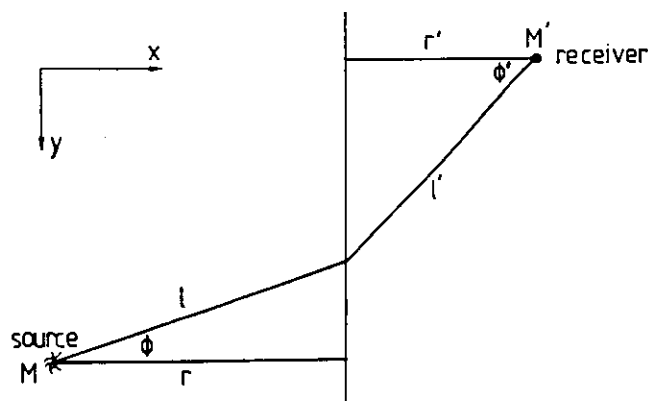


Fig. 4. Geometrical scheme, in the horizontal plane, of the source-receiver system. The source, located in M , and the receiver, located in M' , have the distances r and r' from the vertical boundary. The ray paths, l and l' , depend on the angles of incidence ϕ and ϕ' , defined through Snell's law

Since the sections are in reality limited, in this way we neglect the effects arising at the corners between the horizontal interfaces and the vertical one. We can think the corner effects as giving rise to a system of diffracted waves. These arise because it is impossible to satisfy exactly the continuity conditions on the horizontal interfaces, too. With this approximation a reflected and a transmitted stress-displacement system can be determined. They contain a P-component and an SV-component but their combination does not give a Rayleigh wave any longer because the continuity conditions at the horizontal boundaries are not matched. The medium with the receiver is characterized by a set of normal modes corresponding to solutions of the wave equation verifying the continuity conditions on the horizontal interfaces. Our aim is to determine how each of these modes is excited by the modes contained in the incident wave, or in other words, how the transmitted system redistributes among the normal modes existing in the medium with the receiver. To obtain this, the transmitted system is projected on the system of normal modes characteristic of the medium with the receiver, using an appropriate definition of scalar product.

A stress-displacement vector for an incident wave identified by the subscript I can be defined as

$$A_I = (u_I, v_I, w_I, p_{xxI}, p_{xyI}, p_{xzI}) \quad (64)$$

where p_{ij} is the j -th component of the stress acting across the plane normal to the i -th axis. The p_{xj} (with $j = x, y, z$) stress components are considered because for the geometry of the problem, stresses acting on the vertical plane $x = 0$ are only involved. Defining similarly a stress-displacement vector relative to the normal modes system of the medium with the receiver, the projection of vector A_I on the vector A_{II} is performed via a scalar product, suggested by Herrera's (1964)

- a) the Rayleigh modes for the two media are evaluated assuming each medium as a halfspace instead of a quarterspace; this is a reasonable assumption a few wavelengths from the interface;
- b) a system of diffracted waves arising at the corners of the sections is neglected; this is a good approximation for a small contrast in the elastic parameters characterizing the two quarterspaces (Gregersen and Alsop 1974).

The crucial approximation is contained in this last point. Although it is not easy to estimate quantitatively its accuracy a criterion is given by a reversibility theorem that is demonstrated in Section 10.3.

10.2 The coupling integral

To calculate $\langle \mathbf{A}_T, \mathbf{A}_{II} \rangle$ and $\langle \mathbf{A}_{II}, \mathbf{A}_{II} \rangle$, once the explicit expressions for the various displacement and stress components are assigned, the integration indicated in Eq. (65) must be performed over the semiaxis $z > 0$. At this stage we limit our analysis to the 2-dimensional case of the $x-z$ plane, to the Rayleigh modes and to normal incidence. These three assumptions are strictly connected because for non-normal incidence as for the 3-dimensional case there is transmission of P and SV into SH energy, so conversion between Rayleigh and Love modes must be taken into account. The development of the pertinent algorithms and related codes is presently in progress at IGG-ICTP. The integral (65) simplifies in the following way:

$$\langle \mathbf{A}_T, \mathbf{A}_{II} \rangle = \frac{1}{2i} \int_0^{\infty} [u_T \bar{p}_{xxII} + w_T \bar{p}_{xzII} - p_{xzT} \bar{u}_{II} - p_{zzT} \bar{w}_{II}] dz. \quad (69)$$

Bearing in mind that the z -dependence is different in various layers, the integral (69) may be evaluated as a sum of integrals over each section. As it can be seen from Fig. 5:

$$\int_0^{\infty} dz \rightarrow \sum_{s=0}^{S-1} \int_{H_s}^{H_{s+1}} dz + \int_{H_S}^{\infty} dz$$

where H_s is the depth of the s -th section.

These integrals can be computed both numerically and analytically. In the numerical integration the eigenfunctions are sampled with a constant step (for frequencies up to 1 Hz the step used is 250 m) down to the halfspace, where displacements and stresses tend to zero, and of course the integration along z cannot be performed to infinity but to some arbitrary large depth. To treat frequencies higher than 1 Hz one has to reduce the eigenfunctions sampling step to follow correctly the number of oscillations per cycle. Therefore increasing frequency causes the numerical integration to become more and more time consuming along with the limited precision introduced by the approximated integration in the halfspace. These problems of computation time and precision can be overcome by the analytic integration.

$H_0 = 0$							
$\alpha_1^{(1)}$	$\beta_1^{(1)}$	$f_1(z)$	H_1	$\alpha_1^{(2)}$	$\beta_1^{(2)}$	$g_1(z)$	
$\alpha_1^{(1)}$	$\beta_1^{(1)}$	$f_1(z - H_1)$		H_2	$\alpha_2^{(2)}$	$\beta_2^{(2)}$	$g_2(z - H_1)$
$\alpha_2^{(1)}$	$\beta_2^{(2)}$	$f_2(z - H_2)$	H_3	$\alpha_2^{(2)}$	$\beta_2^{(2)}$	$g_2(z - H_2)$	
$\dots H_{m-1}$							
				H_m			
$\alpha_m^{(1)}$	$\beta_m^{(1)}$	$f_m(z - H_m)$		$\alpha_m^{(2)}$	$\beta_m^{(2)}$	$g_m(z - H_m)$	
H_{m+1}							

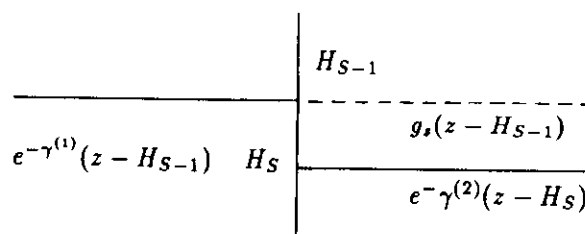


Fig. 7. Depth-dependence of the eigenfunctions in the proximity of the halfspace

value, while the layer constants (e.g. A_m, B_m, C_m, D_m for Rayleigh waves) and the arguments of f and g change according to the new value of H_m (Fig. 6). This occurs because a new interface, though artificial, imposes the introduction of new continuity conditions. In this way any structure can be rearranged to a new one where the layers are lined up. For this last case, as it can be easily verified, the integrals required are simpler, and have the form:

$$J_m = \int_{H_m}^{H_{m+1}} [g_m(z - H_m) f_m(z - H_m)] dz = \int_0^{h_m} [f_m(z) g_m(z)] dz \quad (71)$$

where $h_m = H_{m+1} - H_m$. The integral depends only on the thickness of the section, not on its depth.

The case in which one or both the final sections are a halfspace is characterized by an exponential decay of stresses and displacements at increasing z . In the situation of halfspace $\alpha_S > \beta_S > c$ and consequently $A_S = B_S, C_S = D_S = 1$, the integrals to be evaluated have the form

$$\int_{H_{S-1}}^{H_S} dz [e^{-\gamma^{(1)}(z-H_{S-1})} g_S(z - H_{S-1})] = \int_0^{h_S} [e^{-\gamma^{(1)}z} g_S(z)] dz \quad (72)$$

$$\int_{H_S}^{\infty} dz [e^{-\gamma^{(1)}(z-H_{S-1})} e^{-\gamma^{(2)}(z-H_S)}] = \int_0^{\infty} dz [e^{-\gamma^{(1)}z} e^{-\gamma^{(2)}(z-h_S)}] \quad (73)$$

where $h_S = H_S - H_{S-1}$. The situation is represented in Fig. 7 where, for medium ℓ ($\ell = 1, 2$), $\gamma^{(\ell)} = k^{(\ell)} r_{\alpha^{(\ell)}}^*$ for P-waves and $\gamma^{(\ell)} = k^{(\ell)} r_{\beta^{(\ell)}}^*$ for SV waves. A complete list of the various integrals is given in Appendix B (Vaccari et al. 1989).

10.3 Normalization properties and reversibility theorem

The general scheme to study the dynamics of continuous media is provided by the Lagrangian formulation. In this frame, using variational techniques and Hamilton's principle, it is possible to give a physical meaning to the integral appearing in Eq. (65). In the case of Rayleigh waves propagating in the x -direction the displacement assumes the expression

$$u(x, z, t) = e^{-i(\omega t - kx)} [u(z); 0; iw(z)] \quad 0 \leq z < \infty. \quad (74)$$

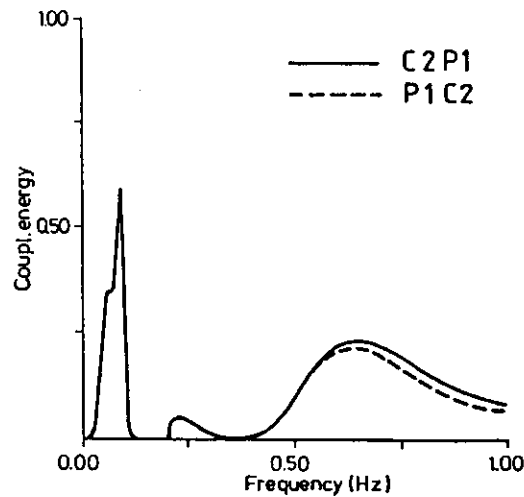


Fig. 8. Energy exchange for both ways of propagation. The second higher mode of model C and the first higher mode of model P are considered. The two curves practically overlap. The reversibility theorem is satisfied for frequencies lower than 0.5 Hz, while the difference visible at larger frequencies, still acceptable, is due to the approximations described at the end of Section 10.1

Looking at the normalization integral, Eq. (69):

$$J_m = \frac{1}{2i} \int_0^\infty [u_m(z) \bar{p}_{xz}(k_m, z) + i w_m \bar{p}_{xz}(k_m, z) - p_{xz}(k_m, z) u_m - i p_{xz}(k_m, z) w_m(z)] dz \quad (75)$$

it can be demonstrated that

$$J_m = \frac{(UE)_m}{\omega} = \frac{1}{\omega} \Psi_m \quad (76)$$

where Ψ_m is proportional to the energy transmitted per unit time across an infinite stripe of unit width in the $y-z$ plane. Thus it is convenient to normalize the surface waves system in such a way that, for each mode n , the energy crossing per unit time an infinite stripe in the $y-z$ plane is equal to ℓ .

$$(u_m, w_m) \text{ to } \left(\frac{u_m}{\sqrt{\omega J_m}}, \frac{w_m}{\sqrt{\omega J_m}} \right). \quad (77)$$

Furthermore, since the main part of the surface displacement can be represented as a superposition of modes

$$u(x, z, t) = \sum_m a_m e^{-i(\omega t - k_m x)} [u_m(z); 0; i w_m(z)] \quad (78)$$

the normalization implies that the energy transmitted per unit time across the stripe is given by

$$UE = \sum_m |a_m|^2.$$

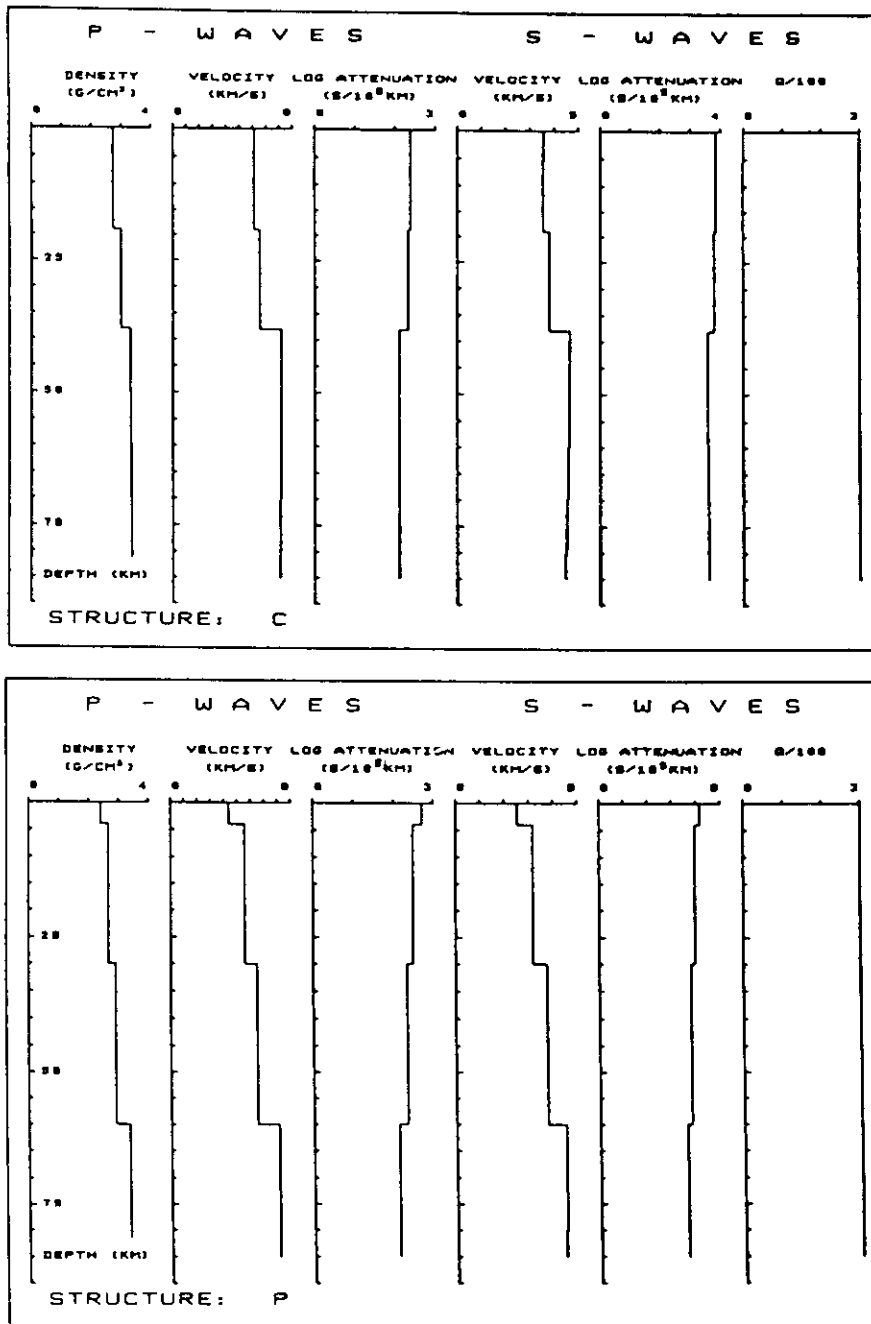


Fig. 9. Elastic and anelastic parameters of the models Continental (C) and Pamir (P)

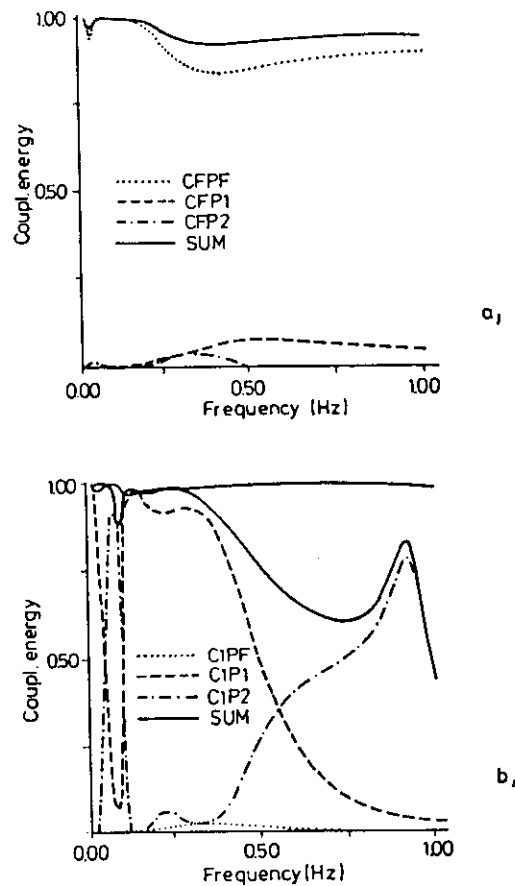


Fig. 10. Transmission of energy from model C to model P. a) incoming fundamental mode: almost all of the energy is transmitted into the outgoing fundamental mode, while mode conversion plays just a secondary role. b) Incoming first higher mode: mode conversion becomes relevant, and to transmit all of the energy more than three outgoing modes must be taken into account. In part b) the thin solid line indicates the sum of the first three outgoing modes, while the heavy solid line is obtained by adding also the contribution of C1P3, C1P4 and C1P5

For a layered structure with a vertical interface along the z -axis (Fig. 1), with our approximations, the displacement on the left of the interface (medium 1) is given by the superposition of N modes

$$u(x, z, t) = \sum_{m=1}^N e^{-i\omega t} [a_m e^{-ik_m x} + b_m e^{ik_m x}] (u_m; O; i\omega_m) \quad x < O. \quad (79)$$

Similarly in the right side medium (medium 2) the displacement

$$u(x, z, t) = \sum_{m'=1}^{N'} e^{-i\omega t} [a_{N+m'} e^{-ik_{m'} x} + b_{N+m'} e^{ik_{m'} x}] (r_m; O; i\omega_{m'}) \quad x > O \quad (80)$$

is given by the superposition of N' modes. The quantities a_i , $i = 1, 2, \dots, N + N'$ are the complex amplitudes of the incoming normal modes; the quantities b_j , $j = 1, 2, \dots, N + N'$ are the complex amplitudes of the outgoing, reflected or transmitted, normal modes. The linearity of equations of motion establishes a linear relation between the two sets of amplitudes. Indicating with $\mathbf{a} = (a_1, \dots, a_{N+N'})$, $\mathbf{b} = (b_1, \dots, b_{N+N'})$ the two vectors of $N + N'$ components there is a matrix $\mathbf{S}$ such that

$$\mathbf{b} = \mathbf{S}\mathbf{a}. \quad (81)$$

The elements of the scattering matrix $\mathbf{S}$ correspond to the reflection and transmission coefficients of the problem studied. From Eq. (76) it follows that the energy incident per unit time on the discontinuity surface is given by:

$$(UE)_{\text{inc}} = \sum_m |a_m|^2 = \mathbf{a} \cdot \bar{\mathbf{a}}. \quad (82)$$

The outgoing energy, transmitted or reflected into the different modes, is

$$(UE)_{\text{out}} = \sum_m |b_m|^2 = \mathbf{b} \cdot \bar{\mathbf{b}}. \quad (83)$$

Since the total energy leaving the vertical boundary is equal to the incident one, $\mathbf{S}$ is a unitary matrix i.e.

$$\mathbf{S}\mathbf{S}^+ = \mathbf{1} \quad S_{ij}\bar{S}_{kj} = \delta_{ik}. \quad (84)$$

When the scattering into body waves is neglected, as we do here, $\mathbf{S}$ is not any more a unit matrix and the relevance of the approximation can be estimated by the difference between $\mathbf{S}$ and the unit matrix.

Another property of the $\mathbf{S}$ matrix follows from the fact that the equations of motion are invariant for complex conjugation and time inversion. This means that if $\mathbf{u}(x, z, t)$ is a solution, also $\bar{\mathbf{u}}(x, z, -t)$ is a solution. The linear relation given by Eq. (81) changes to:

$$\bar{\mathbf{a}} = \mathbf{S}\bar{\mathbf{b}} = \mathbf{S}\bar{\mathbf{S}}\bar{\mathbf{a}} \quad (85)$$

so that

$$\mathbf{S}\bar{\mathbf{S}} = \mathbf{1} \quad S_{ij}\bar{S}_{jk} = \delta_{ik}. \quad (86)$$

The matrix $\mathbf{S}$ is then symmetric, $S_{ij} = S_{ji}$, S_{ij} being the amplitude transmission or reflection coefficient from the mode j into the mode i ($b_i = S_{ij}a_j$). According to the mode normalization given in Eq. (77), the energy crossing per unit time a given surface is equal to 1. Consequently, S_{ij}^2 corresponds to the energy transmitted (or reflected) from mode j to mode i ; from this it follows the reversibility property expressed by the relation

$$S_{ij}^2 = S_{ji}^2. \quad (87)$$

An example of the exchanges of energy between two modes of two different media in welded contact is given in Fig. 8, and shows, for the structural models under consideration (see Fig. 9), the degree of validity of assumption (b) of Section 10.1. In this figure, like in the following ones, the first two characters of each acronym

Table I. Structure FRIUL7A proposed by Panza and Suhadolc (1987)

Thickness [km]	Density [g/cm ³]	P-wave velocity [km/s]	S-wave velocity [km/s]	Q_β
0.04	2.00	1.50	0.60	20
0.06	2.30	3.50	1.80	30
0.20	2.40	4.50	2.50	100
0.70	2.40	5.00	2.90	200
2.00	2.60	6.00	3.30	400
3.50	2.60	6.20	3.45	400
4.50	2.60	6.00	3.35	100
10.00	2.60	5.50	3.30	50
3.50	2.60	6.00	3.50	400
2.50	2.75	6.50	3.75	400
2.50	2.80	7.00	3.85	400
7.50	2.80	6.50	3.75	100
4.00	2.85	7.00	3.85	200
3.00	3.20	7.50	4.25	400
1.50	3.40	8.00	4.50	400
9.00	3.45	8.20	4.65	400

refer to the incident mode, the last two to the outgoing one. For example, the coupling between the second higher mode of structure C (incident mode) and the fundamental mode of structure P (outgoing mode) would be indicated as C2PF. Two further examples, shown in Figs 10a and 10b, allow to appreciate how the incoming energy related to one mode redistributes within the outgoing modes. When the incident fundamental mode is considered (Fig. 10a), almost all of the energy is transmitted into the outgoing fundamental one (CFPF). Looking at the sum of the outgoing energies (thin solid line), it can be seen that in the whole frequency range it is enough to consider just three outgoing modes to transmit across the boundary more than 90 % of the incoming energy. This is no longer true when the first higher mode is incident (Fig. 10b).

11. Examples of computation

11.1 Frequency domain

The layered velocity model in Fig. 11 and Table I is used as an example. It represents an average structure of the Friuli seismic area in the southern pre-Alps, close to the May 6, 1976, Friuli earthquake.

11.11 Phase velocities

The Rayleigh wave dispersion curves for the first 154 modes are shown in Fig. 12a. The modes are well separated for phase velocities less than about 3.35 km/s

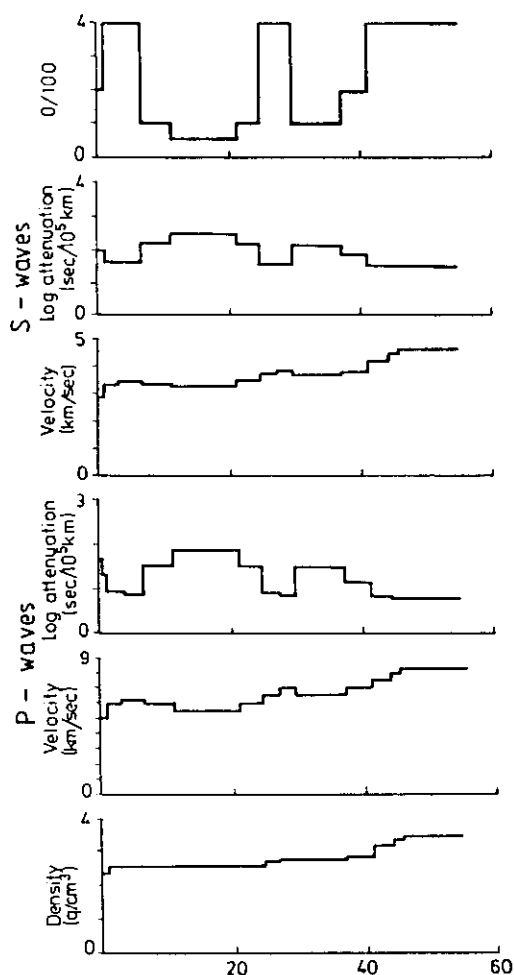


Fig. 11. Structural model FRIUL7A, after Panza and Suhadolc (1987)

what corresponds to the S-wave velocity in the upper part of the upper crustal low-velocity channel. For higher phase velocities, the dispersion curves are closely packed together. The first curves correspond to waves that sample only the first few kilometers of the crust, while the latter ones correspond to waves that sample the entire structure. Essentially three "quasi-osculation" types can be recognized (see Fig. 12b). The first corresponds to horizontal "quasi-osculations" around a constant phase velocity (e.g. 4.25 and 3.50 km/s) and are related to the structural layering. The second is the standard sequence of channel and crustal waves (Panza et al. 1972) due to the presence of a low velocity layer. The presence of two low-velocity layers is clearly recognizable in the range of phase velocities between 3.35–3.45 km/s and 3.75–3.85 km/s. The third is made up of very steep "quasi-osculations" and corresponds to a family of waves mainly sampling the wave guide formed by the sedimentary layers.

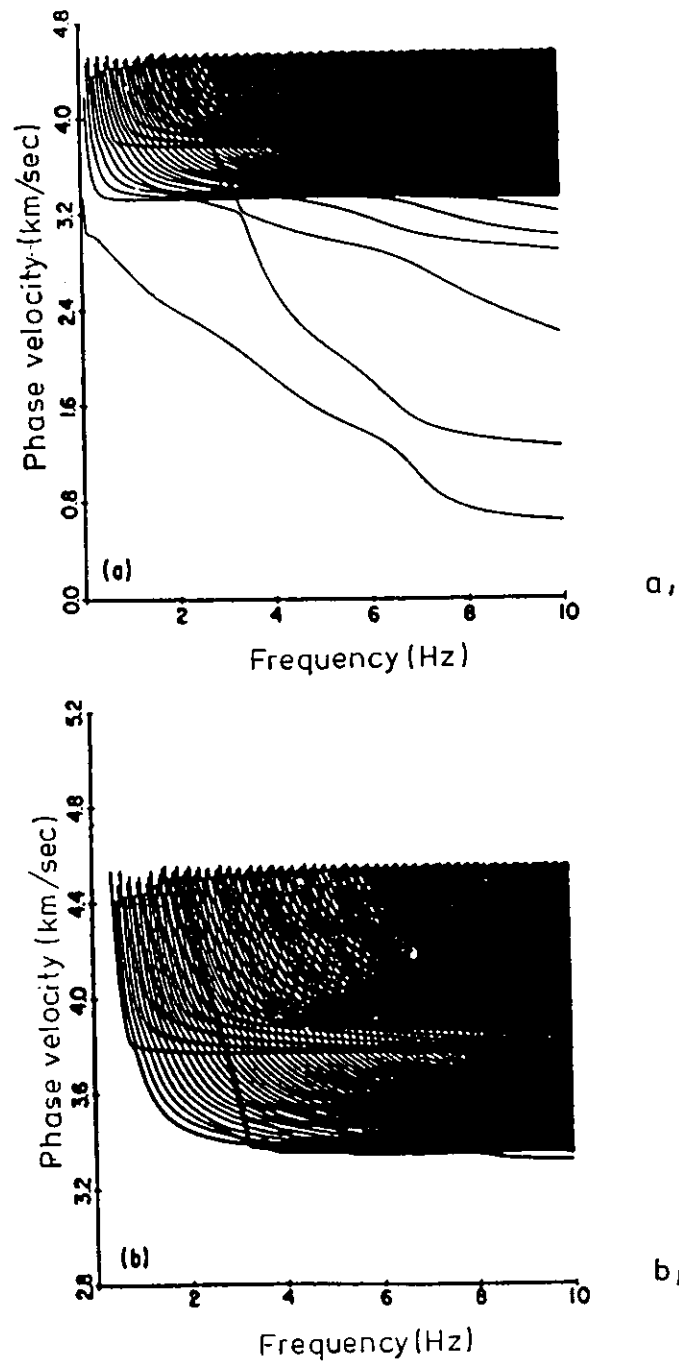


Fig. 12. a) Rayleigh-wave dispersion curves for the structural model FRIUL7A. The mode numbering is the following: 0 for the fundamental mode, 1 for the first higher mode, 2 for the second higher mode, and so on up to 154. b) enlarged portion (modes 6-154) of part (a) showing the effect of low-velocity waveguides

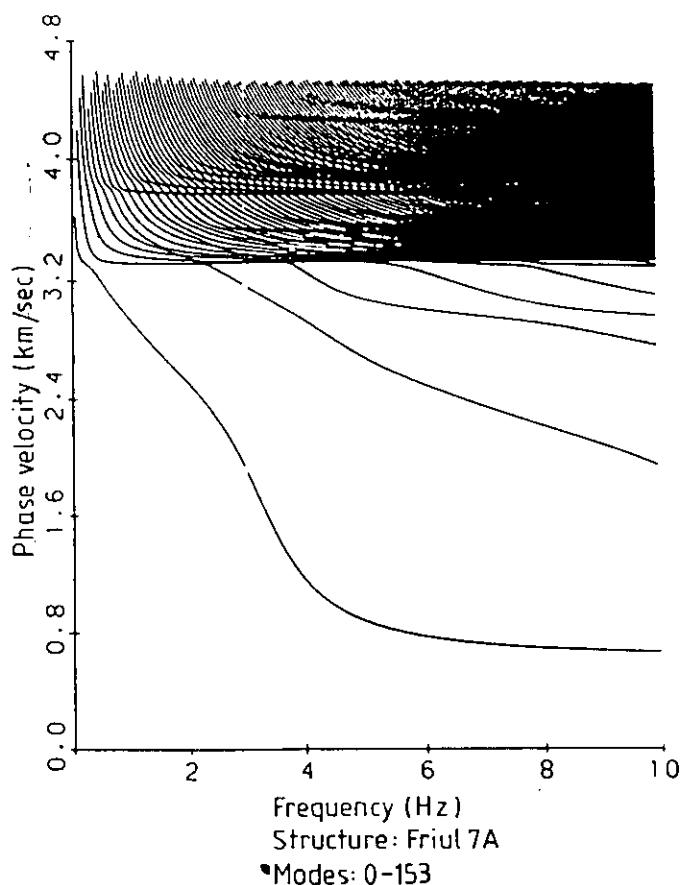


Fig. 13. Love-wave dispersion curves for the structural model FRIUL7A. The mode numbering is the following: 0 for the fundamental mode, 1 for the first higher mode, 2 for the second higher mode, and so on up to 153

The dispersion curves for the first 153 Love modes are shown in Fig. 13. For S-wave velocities less than 3.35 km/s the modes are well separated. This velocity corresponds to the S-wave velocity in the upper part of the crustal low-velocity zone (LVZ). Modes situated in the part of the spectrum below this phase velocity value sample therefore the part of the crust above the uppermost LVZ. In the part of the spectrum with higher phase velocities the dispersion curves are packed together. Since two LVZ's are present in the structural model, areas are seen where the higher Love wave modes decompose into families of low-velocity channel waves and families of waves propagating in the upper crust. A member of the family of upper-crustal waves can be identified at a frequency of about 4 Hz in the phase velocity range 3.35–3.45 km/s. Another type of apparent continuity of the phase velocities for adjacent modes can be related to the structural layering (for example at a phase velocity of about 4.25 km/s). Such parts of the spectrum represent refracted waves

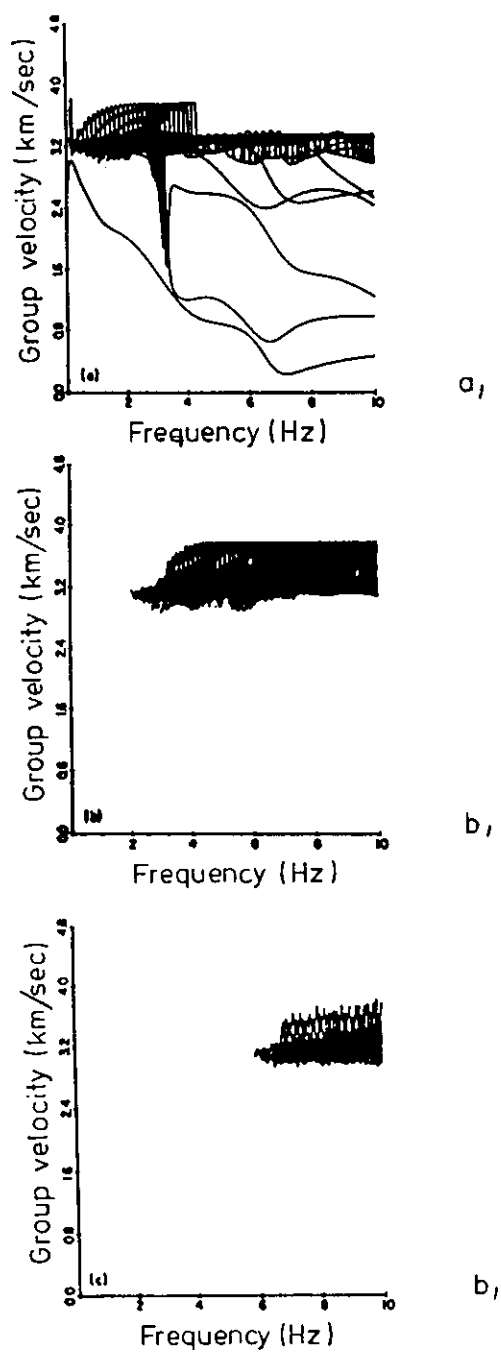


Fig. 14. Rayleigh-wave group velocities for the structure FRIUL7A. The spectrum is divided into three parts: a) Rayleigh modes 0-30, b) Rayleigh modes 31-90, c) 91-154

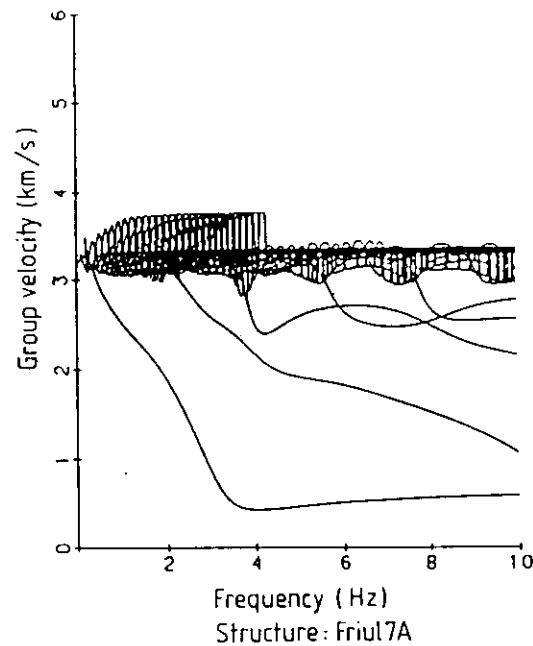


Fig. 15. Love-wave group velocities for the structure FRIUL7A, for the modes 0–30 at strong elastic impedance contrasts. They are characterized by phase velocities which tend to become constant with increasing frequency.

11.12 Group velocities

The group velocities for Rayleigh waves are shown in Figs 14a, b, c. The diagram has been divided into three parts due to the complexity of the pattern. Although it is difficult to follow an individual mode, it is relatively easy to follow the behaviour of channel and crustal waves as well as that of the sedimentary waves. The stationary phases with group velocity between 0.3 and 1.6 km/s, visible for frequencies greater than about 3 Hz, correspond to waves essentially propagating in the low-velocity sediments. For group velocities around 2.4–2.8 km/s, stationary phases are visible at frequencies greater than 5 Hz; these phases can be associated with waves propagating near the bottom of the sediments. The stationary phases, formed by the combination of several higher modes visible in the group velocity interval of 3.0–3.3 km/s, starting from frequencies on the order of 3 Hz, can be considered the high frequency equivalent of Li and Lg phases (Panza and Calcagnile 1974, Schwab and Knopoff 1973, Knopoff et al. 1973, 1974). For frequencies larger than 1 Hz, two flat envelopes corresponding to the two-channel velocities can be easily seen at about 3.75 and 3.35 km/s. Near 3 Hz an envelope of group velocities as low as

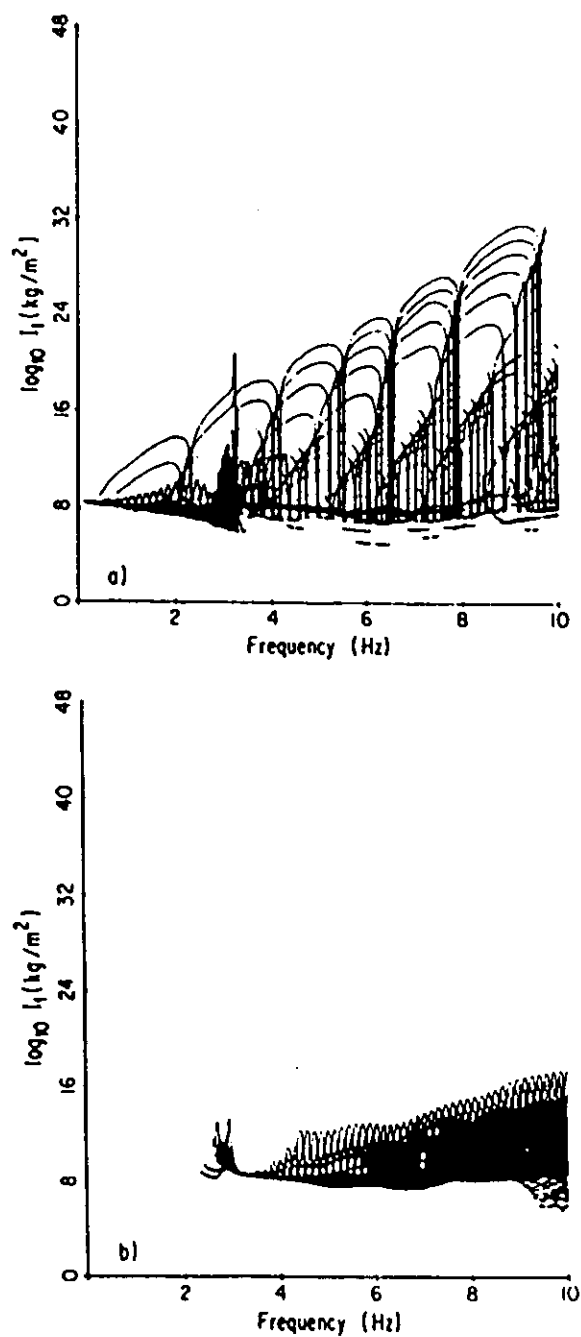


Fig. 16. Rayleigh-wave energy integral I_1 for the structure FRIUL7A. The spectrum is divided into two parts: a) Rayleigh modes 0-30, b) Rayleigh modes 31-154

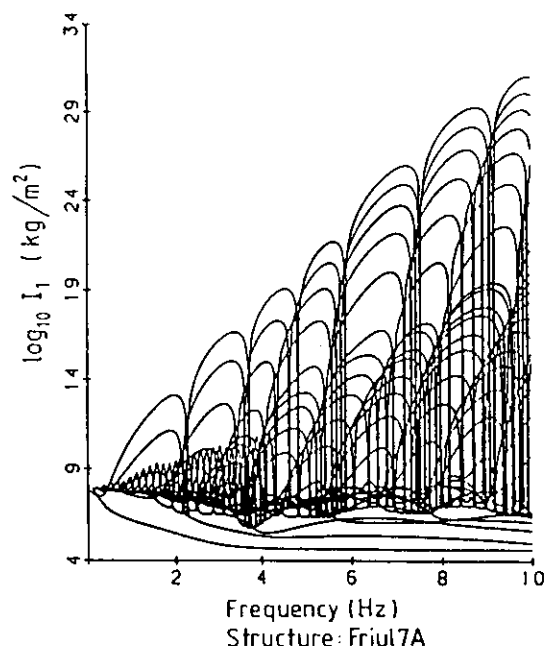


Fig. 17. Love-wave energy integral I_1 for the structure FRIUL7A, for the modes 0–30

1.6 km/s is clearly seen. This corresponds to the waves sampling the sedimentary layers wave guide.

The group velocity spectrum for Love waves is presented in Fig. 15. Modes with group velocities less than about 2.8 km/s correspond to waves propagating in the low-velocity sediments. In the part of the spectrum where group velocities are in the interval 2.8–3.2 km/s, several higher modes form stationary phases. They correspond to families of waves propagating in the upper crust and are characterized by the same type of mode-to-mode continuation as in the phase velocity curves. They can be interpreted as the high-frequency equivalent of Lg phases (Panza and Calcagnile 1974, Schwab and Knopoff 1973, Knopoff et al. 1973, 1974), which are propagating in the upper part of the continental crust. The flat portions of group velocity curves formed by a large number of higher modes at about 3.35 km/s and 3.75 km/s correspond to waves propagating in the upper and lower channel.

11.13 Energy integral

The energy integral can serve as an estimate of the contribution of the different modes to the surface displacement. In general, neglecting the influence of the source depth on the excitation of different modes, small values of the energy integral I_1 correspond to large surface displacements. For Rayleigh (Fig. 16) waves it is interesting to observe that for frequencies smaller than 3 Hz the fundamental mode dominates, while around 3 Hz several higher modes are characterized by small

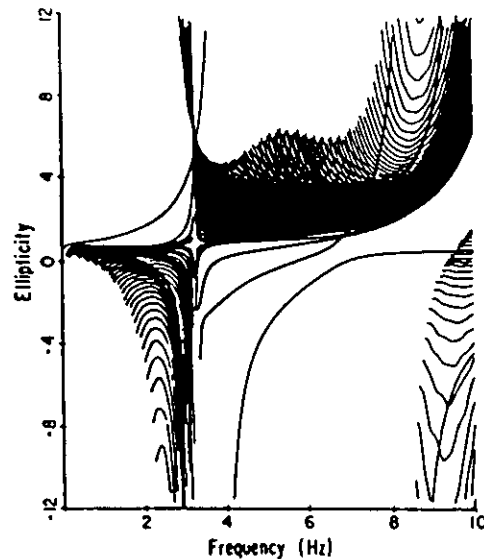


Fig. 18. Ellipticity of Rayleigh modes (0-154). The clearly visible discontinuities (e.g., at about 3 Hz) are due to the crossing of w_0 through zero

values of I_1 . These modes mainly sample the sediments. In the frequency range between 3-6 Hz, the first higher mode is dominant, while for larger frequencies the fundamental mode again dominates the surface displacement. A common feature to all figures are the very narrow peaks around 3 Hz associated with the presence of sediments. If the source is located near the sedimentary layers, one may expect significant surface motion mainly in the horizontal component even if I_1 is quite large. In these portions of the spectrum, the eigenfunctions are characterized by large lobes concentrated in the sedimentary layers, and the ellipticity (see Section 11.14) becomes very large. Also, the fundamental mode is not dominant, i.e., it does not have the smallest I_1 over the entire spectrum due to the sedimentary layers.

For Love waves, on the contrary, in the whole frequency range, the fundamental mode has the lowest values of I_1 (Fig. 17). For a shallow source, the fundamental mode generally dominates the surface displacement. The mode-to-mode continuations in the lower part of the energy integral curves correspond to the high frequency equivalent to Lg waves. The low values of the energy integral indicates that this waves can give rise to significant amplitudes at the surface. Most of the energy of channel waves is concentrated in the channel. Therefore, the energy integral of these families takes higher values than that for upper-crustal waves. For a given member of this family the maximum displacement in the low-velocity zone becomes larger relative to the displacement at the free surface, with increasing frequency. Therefore, the energy integral of this member is characterized by increasing values with increasing frequency. This can be seen in the general pattern of the upper part of Fig. 17.

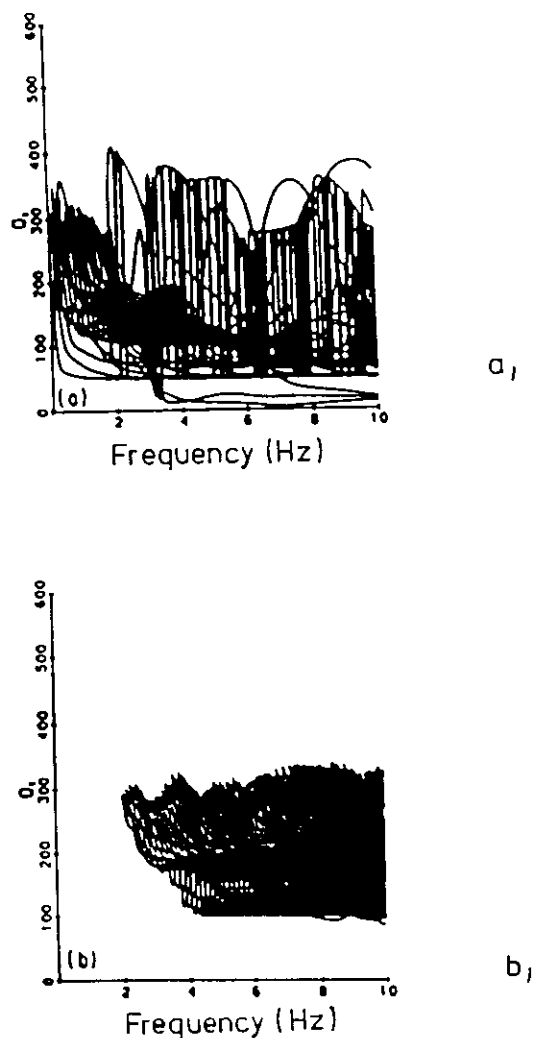


Fig. 19. Rayleigh-wave quality factor Q_z for the structure FRIUL7A. The spectrum is divided into two parts: a) Rayleigh modes 0-30, b) Rayleigh modes 31-154

11.14 Ellipticity

Another important quantity describing Rayleigh-mode particle motion is the ellipticity $\varepsilon_0 = -u^*(0)/w(0)$, i.e. the ratio between the horizontal and vertical components of motion at the free surface. It is very important to observe that ε_0 has abrupt discontinuities. More precisely, at some frequencies $\varepsilon_0 \rightarrow \pm\infty$ as a consequence of the fact that $w(0)$ passes through zero. This is not an obvious property, and it strongly depends upon the elastic properties of the layers closest to the free surface. For frequencies not exceeding 1 Hz and for Earth models without

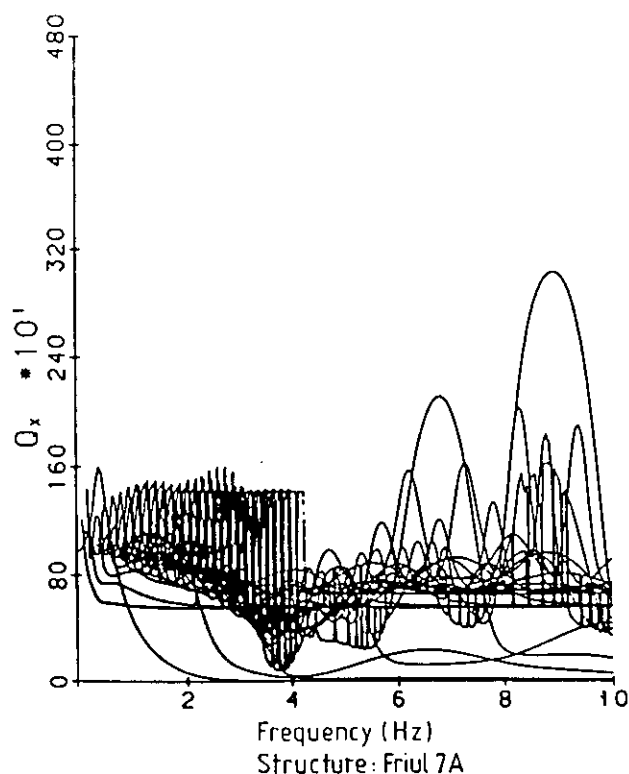


Fig. 20. Love-wave quality factor Q_x for the structure FRIUL7A, for the modes 0–30

sedimentary layers, these discontinuities are present only once in a given mode and only for modes with large order-number (Panza 1985). However, when there are sediments at the top of the model, several discontinuities are also present in each of the first higher modes (Fig. 18). Thus, due to the presence of sediments, the particle motion of several modes is essentially horizontal over a quite wide frequency range.

11.15 Quality factor Q_x

The phase attenuation C_2 is related to the quality factor Q_x by the relation

$$1/Q_x = 2C_1C_2$$

where C_1 is the anelastic phase velocity (Knopoff et al. 1964, Schwab and Knopoff 1971). For Rayleigh waves the quality factor is presented in Fig. 19, while for Love waves in Fig. 20. It is practically impossible to follow individual modes in their entirety, yet it is relatively easy to see the effect of the layering in Q_α and Q_β . For instance, the fundamental mode shows a large peak around 0.5 Hz and this corresponds to wave propagation in the crust outside the low-velocity layers

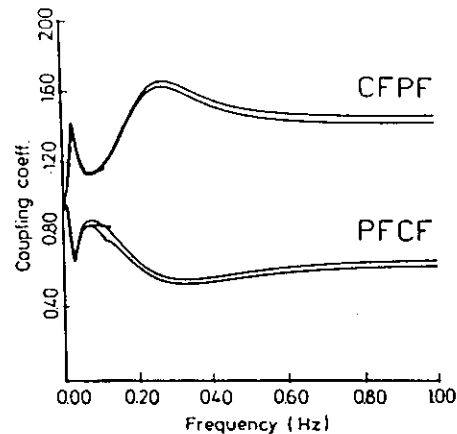


Fig. 21. Coupling coefficients, for both propagation directions (CFPF and PFCF), for the fundamental mode of the continental (C) and Pamir (P) models. Heavy lines indicate the results obtained by Levshin (1985), while the values obtained by Vaccari et al. (1989), following two different approximations, are represented by thin lines. For each direction of propagation, the upper curve is for the approximation where reflections and transmissions are not considered at the vertical boundary, while the lower curve is computed taking reflections and transmissions into account.

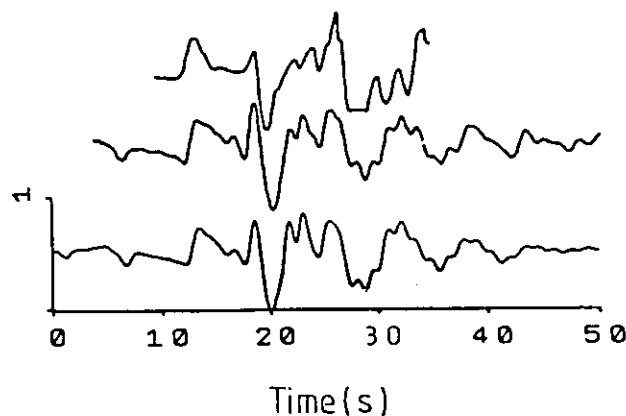


Fig. 22. Comparison between the observed ground displacement (top trace), the synthetic signals (middle trace) computed by Swanger and Boore (1978) and synthetics computed by Florsch et al. (1989) (lowest trace) for the Brawley, 1976 earthquake as recorded at station IVC. For the synthetic signals a vertical right-lateral strike-slip point source with source duration of 1.5 s, placed on a vertical plane at 6.9 km depth, is considered. All amplitudes are normalized to a source with seismic moment of 1 dyn cm.

where $Q_\beta = 400$. For frequencies larger than 3 Hz, several modes are characterized by very low values and this indicates wave propagation in the upper sedimentary layers where Q_β does not exceed 100. The trapping in the low-velocity layers,

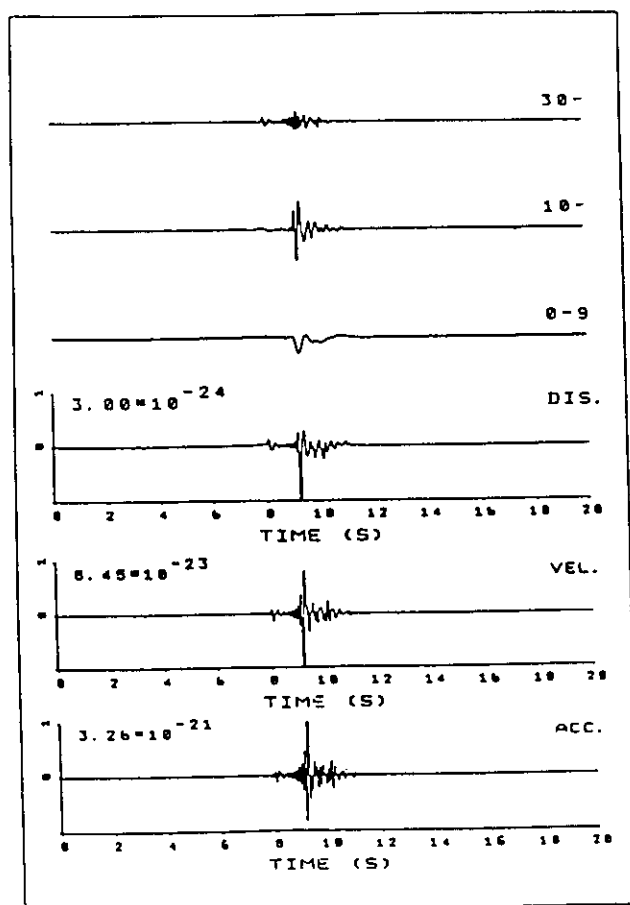


Fig. 23. Displacement, velocity and acceleration (lower three traces), computed for a receiver placed at 30 km distance from the source. The displacement is decomposed in different sets of modes (upper three traces). It shows the contribution of the higher modes to the signal waveform. A point source with source at a depth 7 km is considered (angle strike-receiver $\phi = 280^\circ$, dip $\delta = 30^\circ$ and rake $\lambda = 115^\circ$). The source rupture velocity time-function is modelled by a unit stop function. All amplitudes are normalized to a source with seismic moment of 1 dyn cm. The peak displacement is in units of cm, the peak velocity in units of cm s^{-1} and the peak acceleration in cm s^{-2} (after Vaccari et al. 1989)

characterized by $Q_\beta = 100$ and $Q_\beta = 50$, is clearly visible for several nearby modes that have almost constant Q_x close to 50, 65, and 100. The value 65 represents the weighted average by layer thickness of the values 50 and 100, both present in the upper crustal low velocity zone. Q_β is very low in the sedimentary layers. Modes mainly propagating in these layers are therefore characterized by low Q_x values (e.g. see Fig. 20 for Love modes). This is the case for the first few modes, especially for the fundamental and first higher mode. The effect of layering of Q_β can be observed for several nearby modes that have almost constant Q_x , for example Q_x close to

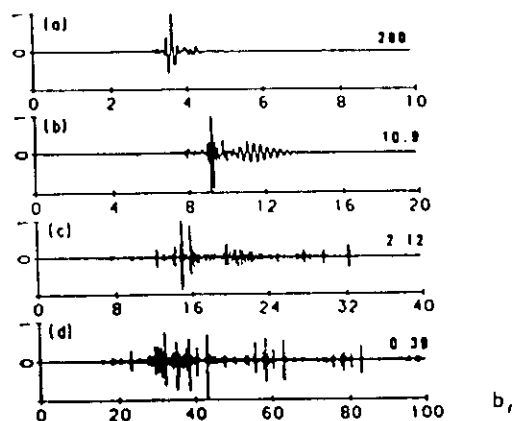
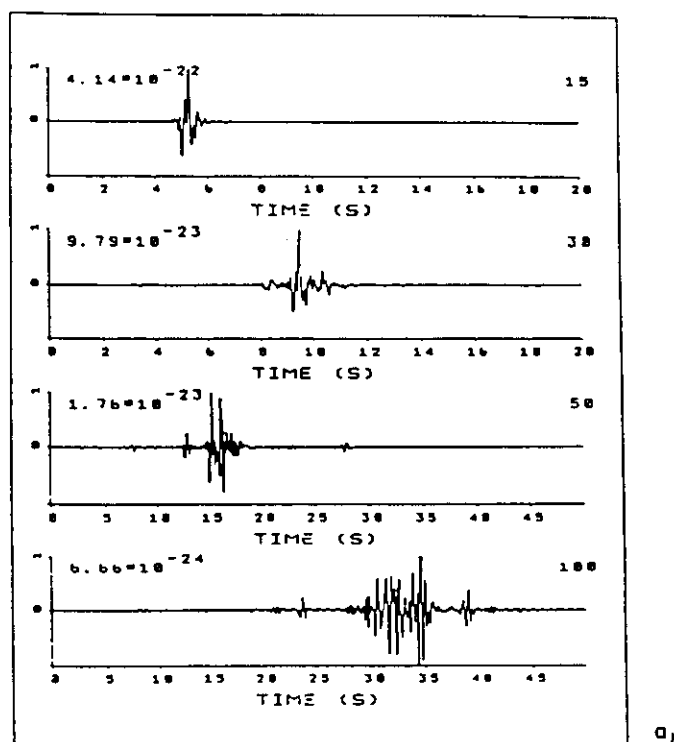


Fig. 24. a) Transverse component of acceleration-time series at different distances from the source (15 km, 30 km, 50 km and 100 km). A point source with source duration of 0.5 s and 7 km depth is considered (angle strike-receiver $\phi = 280^\circ$, dip $\delta = 30^\circ$ and rake $\lambda = 115^\circ$). All amplitudes are normalized to a source with seismic moment of 1 dyn cm. The peak acceleration is in units of cm s^{-2} (after Vaccari et al. 1989); b) Radial component of acceleration for an instantaneous point source ($h = 6$ km, $\phi = 68^\circ$, $\delta = 15^\circ$, $\lambda = 75^\circ$) with seismic moment $|M_0| = 1$ dyn cm as a function of the epicentral distance r ; from top to bottom $r =$ (a) 10, (b) 30, (c) 50, (d) 100 km. The maximum zero-to-peak amplitudes are normalized to one. In this and all subsequent figures, the number above each seismogram gives the peak acceleration (here in units of $10^{-22} \text{ cm/sec}^2$), and on the horizontal axis the time is given in seconds. The structure used is FRIUL7A

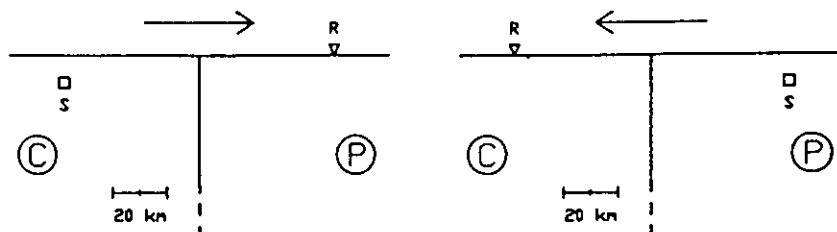


Fig. 25. Geometrical scheme of the source-receiver system used to compute the synthetic seismograms shown in Fig. 27. In this example, the vertical discontinuity is perpendicular to the x - z plane containing the source and the receiver ($\phi = \phi' = 0^\circ$)

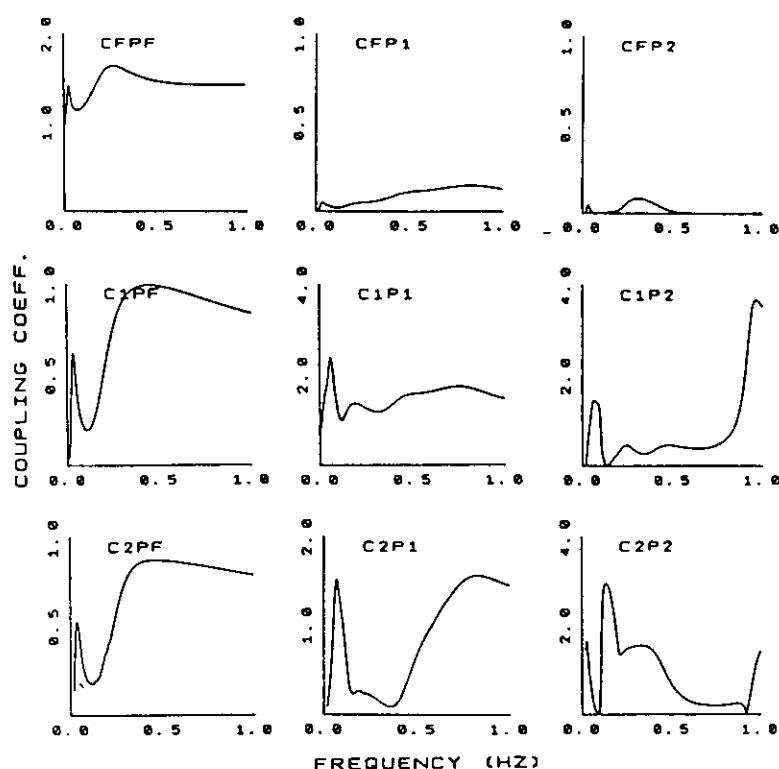


Fig. 26. a. Coupling coefficients used to compute the synthetic seismograms shown in Figs 27a and 27b

65. The resulting Q_z values are close to the values Q_β of the structural model for Love wave modes whose eigenfunction mainly sample the corresponding part of the structure.

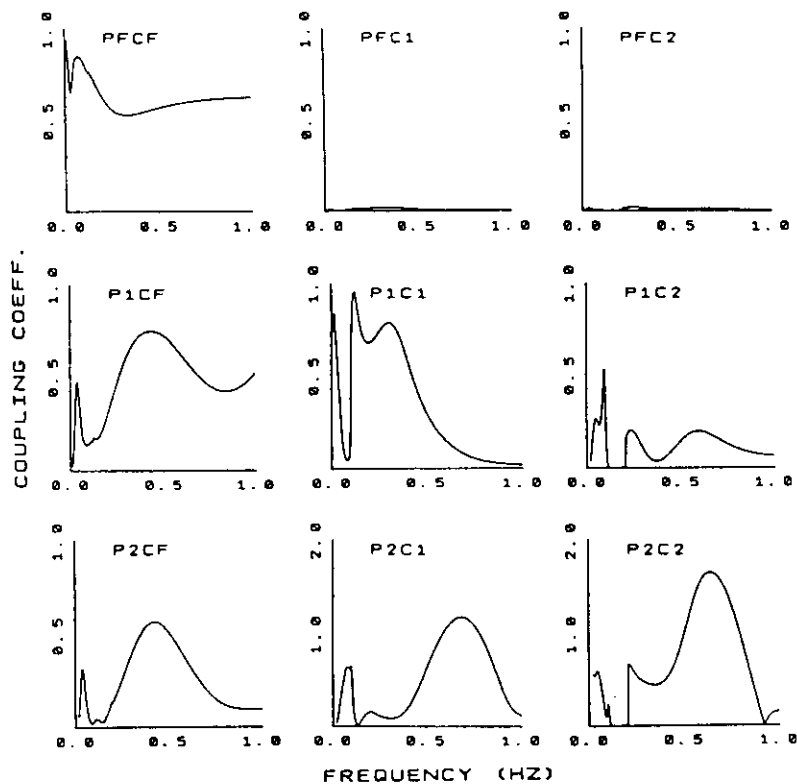


Fig. 26. b. Coupling coefficients used to compute the synthetic seismograms shown in Figs 27c and 27d

11.16 Two-dimensional case

To compare our results with the ones obtained by other authors, the coupling coefficients have been computed for two lithospheric models (Fig. 9), whose elastic properties were proposed by Patton (1980) and used by Levshin (1985). The coupling coefficients obtained by Patton (1980), applying the method proposed by Its and Yanovskaya (1985) to the fundamental modes of the two structures are shown as heavy lines in Fig. 21, while thin lines represent the results obtained by Vaccari et al. (1989). As one can see, in the common frequency range the agreement is very good. For each direction of propagation two thin lines are drawn: the upper one has been computed by simply projecting the stress-displacement system of the first medium on the normal mode system of the second one, without taking into account the local effects introduced by the vertical boundary. This represents a possible simplification of the method and of course has the advantage of requiring less computation time. In this example, the results do not differ very much, and it seems that the coupling between the modes is highly dependent on the difference between the eigenfunctions of the two systems, and that the refraction at the ver-

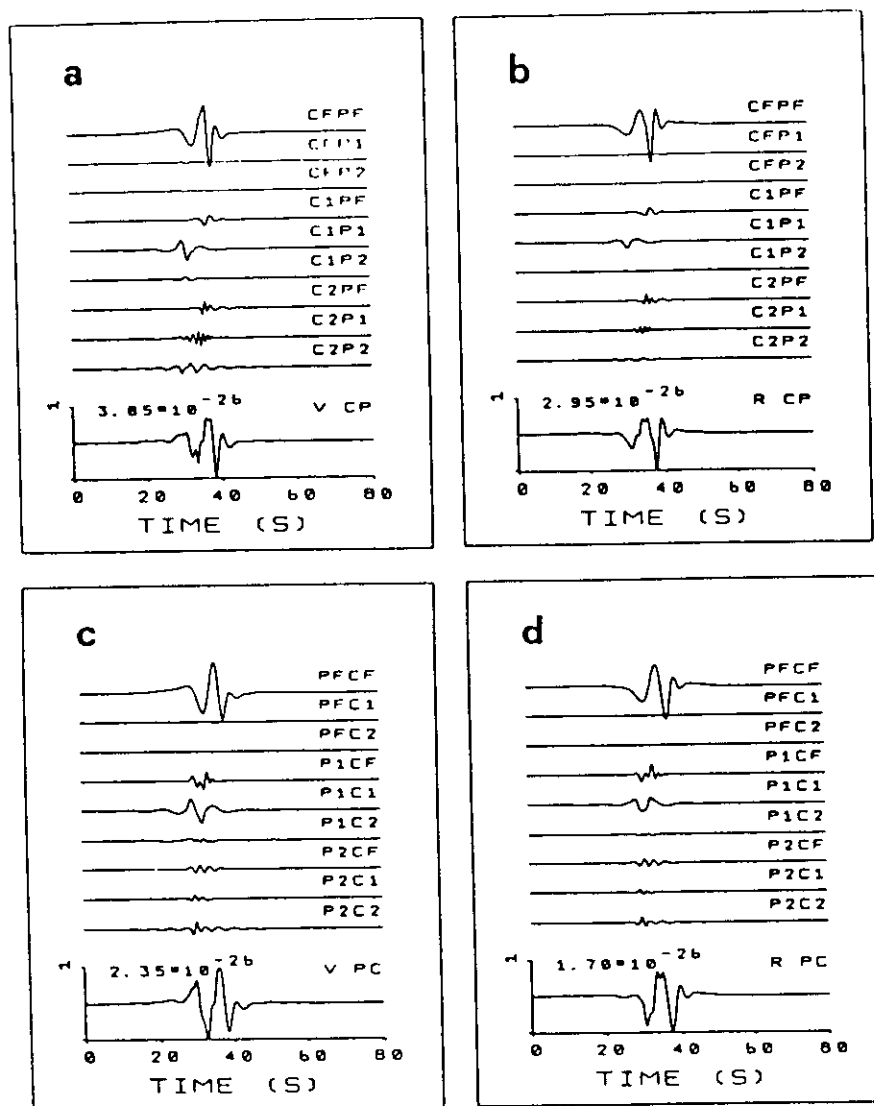


Fig. 27. Synthetic displacements obtained for a source-receiver distance of 100 km, according to the geometrical scheme of Fig. 25. Source in the C model and receiver in the P model: a) vertical component, b) radial component. Source in the P model and receiver in the C model: c) vertical component, d) radial component. The instantaneous point-source used corresponds to a double-couple 10 km deep defined by: strike-receiver angle 60° , dip 90° , rake 0°

tical boundary plays just a secondary role. This may be no longer true for models where the contrast between the elastic parameters of the two structures is very strong, and for different angles of incidence.

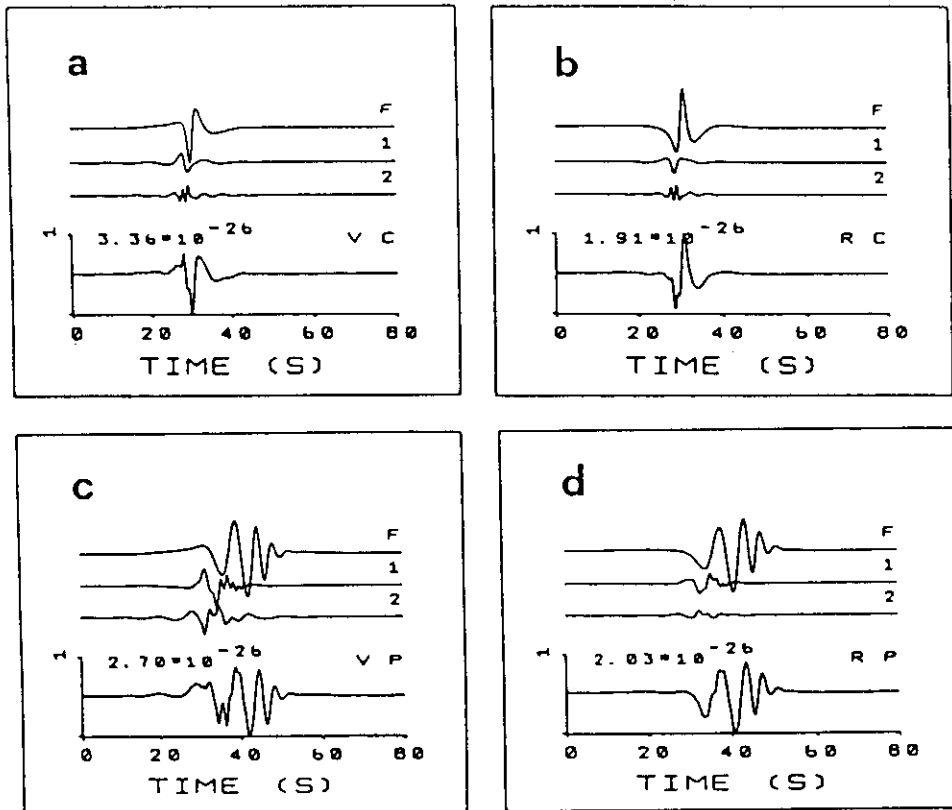


Fig. 28. Synthetic displacements computed for C and P models separately, using the same source parameters of Fig. 27. C model: a) vertical component, b) radial component. P model: c) vertical component, d) radial component

11.2 Time domain

11.21 One-dimensional case

The first example corresponds to the November 4, 1976 Brawley, California earthquake. The structural model and source parameters have been proposed by Heaton and Helmberger (1978). The structure is given in Table II. Since Swanger and Boore (1978) computed synthetic seismograms for this event with the mode-summation technique limited to the perfectly elastic case, their result provides a useful test, even if limited to the perfectly elastic case, for our programs. Therefore, the source parameters used in the synthesis are the same as those given by Swanger and Boore (1978).

A strike-slip point source is placed on a vertical plane at 6.9 km depth. The rupture velocity time-function is a symmetrical triangle with a base of 1.5 seconds. The rupture velocity time-function is given by $(i/\omega) \cdot R(\omega)$, where $R(\omega)$ is the

Table II. Imperial Valley structure proposed by Heaton and Helmberger (1978)

Layer	Thickness [km]	Density [g/cm ³]	S-velocity [km/s]
1	0.95	1.80	0.88
2	1.15	2.35	1.50
3	3.80	2.60	2.40
halfspace	∞	2.80	3.70

Fourier transform of the equivalent point-force time-function. At a distance of 33 km from the source, the displacement consists almost entirely of the fundamental mode and the first higher modes (Fig. 22). The transverse component of the recorded displacements at the station IVC, 33 km from the source is given in the same figure. The upper frequency limit is 1 Hz. It can be seen that there is generally a very good agreement between the two synthetic signals. There is only a slight difference in the value of the peak amplitudes and in the coda, due to the effect of attenuation included in our codes. Similar examples for radial and vertical component of motion are given by Panza and Suhadolc (1987).

In the second example we present synthetic seismograms for the structural model FRIUL7A shown in Table I. The upper frequency limit is 10 Hz. The source parameters correspond to the Friuli, May 6, 1976 earthquake in the point source approximation, with the source parameters taken from Suhadolc et al. (1988). The receivers are chosen in the direction of the dominant lobe of the radiation pattern of SH-waves (north-east direction, with a strike-receiver angle of 235°), resulting from the selected source parameters. Synthetic ground displacements, velocities and accelerations are presented in the lower part of Fig. 23. The signals are filtered with a Gaussian filter (the first filtered frequency is at 9 Hz and the reduction of the amplitude by factor 1/100 at the cutoff frequency of 10 Hz). This filter prevents ringing due to the cutoff frequency. A decomposition of the displacement into different sets of modes is presented in the upper part of the figure. It shows that the higher modes are essential in defining the shape of the waveform, especially in the body wave part of the synthetic seismograms. In Fig. 24a, transverse component synthetics due to a source with a finite rise time are presented. The rupture velocity time-function is a symmetrical triangle with a base of 0.5 seconds. The signals are filtered with the described Gaussian filter. As expected, the energy is shifted to lower frequencies, as the duration of the source is increased. The strong phases at about 35 s, for the signal at 100 km distance from the source (lowest trace), can be identified as the Lg phases. In Fig. 24b an example of the radial component is given.

11.22 Two-dimensional case

Synthetic displacements have been computed putting an instantaneous point-source, corresponding to a double-couple, in one medium and a receiver in the

other one, according to the scheme of Fig. 25. Both the source and the receiver are 50 km away from the vertical boundary, and the parameters defining the source are as follows: depth 10 km, strike-receiver angle 60° , dip 90° , rake 0° . For each model the fundamental and the first two higher modes have been considered, and the contribution of each incident mode to all of the outgoing modes have been computed. The coupling coefficients for both propagation directions are represented in Fig. 26, while the seismograms — vertical (V) and radial (R) components — are shown in Fig. 27. In each frame, from top to bottom, the displacements due to the nine possible combinations between the three modes of each model and their sum are plotted. Each seismogram is normalized to the peak value of the sum, and this allows to fully appreciate the relevance of the energy exchanges between modes of different order number. The synthetic seismograms of Fig. 27, when compared with the seismograms computed for the two models separately (Fig. 28), clearly show the effects of the lateral discontinuity.

The application of the reversibility theorem gives the possibility of evaluating the reliability of the results, even when very complicated models, with high contrasts in the elastic parameters, are in welded contact. Although no comparison with experimental seismograms has been tried yet, the results obtained in the present work encourage us to extend the frequency limits and to take into account the contact of more than two structures.

The analytical integration proved to be accurate and extremely time-saving (up to 80 % faster than the numerical integration, when the eigenfunctions sample the model to the maximum depth).

Appendix A

Derivatives of the matrices for the computation of the group velocities

Three cases have to be distinguished: $c > \beta_m$, $c < \beta_m$ and $c = \beta_m$. To avoid too heavy notations, layer indices m and n are dropped. The S-wave velocity is denoted by β . Introducing body wave dispersion, we have:

$$\beta = \frac{B_1(\omega_0)}{1 + \frac{2}{\pi} \cdot B_1(\omega_0) \cdot B_2(\omega_0) \cdot \ln\left(\frac{\omega_0}{\omega}\right)}.$$

$B_1(\omega_0)$ and $B_2(\omega_0)$ are the phase velocity and the phase attenuation at the reference angular frequency ω_0 . The rigidity is $\mu = \rho\beta^2$, ρ is the density and d is the thickness. Derivatives with respect to c are denoted using a dot symbol ($\cdot$), while those with respect to ω are denoted with a prime symbol ($'$).

If the halfspace is solid, we use the quantity s (Schwab and Knopoff 1972):

$$s = -\mu \cdot \left(1 - \left(\frac{c}{\beta}\right)^2\right)^{1/2}$$

$$\dot{s} = \frac{\delta s}{\delta c} = \frac{\mu \cdot c}{\beta^2 \cdot \left(1 - \left(\frac{c}{\beta}\right)^2\right)^{1/2}}$$

$$s' = \frac{\delta s}{\delta \omega} = \frac{2 \cdot s \cdot B_2(\omega_0) \cdot \beta}{\pi \cdot \omega} \cdot \left(2 + \frac{c^2}{\beta^2 - c^2} \right)$$

first case: $c > \beta$

we use here the following notations:

$$r = \left(\left(\frac{c}{\beta} \right)^2 - 1 \right)^{1/2}$$

$$Q = \omega \cdot r \cdot \frac{d}{c} = k \cdot r \cdot d.$$

The layer matrix is:

$$b_{ij} = \begin{bmatrix} \cos Q & \frac{\sin Q}{\mu \cdot r} \\ \mu \cdot r \cdot \sin Q & \cos Q \end{bmatrix}.$$

We have:

$$\begin{aligned} b_{11} &= -\frac{Q \cdot \beta^2 \cdot \sin Q}{c \cdot (c^2 - \beta^2)} \\ b_{12} &= \frac{1}{\mu} \cdot \left(-\frac{c \cdot \sin Q}{\beta^2 \cdot r^3} + \frac{Q \cdot \beta^2 \cdot \cos Q}{c \cdot r \cdot (c^2 - \beta^2)} \right) \\ b_{21} &= \mu \cdot \left(\frac{c \cdot \sin Q}{\beta^2 \cdot r} + \frac{Q \cdot \beta^2 \cdot r \cdot \cos Q}{c \cdot (c^2 - \beta^2)} \right) \\ b'_{11} &= -d \left(\frac{r}{c} - \frac{2 \cdot B_2(\omega_0) \cdot c}{\pi \cdot \beta \cdot r} \right) \sin Q \\ b'_{12} &= \frac{d}{\mu \cdot r} \left(\frac{r}{c} - \frac{2 \cdot B_2(\omega_0) \cdot c}{\pi \cdot \beta \cdot r} \right) \cos Q - \frac{2 \cdot B_2(\omega_0)}{\pi \cdot \beta \cdot r \cdot \varrho \cdot \omega} \left(2 - \frac{c^2}{\beta^2 \cdot r^2} \right) \sin Q \\ b'_{21} &= \mu \cdot r \cdot d \left(\frac{r}{c} - \frac{2 \cdot B_2(\omega_0) \cdot c}{\pi \cdot \beta \cdot r} \right) \cos Q + \frac{2 \cdot B_2(\omega_0) \cdot \mu \cdot \beta \cdot r}{\pi \cdot \omega} \left(2 - \frac{c^2}{\beta^2 \cdot r^2} \right) \sin Q \end{aligned}$$

second case: $c < \beta$

we use here the following notations:

$$r = -i \cdot \left(1 - \left(\frac{c}{\beta} \right)^2 \right)^{1/2} \quad \text{with} \quad r = i \cdot r^*$$

$$Q = \omega \cdot r \cdot \frac{d}{c} = k \cdot r \cdot d \quad \text{with} \quad Q = i \cdot Q^*$$

$$Q^* = \omega \cdot r^* \cdot \frac{d}{c}$$

subsequently:

$$\begin{aligned} \sin Q &= i \cdot \sinh Q^* \\ \cos Q &= i \cdot \cosh Q^*. \end{aligned}$$

The layer matrix is:

$$b_{ij} = \begin{bmatrix} \cosh Q^* & \frac{\sinh Q^*}{\mu \cdot r^*} \\ -\mu \cdot r^* \cdot \sinh Q^* & \cosh Q^* \end{bmatrix}.$$

We have:

$$\begin{aligned} \dot{b}_{11} &= -\frac{Q^* \cdot \beta^2 \cdot \sinh Q^*}{c \cdot (c^2 - \beta^2)} \\ \dot{b}_{12} &= \frac{1}{\mu} \cdot \left(-\frac{c \cdot \sinh Q^*}{\beta^2 \cdot r^{*3}} + \frac{Q^* \cdot \beta^2 \cdot \cosh Q^*}{c \cdot r^* \cdot (c^2 - \beta^2)} \right) \\ \dot{b}_{21} &= \mu \cdot \left(\frac{c \cdot \sinh Q^*}{\beta^2 \cdot r^*} - \frac{Q^* \cdot \beta^2 \cdot r^* \cdot \cosh Q^*}{c \cdot (c^2 - \beta^2)} \right) \\ b'_{11} &= d \left(\frac{r^*}{c} + \frac{2 \cdot B_2(\omega_0) \cdot c}{\pi \cdot \beta \cdot r^*} \right) \sinh Q^* \\ b'_{12} &= \frac{d}{\mu \cdot r^*} \left(\frac{r^*}{c} + \frac{2 \cdot B_2(\omega_0) \cdot c}{\pi \cdot \beta \cdot r^*} \right) \cosh Q^* - \\ &\quad - \frac{2 \cdot B_2(\omega_0)}{\pi \cdot \beta \cdot r^* \cdot \varrho \cdot \omega} \left(2 + \frac{c^2}{\beta^2 \cdot r^{*2}} \right) \sinh Q^* \\ b'_{21} &= \mu \cdot r^* \cdot d \left(\frac{r^*}{c} + \frac{2 \cdot B_2(\omega_0) \cdot c}{\pi \cdot \beta \cdot r^*} \right) \cosh Q^* - \\ &\quad - \frac{2 \cdot B_2(\omega_0) \cdot \mu \cdot \beta \cdot r^*}{\pi \cdot \omega} \left(2 - \frac{c^2}{\beta^2 \cdot r^{*2}} \right) \sinh Q^*. \end{aligned}$$

third case: $c = \beta$

Here, $r = Q = 0$. Calculations at the limit $c \rightarrow \beta$ are required. The layer matrix becomes:

$$b_{ij} = \begin{bmatrix} 1 & \frac{\omega \cdot d}{\mu \cdot c} \\ 0 & 1 \end{bmatrix}.$$

We have:

$$\begin{aligned} \dot{b}_{11} &= -\frac{\omega^2 \cdot d^2}{c^3} \\ \dot{b}_{12} &= -\frac{\omega \cdot d}{\mu \cdot c^2} \cdot \left(1 + \frac{\omega^2 \cdot d^2}{3 \cdot c^2} \right) \\ \dot{b}_{21} &= \frac{2 \cdot \mu \cdot \omega \cdot d}{c^2} \\ b'_{11} &= \frac{2 \cdot d^2 \cdot \omega \cdot B_2(\omega_0)}{\pi \cdot c} \\ b'_{12} &= \frac{d}{\mu} \cdot \left(\frac{1}{c} - \frac{4 \cdot B_2(\omega_0)}{\pi} \right) + \frac{2 \cdot B_2(\omega_0) \cdot \omega^2 \cdot d^3}{3 \cdot \pi \cdot \mu \cdot c^2} \end{aligned}$$

$$b'_{21} = -\frac{4 \cdot \mu \cdot d \cdot B_2(\omega_0)}{\pi}$$

Appendix B

Computation of the integral quantities I_1 and C_2

The notation S-L denotes the transition from the solid to the liquid or rigid halfspace, while the classical solid halfspace will be denoted S-S. The dot symbol ($\dot{}$) is used here for the derivatives with respect to time. Let us first define the integrals J_m^1, J_m^2 and J_m^3 .

With

$$q_m = k \cdot r_{\beta_m} \cdot (z - z_{m-1})$$

we get:

$$J_m^1 = \int_{z_{m-1}}^{z_m} \cos^2 q_m dz$$

$$J_m^2 = \int_{z_{m-1}}^{z_m} \sin^2 q_m dz$$

$$J_m^3 = \int_{z_{m-1}}^{z_m} \sin q_m \cos q_m dz.$$

We shall also use J_m^4, J_m^5, J_m^6 and J_m^7 which are defined below. We have to distinguish three cases: $c > \beta_m$, $c < \beta_m$ and $c = \beta_m$.

For $c > \beta_m$:

$$J_m^1 = \frac{d_m}{2} + \frac{c \cdot \sin(2Q_m)}{4 \cdot \omega \cdot r_{\beta_m}}$$

$$J_m^2 = \frac{d_m}{2} - \frac{c \cdot \sin(2Q_m)}{4 \cdot \omega \cdot r_{\beta_m}}$$

$$J_m^3 = \frac{c \cdot \sin^2 Q_m}{2 \cdot \omega \cdot r_{\beta_m}}$$

$$J_m^4 = \frac{c \cdot \sin^2 Q_m}{2 \cdot \omega \cdot r_{\beta_m}^2} = \frac{J_m^3}{r_{\beta_m}}$$

$$J_m^5 = r_{\beta_m}^2 \cdot J_m^2; \quad J_m^6 = r_{\beta_m}^2 \cdot J_m^4; \quad J_m^7 = \frac{J_m^2}{r_{\beta_m}^2}.$$

For $c < \beta_m$:

$$J_m^1 = \frac{d_m}{2} + \frac{c \cdot \sinh(2Q_m^*)}{4 \cdot \omega \cdot r_{\beta_m}^*}$$

$$J_m^2 = \frac{d_m}{2} - \frac{c \cdot \sinh(2Q_m^*)}{4 \cdot \omega \cdot r_{\beta_m}^*}$$

$$J_m^3 = \frac{i \cdot c \cdot \sinh^2 Q_m^*}{2 \cdot \omega \cdot r_{\beta_m}^*}$$

$$J_m^4 = \frac{c \cdot \sinh^2 Q_m^*}{2 \cdot \omega \cdot r_{\beta_m}^{*2}} = \frac{J_m^3}{r_{\beta_m}^*}$$

$$J_m^5 = -r_{\beta_m}^{*2} \cdot J_m^2; \quad J_m^6 = -r_{\beta_m}^{*2} \cdot J_m^4; \quad J_m^7 = -\frac{J_m^2}{r_{\beta_m}^{*2}}.$$

For $c = \beta_m$:

$$J_m^1 = d_m; \quad J_m^2 = 0; \quad J_m^3 = 0$$

$$J_m^4 = \frac{\omega \cdot d^2}{2c}; \quad J_m^5 = 0; \quad J_m^6 = 0; \quad J_m^7 = \frac{\omega^2 \cdot d^3}{3c^2}.$$

B.1. Energy integral

The energy integral is:

$$I_1 = \int_0^\infty \varrho \cdot \left(\frac{\dot{v}(z)}{v_0} \right)^2 dz.$$

For a layered medium, the energy integral can be written:

$$I_1 = \begin{cases} \left(\frac{c}{v_0} \right)^2 \cdot \sum_{m=1}^n I_{(m)} & \text{for the (S-L) case} \\ \left(\frac{c}{v_0} \right)^2 \cdot \left(\left(\sum_{m=1}^n I_{(m)} \right) + I_{(S-S)} \right) & \text{for the (S-S) case} \end{cases} \quad (B1)$$

with:

$$I_{(m)} = \int_{z_{m-1}}^{z_m} \varrho_m \cdot \left(\frac{\dot{v}(z)}{c} \right)^2 dz \quad (B2)$$

$$I_{(S-S)} = \int_{z_{n-1}}^\infty \varrho_n \cdot \left(\frac{\dot{v}(z)}{c} \right)^2 dz \quad (B3).$$

The integrals can be written:

$$I_{(m)} = -\varrho_m \cdot k^2 \cdot \left(v_{m-1}^2 \cdot J_m^1 + \frac{(y_{z_{m-1}})^2 \cdot J_m^7}{k^2 \cdot \mu_m^2} + \frac{2v_{m-1} \cdot y_{z_{m-1}} \cdot J_m^4}{k \cdot \mu_m} \right)$$

$$I_{(S-S)} = \frac{\varrho_n \cdot v_{n-1}^2 \cdot k}{2 \cdot r_{\beta_n}^*}$$

where v_m is the displacement in the m 'th layer and y_{z_m} is the stress in the m 'th layer.

B.2. Phase attenuation

The coefficient C_2 is given by:

$$C_2 = \frac{\int_0^\infty \mu \cdot B_1 \cdot B_2 \cdot \left(\frac{y_z^2}{\mu^2 \cdot k^2} + v^2 \right) dz}{c \int_0^\infty \mu \cdot v^2 dz}.$$

C^2 can also be written:

$$C_2 = \frac{I^1 + I^2}{c \cdot I^3}$$

with:

$$\begin{aligned} I^1 &= - \int_0^\infty \mu \cdot B_1 \cdot B_2 \cdot \left(\frac{\dot{v}}{c} \right)^2 dz = k^2 \cdot v_0^2 \cdot \int_0^\infty \mu \cdot B_1 \cdot B_2 \cdot \left(\frac{\dot{v}(z)}{\dot{v}_0} \right)^2 dz \\ I^2 &= \int_0^\infty \mu \cdot B_1 \cdot B_2 \cdot \frac{y_z^2}{\mu^2} dz \\ I^3 &= - \int_0^\infty \mu \cdot \left(\frac{\dot{v}}{c} \right)^2 dz = k^2 \cdot v_0^2 \cdot \int_0^\infty \mu \cdot \left(\frac{\dot{v}(z)}{\dot{v}_0} \right)^2 dz. \end{aligned}$$

To compute I^1 and I^3 we can use the same scheme as for I^2 , assuming simple substitution in multiplicative coefficients:

for I^1 we shall use $(\mu_m B_{1_m} B_{2_m} k^2 v_0^2)$ instead of μ_m and

for I^3 we shall use $(\mu_m k^2 v_0^2)$ instead of μ_m .

Let us now consider I^2 for the S-L case. We obtain:

$$I^2 = k^2 \cdot \sum_{m=1}^n \mu_m \cdot B_{1_m} \cdot B_{2_m} L_m$$

while for the S-S case, we have:

$$I^2 = \left(k^2 \cdot \sum_{m=1}^{n-1} \mu_m \cdot B_{1_m} \cdot B_{2_m} L_m \right) - \frac{k \cdot \mu_n \cdot B_{1_n} \cdot B_{2_n} \cdot v_{n-1}^2 \cdot r_{\beta_n}^*}{2}.$$

The quantity L_m is given by:

$$L_m = v_{m-1}^2 \cdot J_m^5 + \frac{y_{z_{m-1}}^2 \cdot I_{(m)}}{k^2 \cdot \mu_m^2} - \frac{2 \cdot v_{m-1} \cdot y_{z_{m-1}} \cdot J_m^6}{k \cdot \mu_m}.$$

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