

SMR: 1059/8

***FOURTH WORKSHOP ON THREE DIMENSIONAL MODELLING
OF SEISMIC WAVES GENERATION.
PROPAGATION AND THEIR INVERSION***

28 September - 9 October 1998

Receiver Function Techniques

presented by:

Lev VINNIK

Institute of Physics of the Earth

Bolshaya Gruzinskaya 10

D-242

123810 Moscow

Russia

These are preliminary lecture notes, intended only for distribution to participants.

RECEIVER FUNCTION TECHNIQUES

Lev Vinnik

Institute of Physics of the Earth, Moscow, Russia

Introduction

Most techniques for investigating subsurface structures, like seismic tomography, require a network of seismograph station. Receiver function techniques are exceptions: they can be used with the data of a single station. The idea is to replace the network of stations by the network of seismic events. Seismic phases which are present in the seismogram can be classified into primary and secondary phases. For example, the primary P wave is followed by reflected and converted phases generated in the crust and mantle in a vicinity of the station. In order to detect the secondary phases and extract the information contained in them, one should inspect a number of recordings of the same station. This is difficult, because these phases are usually weak, the P waveform of each event is different from the others, and this is true with respect to the secondary phases. However, using a special kind of filtering (deconvolution), the P waveform of each event can be transformed into a standard 'spike' or 'bump'. This deconvolution transforms into a standard form every secondary phase, as well, and then the phases of interest, if they are strong, can be easily traced from one event to the other, or, if they are weak, detected by stacking the deconvolved traces with appropriate time-shift (moveout) corrections.

Receiver functions in a narrow sense

The term 'receiver function' was introduced for the radial component of the initial part of the seismogram $R(t)$, deconvolved by the vertical component $Z(t)$. Instead of these two components, we use the L component corresponding to the principal motion direction of the P wave, and the H (or SV) component, perpendicular to L in the wave propagation plane. H deconvolved by L can also be termed receiver function.

Assuming that the P wave propagates in a horizontally layered and isotropic medium, for every discontinuity there are three secondary phases in the H component, with comparable amplitudes: Ps (converted from P to S), $Ppps$ (transmitted as P , reflected from the free surface as P , and reflected from the discontinuity as S) and $Ppss$ (transmitted as P , reflected from the free surface as S and reflected from the discontinuity as S). The multiple reflections of the higher order are much weaker and can be neglected. Ps , $Ppps$ and $Ppss$ are strong in the H

component and missing in the L component. The only significant secondary phase in $L(t)$ is P_{ppp} (P , once reflected from the free surface and once from the discontinuity), but it is usually much weaker than the parent P .

Deconvolution

Deconvolution can be carried out either in time or frequency domain. Frequency domain deconvolution can be carried out as follows. For the L and H components, neglecting the instrument response, we can write

$$L(t) = S(t) \quad (1)$$

$$R(t) = S(t) * E(t) \quad (2)$$

where $*$ is convolution, $S(t)$ is source function, and $E(t)$ can be written as

$$E(t) = \alpha_1 \delta(t - \tau_1) + \alpha_2 \delta(t - \tau_2) \dots \quad (3)$$

Here α_i are amplitudes of the secondary phases, and τ_i are their delays relative to P . In spectral domain we can write

$$L(\omega) = S(\omega) \quad (4)$$

$$H(\omega) = S(\omega)E(\omega) \quad (5)$$

$$E(\omega) = H(\omega)/L(\omega) \quad (6)$$

Dividing the spectrum of the H component by the spectrum of the L component for extracting information on the crustal structure is the essence of the spectral ratio technique, which was known prior to the receiver functions. The idea of the receiver function approach is to bring the spectral ratio into time domain by inverse Fourier transformation. To avoid large errors caused by small values of the denominator, the spectral ratio to be Fourier transformed is modified as

$$E'(\omega) = \frac{H(\omega)\overline{L(\omega)}}{\Phi(\omega)}G(\omega) \quad (7)$$

where

$$\Phi(\omega) = \max\{L(\omega)\overline{L(\omega)}, c \cdot \max[L(\omega)\overline{L(\omega)}]\}, \quad (8)$$

the bar over L denotes the complex conjugate, and $G(\omega) = \exp(-\omega^2/4a^2)$.

The constant c is called 'water level' and is selected empirically. $G(\omega)$ is required to suppress high frequencies and is also determined empirically. $E'(t)$ is recovered by inverse Fourier transformation.

The time-domain deconvolution can be carried as follows. The vertical component of the P wave can be presented in discrete form as:

$$Z_k = s_k + n_k = \sum_{i=0}^{N-1} r_i u_{k-i} \quad (9)$$

Here s and n denote signal and noise, and u is a function which is equal to 1 for $k = K$ and 0 for $k \neq K$. We look for such a filter g , that, if it is applied to Z , the result has the minimum rms deviation from u . The solution for g is provided by the system

$$[C][G] = [R] \quad (10)$$

Here $[G]$ is the column vector with elements g , $[R]$ is the column vector with elements r , and $[C]$ is the matrix

$$\begin{array}{cccc} C_0^s + C_0^n & C_1^s + C_1^n & \dots & C_{N-1}^s + C_{N-1}^n \\ C_1^s + C_1^n & C_0^s + C_0^n & \dots & C_{N-2}^s + C_{N-2}^n \\ \dots & \dots & \dots & \dots \\ C_{N-1}^s + C_{N-1}^n & C_{N-2}^s + C_{N-2}^n & \dots & C_0^s + C_0^n \end{array}$$

The elements of this matrix are autocorrelations of s and n . In practice, the noise is assumed to be white, which means that its autocorrelation differs from 0 only for the diagonal elements of the matrix. The presence of noise is accounted for by multiplying the diagonal elements by $(1+\lambda)$. The choice of this parameter, like waterlevel parameter, is arbitrary. When it is close to 0, the related 'spike' is broad-band, but the inversion is unstable. When λ is very large (say, 100), the deconvolved waveform is close to the autocorrelation function of the initial waveform. The optimum values of λ are in the intermediate range.

The main purpose of deconvolution is to prepare the individual records for stacking. Instead of deconvolving every record separately, one can find a multichannel deconvolution filter for the set of records. In that case the filter for each record should be found by considering the other records, as well. The calculations of the multichannel deconvolution filter are in principle similar to the single-channel case, but require more algebra. The elements of $[C]$, $[G]$ and $[R]$ for the multi-channel deconvolution are matrices. It was shown that this technique is superior to the single-channel deconvolution, if the spectra of the individual events are strongly different. In that case the 'spike' obtained by multichannel deconvolution is sharper than can be obtained with the same damping from the single-channel deconvolution. The practical advantages of this approach, however, are negligible in the teleseismic range, where the spectra of the P waves don't differ much.

Stacking receiver functions with moveout corrections

To detect weak converted and reflected phases which are present in the receiver function, one should stack receiver functions for events in a broad range of distances. The delay of the converted phase relative to the parent P wave is given approximately by

$$t(Ps) = \int_{r_d}^{r_0} \sqrt{V_S^{-2} - p^2 r^{-2}} - \sqrt{V_P^{-2} - p^2 r^{-2}} dr, \quad (11)$$

where p is ray parameter, r is the radial distance of the discontinuity, r_0 and r_d correspond to free surface and depth of conversion, V_P and V_S are P and S velocities. The time of the converted phase (its delay relative to P) is increasing with the increasing ray parameter value. For 660 km discontinuity, the difference for epicentral distances around 30 and 90 degrees is close to 10 s. To detect the converted phases, the receiver functions should be stacked with travel time corrections, which depend on the ray parameter of P (or distance of event) and the assumed depth of the discontinuity. In practice the stack is calculated for many assumed depths of conversion. The real signals (the converted phases) are focused at depths, which are consistent with the depths inferred from the times of these phases. This is an important criterion for discriminating between the signals and noise.

Contrary to the converted phases, multiply reflected phases have a different dependence on ray parameter: the delay relative to P is increasing with increasing epicentral distance. In principle, for detecting the multiples, one should calculate their theoretical delays and to introduce the moveout corrections in the same way, as it was made for the converted phases. Instead of this, one can apply slowness stacking. The moveout correction in the slowness stack for each receiver function is calculated as a differential slowness multiplied by differential distance. Differential slowness and differential distance are determined relative to the P wave and the reference distance, respectively. The best choice of the reference distance is in the middle of the available distance interval. Calculations of the slowness stack imply that the delays of the signal with respect to P depend linearly on the epicentral distance. Strictly speaking, this is not correct, but can be accepted as an approximation. Then the converted phases and the multiply reflected phases can be detected in the negative and positive differential slowness range, respectively. Since slowness is proportional to wavenumber, stacking of receiver functions is very similar to the conventional wavenumber-frequency filtering with receiver arrays. The properties of the wavenumber filter are determined by the distribution of the available events with epicentral distance in about the same way, as the performance of a receiver array depends on the array aperture. If the events are concentrated in a narrow distance range, this implies filtering with a poor resolution, which can be improved by suitable weighting.

While the receiver function technique was developed for a single station, it can also be applied

to the seismic array. In that case, the receiver functions can be stacked not for separate stations, but for separate conversion points.

Signal/noise ratio enhancement by stacking depends on the degree of correlation between the signals in the individual traces. The effect of scatter in the traveltimes is quantified with the expression:

$$E/E_0 = 1/n + [(n-1)/n] \exp(-\sigma^2 \omega^2), \quad (12)$$

where E is the actual signal energy in the stack, E_0 is the maximum possible energy, n is the number of stacked traces, ω is angular frequency of the signal, and σ is rms value of the traveltime fluctuations. There are examples of detection of phases converted from discontinuities at a depth of several hundred kilometers with a frequency of 1 Hz. Such observations are only possible if σ is not much larger than 0.1 s. This is equivalent to 1 km in depth of the discontinuity.

While stacking the receiver functions is essentially a linear procedure, there are nonlinear detection techniques, like n -th root method. Instead of the initial trace, the N -th root of it with the sign preserved is taken, and the transformed traces are stacked:

$$E_N(t) = \frac{1}{M} \sum_{i=1}^M |e_i(t)|^{1/N} \cdot \text{signum}\{e_i(t)\} \quad (13)$$

The stack is raised to the N -th power, with the sign preserved

$$E'_N(t) = |E_N(t)|^N \cdot \text{signum}\{E_N(t)\} \quad (14)$$

There are examples when this method or equivalent techniques were used for stacking the receiver functions, but the benefits of this are not evident. First, this method distorts waveforms, and since it has no strong theoretical background, the range of distortion is hard to estimate. And second, like other nonlinear techniques, it suppress weak signals and enhances strong ones. The interest of the seismologist is usually opposite.

Inverse problem for crustal receiver function

The secondary phases, which are present in receiver function have different sensitivities to the properties of the earth's medium. Multiply reflected phases are sensitive to both velocities and density contrasts. The reflection coefficient is very small, if the width of the discontinuity is around a quarter of the wavelength or larger. The converted phases are sensitive practically only to the S velocity contrast, and weakly sensitive to the width of the discontinuity. The transmission coefficient of the converted phase is reduced significantly relative to the maximum value, if the width of the discontinuity is around the S wavelength or larger. The optimum

way of interpreting receiver functions in terms of subsurface structure is, first, to identify the secondary phases by inspecting the receiver functions, and second, to apply a formal inversion algorithm.

Inverse problem for receiver functions was approached in several studies dealing with crustal structures. Meaningful results could be obtained for plane-layered models. At high frequencies (around 1 Hz), the wavefield is dominated by scattered waves rather than reflections and conversions, and the technique is applicable only starting from about 0.3 Hz. Stacking of the receiver functions is necessary in order to suppress the random component of the wavefield. In the algorithm developed in the Institute of physics of the Earth (Moscow), the synthetic SV component is calculated by using the expression

$$SV_{syn}(t, v(d), c) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{H_{SV}(\omega, v(d), c)}{H_L(\omega, v(d), c)} L(\omega) \exp(i\omega t) d\omega, \quad (15)$$

where $v(d)$ is a vector of the variable model parameters, c is the apparent velocity, $H_{SV}(\omega, v(d), c)$ and $H_L(\omega, v(d), c)$ are the SV and L components of the theoretical frequency response of the layered structure, and $L(\omega)$ is the spectrum of the stack of deconvolved L components. The theoretical response is computed using Thomson-Haskell matrix method. The model is tested by comparing the synthetic SV component with the stack of the receiver functions. The inversion procedure is based on the general method of solving ill-posed inverse problems. The optimum parameters can be found by iterative minimization of the smoothing functional:

$$F(v(d), c) = \|SV_{obs}(t) - SV_{syn}(v(d), t, c)\| + \alpha q(d) \|v(d) - v_0(d)\|, \quad (16)$$

where $v_0(d)$ is the starting velocity model, $q(d)$ is the weight function and α is the damping parameter. The parameter α changes during the inversion procedure as $\alpha_{p+1} = \alpha_p \Delta\alpha$, where α_p is the value of α in the preceding iteration, and $\Delta\alpha$ is less than 1.0. The second term of this expression keeps the solution near the starting model. This expression implies that the receiver functions are stacked in a narrow range of apparent velocities. Usually the result depends on the starting function, and in order to detect robust features of the solution, many experiments with different starting models are required.

The highest accuracy of inversion is achieved in the high gradient zones, because these zones produce the converted phases, and the solution appears to be well constrained. Therefore, the best results are often obtained for the upper and the lower crust, but not for the middle crust with a slowly varying velocities. There is a trade-off between the average velocity above the discontinuity and its depth. Theoretically, the uncertainty can be decreased by using receiver functions in a broad range of epicentral distance, but practical evidence of this so far is missing.

Other kinds of receiver functions

There are other other seismic techniques with a very similar approach to the data processing. For example, studies of the mantle discontinuities based on the observations of precursors to SS use a similar technique. Teleseismic SS phases (S waves reflected from the Earth's surface in the midpoint between the source and the receiver) are preceded by the weak phases (precursors), reflected from the underside of the mantle discontinuities. In this case, the parent phase is SS , and its precursors are detected with the technique very similar to stacking the receiver functions. SH component of the record is deconvolved by the SH component of SS , and the deconvolved records of many events at many stations with the bouncing points within certain regions are stacked. The times of the precursors relative to SS depend on the S velocity in the upper mantle and depths of the discontinuities in the region. Resolution of this technique is disputable, and at the end of the lecture, I will compare the results of global mapping of the mantle transition zone with the precursors to SS and with the Ps converted phases.

A similar approach is used in the analysis of multiple ScS reverberation. The coefficients of reflection of SH at the surface of the Earth and the core-mantle boundary are close to 1.0, and the records of strong deep events contain a sequence of clearly visible SH waves, reflected a few times from the free surface and the core-mantle boundary. These strong phases are accompanied by weaker ones, reflected from the mantle discontinuities. To detect phases reflected from the mantle discontinuities, the records are deconvolved by ScS and the deconvolved traces are stacked with appropriate moveout corrections.

The best data on discontinuities in subduction zones are provided by the observations of the phases converted from S to P in the source region. They arrive in the tail of P . Usually, these phases are detected in short-period array recordings of deep events. Now, the receiver function technique is modified for detecting these phases in the broad-band recordings of conventional digital stations. The idea is to deconvolve the vertical component of the teleseismic record by the S waveform in the same record and to stack the deconvolved vertical components of several events at several stations with moveout corrections.

Receiver functions for anisotropic media

In the isotropic, laterally homogeneous medium the P wave is coupled only with SV . In anisotropic medium, all three components of motions (vertical Z , radial R and transverse T) are coupled.

One of the most useful techniques for measuring anisotropy in the mantle is closely related to

the receiver function technique. In azimuthally anisotropic medium, S wave splits into two quasi-shear waves, whose polarizations are perpendicular to the wave propagation direction and to each other. For simplicity we will consider hexagonal anisotropy with a horizontal axis of symmetry. This is the simplest kind of azimuthal anisotropy, which is described by the minimum number of elastic parameters. If the SV wave with a small angle of incidence enters the anisotropic layer, the effect of propagation can be described by the two parameters: β , the angle in the horizontal plane between the fast velocity direction and back azimuth of the event, and δt , the delay of the slow split wave relative to the fast one. It can be shown that at long periods ($\omega\delta t \ll 1$) for $R(t) = \cos \omega t$, we have

$$T(t) \approx 0.5\omega\delta t \sin 2\beta \sin \omega t \quad (17)$$

This means that the T component is shifted in time with respect to the R component by a quarter period, and its amplitude is proportional to the frequency. In other words, the T component is proportional to the derivative of the R component. The term $\sin 2\beta$ means that the T/R amplitude ratio is a periodic function of angle β with a period of 180° .

Initial SV polarization is the property of SKS and $SKKS$, the phases generated as S , propagated as P through the liquid Earth's core, and converted to SV at the core/mantle boundary. The presence of the T component in SKS with the properties described by eq. 17 would indicate the presence of anisotropy. Contribution of anisotropy to the T component can be retrieved by deconvolving the T component by the R component and stacking the deconvolved T components of many records with weights depending on back azimuths of the events. The weights W_i are defined as

$$W_i = \cos 2(\phi_i - \psi) / \sum_{i=1}^n \cos^2 2(\phi_i - \psi) \quad (18)$$

where ϕ_i is back azimuth of the i -th event, n is the number of events and ψ is a variable parameter. The maximum amplitudes of the stack are obtained for the values of ψ which differ from the fast direction of anisotropy by $\pm 45^\circ$. The value of δt can be found from the maximum amplitude of the stack. There are other algorithms for extracting the parameters of anisotropy from the SKS recordings.

Due to the near-vertical propagation, SKS is sensitive to the lateral variations of anisotropy, but insensitive to the variations of anisotropy with depth. The receiver function approach is helpful in retrieving variations of anisotropy with depth. When the P wave interacts with the discontinuity between anisotropic media, or isotropic and anisotropic media, both SV and SH

are generated by conversion and reflection. At this boundary, P wave can be generated by the incoming SV and SH . Two methods of processing the data, both based on the receiver function approach are described in the literature.

One method deals with the P precursors to the teleseismic S , which are thought to be the phases converted from S to P at the discontinuities beneath the station. Let $[A(\omega)]$ be the column vector with the elements $R(\omega)$, $T(\omega)$ and $Z(\omega)$, the spectra of the recorded radial, transverse and vertical components. Let $[S(\omega)]$ be the column vector with the elements $SV_0(\omega)$ and $SH_0(\omega)$, the spectra of the SV and SH components of the S wave, which comes from below to the bottom of the layered anisotropic medium. Then

$$[A] = [F(\omega)][S], \quad (19)$$

where matrix $[F(\omega)]$ contains the transfer functions, which can be obtained by anisotropic Thomson-Haskell method:

$$\begin{array}{cc} R_{SV}(\omega) & R_{SH}(\omega) \\ T_{SV}(\omega) & T_{SH}(\omega) \\ Z_{SV}(\omega) & Z_{SH}(\omega) \end{array}$$

From the first two equations of the system (19), we can obtain $SV_0(\omega)$ and $SH_0(\omega)$ as a function of observed R and T . This solution can be used in the third equation of system (19) to obtain the synthetic vertical component:

$$Z_{syn}(\omega) = Z_{SV}(\omega)SV_0(\omega) + Z_{SH}(\omega)SH_0(\omega) \quad (20)$$

The search for the optimum model is carried out by minimizing the penalty function:

$$E(m) = \frac{1}{n} \sum_{events} \frac{\int [Z(t) - Z_{syn}(t, m)]^2 dt}{\int [R^2(t) + T^2(t) + Z^2(t)] dt}, \quad (21)$$

where m represents the variable parameters of the model and n is the number of events. An obvious possibility to develop further this technique is to invert the stacked receiver functions, instead of the individual records.

In the other method, to retrieve anisotropic parameters from the P to S converted phases the receiver functions are stacked with weights given by eq. (18) and moveout corrections given by eq. (11). The optimum model minimizes the discrepancies between the theoretical and observed SV and T components of the stacked receiver functions. The theoretical SV and T components are computed with anisotropic Thomson-Haskell technique, using the deconvolved and stacked P components as the input. A search for the optimum model is conducted by using a genetic algorithm. The main problem of inverting the three-component records for receiver anisotropy

is caused by a trade-off between effects of anisotropy and lateral heterogeneity: the signals in the T component can be generated by topography on the discontinuities.

Examples of applications of receiver function techniques

Applications of the receiver function techniques will be illustrated mainly with results of a few recent studies. The geophysical problems to be addressed are (1) topography of the major upper mantle discontinuities on a global scale, (2) improved estimates of the scaling coefficient for the teleseismic P and S residuals, (3) evidence of midmantle (800 - 1300 km depth) discontinuities, and (4) evidence of flow at the base of the upper mantle.

Literature

- Ammon, Ch. J., G. E. Randall and G. Zandt, On the nonuniqueness of receiver function inversions, *J. Geophys. Res.*, 95, 15303-15318, 1990.
- Berkhout, A. J., Least square inverse filtering and wavelet deconvolution, *Geophysics*, 42, 1369-1383, 1977.
- Cassidy, J. F., Numerical experiments in broadband receiver function analysis, *Bull. seism. Soc. Am.*, 67, 677-691, 1992.
- Chevrot, S., L. Vinnik, and J.-P. Montagner, Global scale analysis of the mantle *Pds* phases, submitted to *J. Geophys. Res.*, 1998.
- Duecker, K. G., and A. F. Sheehan, Mantle discontinuity structure from midpoint stacks of converted P to S waves across the Yellowstone hotspot track, *J. Geophys. Res.*, 102, 8313-8327, 1997.
- Farra, V., L. P. Vinnik, B. Romanowicz, G. L. Kosarev, and R. Kind, Inversion of teleseismic S particle motion for azimuthal anisotropy in the upper mantle: a feasibility study, *Geophys. J. Int.*, 106, 421-431, 1991.
- Flanagan, M., and P. Shearer, Global mapping of topography on transition zone velocity discontinuities by stacking *SS* precursors, *J. Geophys. Res.*, 103, 2673-2692, 1998.
- Kanasevich, R. E., C. D. Hemmings, and T. Alpaslan, Nth-root stack non-linear multichannel filter, *Geophysics*, 38, 327-338, 1973.
- Keith, C. M. and S. Crampin, Seismic body waves in anisotropic media: Propagation through a layer, *Geophys. J. R. astr. Soc.*, 49, 209-223, 1977.
- Kosarev, G. L., L. I. Makeyeva, and L. P. Vinnik, Anisotropy of the mantle inferred from observations of *P* to *S* converted waves, *Geophys. J. R. astr. Soc.*, 76, 209-220, 1984.
- Kosarev, G. L., N. V. Petersen, L. P. Vinnik, and S. W. Roecker, Receiver functions for the Tien Shan analog broadband network - contrasts in the evolution of structures across the Talasso-Fergana fault, *J. Geophys. Res.*, 98, 4437-4448, 1993.
- Kosarev, G., N. Petersen, and L. Vinnik, Comment on 'The use of velocity spectrum for stacking receiver functions and imaging upper mantle discontinuities' by H. Gurrola, J. B. Minster, and T. Owens, *Geophys. J. Int.*, 124, 637-641, 1996.
- Langston, C. A., Structure under mount Rainier, Washington, inferred from teleseismic body waves, *J. Geophys. Res.*, 84, 4749-4762, 1979.
- Levin, V., and J. Park, *P-SH* conversions in a flat-layered medium with anisotropy of arbitrary orientation, *Geophys. J. Int.*, 131, 253-266, 1997.

- Petersen, N., and L. Vinnik, Detection of crustal converted phases as a problem of multichannel filtering, Proc. Acad. sci. Russ., Physics of the Earth, 4, 37-44, 1991 (in Russian).
- Revenaugh, J., and T. H. Jordan, A study of mantle layering beneath the Western Pacific, J. Geophys. Res., 94, 5787-5813, 1989.
- Vinnik, L. P., Detection of waves converted from P to SV in the mantle, Phys. Earth Planet. Inter., 15, 39-45, 1977.
- Vinnik, L. P., R. Kind, G. L. Kosarev, and L. I. Makeyeva, Azimuthal anisotropy in the lithosphere from observations of long-period S waves, Geophys. J. Int., 99, 549-559, 1989.
- Vinnik, L., S. Chevrot and J.-P. Montagner, Seismic evidence of flow at the base of the upper mantle, Geoph. Res. Lett., 25, 1995-1998, 1998.
- Vinnik, L., F. Niu, and H. Kawakatsu, Broad-band converted phases from midmantle discontinuities, Earth, Planets and Space, in press, 1998.