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PROPAGATION AND THEIR INVERSION**

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***Study of Explosive Seismic Source by Joint
Inversion of Surface Wave Spectra and
Signs of the First Motion***

presented by:

Boris BUKCHIN

International Institute of Earthquake Prediction Theory
and Mathematical Geophysics
Warshavskoye Sh. 79, Kor 2
113556 Moscow
Russia

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STUDY OF EXPLOSIVE SEISMIC SOURCE BY JOINT INVERSION OF SURFACE WAVE SPECTRA AND SIGNS OF THE FIRST MOTION

B.G. Bukchin, A.Z. Mostinsky

International Institute of Earthquake Prediction Theory and Mathematical Geophysics, Moscow, Russia,

A combination of isotropic tensor, modelling an explosion, and pure double couple, modelling the tectonic moment release, is considered as the moment tensor of seismic source. The explosion is located at free surface, and the depth of double couple is a varying parameter. Such a source can be characterised by six parameters - seismic moments of double couple and explosion, three parameters determining the double couple focal mechanism, and its depth.

We determine these source characteristics by minimising the misfit between synthetic and observed Love and Rayleigh surface waves spectra and polarities of P-waves first arrivals. The optimal values of the parameters that minimise the misfit we consider as estimates of these parameters. We search them by a systematic exploration of parameter space. To characterize the degree of resolution of every of these source characteristics, we calculate partial residual functions in the following way. For every value of a given parameter we find the minimum residual on the set of all possible values of other parameters. The result function of the residual by the selected parameter characterizes its resolution.

The results of application of this technique for comparative study of the earthquake (27.11.92) and two explosion (21.05.92 and 10.06.94) at Lop Nor (China) are presented.

Description of technique

Instant point source can be described by the moment tensor - a symmetric 3x3 matrix $\mathbf{M}$. Seismic moment M_0 is defined by equation

$$M_0 = \sqrt{\frac{1}{2} \text{tr}(\mathbf{M}^T \mathbf{M})}, \text{ where } \mathbf{M}^T \text{ is transposed moment tensor } \mathbf{M}, \text{ and } \text{tr}(\mathbf{M}^T \mathbf{M}) = \sum_{i,j=1}^3 M_{ij}^2.$$

Moment tensor of any event can be presented in the form $\mathbf{M} = M_0 \mathbf{m}$, where matrix $\mathbf{m}$ is normalised by condition $\text{tr}(\mathbf{m}^T \mathbf{m}) = 2$.

We consider the event under study as a sum of earthquake (0-trace moment tensor $\mathbf{M}_{qu}$) and explosion (moment tensor $\mathbf{M}_{ex}$). Moment tensor $\mathbf{M}$ of such an event is given by sum $\mathbf{M} = \mathbf{M}_{qu} + \mathbf{M}_{ex}$.

Let $\mathbf{I}$ be an identity 3x3 matrix. Then $\mathbf{M}_{ex} = \sqrt{\frac{2}{3}} M_{0ex} \mathbf{I}$, where M_{0ex} is the seismic moment of the explosion. For the earthquake $\mathbf{M}_{qu} = M_{0qu} \mathbf{m}$, where M_{0qu} is the seismic moment of the earthquake, and $\mathbf{m}$ is a normalised moment tensor, such that $\text{tr} \mathbf{m} = 0$ and $\text{tr}(\mathbf{m}^T \mathbf{m}) = 2$.

Let us consider a 6-D linear Euclidean space of symmetric 3x3 matrixes $\mathbf{M}$, and let the scalar product of two vectors $(\mathbf{M}, \mathbf{N})$ is defined by formula

$$(\mathbf{M}, \mathbf{N}) = \sum_{i,j} M_{ij} N_{ij} = \text{tr}(\mathbf{M}^T \mathbf{N}).$$

Isotropic tensors $\mathbf{M}_{ex}$ form a 1-D subspace which is orthogonal to 5-D linear subspace of zero trace tensors $\mathbf{M}_{qu}$. This follows from relations:

$$(\mathbf{M}_{qu}, \mathbf{M}_{ex}) = \sqrt{\frac{2}{3}} M_{0qu} M_{0ex} \text{tr}(\mathbf{m}^T \mathbf{I}) = \sqrt{\frac{2}{3}} M_{0qu} M_{0ex} \text{tr} \mathbf{m} = 0.$$

Then the scalar moment of combined event can be expressed by formula

$$\begin{aligned} M_0 = \sqrt{\frac{1}{2}(\mathbf{M}, \mathbf{M})} &= \sqrt{\frac{1}{2}((\mathbf{M}_{qu} + \mathbf{M}_{ex}), (\mathbf{M}_{qu} + \mathbf{M}_{ex}))} = \sqrt{\frac{1}{2}(\mathbf{M}_{qu}, \mathbf{M}_{qu}) + \frac{1}{2}(\mathbf{M}_{ex}, \mathbf{M}_{ex})} = \\ &= \sqrt{M_{0qu}^2 + M_{0ex}^2} \end{aligned}$$

So 6-D vector $\mathbf{M}$ is a sum of two orthogonal vectors $\mathbf{M}_{qu}$ and $\mathbf{M}_{ex}$, and seismic moments of the explosion component M_{0ex} and of the earthquake component M_{0qu} can be expressed by total scalar moment M_0 and the angle φ between 6-D vectors $\mathbf{M}_{qu}$ and $\mathbf{M}$:

$$M_{0qu} = M_0 \cos \varphi \quad \text{and} \quad M_{0ex} = M_0 \sin \varphi.$$

$\varphi=0$ corresponds to a pure earthquake, $\varphi=90^\circ$ corresponds to a pure explosion.

Let us consider a seismic source as a combination of isotropic tensor, modelling an explosion located at a zero depth, and pure double couple point source at a depth h , modelling the tectonic moment release. Both explosion and earthquake are considered as an instant sources. Such a source can be given by 6 parameters: described above angle φ , double couple depth, its focal mechanism which is characterising by three angles: strike, dip and slip or by two unit vectors (direction of principal tension $\mathbf{T}$ and direction of principal compression $\mathbf{P}$) and seismic moment M_0 . Five of these parameters we determine by a systematic exploration of the five dimensional parametric space, and the 6-th parameter M_0 - solving the problem of minimisation of the misfit between observed and calculated surface wave amplitude spectra for every current combination of all other parameters.

Under assumptions mentioned above the relation between the spectrum of the displacements $u_i(\mathbf{x}, \omega)$ in any surface wave and the total moment tensor $\mathbf{M}$ can be expressed by following formula

$$u_i(\mathbf{x}, \omega) = \frac{1}{i\omega} [M_{qujl} \frac{\partial}{\partial y_{quj}} G_{ij}(\mathbf{x}, \mathbf{y}_{qu}, \omega) + M_{exjl} \frac{\partial}{\partial y_{exj}} G_{ij}(\mathbf{x}, \mathbf{y}_{ex}, \omega)] \quad (1)$$

$i, j = 1, 2, 3$ and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x}, \mathbf{y}, \omega)$ in equation (1) is the spectrum of Green function for the chosen model of medium and wave type (see Levshin, 1985; Bukchin, 1990), $\mathbf{y}$ - source location (we assume that the explosion and earthquake have this same horizontal coordinates, but different depth: h for earthquake and 0 for explosion). We assume that the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model (Woodhouse, 1974; Babich *et al.*, 1976). The surface

wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. The amplitude spectrum $|u_i(\mathbf{x}, \omega)|$ defined by formula (1) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum (Bukchin, 1990). The average phase velocities of surface waves are usually not well known. For this reason, we use only amplitude spectrum of surface waves for determining source parameters under consideration.

If all characteristics of the medium are known the representation (1) gives us a system of equations for parameters defined above. Let us consider now a grid in the space of these 5 parameters. Let the models of the media be given. Using formula (1) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω . Let $u^{(i)}(\mathbf{x}, \omega)$ be any observed value of the spectrum, $i = 1, \dots, N$; $\varepsilon^{(i)}$ - corresponding residual of $|u^{(i)}(\mathbf{x}, \omega)|$. We define the normalised amplitude residual by formula

$$\varepsilon(h, \varphi, \mathbf{T}, \mathbf{P}) = \left[\left(\sum_{i=1}^N \varepsilon^{(i)^2} \right) / \left(\sum_{i=1}^N |u^{(i)}(\mathbf{x}, \omega)|^2 \right) \right]^{1/2} \quad (2)$$

The optimal values of the parameters that minimize ε we consider as estimates of these parameters. We search them by a systematic exploration of the five dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions. Fixing the value of one of varying parameters we put in correspondence to it a minimal value of the residual ε on the set of all possible values of the other parameters. In this way we define two residual functions on scalar argument and two residual functions on vector argument corresponding to the two scalar and two vector varying parameters: $\varepsilon_h(h)$, $\varepsilon_\varphi(\varphi)$, $\varepsilon_{\mathbf{T}}(\mathbf{T})$ and $\varepsilon_{\mathbf{P}}(\mathbf{P})$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters. From geometrical point of view these functions describe the lower boundaries of projections of the 5-D surface of functional ε on the coordinate planes.

To improve the resolution of focal mechanism we use in the inversion polarities of P wave first arrivals. Calculating radiation pattern of P waves for every current combination of parameters we compare it with observed polarities. The misfit obtained from this comparison we use to calculate a joint residual of surface wave amplitude spectra and polarities of P wave first arrivals. Let ε_{sw} be the residual of surface wave amplitude spectra, ε_p - the residual of P wave first arrival polarities (the number of wrong polarities divided by the full number of observed polarities), then we determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_p)(1 - \varepsilon_{sw}) \quad (3)$$

Before inversion we apply to observed polarities a smoothing procedure which we will describe here briefly.

Let us consider a group of observed polarities (+1 for compression and -1 for dilatation) radiated in directions deviating from any medium one by a small angle. This group is presented in the inversion procedure by one polarity prescribing to this medium direction. If the number of one of two types of polarities from this group is significantly larger than the number of opposite polarities, then we prescribe this polarity to this medium direction. If no one of two polarity types can be considered as preferable, then all these polarities will not be used in the inversion. To make a decision for any group of n observed polarities we calculate the sum $m = n_+ - n_-$, where n_+ is the number of compressions and $n_- = n - n_+$ is the number of dilatations. We consider one of polarity types as preferable if $|m|$ is larger than its standard deviation in the case when +1 and -1 appear randomly with this same probability 0.5. In this case n_+ is a random value distributed following the binomial law. For its average we have $M(n_+) = 0.5n$, and for dispersion $D(n_+) = 0.25n$. Random value m is a linear function of n_+ such that $m = 2n_+ - n$. So following equations are valid for the average, for the dispersion, and for the standard deviation σ of value m

$$M(m) = 2M(n_+) - n = n - n = 0, \quad D(m) = 4D(n_+) = n, \quad \text{and} \quad \sigma(m) = \sqrt{n}.$$

As a result, if the inequality $|m| \geq \sqrt{n}$ is valid then we prescribe +1 to the medium direction if $m > 0$, and -1 if $m < 0$.

Description of applications

Using the technique described above we estimated the characteristics of the earthquake, 27.11.92 and two explosion (21.05.92 and 10.06.94) at Lop Nor, China. The estimation of source parameters was done by using spectra of Love and Rayleigh fundamental modes in the spectral domain for periods from 20 s to 40 s for the earthquake, from 25 to 50 s for the first explosion, and from 20 s to 30 s for the second explosion. We used records of IRIS and GEOSCOPE networks. Love and Rayleigh fundamental modes were extracted by using a frequency-time analysis program (FTAN) and floating filtering, described by Lander, 1989, Levshin *et al.*, 1994. Amplitude spectrum of the signal extracted by floating filtering is much more smooth than spectrum of the record just filtered in the same band. It makes the inversion much more stable.

Analyzing the surface wave spectra and polarities of P wave first arrivals we determined the following best estimates for focal mechanisms of the double couples for these events: strike 285°, dip 30°, and slip 60° for the earthquake, strike 330°, dip 75°, and slip 90° for the tectonic release accompanying the 21.05.92 explosion, and strike 349°, dip 41°, and slip 131° for the double couple component of the 10.06.94 explosion. The stereographic projections of nodal planes on the lower hemisphere for these events are shown in Fig. 1. The estimates of seismic moment are $0.78 \cdot 10^{16}$ n·m for the earthquake, $1.4 \cdot 10^{17}$ n·m for the 21.05.92 explosion, and $2.8 \cdot 10^{15}$ n·m for the 10.06.94 explosion. For the double couple depth we obtained following values: 24 km for the earthquake, 6 km for the 21.05.92 explosion, and 4 km for the 10.06.94 explosion. The residual functions for the angle between

(a)

Seismic moment: $1.4 \cdot 10^{17}$ n.m

First nodal plane:

Strike = 150°

Dip = 15°

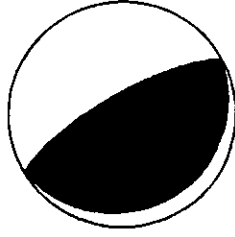
Slip = 90°

Second nodal plane:

Strike = 330°

Dip = 75°

Slip = 90°



(b)

Seismic moment: $2.8 \cdot 10^{15}$ n.m

First nodal plane:

Strike = 349°

Dip = 41°

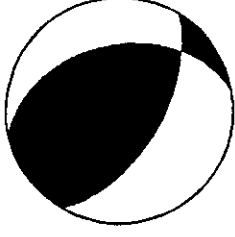
Slip = 131°

Second nodal plane:

Strike = 120°

Dip = 60°

Slip = 60°



(c)

Seismic moment: $0.78 \cdot 10^{16}$ n.m

First nodal plane:

Strike = 285°

Dip = 30°

Slip = 60°

Second nodal plane:

Strike = 139°

Dip = 64°

Slip = 106°

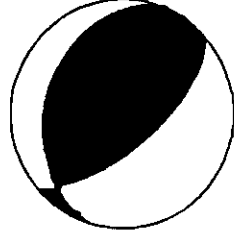


Fig. 1. Double couple solutions and stereographic projections of nodal planes on the lower hemisphere
(a) explosion, 21.05.92, (b) explosion, 10.06.94
(c) earthquake, 27.11.92.

moment tensor and zero trace subspace are given in Fig. 2 for all three events. The optimal value of the angle in the case of the earthquake is equal to 0° as it should be. For the 21.05.92 explosion it is about 15° , which corresponds to the seismic moments $M_{0qu} = 0.96M_0$ and $M_{0ex} = 0.26M_0$. For the 10.06.94 explosion the optimal angle is 20° , which corresponds to the seismic moments $M_{0qu} = 0.93M_0$ and $M_{0ex} = 0.34M_0$.

To compare the residual functions for the angle between moment tensor and zero trace subspace for the earthquake and explosions, we present them in the Fig. 3 in the angle band between 0° and 45° . The residual is varying in this band quite significant. The difference of these curves is not so big, but obvious.

The results show that at least for studied explosions the tectonic release is much more intensive then the explosion caused it. In such a case we can not expect a good resolution of the angle ϕ .

The depth of tectonic release was obtained much bigger then usually assumed. In some of techniques the assumption of zero depth of the accompanying earthquake is an important one.

Under assumption that the explosion can trigger an earthquake at a prepared fault, the non zero depth of the double couple component looks reasonable. If such a prepared faults exists their orientation could be displayed in the focal mechanisms of the earthquakes in the test site. Gao and Richards, 1994 studied the earthquakes at Lop Nor. Focal mechanisms and locations for four earthquakes taken from their report are presented at the Fig. 4. At the same figure we present focal mechanisms for the earthquake and tectonic release in explosions described above. The similarity of focal mechanisms of the earthquakes in the test side and of the tectonic releases in the explosions can be considered as a confirmation of our assumption that the explosion triggers the earthquake at a prepared fault.

We presented here the first application of developed technique. It has to be tested on explosions at another test sites which were successfully studied by another techniques making restricted assumptions such as zero depth of tectonic release (see for example Ekstrom and Richards, 1994).

We started to modify this technique by a generalisation of the source model. It will search for the earthquake component solution among all zero trace tensors instead of double couples. It means that the fault is allowed to be not plane, what can happen as a result of explosion.

Two versions of software for the technique described above are developed: UNIX version and Windows 95 application.

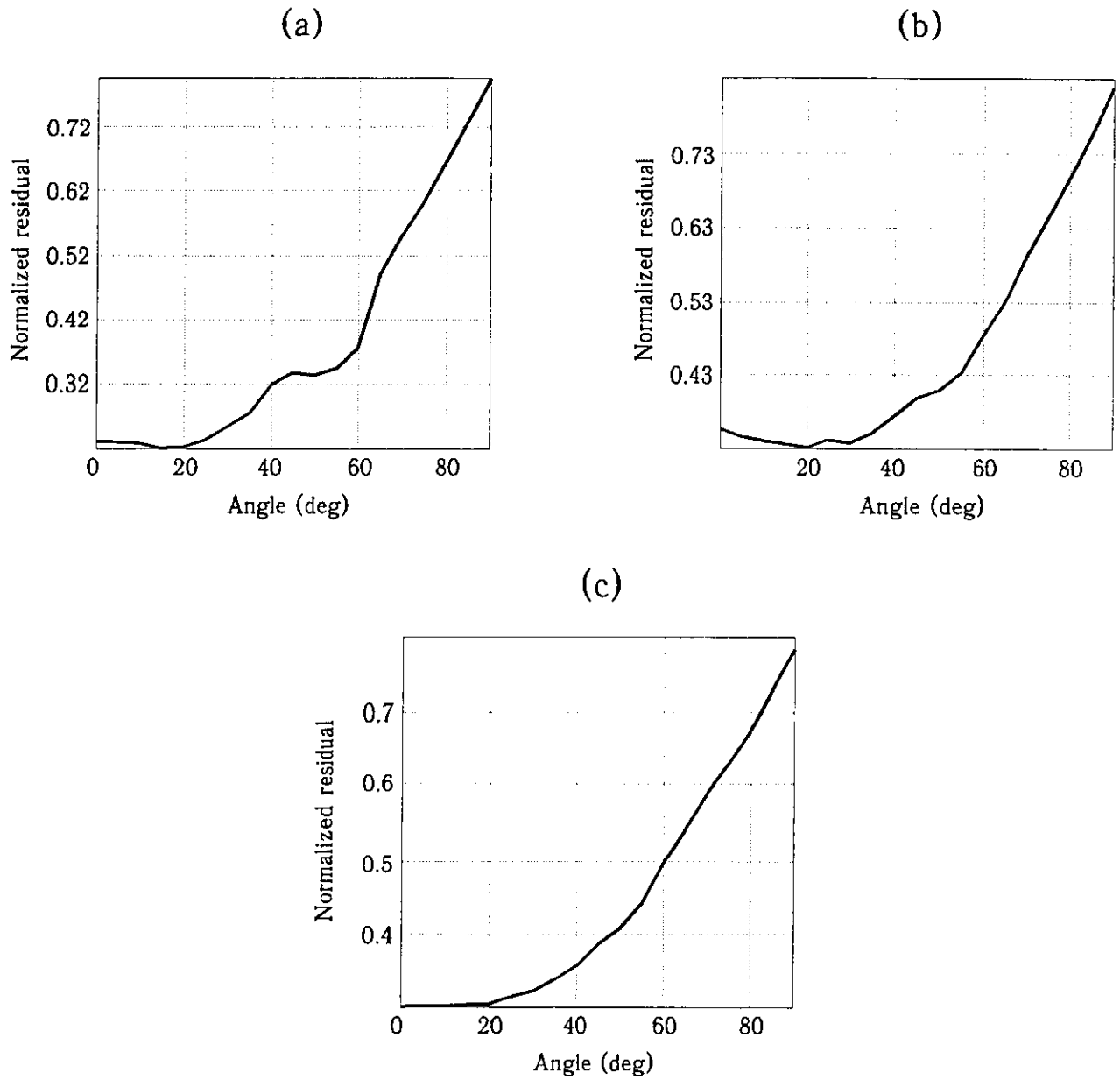


Fig.2. Residual as functions of the angle between moment tensor and subspace of zero trace tensors
 (a) explosion, 21.05.92, (b) explosion, 10.06.94
 (c) earthquake, 27.11.92.

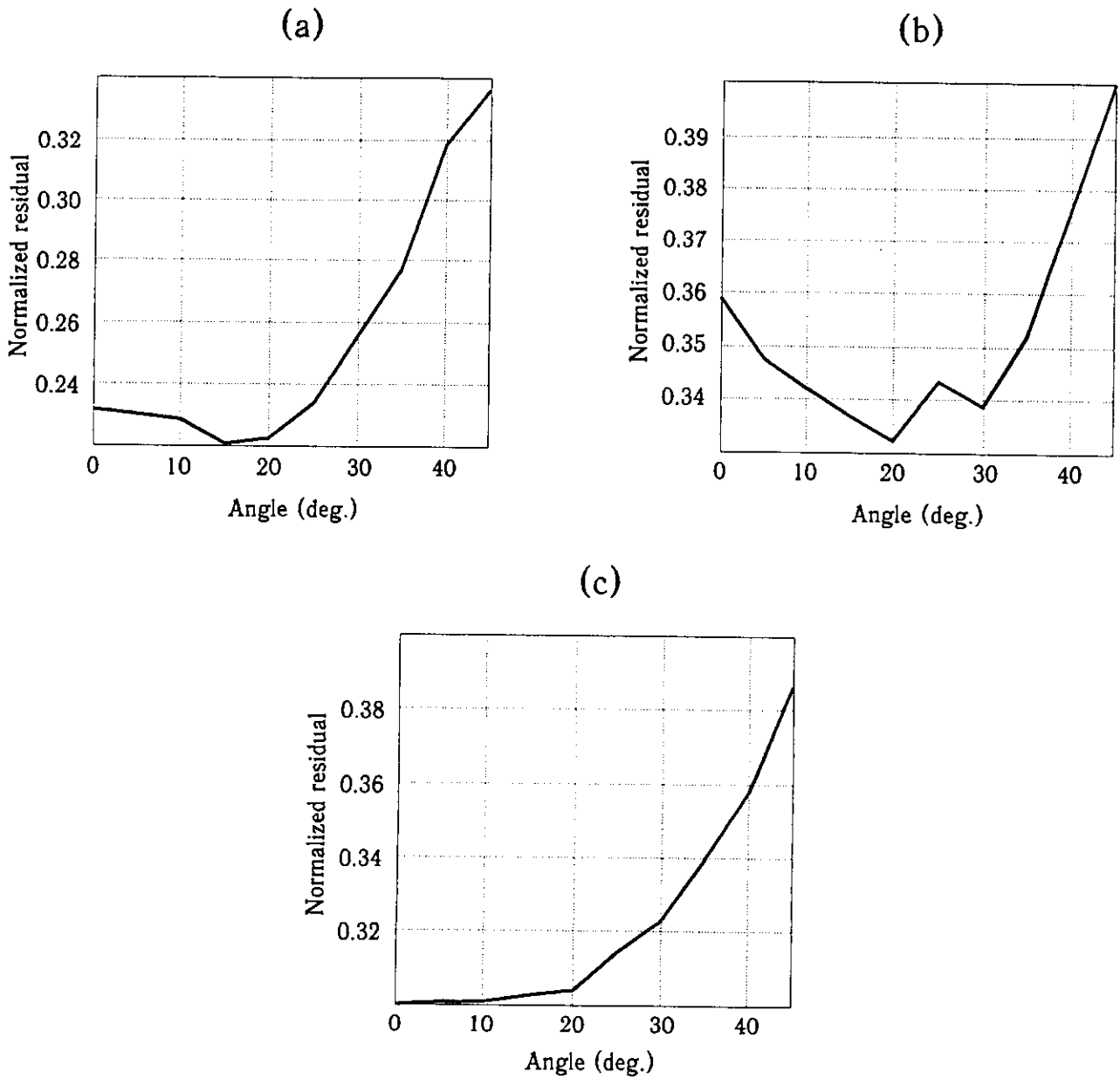


Fig.3. Residual as functions of the angle between moment tensor and subspace of zero trace tensors (fragment)

(a) explosion, 21.05.92, (b) explosion, 10.06.94
(c) earthquake, 27.11.92.

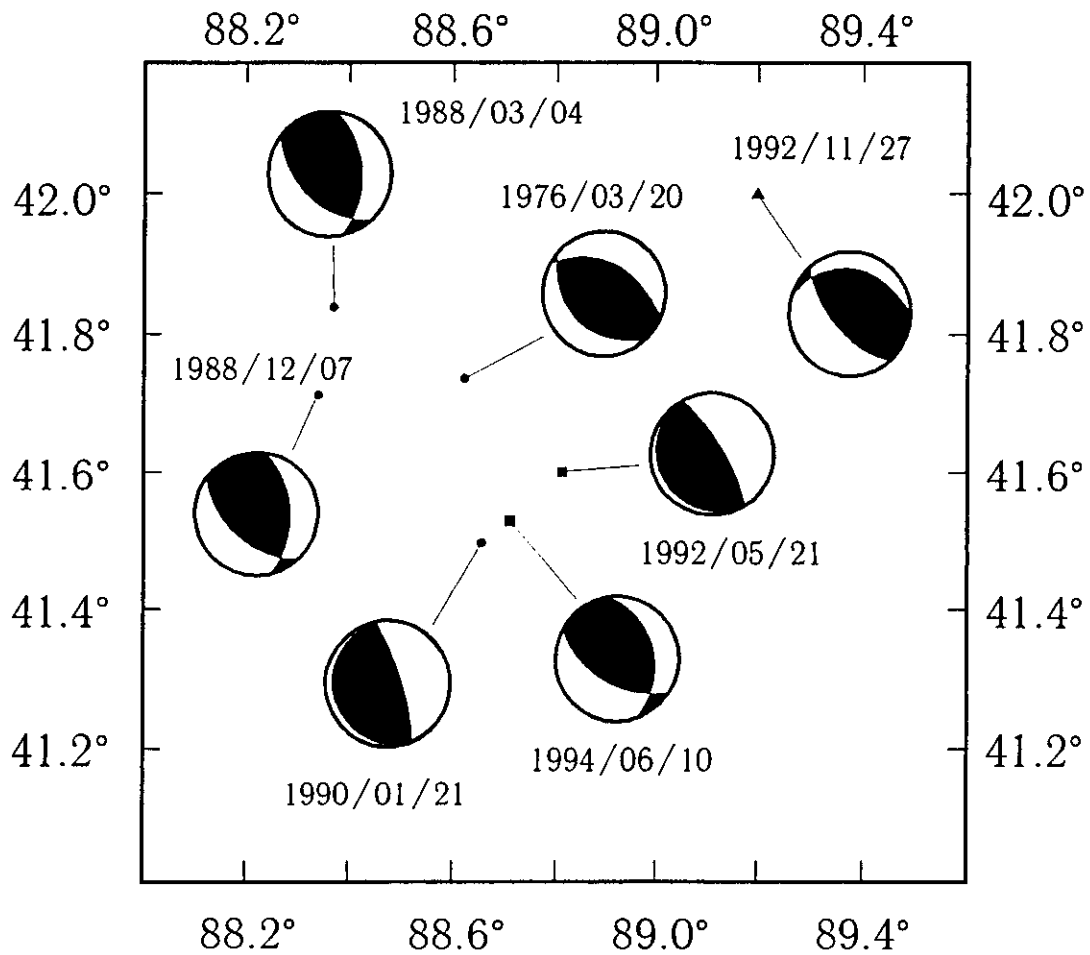


Fig.4. Focal mechanisms of earthquakes in Lop Nor area.
 Squares - inversion results for the explosions;
 triangle - earthquake on 1992/11/27;
 circles - earthquakes reported by Gao and Richards.

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