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**FOURTH WORKSHOP ON THREE DIMENSIONAL MODELLING
OF SEISMIC WAVES GENERATION.
PROPAGATION AND THEIR INVERSION**

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Surface Wave Analysis and Tomography

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These are preliminary lecture notes, intended only for distribution to participants.

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1. Surface Wave Analysis

The surface wave analysis scheme developed by the Seismology Group at University of Colorado, Boulder is designed to measure surface wave characteristics discussed at my previous lecture: phase and group velocities, polarization, amplitude spectrum, and to extract 'cleaned' wanted signals.

The measurements themselves are preceded by several preprocessing procedures which are worthwhile to mention.

1.1 Preprocessing

Data obtained from different data centers around the world are usually presented as SEED volumes (Standard for the exchange, 1993). Preprocessing includes:

- 1) reformatting, by converting data from SEED volumes to the CSS v 3.0 relational database (Anderson *et al.*, 1993; Quinlan, 1994). The important advantage of this database in comparison with the often used SAC is the separation of seismic waveforms from the information related to the recording stations and channels and seismic events. This allow to us to correct or update this information without dealing with records themselves. Records of all stations for a given event are combined into one event volume;
- 2) editing, i.e. removal of isolated glitches and zero-line shifts;
- 3) rejection of all defective records containing unrepairable glitches, gaps or clipping;
- 4) decimation of broadband or intermediate-band channel traces to the sampling rate 1 Hz using decimating filters;
- 4) transformation of horizontal components into radial and transverse components according to the great circle backazimuth to the epicenter.

1.2 Frequency-Time Analysis

Our aim is to extract the signals we desire, related to nearly directly arriving waves that can be interpreted deterministically, from the potentially interfering multipaths and coda that are essentially stochastic in nature. Usually only direct wave trains of fundamental Rayleigh and Love modes are considered as desired signals. Unwanted signals, in particular surface wave coda, overtones, and body waves are greatly reduced in the filtered seismogram on which measurements are obtained.

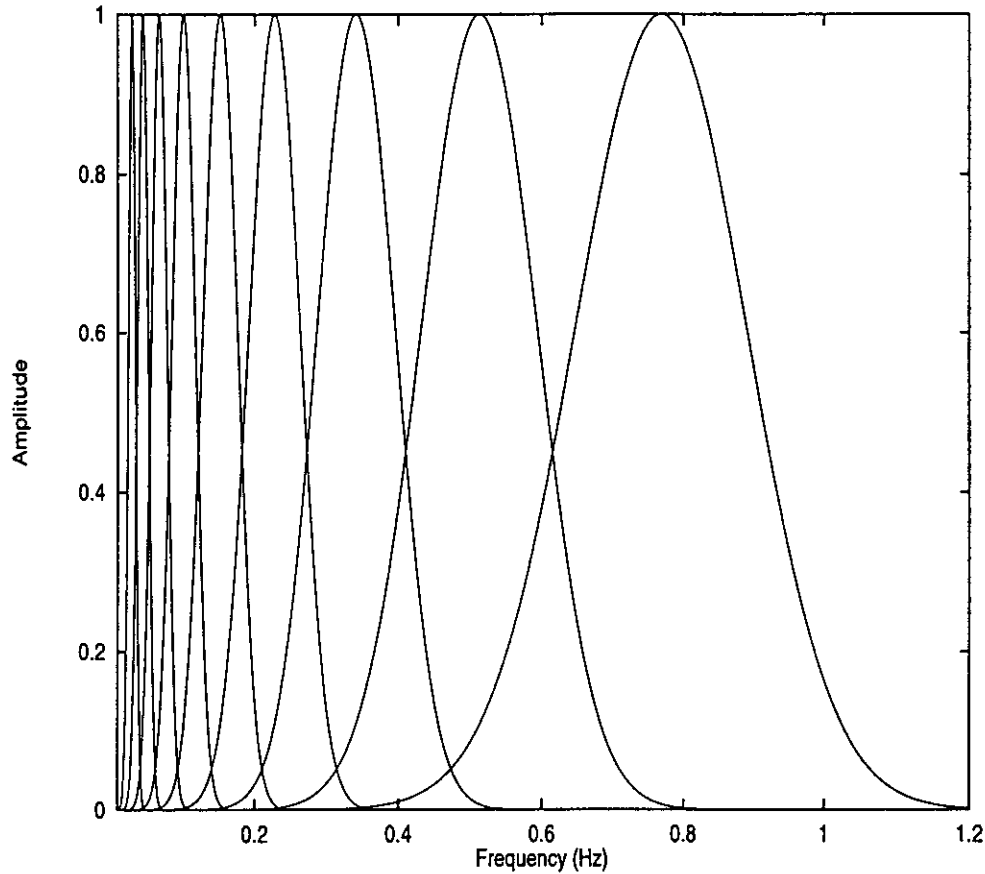


Figure 1. A comb of Gaussian frequency domain filters with constant relative bandwidth used in FTAN.

The basic characteristics of the current measurement procedure is based on a long history of development of surface wave analysis (e.g., Dziewonski *et al.* 1969, 1972; Levshin *et al.*, 1972, 1992, 1994; Keilis-Borok (Ed.), 1989; Cara, 1973; Russell *et al.*, 1988). The recent innovation is that code has been developed which allows measurements to be made rapidly on relatively large volumes of data from heterogeneous networks and a variety of source regions. The innovations have required the development of rational parametric and waveform database structures and the development of relatively rapid graphical routines for human interaction with the data (Ritzwoller *et al.*, 1995; Ritzwoller & Levshin, 1998). The general form of the measurement procedure is as follows. Group velocity - period diagrams for the vertical, radial, and transverse components are constructed using a computer-simulated system of narrow-band Gaussian filters (Figure 1).

An analyst manually traces the apparent group velocity curve for the Rayleigh wave (on the vertical and radial components) and the Love wave (on the transverse component). Time-variable filters are applied around the selected curve to separate the desired signal from the

‘noise’. This results in filtered group velocity - period diagrams for which contamination from interfering signals should be reduced. Group velocity, phase velocity, amplitude, and (optionally) polarization measurements are automatically obtained on the filtered images. Measurements are graded by the analyst according to the quality of a desired signal using the 5-grade scale. All waveform and parametric data as well as surface wave measurements are stored in the CSS v. 3.0 relational database which was significantly extended to accommodate this information.

An unfortunate, but currently still necessary, characteristic of this procedure, is that it depends crucially on direct human interaction with potentially large volumes of seismic waveform data. The success of this method is based on the analyst accurately identifying the main dispersion ridge of the fundamental modes, separating the ‘direct arrival’ from surface wave coda at periods below about 30 seconds, inspecting interpolation near spectral holes, and truncating the measurements appropriately at long periods as the signals weaken. This interaction limits the speed with which the method can be applied, and, therefore, the volume of data that can be processed. Figures 2 and 3 display this procedure.

1.3 Measurements on a Continental Scale

Due to the high average efficiency of surface wave propagation across continents, surface wave measurements can be made at periods up to 100 - 150 seconds for earthquakes as small as $M_s = 5.0$ that propagate across the entire continent. Of course, measurements can be extended to longer periods for substantially larger events.

As an example, group velocity measurements for a single station (KEVO, Finland) for one event (Kuril event, 10/9/94, $M_s = 7.0$) are shown in Figure 2b for the Rayleigh wave (measured on the vertical and radial components) at periods between about 20 and 300 seconds and for the Love wave at periods between about 30 and 250 seconds. Predictions for the spherical model PREM (Dziewonski and Anderson, 1980) are shown for comparison.

A useful by-product of these analyses are ‘cleaned’ or ‘filtered seismograms’. Figure 2c shows a comparison between the raw and filtered seismograms for a single station:event pair. Surface wave coda, overtones and body waves have been greatly diminished from the cleaned seismograms, making them an ideal target for surface wave fitting techniques during a later stage of this research.

1.4 Measurements on a Regional Scale

On a regional scale at shorter periods, smaller events ($M_s < 5.0$) can be analyzed similarly. KNET (Kyrgyzian Digital Network), situated in a complex tectonic setting in Central Asia surrounded to the East, West and South by significant seismicity, is a natural site to focus studies of regional scale measurements. Due to complexity of the region records of KNET stations are usually quite complicated and exhibit a great variance of wave patterns from the same event across array and from one event to other.

Figure 3b presents an example the analysis of KNET data. Seven KNET stations were

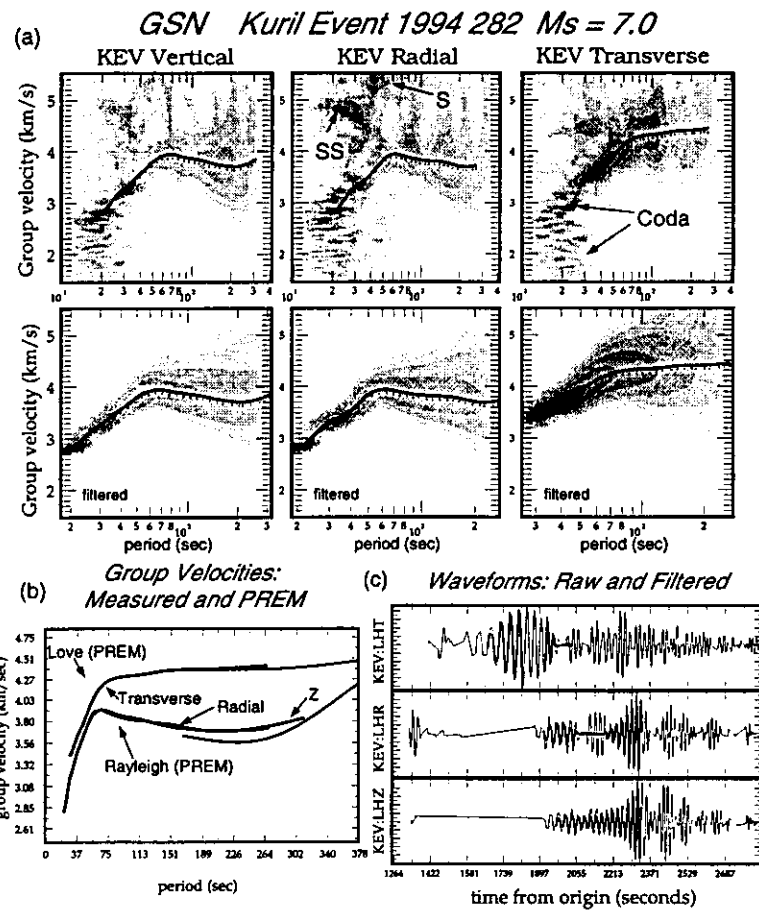
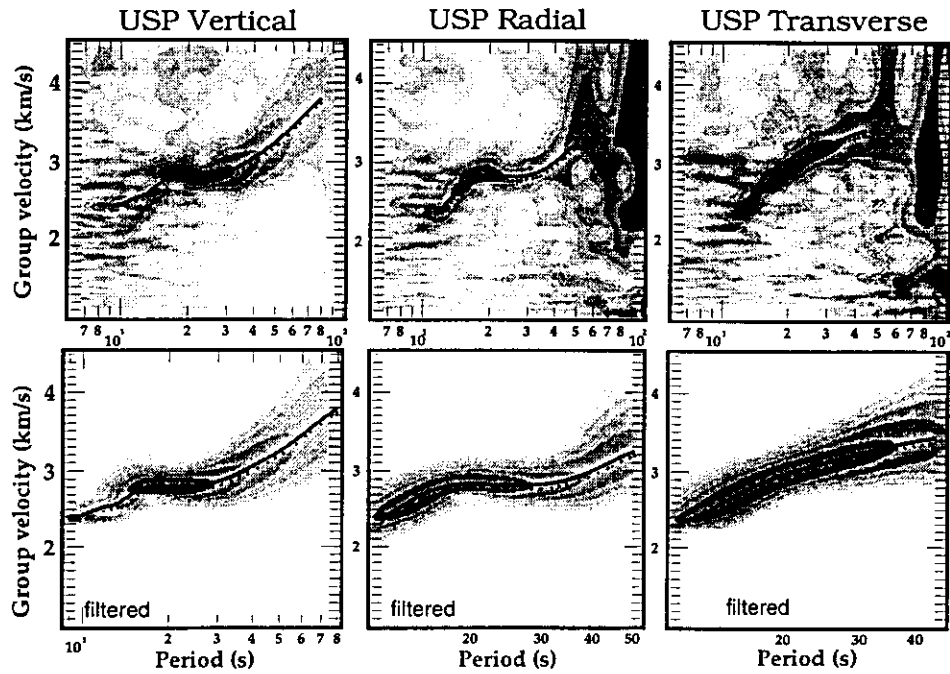
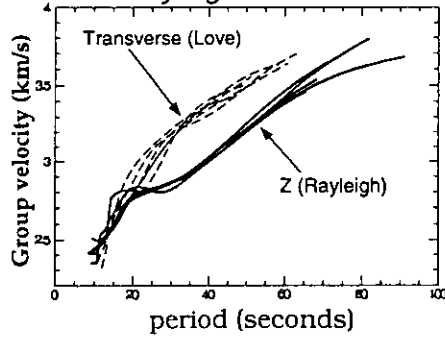


Figure 2. FTAN. Example of a frequency-time analysis for the vertical, radial, and transverse components recorded at the GSN station at Kevo, Finland for an event in the Kurile Islands (October 9, 1994, $M_s = 7.0$, $\Delta = 58.5^\circ$)

(a) *KNET Qinghai Event 1994 27 Ms = 4.8*



(b) *Group Velocities
Rayleigh and Love*



(c) *Waveforms: Raw and Filtered*

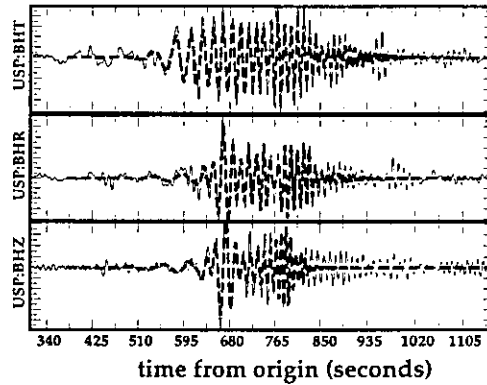


Figure 3. (a) Group velocity - period diagrams for the vertical, radial, and transverse components recorded at the KNET station USP for an event in the Qinghai Province (1/27/94, $M_s = 4.8$, $\Delta = 16$ degrees). Estimated group velocity curves are shown. (b) Rayleigh and Love wave group velocities measured across the array for the same event. (c) Raw and filtered waveforms for the station USP.

operating during the passage of surface waves from an event in the Qinghai Province, China on 1/17/94 ($\Delta \approx 16$ degrees, $M_s = 4.8$). Rayleigh and Love wave group velocity measurements are shown in Figure 3b. Rayleigh wave measurements are quite similar across the array above about 20 seconds period and for Love waves above about 30 seconds period at this azimuth. Variations across the array at shorter periods result both from real differences along the various wave paths near the network and also from Rayleigh - Love interference, which can be significant since the group velocities of the two wave types are similar in this period range. Cleaned and raw waveforms are presented in Figure 3c.

1.5 Data ‘Clustering’ and Rejection of Outliers

The resulting data set of group velocity measurements exhibits considerable redundancy, which allows for consistency tests, outlier rejection, and the estimation of measurement uncertainties. These tests are performed as part of what we call a ‘cluster analysis’ (Figure 4). Measurements whose path-endpoints

lie within 2% of the path length are grouped to produce a ‘cluster’ of dispersion curves. This cluster defines a ‘unique path’. Frequently, these clusters are composed of a large earthquake and its aftershocks recorded at a single station, but in some cases nearby stations allow clustering from a single earthquake. All dispersion curves that are not part of some cluster individually define a unique path.

We produce a single measurement for each cluster at each period and wave type. Clear outliers are identified as part of the cluster analysis because they disagree with other measurements from similar paths.

2. Tomographic Inversion

Several independently developed methods and computer codes for tomographic inversion of surface wave data on phase and group velocity are in use these days.

2.1 Yanovskaya-Ditmar method

I will discuss the applications of an approach developed by Ditmar and Yanovskaya (1987) and Yanovskaya and Ditmar (1990) (see also Keilis-Borok (Ed.), 1989; Ch. 6). We apply this technique to construct the group velocity maps using the group velocity curves that emerge from the cluster analysis applied to measurements made with the frequency-time analysis (FTAN). The method of Yanovskaya and Ditmar is a generalization to two dimensions of the classical one dimensional method of Backus and Gilbert (e.g., Backus and Gilbert, 1968, 1970). There are several features that commend this method. First, it does not require any *a priori* parameterization or truncation of any expansion since the basis functions for the model are superpositions of the kernels of the group travel time integrals. Perhaps more importantly, the method has

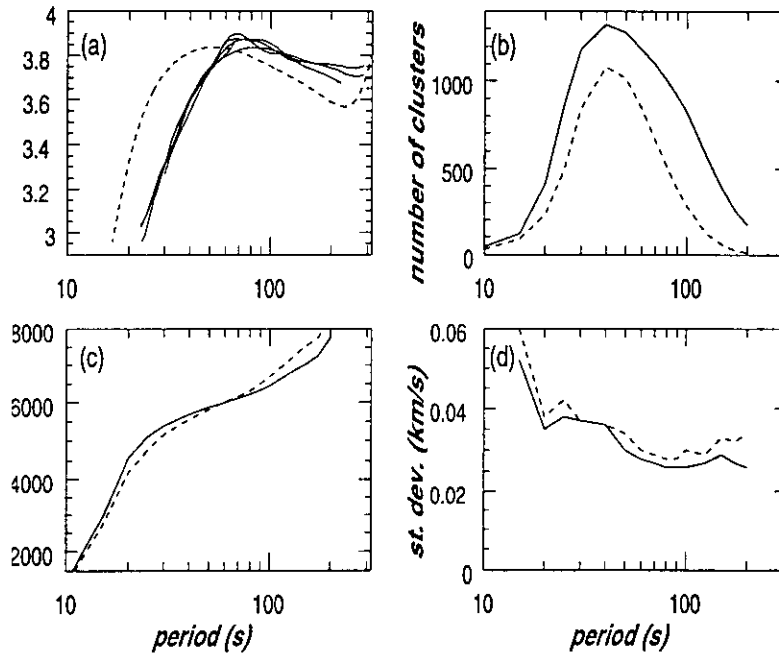


Figure 4. Cluster analysis.

- (a) Example of a cluster of measured group velocity curves.
Estimated Rayleigh wave group velocity curves from a set of five events in the Philippines measured at Eskdalemuir (ESK), Scotland are compared with one another (solid lines) and the group velocity curve predicted from PREM (dashed line). (1989 349, $M_s = 7.4$; 1990 39, $M_s = 6.6$; 1991 49, $M_s = 6.6$; 1991 317, $M_s = 6.4$; 1992 138, $M_s = 7.1$)
- (b) The total number of clusters in the data set plotted as a function of period and wave type (Rayleigh: solid line, Love: dashed line.)
- (c) The average path length versus period (Rayleigh: solid, Love: dashed).
- (d) The average of the standard deviation of the group velocity curves composing all of the clusters. These values are assigned as uncertainties to all measured group velocity curves which are not part of some cluster.

been well tested and provides a well understood foundation for our work. The major disadvantages of the method, as we use it, are that it does not include explicit penalties on model size or the second spatial derivative of the model and the ray basis functions lead to artifacts caused by bad measurements. Thus, outliers must be identified and rejected a priori and the maps must be smoothed in a rational manner a posteriori.

For each frequency and wave type, group velocity is a local function of position, $U(\theta, \phi)$, which we decompose into some reference value (frequently, the average across the studied region), U_0 , and a location dependent perturbation: $U(\theta, \phi) = U_0 + \delta U(\theta, \phi)$. The most significant assumptions of this method are the following. (1) The method seeks a smooth perturbation in group velocity, $\delta U(\theta, \phi)$, relative to a homogeneous model, U_0 , such that $U(\theta, \phi)$ fits the N observed group velocity travel times, t_i^{obs} ($i = 1, \dots, N$), in a weighted least-squares sense. To do this the method attempts to minimize the following penalty function at each period and wave type:

$$\sum_{i=1}^N [w_i(t_i^{obs} - t_i^{pred})]^2 + \lambda \int_S |\nabla U(\theta, \phi)|^2 dA, \quad (1)$$

where

$$t_i^{pred} = \int_{p_i} U^{-1}(\theta, \phi) ds. \quad (2)$$

Here, p_i represents the i -th wave path, w_i is the weight associated with the i th path through the group velocity map $U(\theta, \phi)$, t_i^{pred} is the predicted group travel time along the i -th path, and S is the region under study. Choosing different values of the trade-off parameter, λ , changes the trade-off between the fit to the data and the ‘smoothness’ of the resulting group velocity map. ‘Smooth’ here is defined in terms of the first spatial gradient of the model. The inversion takes place independently for each period and wave type. (2) Relative group velocity variations are assumed to be small in amplitude for each period and wave type: $\delta U(\theta, \phi)/U_0 \ll 1$. This is the justification for a linearized inversion procedure. (3) In equation (2), we currently assume that each wave path is along the great-circle linking the source and receiver and no group time perturbation is introduced by a source phase shift.

Data weights result from three sub-weights. The weight for measurement i is:

$$w_i = \sqrt{m} \frac{g_i}{\sigma_i}, \quad (3)$$

where m is the number of raw measurements that compose the cluster that produced this measurement, σ_i is the uncertainty determined from the cluster analysis for measurement i , and g_i is a weight which depends on the qualitative grade (A - F) assigned to the measurement by the analyst. As discussed in Section 1, if a measurement has resulted from a cluster, the uncertainty, σ_i , is the standard deviation of the dispersion curves composing the cluster. If a measurement has not resulted from a cluster, the average of the standard deviations from all clusters is taken as the uncertainty. The weight g_i is defined as follows. The normative grade is

A and receives a weight of 1.0. Grades of B, C, D, E, and F are given weights of 0.75, 0.4, 0.2, 0.0, and 0.0, respectively. Thus, no measurement receiving a grade less than D is used. The grades of the measurements composing a cluster are averaged to produce a cluster grade which is then subjected to the same weighting criteria. For ungraded measurements, $g_i = 0.5$.

The damping parameter λ in equation (1) is chosen subjectively by analyzing misfit, the visual smoothness, and the physical reasonableness of the resulting group velocity maps. Figure 5 illustrates

the trade-off curve between misfit and λ for the 40 s Rayleigh wave. Misfit is presented, first, as variance reduction relative to the average value across a given map:

$$\text{Variance Reduction} = 1 - \frac{\sum_i (U_i^{obs} - U_i^{pred}(\lambda))^2}{\sum_i (U_i^{obs} - U_0)^2}, \quad (4)$$

where i is the unique path index, $U_i^{pred}(\lambda)$ is the predicted group velocity for path i through the group velocity map constructed with the damping parameter set to λ , U_i^{obs} is the measured group velocity for path i , and U_0 is the reference group velocity. Misfit is also presented as the rms velocity residual:

$$\text{RMS Velocity Misfit} = \left(\frac{1}{N} \sum_{i=1}^N (U_i^{obs} - U_i^{pred}(\lambda))^2 \right)^{1/2}. \quad (5)$$

Three points on the misfit curves are marked with an x: one overdamped, one slightly underdamped, and one severely underdamped. Determination of whether a map is overdamped or underdamped is almost purely subjective. A map is termed overdamped if physically reasonable features are missing from it that appear in maps with less damping. Clearly underdamped maps display aphysical speckling, streaking, and other artifacts. RMS-misfit and variance reduction are, unfortunately, not good indicators of the ‘quality’ of a map, in particular they are not very useful in determining the appropriate damping. In particular, these overall measures of misfit are changed minimally by small-scale features. They are, therefore, poor indicators of the reliability of the estimated small scale anomalies. Thus, although the slightly underdamped map ($\lambda \sim 1$, variance reduction $\sim 90\%$) produces a much better fit to many individual dispersion curves for paths through small scale features than the highly overdamped map ($\lambda \sim 1$), the total variance reduction is only marginally improved from 85%.

As an example Rayleigh wave group velocity maps of Eurasia for each of the dampings in Figure 5 are shown in Figure 6. We typically choose λ to pro

duce a slightly underdamped map. Because the penalty function does not include a spatial second derivative term, we smooth each map a posteriori by applying a Gaussian spatial-smoothing filter with a carefully chosen width. The width chosen depends both on wave type and period and derives from the resolution analyses. This smoothing is designed to be an anti-aliasing filter and we apply it such that the full width at e^{-1} of the maximum height of the filter is half the estimated average resolution for the map. For example, for a 40 s Rayleigh

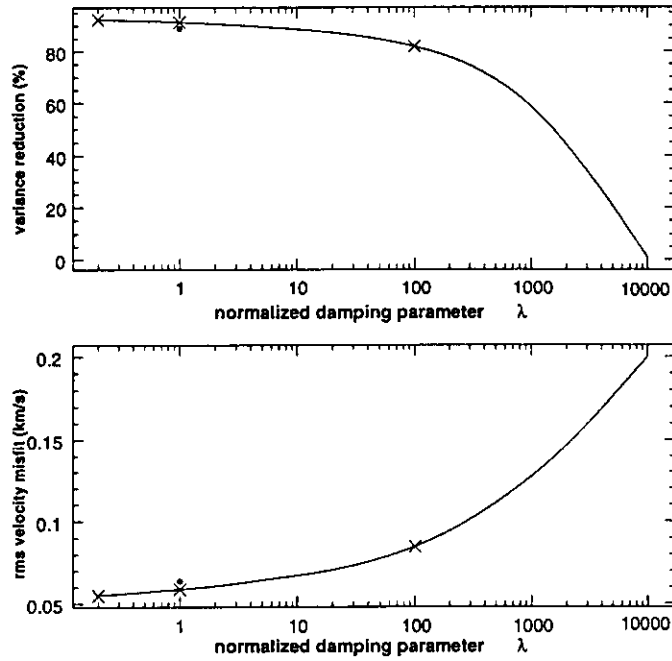


Figure 5. Two measures of misfit as a function of the damping parameter λ in equation (1) for the 40 s Rayleigh wave. The damping parameter has been normalized such that its value is unity for the 'slightly underdamped' case. Three values of the normalized damping parameter are indicated as crosses on the trade-off curves: one severely underdamped ($\lambda \sim 0.2$), one slightly underdamped ($\lambda \sim 1$), and one highly overdamped ($\lambda \sim 100$). The group velocity maps constructed with these three values of λ are shown in Figure 6. The application of the a posteriori smoothing filter (full width at $1/e$ point = 2.5°) to the group velocity map from the slightly underdamped inversion degrades fit to the data by a small amount, as is indicated by the closed circle. (Top) Variance reduction relative to the average group velocity across the slightly underdamped map. (Bottom) Rms velocity difference between the observed group velocities and those predicted from the group velocity maps.

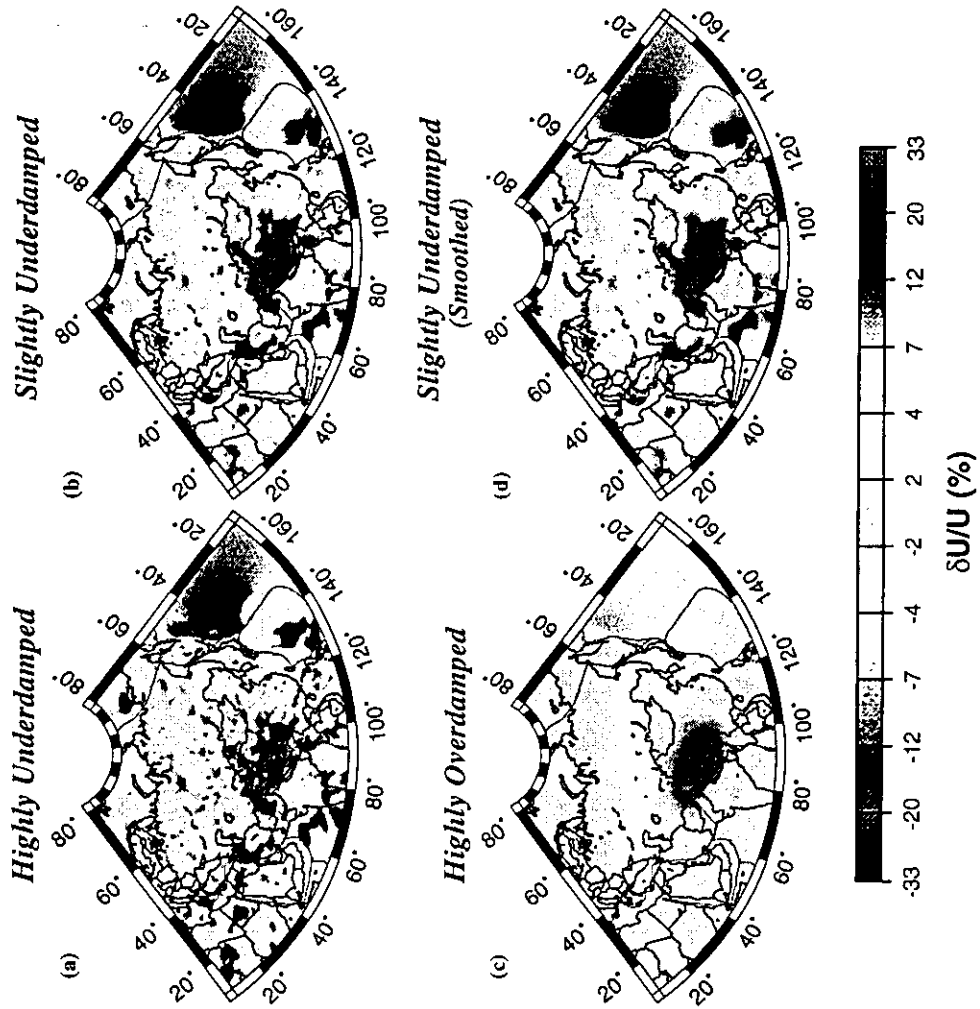


Figure 6. Group velocity maps constructed with the three damping parameters which are indicated with crosses in Figure 5: (top left) highly underdamped ($\lambda=0.2$), (top right) slightly underdamped ($\lambda=1$) and (bottom left) highly overdamped ($\lambda=100$). The application of the *a posteriori* smoothing filter (full-width at the $1/e$ point of 2.5°) to the map at top right produces the smoothed map at bottom right

wave map of Eurasia the Gaussian filter's full width is 2.5 degrees at the e^{-1} point. The effect of this filter is also shown in Figure 6 as the 'smoothed' map. Smoothing reduces artifacts, minimally degrades overall fit to the data (Fig. 5), retains most of the physically reasonable small scale features, and improves coherence between maps with nearly the same periods.

2.2 Other tomographic techniques

As I mentioned before there are numerous approaches to constructing tomographic images (see references). The approach which is developed now by our group is based on an ideology which is quite different from one discussed above. We cover the Earth surface by the regular grid in such a way that rectangles inside each four nodes have approximately equal areas. The unknown variables are values of velocity perturbations at each node. The velocities between nodes are found by interpolation. Besides a usual least square minimization we impose some additional constraints on relative perturbations of variables corresponding to neighboring nodes to obtain relative smooth solutions. In addition we construct the map of the ray density which indicates differences in the ray covering for different parts of the region under study. The damping of solution is made depending on relative density of rays: poorly covered zones are more strongly damped. This approach is now generalized for tomographic studies of azimuthal anisotropy.

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