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"Soil temperature and heat flow"

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SOIL TEMPERATURE AND HEAT FLOW

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Various biological and chemical activities are expressions of heat transfer. Such phenomena may continue in sufficient amounts if certain values of temperature be kept around. Nitrification begins favourably after the soil temperature is of the order of 10-12°C. The optimum temperature for plant growth varies widely. In fact the amount of heat flow in soils affects germination. Even the thermal properties of the seed vary with temperature . While designing roads and buildings, laying pipe lines and underground cables, the need for the thermal characteristics of the concerned soil is felt. Through such data one recognises why and how the annual temperature variation penetrates into the soil at larger depths than diurnal variation. The study of soil temperature tells us what values of maximum and minimum temperature may occur in the dry soil surface and also how these will be affected when the moisture and other environmental conditions are altered. Besides, the thermal insulative nature of dry sand enables one to use it as a solar sand collector and use it to cut down heat dissipation[1] from heat stores (Fig 1.).

1.0 Heat Balance at the Soil Surface[2,3,5]

As topography is uneven, there exists plant cover, there might be different adjoining locations of land and moisture, therefore the term "soil surface" loses its meaning as a mathematical surface. we term it as an "active surface", which means that it is a dynamically active region at which solar insolation is in action. It is at the active surface where heat transfer with the surface air and the underlying soil layers occurs. The temperature oscillations sets to deep soil layer (whose thickness is determined by the thermophysical coefficients and the period of the heat wave), and the lowest of it is the constant temperature level (Fig.2).

The principal source of all thermal effects on soil surface is the direct short-wave solar radiation. Let us call the intensity of this flux as S_{Dir} (cal min^{-1}. cm^{-2}). The next contribution comes from diffuse radiation S_{Dif}. The later reaches the active surface after scattering by dust particles and air components of atmosphere. In addition to the above both we also have long-wave radiation flux E_{LR} from the atmosphere. Now we consider the part given out by the surface. Heat energy is mainly reflected as short-wave radiation by an amount, say, S_{SR}. Also, part of the long-wave radiation is emitted by the active surface into the atmosphere, say, E_{em}. The sum of radiation fluxes (Fig.2) at the active surface during the day time is

$$R_{Day} = S_{Dir} + S_{Dif} + E_{LR} - S_{SR} - E_{em} \qquad (1a)$$

At night as $S_{Dir} = S_{Dif} = S_{SR} = 0$

$$R_{Night} = E_{LR} - E_{em} \qquad (1b)$$

After occurrence of all losses into the active surface the amount of flux remained should be shared by (i) P – the quantity spent on heat exchange with surface air. P is defined as

$$P = A\ C_p \left(\frac{d\theta}{dx}\right)_{x=0} \qquad (2)$$

Here, A is the coefficient of turbulent thermal diffusivity, C_p is the specific heat of air and $d\theta/dx$ is temperature gradient.

(ii) T – an amount utilised in evaporation and is defined as

$$T = A\ L \left(\frac{dq}{dx}\right)_{x=0} \qquad (3)$$

Where, L is the latent heat of vaporization of water and dq/dx is flux gradient.

(iii) H – an amount accounted for the heat transmitted to the underlying soils by conduction.

The heat flux H into the soil is expressed as

$$H = C\,\Delta\theta + \lambda\,\left(\frac{d\theta}{dx}\right)_{x=x_1} \qquad (4)$$

where C is the heat capacity of the surface layer of thickness x with base 1 cm^2, $\Delta\theta$ is the temperature difference on this layer per unit time, and λ is the thermal conductivity of the soil below this layer ($x_1 \rightarrow \infty$).

In summer, during the day, F and T are directed from the soil surface to the atmosphere and H from the soil surface inwards underlying soil. At night in summer, the directions are correspondingly reversed.

Thus, the heat balance equation for a bare soil surface in summer in day time is

$$S_{Dir} + S_{Dif} - S_{SR} + E_{LR} - E_{em} = C\Delta\theta + \lambda\left(\frac{\Delta\theta}{\Delta x}\right)_{x=x_1} - AC_p\left(\frac{\Delta\theta}{\Delta x}\right)_{x=0} - AL\left(\frac{\Delta q}{\Delta x}\right)_{x=0} \quad \ldots\ldots\ldots\ldots(5)$$

The problem is far complicated if a surface is partially covered by vegetation. For summer in day time

$$[\,S_{Dir} + S_{Dif} - S_{SR} + E_{LR} - E_{em} + C\Delta\theta + \lambda\,\frac{\Delta\theta}{\Delta x} - AC_p\,\frac{\Delta\theta}{\Delta x} - AL\,\frac{\Delta q}{\Delta x}\,] +$$

$$[S_{Dir} + S_{Dif} + E_{LR} - \sigma_{leaf}T^4_{leaf} - S^{leaf}_{SR} - a_{airleaf}\Delta\theta_{airleaf} - rX\Delta\theta\,] = 0 \quad \ldots\ldots\ldots\ldots(6)$$

Here, α_{leaf} is the radiation constant of the leaf, S^{leaf}_{SR} is leaf reflectivity, $a_{airleaf}$ is heat transport coefficient, X is the mass transfer coefficient and r is the numerical factor which depends upon moisture content.

2.0. Temperature Regime of Soils[2,3,5]

To investigate the temperature field in soil, we must find the equation $\theta = f(x,y,z,t)$, where θ is the soil temperature, t is the time and x, y, z are spatial cartesian coordinates. Here we assume that the soil is isotropic. This means that θ is assumed to vary with depth x and the horizontal surfaces parallel to the soil surface are considered isothermal. Therefore, $\frac{\delta\theta}{\delta y} = \frac{\delta\theta}{\delta z} = 0$ and we are only left with $\frac{\delta\theta}{\delta x}$ or $\theta = f(x,t)$. We also assume that heat transfer in soil occurs through the mechanism of conduction only, as in a homogeneous solid body. As such, it is the Fick law connecting heat flux density ($\vec{H}$) to the temperature gradient holds :

$$H = - \lambda \frac{\delta\theta}{\delta x} \tag{7}$$

where λ is the coefficient of thermal conductivity (cal/cm/sec/°C) or (W/m/K) of the soil. In fact, the values of various thermophysical coefficients are required to write an exact differential equation. These coefficients form the basis through which we can obtain solutions to the energetics problem of finding, evaluating and analysing the temperature field in a soil.

Let $-(\frac{\delta H}{\delta x})$ be the rate of change of heat flux density with depth (x). Then, in the volume dV (=1x1x dx), heat stored in unit time will be $-(\frac{\delta H}{\delta x})dx$. Also if one defines C as the volumetric heat capacity (cal/cm^3/°c) ,then $C(\frac{\delta\theta}{\delta t})dx$ is the amount of heat stored in the soil. On equating the amount of heat stored in the soil found above by (a) the difference in heat flux density and (b) by the temperature change caused, one gets :-

$$- (\frac{\delta H}{\delta x})\ dx = C\ (\frac{\delta\theta}{\delta t})\ dx$$

$$- (\frac{\delta H}{\delta x}) = C\ (\frac{\delta\theta}{\delta t}) \tag{8}$$

Since H is given by Fick law (7),

$$- \frac{\delta}{\delta x}\ (- \lambda \frac{\delta\theta}{\delta x}) = C \frac{\delta\theta}{\delta t}$$

$$\frac{\delta\theta}{\delta t} = \frac{\lambda}{C} \frac{\delta^2\theta}{\delta x^2} = \alpha \frac{\delta^2\theta}{\delta x^2} \tag{9}$$

α is termed as coefficient of thermal diffusivity ($m^2 s^{-1}$ or $cm^2 s^{-1}$).

The equation (9) is the differential equation for $\theta(x,t)$ which must be obeyed by any possible temperature variation caused by heat transport. To obtain an introductory discussion of temperature variation, we assume that the temperature varies as a pure harmonic function of time around an average value of temperature ($\bar{\theta}$) at all depths.

The surface temperature (x=0) can be written as

$$\theta\ (0,t) = \bar{\theta} + A_{\theta_0} \operatorname{Sin} \omega t \qquad (10)$$

Here, $\bar{\theta}$ the average temperature in the soil which is assumed to be the same at all depths. A_{θ_0} is the amplitude of the periodic wave at the surface and ω is the angular frequency. For diurnal variation $\omega = 2\pi / 24\times60\times60\ s^{-1}$. The temperature at any arbitrary depth would similarly be

$$\theta\ (x,t) = \bar{\theta} + A_{\theta_0} \operatorname{Sin} [\omega t + \phi(x)] \qquad (11)$$

The amplitude and phase term ϕ (x) can be determined by using the above in the differential equation. Thus, we get the temperature variation at depth in the soil as

$$\theta\ (x,t) = \bar{\theta} + A_{\theta_0} \exp\left(\frac{-x}{d}\right) \operatorname{Sin}\left(\omega t - \frac{x}{d}\right) \qquad (12)$$

Here, d is defined as the damping depth. On comparing the equation of temperature at the surface [eq.(10)] we find that in case of the later, the amplitude is smaller by a factor of $\exp\left(\frac{-x}{d}\right)$ as also a phase shift of. $\left(\frac{-x}{d}\right)$ occurs. Thus, temperature diminishes at depths and maximum value will be attained at different times. The soil constant d is determined through the thermal characteristic of the soil and the frequency of temperature variation by

$$d = (2 \lambda / \omega c)^{1/2} \qquad (13).$$

For annual variation it is $(365)^{1/2} = 19$ times larger than diurnal variation. The diurnal variation becomes unperceptible at the depth of 0.3-0.5 m. depending on the thermal characteristic of the soil. The annual variation is not found below 10 m.

Now, we write the expression for heat flux density H ($cal/cm^2/s$) for a sinusoidal variation :

$$H(x,t) = -\lambda \left(\frac{\delta\theta \; ^{ax}}{\delta x}\right) + A_{\theta_0} (\lambda C \omega)^{1/2} \exp \left(\frac{-x}{d}\right) \text{Sin} \left(\omega t + \phi_0 - \frac{x}{d} + \frac{\pi}{4}\right) \qquad (13)$$

$(\lambda C)^{1/2}$ is called the coefficient of heat storage ($W \; m^{-2} s^{1/2} \; {}^{o}C^{-1}$) β of the soil.

What we note from the above equation is that heat flux is also a harmonic function of time with $\pi/4$ phase advanced as compared with the temperature variation at a given depth. This phase shift of $\pi/4$ between H and θ corresponds to a time shift of 3 hours for diurnal variation and 1.5 months for annual variation. As an example, the maximum temperature at a surface with grass is approximately 13 hrs.. The maximum heat flux density in a soil does not occur at the time of maximum insolation, but earlier. This is due to the interaction of various components in the energy balance at the surface.

3.0. The Influence of Soil Thermal Properties on the Temperature Regime[2,3,5]

We have noticed that the heat flux density in the soil is proportional to $(\lambda C\omega)^{1/2}$. The values of coefficients for sand are $\lambda = 0.0042$ cgs units and $c = 0.5$ cgs units. For peat, $\lambda = 0.0007$ and $c = 0.75$. For the sand, we get $d = 15.2$ cm. and for peat, d =

5.1 cm.. As the surface amplitude of heat flux is inversely proportional to $\beta = (\lambda C)^{1/2}$. We get $\beta_{sand} = 0.04$ cgs units and $\beta_{peat} = 0.02$. Therefore, we conclude that the surface amplitude for peat is twice that of sand. We should also note here that the decrease of amplitude is more rapid in the peat soil which is having a smaller value of d.

The above example shows clearly how the thermal coefficients of the soil influence the temperature and heat flux values in the soil. We shall now consider this subject in detail. In fact, it is necessary to know a set of four thermophysical coefficients: λ, C, α and β. This set gives us a comprehensive representation of the thermal properties of the soil.

The thermal conductivity (λ) characterises the soil from its ability to conduct heat and is the amount of heat transferred by the soil per unit time through unit area under unit temperature gradient.(W /m / K)

The volumetric specific heat (C) characterises the soil from the viewpoint of its capacity to heat up or cool down. It is the amount of heat necessary to change the temperature of one cubic centimeter of soil by one degree.($Cal / cm^3 / {}^0C$)

The thermal diffusivity (α) is numerically equal to the thermal conductivity of a soil sample with unit volumetric specific heat.(m^2 / s)

The heat storage coefficient (β) characterises the soil from the viewpoint of its storage qualities and is numerically equal to the thermal conductivity of the soil sample whose thermal diffusivity is equal to unity.($W\ m^{-2}\ s^{1/2}\ {}^0C^{-1}$)

As the soil has capillary-porous structure, heat exchange in it should be considered to take place by a combination of the following processes :-

(i) Thermal conduction over the mass of individual granules of solid soil skeleton,

(ii) heat conduction from particle to particle at the contact points,

(iii) the molecular heat conduction through the water and air present in the pores,

(iv) convection by the gas or liquid in the pores, &

(v) radiation from particles.

Using the analogy of thermal conductivity (λ_{cond}) we may express the coefficients of convective λ_{conv} and radiative heat transfer λ_{rad} as well as a coefficient of moisture conductivity λ_{mois}. We can write the respective fluxes in the form of Ficks equation(7) as

$$H_{cond} = \lambda_{cond} \frac{\delta \theta}{\delta x} \tag{14}$$

Each of the coefficients λ_{cond}, λ_{conv}, λ_{rad}, λ_{mois} is a complicated funtion of various parameters characterising the thermophysical properties and state of the soil. We give here in brief the factors affecting the thermophysical characteristics of the soil.

Conduction The conductivity of the soil depends on the configuration of the constituent phases, packing density or porosity, the nature of contact between the particles, the dimensions of the particles, temperature, water content etc.. We shall be taking this topic in detail in the next section.

Convection The natural convection in soil may occur due to pressure difference. Under the influence of this, the flow of gas or liquid may occur accompanied by the flow of heat. Analysis shows that in natural thick layers and in the presence of small temperature drops, convection does not occur at all^{4}. For layers up to 0.5 cm. in thickness and at temperature drop of 100°C convection is not observed. As such the transport of heat in soils by the mechanism of convection may be ignored in natural conditions. For particles and size of soil pores of ~ 0.1-0.2 mm. convection amounts to 0.15-0.3% of the total heat transfer. Only for very large particles and pores of ~ 3.0 mm. this percentage reach 5%.

Radiation At relatively low temperatures and for samples not having large sized granules, λ_{rad} is also negligibly small. At soil temperatures up to 60^{0} C the radiative flux density is 0.5 X 10^{-5} cal/cm^{2}/sec. For pores of size 0.1 mm. and 1.00 mm. the heat flux due to radiation will be roughly 0.2% to 2.5% respectively of the total flux due to all types of heat transfer. In anamously aggregated soil, when the size reaches 6 mm., H_{rad} may be ~ 7%.

Mass Transfer Moisture movement in soil occurs both as vapour and liquid. The moisture flow carries heat with it, which modifies the conditions of heat transfer and also affects the value of thermal coefficient of the soil. As long as the soil temperature does not exceed 50^{0}C, the heat transport by vapour flow is not more than 10% of the total. The effect of liquid flow is even smaller. The average value of $\lambda_{vap} \approx 0.137$ X 10^{-3} cgs units. The measured values of the soil's thermal conductivity are 20 to 40 times higher than λ_{vap}. For natural soils (i.e. not highly eroded, nor are anomalously wet or overheated) it is reasonable to neglect the effect of mass transfer on the values of thermophysical characteristics of the soil.

3.1.Thermal Conductivity of Soils[2,3,5]

Let us first consider the factors which influence the magnitude of equivalent thermophysical characteristics of the soil.

a) Structure and influence of aggregation The mutual positions of soil particles and pores constitute the system and, as such, may give highly diversified values of thermal conductivity. As an example, we consider the idealized situation where solid grains are assumed to be spheres packed symmetrically. A cubic packing (porosity 47.64 %) will result into a loose system and a hexagonal packing (porosity 25.95 %) will yield a compact system. With the varied size of particles and random packing the complexity of the problem increases. In the figure 5. we have shown how the conductivity values may vary if we consider different structures of the soil, other factors remaining the same.

An increase in the size of particles and pores should alter overall heat transfer. When soil aggregates are crushed, the number of contacts between the many resulting particles sharply increase. These contacts are loose and the interstitial air phase is less conductive; this results in decrease of effective conductivity. On the other hand, experiments have shown a rise of 50% in λ_e of enlarged granules. For soil containing many large paricles like sand, the thermal conductivity increases as compared with fine textured clay.'

(b) Chemical-mineralogical composition Different soils have difference in the chemical mineralogical composition of their skeletons. They have silicon, feldspar, quartz, mica, metal oxides or marble, granite, schists, limestone etc.. Therefore, the thermal conductivity of the solid phase (λ_s) becomes a crucial factor. The conduction of heat through interstitial medium also occurs. Therefore, the value of the thermal conductivity of the fluid phase (λ_f say dry air or vapour) is also an influential factor.

(c) Temperature A change in the temperature affects the thermal process in the soil pore. The thermal conductivity of soil increases with the rise in temperature (Fig.6). For calculating the rise in thermal conductivity of soils per degree with reasonable accuracy, the following empirical formula is also used :

$$A_\theta = \frac{\delta\lambda}{\lambda\delta\theta} = 0.65\ [2.45\ \rho + 1] + 9.5(D-0.66)^{1.1}\ (\ \mathrm{deg}^{-1}\ ) \qquad (15)$$

where ρ is the specific weight and D is the particle size.

(d) Porosity and Specific Weight , Experimental study on a variety of dispersed media has shown that porosity of the medium strongly affects the thermophysical characteristics. There is an appreciable increase in equivalent conductivity due to an increase in specific weight. Specific weight and porosity are related through a simple expression. It is instructive to have expressions of conductivity estimation in terms of the porosity of the sample. It has been found that the conductivity of the soil increases as porosity decreases (Fig.7). This is because non-conducting air in the pores is replaced by a fraction of good conductor solids.

(e) **Soil Water** Water content is the most influential factor in modifying the thermal coefficient of the soil. The physical reasoning for this is that, when soils are wetted, air, a poor conductor is removed and replaced by water, a good conductor. For sandy soils, over the possible range of water content variation, the effective conductivity (Fig.8) may vary by a factor of 5 or even 10.

Like thermal conductivity, other thermal characteristics α, β and C also depend on the above factors. It may be noted that vegetation, mulches, irrigation and drainage also change the thermal characteristics of the soil surface. The dark soil will absorb more energy. Red and yellow soil will show a more rapid temperature rise than white soil. The thermal conductivity of different soils follow the order λ sand $>\lambda$ loam $>\lambda$ clay $>\lambda$ peat.

4.0. Estimation of the thermophysical coefficients[2,3,4,6,7]

We have seen that two independent thermal parameters exist in the expression when we are concerned with the quantitative description of heat transfer. These are : thermal conductivity λ and heat capacity per unit volume C of the material. The diffusivity α and heat storage coefficint β of the soil material can be determined through the knowledge of λ and C of the soil. Here, we wish to deal with the problem of theoretical prediction of these values in terms of the composition, property, porosity and other effective factors.

4.1. Heat capacity (C). We have already defined the volumetric heat capacity of soil as the amount of heat needed to raise the temperature of unit volume of soil by one degree. To obtain an idea of the capability of heating or cooling of soil we need the value of heat capacity per unit volume or specific heat of the soil. Quartz has the lowest value of specific heat and humus the largest, excepting water. Therefore, it is an indication that addition of humus and water will affect the heat capacity of the soil greatly. Wet soils have much higher heat capacity than dried ones. This explains the cold nature of wet soils. However, it may be noted that in the range of temperatures occuring in the field, the specific heat values vary little. Therefore, for calculation purposes, we take a constant value of specific heat for a given soil. The variation of specific heat of dry sand with temperature is shown in the fig.9.

The heat capacity per unit volume of soil may be calculated by adding the heat capacities of the different constituents of the soil in a cubic centimeter. If ϕ_s, ϕ_w and ϕ_a are the volume fractions of the solid material, water and air respectively, then

$$C = \phi_s C_s + \phi_w C_w + \phi_a C_a$$

For most soil minerals at normal temperature, $C_s = 0.46$ cal/g/°C and density ρ is $2.7\ g/cm^3$. For organic materials, $C_s = 0.6$ cgs units and $\rho = 1.3$ cgs units. If water is present, in the above expression, the contribution of air is negligibly small. If ϕ_m denotes the volume fraction of soil mineral, ϕ_o that of the organic material and since $C_w = 1$, we can calculate the heat capacity of soil through

$$C = 0.46\,\phi_m + 0.6\,\phi_o + \phi_w \tag{16}$$

Mathematically, the above expression is a weighted arithmetic mean of the constituent specific heats[2,7].

4.2. Thermal conductivity of soils. The thermal conductivity of soil can not be calculated by taking the weighted arithmetic mean of the constituent conductivities. An account of different influential factors has to be made in a real good expression for the estimation of the thermal conductivity of soil. The thermal conductivity of soil solids λ_s can be obtained through

$$\lambda_s = \lambda_q^{\phi_q} \lambda_o^{(1 - \phi_q)} \qquad (17)$$

where λ_q= 7.7 W/m/K is the value of thermal conductivity of quartz and ϕ_q is the volume fraction of quartz present in the soil; λ_o = 2.0 W/m/K is the thermal conductivity of soil minerals other than quartz having volume fraction $(1-\phi_q)$. The thermal conductivity of water λ_w= 0.6 W/m/K and that of air λ_a= 0.026 W/m/K.

We know that heat flux density H is proportional to the temperature gradient existing in the soil and the proportionality constant is termed as thermal conductivity. As the soil is made up of many components, an appropriate value of conductivity should be assigned to the considered homogeneous system. This is termed as the effective thermal conductivity (λ_e) which depends on the mineral composition, phase orientation, texture, amount of air etc.. A workable expression for the estimation of λ_e is obtained by considering simple models for a real system. Here, we consider the soil to be made up of two components : a continuous medium of conductivity λ_f with volume fraction ϕ, and dispersed granules of conductivity λ_s with volume fraction $(1-\phi)$. This scheme describes dry sand or fully saturated sand.

When one assumes that the flux lines are not markedly altered through bending towards or away from grains, one arrives at semi-empirical expressions for calculating the effective conductivity of the soil. Considering the soil as a medium made up of successive flat layers of solid and fluid, two plans are visualised as shown in fig 10.

(i) A parallel distribution of phases giving maximum λ_e. Hence, this scheme behaves as a good conductor of heat. Mathematically, it is the weighted arithmetic mean of the constituent conductivities.

$$\lambda_e = \phi \lambda_f + (1-\phi) \lambda_s \qquad (18)$$

(ii) A series distribution of phases which yields minimum λ_e. Hence, this scheme behaves as a better insulator of heat. It is the weighted harmonic mean of the constituent conductivities.

$$\lambda_e = [\frac{\phi}{\lambda_f} + \frac{(1-\phi)}{\lambda_s}]^{-1} \qquad (19)$$

Mathematically, one then has an intermediate mean which will correspond to the randomisation of layers.

$$\lambda_e = \lambda_f^{\phi} . \lambda_s^{(1-\phi)} \qquad (20)$$

A plot for these schemes is shown in the figure 11. The cross-hatched area indicates the realisable region on sands and soils.

In order to obtain an agreement with the observed values on soils we might take the weighted geometric mean of the maximum and minimum values of thermal conductivities. Calling the A.M. (Eq.18) as λ_{11} and H.M. (Eq 19) as $\lambda_{\perp}$ we then have

$$\lambda_e = \lambda_{11}^{n} \lambda_{\perp}^{1-n} \qquad (21)$$

In the above scheme n^{th} fraction of the sample is taken to be made up of alternate zones of solid and fluid oriented parallel to the direction of heat flow (giving λ_{11}) and the remaining $(1-n)^{th}$ fraction of the sample zones is oriented perpendicular to the direction of heat flow (giving $\lambda_{\perp}$). Using measured values of λ_e on various soils and $\lambda_s/\lambda_a \sim 130$ one semi-empirically gets

$$n = \frac{0.5 (1-\log \phi)}{\log (\frac{\lambda_s}{\lambda_f} . \phi (1-\phi))} \qquad (22)$$

Here, as an illustration; we give experimental values of silt loam and those computed from the above expressions.

ϕ	Experimental* ($\lambda_e X 10^4$) cal/cm/sec/^oC	n	Calculated ($\lambda_e X 10^4$) cal/cm/sec/^oC
0.568	4.15	0.421	4.20
0.401	6.88	0.475	7.02
0.332	9.18	0.515	9.26

*To convert cal/cm/sec/^oC into W/m/K multiply by 418.7

The above table also gives us the idea that, in a sand-like system, about 50 percent of the sample sets parallel to the direction of heat flow and an almost similar amount perpendicular to the direction of heat flow.

A rigorous mathematical formulation for the conductivity of two-phase systems requires restrictions on the shape of the grains and the orientation of phases. If one considers a regular arrangement of spheres of the dispersed phase forming a cubic lattice, we obtain on considering third order interaction

$$\lambda_e = \lambda_s \; [\; 1 - 2 \; X \; 2.309 \; (\frac{\lambda_s - \lambda_f}{2 \lambda_s + \lambda_f}) \; \phi^{2/3} \;] \qquad (23)$$

In case of dry sand we may use

$$\lambda_e \approx \lambda_s \; (1 - 2.309 \; \phi^{2/3}) \qquad (24)$$

By empirically introducing the coefficient of packing which in turn depends upon mutual packing, structural element and distribution element and distribution of air one finds for cubic packing

$$\lambda_e = 1.5 \; \pi \; \lambda_s \; \frac{0.9 - \phi}{(2.1 - \phi)^2} \qquad (25)$$

and for tetraheral packing of particle.

$$\lambda_e = 3\pi\lambda_a \ln \frac{0.43 + 0.31\phi}{\phi - 0.26} \qquad (26)$$

The geometric mean may be applied to frozen soils for the estimation of λ_e,

$$\lambda_e = \lambda_s^{(1-\phi)} \lambda_w^{\psi} \lambda_i^{(\phi-\psi)} \qquad (27)$$

where ψ is the volume fraction of the unfrozen water and λ_i is the thermal conductivity of ice.

The temperature dependence of thermal conductivity of air and that of λ_w above the freezing point of water are given by empirical expressions:

$$\lambda_a = 0.237 + 6.41 * 10^{-5}\,\theta \qquad (28)$$

$$\lambda_w = 0.55 + 2.34 * 10^{-3}\theta - 1.1 * 10^{-5}\,\theta^2 \qquad (29)$$

$$\lambda_{ice} = \frac{488.19}{273.13 + \theta} + 0.4685 \qquad (30)$$

An interesting feature is the variation of thermal conductivity of sand at interstitial pressures (Fig 12). The sample becomes much insulative at low pressures. An envelope of such a sample around a heat store will reduce the heat dissipation from the latter.

An expression for thermal conductivity of sand at interstitial air pressure embodies terms p - the interstitial air pressure and p_c - characteristic value of air pressure of the sample. The characteristic pressure is that value of interstitial air pressure at which thermal conductivity of the sample reduces to half its value at the normal air pressure. If λ_o is the thermal conductivity of the sand at normal air pressure then its thermal conductivity at interstitial air pressure is given by

$$\lambda_{ap} = \lambda_a \left(\frac{p}{p + p_c} \right) \qquad (31)$$

4.3.Effective thermal diffusivity: We defined the effective thermal diffusivity of a soil sample as the ratio of the effective thermal conductivityy, divided by the effective volumetric specific heat of the soil. The volumetric specific heat C (cal/cm^3/$^\circ$C) is given by

$$C_e = \sum_i C_i \, \rho_i \, \phi_i \qquad (32)$$

For the sample whose constants are $\rho_s = 2.58$ g/cm^3, $C_s = 0.184$ cal/g/$^\circ$C, $\rho_a = 0.0012$ g/cm^3, $C_a = 0.24$ cal/g/$^\circ$C, $\phi = 0.36$,

$$C_e = \rho_a C_a \phi + \rho_s C_s (1-\phi) = 0.304 \text{ cal/cm}^3/^\circ\text{C} \qquad (33)$$

For such a soil whose $\lambda_e = 6.11 \times 10^{-4}$ cal/cm/sec/$^\circ$C, α_e can be obtained

$$\alpha_e = \frac{\lambda_e}{C_e} = 2.0 \times 10^{-3} \text{ cm}^2\text{/sec} \qquad (34)$$

The prediction of effective thermal diffusivity is a much more difficult problem than the prediction of conductivity. In literature one finds various attempts and some of these are on the lines of thermal conductivity predictions. Here, we give one expression which has been obtained for sand-like systems. In deriving the formula it has been assumed that sand is a regular dispersion of spheres in the continuous phase and the display is like a cubic lattice. On accomodating the third order interaction the expression for thermal diffusivity is

$$\alpha_e = \alpha_a' \left[1 + 3.6(1-\phi)^{2/3} \left(\frac{\alpha_s' - \alpha_a'}{\alpha_s' + 2\alpha_a'} \right) + 6.298(1-\phi) \left(\frac{\alpha_s' - \alpha_a'}{\alpha_s' + 2\alpha_a'} \right) \right] \qquad (35)$$

As an example let us take dune sand. We have $\alpha_s' = 26.3$cm^2/s and $\alpha_a' = 2.04 \times 10^{-4}$cm^2/s. The above expression predicts $\alpha_e = 1.84$

cm^2/s. Experimental value of the sample is 2.13×10^{-3} cm^2/s.

4.4. Heat storage coefficient of soil Earlier we defined the heat storage quality of a soil sample by the heat storage coefficient β. It is mathematically $\beta = (\lambda_e C_e)^{1/2} = \lambda_e / (\alpha_e)^{1/2}$. Once we predict the values of this coefficient through the expression of thermal conductivity and specific heat or diffusivity we can calculate the heat storage coefficient of the soil material. For the stated value of the dune sand sample the heat storage coefficient $\beta = 0.014$ $cal/cm^2/s/^oC$.

Like thermal diffusivity, obtaining the heat storage coefficient of a system made of many components on theoretical basis is a difficult task. However, a simple expression can be obtained if we assume that the expressions for predicting the heat storage coefficient are identical to the thermal conductivity expressions. We can proceed as

$$(\sqrt{\lambda_e})^2 = (\sqrt{\lambda_f})^{2\phi} (\sqrt{\lambda_s})^{2(1-\phi)} \qquad (36)$$

$$\text{or} \quad \left[\left(\frac{\lambda_e C_e}{C_e} \right)^{1/2} \right]^2 = \left[\left(\frac{\lambda_f C_f}{C_f} \right)^{1/2} \right]^{2\phi} \left[\left(\frac{\lambda_s C_s}{C_s} \right)^{1/2} \right]^{2(1-\phi)} \qquad (37)$$

Since $\beta_s = (\lambda_s C_s)^{1/2}$, $\beta_f = (\lambda_f C_f)^{1/2}$ and $\beta_e = (\lambda_e C_e)^{1/2}$,

$$\frac{\beta_e^2}{C_e} = \frac{\beta_f^{2\phi}}{C_f^{\phi}} \frac{\beta_s^{2(1-\phi)}}{C_s^{(1-\phi)}} \qquad (38)$$

We assume that C_e is predicted through the weighted geometric mean of the constituent specific heats i.e.

$$C_e = C_f^{\phi} C_s^{(1-\phi)} \qquad (39)$$

Hence,

$$\beta_e = \beta_f^{\phi} \beta_s^{(1-\phi)} \qquad (40)$$

Thus, knowing heat storage coefficients of the constituent phases and the porosity of the system, the heat storage coefficient of the sample can be predicted with a percentage error of 5 - 7 %.

5.0.Measurement of thermal properties of soil

To have a knowledge of temperature field in a sample we should measure its thermophysical coefficients. Experimental values are the true representatives of the real situation; therefore, theoretical estimations are encouraged where the measurements are not possible. It is often required to evaluate those simultaneously not only at a single point but through depths and even at times. In some cases one needs the dependence of these values upon temperature, water content and strucuture. The methods for measurement can be divided into three cateogries.

(i) Computational methods In a computational method we do analysis of the temperature variation at soil depths and time. We can obtain by this technique the thermal diffusivity of the soil only. Here a periodic variation in temperature is assumed. The advantage of such a method is that one requires recording of the temperature only. The disadvantages are many :(i) the value obtained is an average and approximate, (ii) we can find only one coefficient.

(ii) Field methods Here we use the technique through which we obtain coefficients in the natural form of the system, the structure is not destroyed. We may evaluate the parameter with depth and time too. The results obtained are of course less accurate and we cannot study the property under controlled situation.

(iii) Laboratory methods These are the methods which have flexibility and with them we can make wide range of investigations. The values obtained are also accurate. However, the main drawback with them is that, as the sample is brought from the location to the laboratory, structure, mositure and temperature conditions are disturbed.

In the following paragraphs we shall be discussing various laboratory methods and in particular those which can be adapted for field measurements. It is dynamical methods, which take lesser time than steady state methods, are preferred for most of the measurements. However it should be noted that, the steady state methods are the exact ones but adaptation of them to various practical conditions is limited.

5.1.Thermal probe for the determination of λ_e[2,7] Theoretically when a line source is buried in an infinite sample, the change in temperature with time at a point on the source determines thermal conductivity by

$$\lambda_e = \frac{Q}{4\pi(\theta_2 - \theta_1)} \cdot \ln\left(\frac{t_2 - t_0}{t_1 - t_0}\right) \qquad (41)$$

Here, θ_2 and θ_1 are two values of temperature at times t_2 and t_1; t_0 is the correction factor in time to be obtained with a standard material. Q is the power per unit length supplied to the line source.

On this theory an instrument for the determination of thermal conductivity of soil is based and is known by thermal probe or needle method.

The construction of probe is simple and it can be fabricated in the laboratory workshop. In a hollow steel needle an electric heater wire is spread throughout the length. A thermocouple is placed in the middle of the heater. The thermocouple and heater leads are brought out through a plastic handle. Through a reference point device thermocouple ends are brought to a potentiometric device for measuring thermo-emf (Fig.13). The heater wire circuitory consists of a power supply (or storage battery) and a power measuring unit. The thermal probe (or the needle) is inserted in the soil and observations are taken after thermal equilibrium is established. The experiment is conducted in 10 to 15 minutes. Thermal conductivity of the soil is calculated using

$$\lambda = \text{constant } I^2 \frac{\ln (t_2/t_1)}{\Delta V} \qquad (42)$$

where I is the current (in mA) supplied to the heater, ΔV is the rise in emf (in μV) in the duration t_1 to t_2. The constant in the expression is determined through the knowledge of resistance of the heater wire, length of the needle and the temperature-emf conversion factor for the thermocouple used.

5.2.Hot wire method for α[7] The hot wire method for the determination of thermal diffusivity of the soil is based on theory similar as that of the probe. An experimental arrangement for the hot wire technique is shown in the figure 14. As in the probe, here we have a heater encased in a sheath and a thermocouple at a certain distance r from the heater (note, not adjacent to the heater as in the case of the probe). This assembly is inserted in the soil sample under investigation. Heating of a sample by parallel wire is illustrated by the adjoining figure. From the plot of $\frac{\delta\theta}{\delta t}$ versus t, the value of t_{max} is determined. The thermal diffusivity of the sample may then be evaluated through

$$\alpha = r^2/4\ t_{max} \qquad (43)$$

Acalorimeter method for finding diffusivity[7] This method is based on the theory of studying thermally a soil sample, contained in a metallic container (Fig.15), in the stage of regular thermal regime. Knowing (i) the shape factor B of the container which in the case of cylinder depend on its radius R and length l and (ii) using slope of log deflection of the galvanometer versus time curve one may determine the diffusivity of the soil using the expression

$$\alpha = \frac{\delta}{\delta t} \ln \theta \frac{1}{(\frac{2.405}{R})^2 + (\frac{\pi}{l})^2} \qquad (44)$$

Continuous fall electrical method for determining specific heat[8]

One can modify the well known Calendar and Barne's Continuous flow method which is commonly used in various laboratories to find the specific heat of liquids. Here the loose sand/soil is allowed to fall through a double-walled glass tube (Fig.16). A steady state electric current flows through a centrally stretched heater wire. A thermocouple is employed to measure the difference in the temperatures of the incoming and outgoing materials. To vary the rate of fall nozzles of different diameters are employed. The specific heat C is calculated from

$$C = (i_1^2 - i_2^2)\ R \times 10^7 \ / \ J\ (m_1 \Delta\theta_1 - m_2 \Delta\theta_2) \qquad (45)$$

Here, i_1 and i_2 are the current values when the rates of material fall are m_1 and m_2 and the corresponding steady state temperature differences at the ends of the glass tube are $\Delta\theta_1$ and $\Delta\theta_2$ respectively.

Plane heat source method for the determination of heat storage coefficient β[9]

In this method, in principle, the plane heat source is placed in a sample of soil and its temperature rise with time is noted. For such a source the rise in temperature at the centre of the source is proportional to the square root of time and is expressed as

$$\theta = \frac{q}{\beta} \left(\frac{t}{\pi} \right)^{1/2} \qquad (46)$$

The graph of θ vs $(t)^{1/2}$ will be a straight line whose slope B_0 will be $\dfrac{q}{\beta (\pi)^{1/2}}$.

Thus, from the known values of power generated per unit area of the source q and slope B_0 the heat storage coefficient can be determined as

$$\beta = q \;/\; E_0 (\pi)^{1/2} \qquad (47)$$

The construction of the plane heat source is simple and it can also be fabricated in a laboratory workshop. A heater element is stretched between two copper foils (Fig.17). In the centre of the heating element one end of a thermocouple is fixed. This assembly is placed inside the sample. The temperature at the centre of the heat source is recorded after every 10 s. The observations are taken for 250 s (Fig.18). The slope of the straight line between temperature θ and $(t)^{1/2}$ can be obtained by least square fitting using a PC-XT computer.

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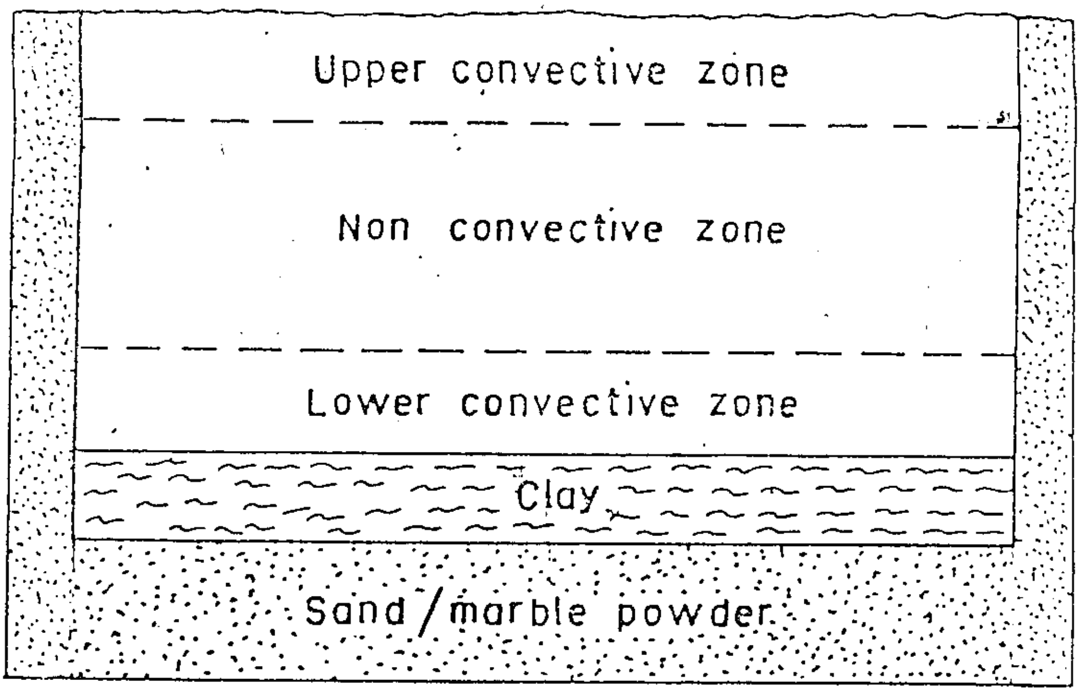

Upper convective zone
Non convective zone
Lower convective zone
Clay
Sand / marble powder

FIG. 1

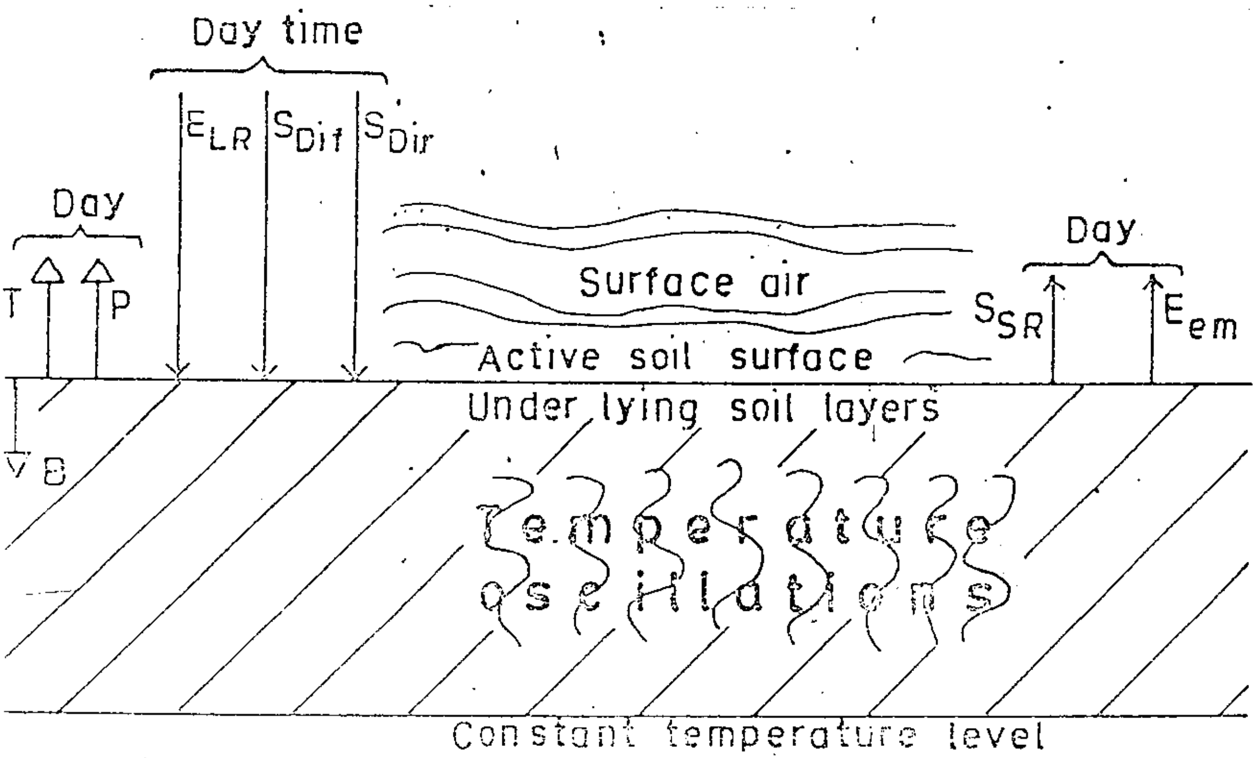

Day time
E_{LR}
S_{Dif}
S_{Dir}
Day
T
P
B
Surface air
Active soil surface
Under lying soil layers
Temperature oscillations
Constant temperature level
Day
S_{SR}
E_{em}

FIG. 2

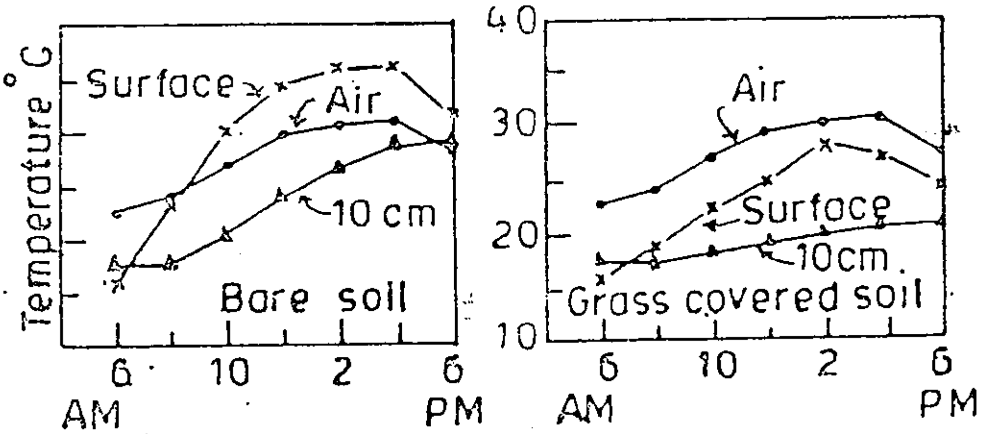


FIG - 3

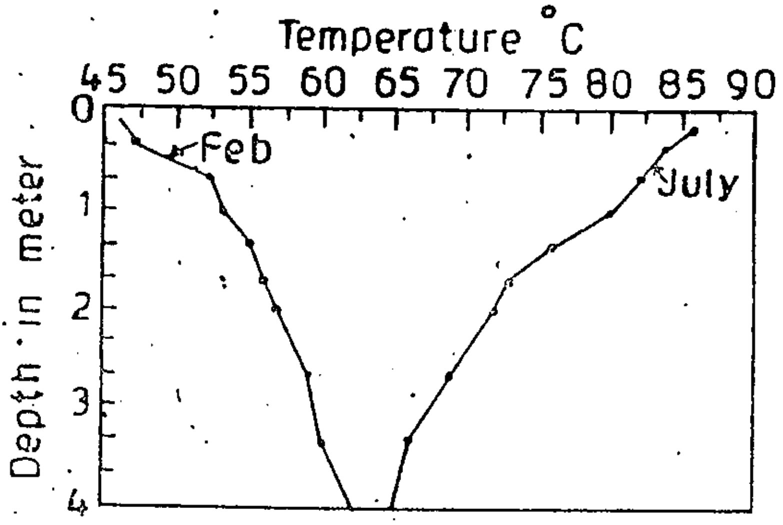


FIG. 4

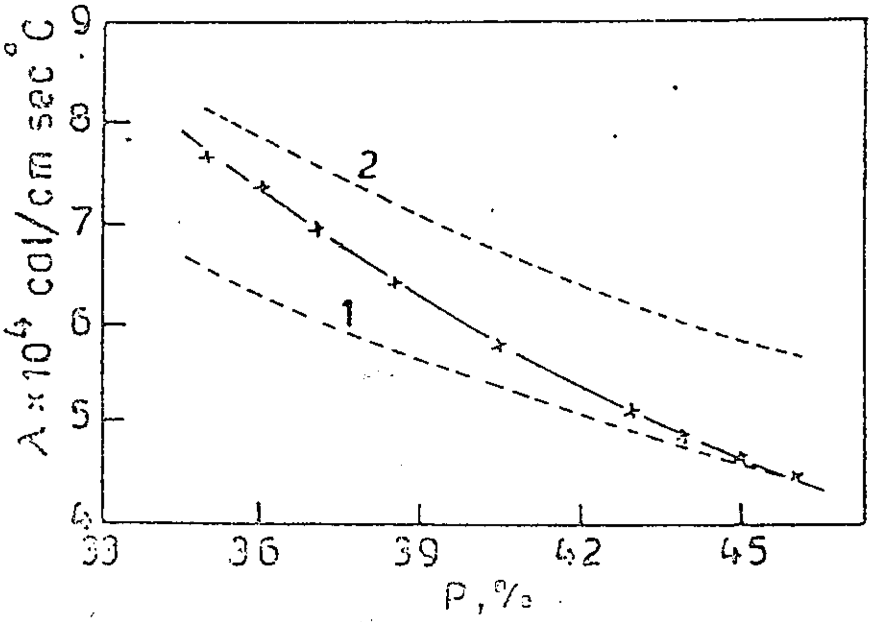


FIG. 5

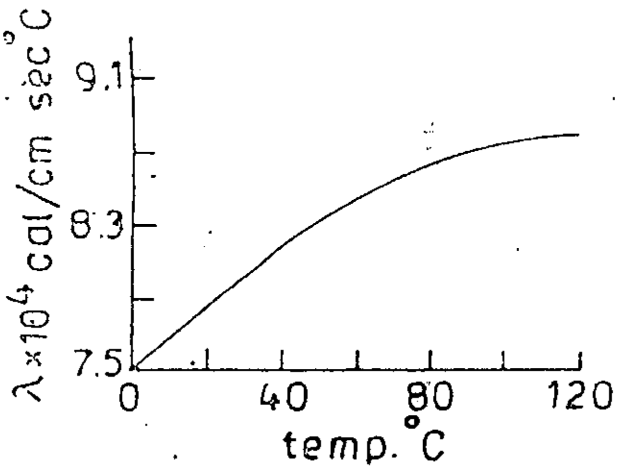


FIG. 6.

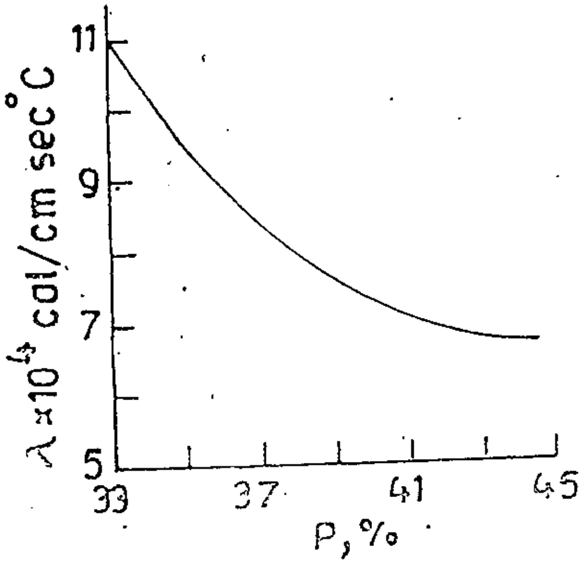


FIG. 7

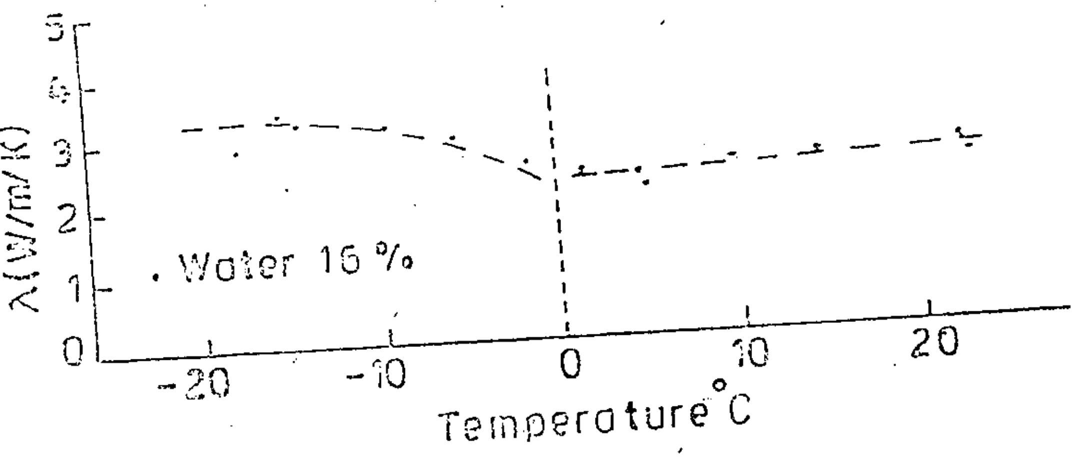


FIG 8

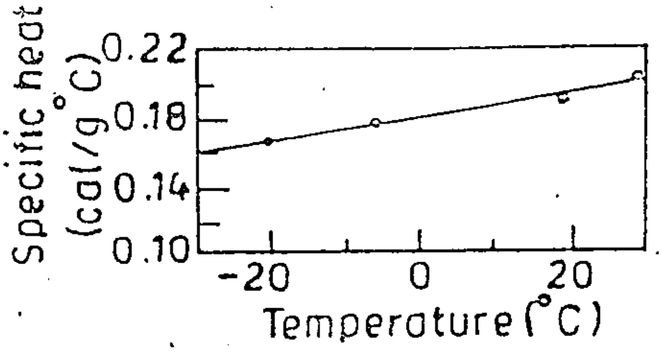


FIG. 9

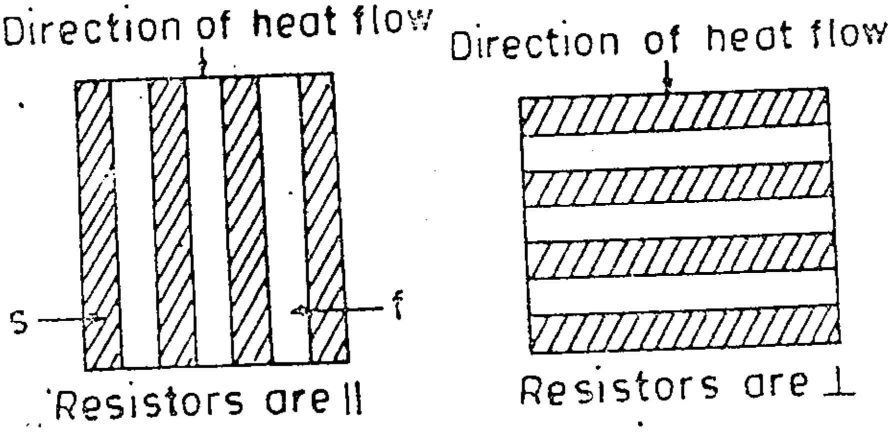


FIG. 10

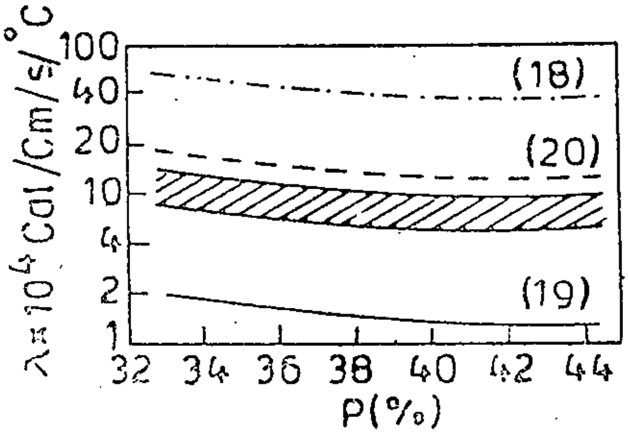


FIG 11

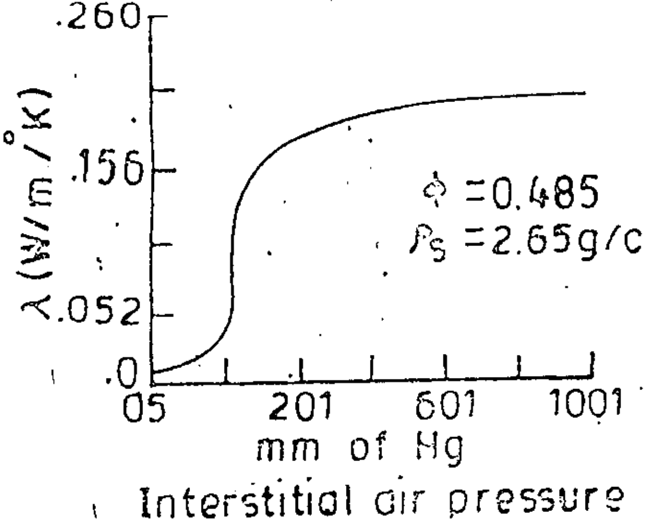
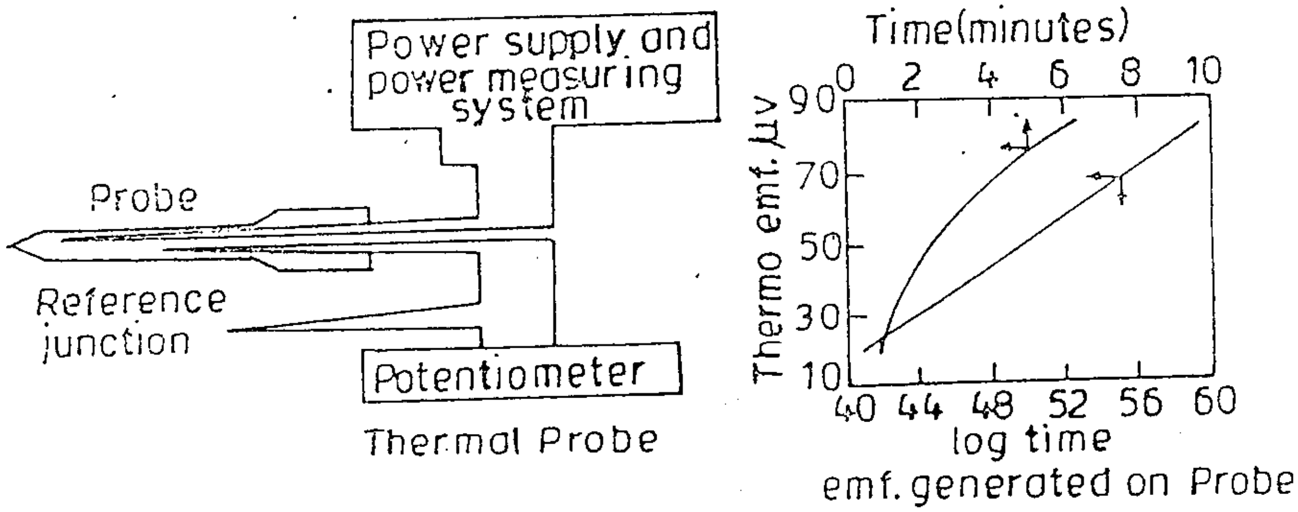


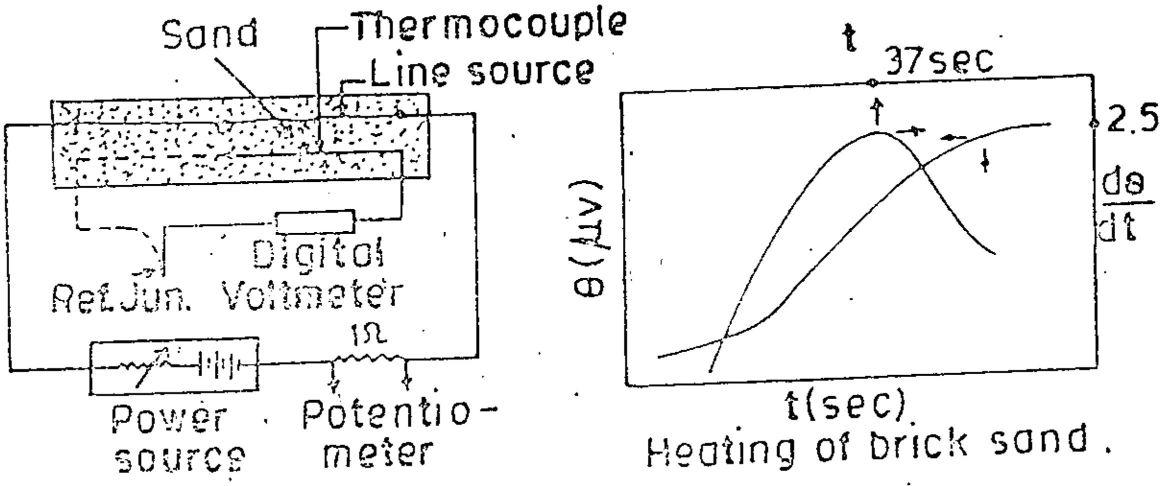
FIG. 12



Thermal Probe

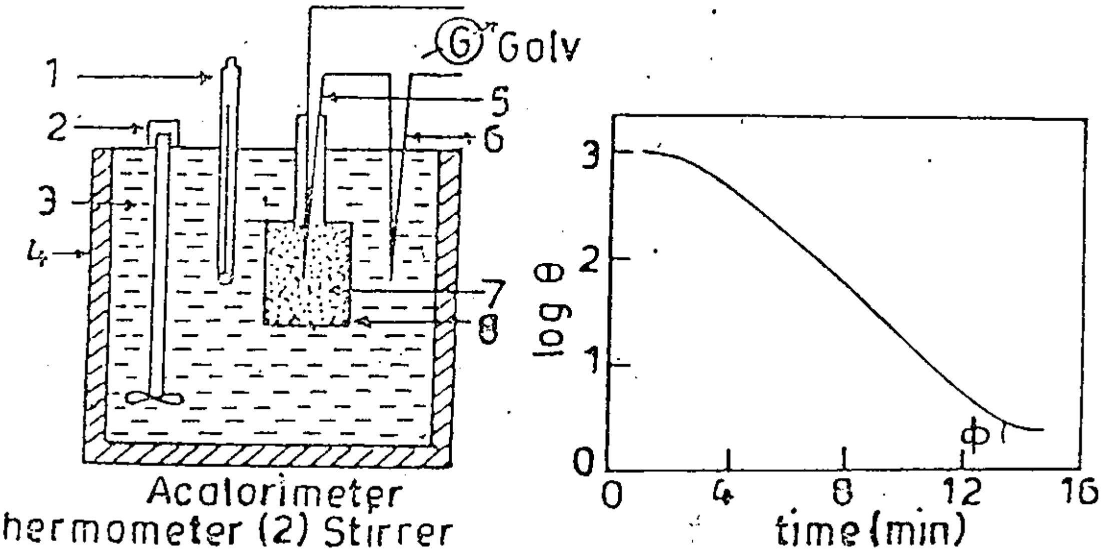
emf. generated on Probe

FIG. 13



Heating of brick sand.

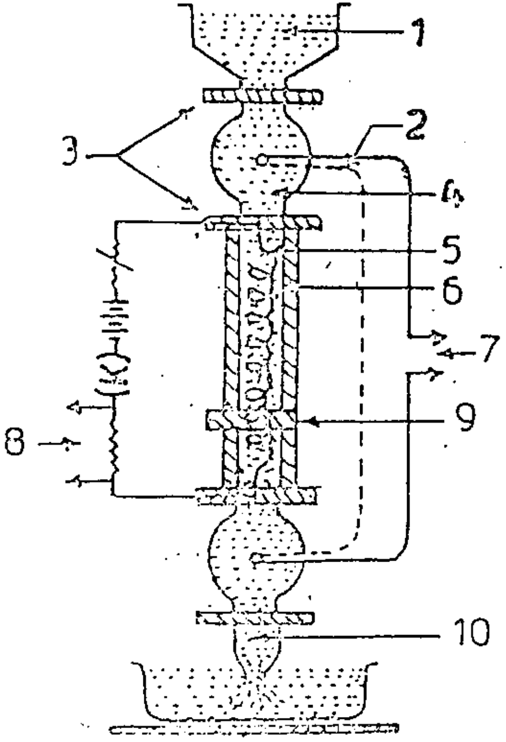
FIG. 14



Acalorimeter
(1) Thermometer (2) Stirrer
(3) Water (4) Thermostat
(5) & (6) Thermocouple (7) Soil
(8) Acalorimeter.

Heating curve for dry sand

FIG. 15



(1) Reservoir (2) Thermocouple
(3) Cork (4) Sample (5) Heater
(6) Evacuated and silvered double walled glass tube
(7) to potentiometer for the measurement of i (9) Glass tube coupled to a vibrator and (10) Nozzle

FIG. 16

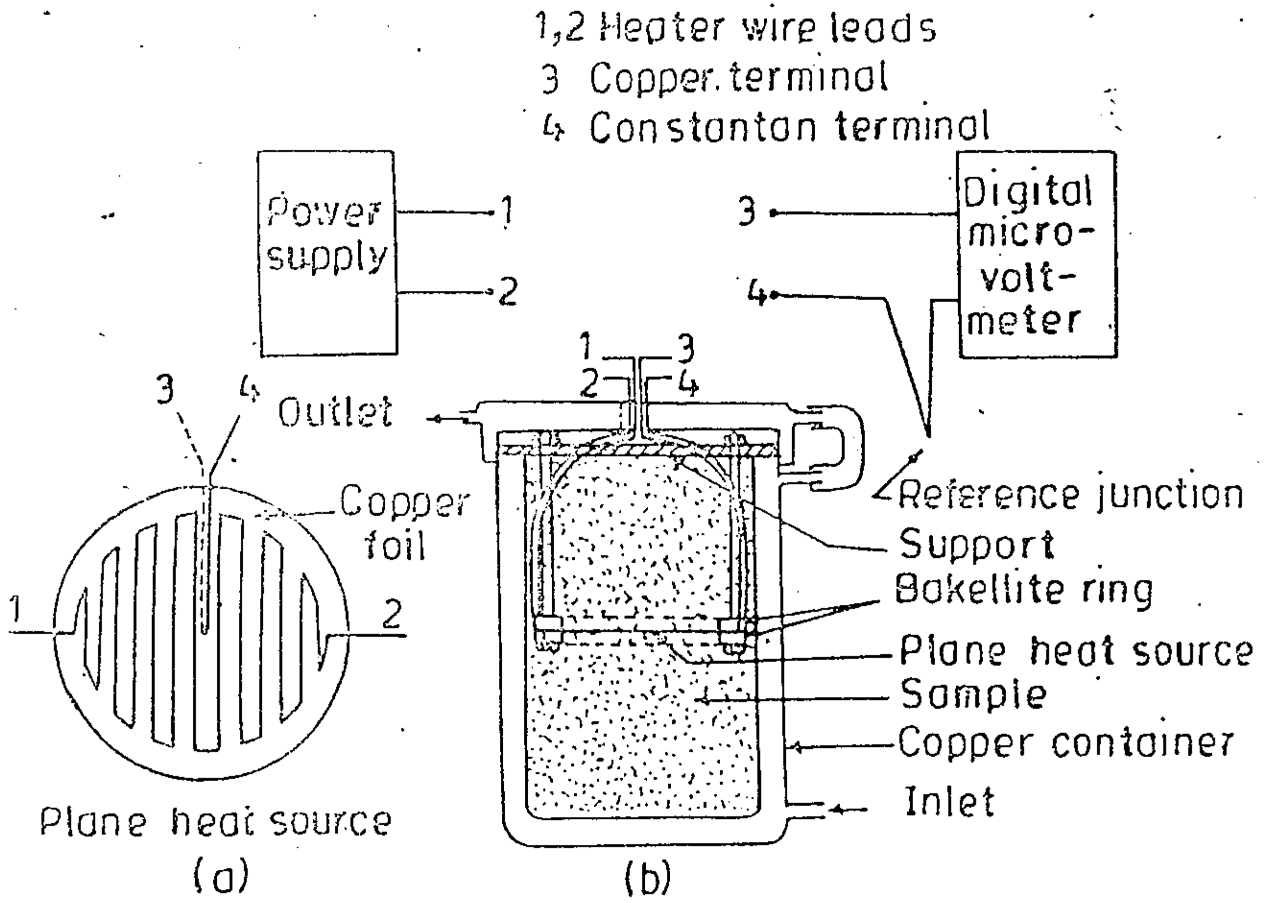

1,2 Heater wire leads
3 Copper terminal
4 Constantan terminal
Power supply
1
2
3
4
Digital micro-volt-meter
Outlet
Copper foil
Reference junction
Support
Bakellite ring
Plane heat source
Sample
Copper container
Inlet
Plane heat source
(a)
(b)

FIG. 17

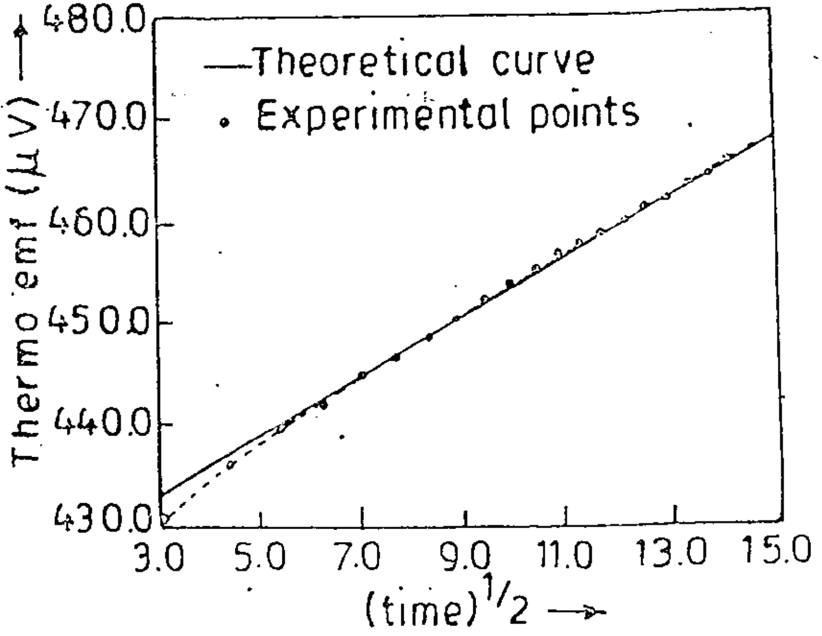

Theoretical curve
Experimental points
Thermo emf (µV)
480.0
470.0
460.0
450.0
440.0
430.0
3.0
5.0
7.0
9.0
11.0
13.0
15.0
(time)^1/2

FIG. 18