INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONES: 224281/2/3/4/5/6
CABLE: CENTRATOM - TELEX 460392-I

SMR/107 - 18

WORKSHOP ON PATTERN RECOGNITION AND ANALYSIS OF SEISMICITY

(5 - 16 December 1983)

STATISTICAL ANALYSIS

L. KNOPOFF

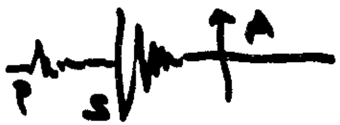
STATISTICAL ANALYSIS

Descartes: List your assumptions

1. DATA
2. ANALYSIS

1. DATA. (t; LAT., LONG.; h; M) (much data processing)

Def. of Magnitudes

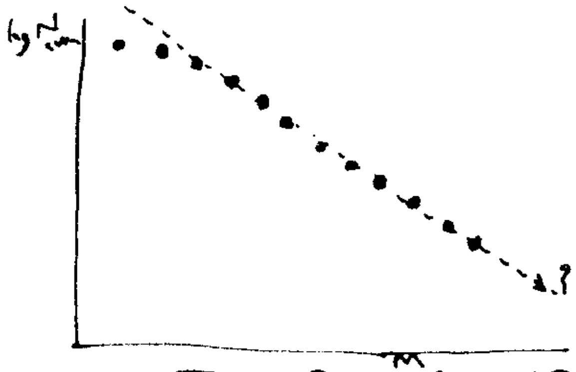


LOCAL MAGNITUDES M_L Richter p342

Teleseisms : M_s (20 sec. surface waves) p346

m_b (1 sec. body waves) p343

$M_s = m_b$ at $M \approx 6.75$



Richter: Elementary Seismology (Freeman, 1958)

Table 22-3 Logarithms of the Maximum Combined Horizontal Ground Amplitude A (in microns) for Surface Waves with Periods of 20 Seconds Produced at the Given Distance by a Standard Shock Taken as Magnitude Zero. (Correlation with Table 22-1 for short distances imperfect and now under investigation)

Δ (degrees)	−log A	Δ (degrees)	−log A
20	4.0	90	5.05
25	4.1	100	5.1
30	4.3	110	5.2
40	4.5	120	5.3
45	4.6	140	5.3
50	4.6	160	5.35
60	4.8	170	5.3
70	4.9	180	5.0
80	5.0		

Table 22-1 Logarithms* of the Amplitudes A_0 (in millimeters) with which a Standard Torsion Seismometer ($T_0 = 0.8$, $V = 2800$, $h = 0.8$) Should Register an Earthquake of Magnitude Zero

Δ (km)	−log A_0	Δ (km)	−log A_0	Δ (km)	−log A_0
0	1.4	150	3.3	390	4.4
5	1.4	160	3.3	400	4.5
10	1.5	170	3.4	410	4.5
15	1.6	180	3.4	420	4.5
20	1.7	190	3.5	430	4.6
25	1.9	200	3.5	440	4.6
30	2.1	210	3.6	450	4.6
35	2.3	220	3.65	460	4.6
40	2.4	230	3.7	470	4.7
45	2.5	240	3.7	480	4.7
50	2.6	250	3.8	490	4.7
55	2.7	260	3.8	500	4.7
60	2.8	270	3.9	510	4.8
65	2.8	280	3.9	520	4.8
70	2.8	290	4.0	530	4.8
80	2.9	300	4.0	540	4.8
85	2.9	310	4.1	550	4.8
90	3.0	320	4.1	560	4.9
95	3.0	330	4.2	570	4.9
100	3.0	340	4.2	580	4.9
110	3.1	350	4.3	590	4.9
120	3.1	360	4.3	600	4.9
130	3.2	370	4.3		
140	3.2	380	4.4		

* Since A_0 is less than 1, its logarithm is negative, and the table shows values for −log A_0.

M_L

I-5

log E = α + β M

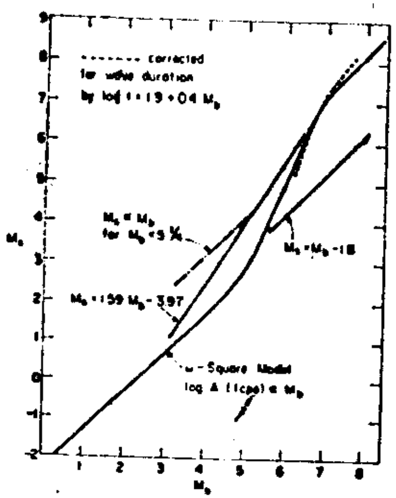


FIG. 8. Theoretical $M_s - M_b$ relation for the ω-square model as compared with empirical formulas of Gutenberg-Richter (1956) and Evernden *et al.* (1971) for earthquakes and that of Thirlaway-Carpenter (1966) for explosions.

3

where LD is in cm². This result is essentially the same as earlier results obtained by Tocher (1958) and Iida (1959, 1965). The uncertainties in (6) are large but not surprising in view of the differences in location and type among the faults.

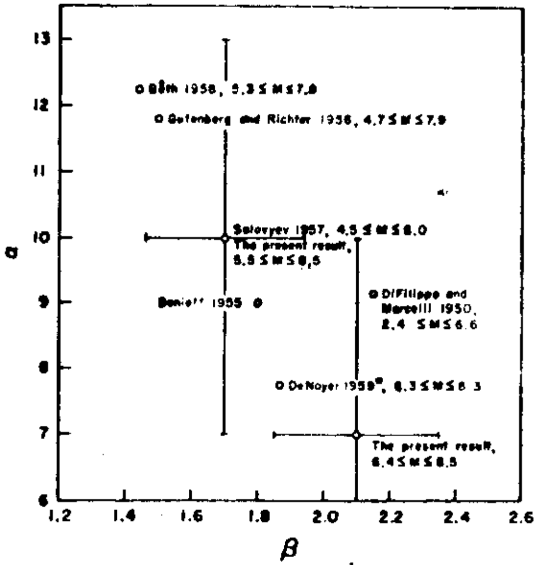


FIG. 2. Coefficients in the magnitude energy relation. Bars indicate uncertainties in the present results. We arbitrarily take the uncertainty in α to be 3 units. *See Tocher (1958).

Comparing (5) and (6), we obtain

$$\beta = 1.70 \pm 0.24$$

$$\alpha = \log [\eta k \mu (D_1 + D_2)] - (3.47 \pm 2.04). \tag{7}$$

This value of β agrees approximately with other estimates (Figure 2).

We note that, if the two smallest shocks are omitted from the above calculation, then

$$\beta = 2.14 \pm 0.25$$

$$\alpha = \log [\eta k \mu (D_1 + D_2)] - (6.77 \pm 2.11)$$

The value of α can, in principle, be determined from (7). However, because of the lack of knowledge of the parameters in (7) and particularly of $(D_1 + D_2)$ and η, α can at best be estimated only very roughly at the present time. In the laboratory model study referred to above, we observed the fractional stress-drop $\delta = D/D_1$ for the

4

$\log N_{cum} = a - bM$ N_{cum} = no of eqs. p.u. time (usually 1 year) with mag ≥ M.

for So. Calif.	$a \approx 4.77$	$b \approx 0.85$	
New Zealand	$a \approx 5.2$	$b \approx 0.90$	± ?
Japan	$a \approx 7.2$	$b \approx 1.00$	
Worldwide	$a \approx 8.56$	$b = 1.0$	

Richter p. 359

Magnitude-energy

$\log E = \alpha + \beta M$

$\alpha = 11.8$ $\beta = 1.5$ Gutenberg & Richter 1956
≈ 10.0 $1.7 \pm .2$ to .3 King & Knopoff (BSSA, 59, 1969, 269)

correlated?

Magnitude-fault length

$M = c_0 + d_0 \log L$

$c_0 = .75$ $d_0 = .98$ L = cm Tocher (1958)
2.27 $.76$ Iida (1959)
± (large!)

Seismic Moment

$M_0 = \mu \times$ average slip $\times$ fault area (energy)

(Aki, 1962)(Niigata)

$M_0 \approx 10^{30}$ dyn cm 1960 Chile, 1964 Alaska
10^{12} μearthquakes
10^{8} μfractures in lab

Keiiti Aki

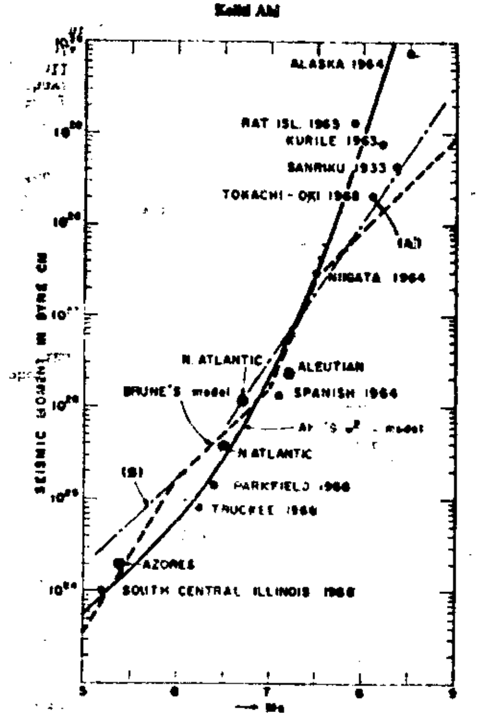


FIG. 4. Seismic moment as a function of magnitude reproduced from Aki (1972) with additional lines (A') and (B) for the ω-model of Brune & King (1967) as described in text.

$$\log M_0 = 1.5 M_s + 16.0 \qquad M_s > 7$$

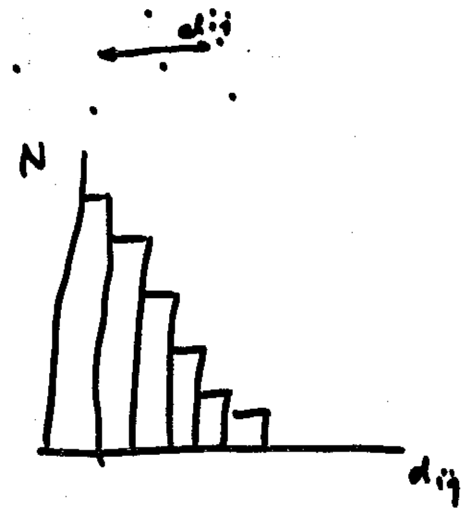
$$= M_s + 19.2 \qquad 7 > M_s > 6$$

$$= 1.4 M_L + 17.0 \qquad 6 > M_L > 3$$

Aki, Geophys. J. R.A.S.
31 (1972) 3-25

Seismic Gaps (Ohtake, Matsumoto & Latham)

Second order moments



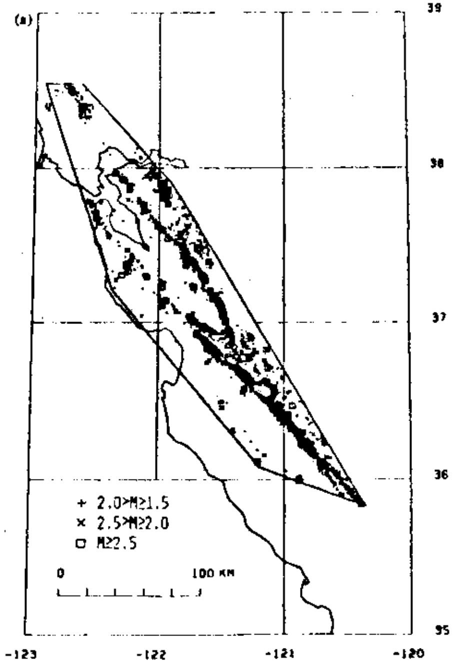
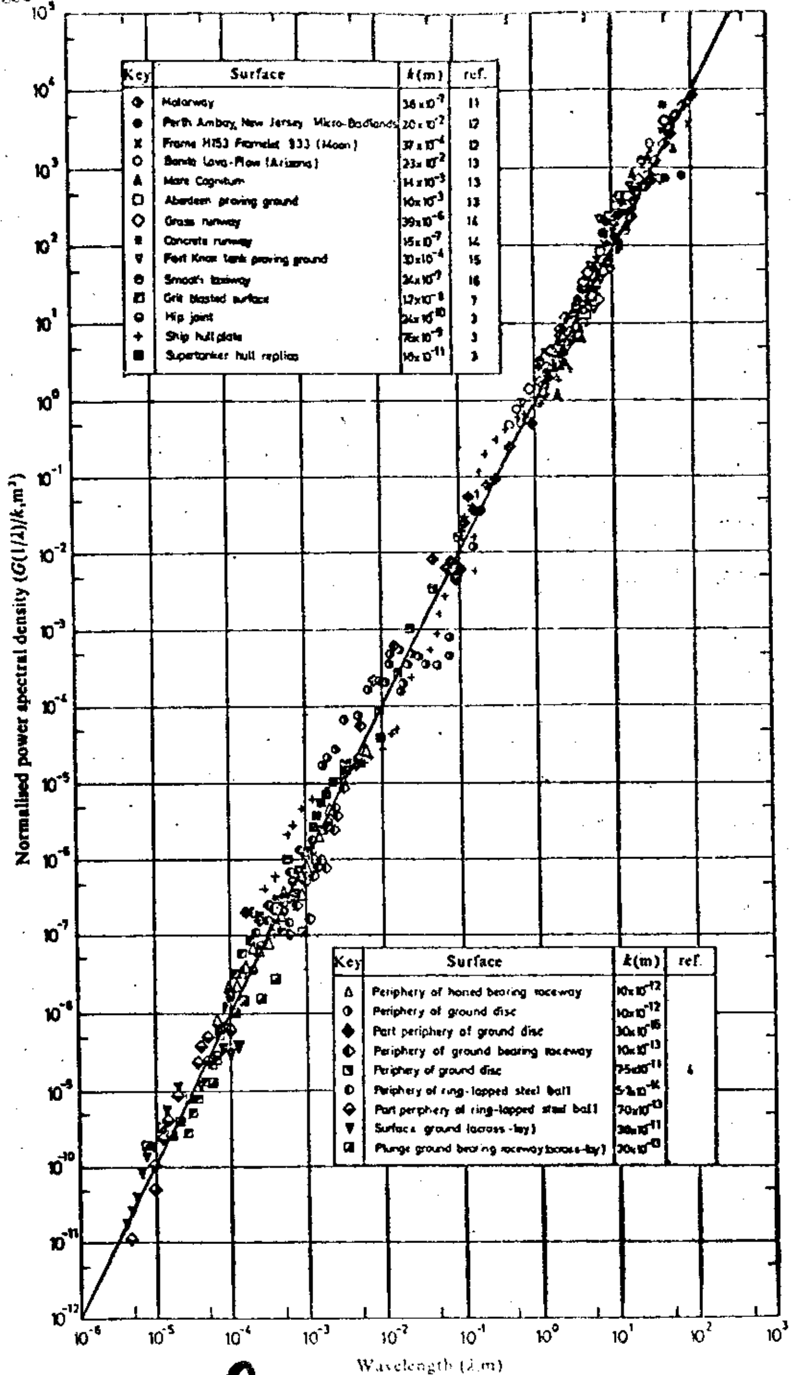


Figure 2. (a, b, c) Seismicity maps for USGS region (1971–1975).

9.04

Sayles + Thomas



a

Fig. 2 Variation of normalised power spectral density ($G(1/\lambda/k$, m^2) with wavelength (λ, m). The graph shows that many different surface topographies existing in the physical universe have a similar form of power spectrum. Note that the spectra available cover almost eight decades of surface wavelength and throughout this range the r.m.s. power increases, to a good approximation, as the square of the wavelength (solid line, equation (2)).

Spatial Distribution

2-point correlation function:

Distribution ~ $\frac{1}{x}$

3-point correlation fn.: Dist. ~ $1/A$ SCALE 2km ↔ 2000km

4-point correlation fn.: Dist. ~ $1/V$

(Kagan & Knopoff, 1980; Kagan, 1982a,b)

Hence faults are not planes

On shorter distance scales → roughness (elevation) studies

Temporal Distributions

2-point correlation fn: Dist. ~ $1/t$ scale 2hrs ↔ 10yrs

(Omori, 1892)

(Kagan & Knopoff, 1980)

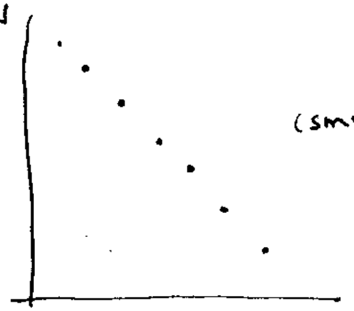
Hence earthquakes not Poisson Independent

(if P. Ind. Dist ~ $e^{-\lambda t}$)

STATISTICAL ANALYSIS (determination of a, b) and variances

[redacted] Goals:
determine a, b
AND VARIANCES !!!

I. $\log N$ cum



(smoothing!)

$\log_{10} N = a - bM$

(ordinary) Least squares?

Assume
1. Normal distrib. of errors
2. statistical independence of each measurement
3. equal weights

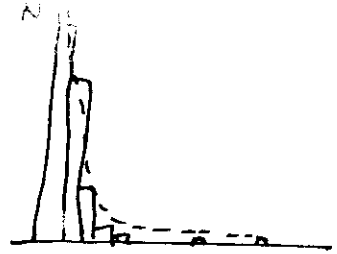
II $\log N$ diff

$$\Delta N = \frac{b\Delta M}{\log_{10} e}\, e^{\left(\frac{a-bM}{\log_{10} e}\right)} ?$$

$$\log_{10} \Delta N = \log(b\Delta M) + a - bM$$

Least squares? Assumpt. 1 + 2. But 3?

III



Diff

Theory of earthquakes?

IV. Maximum likelihood

If $\log_{10} N_{cum} = a - bM$

$$\Delta N_i = N_{cum_i} \frac{b\Delta M}{\log_{10} e}$$

normalizing factor

$$\text{Prob of 1 eq. in } i\text{th interval} = \frac{b\Delta M}{\log_{10} e}\, e^{\frac{a-bM_i}{\log_{10} e}} \left\{\frac{1}{N.F.}\right\}$$

Prob of observing n_i eqs. in ith mag. interval

$$P = \prod_{i=1}^{I} \left\{\frac{b\Delta M}{\log_{10} e}\, e^{\frac{a-bM_i}{\log_{10} e}} \frac{1}{N.F.}\right\}^{n_i}$$

$$N.F. = \sum \Delta N_i = \sum \frac{b\Delta M}{\log_{10} e}\, e^{a-bM_i}$$

$$\text{Take } \log_e P = \left[\frac{a}{\log_{10} e} - \log_e N.F. + \log_e b\Delta M - \log_e \log_{10} e\right] \sum n_i - \frac{b \sum M_i n_i}{\log_{10} e}$$

$$\text{Thus Max} \left\{(a + \log b\Delta M) \sum_i^I n_i - b \sum_i^I n_i M_i - \sum_i^I (b\Delta M)\, 10^{a-bM_i}\right\}$$

see Kantorovich, Molchan, Vilkovich & Keilis-Borok - *Computational Seismology*, vol. 5, pp 80-108, 1971

IV Extreme-value statistics

ref. Gumbel: ~~[illegible]~~ Statistics of Extremes
Columbia Univ. Press, 1958

Lomnitz & Epstein Nature, 211, 1966, 954.

(no upper bound)
(Poissonian)

1. Take largest annual eq. in a year of data
2. Rank order
let

$$G(m) = \frac{m - 1/2}{n}$$

m = mth rank-ordered event

$$M - M_0 = \frac{1}{b \ln 10} \left(-\ln\left(-\ln G(m)\right)\right)$$

$$\log_{10} N_{cum} = a - bM$$

M = Max Likelihood
C = Cum L.S.
D = Diff. L.S.
E = Extreme-value

50 years per sub catalog
15 simulations
Poissonian assumption

$a = 4.77$, $b = 0.85$: M, C, E have mean estimate of a, b within 2σ of theoretical value
M has least σ.

$a = 3.77$ $b = 0.85$ M, C, E
M, C about equal

In both cases
E gives largest σ

Max Magnitude and the infrared catastrophe.

If $\log \dot{N}_{cum} = a - bM$

and $\log E = \alpha + \beta M$

$$d\dot{N} = \text{const.}\ E^{-(b/\beta + 1)}\, dE$$

So Energy released per time

$$\int_{E_{min}}^{E_{max}} E\, d\dot{N} = \text{const.}\ E^{1 - b/\beta} \Big|_{E_{min}}^{E_{max}}$$

if $b \sim 1$ so $b/\beta \sim 0.6$
$\beta \sim 1.5$

Thus E_{max} is finite

$(E = \frac{1}{2}\mu\sigma^2)$

If strain $\epsilon \sim E^{1/2}$
then $\log \epsilon = \alpha + \frac{\beta}{2} M$

a similar argument says
strain released per time is

$$\text{const}\ \epsilon^{1 - 2b/\beta} \Big|_{\epsilon_{min}}^{\epsilon_{max}}$$

since $\frac{2b}{\beta} \sim 1.2$

ϵ_{min}, i.e. E_{min} is finite

lower cutoff automatic because of finite rock strength
upper cutoff implies finite upper bound fault length.

TABLE 2. 15 Monte Carlo Realizations of Earthquake Sequences with T_{max} = 12

Realization number	$\bar{M}_N$	σ_M	b	σ_b	$\bar{T}_{N>C.S.}$	Multiplicative Uncertainty σ_T
1	5.578	0.028	0.665	0.021	88	1.161
2	5.586	0.033	0.662	0.024	83	1.186
3	5.578	0.018	0.752	0.018	157	1.130
4	5.472	0.028	1.211	0.040	4648	1.416
5	5.312	0.017	0.932	0.025	937	1.209
6	5.467	0.014	1.186	0.034	3946	1.269
7	5.587	0.020	0.993	0.033	783	1.258
8	5.700	0.030	0.718	0.026	102	1.195
9	5.529	0.029	0.739	0.027	156	1.209
10	5.641	0.032	0.957	0.040	544	1.395
11	5.573	0.021	0.678	0.016	92	1.124
12	5.539	0.042	0.845	0.027	318	1.2[illegible]
13	5.626	0.023	0.788	0.024	174	1.178
14	5.461	0.013	0.779	0.013	232	1.100
15 –	5.358	0.016	0.748	0.015	158	1.114
Regression of means	5.347	0.003	0.813	0.003	250	1.022
Statistics of realizations	5.347	0.002	0.842	0.178	306	3.40
Average of reciprocals of b_s			(0.81)	(0.140)	(240)	
M_{max} = ∞	5.33		0.85		334	

try $\log \dot{N} = a' - bM$

$a' = \log b + \log M_0 + \log_{10} \log_e 10 + a$

& let this apply from M_0 to M_{max}

D = Diff. L.S.
C = Cum. L.S.
E = Extreme
M = Max Likelihood

Mmax	Accept. at 2σ	Best
8 a = 4.77 b = .85	DMCE	M
6 a = 4.77 b = .85	DM	M
8 a = 3.77 b = .85	MCE	M
6 a = 3.77 b = .85	MC	M

GEOPHYSICAL RESEARCH LETTERS, VOL. 7, NO. 5, PAGES 279-282, APRIL 1980

TIME-PREDICTABLE RECURRENCE MODEL FOR LARGE EARTHQUAKES

Kunihiko Shimazaki and Takashi Nakata*

Earthquake Research Institute, University of Tokyo, Tokyo, Japan
*Institute of Geography, Faculty of Science, Tohoku University, Sendai, Japan

Abstract. We present historical and geomorphological evidence of a regularity in earthquake recurrence at three different sites of plate convergence around the Japan arcs. The regularity shows that the larger an earthquake is, the longer is the following quiet period. In other words, the time interval between two successive large earthquakes is approximately proportional to the amount of coseismic displacement of the preceding earthquake and not of the following earthquake. The regularity enables us, in principle, to predict the approximate occurrence time of earthquakes. The data set includes 1) a historical document describing repeated measurements of water depth at Murotsu near the focal region of Nankaido earthquakes, 2) precise levelling and ^{14}C dating of Holocene uplifted terraces in the southern Boso peninsula facing the Sagami trough, and 3) similar geomorphological data on exposed Holocene coral reefs in Kikai Island along the Ryukyu arc.

Introduction

We will present three example sequences of large thrust-fault earthquakes derived from a historical document and geomorphological studies in Japan. The data show a regularity in that the larger the coseismic displacement, the longer is the following quiet period. In other words, the time-interval between two successive large earthquakes is approximately proportional to the amount of seismic displacement of the preceding earthquake, and not of the following earthquake.

Similar recurrence behavior has been found by Bufe et al. (1977) for small earthquakes on the Calaveras fault in California. It might be interesting to note a striking contrast of the types of earthquake treated in the present and Bufe et al. (1977)'s studies, an M=8 earthquake with thrust faulting on the one hand, and on the other an M=3 strike-slip event. The common observed feature of earthquake recurrence probably represents some basic nature of earthquake faulting.

On the assumption of constant accumulation rate of tectonic stress, the initial and final stresses of faulting τ_1 and τ_2 respectively, govern the behavior of the cyclic stress relief associated with a sequence of earthquakes (Fig. 1). If we consider cycles of stress relief on the same fault area for simplicity, the difference between the two stresses, or the stress drop is always proportional to the coseismic slip. If the initial and final stresses are time-independent, then the sequence of earthquakes would represent a strictly periodic process (Fig. 1-a). However, this is contrary to the observations. Thus one or both of the two stresses must vary in time.

If the final stress varies in time while the initial stress remains constant, we obtain the same regularity of recurrence behavior as is presented in this study (Fig. 1-b). This regularity enables us, in principle, to predict the occurrence time of the coming earthquake. Thus, this case is called "time-predictable recurrence" in this study. In fact, Bufe et al. (1977) successfully predicted an M=3 earthquake using a method equivalent to this. Another extreme would be the case with constant final stress and varying initial stress (Fig. 1-c). In this case the longer the quiet period is, the larger the earthquake that follows. This may be termed as "slip-predictable recurrence". If, however, the initial and final stresses vary in time, no regularity can be found.

Data

Nankaido. The cyclic occurrence of great Nankaido earthquakes in southwest Japan can be traced back to the 7th century from historical documents (e.g. Usami, 1975). However, the coseismic uplifts repeatedly measured at approximately the same site are available only for the last three events, the 1707, 1854, and 1946 earthquakes. The Nankaido earthquakes are considered to be interplate earthquakes along the Nankai trough (Ando, 1975). The coseismic deformation is characterized by the acute landward tilting of the Muroto peninsula, upheaval at its tip and subsidence near its base, while the Muroto peninsula gradually tilts towards the ocean during the interseismic stage (Sawamura, 1953).

The estimated coseismic upheavals at a small port called Murotsu near the tip of the Muroto peninsula (Fig. 2) are listed in Table 1. A brief description of each estimate is given in the following. The vertical coseismic displacements of the Nankaido earthquake of December 21, 1946 (M=8.1) were determined by Sawamura (1953) along the coastal areas. He obtained a coseismic uplift

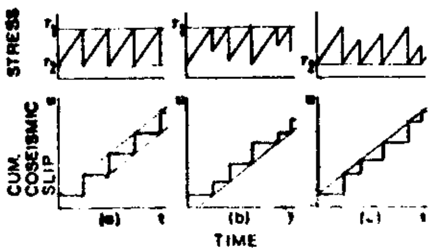


Fig. 1. Schematic recurrence models: (a) strictly periodic, (b) time-predictable and (c) slip-predictable.

-18-

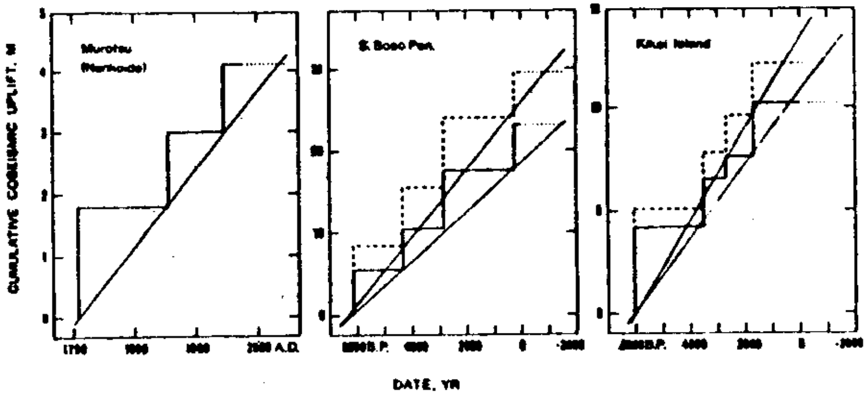
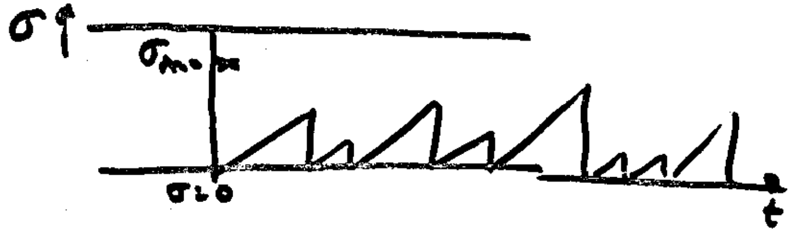
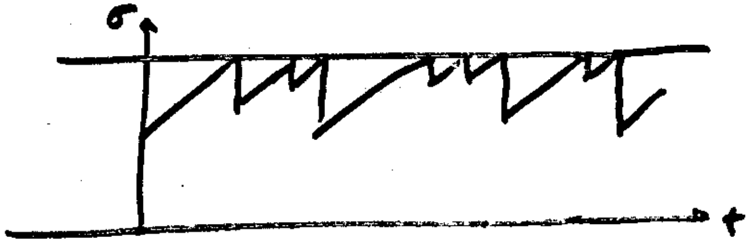


Fig. 4. Cumulative coseismic uplift against date. Some interseismic subsidence is assumed for the broken curves.

(slip)
Stress-prediction



Time Prediction

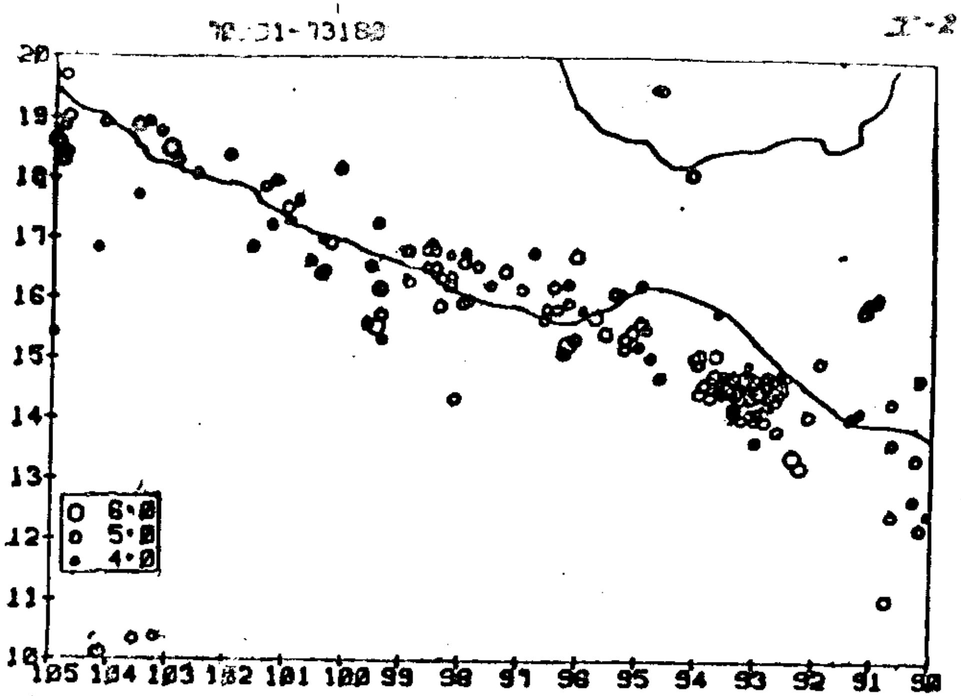


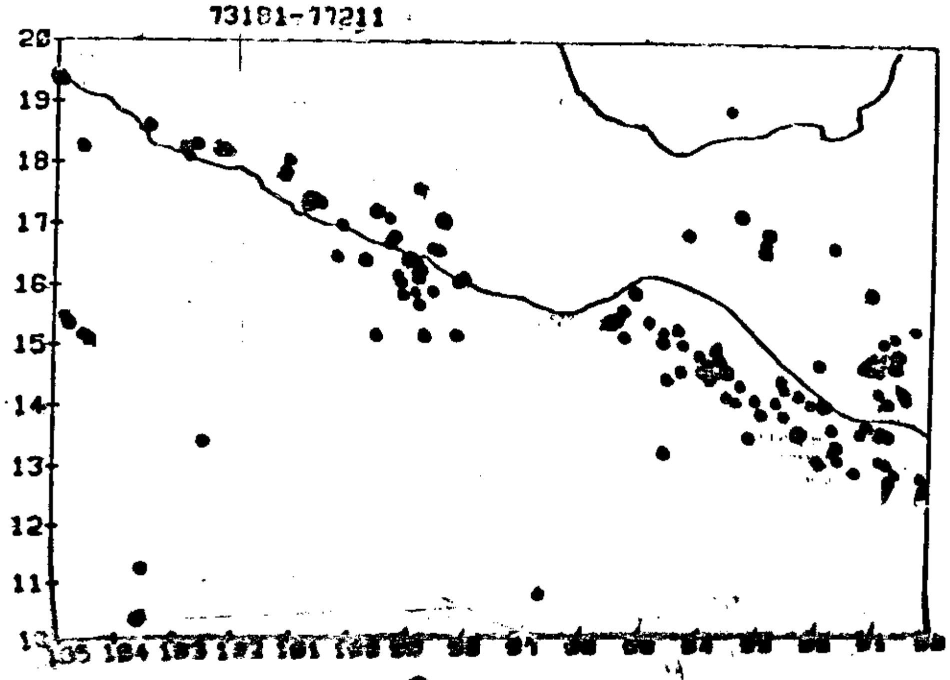
if $\log_{10} \dot{N} = 4.77 - .85 M$

Pred. return time Poisson Process M>8
= 110y.

For Poisson process mean = std. dev.

If $M_{max} = \infty$
$M_0 = 4$
$M_{cutoff} = 8$ $\sqrt{\text{variance}} = 75$ yrs.





References

Statistical Analysis

1. Magnitudes, Magnitude-frequency law

   [1] C.F. Richter, Elementary Seismology, Freeman, 1958 San Francisco

2. Magnitude-energy, Magnitude-fault length formulas

   C.Y. King & L. Knopoff, Bull. Seismol. Soc. Amer. 59 (1969) p.269
   & references therein.

3. Seismic Moment, Magnitude-moment relations

   K. Aki, Geophys. J. Roy. Astron. Soc. 31 (1972) p. 3.

4. Self-similarity in surface topography

   Sayles & Thomas, Nature, 271 (1978) 435

5. Spatial and temporal self-similarity of earthquake distributions

   Y.Y. Kagan & L. Knopoff, Geophys. J. Roy. Astron. Soc. 55 (1978) 67.
   Y.Y. Kagan & L. Knopoff, " " " " " 62 (1980) 303
   Y.Y. Kagan " " " " " 67 (1981) 719, 71 (1982) 659
   F. Omori, J. Coll. of Science, Imper. Univ. of Japan 7 (1894) p. 111.

6. Maximum likelihood method

   Kantorovich, Molchan, Keilis-Borok, Computational Seismology, 5, (1971) p. 80.

7. Theory of Extremes

   Gumbel, Statistics of Extremes, Columbia Univ. Press, 1958
   B. Epstein & C. Lomnitz, Nature, 213 (1966) 954
   L. Knopoff and Y.Y. Kagan, J. Geophys. Research, 82 (1977) 5647

8. Time Predictable model

   K. Shimazaki & T. Nakata, Geophys. Res. Letts. 7 (1980) 279.
   K. Mogi in Earthquake Prediction, Simpson & Richards, eds., A.G.U., p 43.

9. Seismic Gaps

   Ohtake, Matumoto, Latham, PAGEOPH, about 1978.