INTERNATIONAL ATOMIC ENERGY AGENCY
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WORKSHOP ON PATTERN RECOGNITION AND ANALYSIS OF SEISMICITY

(5 - 16 December 1983)

INSTABILITY OF LARGE SYSTEMS

G. PUPPI

• Fields where commonalities emerg dynamical behavior of systems can be found.

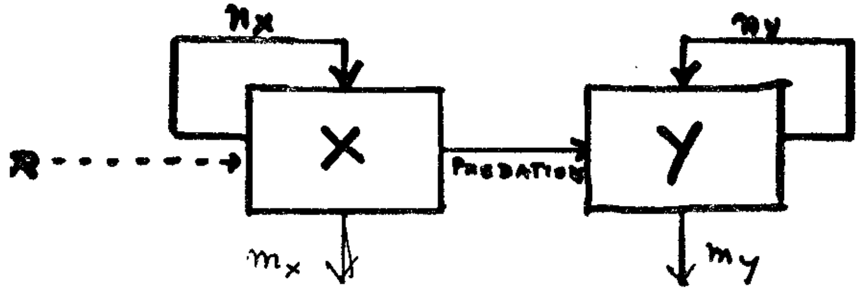
Mechanics
Fluid dynamics
Astrophysics
Chemistry
Electricity
Magnetismus
Optics
Climatology
Ecology
Economy
Sociology
Biology
etc. - - -

<u>Common features</u>:

<u>Systems</u>: Open, Dissipative, Far from Thermodynamical Equilibrium

<u>Interactions</u>: Non linearity, feed-backs

<u>Ecology</u>: System <u>PREY - PREDATOR</u>
(X) — (Y)



• Dynamics regulated by 2 ordinary 1° order differential equations for $\dot{X}$ and $\dot{Y}$

• Discuss only the equation for the Prey considering Y as a parameter. Varying this parameter we will inspect solutions. -

• General Form of Prey equation:

$$\frac{dX}{dt} = G(X) - C(X|Y)$$

G = Term of growth (natural)

C = Term of consumption (predation) depending on level of Predator.

Two assumptions:

$$\begin{cases} G(x) = \varepsilon x\left(1 - \frac{x}{K}\right) \\ C(x|Y) = Y\varphi(x) \end{cases}$$

$\varepsilon = n_x - m_x$ (rate of increase of X)

K = carrying capacity

φ = response to predation

! Resources (R) for the Prey are parameterized in K and ε as well as other causes influencing ε, except predation. —

- In this scheme of evolution of X everything now depend on the choice of $\varphi(X)$. —

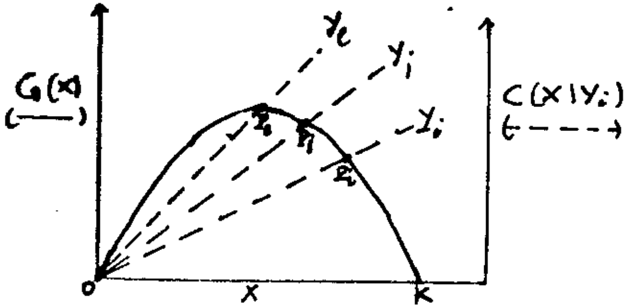
- Once $\varphi(x)$ is chosen the behavior will be inspected graphically.

Case I°

Volterra choice of $\varphi(x) = \gamma x$

so for a particular level Y_i of the Predator

$$C(x|Y_i) = \gamma Y_i X$$



- stability or instability conditions for $\dot{X} = 0$ $G = C$ to be inspected in point of intersection P_i.
For every level of Predation Y_i
$0 \leq Y_i \leq \varepsilon/\gamma$ G and C intersect in $X = 0$ which is an unstable configuration and in a points P_i which are a stable configurations.
The system possess therefore a stable branch

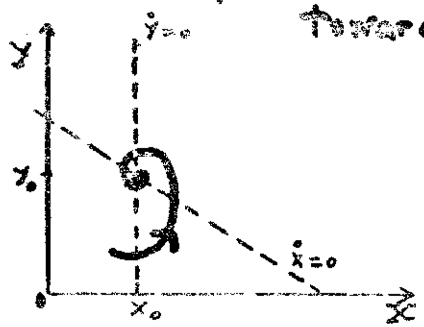
$$\varepsilon\left(1 - \frac{X}{K}\right) - \gamma Y = 0 \quad \begin{cases} x & (0 \to K) \\ y & (\varepsilon/\gamma \to 0) \end{cases}$$

point for different level of Predator is possible only in a controlled experiments

- In natural ecosystems Y is a variable and his evolution governed by a 2° differential equation coupled with that of the Prey.

[In the case of Volterra
$\dot{Y} = -\eta Y + \gamma X Y$]

- There will be only one equilibrium point (X_0, Y_0) solution of set of equations resulting from $\dot{X} = 0$ $\dot{Y} = 0$.
- Basin of attraction all the possible domain of variables and approach to equilibrium via a (damped) spiral in X,Y plane toward $(X_0 Y_0)$

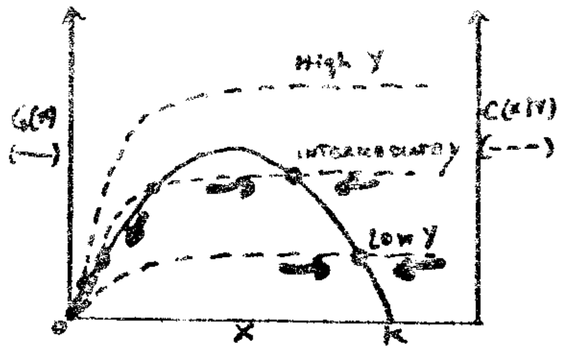


Case II — Choice of $\varphi(X)$ in form of so-called Holling type III
[suited for { Vegetation + Herbivores
Vegetation + Pest
etc. —]

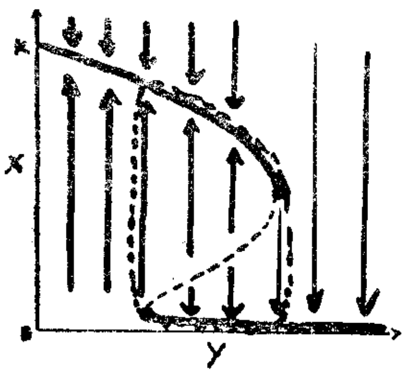
Equation with Predator as parameter

$$\frac{dX}{dt} = \varepsilon X\left(1 - \frac{X}{K}\right) - \gamma Y \frac{X^2}{X_0^2 + X^2}$$

Stationary points for $\dot{X} = 0$, beside the obvious solution $X = 0$ wich represent an unstable point are to be found in the crossing of $G(X)$ and $C(X|Y)$



• stable state
○ instable state



Relaxation oscillation in electrical circuits

Van der Pol Oscillator (see scheme)

Dynamic of the system can be described, with particular choice of variables "voltage" as

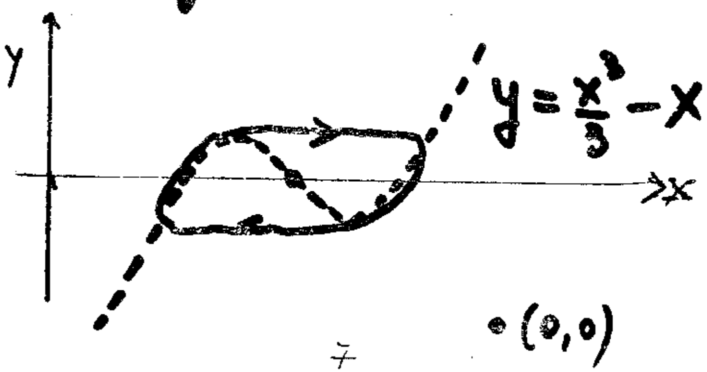
1) $\ddot{x} + \varepsilon(x^2-1)\dot{x} + x = 0$

or the equivalent 1° order autonomous system.

2) $$\begin{cases} \dot{x} = \varepsilon\left[y - \left(\frac{x^3}{3} - x\right)\right] \\ \dot{y} = -\frac{x}{\varepsilon} \end{cases}$$

· Obvious solution of $\dot{x} = \dot{y} = 0$ as $x = y = 0$ unstable.

→ Limit cycle



• (0,0)

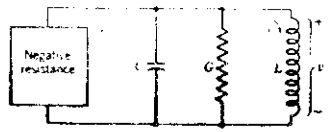


FIG. 4.1. Nonlinear oscillatory circuit.

$$i_1 = F(v),$$

$$F(v) + C(dv/dt) + Gv + i = 0,$$

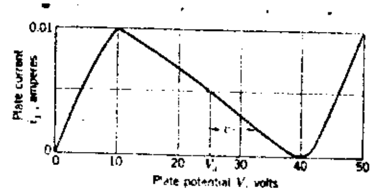


FIG. 4.2. Characteristic of a tetrode as a dynatron (idealized).

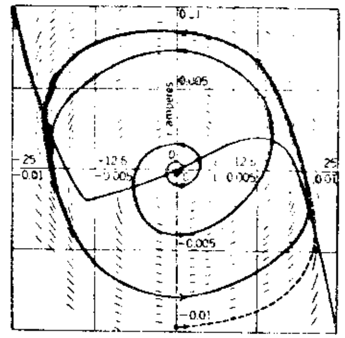


FIG. 4.4. Isocline diagram for harmonic oscillation.

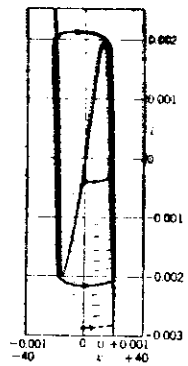


FIG. 4.5. Isocline diagram for relaxation oscillation.

Parametri

Case 4.4		Case 4.5
$2 \cdot 10^{-4}$ mho	G	$2 \cdot 10^{-4}$ mho
$2{,}5 \cdot 10^{-3}$ henry	L	$4 \cdot 10^{-2}$ henry
$4 \cdot 10^{-10}$ farad	C	$2{,}5 \cdot 10^{-11}$ farad
2500.	$k = \sqrt{\frac{L}{C}}$	40'000

The van der Pol oscillator

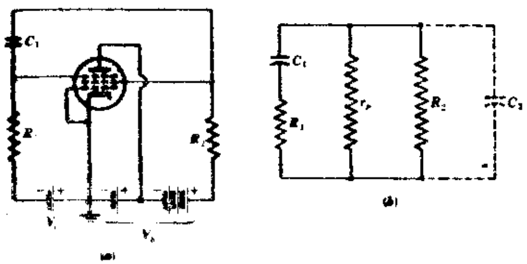


FIG. 12.6. Van der Pol relaxation oscillator: (a) circuit diagram and (b) equivalent circuit.

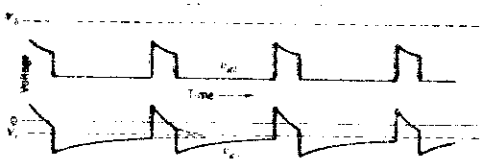


FIG. 12.7. Voltage wave forms in van der Pol oscillator.

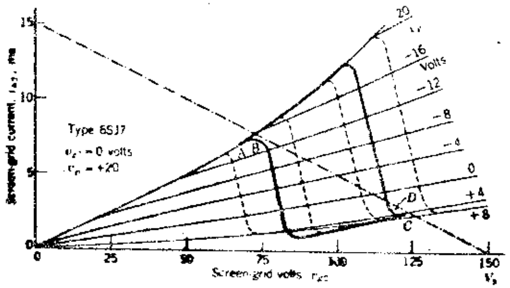


FIG. 12.8. Operation of van der Pol oscillator.

Theorem of Poincaré - Bendixon

- Let consider a system of two ordinary differential equations of autonomous type.

1) $\frac{dx}{dt} = P(x,y) \qquad \frac{dy}{dt} = Q(x,y)$

with $P(x,y)$ and $Q(x,y)$ having continuous first partial derivatives in a region G.

- Let D be a bounded region, such as D and its boundary are contained in G.
- Let D + its boundary do not contain critical point of the system (1)
- Then if $x(t)$ and $y(t)$ is a solution of system (1) that exist and stay in D + its boundary for $t \geq t_0$ (some value t_0)

∴ a) - The solution is periodic, or

b) - The solution spirals toward a periodic solution as $t \to \infty$.

In either case, the system possess a periodic solution.

- Different applications - Inspection of $\vec{V}$

- Other less general, more practical criteria exists for certain classes of equations.
- Example: Liénard equation

$$\frac{d^2x}{dt^2} + f(x)\frac{dx}{dt} + g(x) = 0$$

under certain condition for f and g and their integrals F and G. has a periodic solution

Therefore

- Systems described by equation (4) show a periodic solution in terms of Limit cycle

Other type of stability is a Fixed Point.

- These are the two forms of normal attractors.

Oscillating Chemical Reactions

- Hystorical Background

1828 — Fechner — Oscillation in the current of an electrochemical cell.

1896 — Liesegand — Periodic structures in Precipitation

1899 — Ostwald — Oscillation in the rate of dissolution of chromium in acid.

1920 — Bray — Oscillation in colour in a mixture of H_2O_2, KIO_3, H_2SO_4 diluted. (Variation in concentration of free I_2)

Comment — Little attention and even incredulity. Belief in monotonous approach to equilibrium in homogeneous systems.

1910 and 1920 — Lotka demonstrated with very simple models of a set of reactions including autocatalytic and catalytic ones that oscillation of damped and even permanent character are possible.

— Next 40 years little interest and no development.

1958 — Belousov and afterwards Zhabotinsky change of colour periodically in a mixture of: $Ce(NH_4)_2(NO_3)_5$, $KBrO_3$, $CH_2(COOH)_2$, H_2SO_4, Ferroin. [And variants]

• Last 20 years great interest both experimental and theoretical.

• There are now families of groups of chemical reactions which show oscillations [and much more complicated things.]

• What are general conditions for appearance of persistent oscillations of some character of a mixture of chemically interacting substances?

i) - Open system far from equilibrium TD.

ii) - Presence of autocatalytic reactions

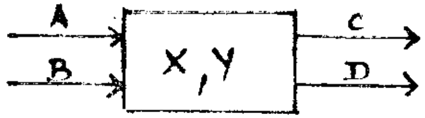
iii) - Existence of more than one stable branch of existence (at least bistability)

To realize point i) for an homogeneous condition best way is to operate in a continuous-flow, stirred-tank, reactor

• In general a Belousov type of mixture reacting implies a great number of simultaneous reactions, too large to be treated analytically. Therefore simplified models has been developed. The simplest one, involving 2 catalytic reactions, is the following, studied in particular by the Bruxelles Group:

System

$A \rightarrow X$	k_1
$2X + Y \rightarrow 3X$	k_2
$B + X \rightarrow Y + D$	k_3
$X \rightarrow E$	k_4



A B, are steady quantities and in particular used as parameters. The kinetics for the species X, Y are

$$
1) \quad \begin{cases} \dfrac{dX}{dt} = k_1 A + k_2 X^2 Y - k_3 B X - k_4 X \\[2ex] \dfrac{dY}{dt} = - k_2 X^2 Y + k_3 B X \end{cases}
$$

• Again a system of 2 autonomous diff equat. Qualitative inspection of 1) under the simplifying assumption of $k_1 = k_2 = k_3 = k_4 = 1$ gives the following results:

1 - Critical relation between B and A as $B_c = A^2 + 1$ which separates two different dynamical domains

2 - For $B < B_c$ the system posses an attractor for $Y_0/X_0 = B$

3 - For $B > B_c$ the system posses a limit cycle

CH-4

Important: Existence of different behaviour in different domain of parameters!

This is not the whole story!
In complex chemically reacting systems in continuous flow-stirred tank reactor, as a function of flow rates of chemicals other periodic states appear in terms of period doubling sequence and of chaotic regime.

To obtain this feature models need to be expanded to at least 3 variables. (Still far from nature compl.) (Reversible Oregonator)

$$A + Y \rightleftarrows X + P \qquad \text{with} \quad X = [HBrO_2]$$
$$X + Y \rightleftarrows 2P \qquad Y = [Br^-]$$
$$A + X \rightleftarrows 2X + Z \qquad Z = 2[Ce^{4+}]$$
$$2X \rightleftarrows A + P \qquad A = [BrO_3^-]$$
$$Z \rightleftarrows fY \qquad P = [HOBr]$$

Results of numerical integration of corresponding system of differential equation: Alternation of periodic and chaotic regimes.

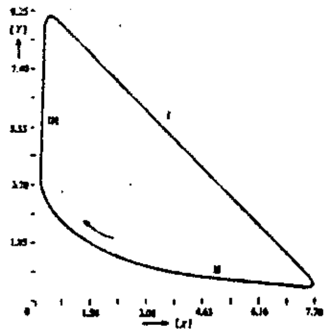


Fig. 2 Limit cycle of the chemical reaction scheme in Fig. 1. The conditions are the same as for Fig. 1. The scaling of concentration of *X* and *Y* is dimensionless. The motion on the limit cycle is discussed in detail by Lavenda *et al.*[16] for a scheme comprising the reactions R_4 through R_8. In part I the autocatalytic step R_{XY} determines the transformation of *Y* into *X*. In part II *X* disappears by the steps R_4 and R_8 while *Y* remains small. In part III, *X* remains small while *Y* is accumulated by the step R_8. The additional step R_Z contributes to the movement only at high concentrations of *Y*, at the end of part III. Here most of the *Y* produced by the step R_8 is consumed by R_Z resulting in a very slow approach to the critical concentration, which activates the autocatalytic step of part I. This analysis clearly demonstrates that in different portions of the limit cycle different chemical processes determine the motion of the system.

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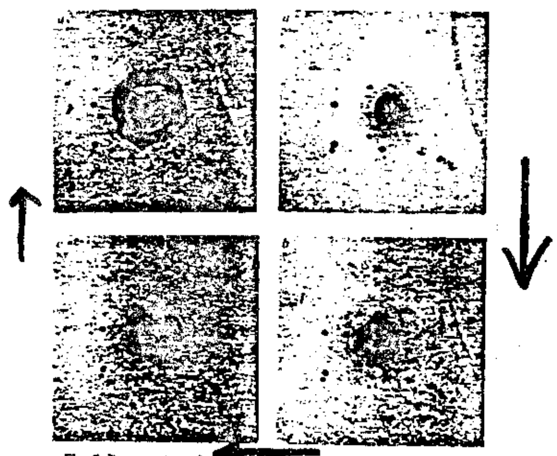
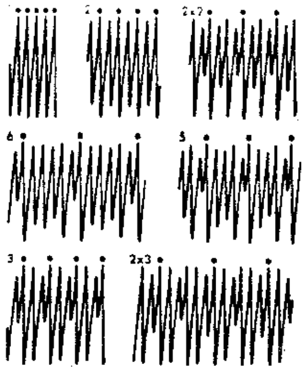


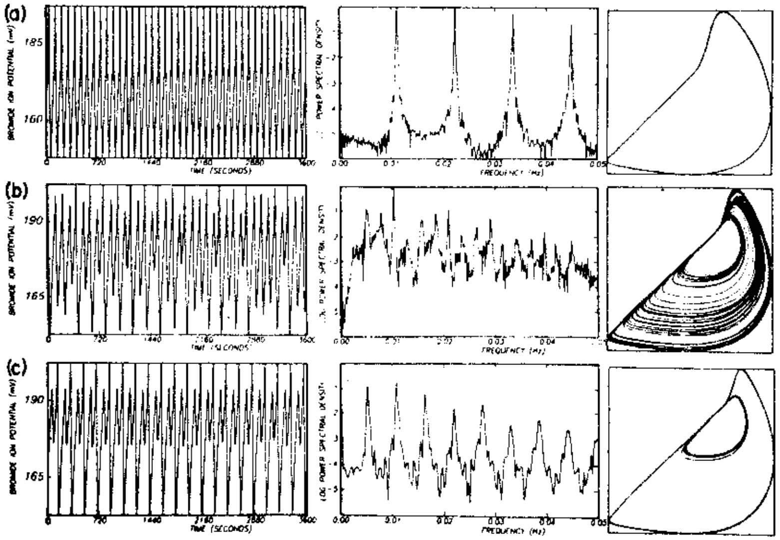
Fig. 5 Propagation of a signal in an oscillatory chemical reaction system. The initial concentrations of the solutions are

$[KBrO_3] = 0.35$ M; $[H_2C(COOH)_2] = 1.2$ M;
$[Ce(SO_4)_2] = 3.9 \times 10^{-3}$ M; $[\text{Ferroin}] = 0.48 - 10^{-3}$ M in 3 N H_2SO_4

About 15 min after initiation of the reaction by addition of the catalyst the ultraviolet pulse is given and shortly afterwards pictures are taken with an interval of 0.5 s. The sequence of the pictures shown is clockwise from (*a*) to (*d*). The white ring represents the blue outward moving wave, the spot in the centre was irradiated. Further explanations see text. In *d*, the following wave is already forming. Other conditions do not show both wave rings simultaneously, but the irradiated spot always turns blue ahead of the bulk solution to form a new wave centre.



[redacted]. Observed bromide-ion potential time series with periods τ (115 s), 2τ, $2\times2\tau$, 6τ, 5τ, 3τ, and $2\times3\tau$; the dots above the time series are separated by one period.



[redacted]. Experimental results for three residence times: (a) regime P_1, $\tau = 0.49$ h, (b) regime C_1, $\tau = 0.90$ h, (c) regime P_2, $\tau = 1.03$ h. For each τ the graphs show the time dependence of the bromide ion potential (proportional to $-\log([Br^-])$) and the corresponding power spectrum and 2-D phase portrait.

If homogeneity condition is relaxed or local perturbations introduced of persistent or transitory character then space-time structure appear indicating self-organization and well defined rules of evolution

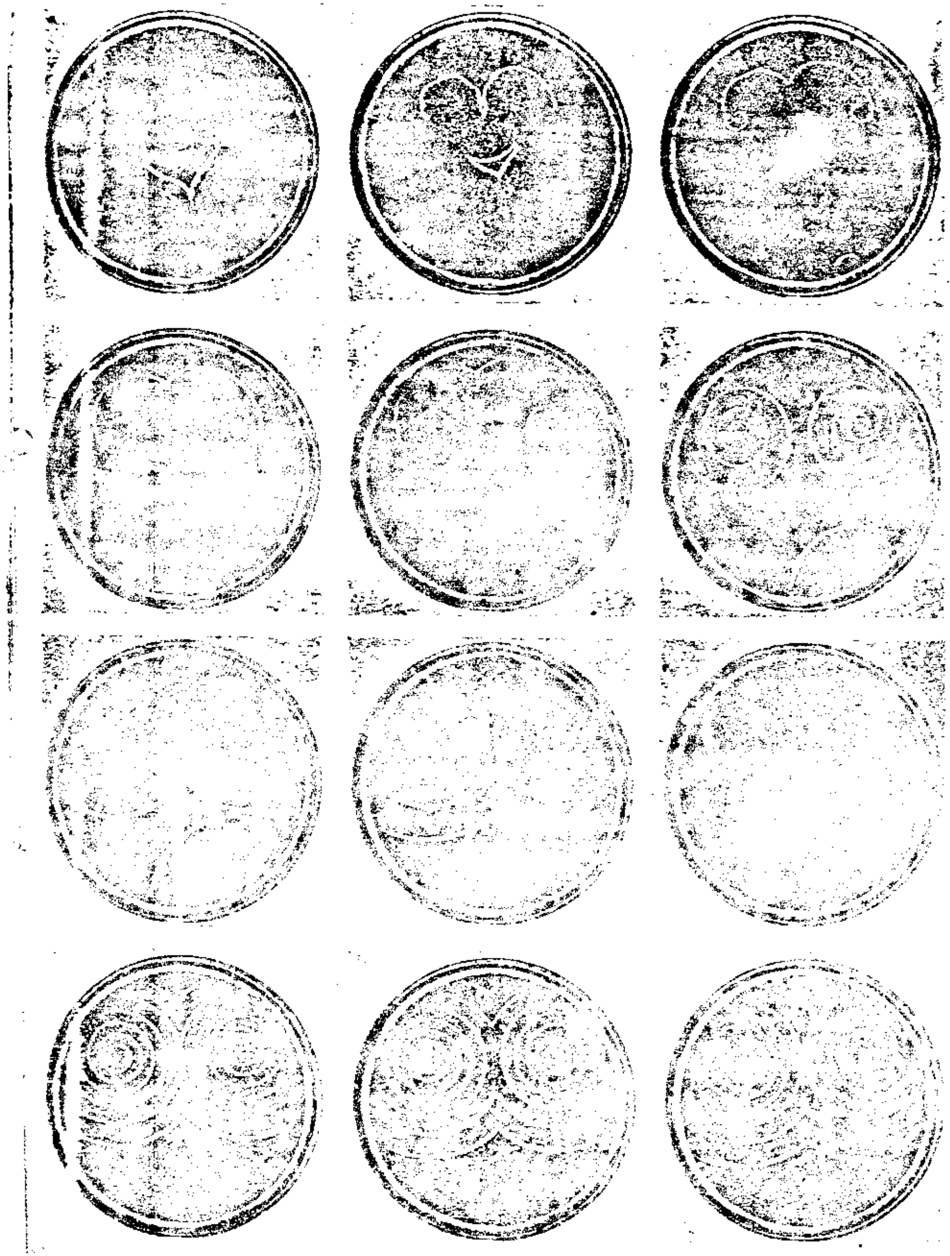
This possibility shown by systems containing autocatalytic reaction being proposed by Turing in 1952 as a way to understand:

Morphogenesis in biology

Breaktrough !

Chaotic behaviour appear in models containing at least 3 differential equations of 1° order

- First found by Lorentz (1962) on the basis of a fluid dynamic example → Used to argue about predictability in weather forecast.



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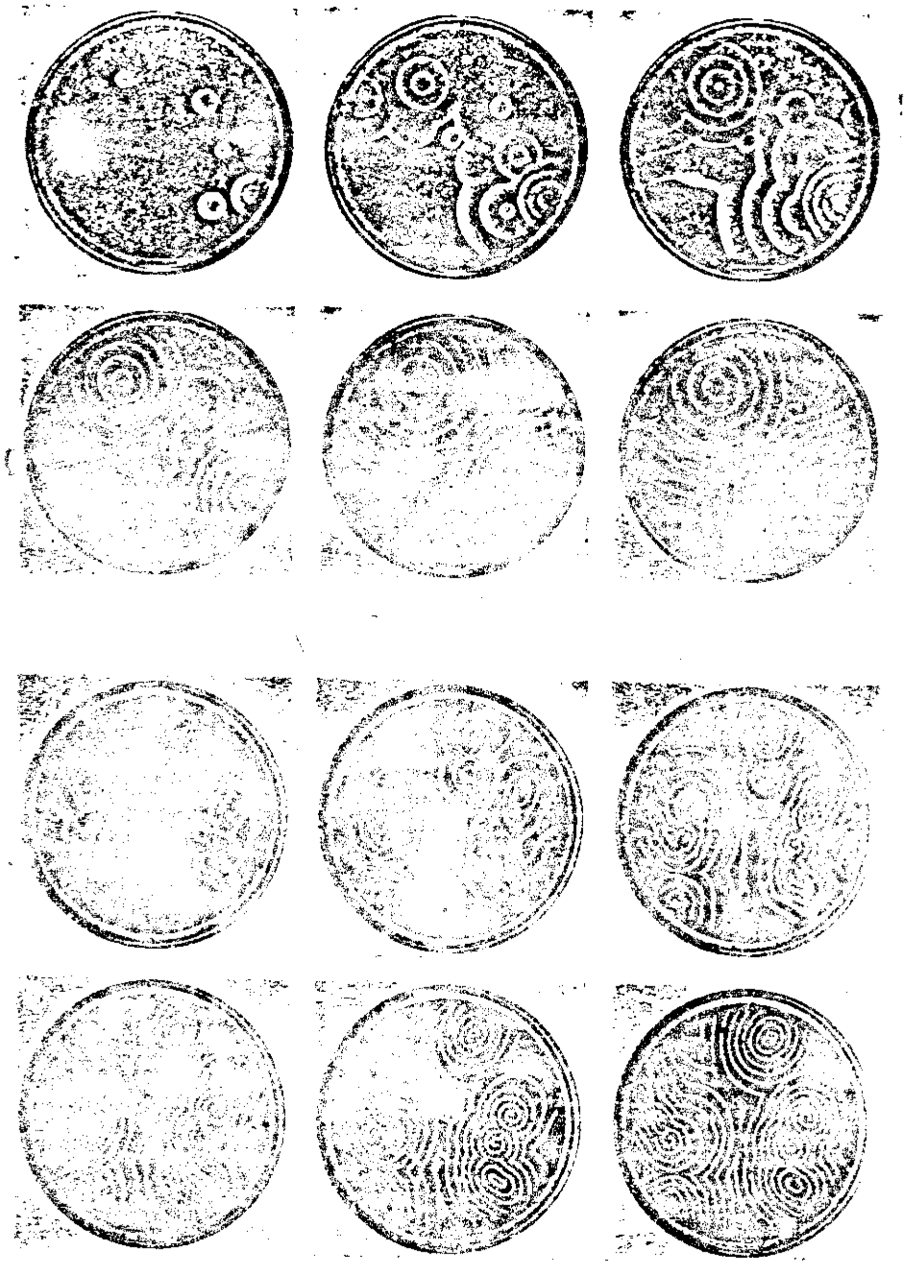

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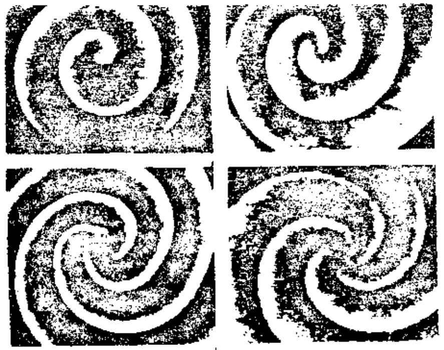


Fig. 1 One-, two-, three- and four-armed vortices in an active excitable chemical medium. A Petri dish 90 mm in diameter was filled with 4.5 ml of reagent similar to that proposed by Winfree: 0.3 M $NaBrO_3$, 0.4 1 M H_2SO_4; 0.1 M $CH_2(COOH)_2$ 0.85 M $CHBr(COOH)_2$; 0.045 M ferroin.

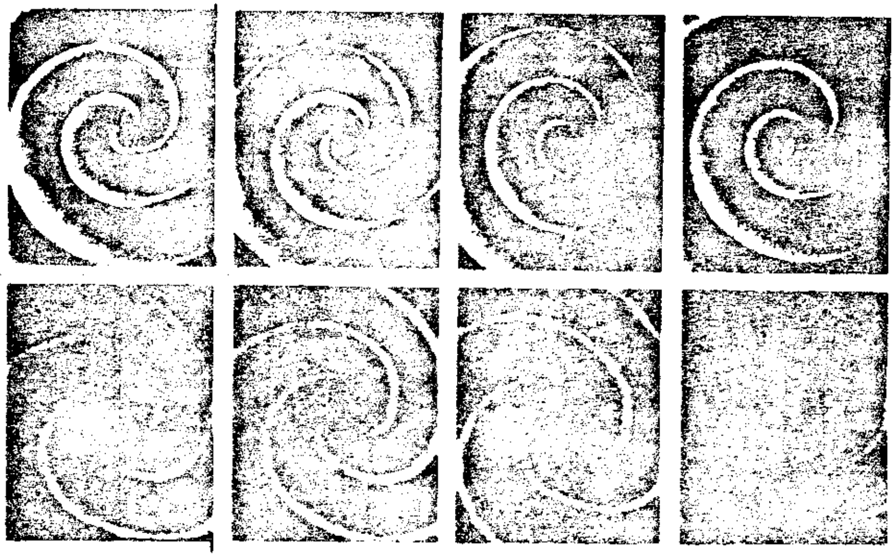
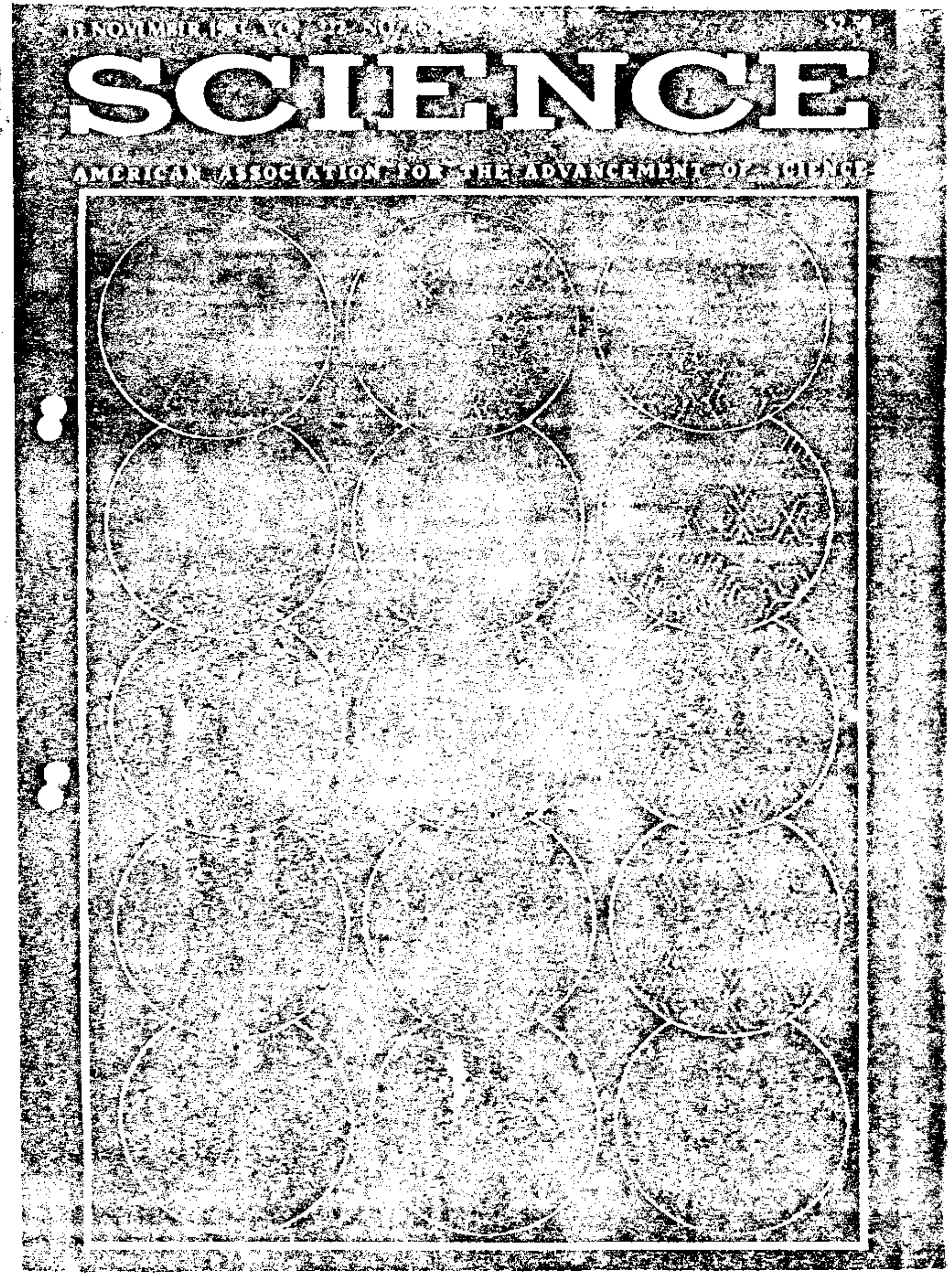


Fig. 2 Wave movement in the core of two- and three-armed vortices. Time interval is 15 s. Other conditions as in Fig. 1

Fig. 3 Tw[illegible] closely spaced [illegible] formed from [illegible] two-armed vortex. The photograph was taken 38 min after mixing the reagents

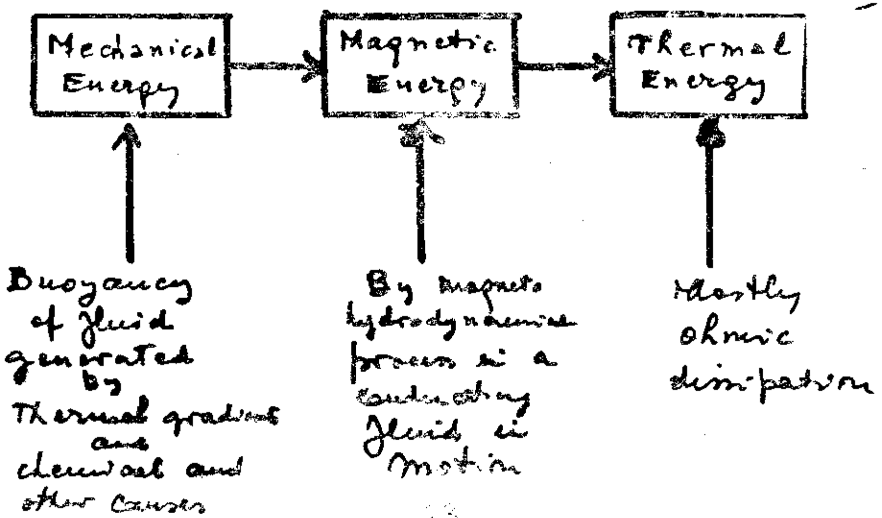


EARTH MAGNETIC FIELD

Target of investigation is understanding of change in **Polarity** and **Intensity** of earth's magnetic field, apparently random in behaviour on time scales $\simeq 10^5 \div 10^6$ y.

Hypothesis - Properties of m.f. determined by magneto-hydrodynamical behaviour of an electrically conducting fluid in relative motion with respect to the rotating earth. —

Energetics:



Model

A simple dynamo's model (Rikitake) regulated by 3 differential equations of autonomous type.
(2 coupled dynamos.)

Symbols:

G = Torque on rotation's axes (cause)
Ω_1, Ω_2 = angular velocities
$I_1\ I_2$ = Currents
{effects}

—

$L_1 = L_2 = L$ self inductances
$R_1 = R_2 = R$ resistances
M mutual inductance
$C_1 = C_2 = C$ moments of inertia

—

4 variables $\Omega_1, \Omega_2, I_1, I_2$ but from

$$\begin{cases} C \dfrac{d\Omega_1}{dt} = G - M I_1 I_2 \\ C \dfrac{d\Omega_2}{dt} = G - M I_1 I_2 \end{cases}$$

$\therefore \Omega_1 - \Omega_2 = A =$ constant

System described by 3 equations.

$$
\begin{cases}
L \dfrac{dI_1}{dt} + R I_1 = M \Omega_1 I_2 \\[2ex]
L \dfrac{dI_2}{dt} + R I_2 = M(\Omega_1 - A) I_1 \\[2ex]
\qquad C \dfrac{d\Omega_1}{dt} = G - M I_1 I_2
\end{cases}
$$

~~[illegible]~~ unstables.

$$\left[I_1 I_2 = \frac{G}{M} \quad \Omega_1 = \frac{R}{G} I_1^2 \quad \Omega_1 = A + \frac{R}{G} I_2^2 \right]$$

- With $X_1 = \sqrt{\frac{M}{G}}\, I_1 \quad X_2 = \sqrt{\frac{M}{G}}\, I_2$

$$\Omega_{1,2} = \sqrt{\frac{GL}{CM}}\, Y_{1,2}$$

governing equation are

$$
\begin{cases}
\dot{X}_1 + \mu X_1 = Y X_2 \\
\dot{X}_2 + \mu X_2 = (Y - A) X_1 \\
\quad \dot{Y} \quad = 1 - X_1 X_2
\end{cases}
$$

Two fixed points with signs of currents reversed. (N and R) unstables – orbits around F.P.

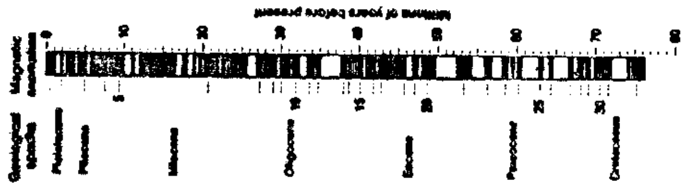


FIG. 8-60 The geomagnetic time scale. This scale is determined by measuring the age of known reversals on land and extrapolating to the reversals detected in the ocean rocks (see Figure 8-58). The numbers on the left correlate with the magnetic anomaly numbers shown on Figure 8-58. The black areas indicate times when the poles were in their normal position, the white areas, when the poles were reversed. (From Heirtzler and others, Journal of Geophysical Research, Fig. 3, 73, 1968, pp. 2119–2136.)

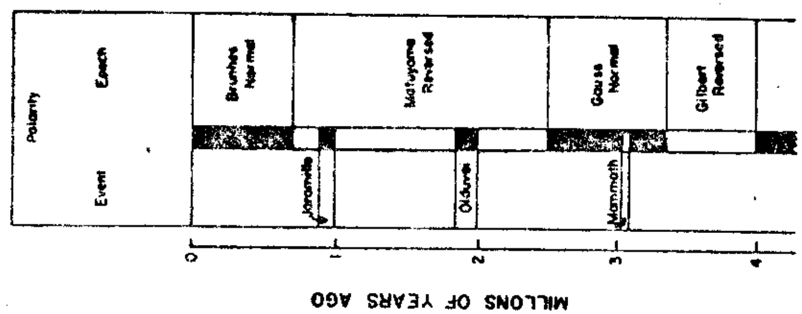


Figure 36. Magnetic history of the earth. Epochs and events of normal polarity are shown as black bars. Epochs and events of reversed polarity are shown as white bars.