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WORKSHOP ON NUCLEAR MODEL COMPUTER CODES

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DESCRIPTION OF THE DWBA CODE DWUCK4.

P.D. KUNZ

Department of Physics
University of Colorado
Boulder, Colorado 80302
U.S.A.

Preface

This manual describes the new export version of the DWBA code DWUCK4. This program has undergone many changes and additions since the previous version 09/AUG/1969 was released. The main features of the program over the older version are 1) coherent summing of amplitudes for different angular momentums, 2) and spin(s) transfers, 3) change in the angular momentum algebra to agree more closely with that of Satchler (Nuclear Physics 55 (1964) 1), 3) first order finite range correction for multi-nucleon transfer, 4) the elastic and inelastic cross sections may be plotted, 5) the program may be divided into two roughly equal sections for overlay purposes, 6) an automatic spin shut-off feature on the distorted waves when no spin orbit potential is given so that only the needed radial wave functions and radial matrix elements are computed, 7) a slightly smaller physical size of the compiled program even though the number of FORTRAN cards has increased considerably, 8) correction of overflow difficulties in the angular momentum subroutine for large angular momentum transfers, 9) conversion cards for use on an IBM 360/370 series computer.

The input data for DWUCK4 is compatible with the previous version except for three instances. The first is that the Q-value convention for the second distorted wave is now the standard option, the second is that the two-nucleon form factor input has been revised, and the last change is that the order for the form factor cards and distorted wave is reversed, the distorted wave cards are now card sets 5 and 6 and the form factor card sets then follow behind as card set 7.

The conversion to other computers other than the CDC 6600 and 7600 series computers should be straightforward except for the double precision cards needed by the IBM 360-370 series machines. The present program contains a subroutine

CUDATE which converts the CDC display code for the TIME and DATE into integer form so that other users may construct a similar decoding routine for their installation. If this is too much trouble you may just replace the subroutine CUDATE with one which sets the TIME and DATE to zeros. One further note is that the program contains an internal storage of 2048 words for a buffer in order to reduce the number of requests to access the mass storage unit (TAPE4) for storage of the distorted wave radial wave functions. This is necessary at the University of Colorado facility because the charging algorithm penalizes heavily for access requests to magnetic disk or tape. This feature is easily removable for a 10% reduction in storage requirements for the program.

The program DWUCK4 is at this time thought to be free of the more easily found errors although there may be some difficulties arising from cases which surpass the testing limits which I have been able to reach with a finite number of test cases.

If there is difficulty with the program usually one can uncover some incompatibility in the data cards, but in case of unfathomable trouble, I would like to know about it in case our error exists in the program. The best way to inform me of your troubles is to send the output and data cards from the run that is causing the trouble along with a short note of what caused you to suspect an erroneous output. If I cannot find the trouble from the output then I can run your test case with my version to see if I can reproduce the symptoms. If this fails, then I will request further information from you. The main value of the program, I feel, is that the user should be able to solve any difficulties that he may have with it.

I wish to thank Professor Ernest Rost for his interest and helpful comments on the program during the seven years of evolution to the present version and I

also am grateful to Professor J. R. Comfort for his suggestions of some of the additions and improvements to the program.

Sincerely yours,

P. D. Kunz

Table of Contents

A. General Description of DWUCK

The computer program DWUCK calculates the scattering differential cross section for a general form of the distorted wave Born approximation (DWBA). The incoming and outgoing waves may be in any combination of spin 0, spin 1/2, or spin 1 particles. The spin dependent parts of the optical potentials for the distorted waves must be diagonal in the total and orbital angular momentum. The calculations are performed in a zero range form between the coordinates of the incoming and outgoing waves. This specialization is not important for the usual inelastic scattering theories. The particle transfer reactions can be treated with this program with a local energy approximation for the finite range effects. However, reactions such as heavy particle stripping and knock-on reactions cannot be calculated with the program since the finite range effects are important and are not included in the present computer code.

The DWBA computer code DWUCK calculates a transition amplitude for the reaction A(a,b)B of the form

$$T = J \int d\underset{\sim}{r}_b \int d\underset{\sim}{r}_a \; \chi_f^-(\underset{\sim}{k}_f, \underset{\sim}{r}_b) \langle bB | V | aA \rangle \chi_i^+(\underset{\sim}{k}_i, \underset{\sim}{r}_a) \quad (1)$$

where χ_f^- and χ_i^+ are the distorted waves, $\underset{\sim}{r}_a$ and $\underset{\sim}{r}_b$ are the relative coordinates for the systems (a,A) and (b,B) respectively and J is the Jacobian of the transformation to these coordinates. The quantity $\langle bB|V|aA\rangle$ is the form factor for the reaction and must contain a delta function for the coordinates $\underset{\sim}{r}_a$ and $\underset{\sim}{r}_b$. In the absence of spin the distorted wave χ_i^+ describes asymptotically a plane wave of momentum k plus an outgoing scattered wave which in the case of no Coulomb potential has the form

$$\chi_i^+(\underset{\sim}{k}, \underset{\sim}{r}) \xrightarrow[r \to \infty]{} e^{i \underset{\sim}{k} \cdot \underset{\sim}{r}} + f(\theta) \frac{e^{ikr}}{r} . \quad (2)$$

The final state distorted wave which has an incoming scattered wave condition is related to the solution with outgoing waves by time reversal

$$\chi^{-*}(\underset{\sim}{k}, \underset{\sim}{r}) = \chi^+(-\underset{\sim}{k}, \underset{\sim}{r}). \quad (3)$$

When the particles a and b have spin, the functions $\chi^{\pm}$ become matrices in spin space $\chi^{\pm}_{m'm}$. The relevant time reversal condition is

$$\chi^{-}_{m'm}{}^{*}(\underset{\sim}{k}, \underset{\sim}{r}) = (-)^{m-m'} \chi^{+}_{-m'm}(-\underset{\sim}{k}, \underset{\sim}{r}) , \quad (4)$$

The factor $\langle bB|V|aA\rangle$ which contains the nuclear structure information is expressed as

$$\langle J_B M_B s_b m_b | V | J_A M_A s_a m_a \rangle = \sum_{\ell s j} B_{\ell s j} \langle J_A j M_A M_B - M_A | J_B M_B \rangle$$
$$\times \langle s_b s m_b \; m_a - m_b | s_a m_a \rangle \langle \ell s m \; m_a - m_b | j \; M_B - M_A \rangle$$
$$\times f_{\ell s j}(r_a) \; \delta\left(r_b - \frac{A}{B} r_a\right) i^{-\ell} Y_\ell^m(\underset{\sim}{r}_a)^* . \quad (5)$$

The quantity $B_{\ell s j}$ is a measure of the strength of the interaction and is equal to $\sqrt{\frac{2J_A+1}{2J_a+1}} A_{\ell s j}$ where $A_{\ell s j}$ is the spectroscopic coefficient used in JULIE.

The distorted waves $\chi_{m'm}(k,r)$ can be written as

$$\chi_{m'm}(k,r) = \frac{\sqrt{4\pi}}{kv} \sum_{JL} i^L \langle L\,\Delta\,M\,m \mid JM' \rangle \sqrt{2L+1}\ \chi_{JL}(k,r)$$
$$\times \langle L\,\Delta\,M'-m'\ m' \mid JM' \rangle\ Y_L^{M'-m'}(\hat{r})\ d^L_{0\,M'-m'}. \qquad (6)$$

The d^L_{om} are the rotation functions[1] for integer spin. The radial part of the distorted waves satisfies the equation.

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{L(L+1)}{r^2} - \frac{2\mu}{\hbar^2}\left(U + U_c + U_L^J\right) \right] \chi_{JL}(k,r) = 0, \qquad (7)$$

where the boundary condition at the origin is $\chi_{JL}(k,0) = 0$ and for large r (where $U + U_L^J$ may be neglected) the $\chi_{JL}(k,r)$ also satisfies the boundary condition

$$\chi_{JL}(k,r) \xrightarrow[r\to\infty]{} \frac{\left[H_L^*(kr) - \eta_L^J H_L(kr) \right] e^{i\sigma_L}}{2i}$$

Here $H_L(kr) = G_L + iF_i$ is the outgoing wave Coulomb function, η_L^J is the reflection coefficient and σ_L is the Coulomb phase shift. The quantity $\frac{\eta_L^J - 1}{2i}$ is the usual elastic partial wave scattering amplitude. The functions χ_{JL} are computed by numerically integrating the differential equations to a value of r where either the potentials $U + U_L^J$ are negligible or where the form factor $f_{\ell sj}(r)$ makes a negligible contribution to the reaction. The χ_{JL} are matched to the Coulomb functions G_L and F_L at the last two integration points to determine η_L^J and the overall normalization constants.

1) A. R. Edmonds, Angular Momentum in Quantum Mechanics, Princeton University Press, Princeton, N.J. (1957), p. 55.

With the definitions of $\langle bB|V|aA \rangle$ and the distorted waves $\chi_{m'm}(kr)$ we can write down the transition amplitude as

$$T = \frac{\sqrt{4\pi}}{k_a k_b} \sum_{\ell s j} \sqrt{2\ell+1}\ B_{\ell s j} \langle J_A\, j\, M_A\ M_B - M_A \mid J_B M_B \rangle$$
$$\times \sum_{J_A L_A J_B L_B} \langle L_a\, s_a\, 0\ m_a \mid J_a m_a \rangle \langle L_b\, s_b\ m_a - m - m_b\ m_b \mid J_b\ m_a - m \rangle$$
$$\times \langle J_b\, j\ m_a - m\ m \mid J_a m_a \rangle\ (2L_b+1) \langle L_b\, \ell\, 0\, 0 \mid L_a\, 0 \rangle$$
$$\times \sqrt{(2s_a+1)(2j+1)(2J_b+1)(2L_a+1)} \begin{Bmatrix} L_b & s_b & J_b \\ \ell & s & j \\ L_a & s_a & J_a \end{Bmatrix}$$
$$\times d^{L_b}_{0\ m_a - m - m_b}\ \frac{C\beta}{A^2}\ I^{\ell s j}_{J_a \ell_a J_b \ell_b}\ i^{L_a - L_b - \ell}$$

In the above expression the usual nine -j symbol[2], $\{\ \}$, appears.

The radial integrals are defined as

$$I^{\ell s j}_{J_a \ell_a J_b \ell_b} = \int dr_c\ \chi_{J_b L_b}\left(k_b, \frac{A}{B} r_a\right) f_{\ell s j}(r_c)\ \chi_{J_a L_a}(k_a, r_a)$$

If we define the beta coefficients as

$$\beta^{m\ m_a\ m_b}_{\ell s j;\, L_b} = \sum_{J_a L_a J_b} i^{L_a - L_b - \ell} \langle L_a\, s_a\, 0\ m_a \mid J_a m_a \rangle$$
$$\times \langle L_b\, s_b\ m_a - m - m_b\ m_b \mid J_b\ m_a - m \rangle \langle J_b\, j\ m_a - m\ m \mid J_a\ m_a \rangle$$
$$\times (2L_b+1) \langle L_b\, \ell\, 0\, 0 \mid L_a\, 0 \rangle \sqrt{(2s_a+1)(2j+1)(2J_b+1)(2L_a+1)}$$
$$\times \begin{Bmatrix} L_b & s_b & J_b \\ \ell & s & j \\ L_a & s_a & J_a \end{Bmatrix} \sqrt{\frac{(\ell - |m|)!}{(\ell + |m|)!}}\ I^{\ell s j}_{J_a L_a J_b L_b}$$

and

$$S^{m\ m_a\ m_b}_{\ell s j} = \sum_{L_b} \beta^{m\ m_a\ m_b}_{\ell s j;\, L_b}\ P_{L_b}^{m_a - m - m_b}(\theta)$$

2) A. R. Edmonds

then we may write the transition amplitude in the form

$$T = \frac{\sqrt{4\pi}}{k_a k_b} \frac{C^2}{AB} \sum_{\ell s j} \sqrt{(2\ell+1)}\, B_{\ell s j} \langle J_A j M_A M_B - M_A | J_A M_A \rangle S_{\ell s j}^{m m_a m_b} .$$

The differential cross section for the A(a,b)B reaction is defined in terms of T by

$$\frac{d\sigma}{d\Omega} = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \frac{1}{2J_A+1} \frac{1}{2\delta_a+1} \sum_{M_A M_B m_a m_b} |T|^2$$

$$= \frac{2J_B+1}{2J_A+1} \frac{1}{4\pi} \frac{1}{E_a E_b} \frac{k_b}{k_a} \left(\frac{C^2}{AB}\right)^2 \frac{1}{2\delta_a+1}$$

$$\times \sum_{j m m_a m_b} \left| \sum_{\ell s j} \sqrt{2\ell+1}\; B_{\ell s j} S_{\ell s j}^{m m_a m_b} \right|^2 ,$$

where E_a and E_b are the center of mass energies for the entrance and exit channels respectively.

The program DWUCK computes the cross sections with the different normalization depending upon whether the case is a particle transfer reaction or inelastic scattering reaction. The two cross sections are

$$\sigma_{DW}^{\ell s j} = \frac{1}{4\pi} \frac{1}{E_a E_b} \frac{k_b}{k_a} \left(\frac{C^2}{AB}\right)^2 \frac{1.0\times10^4}{2\delta_a+1} (2\ell+1) \sum_{m m_a m_b} \left| S_{\ell s j}^{m m_a m_b} \right|^2$$

, $m_a \neq m_b$

$$\sigma_{DW}^{\ell s j} = \frac{1}{4\pi} \frac{1}{E_a E_b} \frac{k_b}{k_c} \left(\frac{C^2}{AB}\right)^2 \frac{1}{2\delta_a+1} \sum_{m m_a m_b} \left| S_{\ell s j}^{1\, m m_a m_b} \right|^2$$

, $m_a = m_b$

in units of $fm^2/Ster$ [$1\ fm^2 = 10\ mb = 10^{-26}\ cm^2$].

Therefore the cross section is

$$\frac{d\sigma^{\ell s j}}{d\Omega} = \frac{2J_B+1}{2J_A+1} \frac{|B_{\ell s j}|^2}{1.0\times10^4} \frac{\sigma_{DW}^{\ell s j}}{2j+1} , \quad m_a \neq m_b$$

$$\frac{d\sigma^{\ell s j}}{d\Omega} = \frac{2J_B+1}{2J_A+1} \frac{2\ell+1}{2j+1} |B_{\ell s j}|^2 \sigma_{DW}^{\ell s j} , \quad m_a = m_b$$

[Note: $m_a = m_b$ also means p=n and ^{3}He=T, etc.]

Card Set 1 (1 card)

ICON(16), ALPHA
FORMAT (16I1, 4X, 15A4)

i	ICON(i)	Function
1	0	Do not read in card set 2 (angle data)
	1,2	Read in card set 2
	9	Calls EXIT to terminate program
2	0	Computes collective or single particle form factors
	1	Use form factors from previous run
	2	Computes two-particle form factors
	3	Computes collective form factor and two-particle form factor. Read in two card sets 7 for each L transfer: collective first; two-particle set second.
3	0	Use same form factor for each L transfer. (Read in only one form factor card set 7.)
	1	Use different form factor for each L transfer. (Read in one set of card set 7 for each L transfer.)
	2	Same as ICON(3)=1 except the cross section is the coherent sum of amplitudes from each L transfer
4	0	Prints form factor before finite range and non-local corrections
	1	Suppresses form factor printing
	2	Suppresses form factor and intermediate print-out from two-nucleon form factor calculation
5	0	Prints elastic S matrix = $(\exp(2i\delta_{\ell j})-1)/2i$
	≠0	Suppresses elastic S matrix print-out
6	0	Suppresses elastic scattering cross section print-out
	≠0	Prints elastic scattering cross sections
7	0	Suppresses radial matrix element print-out
	≠0	Prints radial matrix elements
8	0	Suppresses inelastic scattering amplitude print-out
	≠0	Prints inelastic scattering amplitudes
9	0	No plot of inelastic cross section
	N	Plots inelastic cross section with N-decade log scale
A(10)	0	Usual non-relativistic kinematics
	≠0	Relativistic kinematics
B(11)	0	Suppresses form factor print-out after finite range and non-local corrections
	≠0	Prints form factor after finite range and non-local corrections

C(12)	0	Suppresses radial wave function of DW print-out
	N	Prints out radial wave function of DW every N^{th} point
D(13)	0	Suppresses inelastic cross section punching
	1	Punches inelastic cross section
E(14)	0	Suppresses elastic cross section punching
	1	Punches elastic cross section ratio to Rutherford
F(15)	0	Suppresses print-out or K(r)**2 of distorted waves
	≠0	Prints out K(r)**2 of distorted waves
G(16)	0	No plot of elastic cross sections
	N	Plots elastic cross sections ratio to Rutherford with N-decade log scale

Alpha — Any 60 alpha numeric characters to identify the run, beginning in column 21 of the card

Card Set 2 (1 card)

Angle input

If ICON(1)=1 the input of card set 2 is interpreted as

No. of angles, First angle, Angle increment.
FORMAT (3F8.4)

If ICON(1)=2 the input of card set 2 is interpreted as

Last angle, First angle, Angle increment.
FORMAT (3F8.4)

The program has a set of standard angle data pre-stored for the interval 0° to 180° in 5 degree intervals which will be used until changed by reading in a card set 2. The first angle may be +00.00 since the program will check for a zero angle when computing the elastic scattering cross sections.

Card Set 3

LMAX, NLTR, (LTR(I),I=1, NLTR), (JTR(I),I=1,NLTR)

FORMAT (12I3)

LMAX the maximum partial wave used in the cross section. The limit is given by $300/(2s_a+2s_b+2)-1$, where s_a and s_b are the spins of the initial and final projectiles.

NLTR the number of angular momentum transfers allowed per case. A maximum of 5 is allowed.

LTR(I) the orbital angular momentum transfer

JTR(I) twice the total angular momentum transfer. If the spin transfer on the form factor cards is 0 then JTR(I) can be omitted and JTR(I)=2*LTR(I) is used as a default value.

Card Set 4

DR, RMIN, RMAX, COUEX, FNRNG, AMASS FORMAT (5F8.4)

DR	integration step size
RMIN	lower cutoff on the radial integrals
+RMAX	upper cutoff on the radial integrals
COUEX	Coulomb excitation scale factor
FNRNG	finite range parameter
AMASS	sets the relative DR scale for the form factor and distorted waves

The limitation on DR and RMAX is RMAX/DR ≤ 400. A plus sign for RMAX allows the program to override RMAX by built-in criteria up to the maximum of 400 integration points allowed by storage. The minus sign defeats the override provision and uses |RMAX| as the upper cutoff on the integration of the distorted waves and radial integrals.

If AMASS=0.0, then the mass of the target of the first distorted wave is used in the relative scaling of DR. This may give an unconventional scale of B*DR/A for the radial points of the form factor for pick-up reactions. If you don't mind, let the program use the default option, AMASS=0.0. However, if you do mind, then set AMASS equal to the target mass used in the form factor.

A positive FNRNG will generate a Hulthen form of the finite range correction factor while a negative FNRNG will generate a Gaussian form of this factor except for multinucleon transfer reactions. In this instance only the Gaussian form is allowable.

Card Set 5 (minimum of 2 cards) — Initial distorted wave data set

Card 1 (Kinematic card)

E, MP, ZP, MT, ZT, R_{oc}, AC, PNLOC, 2*FS, QCODE
FORMAT (10F8.4)

E	Laboratory energy of initial projectile (must be non-zero)
MP	Projectile mass (in AMU units)
ZP	Projectile charge
MT	Target mass (in AMU units)
ZT	Target charge
R_{oc}	Coulomb charge radius ($R_c=R_{oc}\ MT^{1/3}$)
AC	Coulomb charge diffuseness (not used)
PNLOC	Non-local range parameter
2*FS	Twice the projectile intrinsic spin
QCODE	Q option (used for card set 6)

Cards (2-N) (Potential cards)

OPT, VR, R_{OR}, AR, VSOR, VI, R_{OI}, AI, VSOI
FORMAT (10F8.4)

OPT	Potential option
VR	Real well depth
R_{OR}	Real well radius ($R_R=R_{OR}\ MT^{1/3}$)
AR	Real well diffuseness
VSOR	Real well Thomas spin orbit factor
VI	Imaginary well depth
R_{OI}	Imaginary well radius ($R_I=R_{OI}\ MT^{1/3}$)
AI	Imaginary well diffuseness
VSOI	Imaginary Thomas spin orbit factor
POWR	Extra variable used for some potential options (described later in potential option descriptions)

Any number of potential cards may be used and the resulting potential will be the sum of potentials defined on the cards. If OPT is a negative option that option will be computed and the potential string will be ended and the next card set will be read in. If OPT is zero, no potential will be computed and the next card set will be read in.

Card Set 6 (minimum of 2 cards)

Final distorted wave data set

Card 1 is the same as in card set 6 except the parameter E is interpreted in three different manners depending upon the QCODE parameter.

QCODE > 0.0	E is ELAB of time reversed reaction
QCODE = 0.0	E is Q of reaction
QCODE < 0.0	E is the partial Q where Q = E + QCODE

This last option allows one to enter the ground state Q in the field for E and allows minus the excitation energy of the state to be entered in the field of QCODE so that the total Q is computed by the program.

The remaining cards of set 6 are defined in the same manner as cards (2-N) for card set 5.

Card Set 7 (minimum of 2 cards)

Card 1 (kinematic card)

Card(2-N) (Potential cards)

Card (N+1) (quantum number card)

Form factor data set for ICON(2)=0. This option will give a single particle or collective type form factor.

E, MP, ZP, MT, ZT, R_{oc}, AC, PNLOC, 2*FS
FORMAT (10F8.4)

E	Binding energy of single particle
MP	Mass of single particle
ZP	Charge of single particle
MT	Mass of core binding single particle
ZT	Charge of core binding single particle
R_{oc}	Coulomb charge radius ($R_c=R_{oc}MT^{1/3}$)
AC	Coulomb charge diffuseness (not used)
PNLOC	Non-local range parameter
2*FS	Twice the spin transfer for this form factor

Same as for card sets 5 and 6 of the distorted waves.

If E ≠ 0.0, the form factor computes a single particle orbital bound by E in the potential defined by the potential cards. In this case an additional card is needed to define the angular momentum quantum numbers of the orbital.

FN, FL, 2*FJ, 2*FS, VTRIAL, FISW, DAMP
FORMAT (10F8.4)

FN	Number of nodes excluding the origin and infinity
FL	Orbital angular momentum of the particle
2*FJ	Twice the total angular momentum of the particle
2*FS	Twice the intrinsic spin
VTRIAL	Scaling factor for the bound state potentials
FISW	Search control for bound state
DAMP	Damping factor for single particle wave function

FISW = 0	Search on well depth, i.e. VTRIAL, for fixed binding energy
1	Search on binding energy, i.e. E, for fixed potentials
2	No search (for E > 0 only)
DAMP≠00	A damping factor exp(-DAMP*r) multiplies the "bound" state function and the function is then renormalized to 1.0.

The total potential is the product of VTRIAL and the real part of the potential defined by cards (2-N). For the usual bound state VTRIAL*VR must be less than zero. If VTRIAL is left blank a standard value of VTRIAL = +60.0 is used. In this case VR should be -1.0.

If E=0.0, a form factor defined by the potential options is computed and card N+1 should be omitted.

Card Set 7

Form factor data set for ICON(2)=2. This option will compute a two-particle type form factor. 13

Card 1
(option card)

CNTRL, QCODE, FLMU, VZERO, FJ2, FJI, FJF
FORMAT (7F8.4)

CNTRL = 0.0 Read zero sets of orbital cards and EXIT from form factor calculation
= 1.0 Read one set of orbital cards
= 2.0 Read two sets of orbital cards

QCODE = 0.0 No option
1.0 Yukawa potential microscopic form factor
2.0 Coulomb potential microscopic form factor
3.0 Tensor potential microscopic form factor
4.0 Not used
5.0 Two nucleon transfer microscopic form factor
6.0 Zero range knockout microscopic form factor

FLMU = $(\text{Range})^{-1}$ of potentials for options 1 and 3 above
= RMS radius of Gaussian wave function for "triton" in option 5 above.

VZERO = Strength of potential for options 1, 2, and 3
= Spectroscopic amplitude for transfer reaction in option 5
= Volume integral of two-body potential in option 6
Here volume integral = $\int V(r)\exp(i\vec{K}\cdot\vec{r})d\vec{r}$

FJ2 = 2* spin of core (j2) to which single particle is coupled in options 1, 2, 3, and 6

FJI = 2* spin of initial nucleus $\vec{j}1 + \vec{j}2 = JI$ in options 1, 2, 3, and 6

FJF = 2* spin of final nucleus $\vec{j}1' + \vec{j}2 = JF$ in options 1, 2, 3, and 6

Cards (2-N)

Following card 1, place one or two sets of cards defining the single-particle orbital. See card set 7 for ICON(2)=0. If the second orbital is identical to the first you may use CNTRL=1.0 and the program will use the results of the first orbital calculation as the second orbital wave function and will not read in the second set. The program will attempt to read in a new card 1 of this data set and coherently add the form factor contributions until a CNTRL=0.0 is detected and the program will terminate the calculation of this form factor. A negative CNTRL will also give a coherent contribution to the form factor but the calculation will terminate similarly to CNTRL=0.0.

A number of variations to the standard options may be made. The list described for QCODE ≤ 10.0 will treat the form factor as a purely real function. If you increase the QCODE by 10.0, i.e. 1.0 becomes 11.0, etc., the program will treat that part of the form factor as purely imaginary. Thus by use of a QCODE number less than 10.0 and another one greater than 10.0 one can generate both real and imaginary parts of the form factor. Further, the use of a negative QCODE will use the single-particle orbitals generated by the previous QCODE to calculate the contribution to the form factor. Hence the single-particle orbital cards can be omitted and in this way one can calculate multiple contributions such as a Yukawa interaction plus a Coulomb term without repunching the orbital cards for the second potential option.

Potential options available for card sets 5, 6, and 7.

OPT = 1.0 Volume Wood-Saxon potential

$$V(r) = VR*f(X_R) + iVI*f(X_I)$$

$$f(X_i) = [1 + \exp(\frac{r-R_{oi}MT^{1/3}}{A_i})]^{-1}$$

Note that for attractive real and absorptive imaginary potentials VR and VI must be negative.

OPT = 2.0 Surface Wood-Saxon potential

$$V(r) = VR*df(X_R)/dX_R + iVI*df(X_I)/dX_I$$

Note that for attractive real and absorptive imaginary potentials VR and VI must be positive. Further, this form has no factors of 4 so that VI=4 W_D where W_D is the other convention of expressing the strength of the surface shape potential.

OPT = 3.0 Second derivative Wood Saxon potential

$$V(r) = VR*d^2f(X_R)/dX_R^2 + iVI*d^2f(X_I)/dX_I^2$$

OPT = 4.0 L·S potential from a volume Wood-Saxon potential

$$V_{LS}(r) = (-VR*\frac{1}{r}\,df(X_R)/dr - iVI*\frac{1}{r}\,df(X_I)/dr)\ L\cdot S$$

Note that the potential is defined in terms of L·S and contains no $(\hbar/m_\pi c)^2 \approx 2$ factor. The strength VR is 4 times the strength used when the $(\hbar/m_\pi c)^2$ factor and L·σ conventions are used for protons and neutrons, but only a factor of two is needed for deuterons since the V_{LS} then is usually given in terms of L·S.

The VSOR parameter used in option 1 will give a spin orbit potential the same as option 4 with the same geometry as used in option 1. The VSOR parameter is then the usual λ factor used in multiplying the Thomas term and has a value of about 25 for protons and neutrons. In this case

$$V_{LS}(r) = -VR*\frac{VSOR}{45.2} * \frac{1}{r}\,\frac{df(X_R)}{dr}\ L\cdot S$$

OPT = 5.0 L·S potential from a surface Wood-Saxon potential

$$V_{LS}(r) = (-VR*\frac{1}{r}\frac{d}{dr}\,df(X_R)/dX_R - iVI*\frac{d}{dr}\,df(X_R)/dX_R)\ L\cdot S$$

OPT = 6.0 Volume Wood-Saxon potential *r**POWR

$$V(r) = [VR*f(X_R) + iVI*f(X_I)]\ r^{**}POWR$$

OPT = 7.0 Surface Wood-Saxon potential *r**POWR

$$V(r) = [VR*df(X_R)/dX_R + iVI*df(X_I)/dX_I]*r^{**}POWR$$

OPT = 8.0 External Form Factor

This option reads in a form factor from the input file. In addition to the option card you must add a separate card to specify the number of points to be read in and whether the form factor is real or imaginary.

F1,F2
FORMAT (2F8.4)

F1 Number of points to be read in
F2 =0.0 places points in real part of form factor
=1.0 places points in imaginary part of form factor

This card is followed by the necessary cards containing the form factor 5 points per card.

FF(I) Form factor points
FORMAT (5E16.7)

The form factor FF(I) is scaled by VR or VI depending upon whether F2 is 0.0 or 1.0. If VR and VI are zero or blank, the form factor is scaled by 1.0.

OPT = 9.0 Normalized Harmonic Oscillator

$$V(r) = VR*N\ L(r/R_{OR})\ \exp(-\tfrac{1}{2}(r/R_{OR})^{**}2)$$

where N=normalizing constant for the harmonic oscillator functions such that

$$\int_0^\infty (V(r))^2 r^2 dr = (VR)^2$$

$L_n^{\ell+\frac{1}{2}}(r/R_{OR}^2)$ is the Laguerre polynomial

Note that the radius parameter is R_{OR}, the quantity input on the potential card and not R_R, which is $R_{OR}MT^{**}(1/3)$.

OPT = 10.0 GAUSSIAN *r**POWR

$$V(r) = VR*\exp(-(r)/R_{OR})^{**}2) * r^{**}POWR$$
$$+iVI*\exp(-(r/R_{OR})^{**}2) * r^{**}POWR$$

OPT = 11.0 Legendre expansion of volume Wood-Saxon potential

$$V(r) = \int_{LTR} [VR * f(X_R(r,\theta)) + iVIf(X_I(r,\theta))]\ Y^\circ_{LTR}(\theta)\ d\Omega_r$$

where $f(X) = 1.0/[1.0 + \exp(X)]$

$$X_i = [r - R_i(1.0 + \beta_{LDRF}\ Y^\circ_{LDRF})]/A_i$$

LTR is the L transfer specified for this form factor from card set 3.

An extra card is needed to follow the OPTION=11.0 card.

BETA, LDRF
FORMAT (2F8.4)

BETA is β_{LDRF}, the deformation parameter

LDRF is the multipole of deformation and can be different from LTR.

OPT = 12.0 Legendre expansion of surface Wood-Saxon potential

$$V(r)_{LTR} = \int [VR * \frac{df}{dX_R}(X_R) - iVI * \frac{df}{dX_I}(X_I)] \, Y^{\circ}_{LTR}(\theta) d\Omega_r$$

where f(X) and X have the same meanings as in OPTION=11.0. The extra card is needed here also to specify β_{LDRF} and LDRF.

In options 11 and 12, VR and VI have the same sign conventions as are used in options 1 and 2. Further DWUCK treats LTR=0 differently from other values of LTR when used as a form factor as a result of options 1,2,3,11, and 12. In these cases the V(r) for LTR=0 is considered to be the coefficient of $P_o(0)$, the zeroth Legendre polynomial, whereas V(r) for LTR>0 is considered to be the coefficient of Y°_{LTR}, the usual assumption of DWUCK angular momentum algebra. Hence the LTR=0 calculations need to be multiplied by an extra 4π factor. This ambiguity does not apply to the single particle, microscopic, or two nucleon transfer form factors.

OPTIONS 11 and 12 may also be used as potential for the distorted waves. In this case, LTR=0 is set by the program.

Form factor options for ICON(2)=0 and E=0.0.

These options have the basic form as denoted in the potential option description except for the following cases.

OPT=1.0

$$F(r) = VR*f(X_R) * (R_R/A_R)^{**}(POWR+1.0) + VI*F(X_I)*(R_I/A_I^{**}(POWR+1.0)$$

OPT=2.0 and OPT=3.0 The real and imaginary parts of the form factor are multiplied by $(R_R/A_R)^{**}(POWR+1.0)$ and $(R_R/A_R)^{**}(POWR+1.0)$ respectively, as in the case of OPT=1.0.

OPT=4.0 and OPT=5.0 These options are not available as form factors.

OPT=6.0 - 11.0 These options give the same form and normalization as in the options for the potential functions.

Coulomb Excitation Addition to the Form Factor

In many cases the inelastic scattering cross section is strongly affected by the Coulomb part of the interaction of the projectile with the nucleus. This may be put in the scattering prescription two ways. The first is the collective form and this is controlled by the parameter COUEX which is entered on card set four. An additional term is added to the form factor during the time of computation of the radial integrals of the form

$$f^{coul}_{\ell o \ell}(r) = COUEX * \frac{3.0*Z*ZT\ e^2}{2\ell+1} \frac{RC^{**}\ell}{r^{**}(\ell+1)} \qquad r > RC$$

$$= 0.0 \qquad r < RC$$

where the parameters Z, ZT, and RC are taken from the kinematic card for the form factor. In this fashion the radius used for Coulomb excitation can be different from the radii for the distorted waves. For the usual collective excitations one uses COUEX = +1.0 although any other scaling may be used that the physics requires.

The second way is via the microscopic inelastic form factor. This method is separate from the collective form described above and COUEX should be set to 0.0. For this method one uses the Coulomb option in the microscopic form factor version and the strength of the interaction must be $ZZ'e^2$ where ZZ' is the effective product of the projectile and target charges and e^2=1.44 MeV-fm. Care must be taken in order to correctly phase the nuclear strength and Coulomb strengths to obtain the correct interference between the two interactions.

Finite Range Correction Factor

If in a stripping reaction

$$a + A \rightarrow b + B \quad \text{where } a=b + c \text{ and } B=A + C$$

one can write the transition amplitude as

$$T_{fi} = \int\int d\vec{R}\, d\vec{x}\ D(x)\ \chi_f^{-*}(R+x)\ \varphi_c^*(R)\ \chi_i^+(R+\frac{m_b}{m_a}x)$$

χ_f^- the distorted wave of the c.m. of particle b

φ_c the bound state function of the transferred particle

χ_i^+ the distorted wave of the c.m. of particle a

D(x) the "overlap" of b with a

where, $D(x) = \int d\tau_b\ \varphi_b^*\ V_{bc}\ \varphi_a$

If we define

$$G(K') = \int e^{i\vec{K}\cdot\vec{x}}\ D(r)\, d\vec{x}$$

then the zero range normalization is given by

$$D_0 = G(0)$$

and the finite range correction parameter R is given by

$$R^2 = -\frac{1}{G(K^2)} \frac{\partial G(K^2)}{\partial (K^2)} \Big|_{K^2=0}$$

The first order correction factor from the local energy approximation for D(x) which multiplies the form factor is

$W_o(r) = [1 + A(r)]^{-1}$ Hulthén form

$= \exp(-A(r))$ Gaussian form

where

$$A(r) = \frac{2}{\hbar^2} \frac{m_b m_c}{m_a} R^2 [E_b - V_b(r_b) + E_c - V_c(r_c) - E_a + V_a(r_a)]$$

where E_a, E_b, E_c, and V_a, V_b, V_c are the energies and potentials for the three light particles of mass m_a, m_b, m_c. The parameter R is the finite range parameter FNRNG from card set 4. A positive FNRNG will select the Hulthén form of W(r) and a negative FNRNG will select the Gaussian form of w(r).

Typical values of the range parameter R are

Reaction	R
(d,p)	.621-.695
(^{3}He,d)	.770
(t,d)	.845
(^{4}He,^{3}He)	≈0.7

Non-Local Correction Factor

The correction needed for the use of an equivalent local potential multiplies the form factor and is of the form

$$W_{NL}(r) = \exp(\frac{\beta_i^2}{8} \frac{2m_i}{\hbar^2} V_i(r))$$

for each of the projectiles and the bound state functions used in the form factor. The variable β_i is the PNLOC parameter on the kinematic card of card sets 5, 6, or 7. The m_i are the masses FM and the V_i are the potentials for the particles. In the case of a bound state the factor W_{NL} multiplies the bound state function and then the function is renormalized to unity. The $V_i(r)$ include any Coulomb potentials for the projectile or particle. Typical values of the β parameter are

Particle	β
p	≈ .85
d	≈ .54
^{3}He	≈ 0.2-0.3
^{4}He	≈0.2

Application to Reactions

1) Collective nuclear excitations by inelastic scattering

To first order in the deformations we can write the interaction term V as

$$V = -\beta_\ell \frac{R_o}{a} V_o \frac{df}{dx} Y_\ell^0(\theta)$$

where $x = (r - R_o)/a$ and θ is the angle between the scattering particle and the nuclear symmetry axis. If we take the strong coupling form of the initial and final state wave function

$$\psi^J_{JMK} = \sqrt{\frac{2J+1}{16\pi^2}} \left[\varphi^J_K D^J_{KM} + (-)^{J-J'} \varphi^{J'}_{-K} D^J_{-KM} \right],$$

we find that

$$\langle J_B M_B \Lambda_b m_b | V | J_A M_A \Lambda_a m_a \rangle = -\beta_\ell \frac{R_o}{a} V_o \frac{df}{dx} \sqrt{\frac{2J_A+1}{2J_B+1}} \times \langle J_A \ell K 0 | J_B K \rangle \langle J_A \ell M_A M_B - M_A | J_B M_B \rangle Y_\ell^M(\hat{r}_a)^*.$$

The form factor computed by DWUCK for form factor option 2 or 3 is

$f_{sox}(r) = -\frac{R_o}{a} V_o \, df/dx$ option 2

$f_{sos}(r) = -R/a \, V_o \, d^2f/dx^2$ option 3

and since s=0 we have

$$B_{sox} = \beta \langle J_A \ell K 0 | J_B K \rangle \sqrt{\frac{2J_A+1}{2J_B+1}}$$

The cross section is

$$\frac{d\bar{J}^2}{d\Omega} = \beta_\ell^2 | \langle J_A \ell K 0 | J_B K \rangle |^2 \bar{J}^2_{pw}(\theta)$$

In option 11 or 12 the form factor to first order in β_ℓ is computed as

$$f_{\ell sj}(r) = -\beta_\ell \frac{R_0}{\alpha} V_0 \, d(\quad)/d r \qquad \text{option 11}$$

$$f_{\ell sj}(r) = -\beta_\ell \frac{R_0}{\alpha} V_0 \, d^2(\quad)/dr^2 \qquad \text{option 12}$$

so that in this case

$$\frac{d\sigma^{\ell}}{d\Omega} = |\langle J_A \; | \; K_0 \; | \; J_B \; K \rangle|^2 \; \sigma_{DW}^{(e)}$$

2) Stripping reactions (d,p)

The matrix element for a (d,p) reaction may be written as

$$\langle J_B M_B s_b m_b | V | J_A M_A s_a m_a \rangle = \sum_{j\ell} S_{j\ell} R_{j\ell}(r_{xA}) (-c)^{\ell}$$
$$\times \langle \ell \, s \, m_j \, \mu - m | j \mu \rangle \langle s_b \, s \, m_b \; m_a - m_b | s_a \, m_a \rangle$$
$$\times \langle J_A \, j \, M_A \; M_B - M_A | J_B \, M_B \rangle \; D(r_{xb}) \; Y_\ell^{m'}(\hat{r}_{xA})$$

where $S_{j\ell}$ is the spectroscopic factor, $R_{j\ell}(r)$ is a normalized radial function, and $D(r_{xb})$ is the product of the neutron-proton potential times the deuteron internal wave function. By making use of the zero range approximation which gives

$$D(r_{xb}) = D_0 \delta(r_x - r_b)$$

we find that

$$B_{\ell sj} = S_{j\ell}^{\frac{1}{2}} \; D_0$$

where the form factor is

$$f_{\ell sj}(r) = R_{j\ell}(r).$$

The cross section then is written as

$$\frac{d\sigma^{(0)}}{d\Omega} = \frac{2J_B + 1}{2J_A + 1} \; S_{j\ell} \; \frac{D_0^2}{1.0 \times 10^4} \; \frac{\sigma_{DW}^{(0)}}{2j + 1}$$

A value of D_0 when the deuteron wave function is described by a Hulthen function

$$\psi_D(r_{np}) = N \left(\exp(-\alpha r_{np}) - \exp(-\beta r_{np}) \right) / r$$

where

$$N = \left\{ \frac{\alpha\beta(\alpha+\beta)}{2\pi(\beta-\alpha)^2} \right\}^{\frac{1}{2}}$$

is

$$D_0 = -\left(1 + \frac{\alpha}{\beta}\right)^{1/2} \frac{\hbar^2}{m} \sqrt{8\pi\alpha}$$

If we take $\beta = 1.48$ F^{-1} then we find

$D_0 = -1.244 \times 10^2$ MeV - F$^{3/2}$

or $(D_0/10^2)^2 = 1.55$ MeV^2 - F^3.

3) Pick-up reaction (p,d)

From time reversal invariance of the (d,p) reaction described above we may write the cross section for the (p,d) reaction in terms of the (d,p) cross section,

$$\frac{d\sigma_{Pd}^{(0)}}{d\Omega} = \frac{2J_A + 1}{2J_B + 1} \; \frac{2s_a + 1}{2s_b + 1} \; \frac{k_a^2}{k_b^2} \; \frac{d\sigma_{dp}^{(0)}}{d\Omega}$$
$$= \frac{3}{2} \; S_{j\ell} \; \frac{D_0^2}{1.0 \times 10^4} \; \frac{\sigma_{DW}^{(0)}(pd)}{2j + 1}$$

where σ_{DW} is calculated with protons in the incident channel and deuterons in the exit channel.

4) Pick-up stripping cross section tables

Reaction	$(d\sigma/d\Omega)\,fm^2$ =	FNRNG Parameter
(d,p)	$1.55\,\frac{2J_f+1}{2J_i+1}\,\frac{\sigma_{DW}(\theta)}{2j+1}$	0.675 → 0.621
(^{3}He,d)	4.42 " " "	0.770
(τ,d)	5.06 " " "	0.845
(^{4}He,^{3}He)	(24 → 46) " " "	≈ 0.7
(^{4}He,τ)	(24 → 46) " " "	≈ 0.7
(p,d)	$2.33\,\frac{\sigma_{DW}(\theta)}{2j+1}$	0.695 → 0.621
(d,^{3}He)	2.95 " "	0.770
(d,τ)	3.33 " "	0.845
(^{3}He,^{4}He)	(12 → 23) " "	≈ 0.7
(τ,^{4}He)	(12 → 23) " "	≈ 0.7

In the above table J_i is the initial target spin, J_f is the final target spin, and j is the total angular momentum transfer.

Users of DWUCK and CHUCK:

There has been a major change in the codes in regard to the phasing of the microscopic inelastic and two-nucleon transfer form factor. This change also carries over in CHUCK to the sequential transfer case. The new two-particle wave functions should carry an extra phase in the amplitude of $(i)^{\ell_1+\ell_2-LTR}$ over the old amplitude convention. This will cause a ground state to ground state L=0 transition to have all positive coefficients, e.g. ^{90}Zr

$$\psi = +.8(p_{1/2})^2 + .6(g_{9/2})^2.$$

Another change has been made in CHUCK for the collective form factor (ICODE=0). When using options 1, 2, or 3, the power of the (R/a) factor starts with one, not zero, so that the usage is now consistent with DWUCK,

Form factor ∝ (R/a)**(POWR + 1.0).

5) Microscopic Interaction Model for Inelastic Scattering

a) Central interactions

The potential between the projectile and one of the target nucleons may be expanded in a Legendre polynomial series

$$V(\underset{\sim}{r}_0 - \underset{\sim}{r}_1) = V_0 \sum_\ell (2\ell+1)\, v_\ell(r_0, r_1)\; P_\ell(\hat{r}_0 \cdot \hat{r}_1)$$

$$= 4\pi V_0 \sum_\ell v_\ell(r_0, r_1) \sum_m Y_\ell^m(\hat{r}_0)^* \, Y_\ell^m(\hat{r}_1) .$$

Using this form we find for the matrix element of V

$$\langle J_B M_B s_b m_b \,|\, V(\underset{\sim}{r}_0 - \underset{\sim}{r}_1) \,|\, J_A M_A s_a m_a \rangle =$$

$$4\pi V_0 \sum_\ell \langle J_A \ell M_A M_B - M_A \,|\, J_B M_B \rangle \langle J_B \,||\, v_\ell(r_0, r_1)\, i^\ell Y_\ell \,||\, J_A \rangle$$

$$\times \langle s_b s m_b, m_a - m_b \,|\, s_a m_a \rangle (-i)^\ell \; Y_\ell^m(\hat{r}_0)^* \frac{1}{\sqrt{2J_B+1}} .$$

The program DWUCK calculates for the form factor the expression

$$FF = 4\pi V_0 \sqrt{2J_A+1}\; \langle j_1' j_2 J_B \,||\, Y_\ell(\hat{r}_1) \,||\, j_1 j_2 J_A \rangle$$

$$\times \int R_{\ell' j_1'}(r_1)\, v_\ell(r_0, r_1)\, R_{\ell j_1}(r_1)\, r_1^2 \, dr_1 ,$$

where the $R_{\ell j}(r_1)$ are the normalized radial functions for the initial and final nucleon states, and the reduced matrix element $\langle j_1' j_2 J_B \,||\, Y_\ell \,||\, j_1 j_2 J_A \rangle$ is defined by Edmonds.[1] The quantity j_2 is the spin for the coupling of the A-1 nucleons. The reduced matrix element $\sqrt{4\pi}\, \langle j_1' j_2 J_B \,||\, Y_\ell \,||\, j_1 j_2 J_A \rangle$ is printed out as RME in the output.

If there is more than one particle in the shell then the strength VZERO in the microcopic form factor option for each configuration is to be multiplied by

the factor accounting for the number of identical particles and amplitudes for the configurations used. Thus

$$B_{\ell v \ell} = a_{j_1 j_2 J_A}\, a_{j_1' j_2 J_B} \sum \langle j_1 j_2 J_B \,||\, Y_\ell(1) \,||\, j_1 j_2 J_A \rangle / \langle j_1' j_2 J_B \,||\, Y_\ell(1) \,||\, j_1 j_2 J \rangle$$

If VZERO includes $B_{\ell v \ell}$, then

$$\frac{d\sigma^\ell}{d\Omega} = \sigma_{\ell\omega}^{\ell v \ell}(\theta)$$

b) $\sigma_i \cdot \sigma_j$ interaction

A central interaction $V(r_0 - r_1)$ with a $\vec{\sigma}_0 \cdot \vec{\sigma}_1$ dependence can be written in the following form

$$V(\underset{\sim}{r}_0 - \underset{\sim}{r}_1)\, \underset{\sim}{\sigma}_0 \cdot \underset{\sim}{\sigma}_1 = 4\pi V_0 \sum_{\ell j \mu} v_\ell(r_0, r_1)(-)^{\ell - 1 + \mu + 1} \mathcal{Y}_{\ell 1 j - \mu}(\hat{r}_0)\, \mathcal{Y}_{\ell 1 j \mu}(\hat{r}_1)$$

where

$$\mathcal{Y}_{\ell 1 j \mu} = \sum_m (\ell s \mu \; \mu - m \,|\, j \mu)\, i^\ell Y_\ell^m(\theta \omega)\, \sigma^{\mu - m}$$

The matrix element for V in the transition amplitude is

$$\langle J_B M_B s_b m_b \,|\, V \,|\, J_A M_A s_a m_a \rangle$$

$$= 4\pi V_0 \sum_{\ell j \mu} \langle \ell s m \; \mu - m \,|\, j \mu \rangle (-i)^\ell \; Y_\ell^m(\hat{r}_0)^*$$

$$\times \langle s_b s m_b \; m_a - m_b \,|\, s_a m_a \rangle \, 2\sqrt{s_a(s_a+1)}\; (-)^{2j}$$

$$\times \langle j_1' j_2 J_B \,||\, v_\ell(r_0, r_1)\, i^\ell \mathcal{Y}_{\ell 1 j}(\hat{r}, \sigma_1) \,||\, j_1 j_2 J_A \rangle$$

$$\times \langle J_A j M_A \; M_B - M_A \,|\, J_B M_B \rangle \frac{1}{\sqrt{2J_B+1}} ,$$

where s=1 and $s_a = s_b$.

The form factor computed by DWUCK4 is

$$f_{\ell s j}(r_0) = 2\sqrt{s_a(s_a+1)}\; i^{\ell} \langle j_1 j_2 \| V_{\ell s j}(r_0, r_1)\, Y_{\ell s j}(\hat{r}_1, \hat{s}) \| j_1 I_2 J_A \rangle \times \sqrt{\frac{(2\ell+1)}{(2j_1+1)(2J_A+1)}}\,.$$

In the above expressions for the form factor we have used

$$\langle s_b \| \sigma_0 \| s_a \rangle = 2\sqrt{s_a(s_a+1)}\,.$$

which is the reduced matrix element appropriate to inelastic scattering $s_a = s_b$. If the spin of the projectile changes such as in (^{6}Li,^{6}He) inelastic scattering then the appropriate matrix element ratio

$$\langle s_a \| T_s \| s_b \rangle / \left(2\sqrt{s_a(s_a+1)(2s_b+1)}\right)$$

must multiply the form factor.

The quantity is

$B_{\ell s j} = -1$ when $s_a = s_b$.

We obtain the cross section

$$\frac{d\sigma}{d\Omega} = \sigma_{DW}^{\ell s j}.$$

Note: It should be pointed out again that the cross section for the microscopic interaction is done in terms of a single particle transition $j_1 \to j_1'$. If there is more than one equivalent particle then the matrix element ratio

$$\sum_i \langle J_B \| Y_{\ell s j} \| J_A \rangle / \langle j_1' j_2 J_B \| Y_{\ell s j}(r_1) \| j_1 I_2 J_A \rangle$$

will have to multiply the $B_{\ell s j}$ factors which are outlined here.

6) Two-nucleon transfer option

The description for the (p,t) reaction is given in the article, H. Baer et al., Annals of Physics (NY) 76, 437 (1973), Appendix.

The usual normalization condition gives the cross section for (p,t) as

$$\frac{d\sigma}{d\Omega} = D_0^2 (\pi\Delta^2/2)^{3/2} (T_B \| N_B 1 \| T_A N_A)^2\, \sigma_{DW}^{\ell s j} / (2j+1),$$

where $(\pi\Delta^2/2)^{3/2} = 9.72$ fm^3 for $\Delta = \Delta' = 1.7$ fm.

For the (t,p) reaction, we have

$$\frac{d\sigma}{d\Omega} = D_0^2 (\pi\Delta^2/2)^{3/2} (T_A \| N_A 1 \| T_B N_B)^2 \frac{2J_B+1}{2J_A+1}\, \sigma_{DW}^{\ell s j} / (2j+1).$$

The value of D_0^2 ranges from 15 - 30 MeV^2-fm^3.