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ON

THE TROPOSPHERE, STRATOSPHERE AND MESOSPHERE

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SUBSURFACE PROPAGATION

Chapter 3 - Modes and mode conversions

Appendices A, B and C

References

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CHAPTER 3 MODES AND MODE CONVERSIONS

3.1. INTRODUCTION

It has been obvious from Chapter 2 that one of the key steps in the theory of subsurface radio wave propagation is to identify the propagation modes. The studies that have been reported show that this task is extremely complex, even in the simplest cases, when we try to solve Maxwell's equations. Furthermore the actual subsurface environment is frequently far more complicated than the simple structures that we were able to solve.

It is therefore necessary to resort to simplified descriptions based on transmission line concepts. The validity of a transmission line model for cables and wires strung parallel to the axis of a tunnel has been justified in section 2.8.4. Quite generally, if we have n wires, we can construct column matrices of the wire currents I and of the wire voltages V, and write transmission line equations in matrix form

$$\frac{d\bar{I}}{dx} = -\bar{y}\,\bar{V} \tag{3.1}$$

$$\frac{d\bar{V}}{dx} = -\bar{z}\,\bar{I} \tag{3.2}$$

Herein x is the coordinate along the tunnel axis.

We have seen that the specific admittance matrix $\bar{y}$ is approximately given by

$$\bar{y} = j\omega\bar{c} \tag{3.3}$$

where $\bar{c}$ is the matrix of capacity coefficients that may be estimated assuming that the tunnel wall is perfectly conducting. Estimating the specific impedance matrix $\bar{z}$ is quite more involved. One may generally write

$$\bar{z} = j\omega\bar{l}_{pc} + \bar{z}_e + \bar{z}_o \tag{3.4}$$

where $\bar{l}_{pc}$ is the matrix of self and mutual specific inductances of the wires assuming that the tunnel wall is perfectly conducting, $\bar{z}_e$ is a correction term to account for the fact that this assumption is not exact, and $\bar{z}_o$ is a diagonal matrix of the specific impedances of the wires as defined in section 2.3. One frequently denotes

$$\bar{z}_{mo} = j\omega\bar{l}_{pc} + \bar{z}_e \tag{3.5}$$

One particular case of coupled transmission lines is that of a leaky feeder strung into a tunnel and for which the coupled transmission line equations are given by eq. (2.40) to (2.43) and illustrated by Figure 2.1, whereon the coupling admittance y_t should be deleted. The equivalent circuit is redrawn on Figure 3.1 where we have introduced quantities

$$z_{co} = z_c - z_t \tag{3.6}$$

$$z_{mo} = z_m - z_t \tag{3.7}$$

which are the specific series impedances of the coaxial and monofilar modes for the case where the leaky shield should be replaced by a non-leaky one : these modes should then obviously be uncoupled.

The coupled transmission lines are

$$\frac{dI_m}{dx} = -y_m V \tag{3.8}$$

$$\frac{dI_c}{dx} = -y_c V_c \tag{3.9}$$

$$\frac{dV_m}{dx} = -z_m I_m - z_t I_c \tag{3.10}$$

$$\frac{dI_c}{dx} = -z_t I_m - z_c I_c \tag{3.11}$$

where we admit that z_t is of the form

$$z_t = j\omega m_t \tag{3.12}$$

Before continuing this study, we shall discuss some questions of terminology. In coupled-line theory the terms monofilar mode and coaxial mode are used for the pairs (V_m, I_m) and (V_c, I_c), and for related quantities like fields, impedances, admittances and so on. In chapter 2 however we used the same names for solutions characterized by a propagation factor $\exp(-\Gamma x)$. The terminology is thus misleading, since the existence of a propagation factor $\exp(-\Gamma x)$ necessarily

implies that of both pairs. In order to avoid any confusion in the remaining of this text, we will refer to the pairs (V_m, I_m) and (V_c, I_c) as the ***coupled modes*** and to the exponential solutions as ***eigenmodes***. Similar considerations apply to the case of multiple wires. We will now start the study of matricial transmission line equations.

3.2. COUPLED-LINE THEORY

3.2.1. General theory

We consider the transmission line equations in matrix form

$$\frac{d\overline{I}}{dx} = -\,\overline{y}\,\overline{V} \qquad (3.13)$$

$$\frac{d\overline{V}}{dx} = -\,\overline{z}\,\overline{I} \qquad (3.14)$$

where $\overline{I}$ and $\overline{V}$ are column vectors with n elements, and $\overline{y}$ and $\overline{z}$ are square matrices of order n. This model applies to a system of n conductors strung parallel to the axis of a tunnel, including the leaky coaxial cable (n = 2) described by eq. (3.8) to (3.11). After derivation and elimination, these equations transform into

$$\frac{d^2\overline{I}}{dx^2} = \overline{y}\,\overline{z}\,\overline{I} \qquad (3.15)$$

$$\frac{d^2\overline{V}}{dx^2} = \overline{z}\,\overline{y}\,\overline{V} \qquad (3.16)$$

These equations have a fundamental set of n linearly independent solutions of the form

$$\overline{I}(x) = \overline{I}_o \exp(\pm\Gamma x) \qquad (3.17)$$

$$\overline{V}(x) = \overline{V}_o \exp(\pm\Gamma x) \qquad (3.18)$$

where $\overline{I}_o$, $\overline{V}_o$ and Γ are solutions of the eigenvalue problems

$$\overline{y}\,\overline{z}\,\overline{I}_o = \Gamma^2\,\overline{I}_o \qquad (3.19)$$

$$\overline{z}\,\overline{y}\,\overline{V}_o = \Gamma^2\,\overline{V}_o \qquad (3.20)$$

The fundamental solutions are called eigenmodes. We implicitely assume that the eigenvalues are non-degenerate.

As results from reciprocity, $\overline{y}$ and $\overline{z}$ are symmetric matrices and, consequently, $\overline{y}\,\overline{z}$ and $\overline{z}\,\overline{y}$ are the transposed of each other. They thus have the same eigenvalues Γ^2, while the eigenvectors $\overline{I}_o$ and $\overline{V}_o$ are left-hand and right-hand eigenvectors of $\overline{y}\,\overline{z}$, respectively. Constructing two modal matrices $\overline{J}$ and $\overline{U}$, the columns of which are the eigenvectors $\overline{I}_o$ and $\overline{V}_o$, respectively, (3.19) and (3.20) may be written

$$\overline{y}\,\overline{z}\,\overline{J} = \overline{J}\,\overline{\mathrm{Diag}}\,\Gamma^2 \qquad (3.21)$$

$$\overline{z}\,\overline{y}\,\overline{U} = \overline{U}\,\overline{\mathrm{Diag}}\,\Gamma^2 \qquad (3.22)$$

where $\overline{\mathrm{Diag}}\,\Gamma^2$ denotes the diagonal matrix of eigenvalues.

However $\overline{J}$ and $\overline{U}$ may not be chosen independently since the voltages and currents are related by (3.13) and (3.14). This imposes the constraint

$$\overline{J}\,\overline{\mathrm{Diag}}\,\Gamma = \overline{y}\,\overline{U} \qquad (3.23)$$

or, equivalently

$$\overline{U}\,\overline{\mathrm{Diag}}\,\Gamma = \overline{z}\,\overline{J} \qquad (3.24)$$

Finally the general solution of the coupled-line equns (3.13) - (3.14) may be written as

$$\overline{V}(x) = \overline{U}\left[\overline{\mathrm{Diag}}\,e^{-\Gamma x}\,\overline{A} + \overline{\mathrm{Diag}}\,e^{+\Gamma x}\,\overline{B}\right] \qquad (3.25)$$

$$\overline{I}(x) = \overline{J}\left[\overline{\mathrm{Diag}}\,e^{-\Gamma x}\,\overline{A} - \overline{\mathrm{Diag}}\,e^{+\Gamma x}\,\overline{B}\right] \qquad (3.26)$$

where $\overline{A}$ and $\overline{B}$ are two column matrices of arbitrary constants to be determined by the boundary conditions. This very compact form of the solutions shows progressive and regressive eigenmodes with generalized matrix amplitudes $\overline{A}$ and $\overline{B}$.

It is important to understand the meaning of the various elements of this solutions. Let us for instance assume that the boundary conditions are such that $A_k \neq 0$, all other elements of $\overline{A}$ and all elements of $\overline{B}$ being zero. One has then

$$\overline{V}(x) = \overline{U}_k\,A_k \exp(-\Gamma_k x) \qquad (3.27)$$

$$\overline{I}(x) = \overline{J}_k\,A_k \exp(-\Gamma_k x) \qquad (3.28)$$

It is seen that A_k is a generalized amplitude factor for a progressive wave of the k-th eigenmode. The wire voltages and currents for this mode are A_k times the elements of the k-th columns of $\overline{U}$ and $\overline{J}$. A given eigenmode is thus characterized by a well-defined distribution of the line voltages and currents. As results from (3.13) and (3.14), we may define the characteristic impedance matrix $\overline{z}/\Gamma_k$ and the characteristic admittance matrix $\overline{y}/\Gamma_k$ of the k-th eigenmode.

The eigenvectors are not yet completely defined by (3.21) and (3.22). Since these equations are homogeneous, we are free to choose (2n) normalizing constants for the (2n) eigenvectors. But as $\overline{J}$ and $\overline{U}$ are related by (3.23) or (3.24), n of these constants are determined. It can easily be shown that the eigenvectors $\overline{U}_i$ and $\overline{J}_j$ corresponding to different eigenvalues Γ_i and Γ_j are mutually orthogonal. Consequently the matrix $\overline{U}^T\overline{J}$, where T indicates transposition, is diagonal and the n remaining constant may be chosen so as to impose the constraint

$$\overline{U}^T\overline{J} = \overline{E} \tag{3.29}$$

where $\overline{E}$ is the unit matrix. This is the usual scattering matrix normalisation. It is such that the scalars $\overline{A}^T\overline{A}/2$ and $\overline{B}^T\overline{B}/2$ give the complex power carried through the transverse plane x = 0 by the progressive and regressive waves, respectively. Thus $A_i^2/2$ and $B_i^2/2$ are relative to the i-th eigenmode.

3.2.2. Application to leaky coaxial cables

A first application of the general coupled-line theory developed hereabove is to leaky coaxial cables. Before starting the analysis, we first consider a perfectly shielded cable with exactly the same internal parameters as the leaky cable under study and located at the same place in the same tunnel. This rather trivial case will be referred to by a subscript o, as was already done in section 3.1. Matrices $\overline{z}$ and $\overline{y}$ are now given by

$$\overline{z}_o = \begin{bmatrix} z_{co} & 0 \\ 0 & z_{mo} \end{bmatrix} \qquad \overline{y}_o = \begin{bmatrix} y_{co} & 0 \\ 0 & y_{mo} \end{bmatrix} \tag{3.30}$$

It is easy to verify that the modal matrices are given by

$$\overline{U}_o = \begin{bmatrix} Z_{co}^{1/2} & 0 \\ 0 & Z_{mo}^{1/2} \end{bmatrix} \qquad \overline{J}_o = \begin{bmatrix} Z_{co}^{-1/2} & 0 \\ 0 & Z_{mo}^{-1/2} \end{bmatrix} \tag{3.31}$$

where

$$Z_{co} = \sqrt{z_{co}/y_{co}} \qquad Z_{mo} = \sqrt{z_{mo}/y_{mo}} \tag{3.32}$$

are the characteristic impedances of the - obviously uncoupled - coaxial and monofilar modes, whereas

$$\Gamma_{co} = \sqrt{z_{co}\, y_{co}} \qquad \Gamma_{mo} = \sqrt{z_{mo}\, y_{mo}} \tag{3.33}$$

are their propagation constants.

As was explained previously, $y_{mo} = j\omega c_{mo}$ where the specific capacity may be evaluated from electrostatics. The series impedance

$$z_{mo} = j\omega l_{mo} + r_{mo} \tag{3.34}$$

is nearly reactive ($r_{mo} << \omega l_{mo}$), but r_{mo} and l_{mo} are functions of frequency. Similar considerations apply to the coaxial mode, except that l_{co} may be considered as constant. We may thus use classical approximations for low-loss transmission lines :

$$\begin{gathered} Z \simeq (l/c)^{1/2} \\ \Gamma \simeq \alpha + j\beta \quad ; \quad \alpha \simeq r/(2Z) \quad ; \quad \beta \simeq \omega\sqrt{lc} \\ \beta >> \alpha \end{gathered} \tag{3.35}$$

with adequate subscripts, either mo or co. We define the velocity ratio of the two modes by

$$\rho = \frac{\beta_{co}}{\beta_{mo}} = \frac{v_{mo}}{v_{co}} \tag{3.36}$$

Actually $v_{co} = 3\ 10^8\ \kappa^{-1/2}$ m/s, where κ is the dielectric constant of the cable insulation, whereas v_{mo} is a slowly increasing function of frequency. Above a few MHz however, v_{mo} is close to $3\ 10^8$ m/s and one has

$$\rho \simeq \sqrt{\kappa} \tag{3.37}$$

Once the numerical values of $\overline{y}$ and $\overline{z}$ are available, it is an elementary task to compute the eigenvalues and the modal matrices. Instead, we will concentrate on

approximate formulas valid for a weak coupling. Derivation of these approximations is a straightforward task which is left to the reader. We will restrict ourself to the presentation of the results.

We define two coupling coefficients

$$C_1 = \frac{m_t}{2\sqrt{l_m l_c}} \frac{\sqrt{\rho}}{\rho - 1} \tag{3.39}$$

$$C_2 = \frac{m_t}{2\sqrt{l_m l_c}} \frac{\sqrt{\rho}}{\rho + 1} \tag{3.40}$$

where $l_m = l_{mo} - m_t$ and $l_c = l_{co} - m_t$ are the specific inductances of the monofilar and coaxial modes. In the weak-coupling assuption, we suppose that

$$C_1^2 \text{ , } C_2^2 \text{ and } C_1C_2 << 1 \tag{3.41}$$

In practice, the values of l_{co} range from 250 to 400 nH/m, those of l_{mo} from 800 to 4000 nH/m, while m_t = 40 nH/m must be considered as an upper limit for specific transfer inductances. The weak-coupling assumption may thus be considered as generally valid, excepted when the velocity ratio ρ is close to unity.

The approximations which are found for the propagation constants of the two eigenmodes are then

$$\Gamma_1 = \alpha_1 + j\beta_1 \quad ; \quad \alpha_1 \approx \alpha_{co} + (C_1 - C_2)^2 \alpha_{mo} \tag{3.42}$$

$$\beta_1 \approx \beta_{co} \tag{3.43}$$

$$\Gamma_2 = \alpha_2 + j\beta_2 \quad ; \quad \alpha_2 \approx \alpha_{mo} \tag{3.44}$$

$$\beta_2 \approx \beta_{mo} \tag{3.45}$$

It should already be clear that the subscripts 1 and 2 refer to the coaxial and monofilar eigenmodes, respectively, whereas the subscripts c and m are used for the coupled modes, i.e. for signals propagating inside and outside the cable. These signals are of course a linear combination of the two eigenmodes.

The modal matrices are approximately given by

$$\overline{U} \approx \begin{bmatrix} Z_{co}^{1/2} & -Z_{co}^{1/2}(C_1-C_2) \\ Z_{mo}^{1/2}(C_1+C_2) & Z_{mo}^{1/2} \end{bmatrix} \tag{3.46}$$

$$\overline{J} \approx \begin{bmatrix} Z_{co}^{-1/2} & Z_{co}^{-1/2}(C_1+C_2) \\ Z_{mo}^{-1/2}(C_1-C_2) & Z_{mo}^{-1/2} \end{bmatrix} \tag{3.47}$$

The meaning of the modal matrix elements, given by (3.27) and (3.28) should be kept in mind when interpreting these results. For instance, it is seen that when the coaxial eigenmode propagates, for a unit voltage inside the cable, there is a wall voltage equal to $(C_1 + C_2)\sqrt{Z_{mo}/Z_{co}}$, and for a unit current flowing along the inner conductor, there is a current $(C_1 - C_2)\sqrt{Z_{co}/Z_{mo}}$ flowing along the wall. Of course, the current I_m flowing along the wall is equal to the opposite of the total current carried by the cable, as is obvious from Figure 3.1.

The coaxial eigenmode obviously has the main part of its power propagating inside the cable. Some power is however also carried by the leakage fields outside the cable. These powers are given by

$$\begin{aligned} \frac{1}{2} V_c I_c^* &= \frac{1}{2} |A_1^2| \\ \frac{1}{2} V_m I_m^* &= \frac{1}{2} |A_1^2| \; (C_1^2 - C_2^2) \end{aligned} \tag{3.48}$$

where A_1 is the normalised wave amplitude of the coaxial eigenmode. Similar considerations and formulas apply to the monofilar eigenmode. It thus appears that $(C_1^2 - C_2^2)$ is the relative power of the leakage fields. This explains also in a very simple way the increase of specific attenuation for the coaxial eigenmode given by (3.42). A similar perturbation exists for α_m but it has been neglected since it has been assumed that α_{mo} is significantly higher than α_{co}. Otherwise a leaky coaxial cable would not perform better than a simple monofilar wire conductor.

We may further comment on the coupling coefficients C_1 and C_2. The specific transfer inductance m_t is only a shield parameter and it does not by itself give full

information on the intensity of the leakage fields. The relevant parameters are the coupling coefficients C_1, C_2. They also depend on the internal and external parameters through l_c, l_m and the velocity ratio ρ as shown by (3.39) and (3.40). In particular, a value of ρ close to unity may have more ingluence on the leakage intensity than a high transfer inductance. At high frequencies where (3.37) is valid, this result is obtained by using a cable insulation with a small dielectric constant. For $\rho \approx 1$, we have $C_1 \gg C_2$, but the weak-coupling assumption (3.41) does not necessarily hold any longer. Finally it should be noted that a too high value of C_1 may have drawbacks in continuous leaky feeders, since it may result in an important increase of α_1, as shown by (3.42).

3.2.3. Discontinuities along a leaky feeder

The theory developed in the previous section apply to a homogeneous leaky feeder in the sense given to this term in tranmission line theory, i.e. when the transmission line parameters are independent of the axial coordinate x. Inhomogeneities can be treated by several methods which are standard practice in transmission line theory. In particular, local inhomogeneities or discontinuities may be represented by lumped circuit elements inserted in the line and give rise to boundary conditions to be used in the general solution (3.25) - (3.26). Quite generally, a discontinuity acts on both eigenmodes. This is always true for a discontinuity which acts on only one of the coupled modes, either on (V_m, I_m) or on (V_c, I_c). Consequently, if one eigenmode is incident on the discontinuity, this eigenmode will be partly reflected, partly transmitted beyond the discontinuity, but also partly converted into the other eigenmode. These mode conversions, which also occur along multiconductor lines, may be intentional or inadvertent. Among these discontinuities we may for instance name the ends of the cable, the coupling of a generator, et. It is therefore essential to dominate the subject, in order either to minimize inadvertent conversions or to deliberately create useful ones.

We will now examine with some detail a particular type of discontinuity that will find application in section 3.5. We consider the transition from a non-leaky coaxial cable to a leaky one. The transition is located at x = 0 : waves (A_{co}, A_{mo}, B_{co}, B_{mo}) of the perfectly shielded cable are flowing for x < 0 and waves (A_1, A_2, B_1, B_2) of the leaky cable are flowing for x > 0. The problem can be solved by using equations (3.25) and (3.26) with adequate subscripts on either side of the transition and expressing the discontinuity of the voltages and currents at the transition. The resultant matrix equations

$$\overline{U}_o(\overline{A}_o + \overline{B}_o) = \overline{U}(\overline{A} + \overline{B}) \tag{3.49}$$

$$\overline{J}_o(\overline{A}_o - \overline{B}_o) = \overline{J}(\overline{A} - \overline{B}) \tag{3.50}$$

can be solved for $\overline{B}_o$ and $\overline{A}$ to obtain the scattering matrix of the transition. Under the weak-coupling assumption (3.41), this yields

$$\begin{bmatrix} B_{co} \\ B_{mo} \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} -C_1C_2 & C_2 & \sqrt{1-C_1^2-C_2^2} & -C_1 \\ C_2 & C_1C_2 & C_1 & \sqrt{1-C_1^2-C_2^2} \\ \sqrt{1-C_1^2-C_2^2} & C_1 & -C_1C_2 & -C_2 \\ -C_1 & \sqrt{1-C_1^2-C_2^2} & -C_2 & C_1C_2 \end{bmatrix} \begin{bmatrix} A_{co} \\ A_{mo} \\ B_1 \\ B_2 \end{bmatrix} \tag{3.51}$$

It can be seen that an eigenmode incident on the transition is :

- reflected with a small reflection coefficient $\pm C_1C_2$
- reflected into the other eigenmode with a reflection coefficient $\pm C_2$
- transmitted through the transition in the corresponding mode with a very small loss
- transmitted through the transition into the other mode with a transmission coefficient $\pm C_1$.

3.2.4. The dedicated-wire technique

Electric traction is frequently used in mines and the trolley wire is sometimes used as a monofilar wire conductor to provide radio communications at a few hundreds of kHz in the haulageways. Actually the rails act as a return conductor; they may be modelled as a metallic ground plane (Wait and Hill, 1977a). The trolley wire is primarily designed for power transmission and it is not a very efficient monofilar wire for radio transmission. Devices like locomotives, rectifiers and so on act as shunt loads with rather unpredictable impedance at radio frequencies. Attempts have been made to use radio frequency chokes in series with these loads. Alternatively an additional monofilar wire may be strung into the tunnel to guide electromagnetic waves. This method was proposed in the United States under the name of dedicated-wire technique (Emslie et al., 1978).

Although there is no reason why it should have more influence than any inadvertent conductor that may exist in the tunnel, particular attenuation was devoted to the trolley wire, and namely to the effect of its shunt loading on the propagation of the low-loss mode guided by the dedicated-wire (Hill and Wait, 1977b; Wait and Hill, 1978). The problem can be treated by multiconductor transmission line theory. In the first instance the metallic ground plane modelling the rails is taken into account by considering a doubled-size tunnel containing image wires, as suggested by Figure 3.2. Attention should obviously be restricted to odd modes. Solving the problem of a shunt load then reduces to a rather elementary problem.

Let us assume that the shunt load is located at x = 0 and that eigenmodes with matrix wave amplitudes $\overline{A}^-$ and $\overline{B}^+$ are incident on the load for x < 0 and x > 0, respectively. Using (3.25) and (3/26), we may write the solution

$$\overline{V}(x) = \begin{cases} \overline{U}\left[\overline{\mathrm{Diag}}\ e^{-\Gamma x}\,\overline{A}^- + \overline{\mathrm{Diag}}\ e^{\Gamma x}\,\overline{B}^-\right] \\ \overline{U}\left[\overline{\mathrm{Diag}}\ e^{-\Gamma x}\,\overline{A}^+ + \overline{\mathrm{Diag}}\ e^{\Gamma x}\,\overline{B}^+\right] \end{cases} \tag{3.52}$$

$$\overline{I}(x) = \begin{cases} \overline{J}\left[\overline{\mathrm{Diag}}\ e^{-\Gamma x}\,\overline{A}^- - \overline{\mathrm{Diag}}\ e^{\Gamma x}\,\overline{B}^-\right] \\ \overline{J}\left[\overline{\mathrm{Diag}}\ e^{-\Gamma x}\,\overline{A}^+ - \overline{\mathrm{Diag}}\ e^{\Gamma x}\,\overline{B}^+\right] \end{cases} \tag{3.53}$$

wherein $\overline{B}^-$ and $\overline{A}^+$ are the still unknown matrix amplitudes of the reflected and transmitted waves. Quite generally, the shunt load may be characterized by an admittance matrix $\overline{Y}$ and we have the continuity equations

$$\begin{aligned} \overline{V}(-0) &= \overline{V}(+0) \\ \overline{I}(-0) &= \overline{Y}\,\overline{V}(+0) + \overline{I}(+0) \end{aligned} \tag{3.54}$$

Using these as boundary conditions for (3.52) and (3.53), we obtain the scattering matrix of the shunt load

$$\begin{bmatrix} \overline{B}^- \\ \overline{A}+ \end{bmatrix} = \begin{bmatrix} \overline{K} & \overline{T} \\ \overline{T} & \overline{K} \end{bmatrix} \begin{bmatrix} \overline{A}^- \\ \overline{B}^+ \end{bmatrix} \tag{3.55}$$

where

$$\overline{T} = \left[\overline{E} + \frac{1}{2}\,\overline{J}^{-1}\,\overline{Y}\,\overline{U}\right]^{-1} \tag{3.56}$$

$$\overline{K} = \overline{T} - \overline{E} \tag{3.57}$$

The submatrices $\overline{K}$ and $\overline{T}$ are of order n if there are n wires and are generalized reflection and transmission matrices. They can easily be calculated if the modal matrices $\overline{J}$ and $\overline{U}$ are known. In general, mode conversion occurs in reflection as well as in transmission, in the sense that, if one eigenmode is incident on the shunt load, all eigenmodes are generated in the two directions away from the discontinuity. This conclusion is of practical importance and provides a good introduction to mode conversion techniques. Note that reciprocity requires $\overline{K}$ and $\overline{T}$ to be symmetric. This example shows how powerful the coupled line theory developped in section 3.2.1 can be.

3.3. GENERAL PROPERTIES OF MODE CONVERTERS

By comparison with the simple monofilar-wire technique, the main idea in the use of continuous leaky coaxial cables is to benefit by the low specific attenuation of the coaxial eigenmode. This attenuation remains small because only a small part of the electromagnetic energy is released in the tunnel space in the form of leakage fields. An alternative solution to the continuous leaky feeder is to use a well-shielded coaxial cable in which mode converters are inserted at discrete places. This principle can also be used with a two-wire line like a ribbon feeder. The function of the mode converters is to convert a small part of the coaxial or bifilar mode power into the monofilar wire supported by the non-leaky transmission line. An obvious requirement for the mode converters is to provide a good impedance match and a low insertion loss for the coaxial or bifilar mode : ideally the loss should be due to the mode conversion only.

This method provides an excellent flexibility in the design of a subsurface communication system because the converter parameters and spacing can be varied along the path in function of the tunnel cross-section, acceptable cable location, distance to the base station, and so on. Mode converters are now widely used in subsurface radio communications (Delogne, 1970; De Keyser, 1972; Delogne, 1972; Deryck, 1972; Delogne, 1973; De Keyser, 1973; Delogne et al., 1973; Delogne, 1974; De Keyser, 1974; Deryck, 1975; De Keyser et al., 1978; Seidel and Wait, 1978c; Delogne, 1979).

A detailed description of several types of mode converters will be given in subsequent sections. Presently we will show that the operational parameters of mode converters are subject to some limitations. The analyses carried out in the present chapter are restricted to frequencies where only transmission line type modes

are significant, but it can be shown that the same devices can be used as efficient launchers for waveguide modes.

Considering the existence of two modes on either side, a mode converter should be seen as a four-port device. We will use the subscripts 1 and 2 to denote the two sides of the converter and the subscripts c and m to indicate the coaxial (or bifilar) and monofilar modes, respectively. We use the normalized waves A and B which are flowing into and emerging out from the device, respectively. By reciprocity, the scattering matrix which relates them should be symmetric. It has thus the following most general form

$$\begin{bmatrix} B_{c1} \\ B_{m1} \\ B_{c2} \\ B_{m2} \end{bmatrix} = \begin{bmatrix} K_{c1} & K_{cm1} & T_c & T_{c1m2} \\ K_{cm1} & K_{m1} & T_{c2m1} & T_m \\ T_c & T_{c2m1} & K_{c2} & K_{cm2} \\ T_{c1m2} & T_m & K_{cm2} & K_{m2} \end{bmatrix} \begin{bmatrix} A_{c1} \\ A_{m1} \\ A_{c2} \\ A_{m2} \end{bmatrix} \qquad (3.58)$$

The meaning of the scattering matrix elements is as follows and is illustrated on Figure 3.3 :

K_{c1} , K_{c2} : reflection factors of the coaxial mode at sides 1 and 2.

K_{m1} , K_{m2} : reflection factors of the monofilar mode at sides 1 and 2

K_{cm1}, K_{cm2} : mode conversion factors in reflection at sides 1 and 2

T_{c1m2} : mode conversion factor in transmission between the coaxial mode at side 1 and the monofilar mode at side 2

T_{c2m1} : same, between the coaxial mode at side 2 and the monofilar mode at side 1.

The mode converters are most frequently lossless devices and the scattering matrix must be self-adjoint. Most types have moreover a transverse symmetry plane for the monofilar mode and either a symmetry or an antisymmetry plane for the coaxial mode. When combined, these properties yield :

$$\begin{aligned} |K_{c1}| &= |K_{c2}| = K_c \\ |K_{m1}| &= |K_{m2}| = K_m \\ |K_{cm1}| &= |K_{cm2}| = K_{cm} \\ |T_{c1m2}| &= |T_{c2m1}| = T_{cm} \end{aligned} \qquad (3.59)$$

$$|K_c^2| + |T_c^2| + |K_{cm}^2| + |T_{cm}^2| = 1 \qquad (3.60)$$

$$|K_m^2| + |T_m^2| + |K_{cm}^2| + |T_{cm}^2| = 1 \qquad (3.61)$$

and some phase relationships. The transverse symmetry plane thus not necessarily implies the equality of the mode conversion factors in reflection and in transmission, nor the equality of the reflection factors or of the transmission factors for the two modes.

For obvious practical reasons, mode converters should have no connection to the tunnel wall. If the mode converter is made with lumped elements, this implies $I_{m1} + I_{m2} = 0$, or

$$A_{m1} - B_{m1} = A_{m2} - B_{m2} \qquad (3.62)$$

As this condition must be satisfied for all combinations of incident waves, one has

$$K_{cm1} = - T_{c1m2} \qquad (3.63)$$

$$K_{cm2} = - T_{c2m1} \qquad (3.64)$$

$$K_{m1} + T_m = K_{m2} + T_m = 1 \qquad (3.65)$$

In particular, it is seen that a coaxial wave incident on the converter always excite two equal monofilar waves in opposite directions. From this we conclude that a directional mode conversion can only be realised by mode converters with a non-zero electrical length.

Finally it the mode converter is lossless, has a transverse symmetry plane, no ground connection and a zero electrical length, it is described by reflection factors K_c, K_m, transmission factors T_c, T_m and a single mode conversion factor S, with the property

$$K_c^2 + T_c^2 + 2S^2 = K_m^2 + T_m^2 + 2S^2 = 1 \qquad (3.66)$$

3.4. ANNULAR-SLOT MODE CONVERTERS

3.4.1. Principle

A short annular slot realising a complete interruption of the outer conductor of a non-leaky coaxial cable has been proposed as the main ingredient of a mode converter. The relevant electromagnetic aspects are available in the book mentioned in section 1.1. A simplified quasi-static analysis can be obtained by considering the cable and the tunnel as two transmission lines having a common conductor. The latter is interrupted over a short length. The problem thus reduces to an elementary circuit calculation suggested by Figure 1.10 where Z_m and Z_c are the characteristic impedances of the monofilar and coaxial modes, respectively. The only difficult point in this respect is to estimate the value of Z_m, but we have seen that this quantity fortunately varies in a very limited range with the electrical parameters of the ground, with the geometrical parameters like tunnel cross-section and cable location, and with the frequency. A comparison with the few exact electromagnetic solutions available between 1 and 50 MHz shows that calculations based on the value $Z_m = 377\ \Omega$ never yield an error larger than 1 dB on the mode conversion factor. This value has been selected for convenience only.

A naked slot does not provide a good impedance match and a low insertion loss for the coaxial mode. The external load impedance "seen" by the slot ($2Z_m$) is indeed rather large, so that most power flowing inside the coaxial cable is reflected back inside the cable. It is thus necessary to add some lumped circuit elements in order to improve the impedance match and to lower the insertion loss.

3.4.2. Resonant matching

In this type of mode converter, the slot impedance is deliberately lowered by connecting a reactance across it. The residual reactive effect can further be compensated at the design frequency by inserting a dual reactance in the inner conductor. Figures 3.4a and 3.4b show two such designs. Eqns (3.58) to (3.66) all apply to this circuit. The elements of the scattering matrix can easily be obtained from the circuit equations, but the calculations are somewhat tedious. The results are shown on Figure 3.5 for a current value of the ratio Z_c/Z_m. The curves are drawn as functions of a reduced frequency ν and of a Q-factor. These parameters are defined as follows :

$$\omega_c = (LC)^{-1/2}$$

and

- Figure 3.4a : $Q = 2Z_m/(\omega_o L)$; $\nu = \omega_o/\omega$
- Figure 3.4b : $Q = 2Z_m\omega_o C$; $\nu = \omega/\omega_o$

The choice of the circuit elements allows some control on the converter performance. Typical values for Q are 6 to 8. the circuit provides a moderate bandwidth. The parameters K_m and T_m have not been shown because they are not very important in most applications, but it may be useful to know that K_m is very small and T_m close to unity.

3.4.3. Wideband matching

When mode converters have to work over very wide frequency bands, a transformer may be integrated in the annular slot. In these types of converters, the bandwidth is only limited by the transformer frequency response.

A first example is shown on Figure 3.6a. The winding sense is such that the coaxial mode would not develop any voltage across the slot if n_1 and n_2 were equal ; hence no mode conversion would occur. A small imbalance allows to create some mode conversion. The device has all the properties considered in the last paragraph of section 3.3. It is easy to show that

$$K_c = T_m = \frac{\xi}{2 + \xi}$$

$$K_m = T_c = \frac{2}{2 + \xi} \qquad (3.67)$$

$$S = \frac{(2\xi)^{1/2}}{2 + \xi}$$

where

$$\xi = \frac{2Z_m}{Z_c}\,(1 - n_1/n_2)^2 \qquad (3.68)$$

A small imbalance will thus yield a low value for the reflection factor and insertion loss of the coaxial mode, but a high value of the same parameters of the monofilar mode.

When this is not desirable, the converter of Figure 3.6b may be used. In spite of the apparent simplicity of the circuit, the calculation of the scattering matrix

elements is extremely tedious, for there is no transverse symmetry plane. The general form (3.58) together with the properties (3.63) to (3.65) apply. One finds :

$$K_{c1} = \left[2(n_1^2 - n_2^2)x - (n_1 - n_2)^2x^{-1}\right] /D$$

$$K_{c2} = \left[2(n_2^2 - n_1^2)x - (n_2 - n_1)^2x^{-1}\right] /D$$

$$K_{m1} = K_{m2} = (n_1 - n_2)^2x^{-1} /D$$

$$T_c = 4n_1n_2/D \tag{3.69}$$

$$T_m = 2(n_1^2 + n_2^2)x/D$$

$$K_{cm1} = - T_{c1m2} = 2n_1(n_1 - n_2) /D$$

$$K_{cm2} = - T_{c2m1} = 2n_2(n_2 - n_1) /D$$

with

$$x = (Z_m/Z_c)^{1/2}$$
$$D = 2(n_1^2 + n_2^2)x + (n_1 - n_2)^2x^{-1} \tag{3.70}$$

A small imbalance allows here to have small values of the reflection factor and of the insertion loss for both modes. It is thus easy to fit the characteristics of mode converters to the requirements of a specific application.

3.5. LEAKY SECTION AS A DIRECTIVE MODE CONVERTER

The mode converters described sofar are non-directive in the sense that the mode conversion factors in reflection and in transmission are equal. A problem may result from this property when several mode converters are inserted in a cable. Standing waves are observed in the tunnel space because of monofilar mode waves travelling in opposite directions. In the early stages of the use of annular-slot

mode converters (De Keyser et al., 1970; Delogne, 1970; De Keyser, 1972; Delogne, 1972), this problem was considered as a serious one and it was solved by a classical technique used in waveguide directional couplers : the slots were used in pairs with a quarter-wavelength spacing between the elements of a pair. Later on, experience showed that standing waves anyway frequently arise because of inadvertent mode conversions in tunnels containing other conductors than the coaxial cable, and this method was given up.

We will now retrieve directional mode conversion as automatically provided in converters consisting of a short section of a leaky coaxial cable inserted in a non-leaky cable. The transition from a non-leaky cable to a leaky one was investigated in section 3.2.3 and is described by the scattering matrix (3.51). Mode conversion occurs in the two directions at the transition but, of course, the coaxial and monofilar modes are not identical on either side of the transition. A non-zero length mode converter can be build by inserting a leaky section of length L inside a non-leaky cable. In this case, the two transitions participate in the mode conversion process. The scattering matrix of the leaky section considered as a whole can easily be calculated, referring to Figure 3.7 and eq. (3.51). Neglecting the second-order terms as permitted by the weak-coupling assumption (3.41), we find (*)

$$|K_c| = |K_m| = 0 \tag{3.71}$$

$$|K_{cm}| = 2C_2 \sin\left[(\beta_{co} + \beta_{mo}) L/2\right] \tag{3.72}$$

$$|T_{cm}| = 2C_1 \sin\left[(\beta_{co} - \beta_{mo}) L/2\right] \tag{3.73}$$

$$|T_c^2| = |T_m^2| = 1 - |K_{cm}^2| - |T_{cm}^2| \tag{3.74}$$

The non-equality of the mode conversion factors in reflection K_{cm} and in transmission T_{cm} is the result of the non-zero electrical length of the device. As the phase constants β_{mo} and β_{co} are proportional to the frequency, the bandwidth of T_{cm} around a maximum of the sine function is larger than that of K_{cm}. It is

(*) Here we take account of the fact that, in transmission line theory, the A and B wavesare progressive and regressive waves, respectively. In network theory however they are ingoing and outgoing waves, respectively, and the scattering matrix relates the latter to the former.

moreover maximum for

$$(\beta_{co} - \beta_{mo})L = \pi \qquad (3.75)$$

The 3-dB bandwidth of T_{cm} thus extends from $f_o/2$ to $3f_o/2$, where f_o is the design frequency yielding (3.75). The choice of L is thus not at all critical. For the design of a mode converter, we may consider that

$$\beta_{mo} \simeq k_o = \omega/(3\ 10^8) \qquad (3.76)$$

$$\beta_{co} \simeq k_o\ \kappa^{1/2} \qquad (3.77)$$

where k_o is the free-space wavenumber and κ is the dielectric constant of the cable insulation. It is seen from (3.39) and (3.40) that $C_1 > C_2$, and thus $T_{cm} > K_{cm}$. The mode conversion thereby exhibit an intrinsic directivity which can be enhanced by chosing a low value for κ; this yields an increase of T_{cm}.

Several interesting properties of a coaxial cable containing periodic leaky sections are worth mentioning. As we have seen in section 3.23, for a continuous leaky cable, the relative power of the leakage fields of the coaxial eigenmode is C_1^2 (we assume that $C_1 >> C_2$). A leaky section excites the monofilar mode with a power 4 C_1^2. We have thus a gain of 6 dB. This property as well as those of a continuous leaky cable are nicely illustrated by Figure 3.8. Another useful characteristic is that the insertion loss of a leaky section for the coaxial mode, given by Eq. (3.74), remains small : for instance C_1 = 0.15 yields a conversion factor T_{cm} of - 9 dB and an insertion loss T_c of 0.5 dB. As a matter of fact the global specific attenuation of a coaxial cable containing a leaky section every d meters will be

$$\alpha_c = \alpha_{co} + \frac{10}{d}\ \log \frac{1}{1 - 4\ C_1^2} \qquad (3.78)$$

This may be significantly smaller than the attenuation (3.42) of a continous leaky cable. This property, together with the 6-dB gain mentioned above, should overcompensate the fact that the radiated field suffers the attenuation α_{mo} of the monofilar mode between two successive leaky sections. The question however requires a closer examination since the optimum value of C_1 for a continuous leaky cable and for a leaky section are not necessarily equal.

3.6. MODE CONVERTERS FOR TWO-WIRE LINES

By lack of place we will simply mention that mode converters exist for this type of line and refer the interested reader to the book mentioned in section 1.1.

3.7. INADVERTENT MODE CONVERSION

We mentioned previously that any discontinuity along a tunnel containing axial conductors is likely to cause mode conversion. Numerous discontinuities may exist in any practical application. Though it is in general impossible to evaluate them quantitatively, it is nevertheless important and sometimes vital for predictions to have a clear picture of the possible effects.

A first type of discontinuity which always exists consist of the hanging devices used to support monofilar and bifilar lines or leaky coaxial cables. In general these devices act as small local shunt capacitors and they can in principle be studied by the method outlined in section 3.2.3. If the spacing is much smaller than the wavelength, the effect is equivalent to an increase of the specific capacity and has negligible consequences, apart if it disturbs symmetry. It was the merit of Deryck (1970, 1972b, 1973) to show that the apparent leaky character of the ribbon feeder was actually an inadvertent mode conversion process due to numerous slight asymmetries, among which are the hanging devices. Periodic spacing, even of small discontinuities, may however be a source of serious perturbation when the spacing is close to a multiple of the half wavelength, because of a filtering effect.

Objects with a small axial size are another type of discontinuity. They in general may be modelled as local capacities. The disturbance therefore increases with frequency. Experience has shown and theory confirms that the effect of an isolated object is mainly local. Indeed the shunt impedance due to the object is generally very small compared with the characteristic impedance of the modes. It has a negligible effect on propagation, although the local disturbance of the electromagnetic fields may be important. This does not remain true when numerous objects exist along the path, because of the resulting accumulation. For a large number of random objects with random spacing, the fields in the tunnel space become rapidly quite irregular.

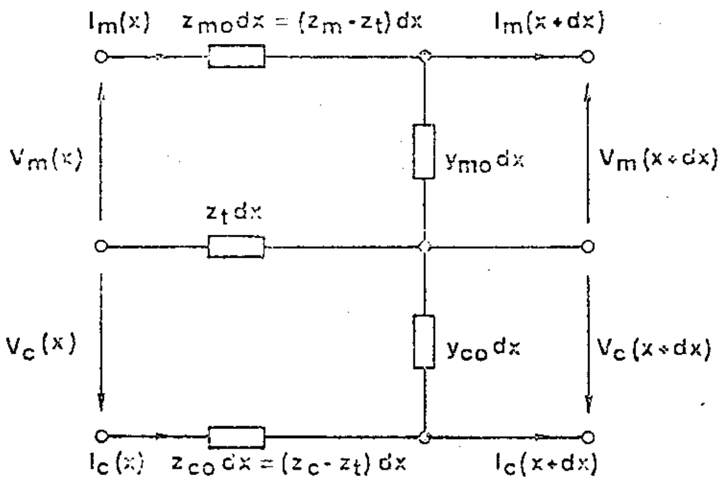

Fig. 3.1 Equivalent circuit of an elementary length dx of a leaky coaxial cable inside a tunnel.

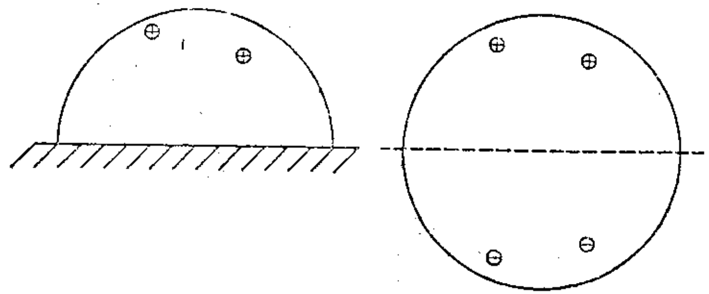

Fig. 3.2 Equivalence of tunnel with metallic ground plane and tunnel with doubled size and image wires.

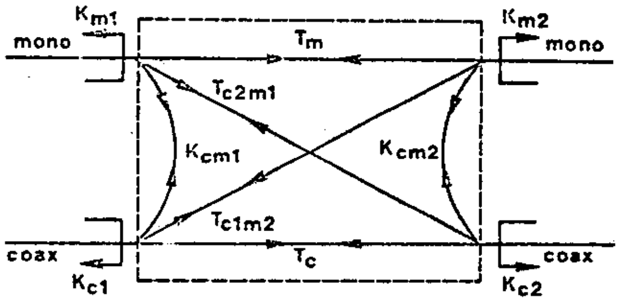

Fig. 3.3 Flow graph to illustrate the meaning of the scattering matrix elements of a mode converter.

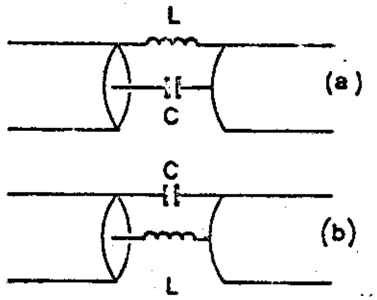

Fig. 3.4 Resonant matchning of an annular-slot mode converter.

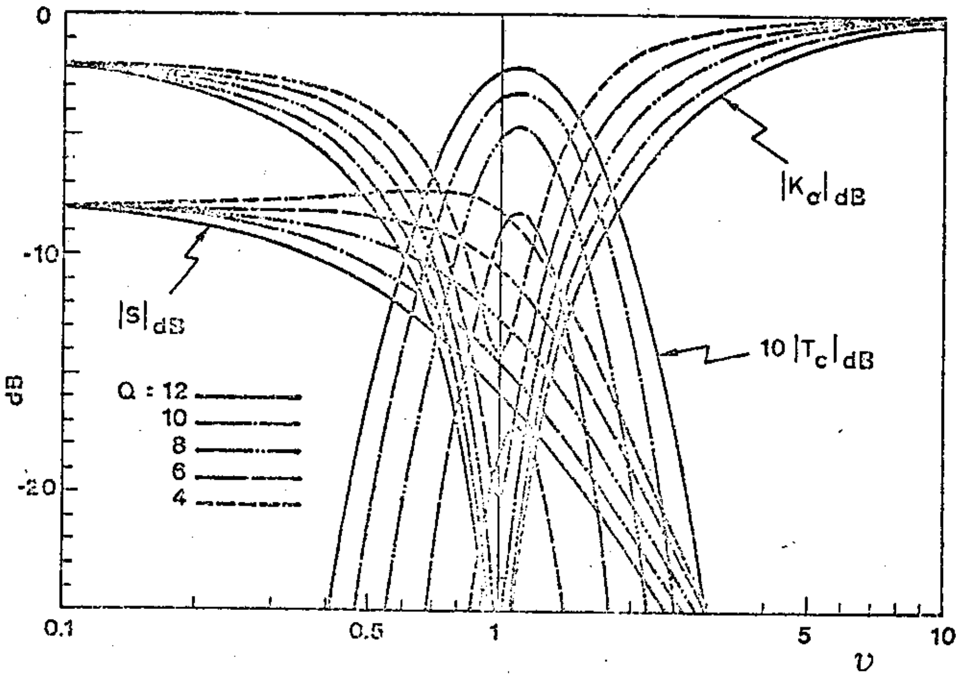

Fig. 3.5 Performance curves of selective annular-slot mode converter.

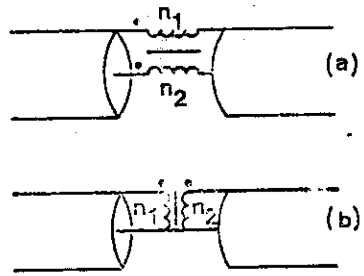

Fig. 3.6 Wideband annular-slot mode converters.

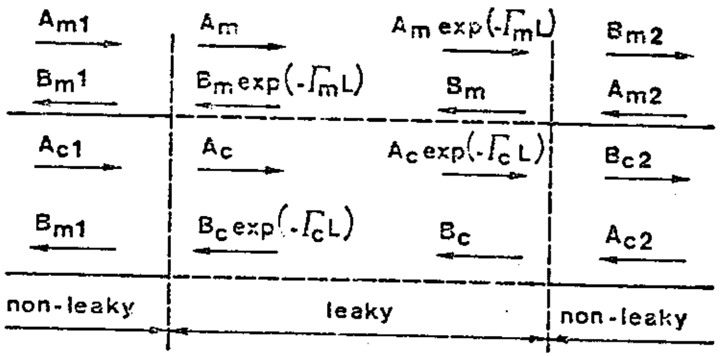

Fig. 3.7 Normalized waves in a leaky-section type mode converter.

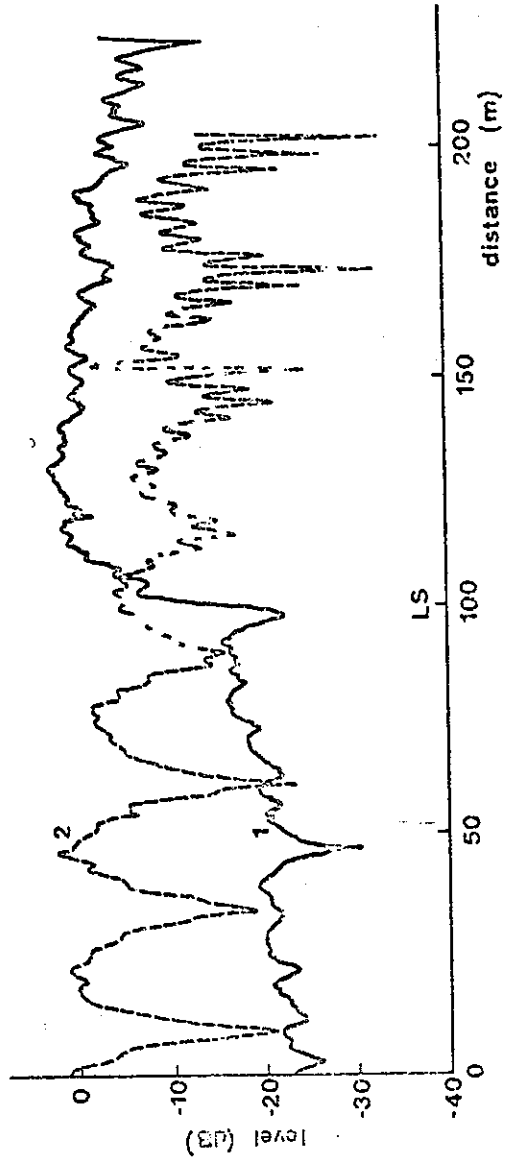

Fig. 3.8 Field level measured along a continuous leaky feeder (curve 2) and along a non-leaky cable with a leaky section of the same type located at 100 m from the generator (curve 1). The generator power has been adjusted to have the same power inside the cable at this point. The frequency is 35 MHz. The dielectric constant of the cable insulation is 1.89.

CALCULATION OF FIELDS FROM POTENTIALS IN THE CARTESIAN COORDINATE SYSTEM

The sourceless $(\overline{J}_a = 0 \; , \; \overline{J}_{ma} = 0)$ part of (2.8) and (2.9) yields, for each cartesian component of the potentials

$$\pi_x' \; : \; \overline{E} = \left(-\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \; , \; \frac{\partial^2}{\partial x \partial y} \; , \; \frac{\partial^2}{\partial x \partial y} \right) \pi_x' \qquad \text{(A.1)}$$

$$\overline{H} = j\omega\varepsilon \left(0 \; , \; \frac{\partial}{\partial z} \; , \; -\frac{\partial}{\partial y} \right) \pi_x' \qquad \text{(A.2)}$$

$$\pi_y' \; : \; \overline{E} = \left(\frac{\partial^2}{\partial x \partial y} \; , \; -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2} \; , \; \frac{\partial^2}{\partial y \partial z} \right) \pi_y' \qquad \text{(A.3)}$$

$$\overline{H} = j\omega\varepsilon \left(-\frac{\partial}{\partial z} \; , \; 0 \; , \; \frac{\partial}{\partial x} \right) \pi_y' \qquad \text{(A.4)}$$

$$\pi_z' \; : \; \overline{E} = \left(\frac{\partial^2}{\partial x \partial z} \; , \; \frac{\partial^2}{\partial y \partial z} \; , \; -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \pi_z' \qquad \text{(A.5)}$$

$$\overline{H} = j\omega\varepsilon \left(\frac{\partial}{\partial y} \; , \; -\frac{\partial}{\partial x} \; , \; 0 \right) \pi_z' \qquad \text{(A.6)}$$

$$\pi_x'' \; : \; \overline{E} = -j\omega\mu_o \left(0 \; , \; \frac{\partial}{\partial z} \; , \; -\frac{\partial}{\partial y} \right) \pi_x'' \qquad \text{(A.7)}$$

$$\overline{H} = \left(-\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \; , \; \frac{\partial^2}{\partial x \partial y} \; , \; \frac{\partial^2}{\partial x \partial z} \right) \pi_x'' \qquad \text{(A.8)}$$

$$\pi_y'' \; : \; \overline{E} = -j\omega\mu_o \left(-\frac{\partial}{\partial z} \; , \; 0 \; , \; \frac{\partial}{\partial x} \right) \pi_y'' \qquad \text{(A.9)}$$

$$\overline{H} = \left(\frac{\partial^2}{\partial x \partial y} \; , \; -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2} \; , \; \frac{\partial^2}{\partial y \partial z} \right) \pi_y'' \qquad \text{(A.10)}$$

$$\pi_z'' \; : \; \overline{E} = -j\omega\mu_o \left(\frac{\partial}{\partial y} \; , \; -\frac{\partial}{\partial x} \; , \; 0 \right) \pi_z'' \qquad \text{(A.11)}$$

$$\overline{H} = \left(\frac{\partial^2}{\partial x \partial z} \; , \; \frac{\partial^2}{\partial y \partial z} \; , \; -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \pi_z'' \qquad \text{(A.12)}$$

For the calculation of a mode with a propagation factor $\exp(-\Gamma z)$ one has, taking into account that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \Gamma^2 + k^2 \right) \pi = 0$:

$$\pi_x' \; : \; \overline{E} = \left(-\frac{\partial^2}{\partial y^2} - \Gamma^2 \; , \; \frac{\partial^2}{\partial x \partial y} \; , \; -\Gamma \frac{\partial}{\partial x} \right) \pi_x' \qquad \text{(A.13)}$$

$$= \left(\frac{\partial^2}{\partial x^2} + k^2 \; , \; \frac{\partial^2}{\partial x \partial y} \; , \; -\Gamma \frac{\partial}{\partial x} \right) \pi_x' \qquad \text{(A.14)}$$

$$\overline{H} = j\omega\varepsilon \left(0 \; , \; -\Gamma \; , \; -\frac{\partial}{\partial y} \right) \pi_x' \qquad \text{(A.15)}$$

$$\pi_y' \; : \; \overline{E} = \left(\frac{\partial^2}{\partial x \partial y} \; , \; -\frac{\partial^2}{\partial x^2} - \Gamma^2 \; , \; -\Gamma \frac{\partial}{\partial y} \right) \pi_y' \qquad \text{(A.16)}$$

$$[illegible] \; \partial^2 \; [illegible] \; , \; -\Gamma \frac{\partial}{\partial y} \Big) \pi_y' \qquad \text{(A.17)}$$

$$[illegible] \; , \; 0 \; , \; \frac{\partial}{\partial x} \Big) \pi_y' \qquad \text{(A.18)}$$

$$\pi_z' \; : \; \overline{E} = \left(-\Gamma \frac{\partial}{\partial x} \; , \; -\Gamma \frac{\partial}{\partial y} \; , \; -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \pi_z' \qquad \text{(A.19)}$$

$$= \left(-\Gamma \frac{\partial}{\partial x} \; , \; -\Gamma \frac{\partial}{\partial y} \; , \; k^2 + \Gamma^2 \right) \pi_z' \qquad \text{(A.20)}$$

$$\overline{H} = j\omega\varepsilon \left(\frac{\partial}{\partial y} \; , \; -\frac{\partial}{\partial x} \; , \; 0 \right) \pi_z' \qquad \text{(A.21)}$$

$$\pi_x'' \;:\; \overline{E} = -j\omega\mu_o \left[0 \quad , \quad -\Gamma \quad , \quad -\frac{\partial}{\partial y} \right] \pi_x'' \qquad (A.22)$$

$$\overline{H} = \left[-\frac{\partial^2}{\partial y^2} - \Gamma^2 \quad , \quad \frac{\partial}{\partial x \partial y} \quad , \quad -\Gamma \frac{\partial}{\partial x} \right] \pi_x'' \qquad (A.23)$$

$$= \left[\frac{\partial^2}{\partial x^2} + k^2 \quad , \quad \frac{\partial}{\partial x \partial y} \quad , \quad -\Gamma \frac{\partial}{\partial x} \right] \pi_x'' \qquad (A.24)$$

$$\pi_y'' \;:\; \overline{E} = -j\omega\mu_o \left[\Gamma \quad , \quad 0 \quad , \quad \frac{\partial}{\partial x} \right] \pi_y'' \qquad (A.25)$$

$$\overline{H} = \left[\frac{\partial^2}{\partial x \partial y} \quad , \quad -\frac{\partial^2}{\partial x^2} - \Gamma^2 \quad , \quad -\Gamma \frac{\partial}{[illegible]} \right] [illegible]_y$$

$$= \left[\frac{\partial^2}{\partial x \partial y} \quad , \quad \frac{\partial^2}{\partial y^2} + k^2 \quad , \quad [illegible] \right] [illegible]$$

$$\pi_z'' \;:\; \overline{E} = -j\omega\mu_o \left[\frac{\partial}{\partial y} \quad , \quad -\frac{\partial}{\partial x} \quad , \quad 0 \right] \pi_z'' \qquad (A.28)$$

$$\overline{H} = \left[-\Gamma \frac{\partial}{\partial x} \quad , \quad -\Gamma \frac{\partial}{\partial y} \quad , \quad -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right] \pi_z'' \qquad (A.29)$$

$$= \left[-\Gamma \frac{\partial}{\partial x} \quad , \quad -\Gamma \frac{\partial}{\partial y} \quad , \quad k^2 + \Gamma^2 \right] \pi_z'' \qquad (A.30)$$

APPENDIX B

CALCULATION OF FIELDS FROM POTENTIALS $U = \pi_z'$ and $V = \pi_z''$ IN THE CYLINDRICAL COORDINATE SYSTEM

We recall that U and V satisfy the equations

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) U = (j\omega\varepsilon)^{-1} J_{az} \qquad (B.1)$$

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) V = (j\omega\mu)^{-1} J_{maz} \qquad (B.2)$$

The sourceless $(J_a = 0 , J_{ma} = 0)$ part of (2.8) and (2.9) yields

$$[illegible] \; \rho \; \partial\phi \qquad (B.3)$$

$$E_\phi = \frac{1}{\rho} \frac{\partial^2 U}{\partial \phi \partial z} + j\omega\mu_o \frac{\partial V}{\partial \rho} \qquad (B.4)$$

$$E_z = \left(\frac{\partial^2}{\partial z^2} + k^2 \right) U \qquad (B.5)$$

$$H_\rho = \frac{j\omega\varepsilon}{\rho} \frac{\partial U}{\partial \phi} + \frac{\partial^2 V}{\partial \rho \partial z} \qquad (B.6)$$

$$H_\phi = -j\omega\varepsilon \frac{\partial U}{\partial \rho} + \frac{1}{\rho} \frac{\partial^2 V}{\partial \phi \partial z} \qquad (B.7)$$

$$H_z = \left(\frac{\partial^2}{\partial z^2} + k^2 \right) V \qquad (B.8)$$

For a mode with a propagation factor $\exp(-\Gamma z)$, one has :

$$E_\rho = -\Gamma \frac{\partial U}{\partial \rho} - \frac{j\omega\mu_o}{\rho} \frac{\partial V}{\partial \phi} \qquad (B.9)$$

$$E_\phi = \frac{-\Gamma}{\rho} \frac{\partial U}{\partial \phi} + j\omega\mu_o \frac{\partial V}{\partial \rho} \qquad (B.10)$$

$$E_z = (\Gamma^2 + k^2)\, U \qquad (B.11)$$

$$H_\rho = \frac{j\omega\varepsilon}{\rho} \frac{\partial U}{\partial \phi} - \Gamma \frac{\partial V}{\partial \rho} \qquad (B.12)$$

$$H_\phi = -j\omega\varepsilon \frac{\partial U}{\partial \rho} - \frac{\Gamma}{\rho} \frac{\partial V}{\partial \phi} \qquad (B.13)$$

$$H_z = (\Gamma^2 + k^2)\, V \qquad (B.14)$$

Next we examine the development of modes in cylindrical harmonics. All quantities are expanded in a Fourier series of the type

$$F(\rho,\phi,z) = \sum_{m=-\infty}^{\infty} F_m(\rho)\, e^{jm\phi}\, e^{-\Gamma z} \qquad (B.15)$$

In a sourceless region the solution of (B.1) and (B.2) yields

$$U_m = Q_m\, I_m(u\rho) + B_m\, K_m(u\rho) \qquad (B.16)$$

$$V_m = P_m\, I_m(u\rho) + A_m\, K_m(u\rho) \qquad (B.17)$$

where A_m , B_m , P_m , Q_m are arbitrary constants and I_m , K_m are modified Bessel functions. We defined

$$u = \sqrt{-k^2 - \Gamma^2} = \sqrt{\gamma^2 - \Gamma^2} \quad ; \quad -\frac{\pi}{2} < \arg u < \frac{\pi}{2} \qquad (B.18)$$

On using these expressions in (B.9) to (B.14) we obtain :

$$E_{\rho m} = -\Gamma u \left[Q_m\, I'_m(u\rho) + B_m\, K'_m(u\rho)\right] + \frac{\omega\mu_o m}{\rho} \left[P_m\, I_m(u\rho) + A_m\, K_m(u\rho)\right] \qquad (B.19)$$

$$E_{\phi m} = -\frac{j\Gamma m}{\rho} \left[Q_m\, I_m(u\rho) + B_m\, K_m(u\rho)\right] + j\omega\mu_o u \left[P_m\, I'_m(u\rho) + A_m\, K'_m(u\rho)\right] \qquad (B.20)$$

$$E_{zm} = -u^2 \left[Q_m\, I_m(u\rho) + B_m\, K_m(u\rho)\right] \qquad (B.21)$$

$$H_{\rho m} = -\frac{\omega\varepsilon m}{\rho} \left[Q_m\, I_m(u\rho) + B_m\, K_m(u\rho)\right] - \Gamma u \left[P_m\, I'_m(u\rho) + A_m\, K'_m(u\rho)\right] \qquad (B.22)$$

$$H_{\phi m} = -j\omega\varepsilon u \left[Q_m\, I'_m(u\rho) + B_m\, K'_m(u\rho)\right] - \frac{j\Gamma m}{\rho} \left[P_m\, I_m(u\rho) + A_m\, K_m(u\rho)\right] \qquad (B.23)$$

$$H_{zm} = -u^2 \left[P_m\, I_m(u\rho) + A_m\, K_m(u\rho)\right] \qquad (B.24)$$

Alternatively these formulas may be replaced by

$$U_m = Q_m\, J_m(\Lambda\rho) + B_m\, H^{(2)}_m(\Lambda\rho) \qquad (B.25)$$

$$V_m = P_m\, J_m(\Lambda\rho) + A_m\, H^{(2)}_m(\Lambda\rho) \qquad (B.26)$$

where

$$\Lambda = \sqrt{k^2 + \Gamma^2} \quad ; \quad -\pi < \arg\Lambda \leq 0 \qquad (B.27)$$

We then have

$$E_{\rho m} = -\Gamma\Lambda \left[Q_m\, J'_m(\Lambda\rho) + B_m\, H^{(2)\prime}_m(\Lambda\rho)\right] - \frac{j\omega\mu_o m}{\rho} \left[P_m\, J_m(\Lambda\rho) + A_m\, H^{(2)}_m(\Lambda\rho)\right] \qquad (B.28)$$

$$E_{\phi m} = +\frac{\Gamma m}{\rho} \left[Q_m\, J_m(\Lambda\rho) + B_m\, H^{(2)}_m(\Lambda\rho)\right] + j\omega\mu_o\Lambda \left[P_m J'_m(\Lambda\rho) + A_m\, H^{(2)\prime}_m(\Lambda\rho)\right] \qquad (B.29)$$

$$E_{zm} = \Lambda^2 \left[Q_m\, J_m(\Lambda\rho) + B_m\, H^{(2)}_m(\Lambda\rho)\right] \qquad (B.30)$$

$$H_{\rho m} = -\frac{j\omega\varepsilon m}{\rho}\left[Q_m J_m(\Lambda\rho) + B_m H_m^{(2)}(\Lambda\rho)\right] - \Gamma\Lambda\left[P_m J_m'(\Lambda\rho) + A_m H_m^{(2)\prime}(\Lambda\rho)\right] \tag{B.31}$$

$$H_{\phi m} = -j\omega\varepsilon\Lambda\left[Q_m J_m'(\Lambda\rho) + B_m H_m^{(2)\prime}(\Lambda\rho)\right] - \frac{\Gamma m}{\rho}\left[P_m J_m(\Lambda\rho) + A_m H_m^{(2)}(\Lambda\rho)\right] \tag{B.32}$$

$$H_{zm} = \Lambda^2\left[P_m J_m(\Lambda\rho) + A_m H_m^{(2)}(\Lambda\rho)\right] \tag{B.33}$$

APPENDIX C

SOME USEFUL FORMULAS ON BESSEL FUNCTIONS

UNMODIFIED BESSEL'S FUNCTIONS

The unmodified Bessel's equation

$$\left(\frac{d^2}{dz^2} + \frac{1}{z}\frac{d}{dz} + 1 - \frac{m^2}{z^2}\right) Z_m(z) = 0 \tag{C.1}$$

may be solved using any two of the linearly independent solutions $J_m(z)$, $N_m(z)$, $H_m^{(1)}(z)$ and $H_m^{(2)}(z)$. In the frame of this book, m is an integer and z is a complex variable with $-\pi < \arg z \leq 0$. We use the fundamental set of solutions $J_m(z)$ and $H_m^{(2)}(z)$. The wronskian of these functions is

$$J_m(z)\, H_m^{(2)\prime}(z) - J_m'(z)\, H_m^{(2)}(z) = \frac{-2j}{\pi z} \tag{C.2}$$

We have

$$J_m(z)\, H_{m-1}^{(2)}(z) - J_{m-1}(z)\, H_m^{(2)}(z) = \frac{-2j}{\pi z} \tag{C.3}$$

For any one of the functions $J_m(z)$ and $H_m^{(2)}(z)$:

$$\left(\frac{d}{dz} \pm \frac{m}{z}\right) Z_m(z) = \pm Z_{m\mp 1}(z) \tag{C.4}$$

Asymptotic values :

$$\left.\begin{aligned} &J_o(z) = 1 \\ &H_o^{(2)}(z) = \frac{-2j}{\pi}\ln\left(\frac{Cz}{2}\right) \quad ; \quad C = 1.7810\ldots \end{aligned}\right\} \text{for } |z| \ll 1 \tag{C.5}$$

$$\left.\begin{aligned} &J_m(z) = \frac{1}{m!}\left(\frac{z}{2}\right)^m \\ &H_m^{(2)}(z) = j\,\frac{(m-1)!}{\pi}\left(\frac{2}{z}\right)^m \end{aligned}\right\} \text{for } |z| \ll m \;,\; m \neq 0 \tag{C.6}$$

$$\left.\begin{aligned} J_m(z) &\approx \sqrt{\frac{2}{\pi z}}\cos\left(z - \frac{2m+1}{4}\pi\right) \\ H_m^{(2)}(z) &\approx \sqrt{\frac{2}{\pi z}}\exp\left[-j\left(z - \frac{2m+1}{4}\pi\right)\right] \end{aligned}\right\} \text{for } |z| \gg m \text{ and } |z| \gg 1 \qquad \text{(C.7)}$$

Addition theorem : for a triangle with sides (r, r_o, R) , where ϕ and ψ are the angles opposite to R and r , respectively, i.e. if

$R = \sqrt{r^2 + r_o^2 - 2rr_o \cos\phi}$ and $r = \sqrt{R^2 + r_o^2 - 2Rr_o \cos\psi}$,

we have for any Bessel function Z_m

$$e^{jn\psi} Z_n(kR) = \sum_{m=-\infty}^{\infty} e^{jm\phi} J_m(kr)\, Z_{m+n}(kr_o) \qquad \text{(C.8)}$$

In particular

$$Z_o(kR) = \sum_{m=-\infty}^{\infty} e^{jm\phi} J_m(kr)\, Z_m(kr_o) \qquad \text{(C.9)}$$

[illegible]'s function : the general solution of equation

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \Lambda^2 - \frac{m^2}{\rho^2}\right) G_m(\rho,\rho_o) = \frac{-\delta(\rho-\rho_o)}{\rho_o} \qquad \text{(C.10)}$$

is

$$G_m(\rho,\rho_o) = \begin{cases} A\, J_m(\Lambda\rho) + B\, H_m^{(2)}(\Lambda\rho) & \text{if } \rho < \rho_o \\ A\, J_m(\Lambda\rho) + B\, H_m^{(2)}(\Lambda\rho) + \frac{j\pi}{2}\left[J_m(\Lambda\rho)\, H_m^{(2)}(\Lambda\rho_o) - J_m(\Lambda\rho_o)\, H_m^{(2)}(\Lambda\rho)\right] & \text{if } \rho > \rho_o \end{cases} \qquad \text{(C.11)}$$

MODIFIED BESSEL'S FUNCTIONS

They are solutions of

$$\left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - 1 - \frac{m^2}{x^2}\right) X_m(x) = 0 \qquad \text{(C.12)}$$

In the frame of this book, m is an integer and x is a complex variable with $\frac{-\pi}{2} < \arg x \leq \frac{\pi}{2}$. We use the linearly independent solutions $I_m(x)$ and $K_m(x)$. They are related to the unmodified Bessel's functions by

$$I_m(x) = I_{-m}(x) = j^m J_m(-jx) \qquad \text{(C.13)}$$

$$K_m(x) = K_{-m}(x) = \frac{\pi}{2}(-j)^{m+1} H_m^{(2)}(-jx) \qquad \text{(C.14)}$$

Consequently :

$$I_m(x)\, K_m'(x) - I_m'(x)\, K_m(x) = \frac{-1}{x} \qquad \text{(C.15)}$$

$$K_{m+1}(x)\, I_m(x) + K_m(x)\, I_{m+1}(x) = \frac{1}{x} \qquad \text{(C.16)}$$

$$I_m'(x) \pm \frac{m}{x} I_m(x) = I_{m\mp1}(x) \qquad \text{(C.17)}$$

$$K_m'(x) \pm \frac{m}{x} K_m(x) = -K_{m\mp1}(x) \qquad \text{(C.18)}$$

Asymptotic values :

$$\left.\begin{aligned} I_o(x) &\approx 1 \\ K_o(x) &\approx -\ln\frac{Cx}{2} \quad ; \quad C = 1.7810\ldots \end{aligned}\right\} \text{for } |x| \ll 1 \qquad \text{(C.19)}$$

$$\left.\begin{aligned} I_m(x) &= \frac{1}{m!}\left(\frac{x}{2}\right)^m \\ K_m(x) &= \frac{(m-1)!}{2}\left(\frac{2}{x}\right)^m \end{aligned}\right\} \quad \text{for } |x| \ll m \text{ , } m \neq 0 \qquad \text{(C.20)}$$

$$\left.\begin{aligned} I_m(x) &= \frac{1}{\sqrt{2\pi x}}\, e^{x} \\ K_m(x) &= \sqrt{\frac{\pi}{2x}}\, e^{-x} \end{aligned}\right\} \quad \text{for } |x| \gg m \text{ and } |x| \gg 1 \qquad \text{(C.21)}$$

Addition theorems : with r, r_o, R, ϕ, ψ defined as hereabove

$$e^{jn\phi}\, I_n(uR) = \sum_{m=-\infty}^{\infty} (-1)^m\, e^{jm\phi}\, I_m(ur)\, I_{n+m}(ur_o) \qquad \text{(C.22)}$$

$$e^{jn\psi}\, K_n(uR) = \sum_{m=-\infty}^{\infty} e^{jm\phi}\, I_m(ur)\, K_{n+m}(ur_o) \qquad \text{(C.23)}$$

Green's function : the general solution of equation

$$\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - u^2 - \frac{m^2}{\rho^2}\right] F_m(\rho,\rho_o) = \frac{-\delta(\rho-\rho_o)}{\rho_o} \qquad \text{(C.24)}$$

is

$$F_m(\rho,\rho_o) = \begin{cases} A\, I_m(u\rho) + B\, K_m(u\rho) & \text{if } \rho < \rho_o \\ A\, I_m(u\rho) + B\, K_m(u\rho) - \left[I_m(u\rho)\, K_m(u\rho_o) - I_m(u\rho_o)\, K_m(u\rho)\right] & \text{if } \rho > \rho_o \end{cases} \qquad \text{(C.25)}$$

where A and B are arbitrary constants.

For more complete information on Bessel's functions, see Erdélyi et al. (1953).

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a) NTIS is the National Technical Information Service, 5285 Port Royal Road, Springfield VA 22161, U.S.A.

b) The abbreviation WEGWME is used for the Proceedings of the Workshop on Electromagnetic Guided Waves in Mine Environments, held in Boulder, Colo., USA, 28-30th March 1978, available from NTIS.

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