INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
- 2 OCT 1984
LIBRARY

INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONES: 224281/2/3/4/5/6
CABLE: CENTRATOM - TELEX 460392-I

SMR/113 - 21

AUTUMN COLLEGE
ON
THE TROPOSPHERE, STRATOSPHERE AND MESOSPHERE

10 September - 19 October 1984

ABSORPTION AND SCATTERING FROM HYDROMETEORS: CROSS-POLARISATION

A. PARABONI
Dipartimento di Geofisica
Politecnico di Milano
Via Ponzio 34
Milano
Italy

These are preliminary lecture notes, intended only for distribution to College participants. Missing or extra copies are available from Room 230.

Depolarization phenomena due to hydrometeors

1st PART

- Coherent propagation through a polidispersion of non-spherical particles (hydrometeors)
- Co-polar and cross-polar waves: basic definitions
- The transfer medium

2nd PART

- Natural media
- Measurements
- Applications

What does coherent propagation means?

When a plane wave enters a dispersion of particles in principle a very complicated phenomenon of multiple scattering takes place: as a consequence the E.M. field across the transverse plane is far from being constant in time and space

Coherent propagation is the evolution of the Coherent field defined as the field obtained by <u>averageing in the transverse plane and in time</u>

Fortunately the coherent field influences but is not influenced by the incoherent one (the variation around the average): it is studied then as if it were alone by means of the EXTENDED VAN DE HULST THEORY

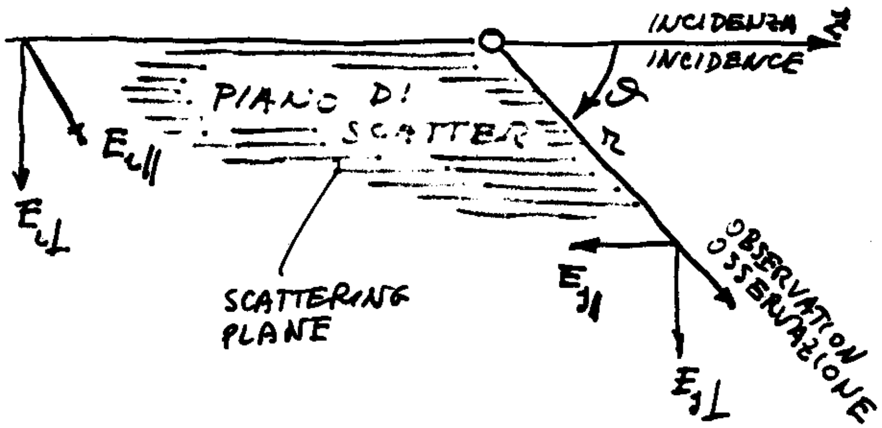
The variable <u>time</u> is no more necessary and a complete PHASOR REPRESENTATION for the field can be used

$$e(t) = \sqrt{2}\, E \cos(\omega t + \varphi) \Rightarrow E\, e^{j\varphi}$$

E effective value

φ phase

3



SPHERICAL DROPS

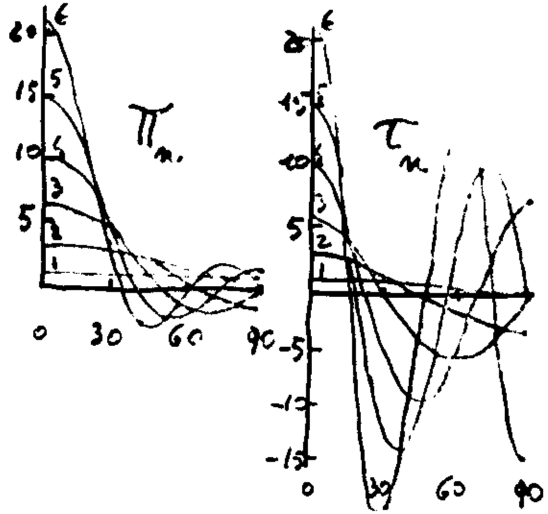
4

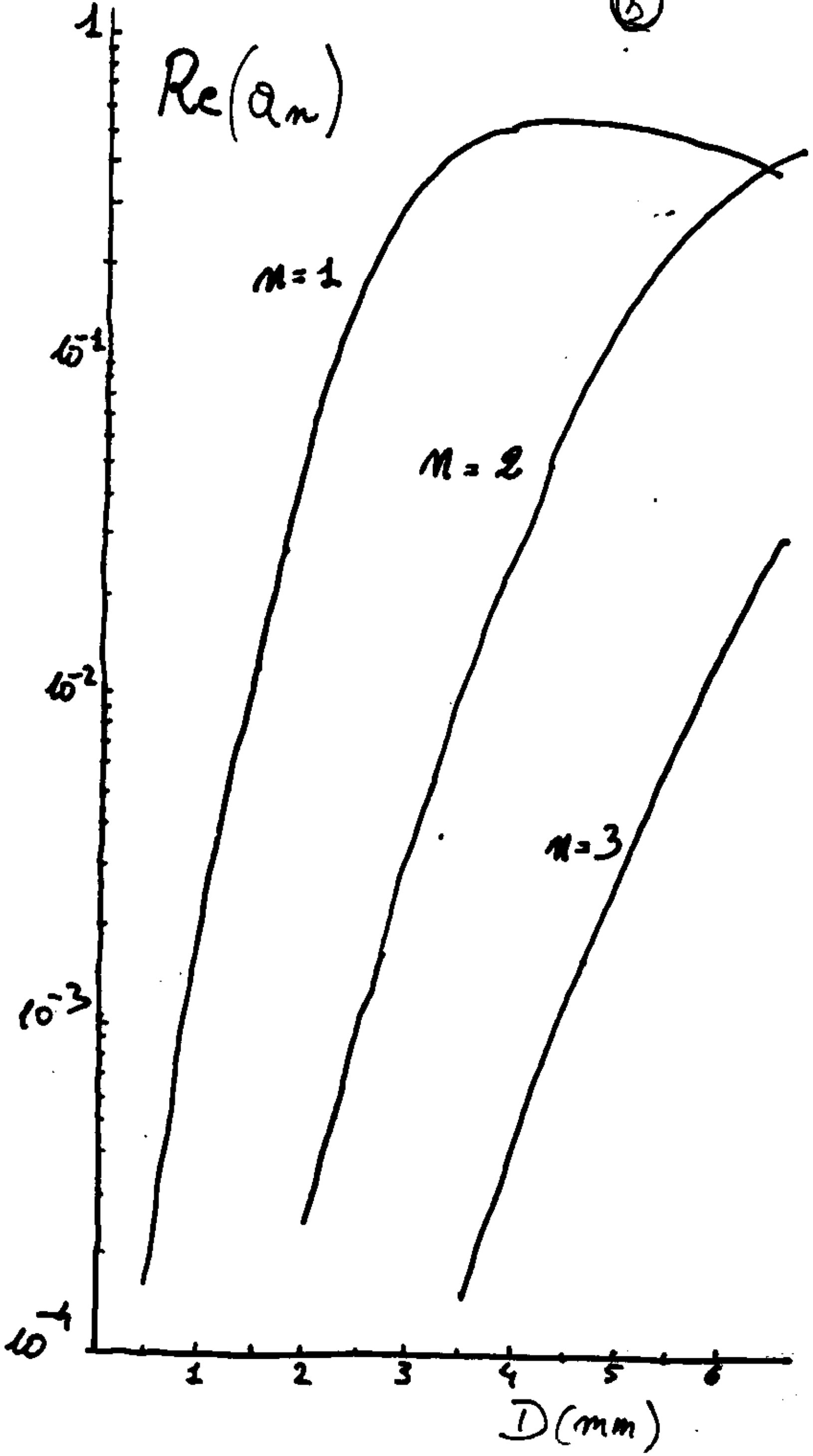
$$E_{s\perp} = E_{i\perp} \cdot S_1(\theta) \cdot \frac{e^{-j\beta r}}{j\beta r}$$

$$E_{s\parallel} = E_{i\parallel} \cdot S_2(\theta) \cdot \frac{e^{-j\beta r}}{j\beta r}$$

$$S_1(\theta) = \sum_1^\infty \frac{2n+1}{n(n+1)} \left(a_n \pi_n + b_n \tau_n\right)$$

$$S_2(\theta) = \sum_1^\infty \frac{2n+1}{n(n+1)} \left(b_n \pi_n + a_n \tau_n\right)$$



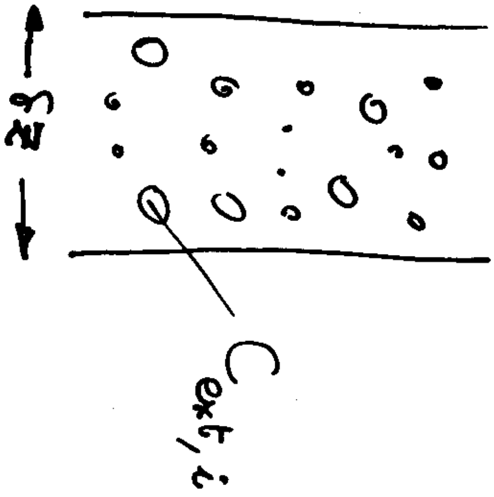


$$C_{scatt} = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1)\left(|a_n|^2 + |b_n|^2\right)$$

$$C_{ext} = \frac{\lambda^2}{2\pi} \sum_{n=1}^{\infty} (2n+1)(a_n + b_n)$$

$$C_{abs} = C_{ext} - C_{scatt}$$

7



$$-\delta\left(\frac{|E|^2}{\eta}\right) = \sum_1^{'} C_{ext,i}\,\frac{|E|^2}{\eta}\,\delta z$$

$$\frac{|E|^2}{\eta} = \frac{|E_0|^2}{\eta}\,e^{-\Upsilon z}$$

$$\Upsilon = \sum_1^{'} C_{ext,i} = \sum_1^{m} C_{ext}(D_i)\,n(D_i)\,\delta D$$

8

EQUIVALENCE BETWEEN THE PARTICLES DISPERSION AND A CONTINUOUS MEDIUM OF SUITABLE CHARACTERISTICS

Logical steps:

a) Substitution of each particle with a pair of orthogonal electric dipoles along x and y

b) Substitution of the concentrated dipoles contained in a transverse thin slab, with a suitable laminar current

c) Evaluation of the elementar variations of the field strength due to rain and to free space propagation

d) Integration of the elementar variations computed in c)

Step 6 : determination of the equivalent electric dipoles imposing complete forward equivalence :

$$E^x_{sc} = \frac{-j\eta}{2\lambda R}(I\ell)^x e^{-j\beta_0 R}$$

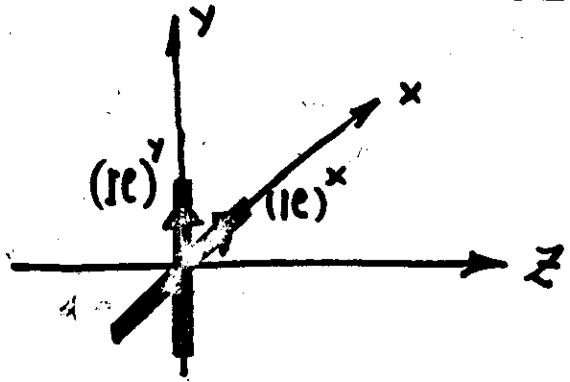
$$E^y_{sc} = \frac{-j\eta}{2\lambda R}(I\ell)^y e^{-j\beta_0 R}$$

⇦ Scattered field by electric small dipoles of value $(I\ell)^x$ and $(I\ell)^y$

$$\begin{vmatrix} E^x_{sc} \\ E^y_{sc} \end{vmatrix} = \frac{e^{-j\beta_0 R}}{j\beta_0 R} \begin{Vmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{Vmatrix} \cdot \begin{vmatrix} E^x_{inc} \\ E^y_{inc} \end{vmatrix} \qquad (9)$$

⇧

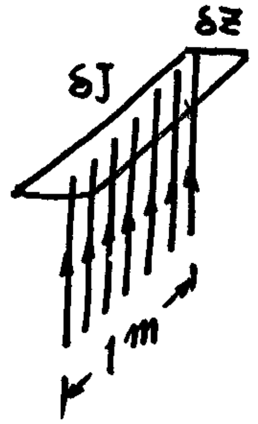
scattered field by asymmetric particles



prop. direction

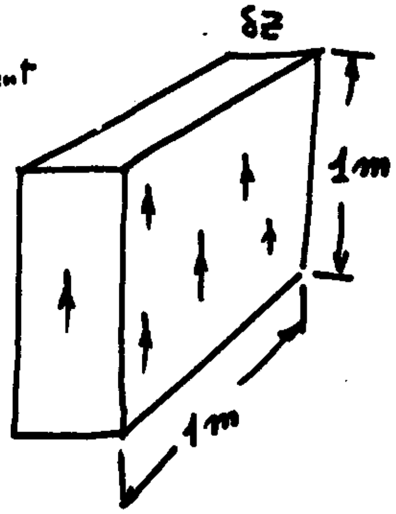
$$\begin{vmatrix} (I\ell)^x \\ (I\ell)^y \end{vmatrix} = \frac{4\pi}{\eta \beta_0^2} \begin{Vmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{Vmatrix} \cdot \begin{vmatrix} E^x_{inc} \\ E^y_{inc} \end{vmatrix} \qquad (10)$$

Equivalent dipoles (amperian)



Step b)

Determination of equivalent laminar currents



laminar electric current

value : .... δJ [A/m]

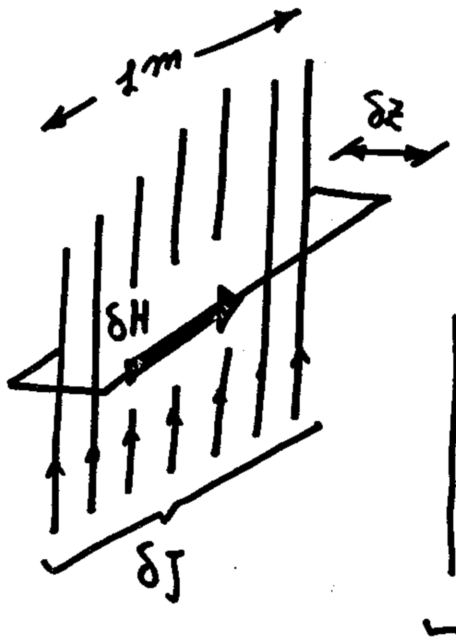
momentum per unit transv. area ...... δJ [A m]

droplets equivalent dipoles momentum : $\sum_{1m^3} (I\ell)_i \, \delta z$ [A m]

$$\delta J = \sum_{1m^3} (I\ell)_i \, \delta z$$

11

Step c): determination of the field strength variation due to rain using the 1st Maxwell equation



$$\delta H = \frac{1}{2} \delta J$$

$$\underbrace{\begin{vmatrix} \delta E^x_{RAIN} \\ \delta E^y_{RAIN} \end{vmatrix} = \frac{2\pi \, \delta z}{\beta_0^2} \begin{Vmatrix} -\sum_1 S_{xx} & -\sum_1 S_{xy} \\ -\sum_1 S_{yx} & -\sum_1 S_{yy} \end{Vmatrix} \cdot \begin{vmatrix} E^x_{INC} \\ E^y_{INC} \end{vmatrix}}_{\text{FIELD VARIATION DUE TO RAIN}}$$

12

$$\underbrace{\begin{aligned}\delta E^{x}_{F.S.} &= -j\beta_0 E^{x}_{inc}\,\delta z \\ \delta E^{y}_{FS} &= -j\beta_0 E^{y}_{inc}\,\delta z\end{aligned}}_{\text{FIELD VARIATION DUE TO FREE SPACE PROPAGATION}}$$

13

$$\underbrace{\vec{\delta E} = \vec{\delta E}_{RAIN} + \vec{\delta E}_{F.S}}_{\text{OVERALL FIELD VARIATION}}$$

$$\begin{vmatrix} \delta E^{x} \\ \delta E^{y} \end{vmatrix} = \delta z \begin{Vmatrix} -\gamma_x(z) & -M(z) \\ -M(z) & -\gamma_y(z) \end{Vmatrix} \cdot \begin{vmatrix} E^{x} \\ E^{y} \end{vmatrix}$$

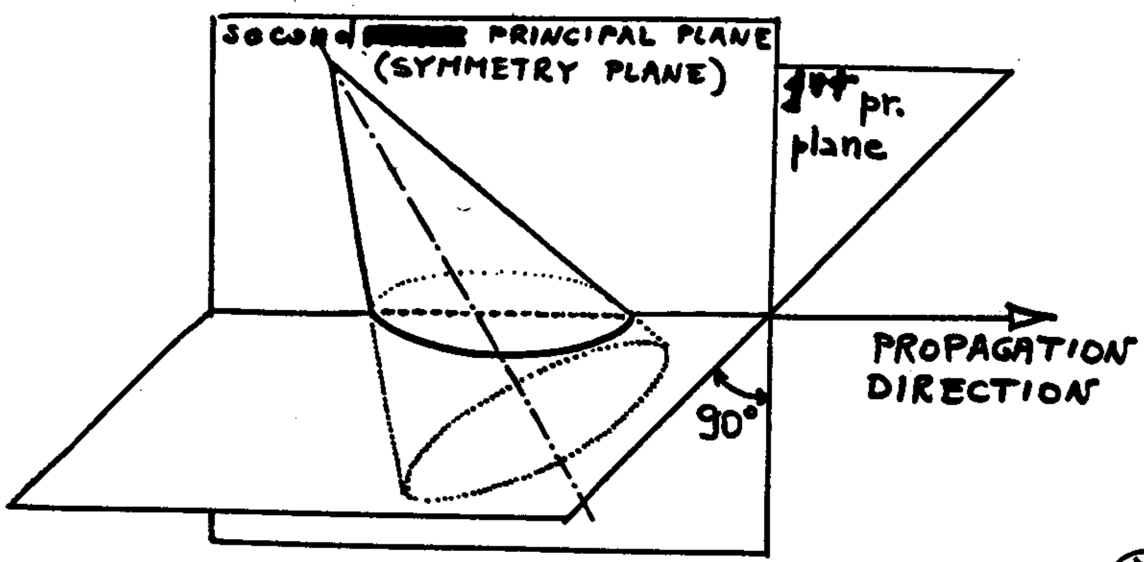
14

$$\gamma_x(z) = j\beta_0 + \frac{2\pi}{\beta_0^2}\sum_1 S_{xx}$$

$$\gamma_y(z) = j\beta_0 + \frac{2\pi}{\beta_0^2}\sum_1 S_{yy}$$

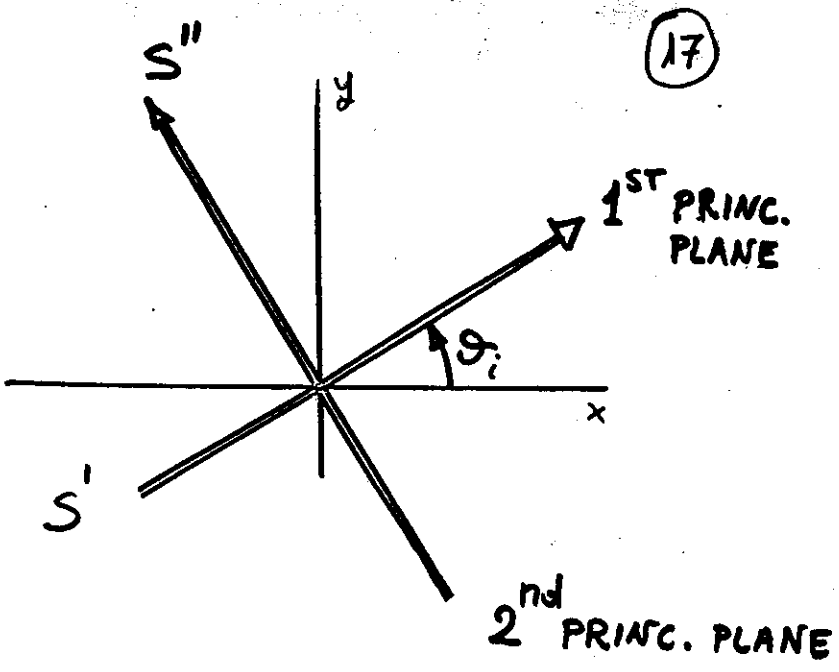
$$\Gamma(z) = \frac{2\pi}{\beta_0^2}\sum_1 S_{xy} \qquad (15)$$

These Terms are depending on the parameters of each particle being the S parameters function of the electric equilibrium s'-s'' and the tilt angle ϑ. The sum has to be extended to a unit volume.



PRINCIPAL PLANES OF A CONIC SCATTERER (16)

(17)



$$-\frac{\pi}{2} < \vartheta < \frac{\pi}{2}$$

(18)

$$\begin{vmatrix} E^x_{sc} \\ E^y_{sc} \end{vmatrix} = \|-\vartheta\| \cdot \frac{e^{-j\beta_0 R}}{j\beta_0 R} \cdot \begin{Vmatrix} S' & 0 \\ 0 & S'' \end{Vmatrix} \cdot \|\vartheta\| \cdot \begin{vmatrix} E^x_{inc} \\ E^y_{inc} \end{vmatrix}$$

INC. FIELD RESOLVED ALONG THE PRINC. PLANES

FORW. SCATTERED FIELD ALONG THE PRINC. PLANES (NON DEPOLARIZED)

$$\|\pm\vartheta\| = \begin{Vmatrix} \cos\vartheta & \pm \operatorname{sen}\vartheta \\ \mp \operatorname{sen}\vartheta & \cos\vartheta \end{Vmatrix}$$

19

$$\begin{vmatrix} E^x_{sc} \\ E^y_{sc} \end{vmatrix} = \frac{e^{-j\beta_0 R}}{j\beta_0 R} \begin{Vmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{Vmatrix} \cdot \begin{vmatrix} E^x_{inc} \\ E^y_{inc} \end{vmatrix}$$

where:

$$S_{xx} = \frac{S' + S''}{2} + \frac{S' - S''}{2} \cos 2\vartheta$$

$$S_{yy} = \frac{S' + S''}{2} - \frac{S' - S''}{2} \cos 2\vartheta$$

$$S_{xy} = S_{yx} = \frac{S' - S''}{2} \operatorname{sen} 2\vartheta$$

20

Foundamental differential system, governing the propagation through the "anisotropic" medium, supposed statistically homogeneous in the transverse plane but general in the longitudinal direction:

$$\begin{vmatrix} \delta E_x \\ \delta E_y \end{vmatrix} = \begin{bmatrix} -\gamma_x(z) & -M(z) \\ -M(z) & -\gamma_y(z) \end{bmatrix} \cdot \begin{vmatrix} E_x \\ E_y \end{vmatrix} \delta z$$

where:

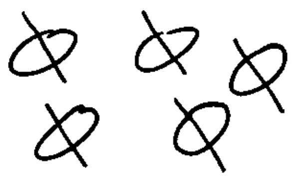
$$\gamma_x(z) = jK_0 + 2\pi K_0^{-2} \sum_1 \frac{s_i' + s_i''}{2} + \frac{s_i' - s_i''}{2} \cos 2\vartheta_i$$

$$\gamma_y(z) \quad '' \quad '' \quad '' \quad - \quad '' \quad ''$$

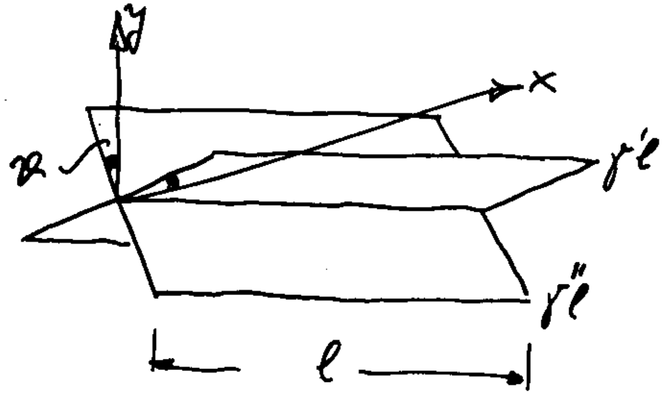
$$M(z) = 2\pi K_0^{-2} \sum_1 \frac{s_i' - s_i''}{2} \operatorname{sen} 2\vartheta_i$$

MEDIUM WITH PRINCIPAL PLANES

- Equialigned raindrops
- Mixed populations composed by equialigned raindrops plus randomly oriented raindrops
- Symmetrical distributions of the orientation of the axes



This medium possesses two ortogonal planes along which linearly polarized waves propagate without being depolarized



RIGOROUS FORMULA FOR THE OVERALL TRANSFER MATRIX IN THE CASE OF PRINCIPAL PLANES

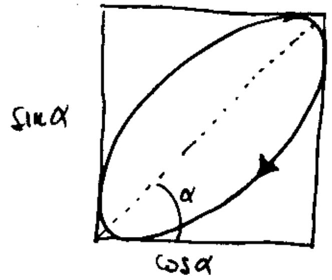
$$\begin{Vmatrix} E^x_{out} \\ E^y_{out} \end{Vmatrix} = \begin{Vmatrix} \frac{e^{-\gamma' l}+e^{-\gamma'' l}}{2}+\frac{e^{-\gamma' l}-e^{-\gamma'' l}}{2}\cos 2\vartheta & \frac{e^{-\gamma' l}-e^{-\gamma'' l}}{2}\,\mathrm{sen}\,2\vartheta \\ \frac{e^{-\gamma' l}-e^{-\gamma'' l}}{2}\,\mathrm{sen}\,2\vartheta & \frac{e^{-\gamma' l}+e^{-\gamma'' l}}{2}+\frac{e^{-\gamma'' l}-e^{-\gamma' l}}{2}\cos 2\vartheta \end{Vmatrix} \begin{Vmatrix} E^x_{in} \\ E^y_{in} \end{Vmatrix}$$

γ', γ'' propagation constant along the two planes

ϑ tilt angle with respect to the x axis

POLARIZED WAVES

Generally polarized unit vector



$$\vec{u} = \cos\alpha\, \vec{u}_x + e^{j\psi} \sin\alpha\, \vec{u}_y$$

$0 \le \alpha \le \pi/2$ $\quad \psi$ whichever

Linear pol: hor. $\alpha = 0$ $\quad \psi$ whichever

vert $\alpha = \pi/2$ $\quad \psi$ whichever

circular pol: left $\alpha = \pi/4$ $\quad \psi = \pi/2$

right $\alpha = \pi/4$ $\quad \psi = -\pi/2$

COUPLES OF ORTHOGONAL UNIT VECTORS

$$\vec{u}^{I} = \cos\alpha_1 \vec{u}_x + e^{j\psi_1} \sin\alpha_1 \vec{u}_y$$
$$\vec{u}^{II} = \cos\alpha_2 \vec{u}_x + e^{j\psi_2} \sin\alpha_2 \vec{u}_y$$

Ortogonality condition

$$\alpha_1 + \alpha_2 = \pi/2$$
$$\psi_2 - \psi_1 = \pi$$

General electric field

$$\vec{E} = E' \vec{u}' + E'' \vec{u}'' = E_x \vec{u}_x + E_y \vec{u}_y$$

Alternative representations

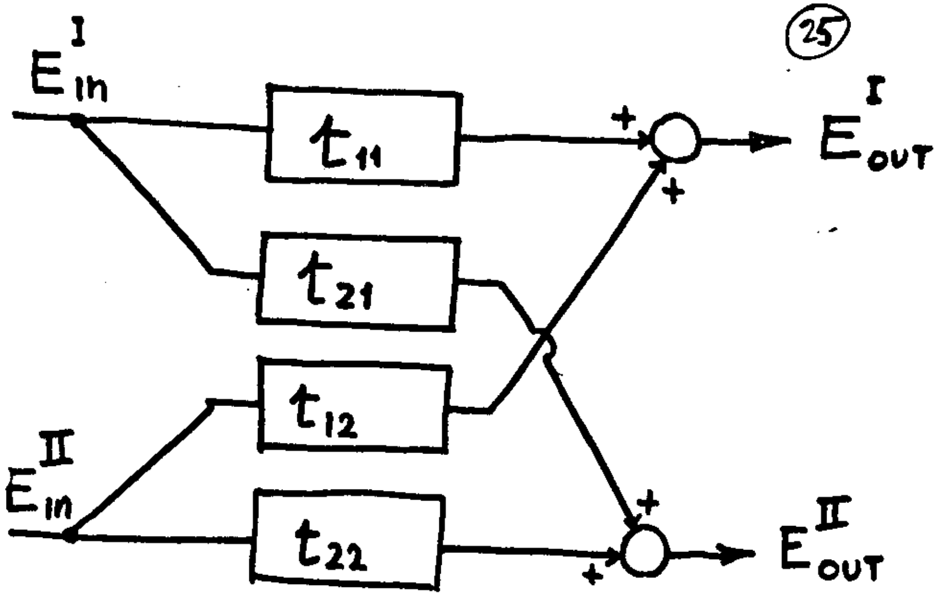
- Polarization factor $\quad p = \dfrac{E_y}{E_x}$

- Poincaré sphere

- Stokes parameters

- ⋮

25



$$\begin{Vmatrix} E_{OUT}^{I} \\ \\ E_{OUT}^{II} \end{Vmatrix} = \underbrace{\begin{Vmatrix} t_{11} & t_{12} \\ \\ t_{21} & t_{22} \end{Vmatrix}}_{\| T_{12} \|} \cdot \begin{Vmatrix} E_{in}^{I} \\ \\ E_{in}^{II} \end{Vmatrix}$$

TRANSFER MATRIX FOR GENERAL ORTHOGONAL POLARIZATIONS

26

DEPOLARIZATION RATIO

$$\delta_{12} = \left. \frac{E_{OUT}^{I}}{E_{OUT}^{II}} \right|_{E_{in}^{I} = 0} = \frac{t_{12}}{t_{22}}$$

$$\delta_{21} = \left. \frac{E_{OUT}^{II}}{E_{OUT}^{I}} \right|_{E_{in}^{II} = 0} = \frac{t_{21}}{t_{11}}$$

TRANSFER CHANNEL WITH PRINCIPAL PLANES (27)

$$\begin{vmatrix} E_x \\ \\ E_y \end{vmatrix} = \begin{vmatrix} e^{-\gamma' l}\cos^2\theta + e^{-\gamma'' l}\,\mathrm{sen}^2\theta & \dfrac{e^{-\gamma' l} - e^{-\gamma'' l}}{2}\,\mathrm{sen}\,2\theta \\ \dfrac{e^{-\gamma' l} - e^{-\gamma'' l}}{2}\,\mathrm{sen}\,2\theta & e^{-\gamma' l}\,\mathrm{sen}^2\theta + e^{-\gamma'' l}\cos^2\theta \end{vmatrix} \begin{vmatrix} E_{x_0} \\ \\ E_{y_0} \end{vmatrix}$$

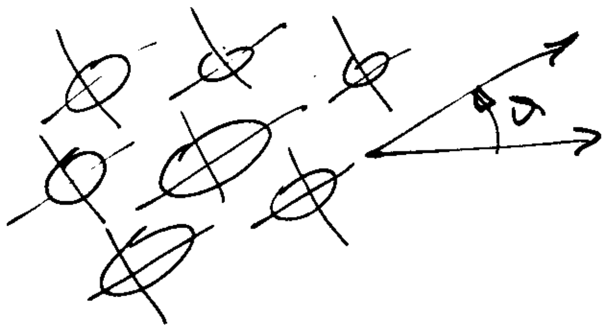
$$\begin{vmatrix} E_l \\ \\ E_r \end{vmatrix} = \begin{vmatrix} \dfrac{e^{-\gamma' l} + e^{-\gamma'' l}}{2} & \dfrac{e^{-\gamma' l} - e^{-\gamma'' l}}{2}\,e^{-j2\theta} \\ \dfrac{e^{-\gamma' l} - e^{-\gamma'' l}}{2}\,e^{+j2\theta} & \dfrac{e^{-\gamma' l} + e^{-\gamma'' l}}{2} \end{vmatrix} \begin{vmatrix} E_{l_0} \\ \\ E_{r_0} \end{vmatrix}$$

$$\delta_{xy} = \frac{\mathrm{sen}\,2\theta}{-\cos 2\theta - \mathrm{Ctgh}\left(\frac{\Gamma' - \gamma''}{2}\,l\right)}$$

$$\delta_{yx} = \frac{\mathrm{sen}\,2\theta}{+\cos 2\theta - \mathrm{Ctgh}\left(\frac{\gamma' - \gamma''}{2}\,l\right)}$$

$$\delta_{lr} = -\mathrm{Th}\left(\frac{\gamma' - \gamma''}{2}\,l\right) e^{-j2\theta}$$

$$\delta_{rl} = -\mathrm{Th}\left(\frac{\gamma' - \Gamma''}{2}\,l\right) e^{+j2\theta}$$



IDENTIFICATION OF THE CHANNEL (WITH RESPECT TO ITS DEPOLARIZATION CAPABILITY)

- <u>Complete identification</u>

  It implyes the knowledge of the ratios between the four t_{ij} terms of the transfer channel

  THREE COMPLEX PARAMETERS are then needed

- <u>Identification of semplifyed models</u>

  Good assumptions on the physical nature of the medium can semplify drastically the problem

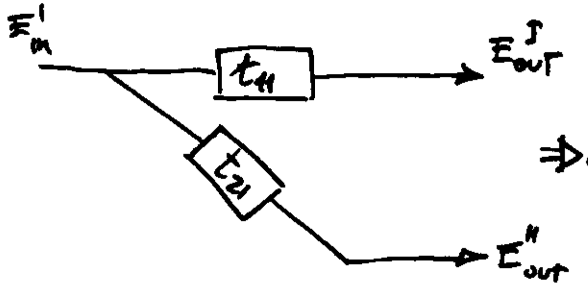
  <u>Typical example</u>: <u>medium with principal planes</u> i.e characterized by two planes which allow the transmission of linearly polarized waves without depolarization

  ONE COMPLEX AND ONE SCALAR PARAMETER are needed:

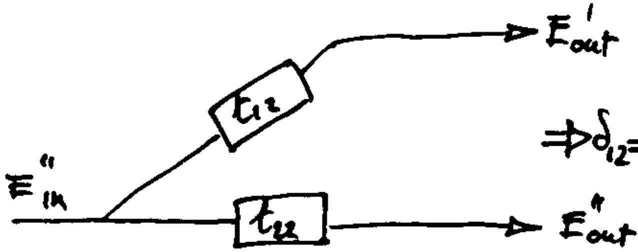
  ANISOTROPY
  (differential attenuation and phase shift)

  AVERAGE CANTING ANGLE

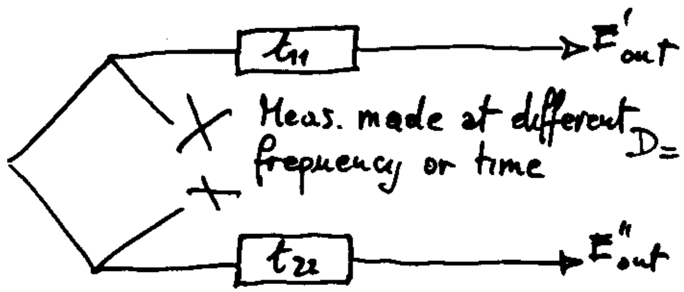
TYPICAL XPD OR XPD-RELATED MEASUREMENTS



$$\Rightarrow \delta_{21} = \frac{E''_{out}}{E'_{out}}\Bigg|_{E''_{in}=0} = \frac{t_{21}}{t_{11}}$$



$$\Rightarrow \delta_{12} = \frac{E'_{out}}{E''_{out}}\Bigg|_{E'_{in}=0} = \frac{t_{12}}{t_{22}}$$



$$D = \frac{E''_{out}}{E'_{out}} = \frac{t_{22}}{t_{11}}$$

CPA = copolar atten.

OBJECTIVES IN DETERMINING THE COMPLETE CHANNEL CHARACTERISTICS

- Prediction of XPD in <u>any</u> polarization (Future satellite systems will operate in linear polarization, whose orientation will depend on the receiving site)
- Efficient link and apparatus design
- Diagnostics of the physical nature of the hydrometeors

TYPICAL MEASUREMENT TECHNIQUES

INCOMPLETE MEASUREMENTS	COMPLETE MEASUREMENTS
1- $\begin{cases} \delta_{lr} \text{ in circ. polariz.} \\ CPA = \text{coherent atten.} \end{cases}$ (OTS, SIRIO, CTS) (Terrestrial links)	1- $\begin{cases} \delta_{xy} \\ \delta_{yx} \\ D = \text{Squilibrium (in lin. pol.)} \end{cases}$
2- $\begin{cases} \delta_{xy} \text{ in lin. polariz.} \\ CPA \end{cases}$ ~~[illegible]~~ (Terrestrial links)	2- $\begin{cases} \delta_{lr} \\ \delta_{rl} \\ D \text{ (in circ. pol)} \end{cases}$
3 $\begin{cases} \delta_{lr} \\ \delta_{rl} \end{cases}$	3- $\begin{cases} \delta_{xy} \\ \delta_{yx} \\ \delta_{rl} \text{ or } \delta_{45} \end{cases}$
4 $\begin{cases} \delta_{xy} \\ \delta_{yx} \end{cases} \dots \begin{cases} \delta_{45} \\ \delta_{135} \end{cases}$	4- $\begin{cases} \delta_{xy} \text{ inclined pol} \\ \delta_{yx} \\ D \end{cases}$ (OLYMPUS, ITALSAT) COMSTAR

CIRC. XPD (Modulus) + CPA

$$|\delta_{\ell r}| = \left|\tanh\left(\frac{\gamma''\ell - \gamma'\ell}{2}\right)\right|$$

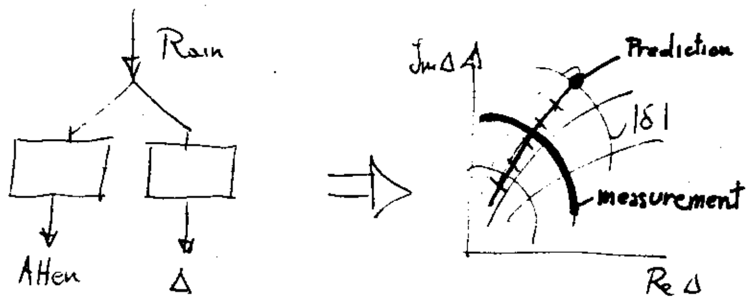
$$\uparrow \frac{\Delta}{2}$$

Diagram $|\delta_{\ell r}| \Rightarrow \Delta$

Theoretical Oguchi model

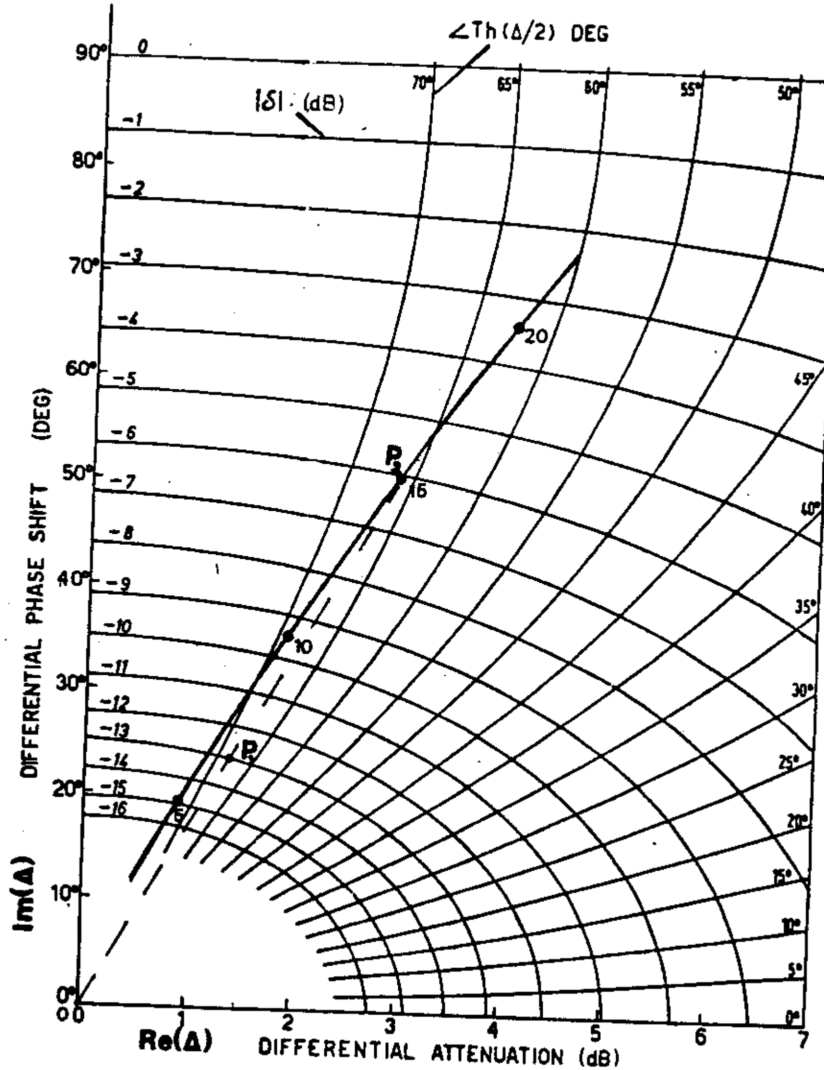
Assumes Equioriented raindrops

- typical Raindropsize (Laws Parsons)
- " shape (ellipsoids or Pruppacher Pitter)
- " eccentricity related to the drop diameter



Why the predicted ... : Anisotropy is noticeably higher than the measured one?

Microphysics of rain structure must be reconsidered



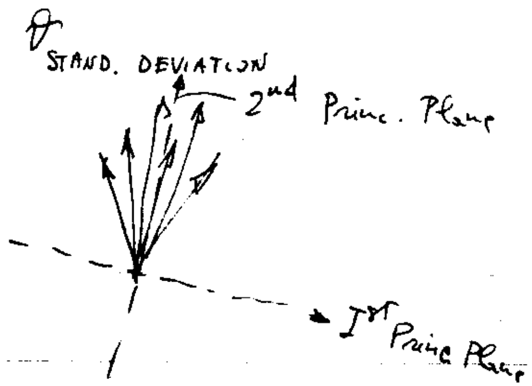
Physical models of the orientations
Physical models of drops vibration

MODEL OF ORIENTATIONS

- TWO POPULATIONS MODEL
  - oriented (weight p)
  - randomly oriented (weight $1-p$) (non-depolarizing)

p = Active fraction

- SPREAD AXES ORIENTATIONS



Active fraction (or population)

$$P = \frac{\Delta}{\Delta_{MAX}} \leqslant 1$$

P can be related to the spread of the orientations of the single drop axes

Experimental results: Terrestrial links (Mognoli)

Spacial links (OTS)

19, NO. 4, JULY 1981 681

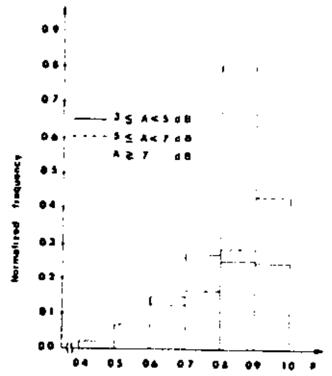


Fig. 3. Conditional histograms of active population p.

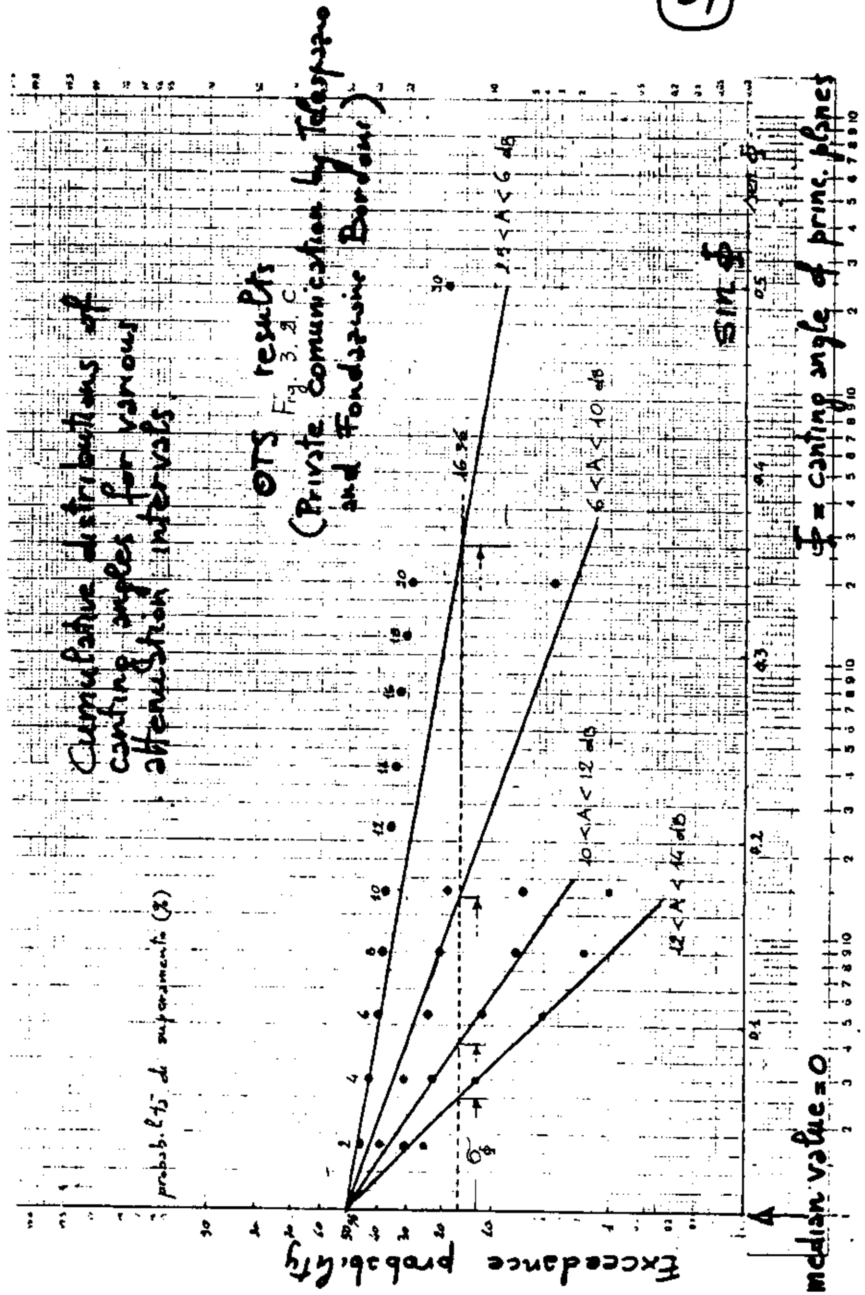
"Monte Magnola" link results

If the argument of δ_{lr} (phase) is measured (i.e the phase shift between cross- and co-polar signals), from

$$\angle \delta_{lr} = \angle -\tanh\left(\frac{\Delta}{2}\right) - 2\vartheta$$

the Principal Planes canting angle ϑ can be derived.

OTS results



LINEAR XPD (modulus) + CPA

Using CPA + Oguchi model + reduction due to imperfect alignement (p) one can deduce an estimate for anisotropy

Then from

$$\left|\delta_{xy}\right| = \left|\frac{\sin 2\vartheta}{\cos 2\vartheta + \operatorname{Cotanh}\left(\frac{\Delta}{2}\right)}\right|$$

ϑ can be obtained

Drawbacks :
- The cross polar signal in linear horizontal (or vertical) is very small and often very near to the system noise level
- The $\left|\delta_{xy}\right|$ has a cuspex (sharp minimum) for $\vartheta = 0$ → Positive angles are undistinguishable from negative ones
- Incoherent detection near the noise level creates biassed measurements : the resulting statistics are then distorted as positive and negative angles are superimposed. (see previous point)

Tilting the polarization planes

It gives many advantages

- Enhances the cross-polar level
- Allows bilateral measurements

Results: Fucino "Fondazione Bordoni" link

Fondazione Bordoni" link at Fucino (ITALY)

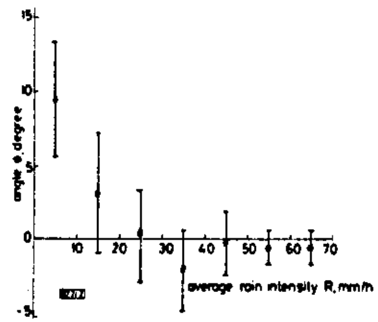


Fig. 2 *Statistics of canting angle ϕ against average rain intensity R*
Stars give the average value of ϕ in each interval of R; vertical bars give the $\pm\sigma$ (standard deviation) interval

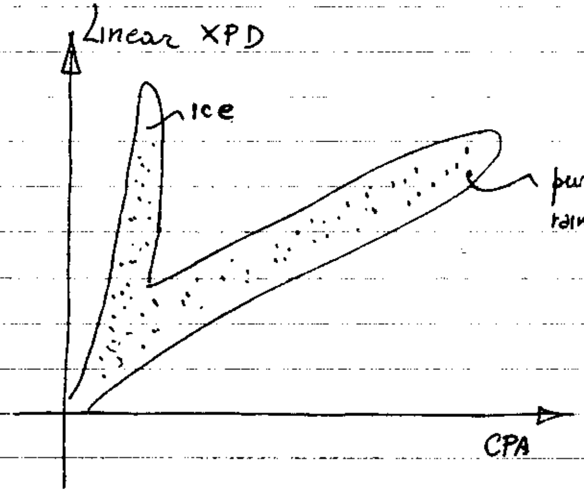
3, Vol. 19 No. 16 pp. 617-619

<u>Basic limitations in measuring just one polarization</u>

If the channel must be fully identified, anisotropy and average canting angles are <u>both</u> needed

As shown before in any case some information must be extracted from CPA through <u>uncertain</u> hypoteses

This is strongly penalizing especially in space-to-earth radio links as the presence of ice aloft produces a large contribution to anisotropy (imaginary part) <u>without CPA</u>.



Typical experimental scatterplot

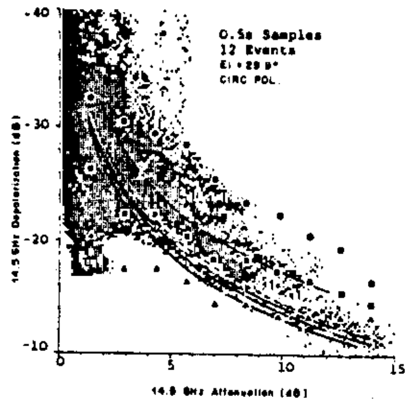


Fig. 29. A scatter plot relating depolarization to attenuation for rain and ice measurements made on the OTS to Martlesham Heath path [*Howell and Thirlwell*, 1980]. The solid circles, squares, and triangles are the CTS to Crawford Hill data from Figure 28 that have been scaled to the parameters of the Martlesham Heath experiment using (8), (9), (12), and (13). The curves on the figure are described by *Howell and Thirlwell* [1980] but will not be discussed here.

OTS results

DOUBLE POLARIZATION (CIRCULAR)

Coherent detection is needed to fully exploit this measurement

In fact

$$\angle \delta_{rl} - \angle \delta_{lr} = 4\vartheta$$

So in addition to the information obtained by $|\delta|$ and CPA (anisotropy and angles spread) we have a rigorous determination of the global canting angle (average)

This measurements is much more accurate than the one using CPA measurements because it does not require to estimate the argument of $\left(\tanh \frac{\Delta}{2}\right)$ making uncertain assumptions on the rain structure

{from CPA measurements

DOUBLE POLARIZATION (LINEAR)

While this system is likely to be the preferred one for future Telecommunication Systems, it must be avoided for channel testing.

Indeed the inversion of the system to extract anisotropy and angle from the δ's is an

ILL-CONDITIONED PROBLEM

Example

$$\vartheta = \frac{1}{2} \tan^{-1}\left[2\left(\frac{1}{\delta_{xy}} - \frac{1}{\delta_{yx}}\right)^{-1}\right]$$

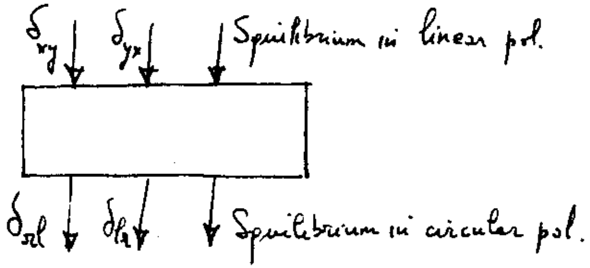
The "core" $\frac{1}{\delta_{xy}} - \frac{1}{\delta_{yx}}$ is a difference between large numbers: to evaluate it with accuracy would require extremely high signal-to-noise ratios (45-50 dB) and unrealistic angles to be measured.

COMPLETE MEASUREMENTS

δ_{xy} δ_{yx} Squilibrium

(COMSTAR, OLYMPUS, ITALSAT)

A "model oriented" description of the channel has been proposed to derive "quasi-physical" parameters in the general case (also if the principal planes do not exist)



Then the parameters

- Effective anisotropy
- Effective canting angle (possibly complex)
- Longitudinal omogeneity index

Apparent anisotropy

$$\Delta = 2\,\mathrm{Tanh}^{-1}\sqrt{\delta_{rl}\,\delta_{lr}}$$

with: $\mathrm{Re}\,\Delta \geq 0$

Apparent canting angle:

$$\vartheta = \frac{1}{2j}\ln\frac{\delta_{rl}}{\sqrt{\delta_{rl}\,\delta_{lr}}}$$

with $-\frac{1}{2}\pi \leq \mathrm{Re}\,\vartheta \leq +\frac{1}{2}\pi$

Longitudinal omogeneity index

$$L = j\,(D_{rl}^2 - 1)/(D_{rl}^2 + 1)$$

where D is the squilibrium

In the case of principal planes the first parameter becomes the true anisotropy, the second becomes neat and the third is null.

- Advantages :
  i - owing their physical likelyhood these parameters should be rather independent of frequency and then useful to extrapulate to whichever frequency or polarization;

  ii - they ought to yeld valuable information about the meteorological nature of the disturbances observed in the link

  iii - they allow to separate XPD contributions due to propagation, from spurious contributions due to malfunctions of the measuring equipment

COMSTAR MEASUREMENTS

Comstar satellite yelded complete measurements (three complex parameters) in linear polarization at 20 GHz.

Tilt angle (or canting angle of principal planes) was identified as the angle yelding minimum XPD.

Anisotropy was not studied as a primary parameter in itself, as XPD was the main interest for its direct involvement in application.

Among the others, interesting results showing the effect of ice needles aligned by the static field in thunderclouds is shown.
The sudden decrease in XPD is due to the lightning discharge which destroys the alignement

51

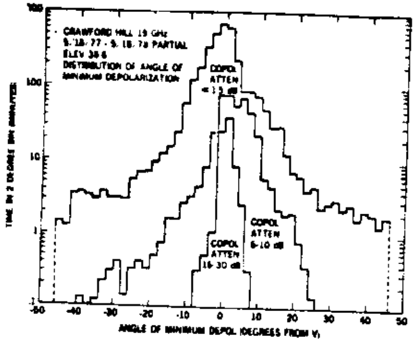


Fig. 17. Angular distributions of incident polarizations yielding minimum depolarization. Measurements were made at Bell Laboratories, Crawford Hill, using a COMSTAR satellite [*Cox et al.*, 1980*a*, *b*]. The data groupings span different attenuation intervals as indicated. The greater angular spread of the ice-dominated events (attenuation of <1.5 dB) is evident.

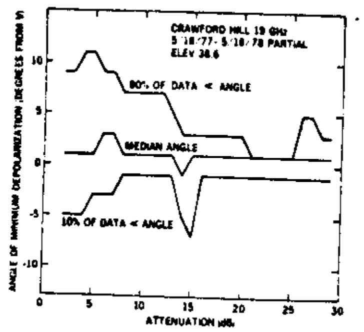


Fig. 18. Angle of more nearly vertical incident polarization yielding minimum depolarization as a function of attenuation for rain and ice events. Median, 10%, and 90% curves indicate the spread in data observed on a COMSTAR to Bell Laboratories, Crawford Hill, path [*Cox et al.*, 1980*a*, *b*]. The median angle was always near vertical.

~~this section shows~~ evidence that the ice and ...

~~source of depolarization. When ice is dominant~~ at ...

COMSTAR results

52

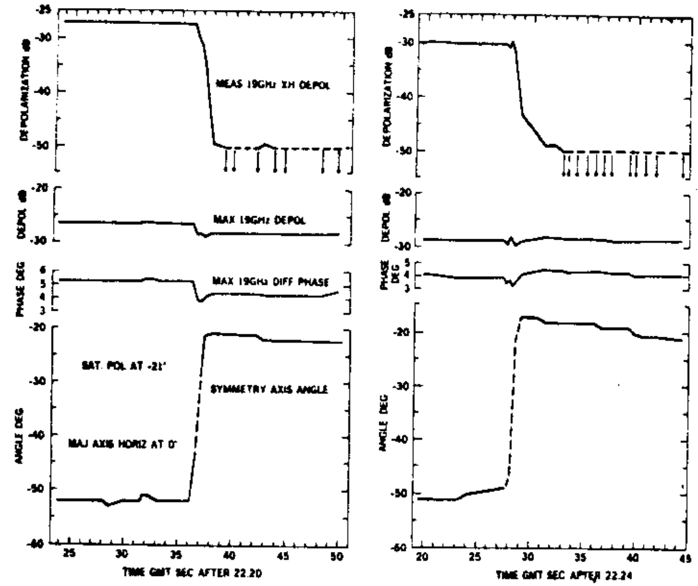


Fig. 6. Signal characteristics and average ice particle axis orientation changes with time for two rapid decreases in depolarization during an ice event measured on a COMSTAR to Bell Laboratories, Crawford Hill, path [*Cox and Arnold*, 1979]. Curves are discussed in the text.

COMSTAR RESULTS

FUTURE ACTIVITY

- Further full-measurements (Olympus 12, 20, 30 GHz, Italsat 20, 40, 50 GHz etc)
- Terrestrial links at millimetric waves
- Theoretical and experimental research on the dynamics of vibrating drops and on ice particles
- Capability of dual-pol. radar in predicting XPD
- Models (on regional and global scale) to predict XPD
- Good data bases to assess prediction methods very effectively (COST 205 activity)

ASSESSMENT OF PRINCIPAL PLANES ORIENTATION IN A TERRESTRIAL LINK AT 18 GHz DURING INTENSE RAINFALL

Indexing terms: Radiowave propagation, Polarisation, Rain-induced attenuation

Simultaneous measurements of attenuation and depolarisation at 18 GHz collected during a propagation experiment on a 9·5 km terrestrial link have been employed to extract information about the angle formed by the principal planes with respect to the vertical-horizontal orientations (canting angle). Tilted polarisation planes have been adopted in order to increase the system sensitivity and to detect the sign of the angle according to a new technique recently devised.

Introduction: Using simultaneous attenuation and depolarisation measurements at 18 GHz, in a 9·5 km terrestrial link, information was inferred about the effective canting angle of the principal planes (better known as the ensemble-average angle of the raindrop axes projected on the transverse plane).

For this purpose a new experimental technique, making use of a tilted linear polarisation (17·5 degrees with respect to the vertical) has been employed in order to achieve a higher cross-polar level for most of the time and to detect the sign of the canting angle even using incoherent detection.[1,2]

Rain intensity measurements along the radio path have also been carried out in order to achieve information on the true rainfall intensity profile and to assess its effect on the canting angle statistics.

Table 1 summarises the main characteristics of the radio link: the experiment lasted about two years; the data were automatically recorded whenever a significant attenuation (>5 dB) was exceeded. The recording time amounted to 1500 s.

Table 1 CHARACTERISTICS OF RADIO LINK

Frequency	17·8 GHz
Path length	9·5 km
Polarisation	Linear
Polarisation tilt angle (with respect to the vertical)	17·5 degrees
Fade-margin	55 dB
Detection	Incoherent

Algorithm: The algorithm employed here consists in extracting the canting angle ϕ from the following expression which gives depolarisation (XPD) in a general medium characterised by principal planes:

$$\mathrm{XPD} = \left| \frac{\sin 2(\phi - \phi_0) \sinh (\Delta/2)}{\cosh (\Delta/2) - \sinh (\Delta/2) \cos 2(\phi - \phi_0)} \right| \qquad (1)$$

In eqn. 1 Δ is the anisotropy and ϕ_0 is the angle of the transmitted polarisation, which, as eqn. 1 shows, plays the role of an angular bias to be chosen so as to optimise the system sensitivity as far as the measurement of ϕ is concerned. Unfortunately the anisotropy is not directly measured as the only measured parameters are XPD and copolar attenuation A.

Then we can only estimate Δ from its relationship with the coherent attenuation A, with a certain degree of uncertainty.

The linkage between these two variables requires a number of additional hypotheses about the rain profile, drop-size distribution, water temperature, spreading of drop orientation and their vibration.

The first set of assumptions made were: uniform rain profile, Laws-Parson drop-size distribution and water temperature of 18°C. With these hypotheses an upper-bound value of anisotropy can be evaluated, and precisely the value which would result if all the drops were equialigned non-oscillating oblate spheroids according to the well known earlier Oguchi calculations:[3]

$$\Delta_{max} = (A_{II} - A_I) + j(B_{II} - B_I) \qquad (2)$$

In eqn. 2 A_I, A_{II}, B_I and B_{II} are the attenuations and phase shifts along the principal planes of the medium, and are related to the average rain intensity R along the path by a power-law relationship of type KR^aL (L = path length). Values of K and a are given by CCIR.[4]

Eqn. 2 overestimates anisotropy because of spreading in drop orientation and their vibration.[5,6] Calling p the reduction factor of anisotropy, we then put

$$\Delta = p\, \Delta_{max} \qquad p < 1 \qquad (3)$$

We assumed here, tentatively, an average value of p as measured in a previous experiment conducted on the same site:[7]

$$p = 0 \cdot 8 \qquad (4)$$

As for the copolar attenuation it is fairly close to A_I independently of $\phi - \phi_0$ and p. With these assumptions, starting from a given value of average rain intensity R, assumed as a free conditioning parameter, a set of curves giving XPD as a function of $A \simeq A_I$ can be drawn for various tilt angles ϕ.

One of these curves has been drawn for $\phi = 0$ and $p = 0 \cdot 8$ in Fig. 1, where the experimental points have also been reported. In the same Figure, the curve for $p = 1$ has been plotted for comparison.

It is seen that the value $p = 0 \cdot 8$, unlike the value $p = 1$, is a better fit to the points on the curve, corresponding to average canting angles, well balanced around zero.

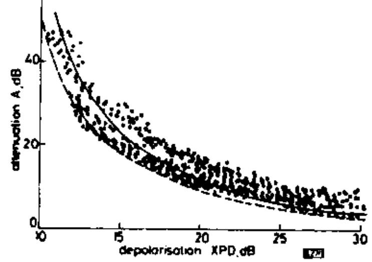


Fig. 1 *Scatterplot of copolar attenuation A against depolarisation XPD*
Curves are for canting angle $\phi = 0°$, reduction factor $p = 1$ (broken line) and $p = 0 \cdot 8$ (solid line)

We thus felt confident about the fact that the assumption $p = 0 \cdot 8$ was adequate to account for the drop dynamics, largely unknown so far, despite their importance in XPD prediction. Values close to $p = 0 \cdot 8$ on the other side have been found by many other researchers.[8] A more sophisticated analysis could be undertaken to find out a value of p as a function of R, in order to better balance the experimental points throughout the whole range of R. Also, an assessment of the apparent variance induced on ϕ by the variance of p and by the nonuniform rain profile could be usefully tried; however,

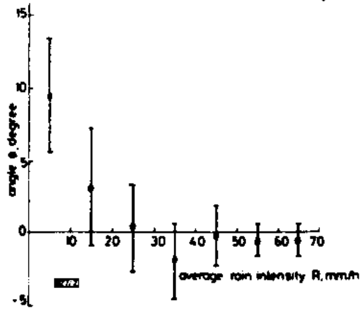


Fig. 2 *Statistics of canting angle ϕ against average rain intensity R*
Stars give the average value of ϕ in each interval of R; vertical bars give the $\pm\sigma$ (standard deviation) interval

(55)

we limited ourselves to the above-mentioned first approximation analysis with a fixed value of p and uniform profile, as we think that with the data so far available more sophisticated hypotheses could not be verified in detail.

Results: Fig. 2 shows, for various classes of average rain intensity, the average value of ϕ and its variance represented by means of vertical bars giving the limits $\pm\sigma$ (σ = standard deviation). Note that this variance includes a contribution induced by the variance of p, which is estimated to be of the order of 50% on average (still utilising information taken by Reference 7).

An apparent bias (average ϕ) exists which can be ascribed to a prevailing wind direction or to an imperfect choice of the average p (which could in principle depend on rain intensity as noticed previously).

A remarkable feature appears to be the clear tendency of the variance to decrease as the rain intensity increases: values of the order of 3 degrees in σ for low rain intensities decrease to less than 1 degree for high rain intensities (> 50 mm/h). This behaviour points to an increased tendency to stability as the drop size increases owing to its increasing inertia.

The average rain intensity as acquired by means of rain gauges alignment beneath the link was found to match the value estimated from attenuation extremely well.

The results found in the present study confirm some analogous ones found in References 9 and 10, even if the more detailed analysis performed here, linking the canting angle statistics to rain intensity, offers a tool, if tentative at the present stage, to generalise the results to sites characterised by different climates.

D. MAGGIORI
P. MIGLIORINI
Fondazione U. Bordoni
Viale Europa 160, Roma EUR, Italy

6th June 1983

A. PARABONI

Politecnico di Milano
Piazza L. da Vinci 32, Milano, Italy

References

1 MAGGIORI, D., MIGLIORINI, P., and PARABONI, A.: 'A technique for improving depolarization measurements beyond 10 GHz: tilting the polarisation planes', *IEEE Trans.*, 1983, **AP-31**

2 MAGGIORI, D., MIGLIORINI, P., and PARABONI, A.: 'Studio di un sistema di misura per la caratterizzazione dei meccanismi fisici causonti degradzione di discriminazione tra polarizzazioni ortogonali'. Fondazione U. Bordoni, Internal Report 37, 1978

3 OGUCHI, T.: 'Scattering properties of oblate raindrops and cross-polarization of radio waves due to rain: calculations at 19·3 and 34·8 GHz', *J. Radio Res. Lab.*, 1973, no. 20

4 CCIR, Doc. WG-5, no. 721, Geneva, 1981

5 DISSANAYAKE, A. W., CHANDRA, M., and WATSON, P. A.: 'Theoretical prediction of differential reflectivity using dual polarization radar for various types of hydrometeors'. URSI Open Symp. on multiple-parameter radar measurements of precipitation, Bournemouth, 1982

6 BEARD, K. V., JOHNSON, D. B., and JAMESON, A. R.: 'Raindrop distortion'. URSI Open Symp. on multiple-parameter radar measurements of precipitation, Bournemouth, 1982

7 MATRICCIANI, E., PARABONI, A., POSSENTI, G., and TIRRÒ, S.: 'Determination of rain anisotropy and effective spreading of the orientation of ellipsoidal raindrops during intense rainfall'. *IEEE Trans.*, 1981, **AP-29**

8 OLSEN, R. L.: 'Cross polarization on terrestrial links—a review', *Radio Sci.*, 1981, **16**

9 NEVES, J., and WATSON, P. A.: 'Cross-polarization, differential attenuation and differential phase-shift measured on a 36·5 GHz terrestrial link'. URSI Open Symp. on effects of the lower atmosphere on radio propagation at frequencies above 1 GHz, Lennoxville, Canada, 1980

10 KOBAYASHI, T.: 'Degradation of cross-polarization isolation due to rain', *J. Radio Res. Lab.*, 1977, no. 24

(56)

IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. AP-29, NO. 4, JULY 1981

Determination of Rain Anisotropy and Effective Spreading of the Orientation of Ellipsoidal Raindrops During Intense Rainfall

E. MATRICCIANI, A. PARABONI, G. POSSENTI, AND S. TIRRÒ

Abstract—Depolarization measurements performed in circular polarization at centimeter wavelength have frequently shown depolarization values lower than those theoretically expected by application of the Oguchi theory, which assumes equioriented ellipsoidal raindrops. This discrepancy can be explained by considering that in reality the drop-axes are oriented at a given instant according to a certain statistical distribution the spreading of which causes a decrease in the anisotropy (differential propagation constants along the principal planes), which is the prime source of rain depolarization (both in circular and in linear polarization). The reduction of anisotropy with respect to the equioriented drops model is experimentally assessed and the spreading necessary in a particular model distribution for the same reduction is determined in this communication. The study is based upon simultaneous measurements of attenuation and depolarization in circular polarization gathered in 1974–1975 during the Magnola experiment (Italy) on a low-elevation terrestrial path (9°) at 11.6 GHz. The results confirmed that the equiorientation of raindrops is too severe an assumption, being the true anisotropy 20–30 percent (on average) less than the expected value in case of equiorientation. This figure corresponds to an average spreading of about 20–30° which is in agreement with the results given by other authors.

INTRODUCTION

The canting angle of raindrops is an important parameter in determining rain depolarization at centimeter and millimeter wavelength. Its determination however is very complicated and expensive, and as a consequence the related experimental activities are very limited. Some experimental results can be found in [1]-[7]. Theoretical models which account for the raindrops orientation have been developed by several authors and may be found in [8]-[14]. Different distributions of the raindrop orientation give different values of rain anisotropy (i.e., the differential attenuation and phase along the principal planes), therefore affecting the rain depolarization both in linear and in circular polarization. It has been found [5] that anisotropy in terrestrial links is rather insensitive to the mean canting angle and therefore depends mainly on the spreading of the distribution. In this communication we suggest a new method for evaluating the rain anisotropy and the related effective spreading based on a particular model distribution. The method uses a single circular polarization and the incoherent detection of the copolar and x-polar signals received.

As an application of the method we analyze some experimental data taken during 1974-1975 in the Magnola experiment performed by Telespazio under an ESRO[1] contract. The method consists of comparing a direct measurement of the rain depolarization in circular polarization with a simultaneous indirect measurement of the rain depolarization, which is extrapolated (through reasonable physical assumptions) from an

Manuscript received September 6, 1979; revised July, 1980.

E. Matricciani and A. Paraboni are with Istituto di Elettrotecnica ed Elettronica, Politecnico di Milano, Italy.

G. Possenti and S. Tirrò are with Telespazio, S.p.A. per la Comunicazioni Spaziali, Corso d'Italia, 43, 00198, Rome, Italy.

[1] Now ESA (European Space Agency).

0018-926X/81/0700-0679$00.75 © 1981 IEEE

attenuation measurement. This comparison allows us to evaluate the decrease in anisotropy [15] with respect to the expected value in case of all equioriented drops. From a bare "system oriented" point of view this decrease can be considered a goal "per se," owing to the possibility it offers to estimate the anisotropy (and hence the circular depolarization) from copolar attenuation measurements only. However communication engineers often require model-oriented information in order to obtain better insight into the physical phenomenon for frequency extrapolation and to pursue connections with other interacting factors as those used for meteorological predictions. For this task a physical model for the oblateness and orientation of the drops must be assumed. Assumptions made may contain some degree of arbitrariness; in particular, allowance can be made for only the main cause of loss of anisotropy (the spreading) to the detriment of possible other ones (i.e., presence of spherical drops).[2] It turns out however that the resulting values of spreading are in good agreement with those given by other authors, indicating that the neglected factors are of a lower importance.

THE MODEL

In the present communication we develop a model based on one first proposed by the Massachusetts Institute of Technology (MIT) Laboratories [10]. MIT suggested the assumption of the coexistance of two subpopulations of raindrops. The first should include all equioriented raindrops and behave according to the Oguchi model. The other should include raindrops with equiprobable orientations which will therefore not affect the wave polarization. This model is referred to as the "bimodal" model. The first population will be called "active population" and is a fraction of the total raindrops number: we call this fraction p, which is assumed to apply to the whole raindrops population, independent of raindrops diameter. The model is described in a standard polar reference by the probability density function

$$f(\theta, \phi) = p \cdot \delta(0) + (1-p) \cdot \frac{1}{2\pi}. \tag{1}$$

In (1) δ represents the impulse of the active population in the origin ($\vartheta = 0$) and the latter term stands for a constant density over a solid angular domain $2\pi(0 \leq \varphi \leq 2\pi, 1 \geq \cos\vartheta \geq 0)$. Since the propagation constant is additive with respect to the single raindrop contributions, we can express the difference of the propagation constants in the two principal planes as follows:

$$\gamma_{II} - \gamma_I = p \cdot (\gamma_{IIo} - \gamma_{Io}) \tag{2}$$

where $\gamma_{IIo} - \gamma_{Io}$ is the maximum value obtained through the Oguchi model, in the case of all equioriented raindrops. According to [9] the depolarization in circular polarization is given by

$$|\delta| = \left|\tanh\left(\frac{\gamma_{II} - \gamma_I}{2} L\right)\right|, \quad (L = \text{path length}). \tag{3}$$

[2] Even the presence of spherical drops may contribute to decrease the anisotropy: however in practice they are confined in the lowest diameter classes which in any case contribute little to the anisotropy.

Then we can write the equation

$$|\delta| = \left|\tanh\left(p\,\frac{\Delta\alpha(A) + j\Delta\beta}{2}\,(A)L\right)\right| \tag{4}$$

where the differential attenuation and phase shift, given by

$$\Delta\alpha(A) = \mathrm{Re}\,(\gamma_{IIo} - \gamma_{Io}), \quad \Delta\beta(A) = \mathrm{Im}\,(\gamma_{IIo} - \gamma_{Io}), \tag{5}$$

can be obtained from the copolar attenuation by means of the Oguchi algorithms [16] and from suitable physical assumptions. If $|\delta|$ and A are measured, (3) can be solved, obtaining the value of the active population p. The parameter p can be directly employed in anisotropy predictions; it can be related to the spreading (root mean square) rms angular deviation from the vertical through the bimodal model defined above. This requires the evaluation of the mean square angle associated with the probability density function (PDF) $f(\theta, \phi)$ of the drop-axes orientation. The rms is given by

$$\vartheta = \left[\int_0^{2\pi} d\phi \cdot \int_1^0 \vartheta^2 f(\theta, \phi)\, d(-\cos\theta)\right]^{1/2}$$

$$= \sqrt{(1-p)\cdot(\pi - 2)} \tag{6}$$

THE ALGORITHM

Equations (2)–(5) have been written for the particular situation of the Magnola radio link (f = 11.6 GHz, L = 8 km, elevation = 9°). The following physical assumptions have been made: a) uniform rainfall intensity along the path; b) raindrop size distributions according to Laws-Parson; c) ellipticity according to Jones [18][3]. $\Delta\alpha(A)$ and $\Delta\beta(A)$ have been evaluated using the values of γ_{Io} and γ_{IIo} obtained in the studies of Bordoni [21] for different rainfall rate and a medium temperature of 20° C. For the same rain intensities the attenuation in circular polarization is computed according to the equation below:

$$A = -20 \log_{10}\left|\frac{e^{-\gamma_{IIo}L} + e^{-\gamma_{Io}L}}{2}\right|. \tag{7}$$

This equation is valid for equioriented raindrops, but it gives a very good approximation also in the case of the model which is proposed here. $\Delta\alpha(A)$ and $\Delta\beta(A)$ have been computed with the following equations:

$$\Delta\alpha \cdot L = 0.1578 \quad A^{(1.0781 - 0.00128 \log_{10} A)}$$

$$\Delta\beta \cdot L = 4.774 \cdot A^{(0.8372 + 0.02384 \log_{10} A)}. \tag{8}$$

Using (8), (4) can be immediately solved and gives a value of p for each pair $|\delta|$, A.

EXPERIMENTAL RESULTS

Measurements performed in circular polarization on the Celano-Magnola path provided many pairs of $|\delta|$, A values [22]. These values have been entered into (8) and (4) and the

[3] The effects of the rain-rate profile along the path and of the raindrop size distribution have been investigated in [19]; they give rise to an overall variation of p of ±6 percent. As for the ellipticity, different shapes, as those given by Pruppacher and Pitter [20], have been found to have very small effect on the overall anisotropy.

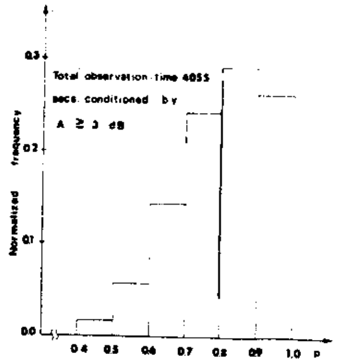


Fig. 1. Histogram of active population p.

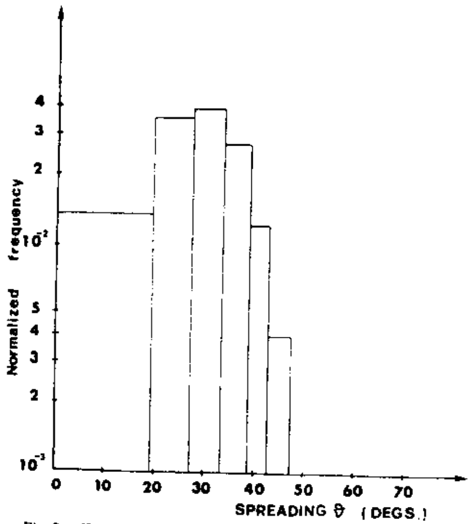


Fig. 2. Histogram of the spreading of rain drop axes orientation.

corresponding values of the active population p have been computed. The values of p have then been statistically classified. Fig. 1 shows the histogram of the active population p. The histogram is normalized to the observation time during which the attenuation exceeded 3 dB. Fig. 2 gives the corresponding histogram of the equivalent spreading θ_S of raindrop axes orientation.

It is found that the mean value of p is 0.80 in good agreement with [3], while the standard deviation is 0.125. Correspondingly the mean value of θ_S is 25° while the standard deviation is 10°. This is in good agreement with [1] and [5]. Fig. 3 shows histograms of the active population conditioned by given levels of attenuation. Fig. 4 shows mean values and standard deviations of p as functions of the attenuation. It can be noted that the mean value of the active population is practically independent of attenuation; conversely the standard

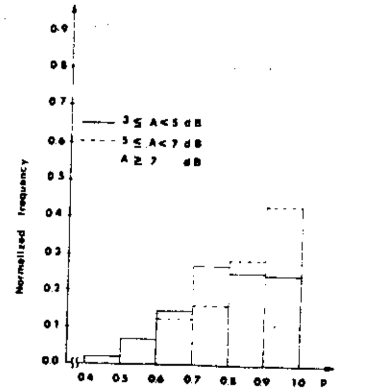


Fig. 3. Conditional histograms of active population p.

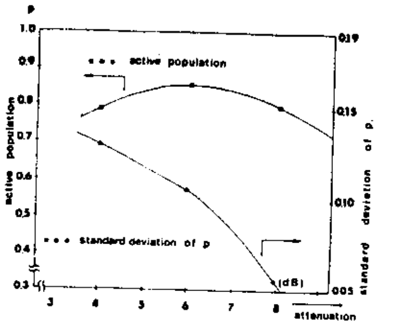


Fig. 4. Mean value and standard deviation of active population p as a function of attenuation level.

deviation tends to decrease as the attenuation increases; the same happens for the drop spreading. These observations are consistent with the well-known fact that large drops, owing to their inertia, are less sensitive to the faster variation of turbulence. As anisotropy is mainly build up by very large drops, the effect of the lowest frequency components of turbulence is then emphasized to detriment of the highest ones. Apparently in the case of moderate rain and lower attenuation a wider spectrum of turbulence gives contributions to the anisotropy, thus enhancing its variance [14].

CONCLUSION

An algorithm for the determination of rain anisotropy and effective spreading of raindrop axes orientation has been proposed. An equivalent parameter expressing the reduction of anisotropy due to spreading has also been suggested.

The results show that equiorientation of raindrops is too severe a hypothesis and implies a value of anisotropy always larger than that measured. It seems necessary to reduce the rain anisotropy by 15-25 percent on average for a direct prediction of the circular depolarization from copolar attenuation measurements. The effective spreading seems in good agreement with the results obtained in [1] and [5].

REFERENCES

[1] M. J. Saunders, "Cross polarization at 18 and 30 GHz due to rain," *IEEE Trans. Antennas Propagat.*, vol. AP-19, pp. 273–277, Mar. 1971.

[2] D. T. Thomas, "Cross polarization distortion in microwave radio transmission due to rain," *Radio Sci.*, vol. 6, pp. 833–839, 1971.

[3] G. C. McCormick, A. Hendry, and L. E. Allan, "Depolarization over a link due to rain. Measurements of the parameters," *Radio Sci.*, vol. 11, pp. 741–749, Aug. Sept. 1976.

[4] P. A. Watson, "Survey of measurements of attenuation by rain and other hydrometeors," *Proc. IEE*, vol. 123-9, pp. 863–871, Sept. 1976.

[5] T. S. Chu, "Rain-induced cross-polarization at centimeter and millimeter wavelength," *Bell Syst. Tech. J.*, vol. 53, pp. 1557–1579, Oct. 1974.

[6] R. A. Semplak, "Effect of oblate raindrops on attenuation at 30.9 GHz," *Radio Sci.*, vol. 5, pp. 559–564, Mar. 1970.

[7] ——, "Simultaneous measurements of depolarization by rain using linear and circular polarizations at 18 GHz," *Bell Syst. Tech. J.*, vol. 53, pp. 400–404, Feb. 1974.

[8] T. Oguchi and Y. Hosoya, "Differential attenuation and differential phase shift calculations at microwave and millimeter wave region," *J. de Rech. Atm.*, vol. 8, pp. 121–128, 1973.

[9] C. Capsoni and A. Paraboni, "Depolarization of an electromagnetic wave travelling through a stratified aerosol of nonspherical scatterers," in *AGARD Proc. Telecommunication Aspects on frequencies between 10 and 100 GHz Conf.*, Gausdal (Norway), 1972, pp. 2.1–2.16.

[10] R. F. Newell, S. G. Geotis, and A. Fleisher, "The Shape of rain and snow at microwave lengths," M.I.T. Res. Rept. no. 28, 1957.

[11] M. Juy and M. Rooryck, "Analyses des mesures de propagation oblique effectuées à Gometz sur le signal due satellite SIRIO," CNET Int. Rept., April 1979.

[12] R. A. Semplak, "Measurements of rain-induced polarization rotation at 30.9 GHz," *Radio Sci.*, vol. 9, pp. 425–429, Apr. 1974.

[13] G. Brussaard, "A meteorological model for rain-induced cross polarization," *IEEE Trans. Antennas Propagat.*, vol. AP-24, pp. 5–11, Jan. 1976.

[14] B. O. Maher, P. J. Murphy, and M. C. Sexton, "A theoretical model of the effect of wind-gusting on rain induced cross-polarization," presented at URSI Symp. on Propagation in Non-Ionized Media, La Baule (France), April 28–May 6, 1977.

[15] A. Paraboni, G. Possenti, and S. Tirrò, "Some experimental results about the spreading of ellipsoidal raindrops orientation during intense rainfall," in *Proc. IEEE Conf. on Antennas and Propagation*, London, 1978, pp. 82–86.

[16] T. Oguchi, "Attenuation of electromagnetic wave due to rain with distorted raindrops," *J. Radio Res. Lab.*, vol. 2-53, Jan. 1964.

[17] D. M. A. Jones, "The shape of raindrops," *J. Meteorol.*, vol. 16, pp. 504–510, 1959.

[18] C. Capsoni, D. Maggiori, E. Matricciani, and A. Paraboni, "Rain anisotropy prediction: Theory and experiment," *Radio Science*, Spring 1981, to be published.

[19] H. R. Pruppacher and R. L. Pitter, "A semi-empirical determination of the shape of cloud and rain drops," *J. Atmos. Sci.*, vol. 28, pp. 86–94, Jan. 1971.

[20] D. Maggiori, "Calcolo dell'attenuazione e dello sfasamento per unità di lunghezza dovuti a pioggia costituita da gocce ellissoidali," Fondazione Ugo Bordoni, Int. Rep. no. 43, 1978.

[21] S. Tirrò, "The Magnola 11.6 GHz depolarization experiment," in *Proc. IEE Symposium on Satellite Communication and System Technology*, London, 1975, pp. 175–187.

A Technique for Improving Depolarization Measurements Beyond 10 GHz: Tilting the Polarization Planes

D. MAGGIORI, P. MIGLIORINI, AND A. PARABONI

***Abstract*—Depolarization measurements in linear polarization aiming at determining the effective canting angle of the principal planes and the related anisotropy can be improved, in many important cases, by adopting the simple expedient of tilting the polarization planes with respect to the vertical/horizontal orientations. Optimum tilt angles depend on the system configuration but the choice is not critical in a range spanning from about 20° to about 30° provided that the disturbance is small enough (say 50 dB under the copolar level) and/or the anisotropy of the medium is large enough (say 2 dB in the differential attenuation).**

I. INTRODUCTION

A model-oriented approach to assess the transfer characteristics of a dual-polarization radio link undergoing severe depolarization by hydrometeors has been recently proposed [1]. The aim of this approach is to achieve information closely related to the physical phenomenon causing depolarization and is therefore advantageous for extrapolation to other polarizations and for the possibility it offers of comparing results obtained in different conditions. This information consist mainly of three "quasi-physical" parameters: the anisotropy, the mean apparent canting angle of the drop axes, and the longitudinal homogeneity index. The first two, which are sufficient for a first approximation description, have received in the past considerable attention by a number of theoretical and experimental researchers [2]–[11]; in addition the CCIR has recently proposed a prediction method which takes them into account [12]. Therefore, it seems justified to think of future depolarization experiments specifically designed to assess them as a primary goal, leaving the determination of depolarization in whichever condition to a subsequent step.

Manuscript received August 5, 1982; revised November 17, 1982.

D. Maggiori and P. Migliorini are with the Fondazione Ugo Bordoni, V. le Europa, 160, Rome, Italy.

A. Paraboni is with the Politecnico di Milano, Dipartimento di Elettronica, Piazza Leonardo da Vinci, 32-20133, Milan, Italy.

This paper is an effort in this direction: it is shown here, both theoretically and experimentally, that the simple expedient of tilting the polarization planes of about 25° with respect to the vertical/horizontal in linear polarization systems allows a substantial improvement in the accuracy and measuring range of most of the envisaged experimental configurations aiming at measuring canting angle and anisotropy of the depolarizing medium. Moreover in case of incoherent detection, the sign of the canting angle is detectable, differently from what would happen otherwise. Accordingly the experiment at 20 and 30 GHz to be performed with the future European satellite L-sat will make use of tilted polarization planes [13], as seen by any experimenter in Europe.

II. MATHEMATICAL BACKGROUND

Even though the considerations in the following apply in general, in this paper reference will be made to a medium which can be described in terms of principal planes, as this model has been proved to be a good simplification in many practical cases [7], [8]. As is known, such medium is completely determined by the anisotropy Δ (differential attenuation and phase shift along the principal planes) and the angle ϕ of the principal planes (perpendicular to each other) with respect to a general reference system (see Fig. 1).

Anisotropy is complex and is related to a number of physical variables such as the drop ellipticity the spreading in the orientations of the drop axes, the raindrop size distribution and the rain intensity profile along the path.

The other parameter ϕ is related to the mean orientation of the drop axes and is real because of the assumption of the existence of the principal planes.

The complex depolarization ratios between the cross-polar and the copolar voltages are given by [10]

$$\delta_{12,21} = \frac{\sin 2(\phi - \phi_0) \tanh\left(\dfrac{\Delta}{2}\right)}{1 \pm \cos 2(\phi - \phi_0) \tanh\left(\dfrac{\Delta}{2}\right)} \tag{1}$$

where the suffix 12 means "transmission on channel one and reception on channel two" and vice versa; ϕ_0 is the tilt angle of the system polarization planes, which is our major concern.

In the present communication we will restrict our consideration to experimental setups able to measure δ_{12} and/or δ_{21} with the aim of determining ϕ and Δ.

Even if there is in principle the possibility to extract polarization-related information also from the copolar voltages complex ratio we will not consider it; it is thought in fact that there will be a significant number of cases in which this parameter will not be measured owing to the experimental difficulty of doing so and the very high related cost of the receiving equipment.

A. Double-Polarization System

From the measured depolarization ratios (1) the canting angle ϕ can be derived directly by the formula

$$\phi = \phi_0 + \tfrac{1}{2} \tan^{-1}\left[2\left(\frac{1}{\delta_{21}} - \frac{1}{\delta_{12}}\right)^{-1}\right]. \tag{2}$$

0018-926X/83/0700-0628$01.00 © 1983 IEEE

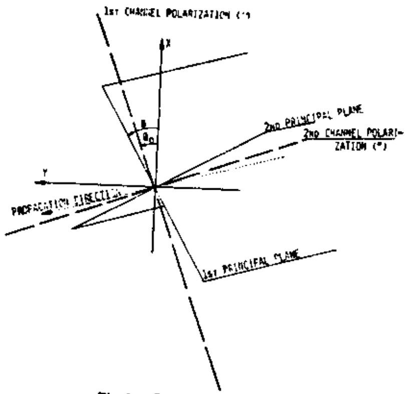


Fig. 1. System geometry.

in which it is expected that only the real part assumes a significant value while the imaginary one, which is related to the departure from the hypothesis of principle planes, is much smaller and of no concern here. The expression $|1/\delta_{21} - 1/\delta_{12}|^{-1}$ in the second member of (2) (kernel) becomes singular when the depolarization ratios approach each other. In practice this means that when the depolarization ratios are very close to each other the system is very vulnerable to any kind of disturbance such as noise, antennas, polarization mismatch, multipath, etc. This happens when $\phi - \phi_0$ tends to 0° or ±45°; because ϕ is ordinarily less than a few degrees it follows that choosing the bias $\phi_0 = 0°$ or $\phi_0 \pm 45°$ would be very detrimental as the system would be lead to work very close to its "blind" zones for most of the time.

It is then clear that an optimum choice for ϕ_0 must exist somewhere inside this range, meaning with this a choice which minimizes some kind of functional related to the statistics of the error in measuring ϕ. A detailed statistical analysis of this error should take into account the nature of the disturbance, which affects both δ_{12} and δ_{21}, and would be a rather complicated exercise owing to the nonlinearity of (2).

Fortunately this is not strictly necessary as an envelope of the maximum error can be easily obtained as shown in Fig. 2 where some curves for various signal-to-disturbance ratio and anistropy have been plotted as function of $\phi - \phi_0$.

These curves have been computed simply by substituting (1) in (2) after having added to the ratios δ a quantity ϵ to account for a disturbance affecting the cross-polar level as follows:

$$\delta_{meas} = \frac{V_x + V_d}{V_c} = \delta + \epsilon, \tag{3}$$

where δ_{meas} is the measured error-affected ratio, V_d is the disturbance superimposed to the cross-polar voltage V_x (copolar voltage V_c is assumed errorless), δ is given by (1) and ϵ is the disturbance to copolar signal ratio.

The difference between the value of ϕ given by (2) and the one inserted in (1) gives the error, the upper bound of which, obtained using opposite values of ϵ in the two δ, is represented in Fig. 2 as function of $\phi - \phi_0$.

The curves show that high values of signal-to-disturbance ratio (≥50 dB) are necessary with low anisotropy to assure error bounds of a few degrees (say 5°). As the anisotropy increases

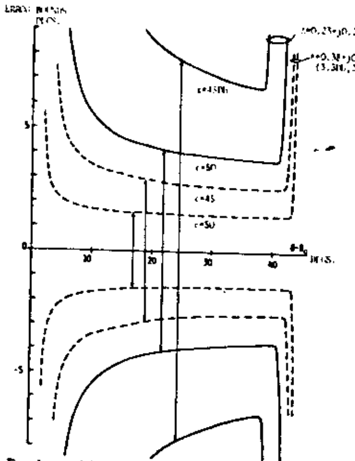


Fig. 2. Envelopes giving the maximum error in the measurement of the effective canting angle versus the canting angle $\phi - \phi_0$; ϵ = level of the maximum disturbance voltage at the cross-polar output with respect to the copolar signal; Δ = anisotropy. Dotted curves: $\Delta = 0.38 + j0.35$ (3.3 dB, 20°). Solid curves: $\Delta = 0.23 + j0.23$ (3 dB, 13°). In case of equi-aligned drop axes the first case corresponds approximately to a 15 dB copolar attenuation at 30 GHz while the second case to a 7.5 dB copolar attenuation.

however the situation improves very quickly and mode rate errors can be made even in the presence of 45 dB or less in the signal-to-disturbance ratio.

Similar conclusions apply to the determination of Δ, which also depends on the kernel of (2).

B. Single-Polarization System

Systems able to perform depolarization measurements in a single polarization can evidently provide less information on the transfer channel than the previous ones, as they allow determination of two parameters (i.e., amplitude and phase of δ) in the case of coherent systems or only one parameter (the amplitude) in the case of incoherent systems. The artifice of tilting the polarization planes is particularly advantageous in this latter case for which a larger degree of uncertainty in the determination of ϕ must be accepted. In this section we will restrict our attention to these systems.

The reason why here the tilt angle ϕ_0 turns out to be advantageous lies on the fact that the modulus of (1) is a nonmonotonic function of the argument $\phi - \phi_0$ owing to a cusp for $\phi - \phi_0 = 0$. The choice $\phi_0 = 0$ would then imply three fundamental disadvantages.

1) It would not allow bilateral measurements of ϕ (i.e., inclusive of the sign).
2) It would lead the envelope detector to work near the zero, thus introducing a drastic deterioration of the output signal-to-noise to ratio with respect to the input one, when signal is comparable with noise [14]; in this case post-detection filtering is of little advantage as the rectified noise has a noticeable dc component.
3) It would introduce an ambiguity which in the common case

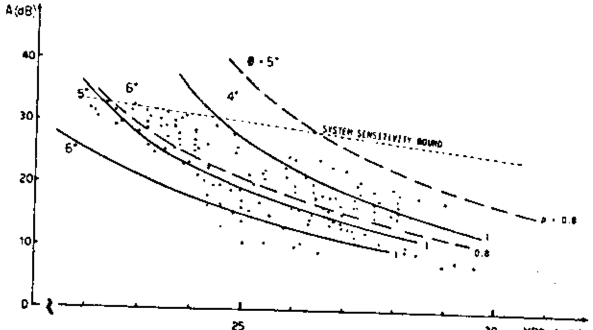


Fig. 3. Scatterplot of copolar attenuation versus depolarization for a terrestrial 9.5-km linearly polarized link at 17.8 GHz. Model-curves refer to an effective canting angle ϕ (with respect to the vertical) as indicated and to an anisotropy derived from an equialigned drop-axes rain model scaling of a factor p to take into account the partial randomization in the drop-axes orientations. No tilting.

of biaslike disturbances (such as the antenna depolarization) would give the same output for couples of asymmetrical values of ϕ, therefore distorting the resulting statistical distributions.

It is then concluded that ϕ_0 must be suitably chosen so as to keep the variation range of ϕ sufficiently far from the ambiguous zones about 0° and 45°. A value around 20° appears appropriate in many cases [15].

III. AN EXPERIMENTAL CHECK

The points exposed in the previous section are experimentally confirmed by the results obtained in a series of depolarization measurements at 18 GHz using linear polarization [15].

The aforementioned experimented was performed by Fondazione Bordoni in two operational phases: in the first phase a vertically polarized wave was transmitted along a 9.5 km terrestrial path and received with a vertically/horizontally dually polarized antenna. Both channels were equipped with incoherent logarithmic amplification/detection chains and, at the end, the depolarization XPD (modulus of δ) was extracted together with the copolar attenuation A. During the second phase the polarization planes were changed by tilting both the transmitting and the receiving antennas of an angle of 17° with respect to the vertical.

In Fig. 3 a scatterplot of copolar attenuation versus XPD taken during the first phase is shown; on the same figure model-curves have been drawn assuming for the anistropy the value given by the Oguchi theory with equi-aligned drop axes ($p = 1$) and partially randomized axes ($p = 0.8$) [10]. The considered tilt angles are 4°, 5°, and 6° with respect to the vertical.

It is clear that the "cloud" of measurements is superiorly bounded by an imaginary borderline which turns out to be approximately represented by the equation:

$$A + \text{XPD} = 55 \text{ dB}, \tag{4}$$

which is near to the minimum level (relative to the clear-air copolar received level) detectable by the system. It follows that canting angles less than about 4° (in absolute value) give rise to an undetectable cross-polar level and could not therefore be measured.

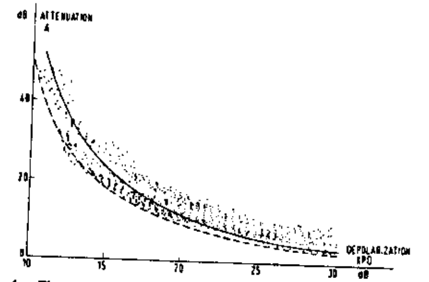


Fig. 4. The same as Fig. 3. Here the model curves refer to a 0° canting angle with respect to the vertical and to the same values of p. Tilting: 17° with respect to the vertical.

In Fig. 4 an analogous scatterplot is represented for the second-phase experiment (17° tilt angle). On the same figure two curves for $\phi = 0°$ and for $p = 1$ and 0.8, as in the previous case, are plotted.

It is clear that no "blind zone" exists. The measurements of ϕ were statistically analyzed and found to fall within a narrow range around the zero.

Fig. 5 gives a cumulative statistical distribution of this variable, based on data collected whenever the copolar attenuation exceeded 6 dB in order to avoid excessive errors due to low anisotropy and considering that depolarization will be particularly important in future applications, when occurring simultaneously with attenuation. The distribution shows that without tilting, the data base would have been reduced to about the 20 percent.

IV. CONCLUSION

Tilting the polarization planes in radio links at centimetric or millimetric waves performing depolarization measurements is an economic expedient allowing increased accuracy, data base enhancement and bilateral detection of the effective canting angle of the depolarizing medium principal planes.

(63)

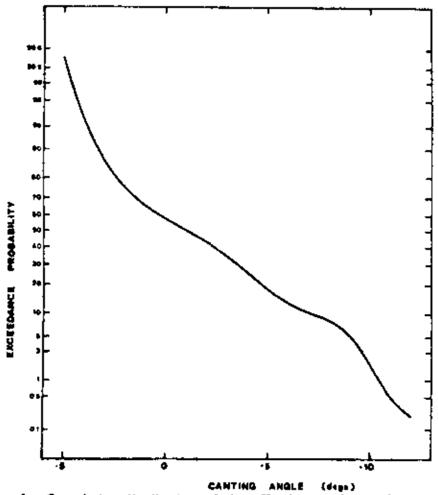


Fig. 5. Cumulative distribution of the effective canting angle ϕ (with the condition that the copolar attenuation exceeds 6 dB).

REFERENCES

[1] C. Capsoni, A. Paraboni, F. Fedi, and D. Maggiori, "A model-oriented approach to measure rain-induced cross-polarization," *Ann des Telecommun.*, vol. 36, no. 1–2, pp. 154–159, Jan.–Feb. 1981.

[2] T. Oguchi and M. Yamada, "Frequency characteristics of attenuation, phase shift and cross-polarization due to rain within communication bands: Calculations at 4, 6, 11 and 14 GHz bands for Intelsat satellite communication systems," *J. Radio Res.*, vol. 28, no. 125/126, pp. 97–131, Mar.–July 1981.

[3] G. C. McCormick and A. Hendry, "Polarization-related parameters for rain measurements obtained by radar," *Radio Sci.*, vol. 11, no. 8–9, pp. 731–740, Aug.–Sept. 1976.

[4] G. Brussaard, "A meteorological model for rain-induced cross polarization," *IEEE Trans. Antennas Propagat.*, vol. AP-24, no. 1, pp. 5–11, Jan. 1976.

[5] I. J. Dilworth and B. G. Evans, "Linear and circular cross polarization on an 11 GHz terrestrial radio link," presented at the URSI Symp. on Effects of the lower atmosphere on radio propagation at frequencies above 1 GHz, Lennoxville, Canada, May 1980.

[6] M. J. Saunders, "Cross polarization at 18 and 30 GHz due to rain," *IEEE Trans. Antennas Propagat.*, vol. AP-19, no. 2, pp. 273–277, Mar. 1971.

[7] J. Neves and P. A. Watson, "Cross-polarization, differential attenuation and differential phase shift measured on a 36.5 GHz terrestrial link" presented at the URSI Symp. on Effects of the lower atmosphere on radio propagation at frequencies above 1 GHz, Lennoxville, Canada, May 1980.

[8] D. C. Cox, "Depolarization of radio waves by atmospheric hydrometers in earth-space paths: A review," *Radio Sci.*, vol. 16, no. 5, pp. 781–812, Sept.–Oct. 1981.

[9] R. L. Olsen, "Cross polarization on terrestrial links—A review," *Radio Sci.*, vol. 16, no. 5, pp. 761–779, Sept.–Oct. 1981.

[10] C. Capsoni, D. Maggiori, E. Matricciani, and A. Paraboni, "Rain anisotropy prediction: Theory and experiment," *Radio Sci.*, vol. 16, no. 5, pp. 909–916, Sept.–Oct. 1981.

[11] T. S. Chu, "Rain-induced cross-polarization at centimeter and millimeter wavelength," *Bell Syst. Tech. J.*, vol. 53, no. 8, Oct. 1974.

[12] CCIR Report 722.

[13] ESA "L-sat user's guide—Propagation package," Doc. UG-6-1, Part 1, Mar. 1981.

[14] W. Davenport and W. Root, *An Introduction of the Theory of Random Signals and Noise*. New York: McGraw Hill, 1958.

[15] D. Maggiori, P. Migliorini, and A. Paraboni, "Studio di un sistema di misura per la caratterizzazione dei meccanismi fisici causante degradazione di discriminazione tra polarizzazioni ortogonali," Fondazione Bordoni Int. Rep. FUB1B5678, 1978.

0018-926X/83/0700-0631$01.00 © 1983 IEEE

(64)

Extrait ANNALES DES TÉLÉCOMMUNICATIONS, tome 36, n°s 1-2
Janvier - Février 1981

A model-oriented approach to measure rain-induced cross-polarization

Carlo CAPSONI *

Aldo PARABONI *

Francesco FEDI **

Dario MAGGIORI **

Abstract

In this paper a new model oriented *approach to measuring rain-induced cross-polarization is outlined : it aims at characterising the physical properties of the depolarising medium rather than at directly measuring the channel depolarization, which is the variable of direct interest for system design. This approach, which allows subsequent determination of depolarization has the advantages of allowing a better understanding of the propagation mechanism, thus yielding information applicable to any system configuration and establishing common procedure to compare cross-polarization data obtained in different circumstances. Different experimental configurations are reviewed together with their ability to identify and assess various possible models for the depolarizing medium.*

Key words : Cross-polarization, Rain, Wave propagation, Microwaves, Physical property, Scattering medium.

MÉTHODE DE MESURE A PARTIR DE MODÈLES DE LA TRANSPOLARISATION DUE A LA PLUIE

Analyse

Cet article présente une nouvelle méthode de mesure, à partir de modèles, de la transpolarisation due à la pluie destinée à caractériser les propriétés physiques du milieu dépolarisant plutôt qu'à mesurer directement la dépolarisation de la voie de transmission. Cette méthode est surtout intéressante au niveau de la conception. Elle permet le calcul ultérieur de la dépolarisation et conduit à une meilleure compréhension du mécanisme de propagation. Les résultats sont applicables à des systèmes de configuration quelconque et permettent d'établir des procédures générales de comparaison des données. Différentes configurations expérimentales sont examinées du point de vue de leur modélisation éventuelle.

Mots clés : Transpolarisation, Pluie, Propagation onde, Hyperfréquence, Propriété physique, Milieu diffusant.

Contents

1. INTRODUCTION

The important role played by rain-induced cross-polarization in adopting frequency reuse techniques for radio systems at frequencies above 10 GHz is well recognized. From an experimental point of view, cross-polarization phenomena can be studied following two different approaches. The first one, which could be described as system design-oriented, consists of directly measuring cross-polarization with an experimental set-up as similar as possible to the envisaged radio system configuration. This approach

* Politecnico di Milano, Piazza Leonardo da Vinci, 32, 20133 Milano (Italia).
** Fondazione U. Bordoni, Viale Europa, 100, 00100 Roma (Italia).

has led to the use of linear polarization to gather information for terrestrial systems, and circular polarization for earth satellite systems. The second approach, which could be termed model-oriented, aims at characterizing the physical properties of the depolarizing medium in order to establish models from which information for system design can be subsequently derived. This approach might lead to an experimental set-up different from the envisaged configuration of the actual system. However it has the advantages of allowing a better understanding of the propagation mechanism and yielding information applicable to any system configuration. Moreover it enables a common procedure to be established for the comparison of cross-polarization data obtained in different locations, based on the parameters of a model which seems to have physical significance.

The purpose of this paper is to emphasize the advantages of the second approach and to review different possible experimental configurations, together with their ability to identify and assess the various models adopted for the depolarizing medium. The paper starts from the most sophisticated experiment, which uses three independent polarizations and coherent detection to characterize the most general linear depolarizing medium. Various degrees of decreasingly complex experiments are then examined in order to study media for which additional hypotheses must be accepted. The paper ends with the simplest experiment which utilizes only one polarization and incoherent detection of copolar and cross-polar signals. Advantages and disadvantages of each solution are illustrated taking into account the conflict between the complexity of the experimental set-up and the accuracy which can be achieved.

2. BACKGROUND

Let us consider two electromagnetic waves $\vec{E}_1$ and $\vec{E}_2$, generically polarized and propagating through a linear medium. The input-output relationship between the complex amplitudes can be expressed, in matrix form, as

$$\begin{bmatrix} E_{10} \\ E_{20} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} E_{1i} \\ E_{2i} \end{bmatrix}, \tag{1}$$

where $T_{ij} = \exp - |A_{ij} + jB_{ij}|$ are general complex parameters, A_{ij} are their associated levels (expressed in logarithmic units), and B_{ij} are their associated phase shifts.

In polarization-related problems, only relative values of levels and phases are involved and so the T_{ij} - parameters can be normalized without loss of information about the depolarization capability of the channel. Among all the possible normalizations, we will assume here the one which makes the terms of the main diagonal the inverse of the other, i.e.

$$\begin{bmatrix} D_{12} & X_{12} \\ X_{21} & 1/D_{12} \end{bmatrix}, \tag{2}$$

where

$$\begin{cases} D_{12} = \exp[(A_{22} - A_{11})/2 + j(B_{22} - B_{11})/2], \\ X_{12} = \exp - [A_{12} - (A_{11} + A_{22})/2 + \\ \qquad\qquad j(B_{12} - (B_{11} + B_{22})/2)], \\ \text{and} \\ X_{21} = \exp - [A_{21} - (A_{11} + A_{22})/2 + \\ \qquad\qquad j(B_{21} - (B_{11} + B_{22})/2)], \end{cases} \tag{3}$$

D is the copolar unbalance associated with the deviation of the copolar levels and phases from their averages. Similarly the X_{ij} parameters are associated with the deviations of the cross-polar levels and phases from the average copolar ones.

The complex parameters (2), (3) will be referred to as the *normalized matrix parameters* ; they fully characterize the depolarizing capabilities of the channel for any polarization. In particular the depolarization ratios considered by communication engineers as primary goals are (Watson [1]) :

$$\begin{cases} \delta_{12} = X_{12} D_{12}, \\ \text{and} \\ \delta_{21} = X_{21}/D_{12}. \end{cases} \tag{4}$$

However, the *normalized matrix parameters* strongly depend on polarization, so a common basis must be established in order to compare measurements made with different polarizations. For this purpose, we propose the use of the *intrinsic parameters*, Δ, φ, L, which seem to have some physical significance. For convenience, we express them as a function of the *normalized matrix parameters* in circular polarization (the relationships are in fact more compact) although they could be expressed in any polarization, i.e.

(5)

$$\Delta = 2 \tanh^{-1} \sqrt{X_{rl} X_{lr}} = 2 \tanh^{-1} \sqrt{\delta_{rl}\, \delta_{lr}} \qquad (\mathrm{Re}\,(\Delta) \geqslant 0),$$

$$\Phi = \frac{-j}{2} \log_e \left(D_{rl} \frac{X_{lr}}{\sqrt{X_{rl} X_{lr}}} \right) = \frac{-j}{2} \log_e \left(\frac{\delta_{rl}}{\sqrt{\delta_{rl}\, \delta_{lr}}} \right) \left(-\frac{\pi}{2} < \mathrm{Re}(\Phi) < \frac{\pi}{2} \right),$$

and

$$L = j(D_{rl}^2 - 1)/(D_{rl}^2 + 1).$$

The following properties hold (see appendix) :

i) if the medium is longitudinally homogeneous and filled with particles with axial symmetry, then $L = 0$;

ii) if the medium has principal planes, that is two perpendicular planes along which linearly-polarized waves propagate without being depolarized, then Φ is real and the real and imaginary parts of the anisotropy become the differential attenuation (in Nepers) $A = \mathrm{Re}\, |\Delta|$ and the differential phase shift (in radians) $B = \mathrm{Im}\, |\Delta|$ along these planes. Φ (in radians) identifies the orientation of the first principal plane with respect to the reference system (*) (Capsoni [2]) ;

iii) Δ and L are invariant with respect to a rotation of the reference system. Φ varies oppositely to this rotation.

Therefore let us call Δ the *generalized anisotropy*, Φ the *generalized canting angle* (Brussaard [3]) and L the *longitudinal homogeneity index*. In an appendix an outline is given as to how the *normalized matrix parameters* (and therefore the depolarization δ in any polarization) may be obtained from the *intrinsic parameters*.

The main advantages of this description are the following :

1) it allows the comparison of measurements made under different conditions ;

2) it allows some testing of the physical properties of the medium ;

3) if the experimental set up does not allow a complete measurement of the medium (three independent complex parameters), it is straightforward to see which assumptions must be considered and how they reflect on the channel description. In the following section it will be shown how the *intrinsic parameters* can be experimentally identified.

3. THREE POLARIZATION SYSTEMS

It is well known that a possible way of identifying a general radio-channel consists in performing separate measurements of the depolarization ratios using three independent polarizations (Beckman [4]). This is achieved using a system which is able to transmit three polarizations with adequate time or frequency division. For instance, for three linear x, y and 45° polarizations, this means the inputs are :

$$\begin{matrix} E & 0 & E \\ 0 & E & E \end{matrix}$$

The *intrinsic parameters* in this case have the expressions :

(6)

$$\Delta = 2 \tanh^{-1} \sqrt{\frac{(D_{xy} - 1/D_{xy})^2 - (X_{xy} + X_{yx})^2}{(D_{xy} + 1/D_{xy})^2 - (X_{xy} - X_{yx})^2}},$$

$$\Phi = -\frac{j}{2} \left(\frac{(D_{xy} - 1/D_{xy}) - j(X_{xy} - X_{yx})}{(D_{xy} + 1/D_{xy}) - j(X_{xy} - X_{yx})} \right) \times \frac{1}{\tanh\,(\Delta/2)},$$

and

$$L = \frac{(X_{xy} - X_{yx})}{(D_{xy} + 1/D_{yx})},$$

with D_{xy}, X_{yx}, X_{xy} related to the measured polarization ratios

$$\begin{cases} D_{xy} = \sqrt{(\delta_{yx} + 1)(\delta_{450} + 1)/(\delta_{xy} + 1)(1 - \delta_{450})}, \\ X_{xy} = \delta_{xy}/D_{xy}, \\ \text{and} \\ X_{yx} = \delta_{yx}\, D_{xy}. \end{cases} \tag{7}$$

Using this measuring system, no assumptions about the physical nature of the medium need be made (but for its linearity). Yet the procedure outlined in the previous section can be applied, thus allowing some basic properties of the medium to be tested, the assessment of the effectiveness of simplified channel models based on these properties (homogeneity, principal planes), and the comparison of cross-polarization data, even if collected for different polarizations and locations.

4. DUAL-POLARIZATION SYSTEM

With a system which transmits only two polarizations but which is also able to measure complex ratios between quantities relative to the two polarizations, the procedures of sections 2 and 3 can be still applied. Measured quantities in this case are δ_{21}, δ_{12}, $A_{22} - A_{11}$, $B_{22} - B_{11}$; the latter two are the differential amplitude and phase of the two copolar signals, which define the parameter D_{12}.

The application of (5) or of (6) and (7) is then straightforward. Conversely, with a simple two-orthogonal-polarization system, only the complex quantities δ_{21} and δ_{12} are measured. This implies that the medium must be supposed longitudinally homogeneous, with particles having rotational symmetry ($L = 0$) in order to simplify the model.

4.1. Linear polarization.

The condition $L = 0$ implies $X_{xy} = X_{yx} = X$ (see eq. (6)) so that, from (7) we get

$$\begin{cases} D_{xy} = \sqrt{\delta_{yx}/\delta_{xy}}, \\ X = \sqrt{\delta_{xy}\, \delta_{yx}}. \end{cases} \tag{8}$$

(*) In circular polarization, the reference system can be identified by the two axes along which a linearly-polarized wave produces two in-phase (x axis) and in-opposition (y axis) signals at the outputs of a dual-circularly-polarized antenna.

Then from (6) we obtain :

$$(9)\quad \begin{cases} \Delta = 2\tanh^{-1}\sqrt{\dfrac{((1/\delta_{xy} - 1/\delta_{yx})^2 + 4)}{(1/\delta_{xy} + 1/\delta_{yx})^2}}, \\ \text{and} \\ \Phi = -\dfrac{j}{2}\log_e\dfrac{((1/\delta_{xy} - 1/\delta_{yx}) - 2j)}{\sqrt{(1/\delta_{xy} - 1/\delta_{yx})^2 - 4}}. \end{cases}$$

The existence of the principal planes can then be tested by checking if Φ is real. This condition is equivalent to

$$(10)\qquad \mathrm{Im}(1/\delta_{xy}) = \mathrm{Im}(1/\delta_{yx}).$$

Notice that equations (9) are very critical because they depend on the differences between large numbers so that small errors in the measurement of δ are greatly emphasized. Moreover the linear polarizations usually adopted are vertical and horizontal, namely the less depolarized ones. This implies that the cross-polar signal levels are very low, thus seriously limiting the measuring range. An improvement could be achieved by tilting the polarizations with respect to the horizontal and the vertical in order to increase the cross-polar levels. A tilting of 45°, however, the one which gives maximum cross-polar level, is not allowed because in this case $\delta_{xy} = \delta_{yx}$ and equations (9) become singular. It has been shown that the best tilt angle is about 15°. This intrinsic criticity of linear polarization does not exist if the copolar unbalance, D_{xy}, is measured in the modulus and argument ; anyway, a tilting of the polarization is still recommended in order to improve the measuring accuracy.

4.2. Circular polarization.

In this case equation (5) can be directly applied. The test about the existence of the principal planes (Φ real) becomes :

$$(11)\qquad |\delta_{rl}| = |\delta_{lr}|.$$

Unlike the case of linear polarization, no criticity exists for circular polarization. Moreover the cross-polar levels are always higher than any other polarization, thus allowing the best signal-to-noise ratio. For this reason, as far as propagation is concerned, circular polarization appears best suited for a complete identification of the medium, whether the copolar unbalance D is measured or not.

5. SINGLE-POLARIZATION SYSTEMS

With a single-polarization system, δ is the only complex quantity measured. This implies that the medium has to be *a priori* supposed linear and homogeneous, with particles having rotational symmetry ($L = 0$). Also, the existence of two principal planes has to be assumed (Φ real). However, such a medium needs 3 scalar parameters to be characterized, and so information obtainable from copolar attenuation, A, has to be utilized and additional hypotheses concerning the medium must be made (shape, raindrop-size distribution and temperature of raindrops, knowledge of rain rate profile along the path). Moreover it is assumed in the present paper that only a fraction, p, of the raindrops contributes to depolarization.

5.1. Circular polarization.

In this case we get

$$(12)\quad \begin{cases} |\delta| = |\tanh(p\Delta_0/2)|, \\ \arg\delta = \arg\tanh(p\Delta_0/2) \pm 2\,\Phi, \\ \text{and}\qquad \Delta = \Delta_0 p, \end{cases}$$

where :

$\pm$ corresponds to left and right respectively ;

$|\delta|$ and $\arg\delta$ are the modulus and argument of the measured depolarization ;

$p(0 < p < 1)$ is the fraction of raindrops contributing to depolarization [5] ;

$\Delta_0 = f(A)$ is the maximum theoretical anisotropy which can be evaluated by using the well known algorithm of Oguchi [6] and assuming a model for precipitation (shape, raindrop-size distribution and temperature of raindrops) and from a knowledge of the rain rate profile along the path (A is the copolar attenuation).

Δ and Φ can be evaluated by inverting (12). An example of the evaluation of Δ and Φ is given in Figure 1. In the chart the values of the constant $|\delta| = |\tanh(\Delta/2)|$ (dB) and $\arg\tanh(\Delta/2)$ (deg) are given as a function of Re $|\Delta|$ and Im $|\Delta|$. Moreover, a curve for copolar attenuation (dB) at 11.6 GHz is shown assuming uniform rain-rate along a path 8 km long. (Obviously the curve changes if another rain rate profile is assumed ; for instance the actual profile, or a uniform one having a length less than 8 km, or others). The points on this curve correspond to Δ_0. If at a certain instant $A = 15$ dB, $|\delta| = -13$ dB and $\delta = 60°$, then the intersection of the straight line joining $A = 15$ dB with the origin with the curve $|\delta| = -13$ dB gives the values of Re $|\Delta| = 1.3$ dB and Im $|\Delta| = 23°$. Moreover, from (12) we obtain $\Phi = (60\text{-}68)/2 = -2°$. It is interesting to note from the figure that $p = \overline{OP}_1/\overline{OP}_2$. This procedure allows a direct evaluation of p. It appears that no information about Φ can be obtained if only $|\delta|$ and A are measured, but evaluation of p is still possible.

5.2. Linear polarization.

In this case

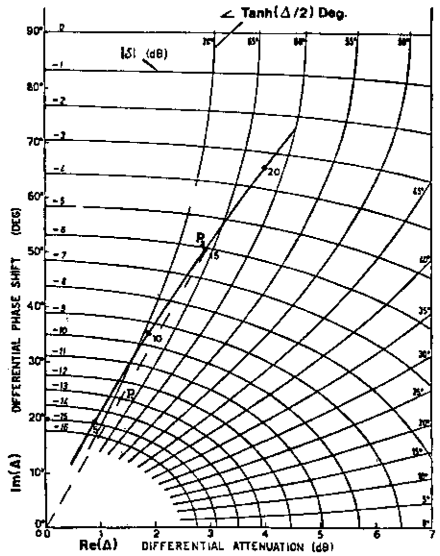


FIG. 1. — Evaluation of Δ and Φ with a single-circular-polarization system.

Evaluation de Δ et Φ dans le cas de polarisation circulaire.

$$(13)\quad \begin{cases} \delta = \sin 2\,\Phi/(\coth(\Delta_0 p/2)\cos 2\,\Phi), \\ \text{and} \\ \Delta = \Delta_0 p. \end{cases}$$

It appears that it is possible to evaluate Δ and Φ on the basis of the measured δ and A. However, it is not possible to draw a general chart similar to that for circular polarization, because the real and imaginary parts of δ depend simultaneously, and in a more complicated way, on Δ and Φ.

If only $|\delta|$ is measured, a certain value of p has to be *a priori* assumed. This can lead to considerable inaccuracy, if (as in the real case) p differs significantly from one [5]. Moreover, information about the sign of Φ is lost. However, it has to be remembered that an angular bias for the polarization with respect to the vertical (obtained by a tilt angle as indicated above) is advantageous not only because the level of cross-polar signal is enhanced, but also because the sign of Φ can be obtained.

6. CONCLUSIONS

A procedure for standardizing the presentation of the cross-polarization data based on a physical model for the depolarizing medium has been suggested.

A triple-polarization system, or a dual-polarization system capable of complex differential measurements, allows a complete characterization of a general depolarizing medium. Moreover the hypotheses of a homogeneous medium filled with particles with rotational symmetry, and the existence of two principal planes can be tested. With decreasing complexity of the experiments, an increasing number of hypotheses regarding the medium has to be *a priori* assumed. These hypotheses have been outlined in various cases of practical interest.

APPENDIX

Relationships between the various polarizations

Let us describe a pair of orthogonal waves $\vec{E}_1$ and $\vec{E}_2$ as

$$(1\,a)\quad \begin{cases} \vec{E}_1 = E_1(\cos\alpha\vec{u}_x + e^{j\Psi}\sin\alpha\vec{u}_y) = E_1\,\vec{u}_1, \\ \text{and} \\ \vec{E}_2 = E_2(\sin\alpha\vec{u}_x - e^{j\Psi}\cos\alpha\vec{u}_y) = E_2\,\vec{u}_2, \end{cases}$$

where $\alpha(0 \leqslant \alpha \leqslant \pi/2)$ is a parameter such that the powers along x and y are proportional to $\cos^2\alpha$ and $\sin^2\alpha$, Ψ is the phase-shift of the y-component with respect to the x-component, and $\vec{u}_1$ and $\vec{u}_2$ are elliptical unit vectors.

Starting from the transfer matrix in circular polarization (left, right), the transfer matrix for a general pair of orthogonal polarization E_1, E_2 is given by

$$(2\,a)\quad \begin{bmatrix} C + \frac{j}{\varepsilon}S & C - \frac{j}{\varepsilon}S \\ S - \frac{j}{\varepsilon}C & S + \frac{j}{\varepsilon}C \end{bmatrix} \begin{bmatrix} 1/D_{rl} & X_{lr} \\ X_{lr} & D_{rl} \end{bmatrix} \times \begin{bmatrix} C - j\varepsilon S & S + j\varepsilon C \\ C - j\varepsilon S & S + j\varepsilon C \end{bmatrix},$$

where $C = \cos\alpha$, $S = \sin\alpha$, and $\varepsilon = e^{j\Psi}$. Matrix (2 *a*) is not normalized as is (2). It allows the evaluation of the depolarization ratio for any polarization, starting from the intrinsic parameters given by (5) after proper inversion. A formula similar to (2 *a*), suitable when starting from linear polarization parameters, is

$$(3\,a)\quad \begin{bmatrix} C & \frac{1}{\varepsilon}S \\ S & -\frac{1}{\varepsilon}C \end{bmatrix} \begin{bmatrix} D_{xy} & X_{xy} \\ X_{yx} & 1/D_{xy} \end{bmatrix} \begin{bmatrix} C & S \\ \varepsilon S & -\varepsilon C \end{bmatrix}.$$

Using (3 *a*) with $\alpha = \pi/4$ and $\Psi = \pi/2$ (linear to

circular), formulae (6) can be derived easily. The invariances iii) of Section 2 can be proved by putting $\Psi = \pi$ in (3 *a*) so that the two external matrices become the usual transformation of coordinates when the reference axes rotate an angle α.

Relationship between the transfer parameters and the scattering properties of the particles

As is shown in [2], the transfer matrix of a longitudinally homogeneous medium for linear polarization is given, ignoring normalizations, by

$$\text{(4 a)} \quad \cosh(\Delta/2)\,\|U\| + [\sinh(\Delta/2)/\Delta/2]\,\|L\|,$$

where $\|U\|$ = unit matrix

and the terms of the $\|L\|$ matrix are

$$\text{(5 a)} \quad l_{xx} = -l_{yy} = 2\pi k^{-2}\, l \sum_i \frac{S_{IIi} - S_{Ii}}{2} \cos 2\Phi_i ,$$

and

$$\text{(6 a)} \quad l_{xy} = l_{yx} = 2\pi k^{-2}\, l \sum_i \frac{S_{IIi} - S_{Ii}}{2} \sin 2\Phi_i ,$$

where l is the path length and k is the wavenumber.

S_{Ii} and S_{IIi} are forward scattering functions along the first symmetry plane of the particle (the one containing the symmetry axis and the propagation direction z), and the second symmetry plane (perpendicular to the first one and containing the propagation direction);

$\sum_i$ means that all the forward scattering functions in a unit volume have to be summed;

Φ_i is the angle of the first symmetry plane with respect to the reference plane xz.

Equation (6 *a*) together with (6) proves the property (*i*) of section 2.

If the medium contains the principal planes, it can be thought of as a cascade of rain slabs the principal planes of which share the same orientations. Then it is possible to assume homogeneity along the path by mixing up all the particles thereby obtaining two effective propagation constants resulting from averaging the true ones along the whole rainy path.

The transfer matrix (4 *a*) can then be used, in equation (9). It is insensitive to any normalization, and gives

$$\text{(7 a)} \quad \Delta = 2\pi k^{-2}\, l \times \sqrt{\left(\sum_i \left(\frac{S_{IIi} - S_{Ii}}{2}\right) e^{j2\Phi_i}\right) \cdot \left(\sum_i \left(\frac{S_{IIi} - S_{Ii}}{2}\right) e^{-j2\Phi_i}\right)} ,$$

and

$$\text{(8 a)} \quad e^{j2\Phi} = \sqrt{\frac{\sum_i (S_{IIi} - S_{Ii})\, e^{j2\Phi_i}}{\sum_i (S_{IIi} - S_{Ii})\, e^{-j2\Phi_i}}} ,$$

(7 *a*) reduces to the well known expression of anisotropy as given by Oguchi [6] for $\Phi_i = 0$. (8 *a*) shows that the effective canting angle can be thought of as resulting from a weighted average of many phasors of the type $e^{j2\Phi_i}$, each corresponding to a particle and having a weight proportional to its electrical asymmetry S_{II}-S_I.

Manuscrit reçu le 10 *août* 1980.

REFERENCES

[1] WATSON (P. A.). Microphysics of hydrometeors and polarization. *J. Rech. atmosph.*, Fr. (1974), VIII, pp. 474-484.

[2] CAPSONI (C.), PARABONI (A.). Depolarization of an E.M. wave travelling through a stratified aerosol of non-spherical scatterers 18th AGARD Tech. Meeting proceedings. *NATO*, USA (1972).

[3] BRUSSAARD (G.). A meteorological model for rain-induced cross-polarization. *IEEE Trans. AP*, USA (*1976*), **24**, n° 1, pp. 5-11.

[4] BECKMANN (P.). The depolarization of electromagnetic waves. The Golem Press, Boulder (1968), 214 p.

[5] PARABONI (A.), POSSENTI (V.), TIRRO (S.). Some experimental results about the spreading of ellipsoidal raindrops orientation during intense rainfall in antennas and propagation. Conf. Publ. IEE, n° 169. Part 2, *IEE*, London (1978), pp. 82-150.

[6] OGUCHI (T.), HOSOYA (Y.). Scattering properties of oblate raindrops and cross-polarization of radio waves due to rain : calculations at microwave and millimeter waves. *IRRL*, (1974), **21**, n° 105, pp. 191-259.