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AUTUMN COLLEGE

ON

THE TROPOSPHERE, STRATOSPHERE AND MESOSPHERE

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Miscellaneous Transparencies

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Potential Temperature

1^{st} law of thermodynamics (adiabatic)

$$c_p \frac{d \ln T}{dt} - R \frac{d \ln p}{dt} = 0$$

Define potential temperature by

$$\ln \theta = \ln T - \frac{R}{c_p} \ln p$$

or $\theta = T \left({}^{p_0}/_{p} \right)^{R/c_p}$

then

$$\frac{d \ln \theta}{dt} = 0 \Rightarrow \frac{d\theta}{dt} = 0$$

An atmosphere is stably stratified if

$$\frac{\partial \theta}{\partial z} > 0$$

& unstable to convective overturning if

$$\frac{\partial \theta}{\partial z} < 0$$

$$\left(N = \sqrt{\frac{g}{\theta} \frac{d\theta}{dz}} \right)$$

Hydrostatic Balance

$$\frac{1}{\rho} \frac{dp}{dz} + g = 0$$

ideal gas law

$$p = R \rho T$$

$$\frac{d(\ln \rho)}{dz} + \frac{g}{RT} = 0$$

for an isothermal atmosphere

$$\rho(z) = \rho(0) e^{-z/H}$$

where

$$H = \frac{RT}{g} \sim 7\text{km}$$

is called the 'atmospheric scale height.

Gravity wave breaking

In the absense of dissipation,

$$\frac{\partial}{\partial z}\left(\rho\overline{u'w'}\right) = 0$$

But if

$$\rho \sim e^{-z/H}$$

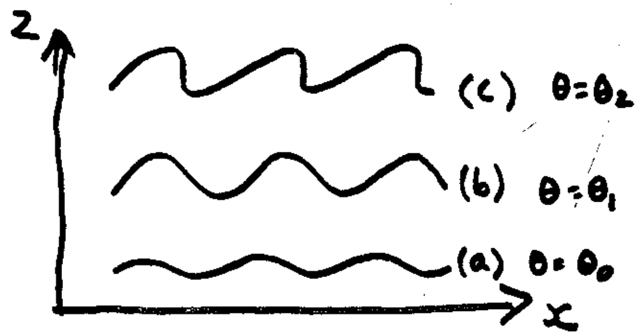
$$\overline{u'w'} \sim e^{z/H}$$

ie $\quad u', w', T' \sim e^{z/2H}$

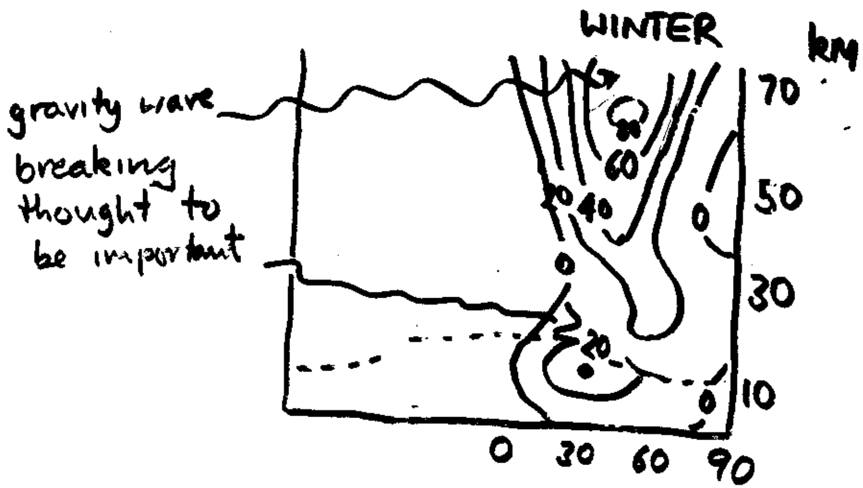
$$\frac{\partial\theta}{\partial z} = \frac{\partial\bar{\theta}}{\partial z} + \frac{\partial\theta'}{\partial z}$$

perturbation static stability $\sim \sin kx$

If amplitude is large enough total static stability can become locally unstable ie $\theta_z = 0$



Red lines depict surfaces of constant θ. At (a) & (b) surfaces are undulating but stable. At (c) surface is unstable at various points. Small scale turbulence is generated & wave is dissipated by turbulent friction. (ie $\partial/\partial z(\overline{u'w'}) \neq 0$ at (c) strong deceleration on mean flow) Important in general circulation of mesosphere (Lindzen JGR, 86, 9707-9714, 1981)



mean zonal winds in winter hemisphere

Momentum equation

$$\frac{d\underline{u}_a}{dt} = -\frac{1}{\rho}\nabla p + \underline{g}$$

Kelvin circulation theorem

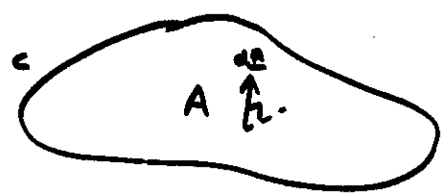
$$\Gamma_a = \int_c \underline{u}_a \cdot \underline{dl} \qquad \text{(circulation)}$$

where c is a closed curve moving with the fluid.

$$\frac{d\Gamma_a}{dt} = \frac{d}{dt}\int_c \underline{u}_a \cdot \underline{dl}$$

$$= \int_c \frac{d\underline{u}_a}{dt} \cdot \underline{dl}$$

$$= -\int_c \frac{1}{\rho}\nabla p \cdot \underline{dl}$$



By Stokes' theorem

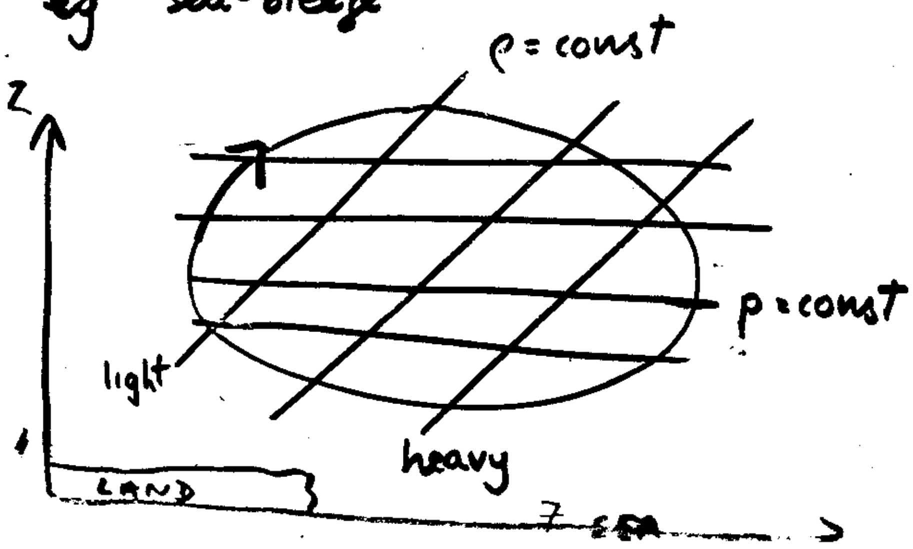
$$\left(\oint_C \underline{a} \cdot d\underline{l} = \int_A (\nabla \times \underline{a}) \cdot d\underline{A} \right)$$

then

$$\Gamma_a \equiv \int (\nabla \times \underline{u}_a) \cdot d\underline{A}$$

$$\frac{d\Gamma_a}{dt} = \int_A \frac{\nabla \rho \times \nabla p}{\rho^2} \cdot d\underline{A}$$

eg 'sea-breeze'



If the fluid is barotropic

ie $p = p(\rho)$

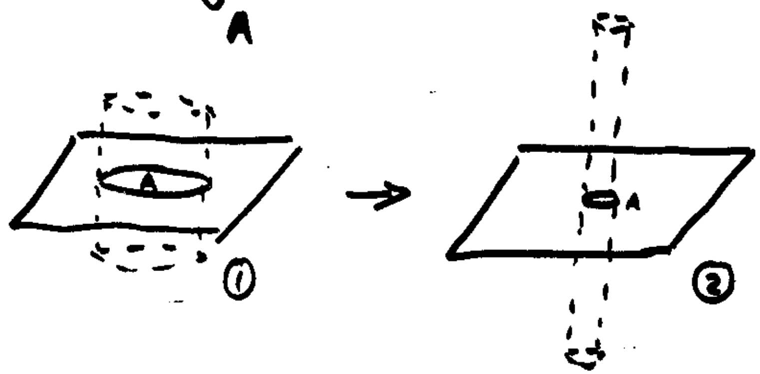
$(\Rightarrow \nabla \rho \times \nabla p = 0)$

then

$$\frac{d\Gamma_a}{dt} = 0$$

$\omega_a = \nabla \times \underline{u}_a$

ie $\int_A \underline{\omega}_a \cdot d\underline{A} = \text{constant}$



$\omega_a \, \delta A = \text{const}$

$\delta A_1 > \delta A_2$

$\omega_{a_1} < \omega_{a_2}$

Return to the baroclinic case

For adiabatic motion

$$\boxed{\frac{d\theta}{dt} = 0}$$

& from the ideal gas law

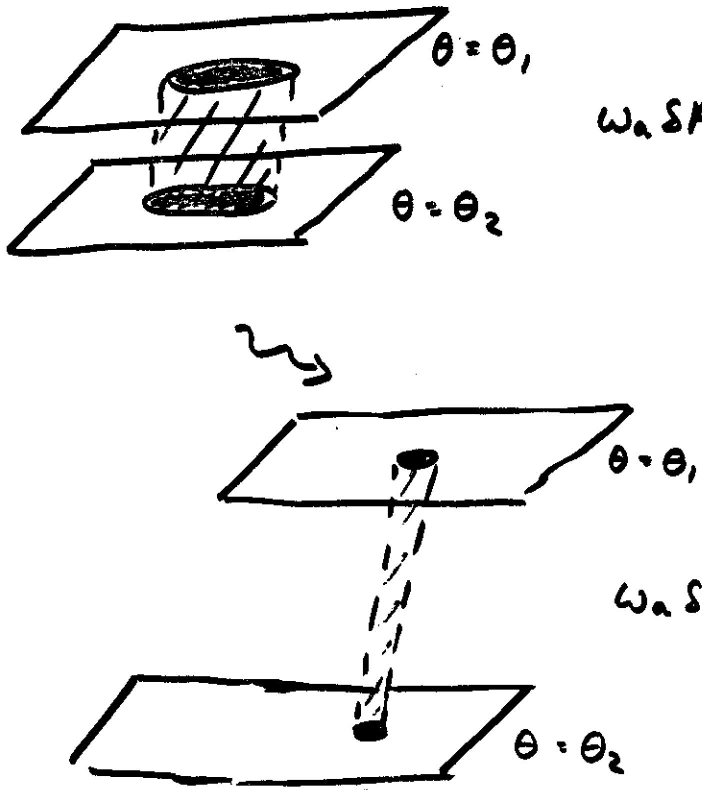
$$\boxed{\theta = \theta(p, \rho)}$$

$\Rightarrow$ that <u>on</u> a θ surface, $\theta = \theta_0$

$$p = p(\rho)$$

ie $(\nabla p \times \nabla \rho) \cdot \nabla\theta = 0$

Hence if A is an area on a θ surface moving with the fluid $\int_A \underline{\omega}_a \cdot \underline{dA}$ is constant in time.



$\omega_a \delta A = \text{const}$

$\omega_a \delta A = \text{const.}$

Can express δA in terms of $\nabla\theta$ & ρ & mass of fluid in column

$$\boxed{\frac{d}{dt}\left(\frac{1}{\rho}\,\underline{\omega}_a \cdot \nabla\theta\right) = 0}$$

THE SINGLE MOST IMPORTANT EQUATION IN LARGE-SCALE DYNAMICAL METEOROLOGY.

$$Q = \frac{1}{\rho}\underline{\omega}_a \cdot \nabla\theta \qquad \frac{dQ}{dt} = 0$$

is called the potential vorticity of the fluid. In the absense of dissipation it is a dynamical tracer.

Now

$$\underline{\omega}_a = \underline{\omega} + 2\underline{\Omega}$$

where $\underline{\omega} = \nabla \times \underline{u}$ is the vorticity in a frame of reference rotating with angular velocity $\underline{\Omega}$.

$$Q = \frac{1}{\rho}(\underline{\omega} + 2\underline{\Omega}).\nabla\theta$$

$dQ/dt = 0$

Questions posed by observations.

1. What causes the zonal asymmetries? Why are there zonal asymmetries in winter but not in summer?
2. Why are there no small scale 'weather systems' such as can be seen on tropospheric charts?
3. What causes sudden warmings? Why are they so intense?
4. Why do sudden warmings tend to occur in late winter?