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ROTATING FLUIDS IN GEOPHYSICS AND PLANETARY PHYSICS

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Rotating Fluids in Geophysics and Planetary Physics [1]

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Introduction

It was with great pleasure that I accepted the President's invitation to present one of the Union Lectures at this 18th General Assembly of the International Union of Geodesy and Geophysics here in Hamburg. This talk will be about fluid motions that are strongly influenced by Coriolis forces due to general rotation relative to an inertial frame of reference. It will outline a variety of experimental and theoretical studies of basic hydrodynamic and magnetohydrodynamic processes in rotating fluids and mention various applications in Earth and planetary sciences and astronomy. The lecture is based on my 1981 Harold Jeffreys Lecture of the Royal Astronomical Society (Quarterly Journal of the Royal Astronomical Society 23, 220–235, 1982).

Several general results concerning "geostrophic" flows characterized by an approximate balance between dynamic pressure gradients and Coriolis forces follow directly from arguments based on the equations of motion and the findings of certain laboratory experiments with rotating fluids (see below, "Geostrophic Flows"). These results provide considerable insight into the nature of many natural phenomena, such as the highly irregular meandering jet-streams that characterize mid-latitude flows in the Earth's atmosphere, the stable closed eddies (including the Great Red Spot and three White Ovals) seen in Jupiter's atmosphere (Fig 1), and the equatorial jet-streams and western boundary currents (such as the Gulf Stream in the Atlantic Ocean and the East-African cross-equatorial atmospheric jet-stream) found in oceans and atmospheres.

I shall also discuss flows that are strongly influenced not only by Coriolis forces due to general rotation but also by Lorentz forces due to the presence of electric currents in the fluid. It is necessary to investigate a wide range of such flows in the study of planetary and stellar magnetism. Of particular interest are the so-called "magnetostrophic" flows, for which Coriolis and Lorentz forces can be comparable with each other in magnitude but act in opposite directions (see below, "Magnetostrophic Flows"). The slow and highly dispersive "magnetohydrodynamic inertial wave", with a frequency which depends on the square of the Alfven speed and inversely on the rotation rate of the system, exemplifies magnetostrophic flow. It is very likely that such waves occur in the electrically-conducting fluid interiors of the Earth and other magnetic planets, and also in the Sun and other magnetic stars, including pulsars.

Finally I shall outline results of recent observational and theoretical work on the distribution of angular momentum within the Earth, including the recognition from analyses of length-of-day data and meteorological wind and pressure observations of an apparently persistent fluctuation in the angular momentum of the Earth's atmosphere on a time-scale of about two months (see below, "The Earth's Non-uniform Rotation"). Transfer of angular momentum between the Earth's solid parts (inner core, mantle and crust) and fluid parts (outer core, oceans and atmosphere) produces polar motion and changes in the length of the day, which are monitored by astronomers with increasing accuracy. The interpretation of these observations presents many fascinating and novel problems in theoretical geophysics, meteorology and oceanography, including several in the study of the hydrodynamics and magnetohydrodynamics of rotating fluids [2].

Hydrodynamics and magnetohydrodynamics are branches of classical physics, but they developed quite slowly until comparatively recently. Useful advances in our knowledge of the behaviour of rotating fluids might have been made in the last century had contemporary scientists appreciated the importance of the discovery, from analyses of meteorological observations, of "geostrophic" motion in the atmosphere. In the event, they evidently played down the discovery if the attitude of R.H. Scott, as revealed in a lecture delivered at the Royal Institution in 1869, can be regarded as typical. "A principle has been much before the

Fig 1 Long-lived anticyclonic eddies in Jupiter's atmosphere, namely the Great Red Spot, which is about 20,000 km long and at least 150 years old (and may have first been seen by Robert Hooke in 1664), and one of the three White Ovals that were seen to form 44 years ago. The arrows give the sense of relative motion. The "wind" speeds, as determined by Dr. R.F. Beebe of the New Mexico State University, are given in kilometres per hour. For comparison note that points on Jupiter's equator rotate at about 35,000 km per hour. The very existence of these long-lived eddies has important implications for theories of atmospheric predictability. There is no generally accepted explanation of the eddies but the experiments illustrated in Fig 2 and 3 indicate one possible line of attack

public of late which was first urged by Professor Buys-Ballot of Utrecht. It may be stated as follows: Stand with your back to the wind and the barometer will be lower on your left than on your right (in the northern hemisphere). No matter how gently the wind blows, the law is found to be true. *This fact, however, is of no use to us in enabling us to judge the coming weather* (my italics). " Scott, who was the second head of the British Meteorological Office (in succession to its founder, Robert Fitzroy), made pioneering contributions to international co-operation in practical meteorology, and the present World Meteorological Organization owes much to Scott's early efforts. But it was left to others to demonstrate, much later, that the concept of geostrophy was of central theoretical importance in dynamical meteorology.

Geostrophic Flows

The motion of a fluid of low viscosity (and negligible electrical conductivity, see on Magnetostrophic Flows, below) that departs but little from solid body rotation with angular velocity $\mathbf{\Omega}$ is usually geostrophic nearly everywhere, with the relative Eulerian velocity $\mathbf{u}$ (as measured in a frame of reference that rotates with angular velocity $\mathbf{\Omega}$ relative to an inertial frame) satisfying

$$2\rho\,\mathbf{\Omega}\times\mathbf{u} = -\nabla p + \rho\mathbf{g}. \qquad (2.1)$$

Here ρ denotes density, p pressure and $\mathbf{g}$ is the acceleration due to gravity and centripetal effects. Equation (2.1) is the leading approximation to the full equation of motion

$$\rho\,[D\mathbf{u}/Dt + 2\mathbf{\Omega}\times\mathbf{u} - \mathbf{r}\times d\mathbf{\Omega}/dt] = -\nabla p + \rho\mathbf{g} - \nabla\times(\nu\rho\nabla\times\mathbf{u}) + \mathbf{j}\times\mathbf{B}. \qquad (2.2)$$

It is valid in regions where the Coriolis term $2\rho\mathbf{\Omega}\times\mathbf{u}$ greatly exceeds the relative acceleration term $\rho\, D\mathbf{u}/Dt \equiv \rho\,(\delta\mathbf{u}/\delta t + (\mathbf{u}\cdot\nabla)\,\mathbf{u})$ (where t denotes time), the precessional term $\rho\,\mathbf{r}\times d\mathbf{\Omega}/dt$ (where $\mathbf{r}$ is the position vector), the viscous term $\nabla\times(\nu\rho\nabla\times\mathbf{u})$ (where ν denotes kinematic viscosity), and the Lorentz term $\mathbf{j}\times\mathbf{B}$ (which vanishes when there is no electric current, density $\mathbf{j}$, or magnetic field $\mathbf{B}$, see on Magnetostrophic Flows below).

Buys-Ballot's law follows immediately from equation (2.1), which shows that the horizontal fluid velocity is directed at right-angles to the horizontal component of the pressure gradient, with an average error of the order of the ratio of the neglected (ageostrophic) terms of the Coriolis term. This amounts to about 5 or 10 % for large-scale motions in the Earth's atmosphere and much less for the oceans.

Equation (2.1) leads to the important result that (2):

> *The hydrodynamical motion of a fluid of low viscosity that departs only slightly from rigid body rotation will not in general be symmetric about the rotation axis even when the boundary conditions are axisymmetric.* (2.3)

The validity of this result, which provides the most direct explanation of the occurrence of large-scale non-axisymmetric disturbances in the Earth's atmosphere and other natural systems, is readily verified by laboratory experiments. The result can be deduced as follows. In cylindrical co-ordinates (r, ϕ, z) where $\mathbf{\Omega} = (o, o, \Omega)$ the second component of equation (2.2) (cf. equation (2.1)) is:

$$u_r = -(2\rho\Omega r)^{-1}\,\delta p/\delta\phi + A_\phi \qquad (2.4)$$

(since $(\mathbf{g})_\phi = 0$ by the assumption of axial symmetry in the boundary conditions), where A_ϕ denotes the sum of all the ageostrophic terms. Now, over any cylindrical surface of radius r the rate of advective transport H (r, t; Q) of any quantity Q (per unit volume), such as heat, angular momentum, etc., is given by

$$H(r,t;Q) = \int_{z_1}^{z_2}\int_{o}^{2\pi} u_r\,Q\,r\,d\phi\,dz = \int_{z_1}^{z_2}\int_{o}^{2\pi}\left[-\frac{1}{2\rho\Omega}\frac{\delta p}{\delta\phi} + r A_\phi\right] Q\,d\phi\,dz. \qquad (2.5)$$

Since the contribution A_ϕ to equation (2.4) decreases rapidly with increasing Ω, advective transport perpendicular to the axis of rotation, as measured by H (r, t; Q), will be negligible unless the flow pattern departs significantly from axial symmetry. In the axisymmetric base we have $\delta p/\delta\phi = 0$ and H (r, t; Q) of the order of the small ageostrophic contribution.

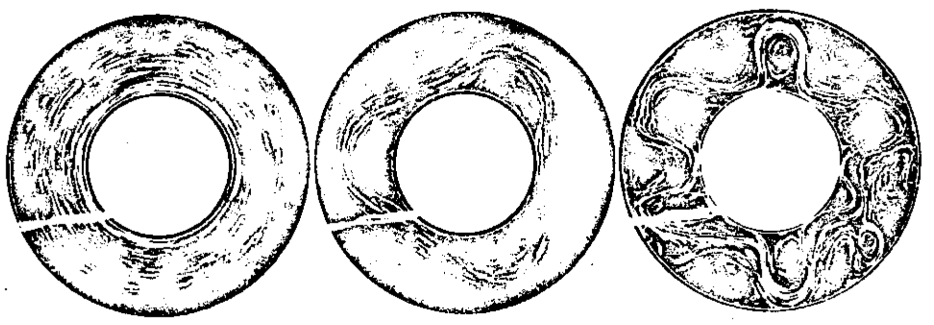
This argument is the basis of (2.3). There may be singular cases when the flow remains axisymmetric and in consequence advective transfer perpendicular to the rotation axis is negligible. Indeed, such cases can be realized in the laboratory by taking certain special precautions, but the general conclusion from laboratory experiments is that (2.3) is a correct inference from the geostrophic equation. Owing to departures from axial symmetry in the properties of the Earth's surface, it is not immediately obvious why large-scale mid-latitude atmospheric motions, which advect heat from tropical to polar regions, are highly non-axisymmetric. But meteorologists now accept that this lack of axial symmetry in atmospheric motions is a direct consequence of the Earth's rapid rotation.

There is a further property of equation (2.1) that leads to a useful general prediction. The equation is mathematically degenerate; being lower in order than the full equation (2.2), to which it is a leading approximation when Ω is large, it cannot be solved under all the necessary boundary conditions. For this to be possible every term in equation (2.2) must be included in the analysis, which implies that (1):

> *Regions of highly ageostrophic flow occurring not only on the boundaries of the system but also in localized regions (detached shear layers, jet-streams etc.) of the main body of the fluid are necessary concomitants of geostrophic motion.* (2.6)

Within these highly ageostrophic regions $\rho\, D\mathbf{u}/Dt + \nabla\times(\rho\nu\nabla\times\mathbf{u})$ is comparable in magnitude with $2\rho\mathbf{\Omega}\times\mathbf{u}$; the corresponding relative vorticity $\nabla\times\mathbf{u}$ can be comparable with or even exceed 2Ω in magnitude. Many examples of such vorticity concentrations are found in the laboratory and in Nature. They are often associated with steep gradients of temperature (thermal fronts), as in jet-streams and western boundary currents found in the atmosphere and oceans. Western boundary currents such as the Gulf Stream, Kuroshio Current and Somali Current in the oceans and the low-level cross-equatorial atmospheric jet-stream off East Africa arise as a result of effects due to the combined influence of the Earth's rotation and curvature. Such effects, which are readily produced in the laboratory, also give rise to strong westerly (i.e. positive) equatorial jet-streams in the atmosphere of Jupiter, Saturn and the Sun and in the terrestrial oceans (1, 2).

We have seen that slow relative hydrodynamical flow in a rotating fluid of low viscosity will in general be non-axisymmetric (see (2.3)). Laboratory studies show that there are two non-axisymmetric regimes of thermal convection in a rotating fluid annulus subject to differential heating in the horizontal, one highly regular (i.e. spatially and temporally periodic) and the other, which is reminiscent of large-scale flow in the Earth's atmosphere, irregular



(I) symmetric ($\Omega = 0{\cdot}341$ rad s^{-1})

(II) steady waves ($\Omega = 1{\cdot}19$ rad s^{-1})

(III) irregular ($\Omega = 5{\cdot}02$ rad s^{-1})

Fig 2 Streak photographs giving one example of the top-surface flow pattern in each of the three main modes of thermal convection in a rotating fluid annulus subject to axisymmetric heating at the outer side-wall and cooling at the inner side-wall, according to experiments first carried out by the author in 1950(1). The character of the motion depends on a number of parameters involving the acceleration of gravity, dimensions of the apparatus, thermal coefficient of cubical expansion, viscosity and thermal conductivity of the working fluid, etc. In an experiment in which Ω the angular speed of rotation of the whole apparatus about its vertical axis of symmetry, is the only quantity varied in this particular experiment, axisymmetric flow is found at low values of Ω (case I). At intermediate and high values od Ω the flow is non-axisymmetric, with welldeveloped je-streams (cases II and III). At intermediate values the flow is regular – i.e. spatially and temporally periodic, with wave-number ranging from 2 to 5 as Ω increases for the geometry shown – as in case II. The wavenumber is 3 and there is little tendency for the flow to "vacillate", so that the pattern is effectively steady apart from an angular drift at constant speed relative to the walls of the apparatus. At high values of Ω (case III) the flow is highly irregular; this chaotic motion is an example of what meteorologists concerned with large-scale motions in the Earth's atmosphere have termed "geostrophic turbulence". A wide variety of fluid dynamical and other non-linear systems are now known to exhibit either ordered or chaotic behaviour depending on the impressed conditions, and important unifying theoretical concepts are now beginning to emerge

(1, 3). Thus, when the basic rotation rate Ω of the fluid annulus exceeds a certain value Coriolis forces inhibit axisymmetric overturning motions in meridian planes and promote a completely different kind of motion, which has been termed "sloping convection". The motion is then non-axisymmetric and largely confined to jet-streams, with typical trajectories of individual fluid elements inclined at small but essentially non-zero angles to the horizontal (Fig 2). The kinetic energy of the non-axisymmetric flow derives from the interaction of slight upward and downward motions in these sloping trajectories with the potential energy field produced by the action of gravity on the density variations associated with the applied differential heating. The kinetic energy of the motion is dissipated by friction arising in boundary layers on the walls of the container and in the main body of the fluid. The critical value Ω_R of the rotation speed is of course dependent on many parameters, including the acceleration of gravity, the shape and dimensions of the apparatus, the coefficients of thermal expansion, thermal conductivity and viscosity of the fluid and its mean density, and the distribution and intensity of applied differential heating. This dependence has been determined by extensive laboratory studies and interpreted on the basis of stability theory (1, 3, 4, 5, 6).

Provided that Ω, though greater than Ω_R, does not exceed a second critical value Ω_I, the main features of the non-axisymmetric motion are characterized by great regularity and the heat flow is virtually independent of Ω and some 20 % less than its value when $\Omega = 0$. This regular flow is either steady (apart from a slow steady drift of the horizontal flow pattern relative to the walls of the container) or it exhibits periodic "vacillation" in amplitude, shape and other characteristics. The number of "waves" m around the annulus is not uniquely determined by the impressed conditions; the flow is found to be "intransitive" owing to the occurrence of what are now called "multiple equilibrium states". But the most likely value of m tends to increase with increasing Ω and when $\Omega = \Omega_I$, m has that value for which the azimuthal scale of the horizontal flow pattern is about 1.5 times the radial scale and the flow undergoes a transition to irregular flow or "geostrophic turbulence". When $\Omega > \Omega_I$ we have the irregular flow regime, for which heat flow decreases with increasing Ω (1).

The importance of these findings for theories of large-scale atmospheric motions and their predictability and for certain astrophysical studies (7, 8) is now recognized. Many laboratory studies of various aspects of sloping convection have been carried out, including determinations of heat

transfer, flow structure and regime transitions over a wide range of mechanical and thermal boundary conditions. Numerical studies based on the governing mathematical equations are now playing an increasingly important role in this work and significant if more limited analytical studies have also been made. It is noteworthy that, despite the essential non-linearity of these equations, the main features of the non-axisymmetric flow patterns can be interpreted by straightforward arguments based on general thermodynamic considerations and the requirement that the flow should be quasi-geostrophic nearly everywhere. Thus, when the distribution of applied heating and cooling is such that the corresponding gradient of the impressed temperature field has the same sign at all radii, the most conspicuous feature of the upper-level flow pattern in the regular non-axisymmetric regime is a continuous jet-stream meandering in a wavy pattern between the bounding cylinders. When, however, the impressed radial temperature gradient changes sign near mid-radius, as in the case when heat is introduced throughout the body of the fluid (by passing a weak alternating electric current through the fluid) and withdrawn at both sidewalls, the corresponding upper surface flow consists of several closed eddies, each circulating "anticyclonically", with the horizontal motion largely confined in a narrow jet-stream at the periphery (1).

These general characteristics have their counterparts in atmospheric flows. The meandering jet-streams within which the upper-level tropospheric flow of air is mainly concentrated in the Earth's atmosphere are manifestations of sloping convection produced by differential solar heating, which maintains a systematic temperature contrast between tropical and polar regions in each hemisphere. These atmospheric motions are highly irregular (and therefore unpredictable in detail over long periods of time), presumably because the Earth's angular speed of rotation exceeds the critical value Ω_I (which depends, amongst other things, on the depth of fluid), although, as has already been mentioned, non-axisymmetric variations in surface conditions introduce complications which are not yet fully understood (1).

Infra-red observations of the planet Jupiter indicate that the Jovian atmosphere is heated from below at about the same rate as its upper reaches are heated by solar radiation. Unlike the terrestrial case (where non-solar atmospheric heating is utterly negligible), north-south temperature gradients in Jupiter's atmosphere change sign several times between the equator and pole and there is no evidence of any significant temperature contrast between equatorial and polar regions (1, 9, 10). There are abundant observations of Jovian atmospheric motions at upper cloud level, some of which go back many decades and even longer (9), and the "Pioneer" and "Voyager" space probes have added further details (11). But our knowledge of what goes on below the cloud level is meagre and this produces difficulties with the interpretation of observations of upper-level atmospheric motions. Indeed, it can be argued that the main task of the "Jovian meteorologist" should perhaps be to use these observations to improve our knowledge of the vertical structure of the planet. But here is not the place to discuss these observations of Jupiter in detail and review the many interesting though largely controversial issues being debated by those of us who take an interest in the interpretation of the observations in terms of basic dynamical processes (12). There is however one striking phenomenon upon which the above mentioned laboratory experiments on sloping convection in a fluid annulus subject to internal heating might have some bearing (1, 13). The highly stable closed anticyclonic eddies, with the mean motion concentrated in a jet-stream at the periphery of each eddy, that are found in the experiments when the temperature gradient changes sign at mid-radius are remarkably similar dynamically to the long-lived eddies to be seen in Jupiter's atmosphere in the southern hemisphere. Of these Jovian eddies, the largest, oldest and most conspicuous is the Great Red Spot in the South Tropical Zone, which may be at least three hundred years old (Fig 3). Next in size and age are the three White Ovals that formed in 1939 at the boundary between the South Temperate Belt and the South Temperate Zone, apparently as the residue of the highly irregular South Tropical Disturbance that was first seen in 1901. The smallest of the long-lived eddies are clearly seen in the magnificent "Voyager" pictures as about a dozen oval markings somewhat closer to the pole (11). The motion in each of these Jovian eddies is anticyclonic and in the case of the Great Red Spot it is largely confined to a narrow region at the edge of the eddy. It has been proposed that the eddies are manifestations of sloping convection in Jupiter's atmosphere, implying that they derive their kinetic energy directly from the potential energy due to the action of gravity acting on density variations produced by internal and solar heating, and that they transport heat from the middle parts towards the edges of the latitudinal bands in which they arise (1, 13).

Preliminary calculations indicate that there is nothing unreasonable about this hypothesis so far as its implications for the vertical structure and other properties of Jupiter's atmosphere are concerned, and a detailed examination of these implications and a critical comparison of the hypotheses with other proposals as to the nature of the long-lived anticyclonic eddies is now being prepared. The hypothesis (which also has implications for motions in the atmosphere of Saturn) raises a number of fluid-dynamical questions to be resolved by further laboratory and numerical work and some of this is now in hand. Amongst these questions is that of the instability of the strongly-sheared flow in the jet-stream itself. There is experimental evidence that when viscous effects are sufficiently small the jet-stream develop local instabilities on one side but not on the other. Pictures of Jupiter show highly irregular flow on a comparatively small scale just outside the Great Red

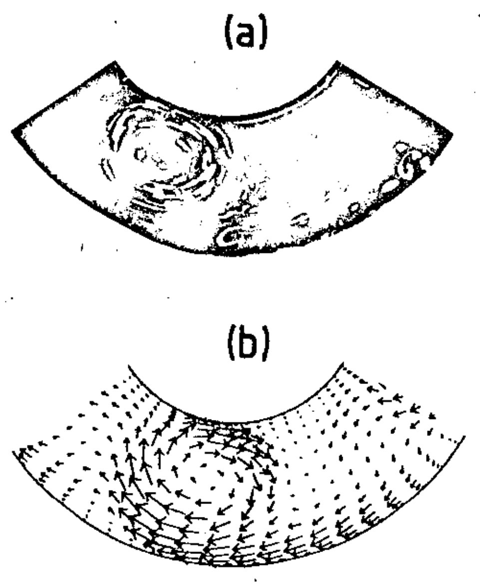


Fig 3 Numerical simulation of the top-surface flow pattern (Fig 3b) of thermal convection in a rotating fluid annulus subject to internal heating, with cooling taking place at both side-walls, as produced in laboratory apparatus (Fig 3a). Conditions are such that the flow is non-axisymmetric and regular with azimuthal wavenumber 4 (cf. Fig 2, case II) and little vacillation; that is to say the flow consists of 4 virtually identical steady eddies each circulating in an anticyclonic sense at the upper level, with horizontal motion largely confined to a peripheral jet-stream. Recent studies of the structure, stability and energetics of such eddies strengthen the proposal that they are dynamically similar to the long-lived eddies in the atmosphere of Jupiter and Saturn, such as the Great Red Spot and three White Ovals illustrated in Fig 1b (1, 13, 14). Of particular importance in this connection is the discovery that close to the transition to axisymmetric flow, the regular non-axisymmetric flow can be characterized by the presence of a single isolated compact eddy (14)

Spot (and other long-lived eddies), but not on the inside (11). It will be important to establish by experiment and theory whether or not this highly irregular flow arises as a result of a "one-sided" instability of the jet-stream at the edge of the main eddy.

Magnetostrophic Flows

The magnetic fields of the Earth and Sun, and of other magnetic planets and stars, are thought to be due to electric currents flowing within their interiors. It is now accepted that these currents are largely maintained against the effects of ohmic dissipation by electromotive forces due to motional induction, as Larmor first pointed out in his pioneering paper on self-exciting fluid dynamos (15, 16, 17, 18). The fluid motions involved are produced in most (if not all) cases by the action of gravity on density variations (19, 20).

Application of the laws of Gauss, Faraday and Ampère, together with Ohm's law, to a moving medium leads to the following differential equation for the magnetic field B at a general point at time t:

$$\delta \mathbf{B}/\delta t = -\nabla \times (\sigma^{-1}\nabla \times (\mu^{-1}\mathbf{B})) + \nabla \times (\mathbf{u} \times \mathbf{B}) \quad (3.1)$$

Here σ is the electrical conductivity and μ the magnetic permeability. This equation shows that a necessary (but not sufficient) conditions for dynamo action is that the magnetic Reynolds number

$$R = U L \bar{\mu} \bar{\sigma} \quad (3.2)$$

should exceed a certain critical value R* (say), where R* depends on the boundary conditions etc. but is typically about 10 or 10^2. Here U is a typical flow speed, L a characteristic linear dimension of the system and μ and σ are typical values of $\bar{\mu}$ and $\bar{\sigma}$ respectively (16, 17).

The mathematical analysis of theoretical dynamo models is greatly complicated by the finding that suitable departures from axial symmetry seem to be required for dynamo action to occur. This follows from the existence theorems for self-exciting dynamos (16, 17) and the recent extension of Cowling's theorem (21) showing quite generally that:

The ohmic decay of a magnetic field that retains an axis of symmetry cannot be prevented by fluid motions. (3.3)

This result (which suggests, incidentally, that palaeomagnetic and archaeomagnetic data might be expected to show evidence that departures from axial symmetry are systematically less during the decay phase of a geomagnetic polarity "reversal" or "excursion" than during the growth or recovery phase (22)) when combined with (2.3) provides a useful starting point for discussing how rotation affects the generation of magnetic fields by the dynamo process. By promoting departures from axial symmetry when the magnetic field B is so weak that the magnitude of the Coriolis force per unit volume $2\rho \Omega \times \mathbf{u}$ far exceeds that of the Lorentz force $\mathbf{j} \times \mathbf{B}$ (see equation (2.1) above and (3.3) below) -- where $\mathbf{j}$ denotes the current density equal, by Ampere's law, to $\nabla \times (\mathbf{B}/\mu)$ -- the field will be amplified by dynamo action (provided, of course, that $R > R^*$, see equation (3.2) above). As B increases in strength, the Lorentz force $\mathbf{j} \times \mathbf{B}$ grows, thereby raising the magnitude of the ageostrophic term A_ϕ in equation (2.4). If Lorentz forces eventually match Coriolis forces in magnitude the flow becomes approximately magnetostrophic, satisfying

$$2\rho \Omega \times \mathbf{u} \doteq -\nabla p + \rho \mathbf{g} + \nabla \times (\mu^{-1}\mathbf{B}) \times \mathbf{B}. \quad (3.4)$$

The study of magnetostrophic flows, particularly of certain classes of wave motions, sheds considerable light on the dynamics of the fluid interiors of planets and stars, including pulsars, and on the origin of planetary and stellar magnetism (4, 8, 16, 23, 24).

The near alignment between the axes of rotation and magnetic dipole moment of the Earth, Jupiter and Saturn is almost certainly a direct consequence of the strong influence of Coriolis forces on the fluid motions producing dynamo action, as several workers have suggestes, but much remains to be done in the way of detailed studies of dynamo action, as several workers have suggested, but much electrodynamics. In an attempt to set useful limits on the strength of the magnetic field produced by dynamo action it has been argued on the basis of general considerations (2.2) and (3.1) that the *magnetic field is unlikely to build up beyond that value for which the Lorentz torque acting on an individual fluid element balances the appropriate acceleration term* (9). Whence, the average strength B_i of the magnetic field within the fluid satisfies

$$B_i \lesssim B_s R^{1/2}, \quad (3.5)$$

where B_s is the "scale magnetic field strength" given by

$$B_s \equiv [\rho(\Omega + U/L)/\bar{\sigma}]^{1/2}. \quad (3.6)$$

Observe that the ratio of the magnetic to kinetic energy implied by equation (3.5) is $< (1 + L\Omega/U)$ and may, therefore, be very large when the basic rotation rate is so rapid that the so-called Rossby number

$$Y \equiv U/L\Omega \quad (3.7)$$

is much less than unity. Equipartition between magnetic energy and kinetic energy is to be expected only when Y is very large; for the Earth's core the precise value of Y is not known but (as for the fluid interiors of other planets and satellites) it is certainly very much *less* than unity, possibly $\sim 10^{-6}$.

Now, although the rate of generation of total magnetic energy by the dynamo mechanism, and hence B_i, is expected to *increase* with increasing electrical conductivity σ, the strength B_e of the *external* magnetic field produced by the dynamo (and this is the only part of the field we are able to observe) could *decrease* with increasing σ when σ is large (remembering that it is impossible to change the magnetic flux linkage of a perfect conductor) (25). It can therefore be supposed that

$$B_e/B_i \doteq R^{q} \quad (3.8)$$

where the index q is essentially positive and possibly close to unity. Hence

$$B_e \lesssim B_s R^{-q+1/2} \equiv \hat{B}_e. \quad (3.9)$$

$\hat{B}_e$ is thus an upper limit to strength of the magnetic field just above the fluid region where dynamo action is taking place. If $q = 1$ then $\hat{B}_e$ satisfies

$$B_s R^{-1/2} \lesssim \hat{B}_e \ll B_s R^{1/2}. \quad (3.10)$$

Corresponding expressions for the equivalent magnetic moment can be obtained from the equations (3.10) and (3.6) by multiplying by the cube of an appropriate length. Taking as typical values for the core of the Earth

$\Omega \doteq 10^{-4}$ rad s^{-1}, $\rho \doteq 10^4$ kg m^{-3},

$\bar{\sigma} \doteq 3 \times 10^5$ S m^{-1}, $U \doteq 10^{-4}$ m s^{-1} and $L \doteq 10^6$ m,

we find that

$Y \equiv U/L\Omega \doteq 10^{-6}$ and $B_s \doteq (\rho\Omega/\bar{\sigma})^{1/2} = 2 \times 10^{-3}$ T (20 gauss).

Accordingly, by equations (3.5) and (3.10) we have:

$B_i \lesssim 10^{-2}$ T (100 gauss), $B_e \lesssim 4 \times 10^{-4}$ T (4 gauss),

if for the magnetic Reynolds number R we take a value suggested by various studies of kinematic dynamos of the Bullard-Gellman type (16, 17) namely $R \doteq 25$. Now, the mainly dipolar field of 5×10^{-5} T (0.5 gauss) at the surface of the Earth implies that the average field strength just outside the core, B_e, is $\lesssim 10^{-3}$ T (10 gauss). This falls within the preferred range of B_e implied by the above discussion. So far as the value of B_i for the Earth's core is concerned, this cannot be inferred directly from geomagnetic observations. But various lines of evidence indicate that the magnetic field throughout the main body of the core might be largely toroidal in configuration, with lines of force lying approximately on horizontal surfaces, and $\sim 10^{-2}$ T (100 gauss) in strength, which is also concordant with the above calculation (9, 26).

Other approaches to the problem of finding a useful expression for the magnetic moment produced by a dynamo in a rotating fluid have been taken by various writers. Levy and Roberts, for example, exploited a specific dynamo model due to Parker in which a key parameter is a "turbulence correlation length" λ. Because the dependence of λ on Ω and other quantities is unknown it is hard to compare this approach with the one described above. Superficially equation (3.10) indicates a weaker Ω-dependence for B_e than the expressions given by Levy and Roberts, but because they involve λ the difference between the two approaches may turn out to be more apparent than real. By *assuming* that B_i is no bigger than B_e, Busse was able to exploit a particular dynamo model based on the assumption that Lorentz forces are small in comparison with Coriolis forces, and thus obtain the result that

$$B_e \doteq B_i \doteq 10^{-2.5} L\Omega(\bar{\kappa}\bar{\mu}\bar{\sigma})^{1/2}$$

where $\bar{\kappa}$ denotes the thermal diffusivity of the fluid (9, 26).

By Faraday's law, the magnetic flux linkage of a perfect conductor cannot change (Fig 4), so that effects due to ohmic dissipation are central to dynamo theories of the generation of the external magnetic field (16, 17, 18, 25). On the

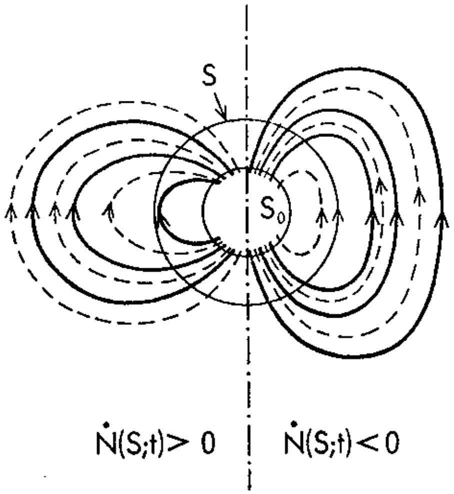


Fig 4 Illustrating the principle of the method introduced by the author in 1978 for finding the radius r_c of the electrically-conducting fluid core of a planet in which magnetohydrodynamic dynamo action is taking place (see (25) and (34)). $N(S;t)$ (see full lines) and $N(S;t+\Delta t)$ (see dashed lines) measure the total number of intersections of magnetic lines of force with a general spherical surface S well above the core at epochs t and $t + \Delta t$ respectively, and $\dot{N}(S;t)$ is the time rate of change of $N(S;t)$. The corresponding quantities evaluated at the surface of the core are $N(S_0;t)$, $N(S_0;t + \Delta t)$ and $N(S_0;t)$, whilst $\dot{N}(S;t)$ varies on the time-scale of the motions in the core that produce self-existing dynamo action, $N(S_0;t)$ varies on the much greater time-scale of ohmic decay, which is about 10^4 years for the Earth and possibly very much longer for Jupiter and Saturn. The value of r_c can therefore be determined from detailed measurements of secular changes in the magnetic field in the accessible region near or well above the surface of the planet by finding, by downward extrapolation from the measurements, that surface S_0 for which temporal changes in $N(S_0;t)$ are effectively negligible (on the secular variation time-scale). The method has been well tested by applying it to the Earth, for which it gives the liquid metallic core radius to within 2 % of the value determined by the more accurate methods of seismology. It will therefore be possible in due course to apply the method with confidence to Jupiter, Saturn and other magnetic planets when more is known about the structure and time variations of their magnetic fields

other hand, such effects may not be of primary importance when dealing with secular changes, which occur on much shorter time scales than the time scale of free decay, and their neglect leads to a considerable simplification of the governing equations. The mathematical difficulties involved are still severe, especially when realistic boundary conditions are taken into account, and their discussion lies beyond the scope of this article (4, 18, 24). Fortunately, some of the main processes are exemplified by the properties of plane waves of angular frequency ω and vector wave number $\mathbf{k}$ propagating relative to a fluid which rotates uniformly with steady angular velocity Ω and is pervaded by a uniform magnetic field $\mathbf{V}(\mu\rho)^{1/2}$ where $\mathbf{V}$ is the Alfven velocity. The dispersion relationship for such waves is the biquadratic equation

$$\omega^4 - \omega^2 (2\omega_V^2 + \omega_\Omega^2) + \omega_V^4 = 0 \qquad (3.11)$$

where

$$\omega_V^2 \equiv (\mathbf{V}\cdot\mathbf{k})^2 \text{ and } \omega_\Omega^2 \equiv (2\Omega\cdot\mathbf{k})^2 / |\mathbf{k}|^2,$$

with solutions

$$\omega^2 = \omega_V^2 + \frac{1}{2}\omega_\Omega^2 \pm \left[\frac{1}{4}\omega_\Omega^4 + \omega_V^2\omega_\Omega^2\right]^{1/2} \equiv \omega_+^2, \omega_-^2 \qquad (3.12)$$

according as the upper or lower sign is taken. For wavelengths so short that $2\omega_V^2 \gg \omega_\Omega^2$ – which for the Earth's core (where $V \lesssim 10^{-1}$ m sec^{-1}) corresponds to motions on scales much less than 0.1 times the core radius – equation (3.12) reduces to the dispersion relationship for ordinary Alfven waves, $\omega^2 = \omega_V^2$, which are nondispersive, linearly polarized and characterized by equipartition between magnetic and kinetic energy. At the other extreme, when $2\omega_V^2 \ll \omega_\Omega^2$ – and this is the case of interest when dealing with waves in the core on scales $\simeq 10^6$ m – Coriolis forces are so strong that the two roots of equation (3.12) have quite different values:

$$\omega_+^2 = \omega_\Omega^2 > \omega_-^2 = \omega_V^4 / \omega_\Omega^2. \qquad (3.13)$$

This extreme "frequency splitting" due to rotation is accompanied by other effects, notably wave dispersion, circular or elliptical polarization of the trajectories of individual fluid elements, and imbalance of kinetic energy (the whole of which is now associated with the fast "inertial mode") and magnetic energy (now entirely in the slow magnetostrophic "magnetic mode").

When equations (3.13) are satisfied, the period of the *inertial mode* $2\pi/\omega_+$ is then typically $\gtrsim \pi/\Omega$ (i.e. a few days), whilst that of the *magnetic mode* $2\pi/\omega_-$ is $\sim 2\pi\,\Omega\,L^2 / V^2$, which for the Earth's core when $L \doteq 10^6$ m is 10^{10} sec (300 years) and, therefore, comparable with the time scale of the geomagnetic secular variation. This is the quantitative basis of the theory of the geomagnetic secular variation that interprets its general time scale and westward drift in terms of magnetohydrodynamic oscillations of the liquid core (24). The electrical conductivity of the overlying "solid" mantle, though weak, might be sufficient to prevent magnetic changes in the core on the short time scale of the inertial modes from penetrating to the Earth's surface.

The Earth's Non-uniform Rotation

In the absence of internal energy sources and mechanical, gravitational or radiative interactions with other bodies, the

solid parts of the Earth (i.e. crust, mantle and inner core) and the fluid parts (atmosphere, hydrosphere and outer core) would rotate together as a rigid body at a constant rate about its fixed axis of maximum moment of inertia through its centre mass. Positional astronomers equipped with perfect telescopes and clocks (and the necessary life-support systems all, together with the astronomers, of zero mass!) would find no variation in the latitude of any station and the longitude of all fixed stars would change at a constant rate.

The successful use of the Earth's rotation as the basis of the earliest attempts to provide a practical unit of time attests to the validity of the above picture as a good "zeroth approximation" to the truth (27). But over the years, as clocks based on other physical phenomena were invented and methods of positional astronomy improved, tiny fluctuations in the "length of the day" (up to a few milliseconds) and very slight movements of the Earth's pole of rotation (up to a few metres) came to light. The interpretation of these variations in the magnitude and direction of the Earth's rotation vector in terms of energetic processes and angular momentum transfer within the Earth-Moon system is one of the most fascinating problems in the whole of the Earth sciences, for it brings together several otherwise diverse areas of study, including solid Earth geophysics, meteorology, oceanography, hydrology, glaciology and geomagnetism.

Such studies of the Earth's rotation go back to the last century. The first thorough review of the subject was presented by Munk and MacDonald in 1960, and Lambeck (28) has given an up-to-date discussion taking into account recent improvements in instrumentation, international co-operation and basic ideas in geodynamics.

The magnitude of the principal moment of inertia of the atmosphere I_a is about 1.42×10^{22} kg m^2, which should be compared with that of the whole Earth $I = 8.4 \times 10^{37}$ kg m^2. During the development of the three-dimensional and non-axisymmetric global atmospheric circulation following the imaginary "switching on" of the main source of energy for atmospheric motions, solar heating, the angular momentum of the atmosphere M_a (say) would increase by an amount δM_a (magnitude $\doteq 1.5 \times 10^{26}$ kg m^2 s^{-1}). This increase is a measure of the "super-rotation" of the densest parts of the atmosphere, the troposphere and lower stratosphere, at an average angular speed

$$|\delta M_a| / I_a \doteq 10^{-6} \text{ rad s}^{-1} \doteq 1.5 \times 10^{-2} |\Omega| \qquad (4.1)$$

relative to the underlying planet, the corresponding linear speed being about 10 ms^{-1}. Here Ω is the angular velocity of the solid Earth, which is very close to the total angular momentum M (magnitude 5.85×10^{33} kg m^{2}s^{-1}) divided by I; thus $|\Omega| \doteq 7.28 \times 10^{-5}$ rad s^{-1}.

Solar radiation provides the kinetic energy of atmospheric motions, but it cannot provide the angular momentum. This comes largely from the underlying planet which, to conserve angular momentum of the whole Earth, would undergo a change $-\delta M_a$, largely manifested as a change

$$|\delta\Omega| \doteq -|\delta M_a| / I \doteq -3 \times 10^{-8} |\Omega|$$

in the magnitude of Ω; the corresponding increase in the length of the day (l.o.d.) would be about 3×10^{-3} s. This simple calculation shows, as Munk and Miller were evidently the first to realize (28), that fluctuations in the general atmospheric circulation could produce changes in the length of the day of up to about l.o.d. It also implies that since the biggest of the observed irregular l.o.d. changes – the so-called "decade variations" – are about 5×10^{-3} s, they cannot originate in the atmosphere. The decade variations are now generally supposed to originate largely in the Earth's liquid metallic core, where the main geomagnetic field is produced by the magnetohydrodynamic dynamo process (see Section on Magnetostrophic Flows above).

Length-of-day fluctuations of meteorological origin are largely associated with displacements in the mean latitude of the major jet-streams in the troposphere. The concomitant vertical transfer of angular momentum at the Earth's surface is due to a slight imbalance between the positive couple exerted on the underlying planet by the average surface westerlies in mid-latitudes and the negative couple exerted by the surface easterlies at low and high latitudes, the net couple being **K**. Monitoring the angular momentum "budget" of the atmosphere with the aid of improved data on l.o.d. is now beginning to interest meteorologists concerned with fluctuations of the atmospheric circulation on time scales upwards of a few days.

Direct determinations of **K** cannot be made with acceptable accuracy. Even with a complete set of surface wind and pressure observations, an understanding of boundary layer processes and airflow over and around mountain ranges that goes well beyond present knowledge would be required (2). The value of **K** is best obtained indirectly from the rate of change of the total angular momentum of the atmosphere, which depends only on the distribution of velocity and pressure throughout the atmosphere. But even this quantity is subject to large uncertainties owing to the paucity of regular meteorological data from the southern hemisphere and tropical regions.

During 1979, meteorologists made a special effort to monitor the whole atmosphere, providing for the first time data sets that could be used with confidence in the assessment of fluctuations in M_a. It has been shown from these data sets that observed short-term changes in the l.o.d. during 1979 can be fully accounted for on the basis of angular momentum exchange with the atmosphere (29). This demonstration improves the meteorological usefulness of the data set on rapid changes in the l.o.d. since 1955, when "atomic" clocks were introduced. This data set

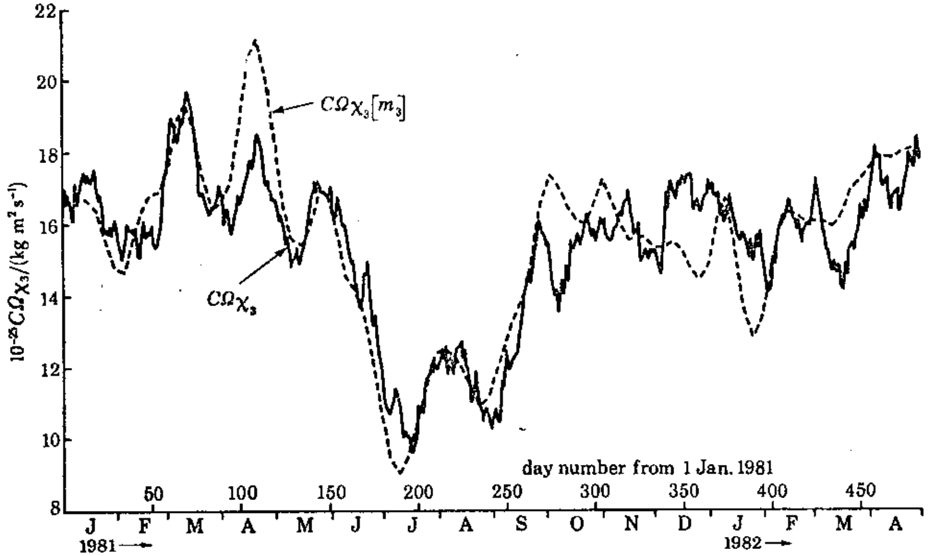


Fig 5 Demonstration that short-term non-tidal changes in the length of the day during the interval from 1 January 1981 to 30 April 1982 can be accounted for in terms of angular momentum exchange between the atmosphere and solid Earth, without having to invoke significant non-meteorological excitation (29, 32). The diagram compares the atmospheric axial effective angular momentum function χ_3 with corresponding fluctuations in the quantity $\chi_3[m_3] = \Delta\Lambda/\Lambda_0 - \langle\Delta\Lambda/\Lambda_0\rangle$, defined as the value of χ_3 implied by the observed changes in length of day $\Lambda \equiv \Lambda_0 + \Delta\Lambda = 2\pi/\Omega(1 + m_3)$, assuming these changes to be due entirely to meteorological processes. The full line gives the daily values of $C\,\Omega\,\chi_3$ and the broken line pentad values of $C\,\Omega\,\chi_3\,[m_3]$, the constant of integration $\langle\Delta\Lambda/\Lambda_0\rangle$ being taken as $0.42 \times 10^{-3}\,\Omega/2\pi$. C is the larger of the two principal moments of inertia of the Earth (7.04×10^{37} kg m^2), Ω is the mean angular velocity of rotation of the Earth (0.7292115×10^{-4} radian per sidereal second), and $\Lambda_0 = 2\pi/\Omega$

reveals a pronounced fluctuation on a time scale of about 50 days with an amplitude comparable with the better known seasonal variations (30, 31). The elucidation of this fluctuation should greatly advance our understanding of the general circulation of the atmosphere and might lead to improvements in long-range weather forecasting by a combination of statistical and dynamical methods (Fig 5).

All components of M_a exhibit fluctuations. The non-axial components give rise to polar motions of a few metres, including a contribution to the so-called Chandlerian wobble with a period of 14 months (28). The excitation and damping of the Chandler wobble are still controversial issues amongst geophysicists, but any attempts to infer mantle motions preceding earthquakes from Chandler wobble observations will require in the first instance a full assessment of atmospheric effects. This should be possible soon when a new collaborative programme involving a number of institutions is fully operational. According to a very recent detailed study of fluctuations in all three components of atmospheric angular momentum during the period 1 January 1981 to 30 April 1982 (32), exchange of angular momentum between the atmosphere and solid Earth during that period (when there were no earthquakes exceeding 7.9 in magnitude) accounted adequately for the observed variations in the length of the day and polar motion (Fig 6).

We have seen that, at several milliseconds in amplitude, the so-called "decade fluctuations" are too great to be accounted for in terms of atmospheric (or oceanic) processes and that geophysicists generally agree that, in the absence of a reasonable alternative, the fluctuations must be largely due to angular momentum transfer between the Earth's 'solid' mantle of thickness ≑ 2900 km and underlying liquid outer core of thickness ≑ 2200 km. The distortion and displacement of the geomagnetic field pattern at the Earth's surface, including the well-known westward drift of the non-dipole field at some 2×10^{-4} ms^{-1}, can be taken as manifestations of core motions, but the accurate determination of all but the broadest features of these motions from geomagnetic observations is an impos-

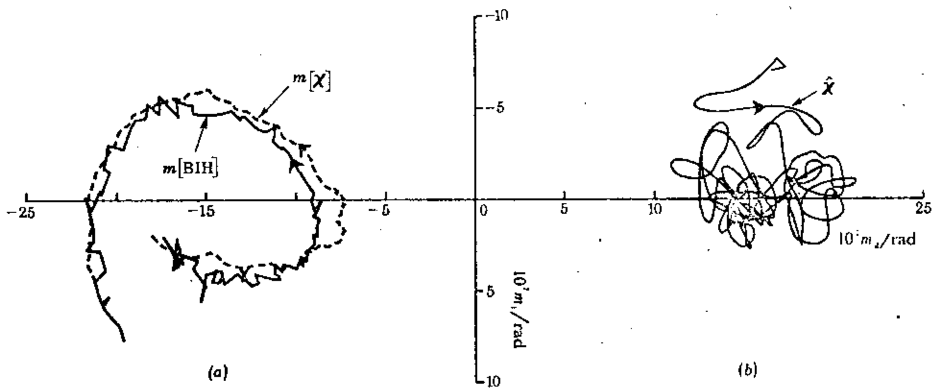


Fig 6 Demonstration that polar motion during the interval 1 January 1981 to 30 April 1982 can be accounted for in terms of angular momentum exchange between the atmosphere and solid Earth, without having to invoke significant non-meteorological excitation (32). Fig 5a compares the observed polar motion m [BIH] (thick full line), as determined by the Bureau International de l'Heure in Paris, with the expected polar motion m [χ] (broken line), assuming the excitation to be due entirely to meteorological processes. m [χ] was calculated with the equatorial effective angular momentum function χ as input. Fig 5b shows the smoothed values χ of the atmospheric effective angular momentum function (see (32) for further details). $m = m_1 + im_2$ and $\chi = \chi_1 + i\chi_2$. The units of both axes are radians; 10^{-6} rad is equivalent to a displacement of about 6.4 m over the Earth surface

sible task (33, 34). Nevertheless, the quantitative requirement that the time scale and r.m.s. value of fluctuations in zonal speed of core motions be generally compatible with the amplitude of the decade variations in the length of the day is not particularly restrictive (28).

The principal quantitative difficulties emerge when the nature of the horizontal stresses that couple the core to the mantle, across the core-mantle interface, are considered (28, 33). These stresses must suffice both quantitatively and qualitatively to account for the fluctuating couple at the core-mantle interface implied by foregoing interpretation of the decade variations in the length of the day. We can write

$$F = F_V + F_E + F_T \doteqdot 0.04\ \text{N m}^{-2}, \qquad (4.2)$$

where F is the average magnitude of these horizontal stresses, the corresponding magnitude of the unbalanced couple between core and mantle being $10^{19} - 10^{20}$ kg m^2 s^{-2}. F_V, F_E and F_T are the contributions to F associated with viscous coupling, electromagnetic coupling and topographic coupling respectively.

If δ is the thickness of the viscous boundary layer at the surface of the core then

$$F_V = C_V\,\nu\,\rho\,\Delta U/\delta \qquad (4.3)$$

where ρ is the density of the core (≑ 10^4 kg m^{-3}), ν is the coefficient of molecular or eddy kinematic viscosity, depending on whether or not the boundary layer is turbulent, ΔU is a typical magnitude of fluctuations in the azimuthal flow speed of the core material relative to the mantle, and C_V is a 'viscous drag coefficient' which on general grounds should be around unity.

The electric currents responsible for the main geomagnetic field leak out of the metallic core into the weakly conducting lower mantle and these, together with currents induced in the lower mantle by fluctuations in the main geomagnetic field give rise to electromagnetic coupling between the core and mantle. If B_1 is the fluctuating horizontal component of the magnetic field at the core-mantle interface and B_2 the vertical component, and μ denotes the magnetic permeability (not significantly different from $4\pi \times 10^{-7}$ Hm^{-1}, that of free space) then the term F_E in equation (4.2) is given by

$$F_E = C_E\,B_1\,B_2/\mu \qquad (4.4)$$

where C_E is the 'electromagnetic drag coefficient' which, like C_V, should be around unity.

The third term in equation (4.2), F_T, arises if irregular topographic features are present on the core-mantle interface and h, a typical value of the vertical dimensions of these features, exceeds the thickness of the viscous boundary layer δ. F_T is related to h by the expression

$$F_T = C_T\,\Omega\,\rho\,\Delta U\,(h - \delta) \qquad (4.5)$$

where Ω is the angular speed of rotation of the mantle (≑ 10^{-4} rad s^{-1}) and C_T is the 'topographic drag coefficient'. C_T is unlikely to exceed unity but (unlike C_V and C_E) it may in some circumstances take on very small values.

Viscous coupling is almost certainly inadequate but electromagnetic coupling (proposed by Bullard) might suffice and topographic coupling (proposed by the writer) could be important with bumps no more than about a kilometre in height (15, 33, 35). But high topography does not necessarily imply strong coupling, owing to the complicated dependence of C_T on h, with C_T decreasing rapidly with increasing h when h exceeds a certain critical value h^*, dropping to values at $h \gg h^*$ that are typically very much smaller than those attained when $h < h^*$. The determination of h^* and the investigation of the detailed dependence of C_T on h and other parameters are matters of considerable importance in the study of the dynamics of atmospheres, oceans and planetary cores, where the influence of topography on fluid flow can in some circumstances be considerable. Preliminary studies of magnetohydrodynamic flow over and around topography in a rapidly rotating fluid have now been made. They indicate that, owing to the presence magnetohydrodynamic inertial waves produced by interactions involving the magnetic field at the core-mantle interface, the appropriate value of C_T might be comparatively high, around unity (36). The corresponding topography coupling would be adequate to account for the observed "decade" variations in the length of the day.

Concluding Remarks

Developments in many areas of basic physics can be stimulated by the demands made upon them by geophysics, planetary sciences and astronomy. An important weapon required in the theoretical armoury of these subjects is a deep and extensive understanding of basic fluid dynamical processes in rapidly rotating fluids. Our knowledge of hydrodynamic processes though far from adequate, still greatly exceeds that of magnetohydynamic processes. Whilst the former can be studied and insights developed through the use of laboratory investigations carried out under controlled and reproducible conditions, laboratory studies of the latter are inevitably much more limited in their scope owing to the difficulty of achieving high values of the magnetic Reynolds number (see equation (3.2)). Moreover, whilst geostrophic flows can be observed directly in the terrestrial oceans and atmosphere, magnetostrophic flows occur in inaccessible regions, the nearest being the Earth's liquid metallic core, where the main geomagnetic field originates. But magnetostrophic flows probably underlie a very wide range of phenomena encountered in the study of stars and planets, the successful interpretation of which will require in the first instance further systematic theoretical and numerical work on the magnetohydrodynamics of rotating fluids.

In conclusion, it remains for me to apologize to experts in this audience, consisting of geophysicists of all kinds, geodesists, seismologists, volcanologists, geochemists, geomagneticians, aeronomers, meteorologists, atmospheric physicists hydrologists and oceanographers, for the somewhat personal selection of topics I have touched upon. But the list of references given in the written version of the talk should serve as a guide to anyone wishing to go further into this fascinating area of classical physics, with so many applications in geophysics, planetary sciences and astronomy.

Notes

1) Union Lecture delivered on 22 August 1983 during the 18th General Assembly of the International Union of Geodesy and Geophysics, Hamburg, Federal Republic of Germany and based on the 1981 Harold Jeffreys Lecture of the Royal Astronomical Society.

2) The lecture was illustrated with many slides and several short films showing the phenomena mentioned in the introduction. It is impossible to reproduce this visual material here, and without the material a verbatim account of the whole lecture would be of little value. This written account presents a few basic ideas and results without attempting to provide a comprehensive and critical review of any of the topics mentioned. References to original work can be found in the bibliographies given in the recent publications cited in the text and listed at the end of this paper.

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