



INTERNATIONAL ATOMIC ENERGY AGENCY  
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION



INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS  
34100 TRIESTE (ITALY) - P.O. B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONES: 224281/2/3/4/5/6  
CABLE: CENTRATOM - TELEX 480392-I

SMR/113 - 7

AUTUMN COLLEGE  
ON  
THE TROPOSPHERE, STRATOSPHERE AND MESOSPHERE  
10 September - 19 October 1984

---

DYNAMICS OF ROTATING FLUIDS AND PLANETARY ATMOSPHERES

R. HIDE  
Geophysical Fluid Dynamics Laboratory  
Meteorological Office (21)  
Bracknell  
Berkshire RG12 2SZ  
U.K.

---

These are preliminary lecture notes, intended only for distribution to College participants. Missing or extra copies are available from Room 230.

- 1 -

DYNAMICS OF ROTATING FLUIDS AND PLANETARY ATMOSPHERES

by

Raymond Hide

Geophysical Fluid Dynamics Laboratory,  
Meteorological Office (Met O 21),  
Bracknell, Berkshire RG12 2SZ,  
England, U.K.

Invited paper presented on 19 October 1983 at a special meeting of the Royal Meteorological Society to mark the centenary of the Society's receipt of permission to adopt the prefix "Royal".

(26 pages long)

1. INTRODUCTION

On this special occasion it is my pleasant duty to greet the Royal Meteorological Society on behalf of an older (by thirty years) sister, the Royal Astronomical Society, of which I have the honour to serve as the current president. Developments in the basic and applied sciences promoted by the Royal Meteorological Society have been particularly rapid and extensive since the year 1950, when Sir Robert Watson-Watt presided over the celebrations of the centenary of the Society's foundation by Samuel Whitbread, its first president, James Glaisher, its long-serving first secretary, and other energetic eminent Victorians. But in 1975 I failed to convince Council that special significance should be attached to the one hundred and twenty-fifth anniversary of the Society's foundation, so I have to congratulate Professor Charnock on his superior powers of persuasion. The centenary of the receipt by the Society in 1883 of permission to adopt the prefix "Royal" provides a good excuse to hold a scientific meeting for the purpose of reviewing recent progress in our general field, and I am delighted to be able to take part.

Reading between the lines of my correspondence with the Society's Meetings Secretary I gained the impression that each of today's invited speakers was being asked and possibly even challenged to say something new and useful about topics chosen for discussion in his presidential addresses. In my own case it seems appropriate that I should outline some of the main findings and new insights stemming from recent work of the Geophysical Fluid Dynamics Laboratory of the Meteorological Office, where we have been engaged for a number of years in experiments with rotating fluids, including numerical modelling, and in various studies

(Gom)

of the dynamics of large-scale motions in the atmospheres of the Earth and other planets, all with the general aim so far as meteorology and oceanography are concerned of providing a sound basis for theories of predictability.

The concept of geostrophy is central to dynamical meteorology, and it is probably most widely known as Buys-Ballot's Law, which I find easy to remember in the unconventional form once put to me by a senior member of the forecasting branch of the Meteorological Office: "If you stand with your back to the wind and the low pressure is on the left, you know you are in the Northern Hemisphere!". A famous British physicist once told a class of students that the subject of electronics was very easy "provided that you know Ohm's Law, but you have to know it thoroughly". One might paraphrase this remark by saying that a thorough knowledge of Buys-Ballot's Law is all that is required by dynamical meteorologists, but it is only comparatively recently that the most direct and yet most profound implications of geostrophy came to be appreciated. Quite straightforward general considerations of the governing equations suffice to show, and laboratory experiments on thermal convection in rapidly rotating fluids amply confirm (see below), that two necessary concomitants of any geostrophic flow are: (a) certain regions of highly ageostrophic flow, often located within the main body of the fluid, and (b) strong departures from axial symmetry of the flow pattern even under axisymmetric boundary conditions when substantial advection of properties such as heat perpendicularly to the rotation axis is occurring (see Hide 1981, 1982). The meandering jet streams of the mid-latitude circulation of the terrestrial atmosphere exemplify these salient characteristics of flows in a rapidly-rotating fluid, and

so do the other large oval eddies in the atmospheres of Jupiter and Saturn, including the Jovian Great Red Spot. More subtle considerations are required in order to understand why, on the one hand, the large oval eddies on Jupiter and Saturn enjoy lifetimes ranging from decades to centuries, whereas, on the other hand, large-scale motions in the Earth's atmosphere are highly chaotic.

The Geophysical Fluid Dynamics Laboratory of the Meteorological Office carries out research on basic hydrodynamical processes in rapidly rotating fluids. Such processes underlie a wide variety of phenomena in the atmosphere and hydrospheres of the Earth and other planets. Laboratory studies play an important role in this research, along with mathematical and numerical work carried out in direct combination with laboratory investigations. Many of the major problems of dynamical meteorology and oceanography require for their satisfactory solution a combined attack involving the analysis and interpretation of observations in terms of basic hydrodynamical processes, and the investigation and exploitation of related systems, such as numerical models, laboratory analogues and the atmospheres of other planets. These include laboratory and numerical studies of thermally and mechanically produced motions in rotating fluids under a wide variety of axisymmetric or non-axisymmetric boundary conditions, the investigation of angular momentum fluctuations of the Earth's atmosphere and associated changes in the length of day and polar motion, and the interpretation of super-rotation of planetary atmospheres and of long-lived eddies in the atmospheres of Jupiter and Saturn.

Parts of the survey given here have already been presented elsewhere, in the proceedings of two recent workshops, one on "Instabilities in continuous media": held in Venice, Italy, in December 1982, and the other on "Predictability of fluid motions" in La Jolla, California, in February 1983. It was prepared with the assistance and advice of my colleagues Mr R. T. H. Barnes, Dr P. Hignett, Dr I. N. James, Dr P. L. Read, Dr A. A. White and Dr C. A. Wilson, to whom I must express my indebtedness.

(Go on)

## 2. ATMOSPHERIC MOMENTUM FLUCTUATIONS, LENGTH OF DAY CHANGES AND POLAR MOTION

Possibly the most striking large-scale dynamical features of the Earth's atmosphere are its average "super-rotation" relative to the solid Earth and the concentration of much of the motion in jet streams. Studies of the complex processes that produce and maintain jet streams are central to any attempt to predict large-scale atmospheric motions.

Variations in the distribution of mass within the atmosphere and changes in the pattern of winds, particularly the strength and location of the major mid-latitude jet streams, produce fluctuations in all three components of the angular momentum of the atmosphere on timescales upwards of a few days. Hide, Birch, Morrison, Shea and White (1980) showed that variations in the axial component of atmospheric angular momentum during the Special Observing Periods in 1979 of the First GARP Global Experiment (FGGE, where GARP is the Global Atmospheric Research programme) are well correlated with changes in length-of-day. This would be expected if the total angular momentum of the atmosphere and 'solid' Earth were conserved on short timescales (allowing for lunar and solar effects), but not if angular momentum transfer between the Earth's liquid core and solid mantle, which is accepted to be substantial and even dominant on timescales upwards of several years, were significant on timescales of weeks or months. Fluctuations in the equatorial components of atmospheric angular momentum should contribute to the observed wobble of the instantaneous pole of the Earth's rotation with respect to the Earth's crust, and it has recently been demonstrated by Barnes, Hide, White and Wilson (1983) that this contribution is considerable.

Barnes et al (1983) found it necessary to re-examine some aspects of the underlying theory of non-rigid body rotational dynamics and angular momentum exchange between the atmosphere and solid Earth. Since only viscous or topographic coupling between the atmosphere and solid Earth can transfer angular momentum, no atmospheric flow that everywhere satisfied inviscid equations (including, but not solely, geostrophic flow) could affect the rotation of a spherical solid Earth. Currently available meteorological data are not adequate for evaluating the usual wobble excitation functions accurately, but it was shown that partial integration leads to an expression involving simpler functions - termed "equatorial angular momentum functions" - which can be reliably evaluated from available meteorological data. The length-of-day problem was treated in terms of a similar "axial angular momentum function", and "effective angular momentum functions" were defined in order to allow for rotational and surface loading deformation of the Earth. Daily values of these atmospheric angular momentum functions were calculated from the 'initialised analysis global database' of the European Centre for Medium-Range Weather Forecasts (ECMWF) for the period 1 January 1981 - 30 April 1982, along with the corresponding astronomically-observed changes in length-of-day and polar motion, published by the Bureau International de l'Heure (BIH) in Paris. Changes in length-of-day during this period could evidently be accounted for almost entirely by angular momentum exchange between the atmosphere and solid Earth, and the existence of a persistent fluctuation in this exchange, with a timescale of about 7 weeks, was confirmed. The successful elucidation of this 7-week fluctuation in the atmospheric angular momentum will constitute a major future advance in our understanding and ability to

predict the behaviour of large-scale features of the general circulation of the atmosphere. The work of Barnes et al (1983) offers a theoretical basis for future routine determinations of atmospheric angular momentum fluctuations for the purposes of meteorological and geophysical research, including the assessment of the extent to which movements in the solid Earth associated with very large earthquakes contribute to the excitation of the Chandlerian wobble. Several institutions in various countries are now cooperating in a systematic effort to pursue the implications of these angular momentum studies.

(92 m)

### 3. DIFFERENTIAL ROTATION PRODUCED BY POTENTIAL VORTICITY MIXING IN A RAPIDLY-ROTATING FLUID

Differential rotation in a partially or wholly fluid astronomical body such as a planet or star is associated with energetic processes involving transformations between gravitational potential energy, kinetic energy and thermal energy. In the absence of the internal or external energy sources required to drive these processes, the body would rotate rigidly at a constant rate  $\Omega_0$  (say) about its fixed axis of maximum moment of inertia through its centre of mass. Relative to that frame of reference, all components of the Eulerian flow velocity  $\underline{u}(R, \theta, \lambda, t) = (u, -v, w)$  would vanish, where  $(R, \theta, \lambda)$  are spherical polar co-ordinates of a general point,  $R$  being distance from the centre of mass,  $\theta$  co-latitude and  $\lambda$  east-longitude. Relative to any other frame which rotates steadily with constant angular speed  $\omega$  with respect to this basic frame about the polar axis, including an inertial frame, for which  $\omega = -\Omega_0$ , we have  $(u, -v, w) = (0, 0, -R\omega \sin \theta)$ .

A major objective in the construction of theoretical models of hydrodynamical motions in planetary and stellar atmospheres and interiors is the determination from first principles of the magnitude and distribution of the mean differential rotation, as specified by

$$\bar{\Omega}(R, \theta) \equiv [\bar{u}](R, \theta) / R \sin \theta$$

$$= (2\pi T^{-1}) \int_0^T \int_0^{2\pi} (R \sin \theta)^{-1} u(R, \theta, \lambda, t) d\lambda dt$$

where the length of time  $T$  over which the average is taken is long in comparison with typical timescales associated with  $\underline{u}(R, \theta, \lambda, t)$  but is otherwise arbitrary. (We are here following a conventional notation of using an overbar to denote time average and square bracket to denote

longitudinal average.) The dependence of  $[\bar{u}]$  on  $R$  and  $\theta$  would of course emerge from a full solution of the governing equations of hydrodynamics, thermodynamics, and (in the case of electrically-conducting fluids electrodynamics, see Hide (1983)), under appropriate boundary conditions. But these equations are highly intractable and have only been solved in simplified cases. Possibly the most advanced work in this connection is that done by dynamical meteorologists in their numerical studies of the general circulation of the Earth's atmosphere, in which are reproduced  $[\bar{u}](R, \theta)$  and other principal features of atmospheric flow. The Earth's atmosphere is the only natural system for which observations are sufficient for the direct determination of  $\bar{\Omega}(R, \theta)$ . On average the atmosphere rotates faster than the solid Earth,  $[\bar{u}]$  (if measured relative to the underlying surface) is found to be positive nearly everywhere, with an average value of  $10 \text{ ms}^{-1}$ , but with negative values in certain regions, including the Trade Winds at low levels in the Tropics. The highest values of  $[\bar{u}]$  in the troposphere, about  $30 \text{ ms}^{-1}$ , are associated with mid-latitude jet streams.

In the case of the atmospheres of Jupiter and Saturn, observations of motions of markings on the visible surfaces of dense cloud going back many decades provide limited information about dependence of  $[\bar{u}]$  at the (horizontally variable) cloud level as a function of  $t$  and  $\theta$ . Both planets have strong equatorial jet streams at their visible surfaces, where the speeds attained are as high as about  $100 \text{ ms}^{-1}$  relative to the deep interior for Jupiter and  $400 \text{ ms}^{-1}$  for Saturn, (the speeds of rotation of these planetary interiors having been determined from radioastronomical observations). The jet streams are positive (ie

westerly) in direction and this implies that they must be produced by non-axisymmetric processes involving the action of local east-west pressure gradients. Comparable information on the dependence of  $[\bar{u}]$  on  $\bar{t}$  and  $\bar{\theta}$  for the solar atmosphere can be obtained from observations of sunspot motions and from spectroscopic data. The visible surface of the Sun rotates most quickly at the equator and exhibits a general decrease with distance from the equator that is more gradual than the corresponding latitudinal variations of zonal flow at the visible surfaces of Jupiter and Saturn. Some theories of the origin of the magnetic fields of planets and stars invoke differential rotation in their electrically-conducting fluid interiors as the main amplification process, but there are no direct observations of  $[\bar{u}]$  in these regions.

As noted above, departures from axial symmetry in the pattern of relative motion of a rapidly-rotating fluid are to be expected even when the boundary conditions are axisymmetric. But the correct quantitative representation of the effects of non-axisymmetric features on the magnitude and form of the differential rotation is by no means straightforward and presents serious technical difficulties. Some of these can be overcome by the introduction of a "mixing hypothesis", which leads to considerable theoretical simplifications without sacrificing essentials. Hide and James (1963) have investigated differential rotation in a rotating spherical shell of incompressible fluid by assuming that non-axisymmetric motions act in such a way as to smooth out latitudinal gradients in potential vorticity. The latitudinal profiles of  $\bar{\Omega}$  thus obtained depend inter alia on the thickness of the shell, exhibiting strong jets near the equator when the

shell is thin and at mid-latitudes when the shell is thick. The model used was developed as an improvement on one proposed much earlier by Rossby (1947), who considered the effects of horizontal mixing of radial filaments of fluid on the profile of mean zonal flow, and derived expressions for such profiles on the assumption that mixing eliminates gradients of the vertical component of absolute vorticity poleward of a certain arbitrary latitude. It is also related to one used in an independent study by Rhines and Young (1982). In keeping with the constraints of the Proudman-Taylor theorem, Hide and James (1963) considered the behaviour of axial filaments of fluid, supposing that each filament retains its coherence and, owing to the weakness of frictional effects, undergoes little change in its potential vorticity over timescales of typical displacements perpendicular to the rotation axis. These displacements are associated with local pressure gradients which, in a rapidly rotating fluid, act at right-angle to the displacements. It is remarkable that such a simple model can reproduce many of the observed features of the differential rotation of the Earth, Jupiter, Saturn and the Sun. Whether or not internal dynamical processes such as those studied in our paper can account for the enormous value of the super-rotation of the atmosphere of Venus, at more than ten times the speed of the underlying planet, is a matter for further investigation. Some workers have argued that such high value cannot be explained without invoking the action of external couples and have developed models based on the action of the Sun's gravitational field on non-axisymmetric density variations associated with thermal tides (see Gold and Soter (1971)).

(90 m)

#### 4. LABORATORY AND NUMERICAL STUDIES OF THERMALLY-PRODUCED MOTIONS IN ROTATING FLUIDS

Many features of the large-scale atmospheric circulation can be reproduced in a liquid filling a cylindrical annulus rotating about a vertical axis, when the inner and outer walls of the annulus are maintained at different temperatures. Laboratory studies over a wide range of impressed conditions have revealed the existence of several possible flow regimes: axisymmetric flow at comparatively low rotation rates (or high temperature differences), regular non-axisymmetric flow at intermediate rotation rates, and irregular non-axisymmetric flow at high rotation rates. Baroclinic waves associated with meandering jet streams are characteristic features of non-axisymmetric flows. Regular flows may take the form of either steady or vacillating waves (in which periodic changes of amplitude or shape occur). Being spatially and temporally periodic, these wave flows are forecastable in the meteorological sense. But different regular flows may be observed in different experiments under the same impressed conditions, implying that regular flows are intransitive. The irregular flows are aperiodic and only poorly forecastable; the extent to which they are intransitive has not yet been fully investigated. (For reference to early work see Hide and Mason (1975).)

Studies based on the joint use of laboratory systems and their counterparts in numerical models make it possible, amongst other things, to "verify" the basic dynamical structure of numerical models of rotating baroclinic flow in a way that is virtually impossible for atmospheric numerical models, in which important small scale processes have to be represented by comparatively crude and uncertain

parametrizations. A high resolution numerical model based on the Navier-Stokes equations for incompressible flow is currently being used in work of this type. Our numerical model reproduces most of the flow phenomena seen in the laboratory systems, namely axisymmetric flow, steady waves, intransitivity, wavenumber transitions, hysteresis, amplitude vacillation and irregular flow (geostrophic turbulence), but a convincing numerical simulation of shape vacillation has not yet been achieved. Several detailed quantitative comparisons between laboratory measurements of steady waves and corresponding numerical simulations have now been carried out, with encouraging results (see Hignett, White, Carter, Jackson and Small (1984)).

An important element of our current programme is the investigation of the nature of steady and vacillating wave flows. Laboratory experiments have demonstrated that vacillation occurs adjacent in parameter space to transitions either to a lower wavenumber flow (amplitude vacillation) or to irregular flow (shape vacillation). Amplitude vacillation is a doubly-periodic flow whose spectral characteristics can be interpreted in terms of an amplitude and frequency modulated wave (see eg Hignett (1984)). Accounting for the precise conditions under which steady or vacillating waves can occur is still an unsolved problem. One hypothesis now being tested using data from laboratory experiments and numerical integrations is that steady waves can arise only when initial wave developments are strong enough to bring about large changes in the mean flow structure.

The ability of numerical models to cover combinations of parameters not readily attainable in the laboratory is being exploited in a study of the axisymmetric flow at very low rotation rates. Of particular



interest here are the magnitudes of the mean azimuthal flow and the total heat transport, which compare well with predictions based on straightforward scaling arguments and boundary layer theory. Experiments at higher rotation rates have been carried using two small annular convection chambers, one with internal heating and one with wall heating. Effects of varying the end-wall boundary conditions have also been investigated. These experiments appear to bear out some new ideas concerning the occurrence of non-axisymmetric flows which do not directly invoke baroclinic instability theory as a starting point (Hilde, Hignett and White (1984)).

Recent work on annulus flows under a variety of impressed temperature fields, obtained by heating or cooling the fluid internally and cooling or heating one or both side walls are mentioned below, in the section on sloping convection in the laboratory and in the atmosphere of Jupiter and Saturn.

(90 m)

## 5. ANALYTICAL STUDIES OF LINEAR AND NONLINEAR WAVES IN ROTATING FLUIDS

Analytical studies play an important role in the formulation and interpretation of the abovementioned laboratory experiments and numerical simulations. Linear theories assist to a limited extent in the interpretation of the observed transitions from axisymmetric to regular wave flow and from regular to irregular wave flow; but non-linear analyses are required for more detailed comparisons with the experimental and simulated flows, and also to guide the formulation of numerical models.

So far as linear mathematical studies are concerned, the theory of baroclinic instability reveals many flows that are unstable, and others which, because of dynamical constraints, that are stable even though they possess "available potential energy". Some of the stable flows are similar in many respects to the mean flows found in the regular waves regime. One possible theoretical model of a steady wave therefore consists of a neutral wave on a stable mean flow. Another is based on the finite amplitude steady waves and associated mean flows (governed nevertheless by linear equations) that arise as exact solutions of the quasi-geostrophic potential vorticity equation. These solutions are of general interest as analytical illustrations of the celebrated "non-acceleration theorem" of wave/mean-flow interaction theory, and they also account for many of the gross features of real and simulated steady waves (White (1984a)).

Both theoretical models are consistent with the distinctive mean flow structure of steady waves and more detailed diagnostic studies will be needed to determine which is the more appropriate model.

Analytical non-linear studies gained impetus during the 1970's, when Pedlosky and Drazin made considerable progress with the mathematically-demanding problem of establishing descriptions of weakly nonlinear baroclinic waves interacting with a mean flow. An important later development was the discovery by members of this laboratory in collaboration with others (for references see Moroz (1981), Moroz and Brindley (1981)) of soliton-type solutions for the propagation of baroclinic wave packets. In subsequent studies the conditions under which such solutions (and various kinds of less ordered behaviour) occur in the weakly nonlinear models were delineated. In view of their implications for the predictability of fluid motions in rotating systems, it is clearly important to determine how far the weakly nonlinear models are applicable to real fluid systems. Our numerical model results imply that the state of marginal stability adopted in the existing theoretical treatments is not the most appropriate, and indicate what further analytical investigations are now required.

The weakly nonlinear models are specialisations of quasi-geostrophic formulations that are widely used in meteorological theory. Another matter for consideration is the applicability of quasi-geostrophic models themselves to the real laboratory flows. Diagnostic studies using numerical data from wave simulations are at present in progress to investigate this question.

The use of quasi-geostrophic equations in our theoretical work is but one example of the application of approximate forms of the Navier-Stokes equations in geophysical fluid dynamics. In meteorological modelling, for instance, approximate formulations (such as the hydrostatic set) are invariably used. In spite of this, no

systematic theory of approximation is yet available, and consequently several important issues are uncertain. In particular, it is not yet clear to what extent the various general properties of the original equations (eg. conservation of mass, energy, angular momentum, etc) should be reproduced by the approximate forms. A theoretical case study based on the quasi-geostrophic equations has recently been completed (see White (1984b)), indicating that accuracy can be significantly improved by retaining conservation properties in approximate formulations.

(Gou)

## 6. SLOPING CONVECTION IN THE LABORATORY AND IN THE ATMOSPHERES OF JUPITER AND SATURN

It is accepted that long-lived prominent markings seen on the visible surface of dense clouds on Jupiter and Saturn, such as the Jovian Great Red Spot and three White Ovals, are manifestations of atmospheric motions, so that their explanation must be given in terms of basic processes in fluid dynamics. The very existence of such features has important implications for theories of atmospheric predictability. There have been several incomplete suggestions as to the nature of the Great Red Spot. According to one idea it is the upper end of a Taylor column produced by the interaction between atmospheric motions and deep-seated topography (which might be a "hydrogen-helium ice floe"). In the so-called "soliton" or "modon" theories the stability of the Spot is accounted for on the basis of a balance between dispersion due to the latitudinal variation of the vertical component of the Coriolis parameter ("beta-effect") and horizontal advection, with the soliton drawing its energy directly from the kinetic energy of the background zonal shear and the modon from the coalescence of smaller eddies. The hypothesis that the Great Red Spot is analogous to a terrestrial hurricane invokes small-scale moist convection as the basic energy source, with friction playing a key role in organizing the flow but with the beta-effect playing only a modifying role. (For reference see Read and Hide (1983,1984).)

According to a recent proposal (Hide 1980, 1981) long-lived anticyclonic eddies in the atmospheres of Jupiter and Saturn, including the Jovian Great Red Spot and White Ovals, might be manifestations of fully developed "sloping" or "slantwise" convection characteristic of

quasi-steady thermally driven flows in a rapidly-rotating fluid of low viscosity subject to internal heating. Baroclinic eddies of this type derive their kinetic energy directly from the potential energy due to gravity acting on the variable density field maintained by differential heating and cooling. They were first discovered in laboratory experiments by Hide and Mason and they are now also being studied with the aid of numerical models. On Jupiter and Saturn they would transport heat from the lower middle parts towards the upper outer parts of the atmospheric zones in which they occur. In such an eddy, the upper level horizontal motion is largely concentrated in a jet stream circulating around the relative quiescent core of rising fluid in an anticyclonic sense, with descending motion occurring in a narrow "collar" surrounding the jet stream. Theory predicts and numerical experiments confirm that stable eddies with cyclonic upper level horizontal circulation, descending motion in the core and ascending motion in a collar surrounding the cyclonic jet stream, would be characteristic of fully developed slantwise convection in a rapidly rotating fluid subject to internal cooling, and it has been suggested that the cyclonic "barges" on Jupiter and Saturn might therefore be manifestations of slantwise convection transporting heat from the lower outer parts towards the upper inner parts of the atmospheric belts in which they occur (Read and Hide (1983)).

The "sloping convection" hypothesis has most in common with some of the numerical general circulation studies of Jupiter's atmosphere, in which latitudinal jet streams and transient baroclinic eddies are produced in integrations of a numerical model similar to those used to investigate the mid-latitude circulation of the Earth's atmosphere. But

it is to laboratory experiments that we owe the demonstration that long-lived baroclinic eddies can exist over a wide range of conditions and that their stability is a consequence of the action of nonlinear advective effects and not directly to viscosity or to the particular geometry of the boundaries, which can modify the eddies in certain details without affecting their main properties.

New laboratory and numerical experiments mentioned in this summary have been undertaken with two distinct but related objectives in mind, namely (a) the extension of knowledge and deepening of insight into sloping convection in rapidly-rotating fluids, and (b) the improvement in our understanding of the structure and dynamics of Jupiter and Saturn. Further experiments bearing on the isolated nature of the long-lived eddies on Jupiter and Saturn have been carried out (Read and Hide (1983), (1984)) and investigations of the nature of transient small-scale atmospheric eddies seen in the vicinity of long-lived eddies are now being planned.

(Go on)

#### 8. CONCLUDING REMARKS

The understanding of interactions between motions on different scales of length and time is of central importance in predictability studies. It is now recognised that such interactions are more complicated than those envisaged by L. F. Richardson when, in parodying Jonathan Swift's lines

So, Nat'ralists observe a flea  
Hath smaller Fleas that on him prey,  
And these have smaller Fleas to bit'em  
And so proceed ad infinitum,

he wrote

Big whorls have little whorls,  
Which feed on their velocity:  
And little whorls have lesser whorls,  
And so on to viscosity (in the molecular sense).

Richardson avoided the alternative statement by de Morgan (see Mandelbrot 1982):

(Go on)

Great fleas have little fleas, upon their backs  
 to bite them,  
 And little fleas have lesser fleas and so  
 ad infinitum,  
 And the great fleas themselves in turn, have greater fleas to  
go on  
 While these again have greater still, and greater still, and  
so on.

The italics are mine, and they draw attention to the "anti-cascade" process implied by Fjørtoft's celebrated theorem whereby large-scale motions can gain energy from smaller scales. In the highly anisotropic fluid systems found in Nature and also in the laboratory systems discussed above, the dynamical constraints are such as to inhibit non-linear energy exchanges and, when such exchanges occur, to promote the "anti-cascade" process. Theoretical meteorologists are now well equipped to attack the problem of understanding such processes in sufficient detail to be able to meet some of the demands of their more practical colleagues. Laboratory studies with rotating fluids and associated numerical and analytical work as well as studies of the atmospheres of other planets will continue to play key roles in the development of theoretical ideas.

Go on

# REFERENCES

- Barnes, R.T.H., Hide, R., White, A.A. and Wilson, C.A., 1983, "Atmospheric angular momentum fluctuations, length of day changes and polar motion", Proc. Roy. Soc. London, A387, 31-74.
- Gold, T. and Soter, S., 1971, "Atmospheric tides and the 4-day circulation of Venus", Icarus 14, 16-20.
- Hide, R., 1980, "Jupiter and Saturn: giant magnetic rotating fluid planets" (Halley Lecture), The Observatory, 100, 182-193.
- Hide, R., 1981, "High vorticity regions in rotating thermally-driven flows", Meteorological Magazine, 110, 335-344.
- Hide, R., 1982, "On the role of rotation in the generation of magnetic fields by fluid motions", Philos. Trans. Roy. Soc. A306, 223-234.
- Hide, R., 1983, "The magnetic analogue of Ertel's potential vorticity theorem", Annls. Geophys. 1, 59-60.
- Hide, R., Birch, N.T., Morrison, L.V., Shea, D., and White, A.A., 1980, "Fluctuations in the angular momentum of the atmosphere and variations in the length of the day", Nature, 286, 114-117.
- Hide, R., Hignett, P. and White, A.A., 1983, "Axisymmetric thermal convection in a rotating fluid", (in preparation).
- Hide, R. and James, I.N., 1983, "Differential rotation produced by large scale potential vorticity mixing in a rapidly-rotating fluid", Geophys. J. Royal Astronomical Soc., 74, 301-312.
- Hide, R. and Mason, P.J., 1975, "Sloping convection in a rotating fluid", Advances in Physics, 24, 47-100.
- Hignett, P., 1984, "Spectral characteristics of amplitude vacillation in a differentially-heated rotating fluid annulus", (in preparation).

- Hignett, P., White, A.A., and Carter, R.D., Jackson, W.D.N., and Small, R.M., 1984, "A comparison of laboratory measurements with numerical simulations of baroclinic waves in a rotating fluid annulus", (in preparation).
- Mandelbrot, B.B., 1982, The fractal geometry of Nature, W. H. Freeman and Co., San Francisco, pp viii + 461.
- Moroz, I.M., 1981, "Slowly modulated baroclinic waves in a three-layer model", J. Atmos. Sci., 38, 600-608.
- Moroz, I.M. and Brindley, J., 1981 "Evolution of baroclinic wave packets in a flow with continuous shear and stratification", Proc. Roy. Soc. London, A377, 379-404.
- Read, P.L., and Hide, R., 1983(a) "On long-lived eddies in the laboratory and in the atmospheres of Jupiter and Saturn", Nature, 302, 126-129.
- Read, P.L., and Hide, R., 1983(b) "Sloping convection in the laboratory and in the atmospheres of Jupiter and Saturn" Annls Geophys. 1, 135-137.
- Read, P.L. and Hide, R., 1984 "On long-lived eddies in the laboratory and in the atmospheres of Jupiter and Saturn: 2, An isolated eddy analogous to the Jovian Great Red Spot", Nature (in the press).
- Rhines, P.B., and Young, W.R., 1982 "Homogenization of potential vorticity in planetary gyres", J. Fluid Mech. 122, 347-367.
- Rossby, C. G., 1947, "On the distribution of angular velocity in gaseous envelopes under the influence of large-scale horizontal mixing", Bull Amer. Meteorol. Soc., 28, 53-68.

- White, A.A., 1984(a), "Finite amplitude steady baroclinic waves and mean flows: analytical illustrations of the Charney-Drazin non-acceleration theorem", Quart. J.R. Met. Soc (submitted).
- White, A.A., 1984(b), "Approximate forms of the equations governing nearly-geostrophic motion: Part II - A case study based on the type 1 formulation", (in preparation).

(End of paper)