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Frequency reuse & power control in wireless telecommunication systems

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Lack of free frequencies hampers many wireless applications. As their number grows, wireless telecommunication systems have to share the available spectrum resources. The frequency reuse, combined with the control of power radiated by transmitting stations, is becoming an increasingly widespread necessity as it allows more systems to share common frequency bands.

Note: The opinions expressed are the author's personal views, and do not engage any entity.

Key words: *Radiocommunications, Frequency reuse, Power control, Optimization*

With the development of mobile communications, rural communication, wireless local loop, and other terrestrial and satellite wireless applications, the problem of spectrum scarcity is becoming increasingly sharp, as the number of wireless systems grows whereas the available spectrum resources do not. Appropriate frequency bands are often occupied, and there is no free spectrum available. Spectrum scarcity is one of the most important problems facing wireless telecommunications of all kinds and the lack of free frequencies hampers the development of many applications. More and more radiocommunication systems have to reuse the available frequency bands and frequency sharing is becoming necessary. In some systems, such as various Code Division Multiple Access (CDMA) systems, or Single-Frequency Networks (SFNs), a number of transmitting stations share a common frequency band by design.

Sharing the radio frequency spectrum raises the question of mutual interference between the systems. On the one hand, the availability of interference-free service must be kept at a high level. On the other hand, radio interference can seriously degrade the quality and availability of service. Sharing frequencies and controlling interference are the two sides of the same issue. When the wireless networks are operated by separate entities, frequency sharing, interference, and electromagnetic compatibility problems are of critical importance. Such a case is becoming typical, in view of current trends towards liberalization and privatization of the telecommunication sector in many countries. In countries, where access to spectrum resources is auctioned or offered at a price, the issue involves also an important economic aspect.

This paper focuses on rational use of radio frequencies by controlling the power radiated by radio transmitters in a communication network. 'Network' is understood here as a collection of radio links, intended or unintended, that may interfere with each other. Each link consists of a transmitter (emitter) and a receiver (receptor) of electromagnetic waves, carrying wanted or unwanted signals, and there is not necessarily any functional relationship between them. The approach is based on elementary geometrical representations, easy to understand. The paper explains why power control is beneficial and how to assign power to the transmitters optimally under the constraints of performance, and realizability as well as those imposed by the environment. The criterion of optimality is minimal total power radiated; the reason why this criterion is applied is explained further in the text.

Interference

In any wireless network, interference level depends on several technical and operational parameters of the systems involved and on the system's environment. It is a challenging task to keep interference below or at an acceptable level in optimal way. To do so, special methods, such as described in this paper, are needed. However, willingness of the involved parties to coordinate their efforts to find mutually acceptable solutions is a necessary ingredient. Moreover, to be efficient, the engineering knowledge has to be complemented by an appropriate legal/ regulatory framework. If the networks sharing a frequency band are deployed on the opposite sides of a state border, they have to be supported by special intergovernmental agreements, such as the famous Vienna agreement, for instance [Vienna '93].

Spectrum used

The radio frequency spectrum is a multi-dimensional concept, with the frequency, geometric/ geographic volume and time as principal dimensions. For a short observation period (snapshot), or for time-independent systems, the time dimension is insignificant. Figure 1 illustrates such a case.

[Figure 1. Space used by transmitter T1 and denied to other radio communications. The axes of the three-dimensional space are x - East distance, y -North distance, z - frequency. The volume denied increases with the power radiated by the transmitter.]

A wireless telecommunication system is said to use, occupy, or deny a frequency band when other systems cannot use that band because of harmful interference. Transmitters deny the spectrum to receivers, and receivers deny the spectrum to transmitters [Berry 77] over specific distances. At small distances, the transmitter can jam other communications. Note that 'distance' involves both the geographic/ geometric/ geographic distance and the distance along the frequency axis. With omnidirectional antennas and regular propagation conditions, the denied space can be approximated by a cylinder. The position of the cylinder depends on the center frequency and geographical location of the transmitter. Its height represents the frequency band occupied, and its diameter - the geographic area denied. The volume denied by transmitter depends on many variables, deterministic and probabilistic and is an increasing function of the transmitter's power. In practice, the figure of denied space is more complex.

Minimizing power: reasons

There are several reasons for minimizing the power:

← ***Economical use of the RF spectrum.*** Minimal power radiated means minimal space used (Figure 1) more radio systems operating over a given area, and increased network capacity.

← ***Enhanced channel reuse.*** In cellular systems, reuse of radio channels throughout a geographic region is widely applied. Each base station is assigned a group of radio channels to be used within a small geographic area or cell, see Figure 2. The same channels can be reused in another cell if the ***co-channel interference*** level is low. This type of interference is possible in all networks, even in those with 'ideal' equipment and perfectly orthogonal signals such as FDMA, TDMA, or CDMA (see Box). The detrimental effects of co-channel interference can be reduced by optimum power adjustment.

← ***Enhanced adjacent-channel protection.*** Due to limited dynamic range of receivers, a strong-signal channel may unintentionally 'leak' into the adjacent channels. In CDMA systems, it is known as 'near-far' problem and 'adjacent code' interference [see e.g. Zander 1994]. The optimal transmitting power control keeps the power of signals at the receiver's input as low as possible and thus limits the 'leakage' effects.

← ***Reduced non-linear effects.*** Non-linear effects in receivers and in vicinity of transmitting antennas lead to unintended generation of harmonics and intermodulation products. Their level decreases with decreasing power.

← ***Enhancement of environmental protection.*** Radio waves interact with the matter and with living organisms, and, in order not to introduce unintended, possibly harmful effects, the power radiated by transmitters should be minimal.

← ***Reduced power consumption.*** In portable hand-held applications, battery power is a scarce commodity. With lower transmitting power, the battery life is longer, and the equipment is lighter.
← ***Reduced cost of frequencies used.*** Where spectrum market exists, that is where the operator has to pay for the spectrum used by intentional and unintended radiation, power minimization has an economic dimension [Struzak, 1996].

[Box. Loud, quiet, and balanced systems
There is some analogy between radio links and talking people. When several persons are speaking at the same time, some of them cannot communicate effectively because of noise from other talks. What is the wanted message for one, is perceived as noise by others. The same applies to a network of radio links and messages carried by radio waves. The RF power radiated by a transmitter and desired at some radio receivers may disturb the operation of other receivers.

'Loud' networks. In the absence of any other mechanism to mitigate disturbance, a speaker instinctively raises the voice and keeps it well above the noise level. Such a strategy works well if only one speaker is raising the voice to improve his communications. It fails, however, when every speaker raises the voice level. Even with all people shouting as loudly as they can, some communications may still be impossible. The reason is that the increase of voice levels by all speakers does not necessarily improve the ratio of wanted-to-unwanted messages. We call it is a 'bazaar' model. Such strategy is known from the early days of radiocommunications. In 1926, in the case of Zenith Radio against the government of the United States, the verdict of the court was that imposing any restriction on the power, frequency, or hours of operation of a radio station was illegal. It opened 'power race' in which each operator wanted his station to be 'louder' than that of his competitor, and nobody cared about interference. As a result, ability to make a reasonable use of the spectrum became severely compromised. It forced the US Congress to modify the law to stop the chaos and to enact the Radio Act in 1927. Today, international agreements and treaties such as Radio Regulations of International Telecommunication Union regulate the use of the radio frequency spectrum [Rosston et al. 97].

'Quiet' networks. A possible strategy would be to lower rather than to rise the voice, aiming at minimizing the interference level, rather than at maximizing the wanted-signal level, as in the 'bazaar' model. We call it a 'temple' model: in temple, people use low voice in order not to disturb the celebrations. In the limit, everybody whispers. However, although the interference level is minimal, for some people the whispers may be below the audibility threshold. The reason is the same as in the previous case: the decrease of voice level by all speakers modifies in the same proportion the strength of the wanted and unwanted messages without changing their mutual relation.

Balanced networks. Better results could be achieved if each speaker adjusts his voice level to the local conditions in order to keep balance between the wanted message and the ambient noise. Such strategy requires some speakers to raise the voice and others to lower it. The same reasoning applies to radiocommunication networks. The problem of optimum sharing the spectrum resources consists in finding a proper balance between the wanted signals and unwanted signals. Determining such a balance is not a trivial problem if the network involves a large number of transmitters and receivers. It cannot be done without involving a systematic optimization method. One such a method is explained further in this text. [The box ends here]

[Box. FDMA, TDMA & CDMA

As the number of radio links increases, multi-access techniques become more significant. Common techniques are known as the Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA), and Code Division Multiple Access (CDMA).

In FDMA, the frequency band is divided in a number of sub-bands or frequency channels, and each channel is assigned to a radio link. The channels need not be all of the same width. It is as if each talking pair in a crowd of people was put into a separate compartment in order to eliminate disturbances due to noise from other conversations. However, as the available radio frequency spectrum is insufficient to assign a separate band for each radio link, some channels have to be reused. The radio links sharing the same frequency band are prone to co-channel interference, level of which decreases with the power and distance. The number of channels and reuse-distance are interdependent (Fig. 2).

[Figure 2. An illustration of the frequency reuse concept in cellular systems. The geographical area is divided into a number of square cells, each with a transmitter using its assigned radio channel. Cells sharing the same channel have the same color and pattern. With four channels (A) the common-channel cells are separated by one cell. With nine channels (B) they are separated by two cells. In practice, the cells have form that is more complex.]

TDMA does in the time domain what FDMA does in the frequency domain. Here, each radio link uses the same frequency band, during repeated periodically short time slots assigned. It is as if people were talking at a business meeting, where the chairperson gives the floor to each speaker one after another.

In CDMA, each radio link uses the same frequency band and the same time, but the signal is coded and neighboring links employ different codes. The code plays here similar role as the frequency in FDMA or time in TDMA. It is as if all people were talking at the same time but using different languages. Although the conversations in foreign language also make noise, the noise is less annoying. [The box ends here]

Two transmitters, two receivers

This section introduces concepts of assignment plan, and its cost, realizability, compatibility, and feasibility. For that purpose, an example of two radio links is considered, see Figure 3. The links numbered '1' and '2' and sharing a common frequency band, consist each of one transmitting station (T) and one receiving station (R). Let P1 and P2 be the power radiated by each transmitting station. Any specific combination of these powers is called ***'power assignment plan'***, ***'power plan'***, or simply ***'plan'***. In geometric interpretation, each plan represents a point on plane (P1, P2).

[Figure 3. Hypothetical example of two radio-links sharing a common frequency band. T1 and T2 are transmitters, and R1 and R2 are receivers. No restrictions are imposed on their types, locations, frequencies, etc.]

Realizability constraints

To be realizable, any plan has to take into account physical constraints. One of such constraints imposed on all systems is that the power radiated by the transmitting station (*P*) must be positive and limited, see Figure 4.

[Figure 4. Realizability constraints imposed on the transmitting power and realizability region. Shadowed area represents unrealizable plans. The limiting values (Pmin, Pmax) are determined by the technology, costs, and other factors.]

Radio noise, unavoidable in any telecommunication network, is another constraining factor. There are several sources of noise possible. Thermal noise is generated in electrical circuits and its power depends upon the temperature of the circuit and its resistance. The noise power transferred to a matched load is $N = k\,T\,B$ [W], where k=1.38x10^{-23} [Joules/Kelvin] (Botzman's constant), T is noise temperature [Kelvin], and B is the equivalent bandwidth of the link [Hz]. The background noise power N is the equivalent in-band RF noise power at the receiver input due to all uncontrolled sources: thermal noise, atmospherics, man-made emissions, etc. Its value may derive from measurement or from prediction calculations.

EMC constraints

Electromagnetic compatibility (EMC) is the ability of a device or system to function satisfactorily in its electromagnetic environment without introducing intolerable disturbance to that environment. It involves two aspects. One is the effect of 'foreign' signals 'in-flowing' to the equipment or system. These signals may seriously degrade the quality of service and even disrupt communications. EMC is thus related to the quality and reliability of radio links and service availability. Another EMC aspect is 'out-flow' of signals from the equipment/ system at hand into its environment (Figure 5). Note that, when considering the influence of T1 and T2 on the environment, appropriate environment-test receivers should be included into consideration.

[Figure 5. The system at hand and its environment make one ensemble. We break it to ease considerations, and model their interactions by in-flowing and out-flowing signals. The shadowed arrows represent out-flowing signals. The white arrows represent in-flowing signals.]

Unwanted signals

Radio links T1R1 and T2R2 are designed to carry ***wanted signals*** C1 and C2, respectively. Due to equipment imperfections and laws of nature, a fraction of the power radiated by transmitter T1 and designed to reach R1, unintentionally reaches receiver also R2 as ***unwanted signal*** (I1). Another unwanted signal (I2) is generated by transmitter T2. The links T2R1 and T1R2 are not designed: they are ***unintended,*** or ***spurious.*** The ***unwanted*** signals they carry (I_1 and I_2) are known as ***unintended, interfering,*** or ***spurious*** signals (Figure 6).

With linear systems and linear propagation medium (the usual case), the signal power at the receiver input is proportional to the power radiated by the transmitters, no matter if it is intended or unintended. The coefficient of proportionality is the ***transmission coefficient*** (*t*). It includes effects of directive antennas, propagation loss, and other relevant factors. Its value may derive from measurements or from prediction equations. Some factors may be vary in time and may have significant random components. In that case, the transmission coefficient also varies and has random components.

Figure 6. Graph representation of the system from Fig. 2-1. Thick blue lines: intended transmissions. Narrow red lines: unintended transmissions. C: wanted signal. I: interfering signal. N: noise. Q: required signal-to-interference-plus-noise ratio. R: receiver. t: transmission coefficient. T: transmitter. P: power radiated by the transmitter. The first index of the transmission coefficient is the receiver number; the second index is the transmitter number.

Compatibility constraints

Two types of constraints are to be taken into account. <u>EMC constraints Type 1</u> result from the necessity to assure the quality and availability of the communication service against the unwanted signals that may distort the messages or even disrupt the wanted communications. Although unwanted signals and thermal noise interact with the intended signals in different ways, their joint effect can usually be approximated by adding their powers. (In more precise analyses, they may need to be treated separately, taking into account probabilities of occurrence and other statistical measures.) To keep the service quality/ availability at the required level, the power ratio of the wanted signal to the interfering signal-plus-noise should be sufficiently high. Each receiver imposes one constraint of Type 1:

Receiver R1: $[C1 / (I1 + N1)] \geq Q1$
Receiver R2: $[C2 / (I2 + N2)] \ Q2$

Here, Q1 and Q2 are the minimum acceptable values of the wanted signal to the interfering signal-plus-noise ratio. Numerical values of Q1 and Q2 depend on various factors, and can be derived from prediction equations or determined experimentally.

These constraints are *regional constraints,* as they define a specific region in plane (P1, P2). Figure 7 is an example of the constraint imposed by receiver R1.

Figure 7. Electromagnetic compatibility (EMC) constraint Type 1 imposed by receiver R1 on the power radiated by transmitters T1 and T2. Shaded area: region of incompatible plans due to unacceptable interference. 'b' is the minimum power of T1 required to overcome the background noise N1 when transmitter T2 is switched off.

Note that the constraint line crosses the OP1 axis at point (b, 0), where 'b' is the minimum power of transmitter T1 required for normal operation of R1 when T2 is switched-off and noise is the only threat to the wanted signal. Note also that, in the limiting case, the increase of T2-power of by one unit must be compensated by 'a' units of the power radiated by transmitter T1 to maintain the required communication quality. A similar diagram can be produced for receiver R2.

<u>EMC constraints Type 2</u> result from the necessity to avoid the receiver's overloading. An excess of RF power may force a nonlinear mode of the receiver's operation, or may lead to a physical damage of the receiver's input circuitry. To avoid such effects, limits have to be imposed on the total power of intended and unintended signals $(C + I)$ at the receiver's input. Each receiver imposes one constraint of Type 2:

Receiver R1: $(C1 + I1) \ E1$
Receiver R2: $(C2 + I2) \ E2$
Here, E_1 and E_2 are the maximum allowable powers at the input of receiver R_1 and R_2, respectively. Requirements of environmental protection result in the same type of constraints.

Figure 8 shows all four EMC constraints imposed by receivers R1 and R2.

Figure 8. Electromagnetic compatibility constraints Type 1 and Type 2 imposed by receivers R_1 and R_2 on transmitters T_1 and T_2. Clear area: compatibility region where the EMC requirements are fulfilled. Shaded area: region of unacceptable reception conditions. Thick vertical line: acceptable range of power radiated by T1 when power of T2 is fixed to P2.*

The borders of the compatibility region are pieces of straight lines connecting vertices A-B-C-D. If the power of T2 is fixed, say to P2* watt, then, for the links to operate correctly, the power of transmitter T1 must be contained within the compatibility region: it cannot be above, or below, the limits shown in the figure. The same reasoning applied to the situation when the power of transmitter T1 is fixed leads to similar restrictions imposed on P1 as shown in the figure.

Extreme case 1: Weak interactions

Interactions between the radio links are weak when the radio noise dominates over the interfering signals N1>>I1 and N2>>I2. In such a case, the performances of the links are ***noise-limited*** and unintended signals can be disregarded. The case is illustrated in Figure 9.

Figure 9. Limiting case of weak interactions. Incompatibility regions are shaded.

Extreme case 2: Strong interactions

Interactions between the radio links are strong when the interfering signals dominate over the background noise, I1>>N1 and I2>>N2. In such a case, the performances of the network are ***interference-limited*** and the noise can be disregarded. The case is shown in Figure 10. The compatibility region degenerate into one slant line that actually is composed of two coinciding lines - borders of the regions of unacceptable signal-to-interference ratio at the inputs of receivers R1 and R2.

Figure 10. Limiting case of strong interactions. Compatible plans are possible only on the slant line and at some distance from the origin (near the origin the assumptions made are not satisfied).

In that case, for compatible plans, the ratio of the powers radiated by transmitters T1 and T2, and the signal to interference plus noise ratio have to keep values determined by the transmission coefficients as shown in the figure. Example: If, for instance, $t_{11}=10^{-7}$, $t_{12} = 10^{-8}$, $t_{21} = 4.10^{-8}$, $t_{22} = 10^{-7}$, and Q1 = Q2 = Q, then the compatible operation of the links would be possible only if P1 / P2 = $(10^{-8}.10^{-7} / 10^{-7}.4.10^{-8})$ = (1/4) = 1/2 and Q = $(10^{-7}\cdot 10^{-7} / 10^{-8}.4.10^{-8})$ = (4.10^{+2}) = 20

Feasibility & optimal plan

A feasible power assignment plan does assure compliance with all the constraints and requirements imposed. The set of all feasible plans forms the *feasible region* - a conjunction of the realizability and compatibility regions discussed earlier. Figure 11 shows an example of such a feasibility region. Its borders, created from pieces of the straight lines that represent the various system constraints, form a convex *feasibility polygon* ABCD. Each plan represented by a point within the feasible region (or on its border) is *feasible*. However, in some cases the region may be empty. It happens, if the constraints are contradictory among themselves. In such a case, no feasible plan is possible, and the radio links cannot operate as required: some parameters of the radio links must be changed.

Each assignment plan is associated a value of the *objective function* F(P1, P2), called also the *value* or *cost* or the plan. In our case, the *cost function* is expressed in terms of the power radiated, and equals F = P1 + P2.

[Figure 11. The cost of a power assignment plan is the value of objective function F associated with the plan - the total power radiated (F = P1 + P2). In geometric interpretation, it is the distance measured on the F-axis.]

The objective is to select the best or *optimal plan* from all feasible plans. The optimal plan does assure the minimal value of the cost function F - minimum of the total power radiated by transmitters. As F is linear function of two variables, P1 and P2, it maps the feasibility region ABCD into a polygon on a slant plane F, as shown in Figure 12. Note that the maximum and minimum values of a linear function with linear constraints occur on one of the vertexes of the feasibility polygon. Instead, therefore, to examine all possible plans represented by point inside the feasibility polygon - whose number is infinite - it is sufficient to check only the plans represented by the vertices of the polygon, whose number is finite. In our case, the optimal assignment plan can be found by simple inspection. It coincides with point D.

Example: If, for instance, $t_{11}=10^{-7}$, $t_{12} = 10^{-8}$, $t_{21} = 4.10^{-8}$, $t_{22} = 10^{-7}$, and Q1 = Q2 = 1, N1 = N2 = 10-6 W then the optimum plan is P1 = 1 (1.10-6.10-8 + 10-6.10-7) / (10-7.10-7 - 1.1.10-

8.4.10-8) 11 W, P2 = 1 (1.10-6.4.10-8 + 10-6.10-7) / (10-7.10-7 - 1.1.10-8.4.10-8) 14 W, and the minimal total power is F 25 W.

Figure 12. All feasible plans are represented by polygon ABCD, and the total power radiated associated with each plan by polygon A'B'C'D' above the feasibility region in plane (P1, P2). D is the optimal plan and F(D) is the minimum total power radiated by transmitters.

Two transmitters, multiple receivers

This section extends the considerations of the previous section (dealing with two radio links) onto a pair of point-to-multipoint systems, with two transmitter delivering signals simultaneously to a number of receivers, as in broadcasting applications. As the transmitter constraints do not depend on receivers, they are the same as previously. However, extra receivers introduce additional constraints. All relations are linear, and the feasibility polygon can be represented on a plane. Although the polygon has more vertices (because of a larger number of constraining inequalities), the polygon it is convex as previously. The convexity property is an important feature, specific to linear programming problems (Figure 13).

Figure 13. A region is convex if a line segment connecting any two points in the region lies entirely in the region. Only the region A is convex. Region C is a set of discrete points. Convexity ensures that any local maximum of a linear objective function is a global maximum.

As discussed in the previous section, the optimum plan coincides with one of the vertices of the feasibility polygon. To evaluate the number of vertices, note that each realizability constraint defines 2 lines that may be the borders of the feasibility polygon, and each compatibility constraint another 2 such lines. With n transmitters and m receivers, there is $l = 2(m + n)$ border lines, and two such crossing lines are needed to create one vertex. If no two lines are parallel, the number of crossings of any two lines is $\{(l!) / [2!\,(l - 2)!]\} = [l!(l - 1) / 2]$ $l^2/2$ for large networks. It should be noted, however, that not all lines are actual edges (borders) of the polygon and not all crossings are its actual vertices. Some lines may cross far away from the polygon. The relation, shown in Figure 14, gives only an estimation of the upper bound of possible number of vertices of the feasibility polygon. Anyway, the number of vertices of the feasible polygon - that is potentially optimal plans - is large even for modest networks, and a systematic optimization method to select the best plan is needed.

Figure 14. Upper bound of the number of the feasibility polygon's vertices versus the number of receivers (n) and transmitters (m).

Box: Optimization

Optimization is the determination of design parameters subject to constraints that yield the best performance of the system in hand. The following necessary elements are implicit in any optimization problem:

there are design parameters subject to control

the system performance to be optimized is described in mathematical terms of these parameters

a measure of the performance (good - bad) is selected

the constraints or requirements imposed on the design parameters are explicitly specified.

The descriptive equation for the system performance written in terms of the variable design parameters is called the ***objective function***, the ***criterion function***, or the ***functional***, and the design parameters are called simply the ***variables.***

There are two types of constraint relations that limit the permissible values of the variables. The first type is equalities or functional constraints. The second type is inequalities or regional constraints. A necessary condition for optimization with functional constraints is that the number of constraining equalities be less than the number of variable design parameters. If this condition is not met, the problem is over-constrained, since none of the parameters could vary. This requirement does not apply to the number of regional constraints. Describing a problem in this form is not always easy, but is required if mathematical optimization techniques are to be applied.

The technique to be used to solve a given problem depends largely on the problem itself, i.e. on the type, form, and complexity of the objective function and constraints that apply. In certain optimization problems, there are no functional constraints and regional constraints can be ignored. In such a case, the optimum parameter for the objective function can often be determined using differential calculus to find optimum i.e. maximum or minimum of the function. When functional constraints must be considered, the Langrangian multiplier technique may be useful for finding optimum parameter values. For linear objective functions with linear constraints, there are several techniques, collectively called linear programming (Box 6). Many problems are not amenable to solutions by the techniques previously mentioned. Systematic search methods can be effective tools in some instances of this kind.

The usual question in optimization is how much time is required to find the solution. The question of uncertainty pervades all numerical methods and controls the number of evaluations computed for a given problem. The interval of uncertainty and the problem of local optima can present serious difficulties. [The box ends here]

Multiple transmitters, multiple receivers

This section deals with general case of multiple transmitters and multiple receivers sharing a common frequency band. We saw earlier that two transmitters generated a two-dimensional space, suitable for geometrical interpretation on a plane. We saw also that the edges of the feasibility region were lines, or one-dimensional spaces. Similarly, three transmitters involve three decision variables, or a three-dimensional space. Inequalities in three variables correspond to half-spaces defined by planes in three-dimensional space. In such a space, the feasible region is no longer a polygon but becomes a three-dimensional convex solid, or ***polytope***, such as shown in Figure 15. The faces of such a polytope are convex polygons in two-dimensional spaces. As previously, maximum and minimum values of linear objective function lie at the vertices of the polytope.

Figure 15. A feasibility region in 3-dimensional space - a crystal-like solid structure, or polytope. Each point of that region corresponds to a particular value of total power radiated. As the total power and constraints are linear functions, the polytope is convex and the minimum value lies at one of its vertices

Box 5. Optimal power control - mathematical formulation

Optimization of the power assignment plan is formulated here as a problem of linear programming [Bock et al.]. A number of efficient algorithms for solving such problems are available (see, for instance, [Cottingham 72, Gass 64], as well as computer programs implementing them [Press et al.]. (*Note: Index i is reserved for the receiving stations that are numbered from 1 to m. Index j is reserved for the transmitting stations that are numbered from 1 to n.*)

The objective function F to be minimized, is $F = \sum_{j=1}^{n} P_j$,

where P_j is the power to be assigned to the j-th transmitting station.

For *j-th* transmitter, the realizability constraints are $0 \le PMin_j \le P_j \le PMax_j$,

where *Pmaxj* and *Pminj* are the maximum and minimum allowable values of power to be assigned.

Power transmission factors t_{ij} (which, when multiplied by the transmitter power P_j gives the effective power at the input of *i-th* receiver) are: $[t_{ij}] = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1m} \\ t_{21} & t_{22} & \cdots & t_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ t_{n1} & t_{n2} & \cdots & t_{nm} \end{bmatrix}$.

Desirability factors d_{ij} are $[d_{ij}] = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ d_{n1} & d_{n2} & \cdots & d_{nm} \end{bmatrix}$.

They differentiate between the wanted and the unwanted transmissions. If the signal from *j-th* transmitter is desired at *i-th* receiver, $d_{ij} = 1$, otherwise $d_{ij} = 0$.
The signal power C_i, from the wanted transmitter at the input of *i-th* receiving station is:

$$C_i = \sum_{j=1}^{n} d_{ij} t_{ij} X_j .$$

The total power of signals I_i from all the unwanted transmitters (to which power is being assigned), at the input of *i-th* receiving station, amounts $I_i = \sum_{j=1}^{n} (1 - d_{ij}) t_{ij} X_j$.

For *i-th* receiver, the EMC constraints type 1 [C / (I + N) Q] are

$$\sum_{j=1}^{n} [d_{ij}(1+Q_i) - Q_i]_{ij} X_j \ge Q_i N_i ,$$

where Q_i denotes the minimal value of signal-to-noise-plus-interference ratio acceptable at the input of the receiver. Coefficients d_{ij}, Q_i, t_{ij} and N_i are positive numbers, or zero.

For *i-th* receiver, the EMC constraints type 2 are $\sum_{j=1}^{n} t_{ij} X_j \le E_i$.

E_i is the maximum acceptable power of wanted and unwanted signals at the input of the receiver. Coefficients E_i are positive numbers, or zero.[The box ends here]

With each additional transmitter, the number of decision variables increases. A system with ***n*** transmitting and ***m*** receiving stations involves ***n*** decision variables and ***2(n+m)*** regional constraints. The geometrical interpretation of a problem with ***n*** decision variables remains valid but it becomes more difficult to visualize the n-dimensional polytope formed by the intersections of (n-1)-dimensional hyperplanes. Vertices, however, retain their special status. With linear objective function, the optimum solution must lie at one of the vertices of the polytope. However, visual inspection is impossible and the solution is to be determined by algebraic methods.

One of the most popular such algorithms is the simplex method, introduced by Georg Dantzig in 1947. In this method, an optimum solution is to be found by moving from one vertex of the feasibility region to another along the edges of a polytope like the one shown earlier. As mentioned earlier, all plans represented by a point within the polytope, on its surface, or by its vertex, are feasible, whereas those represented by a point outside the polytope are unfeasible. With linear constraints, the polytope is convex, and the minimum and maximum values of linear objective function must lie at one of vertices of the polytope. The simplex algorithm examines the vertices selectively, one after other, along the edges of the polytope. At each step along the path, the value of the plan is improved and the process is continued until the optimum is reached. Computer codes for the simplex method have been published, for instance, by Press et al. [Press et al., 1989].

Linear Programming

Linear programming is a mathematical field of study introduced in relation to military operations research dealing with optimization of linear problems. It was developed shortly after World War II, in response to logistic problems of those times. One of the earliest publications on linear programming was devoted to the operation of the 1948 Berlin Airlift. It should be noted, however, the term 'programming' has been used here in the sense of planning, and not computer programming, although the computer has become an indispensable tool here.

'Linear' refers to proportional, or linear, relationships between the variables involved in the problems solved. Linear programming has been developed into a separate branch of applied mathematics and attracted a great deal of attention because of both theoretical and practical interest. It is concerned with the explicit formulation and quantitative analysis of optimization problems.

Problems in linear programming are concerned with the optimum allocation of scarce resources, obeying linear restrictions, and under the proportionality and divisibility conditions, as in the problem of power assignment problem. Usually, the most difficult is expressing a practical set of circumstances in mathematical terms of linear programming. It is not a trivial task, and neither is the collection and organization of the data describing the circumstances.

There are several linear programming algorithms, the most popular being the simplex algorithm. It is iterative algebraic technique that can lead directly to the optimum solution, if such one exists. When variables are integer numbers, the linear programming is called linear integer programming. Other computational methods also exist to find a maximum or minimum of functions, and computer programs implementing them have been available. One of them, called the method of simulated annealing, has recently attracted significant attention as suitable for optimization problems of vary large scale and with local extrema. The method uses analogy with thermodynamics, specifically with the way that metals cool and anneal. A computer code for that method has also been published by Press et al. [Press et al., 1989]. It seems, however, it offers no special gain over the simplex method as the problem is linear, i.e. without local extrema.

Even with relatively small number of transmitters, the feasible region can have an enormous number of vertices. However, methods of linear programming require only a very small fraction of them to be visited to find the solution. Every linear programming problem with **n** variables and **m** constraints has a corresponding *dual problem* with **m** variables and **n** constraints. Sometimes, solving the original, or *primal,* problem and dual problem together can be computationally useful. Problems, involving thousands of decision variables and several thousands of constraints can often be solved in a few minutes on fast computers.

Very large problems can be reduced. One possible approach is to apply the rule of representation. Instead of considering each receiver individually, one can deal with the representatives, in the same way as the members of the Parliament represent social groups in a democratic society. Another approach is to decompose the original (large) problem into two (or more) smaller sub-problems that can be processed separately and that give the same result. However, adequate description of a large system, and collection and organization of a large number of the necessary data might be a serious problem in itself.

Conclusion

Power radiated by a transmitter controls the strength of wanted and unwanted signals. It determines the system performances and spectrum space used. Its importance is comparable to that of the operating frequency of the transmitter. Power control can be used in conjunction with, and independently from, other techniques to achieve better utilization of the spectrum; to limit interference potential inside and outside the system; and to maximize the number of transmitters operating simultaneously over a given area. Optimum power control complements rational frequency assignment, use of directional and adaptive antennas, orthogonal modulations, and other techniques.

The method described in the paper can be applied to a wide class of problems. The transmitters and receivers not necessarily have to be of the same type, or to operate in the same service, and co-channel operation of links is not essential. The approach can be used also in self-adaptive systems and real-time spectrum management systems to improve the use of the frequency spectrum resource and electromagnetic compatibility of the systems involved.

The paper described how to assign optimally power to transmitters in a wireless communication system that contains a number of radio links through application of methods of linear programming. No restrictions were imposed on the system location, frequency, modulation, etc., including satellite radio links. The links may be intended or unintended and may interfere each other. 'System' is understood here as a collection of emitters and receptors of electromagnetic waves carrying wanted and unwanted signals. There is not necessarily any functional relationship between the transmitters and receivers.

The aim of optimization is to minimize the volume occupied in a multidimensional space by the system, under the constraints resulting from the required performances and physical realizability of the system. The performances involve the communication quality, availability and/or reliability of each intended radio link, expressed in terms of signal-to-noise-and-interference ratio and the total power at the receiver input. The radiated power is adjusted in such a way that the performance of the poorest system involved is nevertheless as good as required. The solution, if exists, is the best one and no better power assignment plan is possible with the criterion and constraints imposed.

As the area denied by the transmitter normally increases with the power radiated, the minimum power means minimum geographic space denied to other systems. The minimum space denied means in turn the maximum number of transmitters operating over a given area. In view of introduction of spectrum market where the operator pays for the spectrum and space used, the issue takes a significant economic dimension. The minimum of the power radiated means also minimal power consumption, an important aspects in many applications that also has a cost impact.

The discussion was focused on point-to-point and point-to-multipoint communications. The conclusions, however, can easily be extended to broadcasting or point-to-area systems. For that purpose, a number of test-points can be selected, each representing a small element of the surface area. The paper can serve as an example of how mathematical methods can be applied to solve practical problems of radiocommunications.

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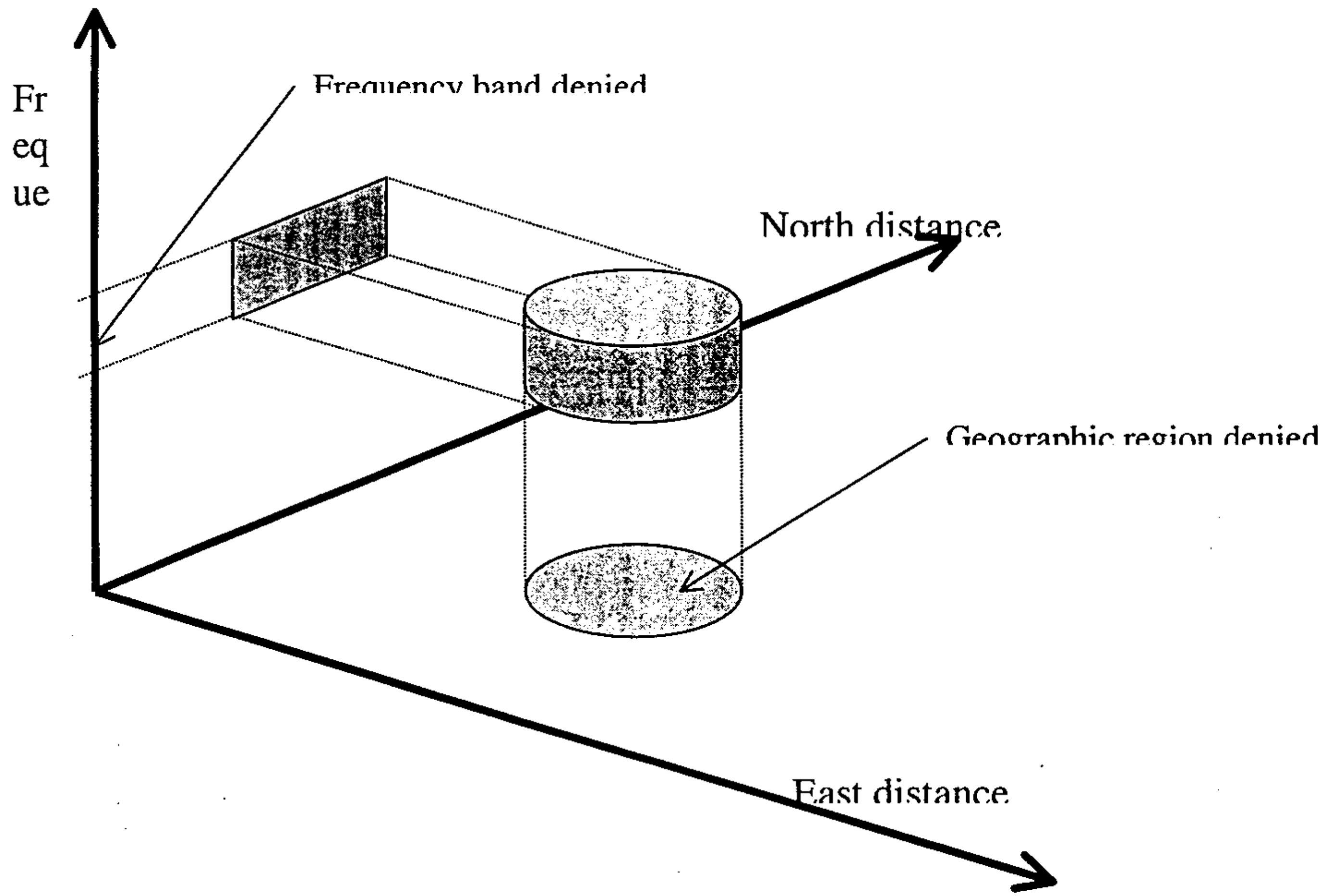
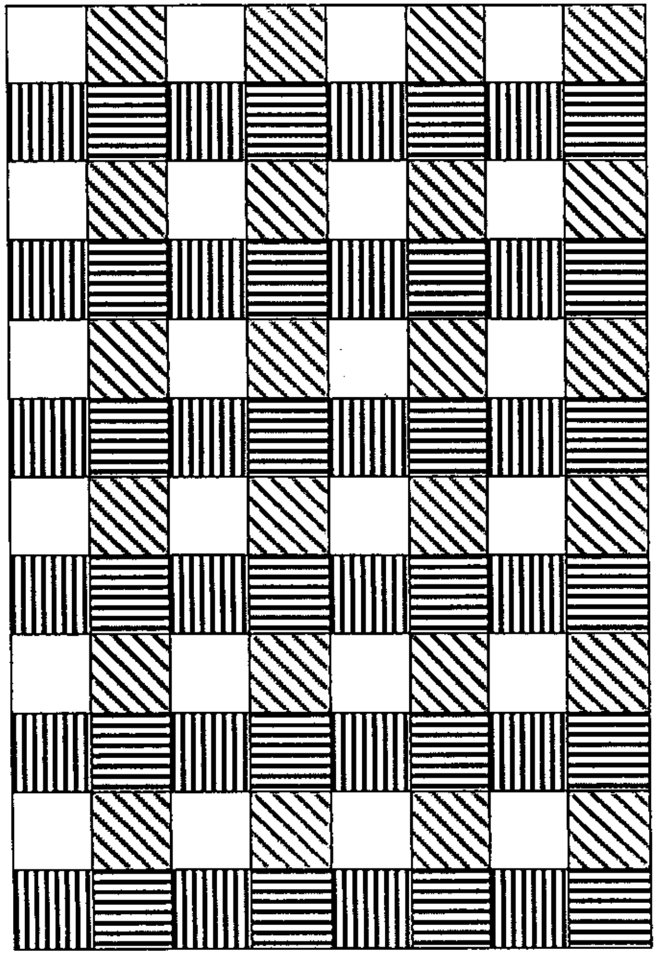
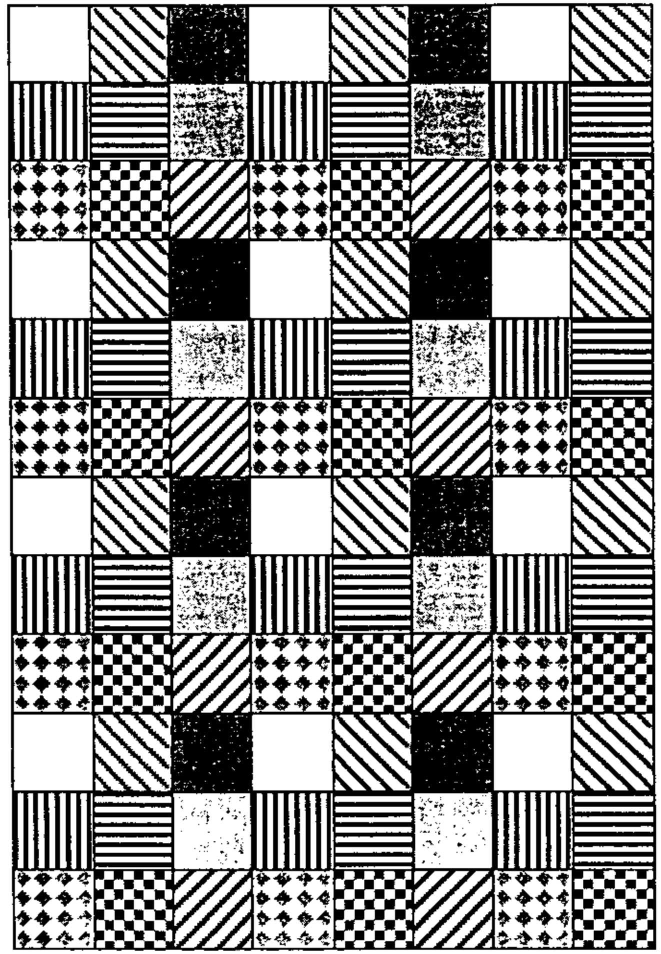
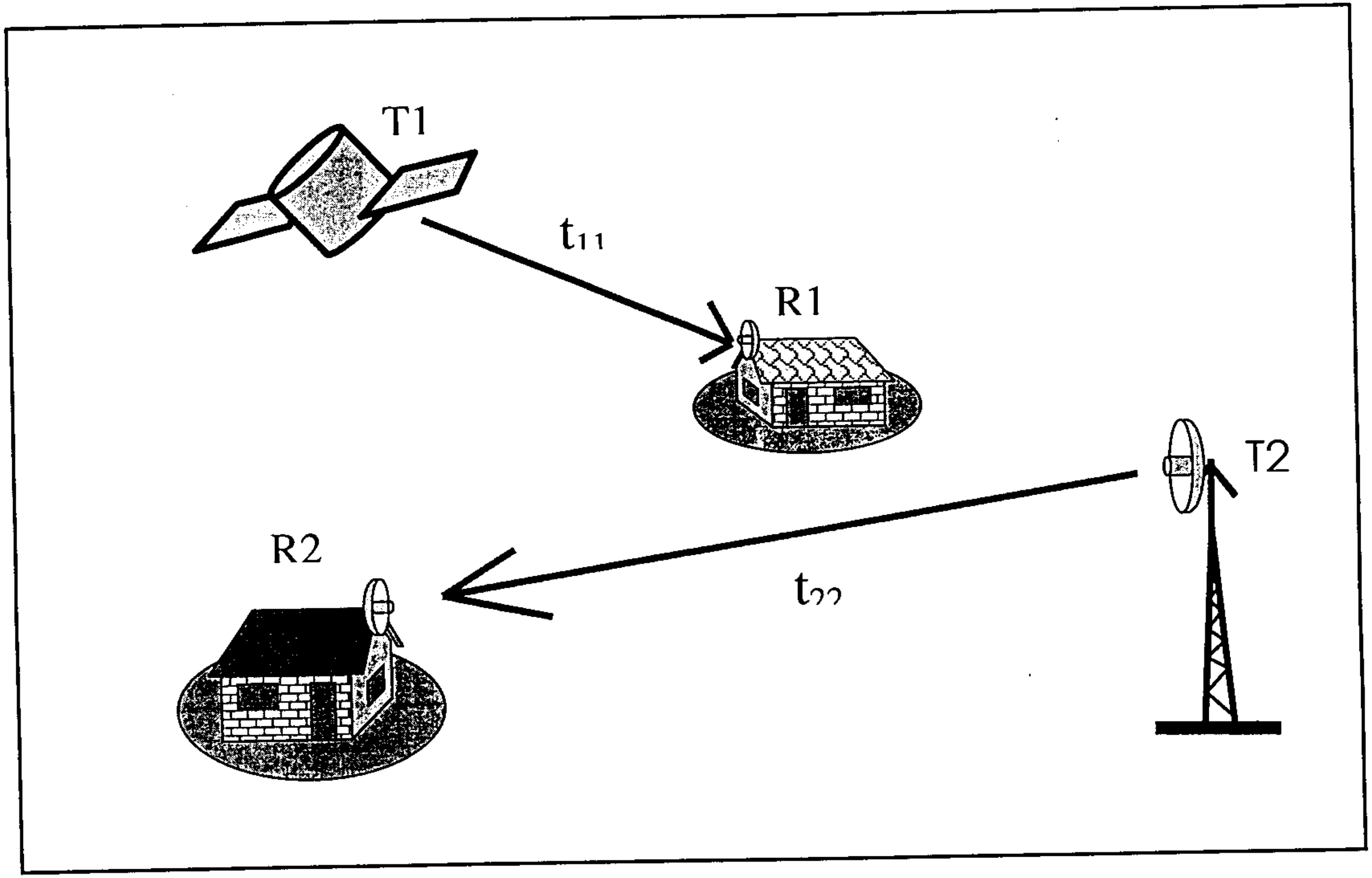

Fr
eq
ue
Frequency band denied
North distance
Geographic region denied
East distance

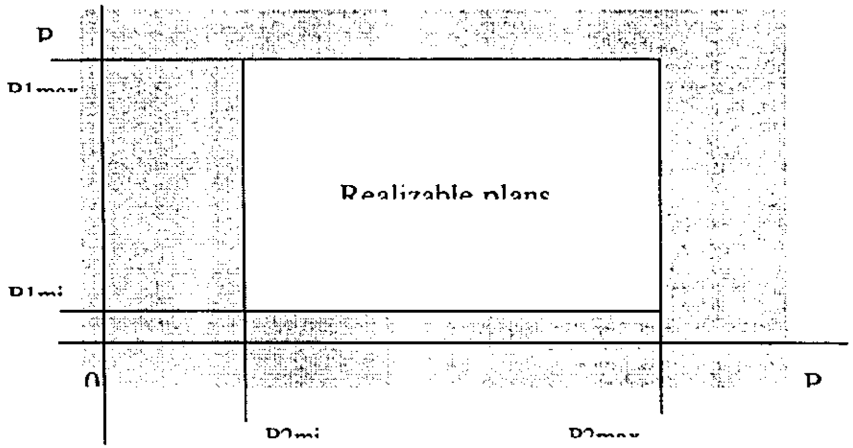
A



B




T1
t11
R1
T2
R2
t22

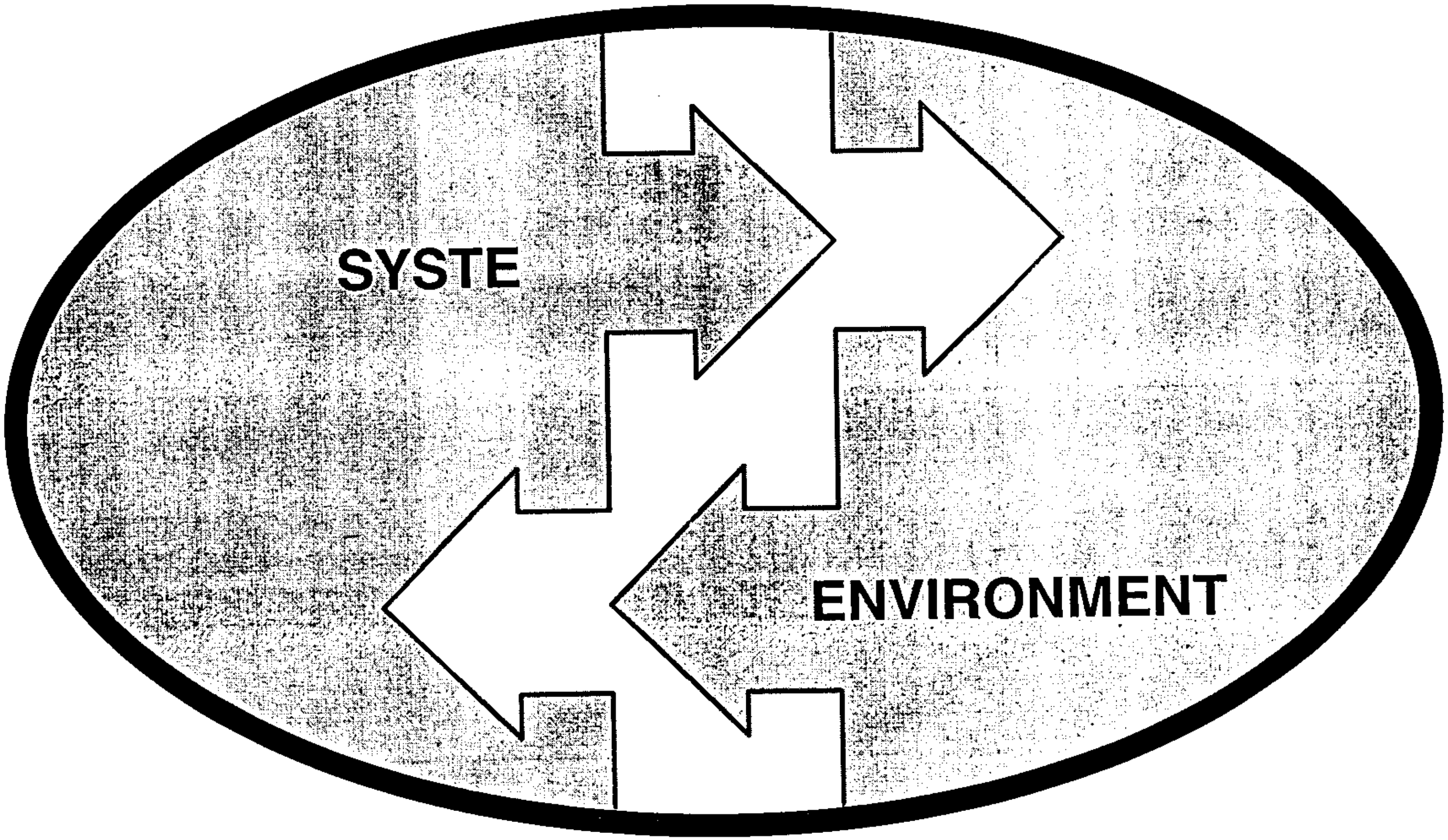


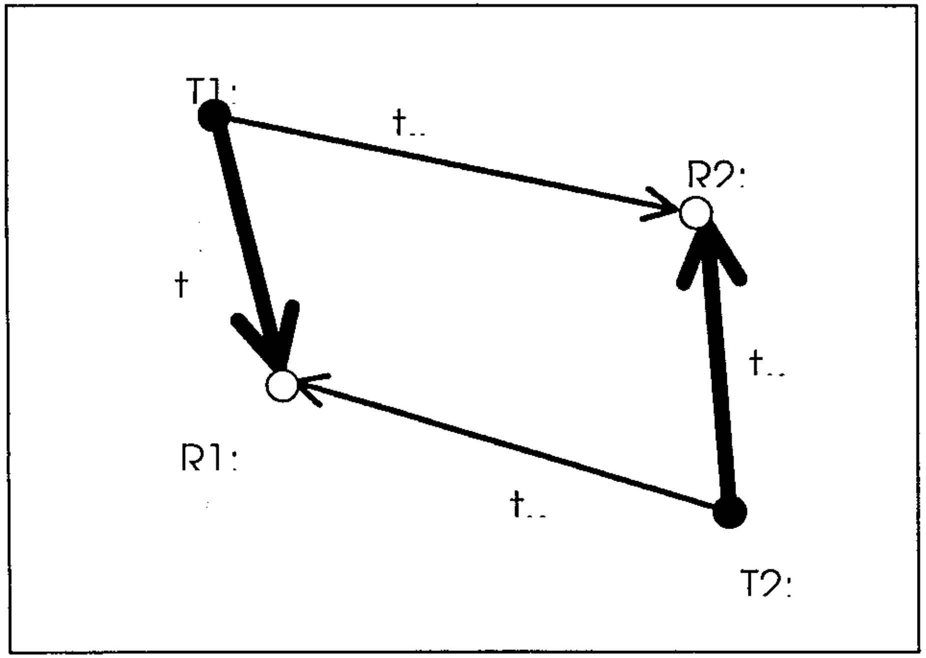
Realizability constraints

$P1 \geq P1min$

$P1 \leq P1max$

$0 \leq P1min \leq P1max < \infty$


SYSTE
ENVIRONMENT



Designed links

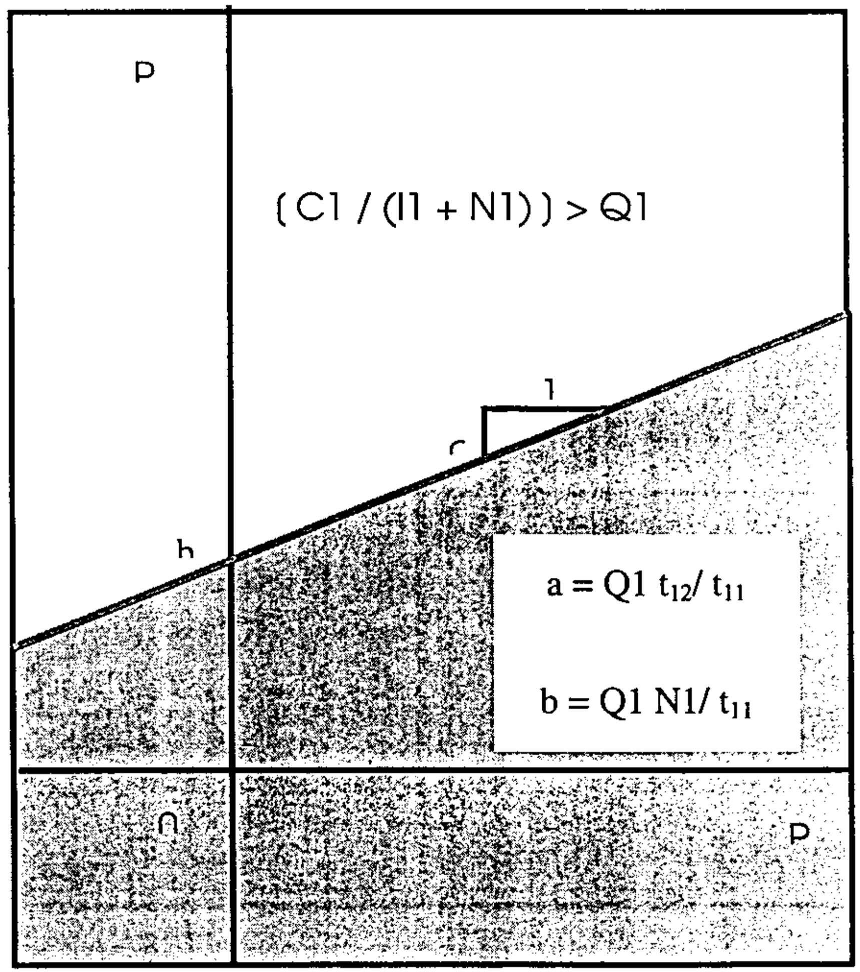
T1→R1 & T2→R2

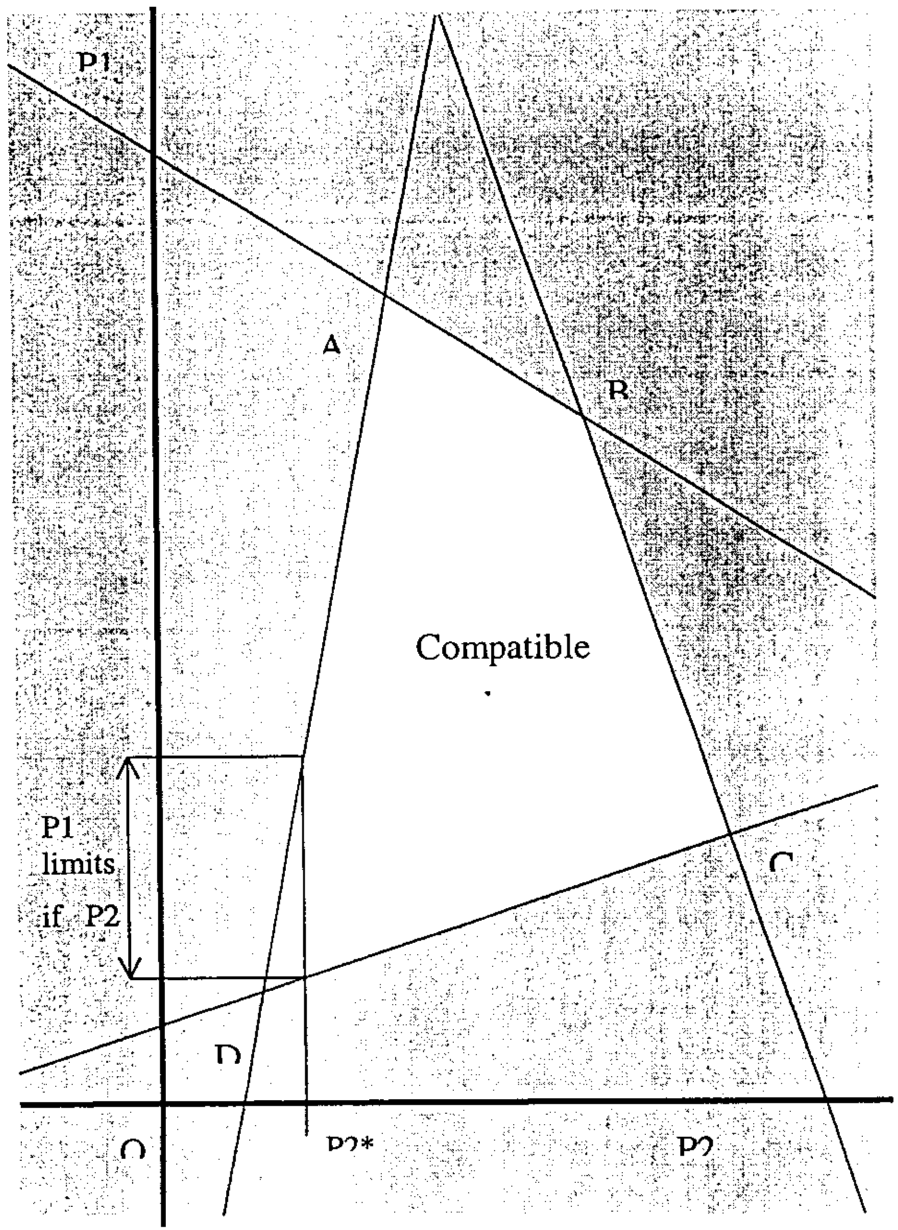
Wanted signals

C1 = t_{11}P1 & C2 = t_{22} P2

Spurious links

T1→R2 & T2→R1


p
(C1 / (I1 + N1)) > Q1
1
r
h
a = Q1 t12/ t11
b = Q1 N1/ t11
n
p



Compatibility constraints

A-B: $C1 + I1 = [t_{11}P1 + t_{12}P2] \leq E1$

B-C: $C2 + I2 = [t_{22}P2 + t_{21}P1] \leq E2$

C-D: $C1 / (I1 + N1) = [(t_{11}P1) / (t_{12}P2 + N1)] \geq Q1$

D-A: $C2 / (I2 + N2) = [(t_{22}P2) / (t_{21}P1 + N2)] \geq Q2$

Crossings OP1 ($P2 = 0$)

A-B: $P1 = E1 / t_{11}$

B-C: $P1 = E2 / t_{21}$

C-D: $P1 = Q1.N1 / t_{11}$

D-A: $P1 = (-N2) / t_{21}$

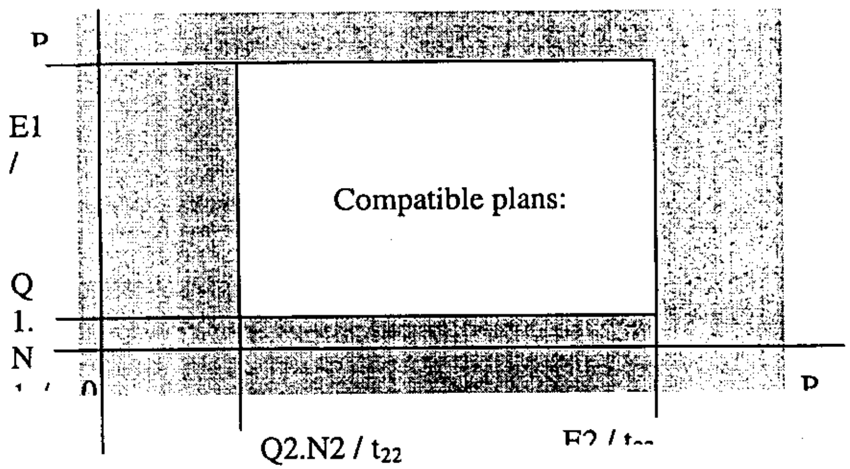
Crossings OP2 ($P1 = 0$)

A-B: $P2 = E1 / t_{12}$

B-C: $P2 = E2 / t_{22}$

C-D: $P2 = (-N1) / t_{12}$

D-A: $P2 = Q2.N2 / t_{22}$



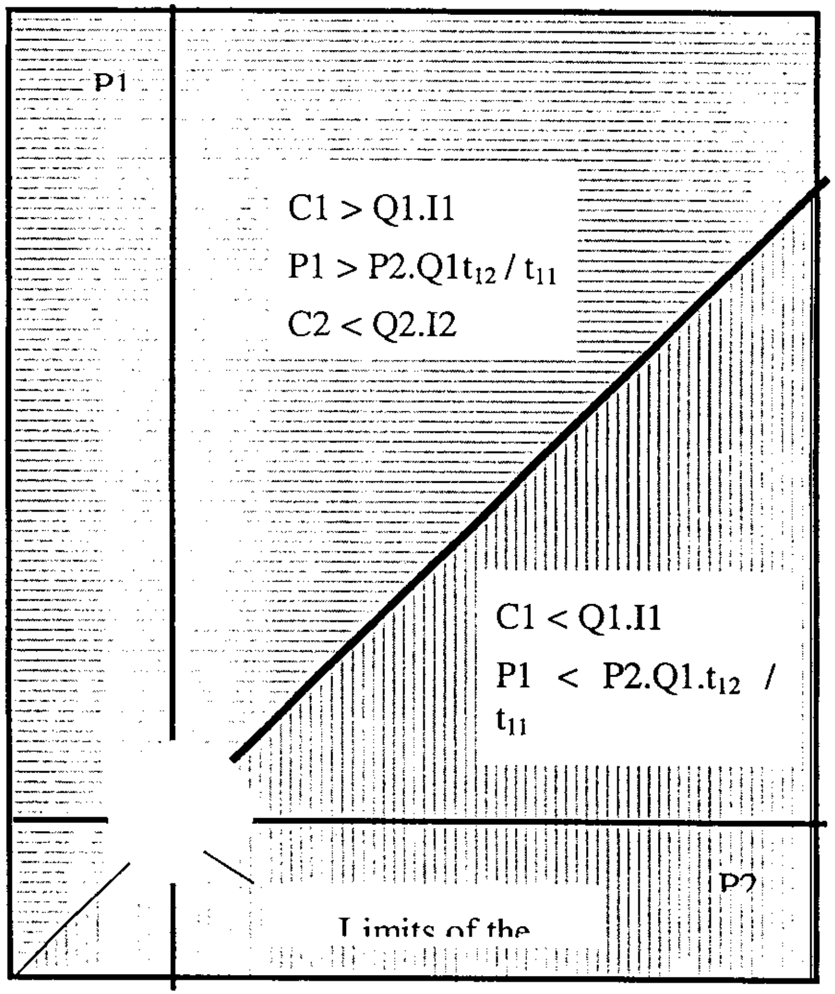
Weak interactions

$P1 << E1 / t_{12}$, $P2 << E2 / t_{21}$

$P1 << N1 / t_{11}$, $P2 << N2 / t_{22}$

Compatibility constraints

$P1 \leq E1 / t_{11}$



Strong interactions

P1 >> N2 / t21

P2 >> N1 / t12

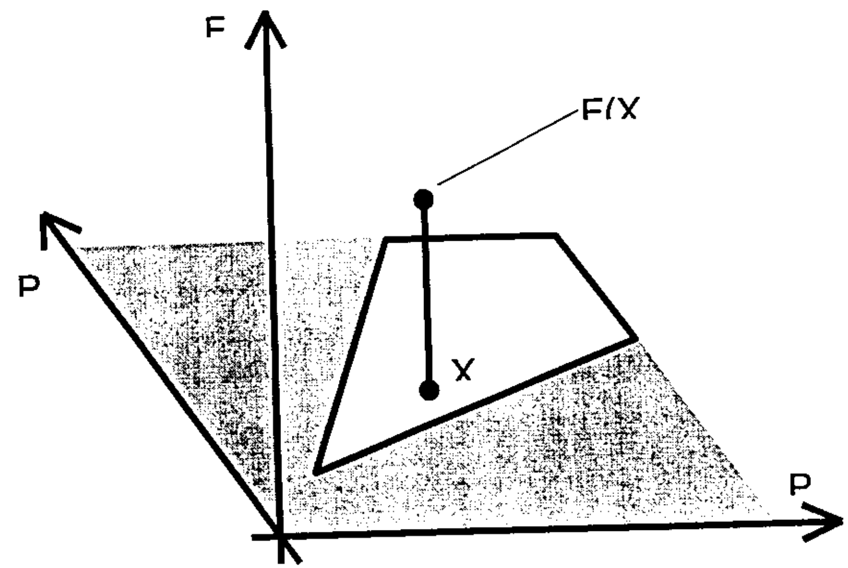
Compatibility constraints

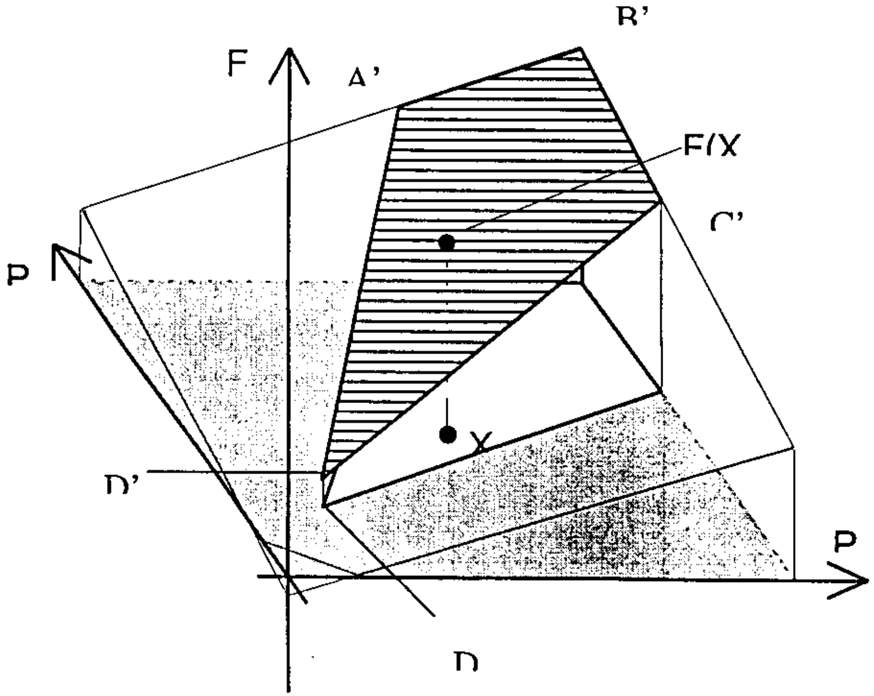
$P1 / P2 \rightarrow \sqrt{(Q1/Q2)}.\sqrt{(t_{12}t_{22} / t_{11}t_{21})}$

$Q1Q2 \rightarrow t_{11}t_{22} / t_{12}t_{21}$

If $Q1 = Q2 = Q$ then $Q = \sqrt{(t_{11}t_{22} / t_{12}t_{21})}$

Feasible region degenerated into line


F
F(X
P
X
P



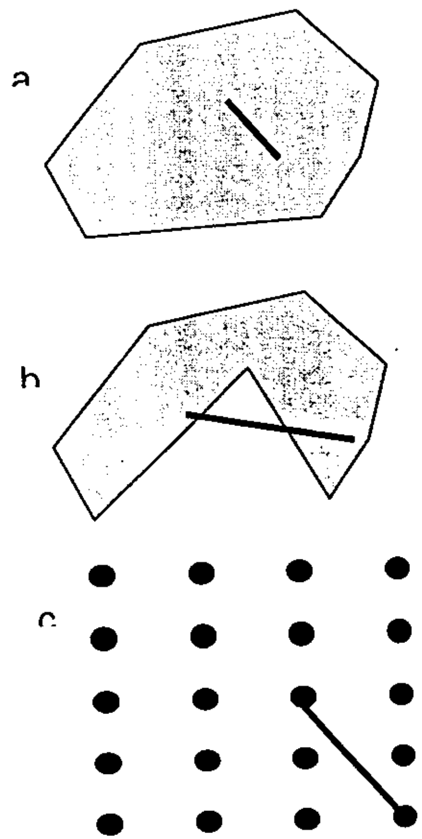
Optimal plan: D

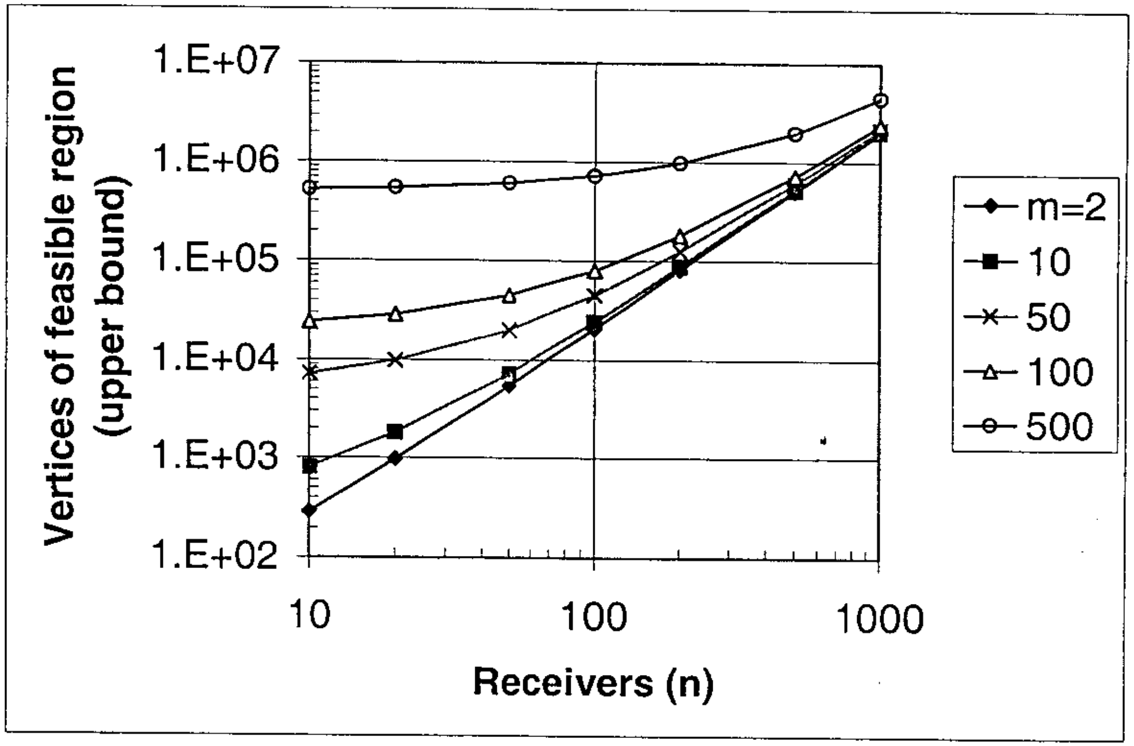
$P1min = Q1\ (Q2.N2.t_{12} + N1.t_{22})\ /\ M$

$P2min = Q2\ (Q1.N1.t_{21} + N2.t_{11})\ /\ M$

$M = (t_{11}.t_{22} - Q1.Q2.t_{12}.t_{21})$

min (F) = P1min + P2min


a
b
c


Vertices of feasible region
(upper bound)
1.E+07
1.E+06
1.E+05
1.E+04
1.E+03
1.E+02
10
100
1000
Receivers (n)
m=2
10
50
100
500