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COLLEGE ON MEDICAL PHYSICS
AND
WORKSHOP ON
NUCLEAR DATA FOR SCIENCE AND TECHNOLOGY:
MEDICAL APPLICATIONS
(20 SEPTEMBER - 15 OCTOBER 1999)

"Overview of Nuclear Reaction Models Used in Nuclear Data Evaluation" Parts I & II

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OVERVIEW OF NUCLEAR REACTION MODELS USED IN NUCLEAR DATA EVALUATION

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IAEA-NDS

AGENDA

- Introduction
- Optical model
- Direct reaction models
- Preequilibrium models
  - exciton model
  - hybrid model
  - quantum models: Multistep Direct & Compound

AGENDA cont.

- Statistical model
- Connections among nuclear models
- Some codes

Why nuclear models?

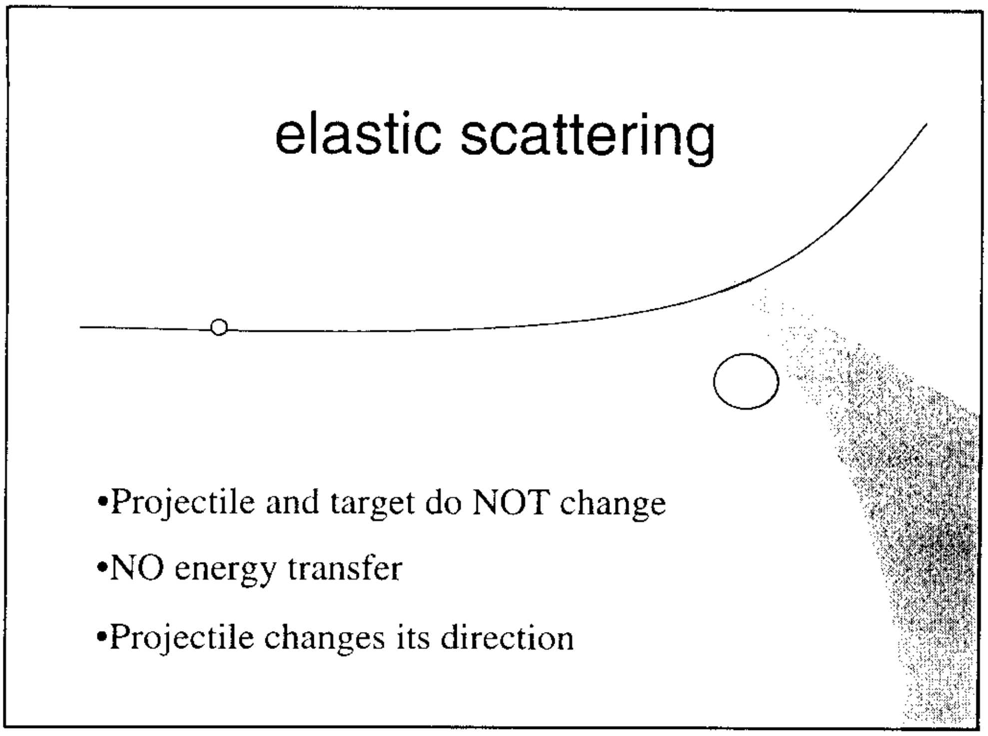
- Provide data if experimental data are not available
- Fill gaps in experimental data
- Discriminate between discrepant measurements
- Reveal wrong data
- Ensure internal consistency of evaluation

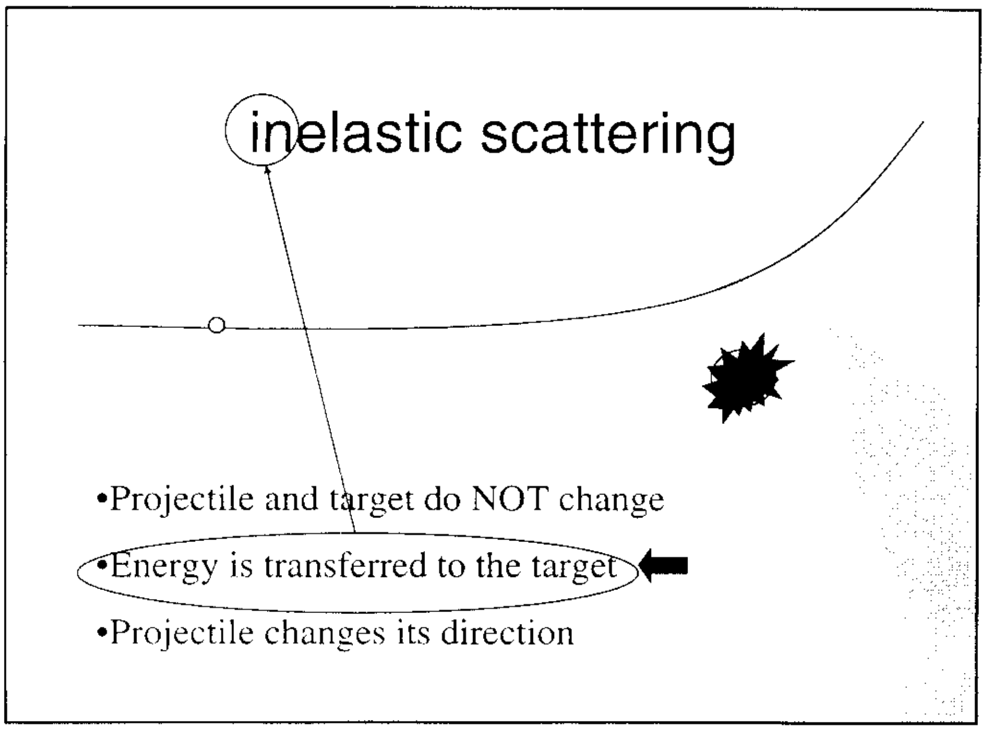
Two rules of evaluation methodology

- Never trust a **single** experiment!
- If **two** experiments agree - they **both** might be wrong!

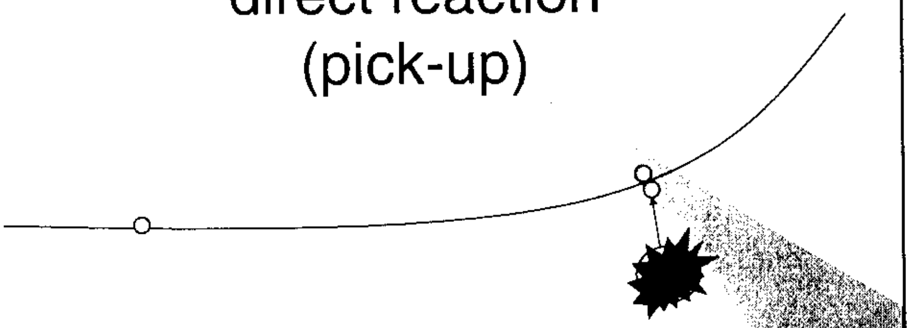
INTRODUCTION

- What happens when a projectile strikes a target?
  - can be elastically scattered
  - can be inelastically scattered
  - can induce direct reaction
  - can be absorbed (composite nucleus is formed)


elastic scattering
•Projectile and target do NOT change
•NO energy transfer
•Projectile changes its direction

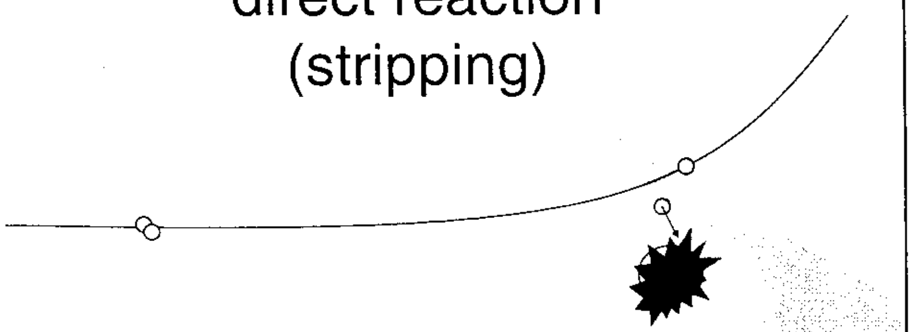

inelastic scattering
•Projectile and target do NOT change
•Energy is transferred to the target
•Projectile changes its direction

direct reaction (pick-up)



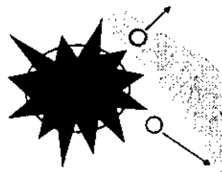
- Projectile and target do DO change
- Energy is transferred to the target
- Projectile becomes ejectile and changes its direction

direct reaction (stripping)



- Projectile and target do DO change
- Energy is transferred to the target
- Projectile becomes ejectile and changes its direction

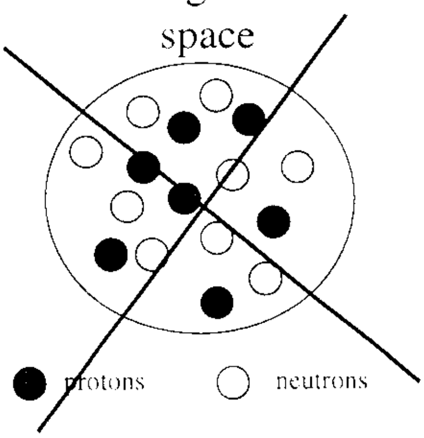
composite nucleus formation
(Compound Nucleus & Preequilibrium)



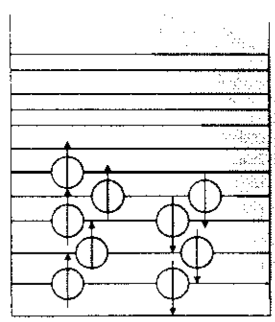
- Projectile disappears
- Energy is transferred to the target
- Ejectiles are emitted

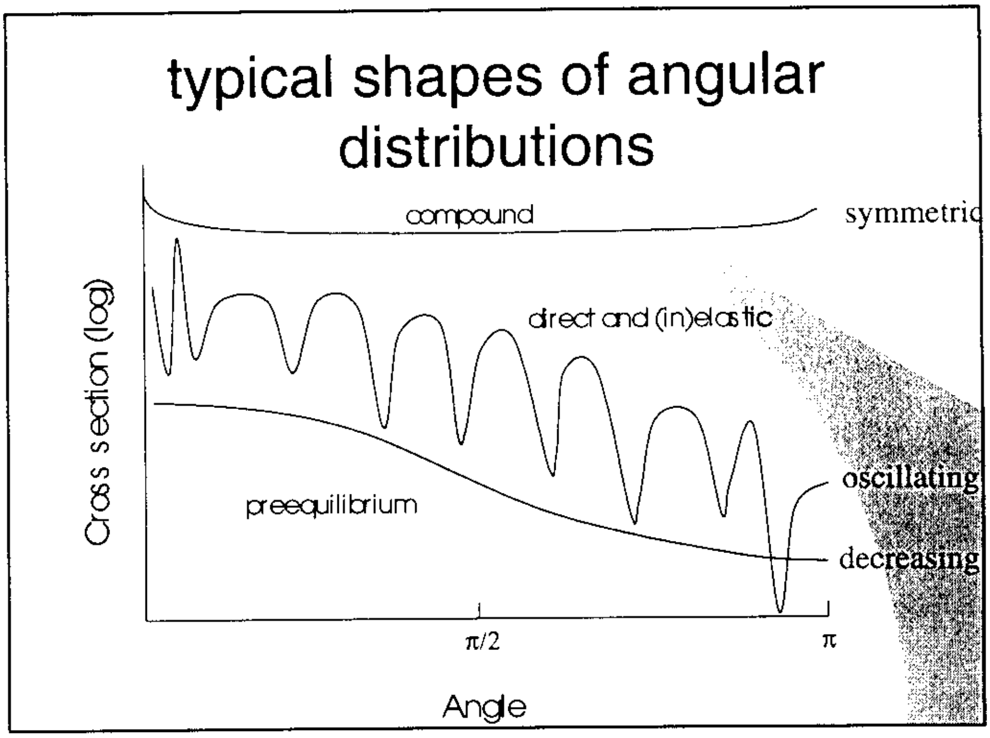
representation of the nucleus

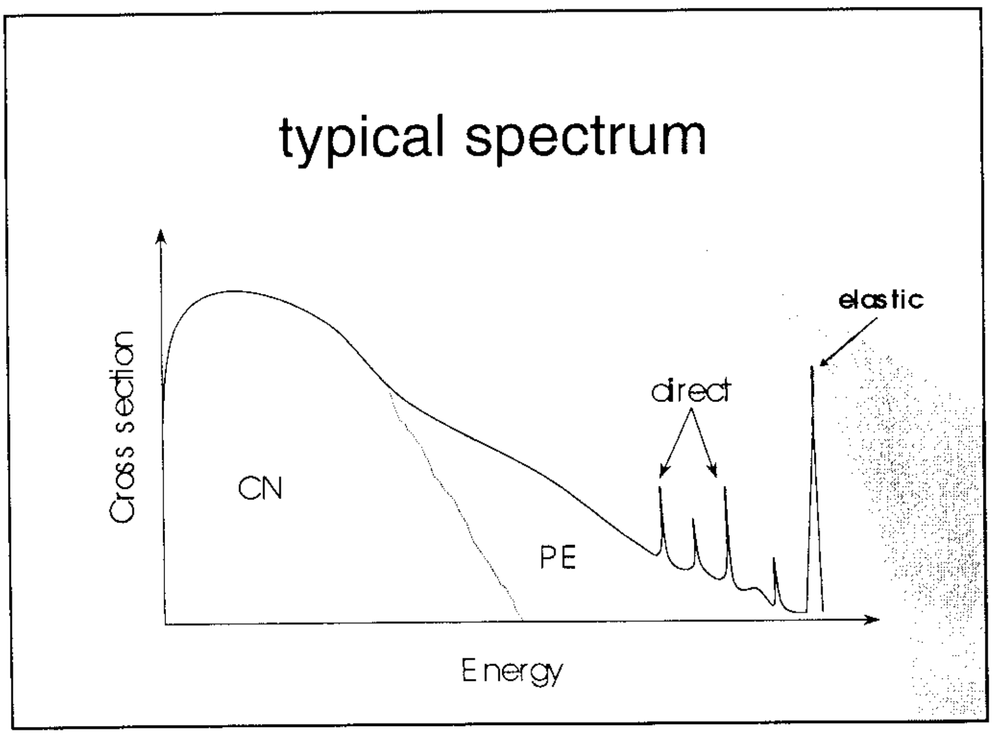
Configuration space



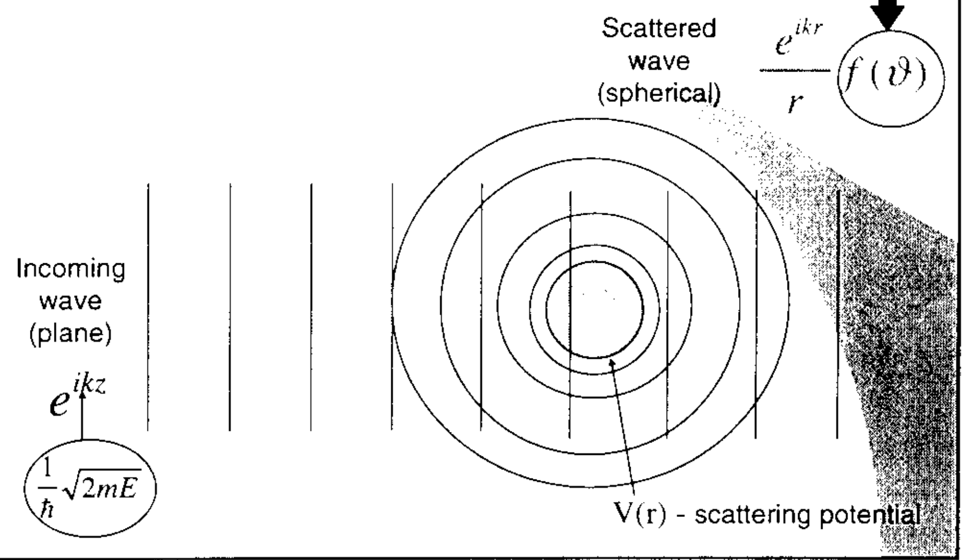
Phase space (shell model)



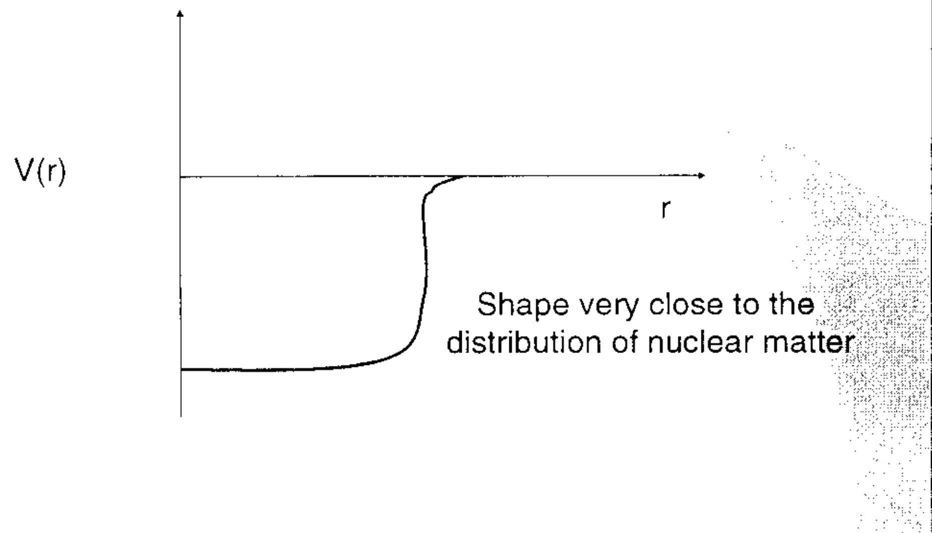

typical shapes of angular distributions
compound
symmetric
direct and (in)elastic
oscillating
preequilibrium
decreasing
Cross section (log)
π/2
π
Angle


typical spectrum
elastic
direct
CN
PE
Cross section
Energy

WAVE SCATTERING



scattering potential



Schroedinger equation (spinless and neutral particle)

$$\nabla^2\Psi + \frac{2m}{\hbar^2}(E - V)\Psi = 0$$ Schroedinger equation

$$\Psi \cong e^{ikz} + \frac{e^{ikr}}{r} f(\vartheta)$$ asymptotic wave function

$$\frac{d\sigma}{d\Omega} = |f(\vartheta)|^2$$ differential cross section

partial wave expansion

$$\Psi(\mathbf{r}) = \sum_l \frac{U_l(r)}{r} P_l(\cos\vartheta)$$

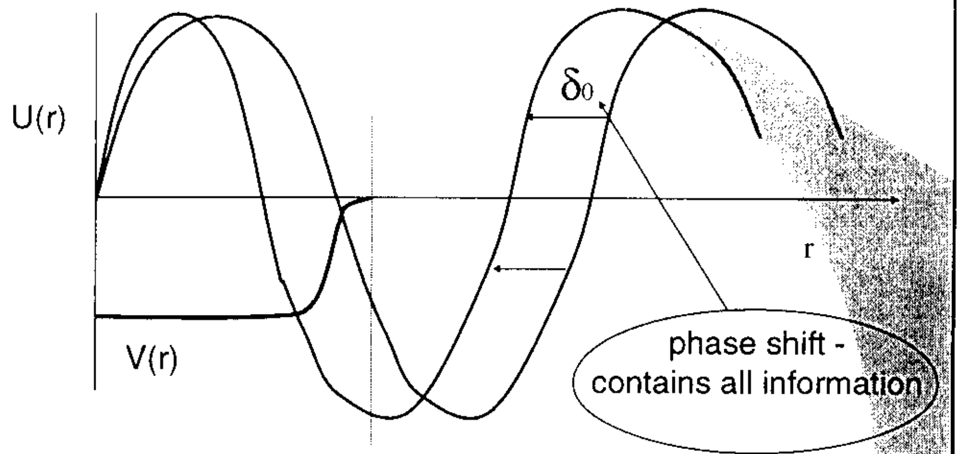
angular momentum

Schroedinger equation can be reduced to

$$\frac{d^2U_l(r)}{d^2r} + \left\{\frac{2m}{\hbar^2}\left(E - V(r)\right) + \frac{l(l+1)}{r^2}\right\}U_l(r) = 0$$

1= 0 case

graphical representation (radial wave function)



elastic cross section

$$f(\vartheta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1)\left(e^{2i\partial_l} - 1\right) P_l(\cos\vartheta)$$

$$\frac{d\sigma_E}{d\vartheta} = \left| f(\vartheta) \right|^2$$

Integrating over ϑ

$$\sigma_E = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \partial_l$$

S-function

OPTICAL MODEL
(introducing absorption)

- Analog of light scattering and absorption by a cloudy crystal ball

Light scattering: complex refractive index

Nuclear scattering: complex scattering potential

- Imaginary potential removes flux from the elastic channel (simulation of absorption)

Optical Model potential

$U_{opt}(r) =$

$+V_C(r)$	a Coulomb term
$-Vf_V(r)$	a real volume term
$+V_s g_V(r)$	a real surface term
$+iW_s g_W(r)$	an imaginary surface term
$-iW_V f_W(r)$	an imaginary volume term
$+d_{so}\vec{l}\bullet\vec{s}V_{so}h_{V_{so}}(r)$	a real spin orbit term
$+id_{so}\vec{l}\bullet\vec{s}W_{so}h_{W_{so}}(r)$	an imaginary spin orbit term

Coulomb potential (uniformly charged sphere)

Charge radius

$$V_C(r) = \left(\frac{3}{2} - \frac{r^2}{2R_C}\right) Z_p Z_T e^2 \Big/ R_C \qquad for \quad r \le R_C$$

$$= Z_p Z_T e^2 \Big/ r \qquad for \quad r > R_C$$

projectile charge

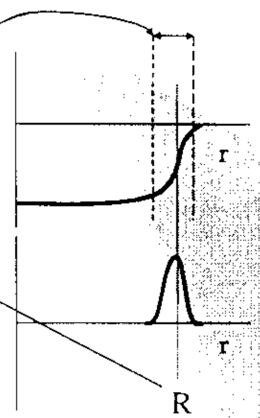
target charge

Optical potential shape

Volume $f_i = \dfrac{1}{1+\exp\left(\dfrac{r-R_i}{a_i}\right)}$ $\quad i = V, W$

Surface $g_i = -4a_i \dfrac{d}{dr} f_i(r)$ $\quad i = V, W$

Spin-orbit $h_i = -\dfrac{1}{r}\dfrac{d}{dr} f_i(r)$ $\quad i = V_{so}, W_{so}$



Optical potential parameters

- Depths: V, V_s, V_{so}, W_s, W_v, W_{so}
- Radii: $R_i = r_i A^{1/3}$
- diffusivities: a_i
- these are fitted to reproduce experimental results (elastic scattering)
- global parameters => valid for a range of energies and nuclei

Optical model (how do we proceed?)

- Fix optical model potential
- Solve Schroedinger equation (numerically!) to obtain phase shifts (S-matrix elements)
- calculate physical observables from phase shifts (cross sections, angular distributions, polarizations)

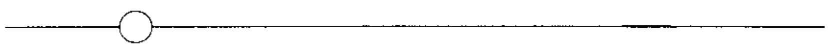
cross section formulae

total $\sigma_{tot} = \frac{2\pi}{k^2}\sum_l (2l+1)\left(1-\operatorname{Re} S_l\right)$

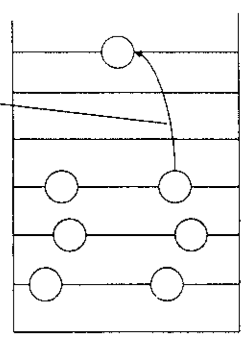
elastic $\sigma_{el} = \frac{\pi}{k^2}\sum_l (2l+1)\left|S_l - 1\right|^2$

reaction $\sigma_r = \frac{\pi}{k^2}\sum_l (2l+1)\left(1-\left|S_l\right|^2\right)$

OPTICAL MODEL (inelastic scattering)

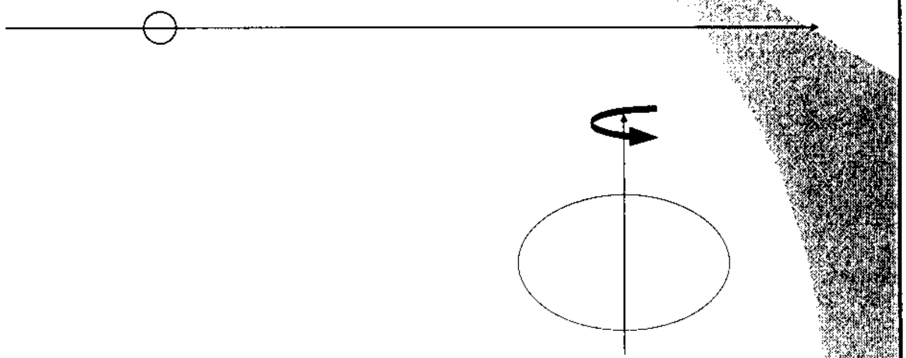


$$M = \left\langle \psi_f \middle| V \middle| \psi_i \right\rangle$$

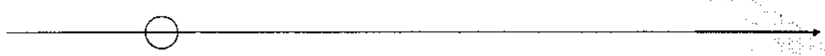


In most cases calculated cross sections are **TOO SMALL !**

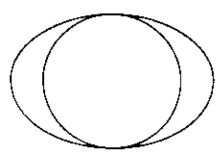
inelastic scattering to collective states (rotation)



inelastic scattering to collective states (vibration)



In both cases many nucleons take part in the excitation - **COLLECTIVE EFFECT** !



generalized (deformed) optical potential

For both rotational and vibrational nuclei the radius can be expressed in terms of spherical harmonics $Y_{\lambda\mu}$

$$R(\hat{r}) = R_0\left(1 + \sum_{\lambda\mu} a_{\lambda\mu} Y_{\lambda\mu}(\hat{r})\right) \qquad \textit{vibrational}$$

angular momentum

vibrational parameters

$$R(\hat{r}) = R_0\left(1 + \sum_{\lambda} \beta_\lambda Y_{\lambda 0}(\hat{r})\right) \qquad \textit{rotational}$$

internal coordinates

deformation parameters

generalized (deformed) optical potential (cont.)

- Potential depends on the distance form nuclear surface
- In first order Taylor expansion:

deformation term

$$U_{opt}(r,\hat{r}) = U_{opt}(r - R(\hat{r})) =$$

$$= U_{opt}(r) + R_0 \frac{dU_{opt}(r)}{dr} \sum_\lambda \beta_\lambda Y_{\lambda 0}$$

spherical o.m.p.

for rotational nuclei and similar for the vibrational ones

Key ingredient

Distorted wave Born approximation (DWBA)

Lippmann-Schwinger equation (integral representation of the Schroedinger equation)

$$\Psi_J(r) = \Psi^+_{0J}(r) + \int_0^\infty dr' G^+_{0J}(r,r') U'_J(r') \Psi_{0J}(r')$$

incoming wave

1-st order DWBA

Green function

$$G^+_{0J}(r,r') = \frac{1}{(2\pi)^3} \int dk' \frac{\Psi_{0J}(k',r)\Psi^*_{0J}(k',r')}{E - E' + i\varepsilon}$$

DWBA (in practice)

We have to evaluate matrix element

$$M = \langle f | U | i \rangle = \int \chi_f^*(\xi) U(\xi) \chi_i(\xi) d\xi$$

final state wave function

initial state wave function

interaction potential

Cross section is proportional to M^2

What if deformation is LARGE?

COUPLED CHANNELS

$$T = -\frac{\hbar^2}{2m}\nabla_r^2$$

$H(\xi)$

Schroedinger equation

$$\left\{T - V(r,\xi) + H(\xi)\right\}\Psi(r,\xi) = E\Psi(r,\xi)$$

Internal nuclear states

$$H(\xi)\chi_a(\xi) = \varepsilon_a\chi_a(\xi)$$

Coupled Channels (cont.)

Total wave function can be expanded

$$\Psi(r,\xi) = \sum_a \psi_a(r)\chi_a(\xi)$$

Relative motion wave function

Rewrite Schroedinger equation

$$\int \chi_b^*(\xi)\left\{T - V(r,\xi) + H(\xi)\right\}\sum_a \psi_a(r)\chi_a(\xi) = \int \chi_b^*(\xi) E \sum_a \psi_a(r)\chi_a(\xi)$$

orthonormality: $\int \chi_b^*(\xi)\chi_a(\xi)d\xi = \delta_{ab}$

Coupled Channels (cont.)

using orthonormality of internal wave functions

$$\left(T - E + \varepsilon_b\right)\Psi_b(r) = \sum_{a=1}^{N} V_{ba}(r)\Psi_a(r)$$

Set of coupled equations

where

$$V_{ba}(r) = \int \chi_b^*(\xi)V(r,\xi)\chi_a(\xi)d\xi$$

Coupled Channels (interaction potential)

IMPORTANT NOTE:

- if all channels were included in the CC calculations => **NO** imaginary part of the potential would be needed
- practically it is impossible and imaginary optical potential is added to account for the excluded channels
- however, inclusion of some channels leads to the **decreased** imaginary optical potential

Coupled Channels (input required)

- Input channel specification: A_p, Z_p, A_T, Z_T, E_{cm},
- optical model potential parameters
  - depths: V, W, W_s, V_{so}, and W_{so}
  - geometry: reduced radii r_i and diffusivities a_i
- states of the target: energy, spin, parity
- oscillation amplitudes for vibrational or deformation parameters for rotational excited states
  - nature of vibrational states (one phonon, two phonon or mixture of the two)

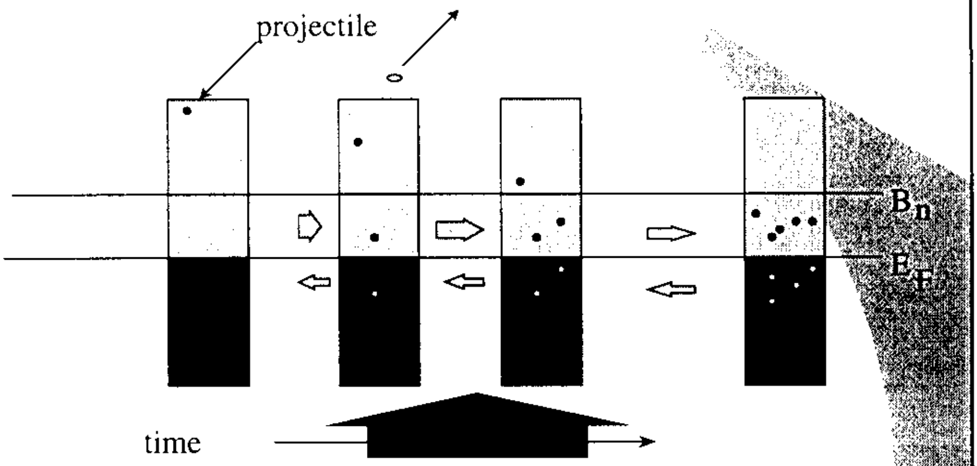
OM (first two items)

CC (last two items)

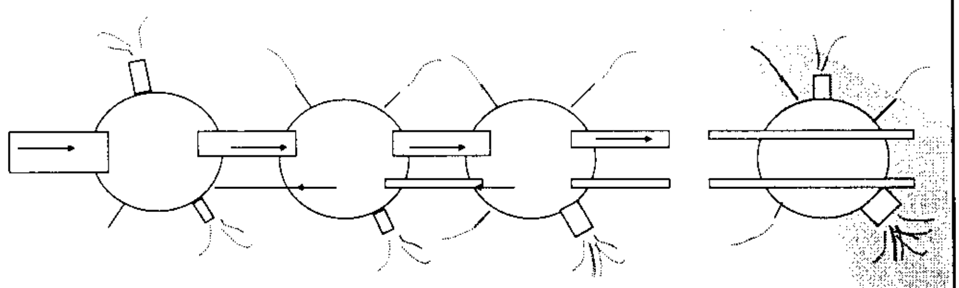
What happens to the absorbed flux?

- Preequilibrium
- Compound Nucleus (Statistical Model)

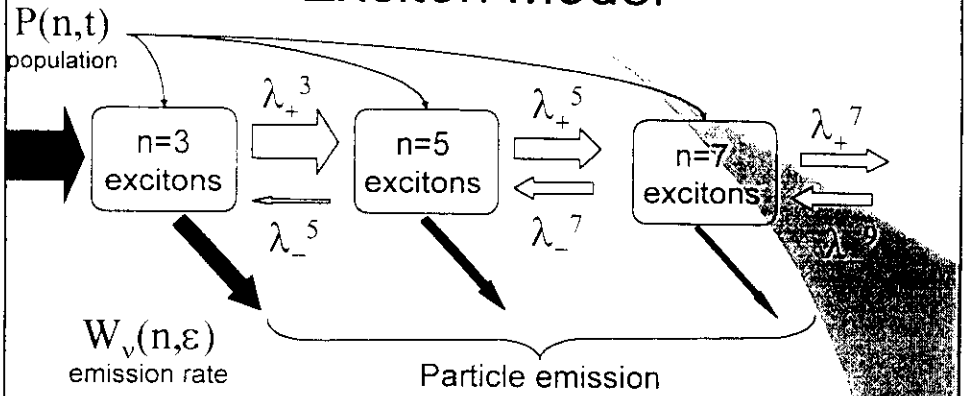
PREEQUILIBRIUM
how does absorption happen?



hydrodynamical analog
(water pumped into a chain of connected tanks)



Exciton Model



$$\frac{d\sigma(\varepsilon)}{d\varepsilon} = \sigma_{abs} \sum_{n}^{\bar{n}} W_\nu(n,\varepsilon) \int_{t=0}^{t_{eq}} P(n,t)\, dt$$

exciton model (cont.)

$$\frac{d\sigma_\nu(\varepsilon)}{d\varepsilon} = \sigma_{abs} \sum_{n}^{\bar{n}} W_\nu(n,\varepsilon) \int_0^\infty P(n,t)dt$$

Probability of particle emission

Emission rate

$$W_\nu(n,\varepsilon) = \frac{2s+1}{\pi^2\hbar^3} \mu\varepsilon\sigma(\varepsilon) \frac{\omega(n-1, E-\varepsilon)}{\omega(n,E)}$$

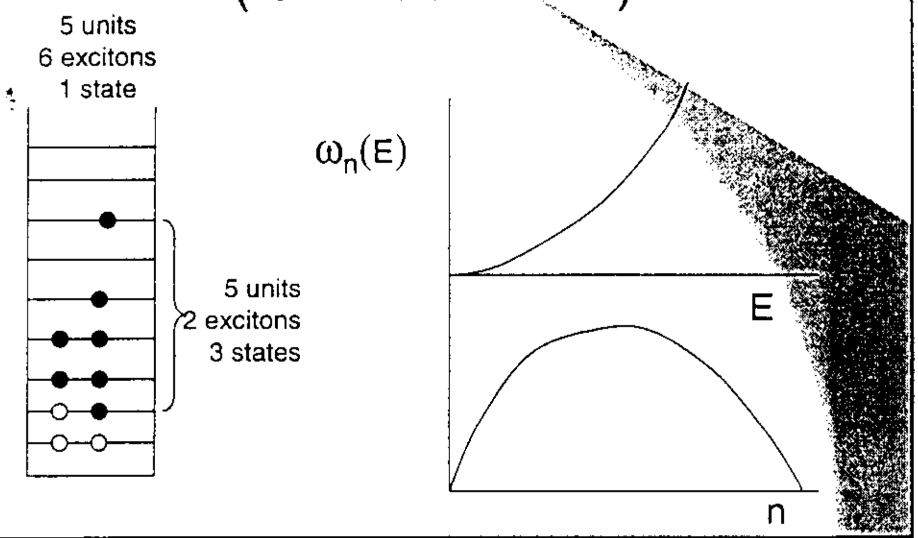
Probability of having an exciton with appropriate energy

Level densities

$$\omega(n,E) = \frac{g(gE)^{n-1}}{p!h!(n-1)!}$$

density of single-particle levels

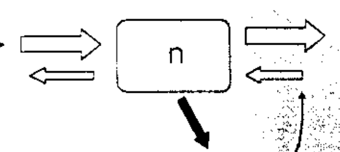
exciton model (level densities)



exciton model (cont.)

$$\frac{d\sigma_\nu(\varepsilon)}{d\varepsilon} = \sigma_{abs} \sum_{n}^{\bar{n}} W_\nu(n,\varepsilon) \int_0^\infty P(n,t)dt$$

Master equation approach



$$\frac{P(n,t)}{dt} = P(n-2,t)\lambda_+(n-2,E)$$
$$+ P(n+2,t)\lambda_-(n+2,E)$$
$$- P(n,t)\left[\lambda_+(n,E) + \lambda_-(n,E) + W(n)\right]$$

exciton model (cont.)

Transition rates

$$\lambda_+(n,E) = \frac{\pi}{\hbar}|\overline{M}^2|\omega_{acc}^+ = \frac{\pi}{\hbar}|\overline{M}^2|\frac{g^3E^2}{n+1}$$

$$\lambda_-(n,E) = \frac{\pi}{\hbar}|\overline{M}^2|\omega_{acc}^- = \frac{\pi}{\hbar}|\overline{M}^2|gph(n-2)$$

Accessible state density

Square of the averaged matrix element (PARAMETER)

exciton model (cont.)

- Set of Master Equations solved numerically
- Never-come-back approximation ($\lambda_- = 0$) => closed form expression
- Parameters:
  - g - density of single particle states
  - $|M^2|$ - square of matrix element
  - n_0 - initial number of excitons

Hybrid model (M. Blann)

Never come-back approximation

$$\frac{d\sigma(\varepsilon)}{d\varepsilon} = \sigma_{abs} \sum_{n_0}^{\bar{n}} D_n \frac{\omega(n-1, E-\varepsilon)}{\omega(n,E)} \frac{\lambda_n^c(\varepsilon)}{\lambda_n^c(\varepsilon) + \lambda_n^+(E)}$$

Depletion factor

Probability of having an exciton with appropriate energy

Branching ratio for particle emission

λ^+_n proportional to OM imaginary potential depth or to nuclear matter free path

no $|M^2|$ parameter !

Exciton <=> Hybrid

- Similar mathematical form but different physical background!
  - Exciton - strong mixing of internal states
  - Hybrid - no mixing of internal states

⬇

- Exciton
  - exclusive cross sections
  - $n_{i+1} = n_i + 2$
- Hybrid
  - inclusive cross sections
  - $n_{i+1} = 3n_i$

Other classical PE models

- Geometry Dependent Hybrid Model
- Random Walk Model
- Boltzmann Master Equation
- Monte Carlo Preequilibrium Model

Extensions of classical PE models

- Treatment of angular momentum
- Angular distributions
  - leading particle model (Agasi, Weidenmueller and Mantzouranis)
  - linear momentum model (Chadwick and Oblozinsky)
  - Kalbach parameterization
- Two-gas model (neutron-proton distinction)

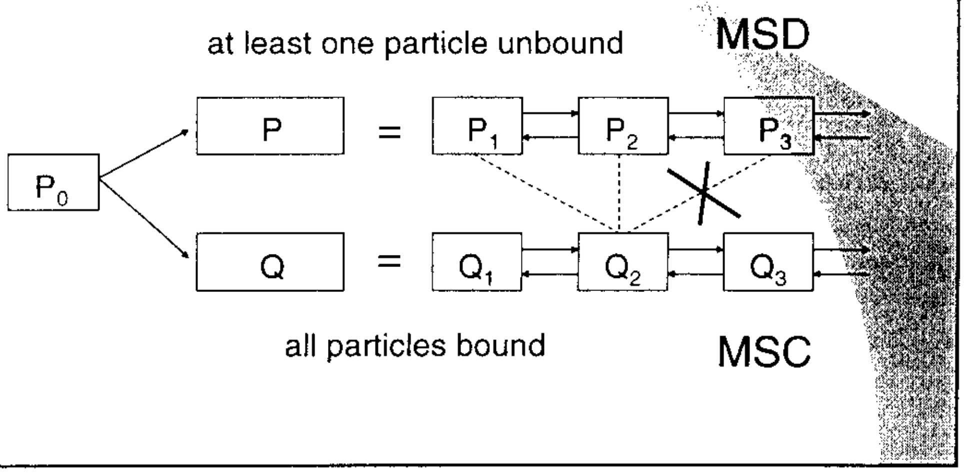
Common features of classical PE models

- Based on semi-classical considerations
- Easy to use (fast!)
- Flexible enough to provide good description of experimental data
- Fail to predict spectra at backward angles
- Ignore nuclear structure and collective effects

Quantum preequilibrium MSD & MSC

- Multistep Direct (MSD)
  - **FKK** - Feshbach, Kerman, Koonin
  - **TUL** - Tamura, Udagawa, Lenske
  - **NWY** - Nishioka, Weidenmueller, Yoshida
- Multistep Compound (MSC)
  - **FKK** - Feshbach, Kerman, Koonin
  - **NVWY** - Nishioka, Verbaarschot, Weidenmueller, Yoshida

Quantum preequilibrium (P & Q spaces)



Quantum preequilibrium (FKK-MSD)

- One-step cross section

$$\left(\frac{d^2\sigma}{dU\,d\Omega}\right)_1 = \sum_L (2L+1)\,\omega(U,L)\left\langle \left(\frac{d^2\sigma}{dU\,d\Omega}\right)_{DWBA}\right\rangle_L$$

- M-step cross section

$$\left(\frac{d^2\sigma}{dU\,d\Omega}\right)_M = \sum_n \sum_m \int \frac{dk_1}{(2\pi)^3}\int \frac{dk_2}{(2\pi)^3}\cdots\int \frac{dk_n}{(2\pi)^3}\,\frac{d^2W_{m,n}(k_f,k_n)}{dU\,d\Omega}$$

$$\times \frac{d^2W_{n,n-1}(k_n,k_{n-1})}{dU_n\,d\Omega_n}\cdots\frac{d^2W_{21}(k_2,k_1)}{dU_2\,d\Omega_2}\left(\frac{d^2\sigma(k_1,k_i)}{dU\,d\Omega}\right)_1$$

Quantum preequilibrium (FKK-MSD cont.)

Transition probability

$$\frac{d^2W_{n,n-1}(k_n,k_{n-1})}{dU_n d\Omega_n} = 2\pi^2 \omega(k_n)\omega_2(U)\left\langle \left| v_{n,n-1}(k_n,k_{n-1}) \right|^2 \right\rangle$$

Density of states of the particle in the continuum

Density of particle-hole states in the residual nucleus at energy U

DWBA matrix element for the transition from a state n-1 to a state n with change of the momentum from k_{n-1} to k_n

Quantum preequilibrium (FKK-MSD - how to do it?)

- Calculate DWBA double differential cross sections for the first step for all possible pairs of initial and final states and angular momentum transfers L
- make average of over these states for each L
- use these averages to define $W_{n,n-1}(k_n,k_{n-1})$
- perform integrals one by one treating the result of the previous integration as a source term for the subsequent one (convolution)

Quantum preequilibrium (FKK-MSD - practical remarks)

- calculations are lengthy but feasible
- standard DWBA codes can be used
- quantum treatment results in a proper reproduction of backward scattering
- due to convolution structure arbitrary number of steps can be considered

Quantum preequilibrium (FKK-MSD - drawback)

- Independent-particle model (no configuration mixing in residual system) => no collective effects
- never come-back approximation
- one free parameter - V_0 interaction strength
- difficult treatment of cluster emission

Quantum preequilibrium (TUL-MSD - highlights)

- configuration mixing in residual nucleus => collective effects (vibrations) taken into account
- non-convolution form => limited to 2 or 3 steps only
- never come-back approximation
- cluster emission and charge exchange reactions difficult to treat
- due to averaged form factors calculations are faster than with FKK

Quantum preequilibrium (FKK-MSC)

- proceeds through Q-space
- chaining hypothesis => transitions between neighboring stages only
- never come-back approximation
- memory of the projectile direction is lost => angular distributions symmetric around 90°

Quantum preequilibrium (FKK-MSC cont.)

Schematic cross section formula

$$\sigma_{MSC} \equiv \left(\frac{2\pi\Gamma_1^{\alpha}}{D_1}\right)\left(\sum_n \frac{\Gamma_1^{\uparrow}}{\Gamma_n}\right)\left(\prod_{k=1}^{k=n-1}\frac{\Gamma_k^{\uparrow}}{\Gamma_k}\right) \qquad \Gamma_n = \Gamma^{\uparrow}_n + \Gamma^{\downarrow}_n$$

Stage-1 formation cross section

Emission probability

Depletion factor

total width

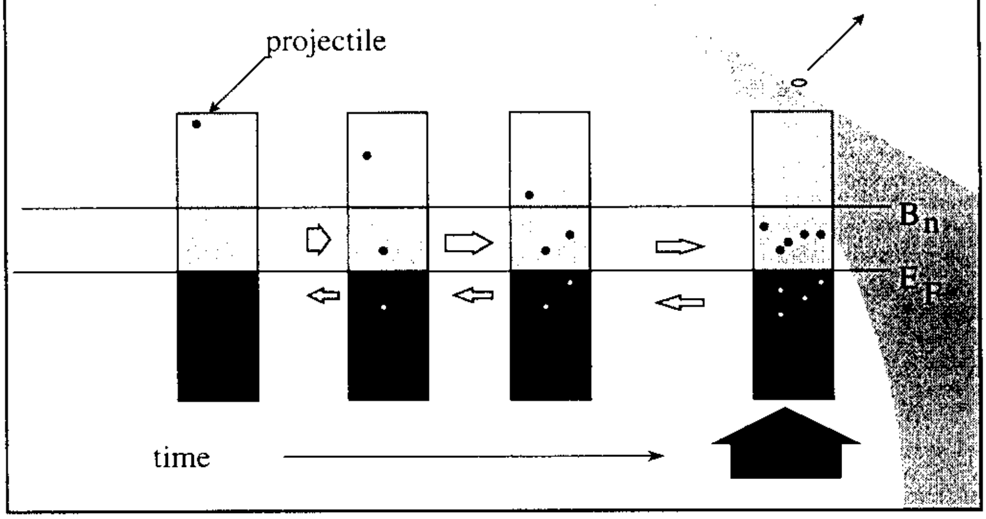
emission width

spreading width

Quantum preequilibrium (FKK-MSC cont.)

- All widths can be expressed through OM transmission coefficients and level densities
- level densities are calculated in the frame of the independent particle model (equidistant) with binding energy limit
- spin coupling is performed strictly
- if OM is used to calculate absorption cross section there is no free parameter in the model (except 'g' and OM parameters)

COMPOUND NUCLEUS
(how is it formed?)



Compound Nucleus features

- long life time (10^{-18} s compared to 10^{-22} s for direct reactions)
- Factorisation of CN formation and decay (Borh's hypothesis)
- Symmetric angular distributions
- Time independence

formal development

Assumptions:

- many open channels
- huge number of resonances
- decay width vary widely
- low energy => reduced phase space => rapid energy dependence => averaged cross sections
- high energy => larger phase space => smooth cross sections => selfaveraging

formal development

S-matrix

$$\Rightarrow \quad < S_{ab}^{fl} >= 0$$

energy averaged cross section

$$< \sigma_{ab} >= \left|\delta_{ab} - < S_{ab} >\right|^2 + <|S_{ab}^{fl}|^2 >=$$
$$\sigma_{ab}^{dir} + \sigma_{ab}^{fl}$$

(kinematical factors neglected)

$<S_{ab}>$ - given by optical model

THE AIM OF THE STATISTICAL THEORY OF NUCLEAR REACTIONS IS TO EXPRESS:

$$<| S_{ab}^{fl} |^2> \quad \text{in terms of} \quad \left| < S_{ab}^{fl} > \right|^2$$

Hauser-Feshbach formula

- transmission coefficient (optical model)

$$T_a = 1 - |< S_{aa} >|^2$$

- unitarity SS⁺=1 (flux conservation) implies

$$T_a = \sum_{b=1}^{\Lambda} \sigma_{ab}^{fl}$$

- Bohr hypothesis: $\sigma_{ab}^{fl} = \xi_a \xi_b \qquad a \neq b$

Hauser-Feshbach formula cont.

using unitarity

$$\sigma^{fl}_{ab} = \frac{T_a T_b}{\sum_c^{\Lambda} T_c}$$ Hauser-Feshbach formula !

However, compound elastic cross section is wrong !

Correlation of the identical entrance and exit channel is not taken into account.

Should be enhanced by factor 2 - 3.

fix to Hauser-Feshbach

several formulae of the type

$$\sigma^{fl}_{ab} = \frac{V_a V_b W_{ab}}{\sum_{c=1}^{\Lambda} V_c}$$

width fluctuation correction (W_{ab})

have been proposed (Moldauer, HRTW).

Relevant at low energies only!

Final solution (Heidelberg triple integral)

$$
\begin{aligned}
&< S_{ab}(E_1) S^*_{cd}(E_2) > = < S_{ab}(E_1) > < S^*_{cd}(E_2) > + \\
&+ \frac{1}{8}\int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda \, \frac{(1-\lambda)\lambda|\lambda_1-\lambda_2|}{((1+\lambda_1)\lambda_1(1+\lambda_2)\lambda_2)^{1/2}(\lambda+\lambda_1)^2(\lambda+\lambda_2)^2} \\
&\times \exp\{-i\pi(E_2^*-E_1)(\lambda_1+\lambda_2+2\lambda)/d\}\prod_{e=1}^{\Lambda}\frac{(1-T_e\lambda)}{((1+T_e\lambda_1)(1+T_e\lambda_2))^{1/2}} \\
&\times \left\{ \begin{array}{l} \delta_{ab}\delta_{cd} < S_{aa} > < S_{cc} > T_a T_c \left[\frac{\lambda_1}{1+T_a\lambda_1}+\frac{\lambda_2}{1+T_a\lambda_2}+\frac{2\lambda}{1-T_a\lambda}\right] \\ \times \left[\frac{\lambda_1}{1+T_c\lambda_1}+\frac{\lambda_2}{1+T_c\lambda_2}+\frac{2\lambda}{1-T_c\lambda}\right] + (\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})T_a T_c \\ \times \left[\frac{\lambda_1(1+\lambda_1)}{(1+T_a\lambda_1)(1+T_b\lambda_1)}+\frac{\lambda_2(1+\lambda_2)}{(1+T_a\lambda_2)(1+T_b\lambda_2)}+\frac{2\lambda(1+\lambda)}{(1+T_a\lambda)(1+T_b\lambda)}\right] \end{array} \right\}
\end{aligned}
$$

application of Hauser-Feshbach formula

$$\sigma_{ab}^{fl} = \frac{T_a T_b}{\sum_c^{\Lambda} T_c}$$

What is a channel?

channel definition

x - particle type
ε - particle energy
l - angular momentum
s, I, J - spin
π - parity

$$a = ((x, \varepsilon, l, s, \pi)(I_T, \pi_T); (J_{CN}, \pi_{CN}))$$

- projectile/ ejectile → $(x, \varepsilon, l, s, \pi)$
- target/ residue → (I_T, π_T)
- Compound Nucleus → (J_{CN}, π_{CN})

coupling

$\vec{s} + \vec{l} + \vec{I}_T = \vec{J}_{CN}$	angular momentum
$\pi \pi_T (-1)^l = \pi_{CN}$	parity
$\varepsilon + B_x = E_{exc}$	energy conservation

experimental channel

incoming channel

$$A = \sum_{l} \sum_{s} \sum_{J_{CN}} \sum_{\pi_{CN}} (x, \varepsilon, l, s, I_T, \pi_T; J_{CN}, \pi_{CN})$$

outgoing channel

$$B = \sum_{\varepsilon} \sum_{l} \sum_{s} \sum_{J_{CN}} \sum_{\pi_{CN}} (x, \varepsilon, l, s, I_R, \pi_R; J_{CN}, \pi_{CN})$$

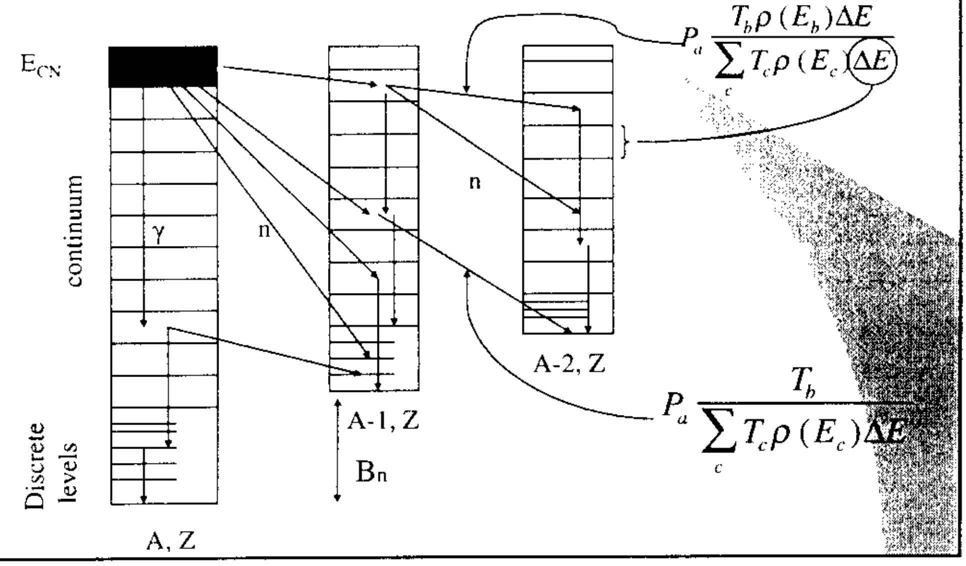
sum over the energy must be dropped if we look for the spectra

moreover:
apart of the discrete region, the levels are so many that we have to use level densities

$$\rho(E_{exc}, J, \pi)$$

and replace sum by the integral over energy.

Compound Nucleus calculation scheme



Compound Nucleus (comments to the figure)

- Spin and parity is not shown, thus the figure refers to the Weisskopf-Ewing model rather than to the Hauser-Feshbach one
- Spin and parity add additional two dimensions to the plot
- Emission of other particles (in addition to neutrons) should be included

Compound Nucleus model parameters

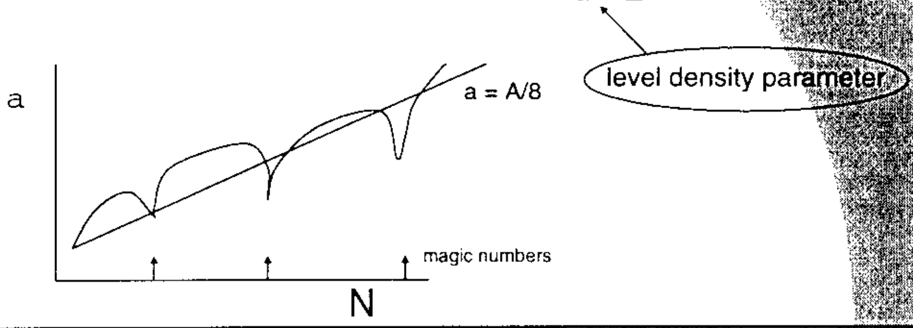
- optical model potential => transmission coefficients
- level densities
- nuclear masses => binding energies
- Giant Multipole Resonance parameters => transmission coefficients for photons
- discrete levels and branching ratios
- fission barriers
- shell corrections => level densities and fission barriers

LEVEL DENSITIES

- describe continuum where nuclear levels are so many that can not be treated individually
- in principle, total level density should be a sum of partial level densities (over all exciton numbers)
- collective effects make them usually much higher than that

level densities (cont.)

- Most striking feature - strong increase with excitation energy
- simplest formula: $\rho(E) \approx \dfrac{\exp\left(2\sqrt{aE}\right)}{a^{1/4}E^{5/4}}$



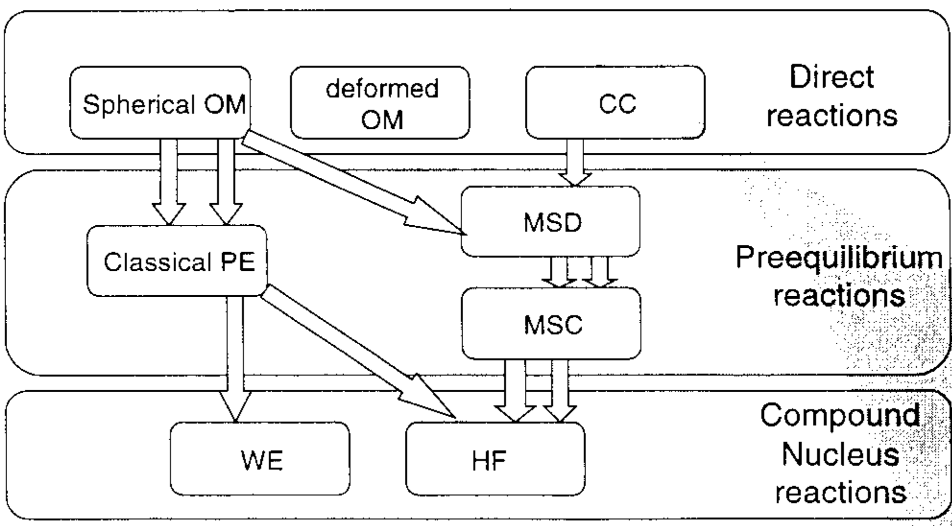
CONNECTIONS BETWEEN NUCLEAR MODELS

- Optical/Direct Model - provide essential ingredients for PE and CN
- Preequilibrium Models - describe CN formation
- Shell Model - provides bases for the formal CN development
- Fermi Gas and BCS Models - provide level densities

connections between nuclear models (cont.)

- Nuclear Structure Models - provide discrete levels, gamma transition probabilities
- Liquid Drop + Shell Model - provide binding energies, shell corrections, fission barriers, and nuclear shape

EVALUATION FLOW-CHART



SOME NUCLEAR REACTION MODEL CODES

- **ECIS** - CEN Saclay (J. Raynal)

  OM, DWBA, and CC, parameter search to fit experimental data, a range of models (e.g. first or second order harmonic or anharmonic vibrational model, symmetric or asymmetric rotational model), statistical model including width fluctuation corrections (Moldauer).

  THE STATE OF ART CODE !

- **DWUCK** - Boulder, Colorado, (P. D.Kunz)

  DWBA, scattering differential cross sections for spin 0, 1/2 or 1 particles.

  Standard DWBA code

- **SCAT-2** - Bruyeres le Chatel (O. Bersillon)

  OM - total cross sections, elastic scattering and angular distributions, and transmission coefficients for a spherical nucleus. Incident particles: neutron, proton, deuteron, ^{3}H, ^{3}He or α.
  Often used for calculation of transmission coefficients for Compound Nucleus

- **ALICE** - LLNL, Livermore (M. Blann)

  Weisskopf-Ewing evaporation, Hybrid model preequilibrium, Monte Carlo preequilibrium (recent version), multiple particle emission, fission, double differential spectra.
  Very useful at high energies, oversimplified at low ones.

- **CASTHY** - JAERI, Tokaj-mura, Japan

OM+HF, Moldauer correction, binary reactions, gamma cascade, postprocessing into ENDF/B format.

Code extensively used for nuclear data evaluations in Japan.

- **HAUSER 5** - Westinghouse (F.M. Mann)

HF+PE+(n,a) pick up, binary and tertiary reactions, fission, double differential cross sections, discrete levels but no gamma cascade.

Designed for calculations up to 50 MeV.

- **TNG** - Oak Ridge (C.Y. Fu)

  OM+HF+PE, Moldauer correction, binary and tertiary reactions, gamma cascade, angular distributions, output in ENDF/B format.
  Designed for calculations up to 20 MeV.

  Code used for nuclear data evaluation.

- **STAPRE** - IRK, Vienna (M. Uhl)

  HF+PE (random walk), Moldauer correction, multiparticle emission (6), gamma cascade, fission.

  Code extensively used in many laboratories.

- **GNASH** - LANL, Los Alamos,
  (P.G. Young, M.B. Chadwick)

  HF+PE+MSC(FKK)+MSD(FKK), Moldauer correction, second chance preequilibrium, multiparticle emission, fission, gamma cascade, double-differential cross sections, discrete levels, variable dimensions, input library, output postprocessing into ENDF/B format. Energy range up to ~200 MeV. The most important code for nuclear data evaluation !

- **EMPIRE II** - IAEA, Vienna (M. Herman)

  OM+HF+MSC(NVWY)+MSD(TUL), multiparticle emission, fission, gamma cascade, double-differential cross sections, discrete levels, neutron or any nucleus as a projectile, dynamical effects, variable dimensions, input library, output postprocessing into ENDF/B format.

  Extremely easy to use, applied in nuclear data evaluations, to be released.

CONCLUSIONS

- Optical model, direct reaction, preequilibrium and statistical theories account for a major part of nuclear reaction
- Theory is in a good shape
- Predictive capability:
  - about 10% for strong reaction channels
  - can be an order of magnitude for weak reaction channels

CONCLUSIONS (cont.)

- Hauser-Feshbach model is a key ingredient but alone it is never enough,
  - at low energies it must be supplemented with width fluctuation correction
  - at incident energies above few MeV preequilibrium emission must be taken into account
  - above ~50 MeV second chance preequilibrium is needed

CONCLUSIONS (cont.)

- reaction channels to collective discrete levels must be treated within DWBA or CC approaches

- New generation of codes is being prepared (EMPIRE -2, new GNASH)
- There is a tendency to use more advanced modeling (CC, MSD&MSC)