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SMR.1148 - 46

COLLEGE ON MEDICAL PHYSICS
AND
WORKSHOP ON
NUCLEAR DATA FOR SCIENCE AND TECHNOLOGY:
MEDICAL APPLICATIONS
(20 SEPTEMBER - 15 OCTOBER 1999)

"Photon and Electron Interactions with Matter"

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Photon and Electron Interactions with Matter

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Workshop on Nuclear Data for Science and Technology
Medical Applications
International Center for Theoretical Physics
Trieste, October 4-15, 1999

Require information on radiation interaction cross sections to determine:

- **conversion quantities used in medical physics (*e.g.*, photon mass energy-absorption coefficient ratios, electron stopping-power ratios)**

- **fate of primary (incident) radiation (*e.g.*, attenuation of photon beams)**

- **production of secondary radiation**

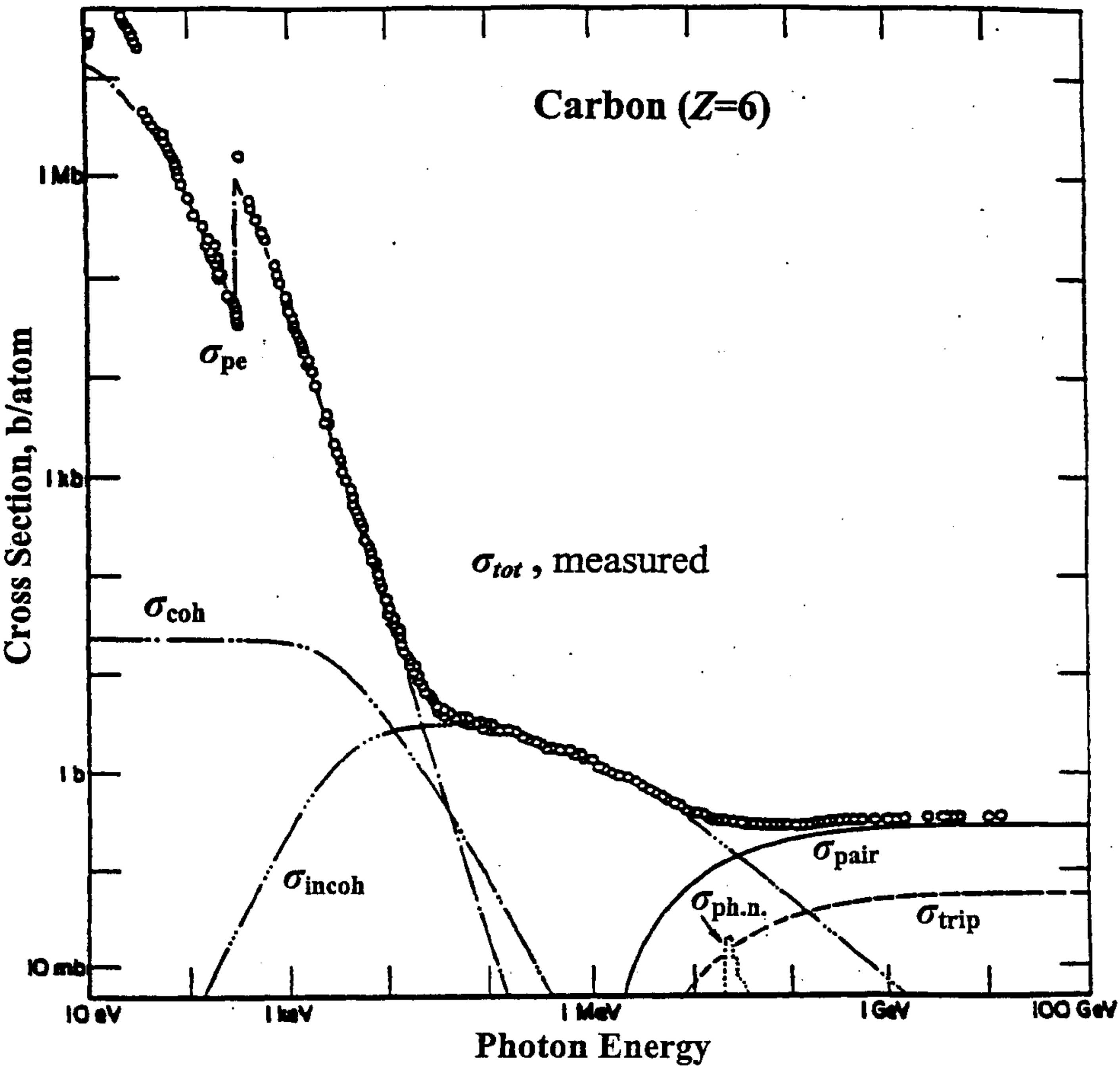
Topics covered:

Photon interaction cross sections
- Elastic (coherent) Rayleigh scattering
- Inelastic (incoherent) Compton scattering
- Photoelectric absorption
- Pair production
- Triplet production
- Total attenuation coefficients
- Energy-transfer coefficients
- Energy-absorption coefficients

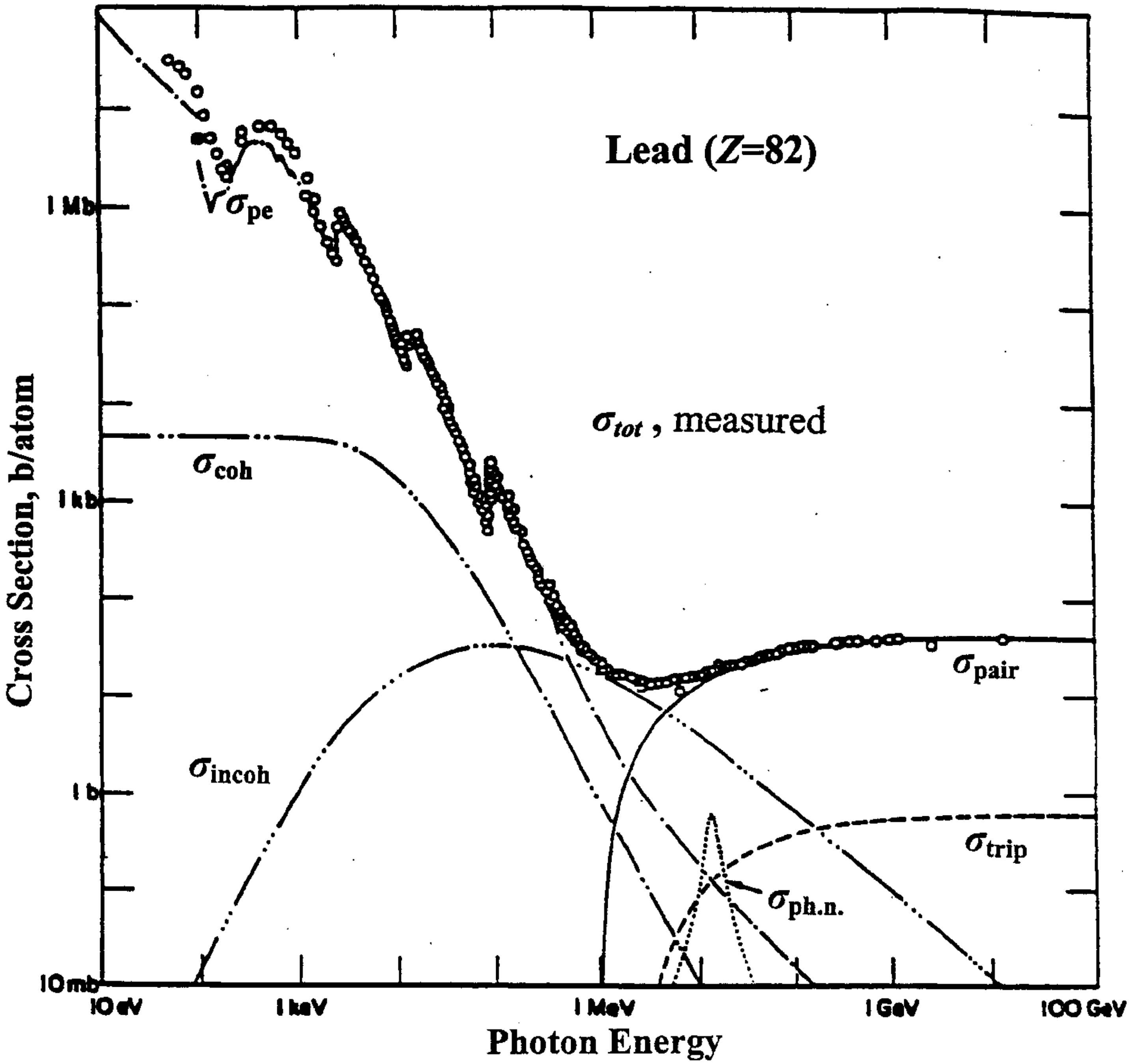
Electron and positron interaction cross sections
- Electronic (collision) stopping power
- Radiative stopping power
- Range
- Bremsstrahlung production
- Elastic scattering
- Ionization

Energies of interest in medical physics applications, $\gtrsim 1\,\text{keV}$

PHOTON INTERACTIONS



Contributions of the partial cross sections σ_{pe} for the atomic photoeffect, σ_{coh} for coherent Rayleigh (elastic) scattering, σ_{incoh} for incoherent Compton (inelastic) scattering, σ_{pair} for electron-positron pair production in the field of the nucleus, σ_{trip} for pair production in the field of the atomic electrons ("triplet" production), and $\sigma_{ph.n.}$ for nuclear photo-absorption. Measured values of the total cross section σ_{tot} are not all shown in regions of high measurement density. From Hubbell (1999).



Contributions of the partial cross sections σ_{pe} for the atomic photoeffect, σ_{coh} for coherent Rayleigh (elastic) scattering, σ_{incoh} for incoherent Compton (inelastic) scattering, σ_{pair} for electron-positron pair production in the field of the nucleus, σ_{trip} for pair production in the field of the atomic electrons ("triplet" production), and $\sigma_{ph.n.}$ for nuclear photo-absorption. Measured values of the total cross section σ_{tot} are not all shown in regions of high measurement density. From Hubbell (1999).

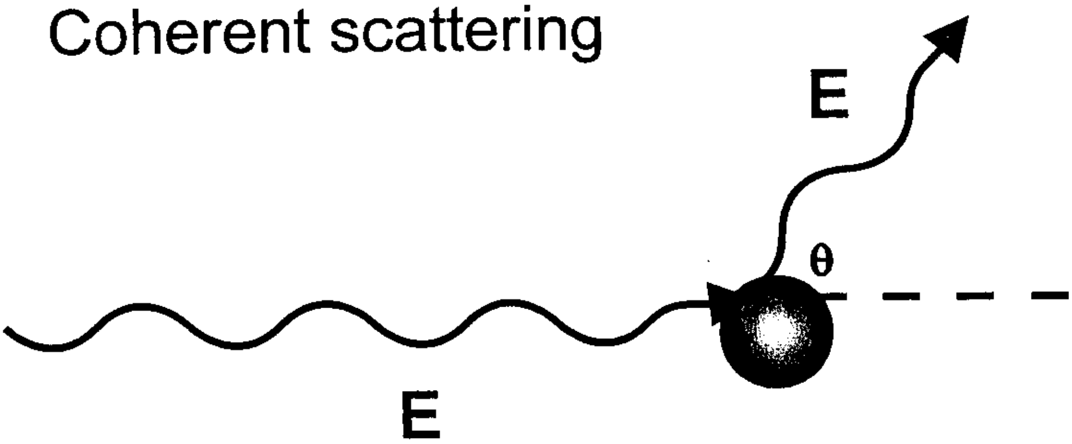
NIST PHOTON CROSS-SECTION DATABASE

- $1 \leq Z \leq 100$
- $1\ \text{keV} \leq E \leq 100\ \text{GeV}$

Process	Interaction
Coherent (Rayleigh) Scatter	atomic electrons
Incoherent (Compton) Scatter	atomic electrons
Photoelectric Absorption	atomic electrons
Pair Production	field of nucleus
Triplet Production	field of atomic electrons

PHOTON INTERACTIONS

Coherent scattering



- Incident photon scattered
- Scattered photon has same energy as incident photon
- $\sigma \sim Z^2$

COHERENT SCATTER

$$\sigma_{coh} = \int_{-1}^{+1} \frac{d\sigma_T}{d(\cos\theta)} \, [F(x,Z)]^2 \, d(\cos\theta)$$

$$\frac{d\sigma_T}{d(\cos\theta)} = \pi r_e^2 (1 + \cos^2\theta)$$ Thomson cross section

$F(x,Z)$: atomic form factor

(from Hartree-Fock calculations)

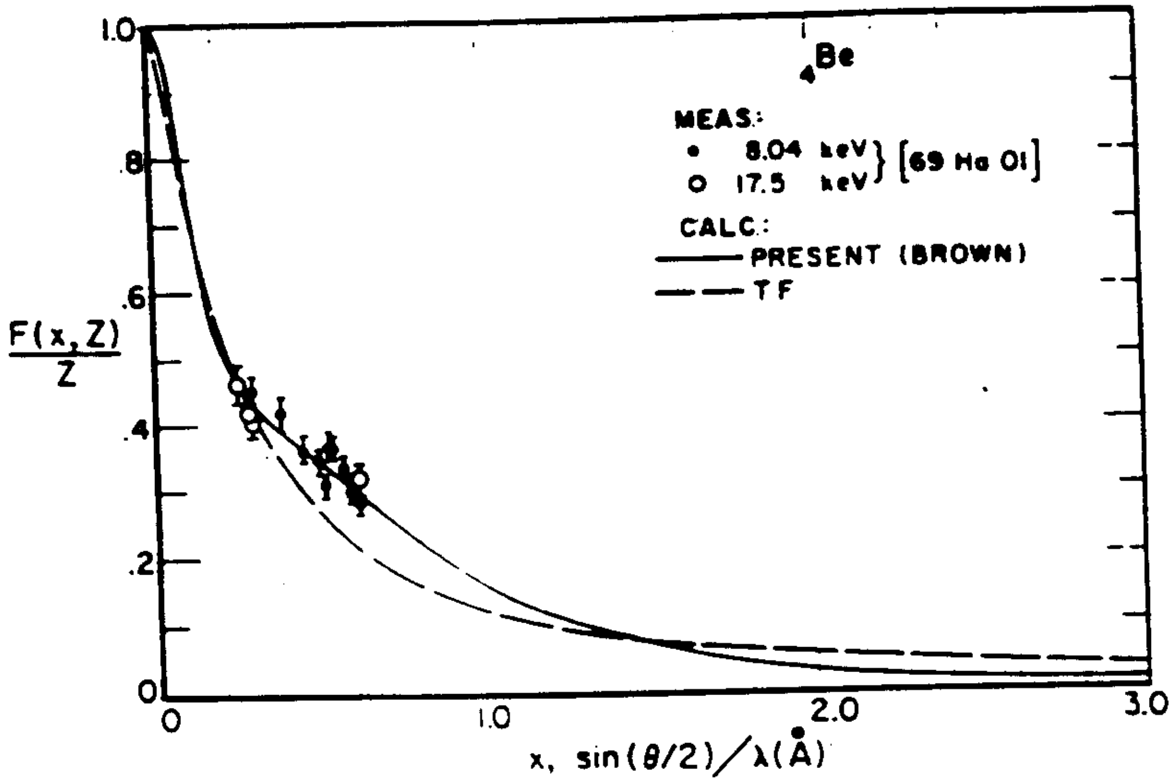
Z = atomic number

$x = C E \sqrt{1 - \cos\theta}$

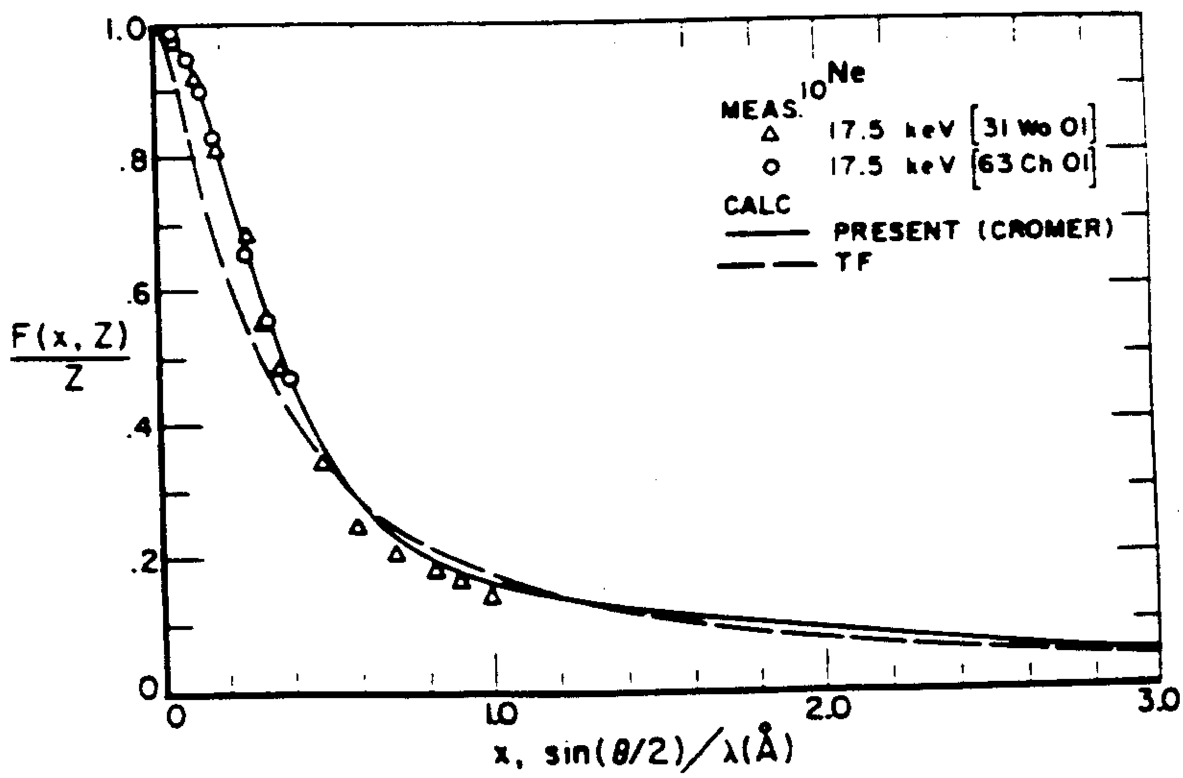
E = energy of incident photon

$C = 57.03 \text{ MeV}^{-1}$

$Z \geq F(x,Z) \geq 0$

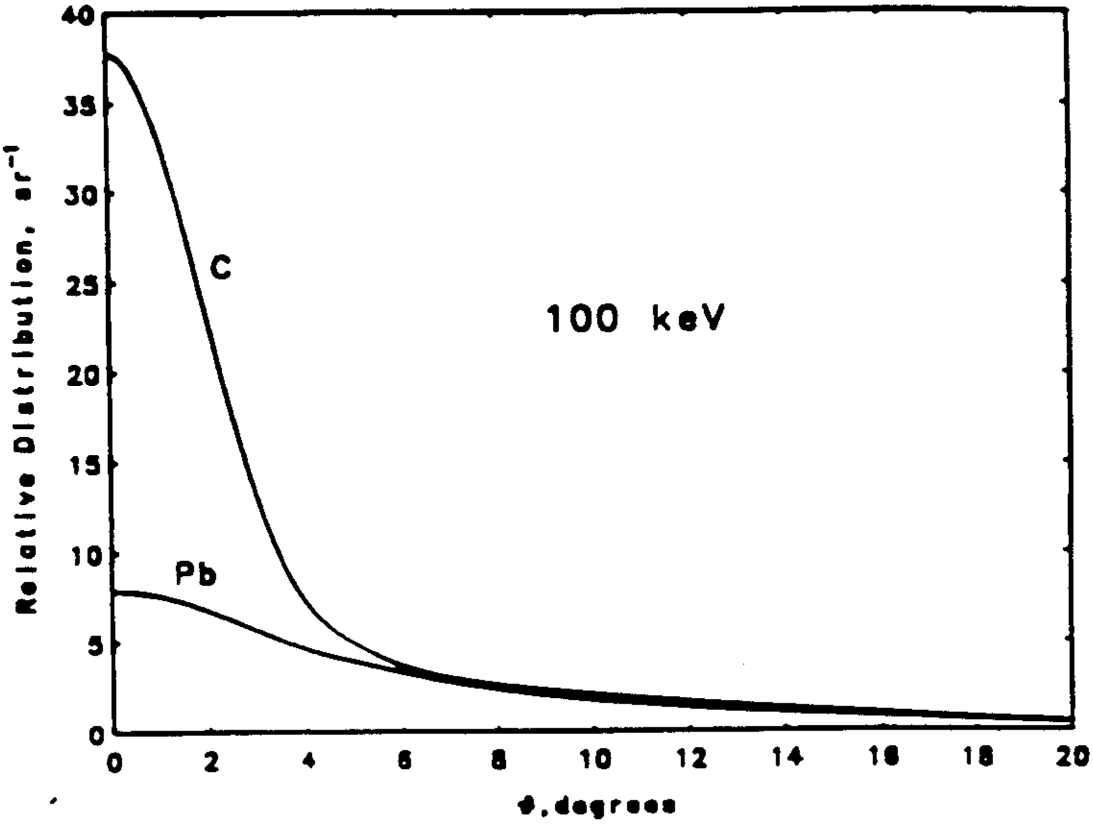


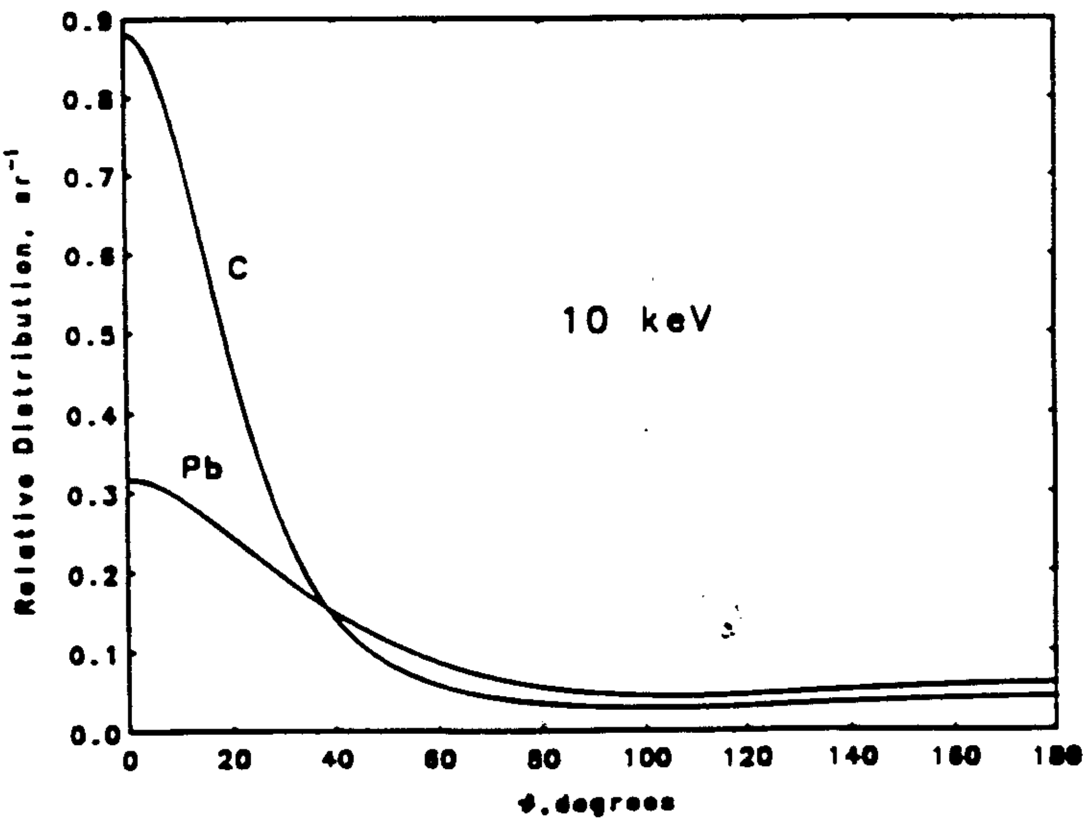
1.0
.8
.6
.4
.2
0
F(x, Z)/Z
0
1.0
2.0
3.0
x, sin(θ/2)/λ(Å)
$_4$Be
MEAS.:
• 8.04 keV
o 17.5 keV
[69 Ha 01]
CALC.:
PRESENT (BROWN)
TF



1.0
.8
.6
.4
.2
0
F(x, Z)/Z
0
1.0
2.0
3.0
x, sin(θ/2)/λ(Å)
$_{10}$Ne
MEAS.
Δ 17.5 keV [31 Wo 01]
o 17.5 keV [63 Ch 01]
CALC
PRESENT (CROMER)
TF

COHERENT SCATTERING





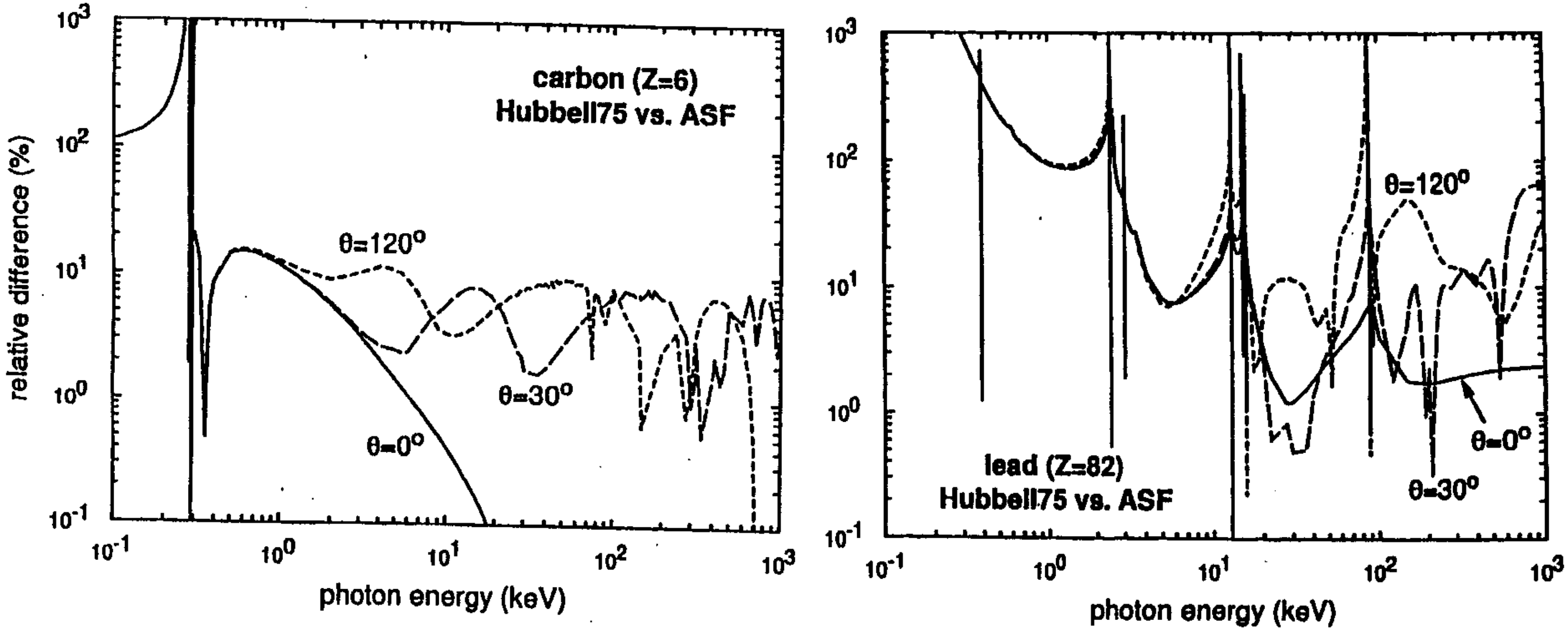
Elastic (Coherent) Rayleigh Scattering

$$\sigma_{coh} = \pi r_e^2 \int_{-1}^{+1} (1 + \cos^2\theta)\,[F(x,Z)]^2\,d(\cos\theta)$$

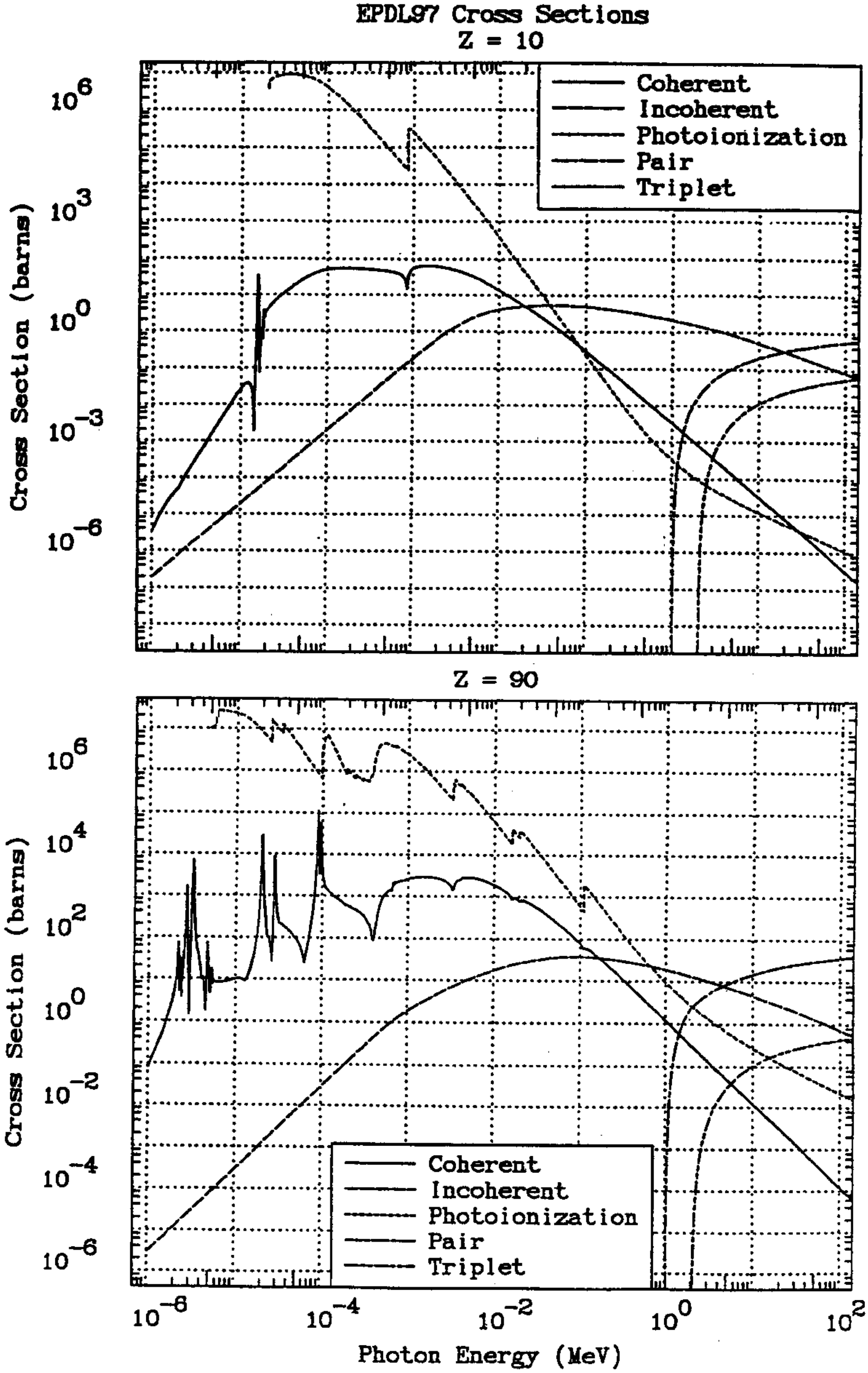
$\uparrow$

form factor

- **Simple form factors (non-relativistic, relativistic): FF**
- **Modified form factors: MFF**
- **Modified form factors plus anomalous scattering factors: MFF+ASF**
- **S-matrix calculations**

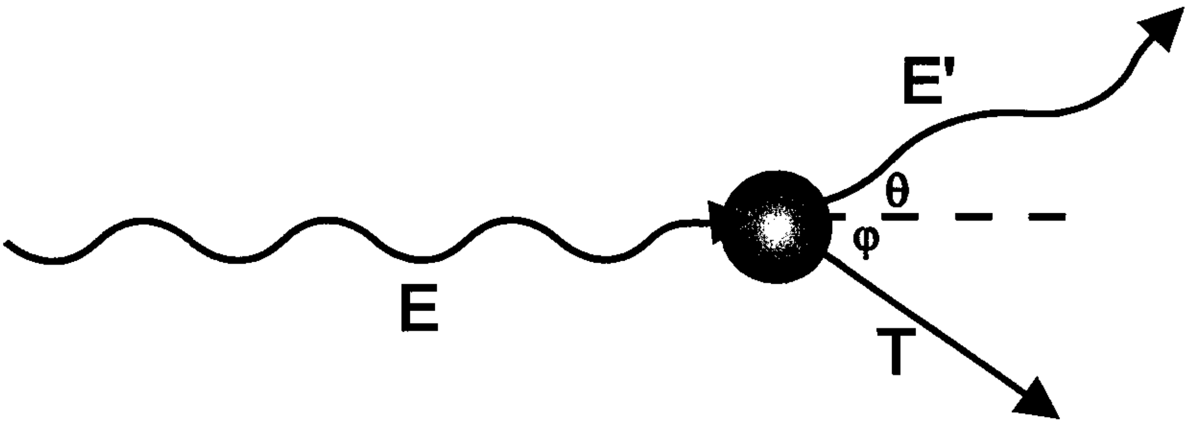


Relative differences in $d\sigma_{coh}/d\Omega$ from Hubbell (1975) FF and from MFF+ASF calculations, as a function of photon energy for scattering angles of 0°, 30° and 120°. (From Bradley et al., 1999)


EPDL97 Cross Sections
Z = 10
Coherent
Incoherent
Photoionization
Pair
Triplet
10^6
10^3
10^0
10^{-3}
10^{-6}
Cross Section (barns)
Z = 90
10^6
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
Cross Section (barns)
Coherent
Incoherent
Photoionization
Pair
Triplet
10^{-6}
10^{-4}
10^{-2}
10^0
10^2
Photon Energy (MeV)

PHOTON INTERACTIONS

Incoherent (compton) scattering



$E' \cong E-T-B$

- Incident photon scattered
- Mostly outer shell atomic electrons ejected ($B \approx 0$)
- X-rays and Auger electrons can be emitted, particularly with inner shell vacancies
- $\sigma \sim Z$

INCOHERENT SCATTERING

$$\sigma_{incoh} = \int_{-1}^{+1} \frac{d\sigma_{KN}}{d(\cos\theta)} S(x,Z)\, d(\cos\theta)$$

$$\frac{d\sigma_{KN}}{d(\cos\theta)} = \pi r_e^2 \left[1 + \cos^2\theta + \frac{k^2(1-\cos\theta)^2}{1+k(1-\cos\theta)} \right] \Big/ [1+k(1-\cos\theta)]^2$$

Klein-Nishina cross section

for scattering from an unbound electron

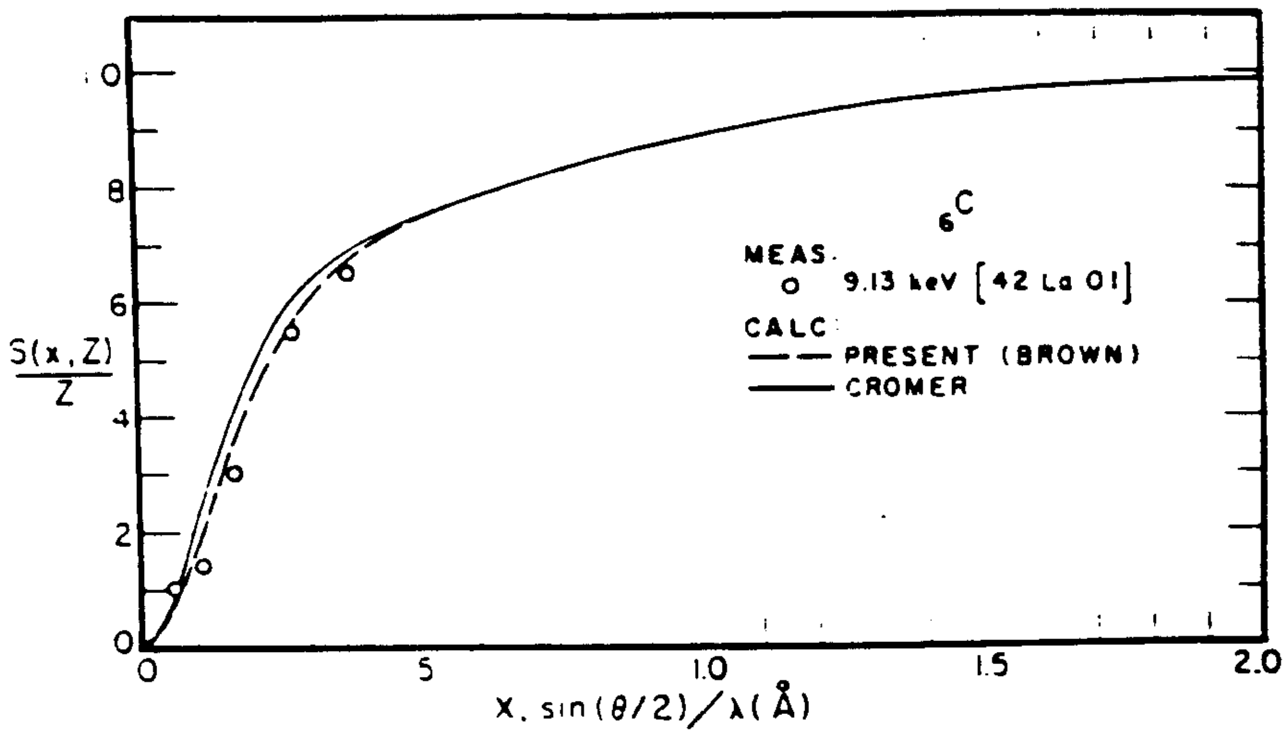
$$k = E/mc^2$$

$$\frac{mc^2}{E'} - \frac{mc^2}{E} = 1 - \cos\theta$$

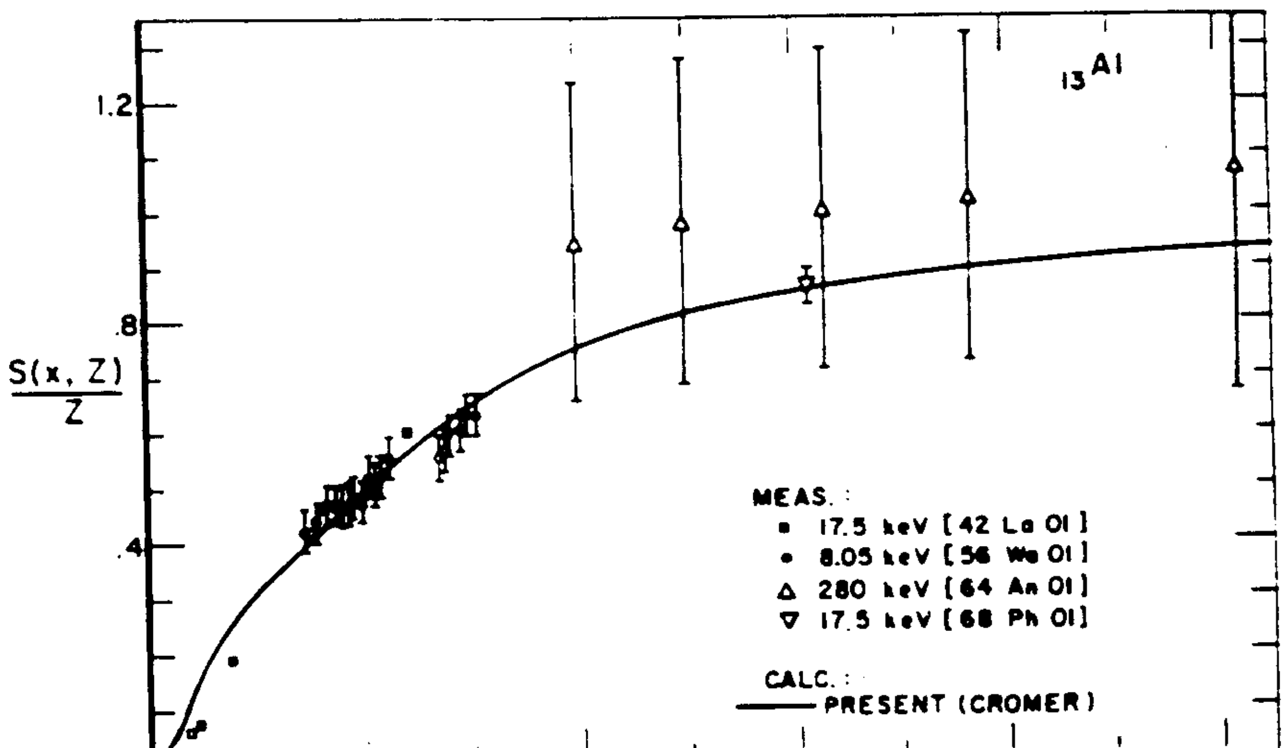
S(x,Z) incoherent scattering function

(from Hartree-Fock calculations)

$$0 \le S(x,Z) \le Z$$

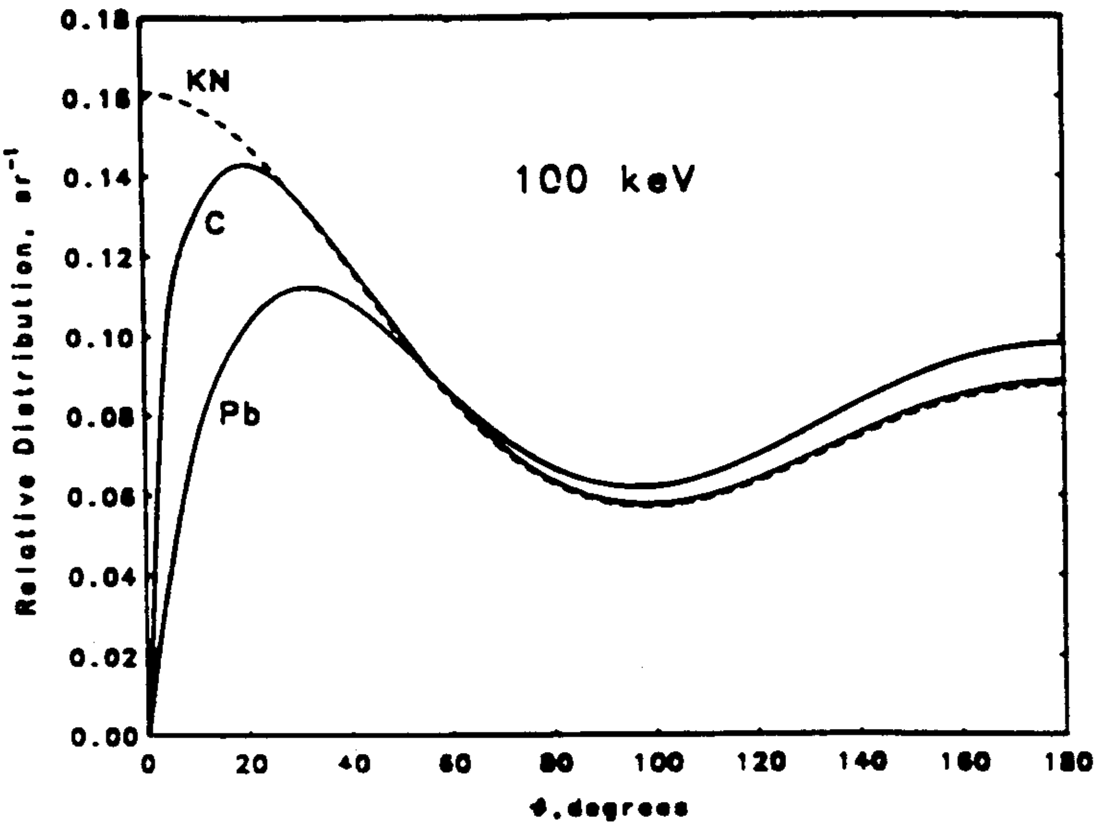


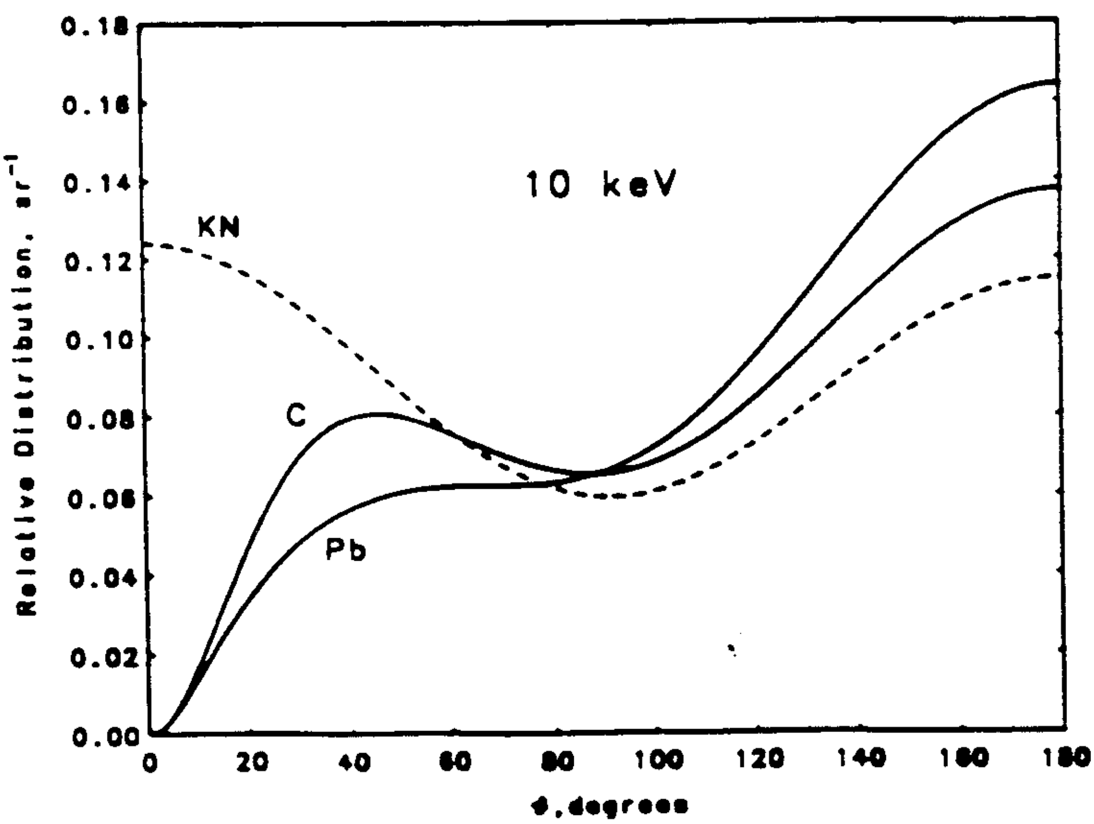
$_6C$
MEAS.
o 9.13 keV [42 La 01]
CALC:
PRESENT (BROWN)
CROMER
S(x, Z)/Z
X, sin(θ/2)/λ(Å)



$_{13}Al$
MEAS.:
17.5 keV [42 La 01]
8.05 keV [56 We 01]
280 keV [64 An 01]
17.5 keV [68 Ph 01]
CALC.:
PRESENT (CROMER)
S(x, Z)/Z
x, sin(θ/2)/λ(Å)

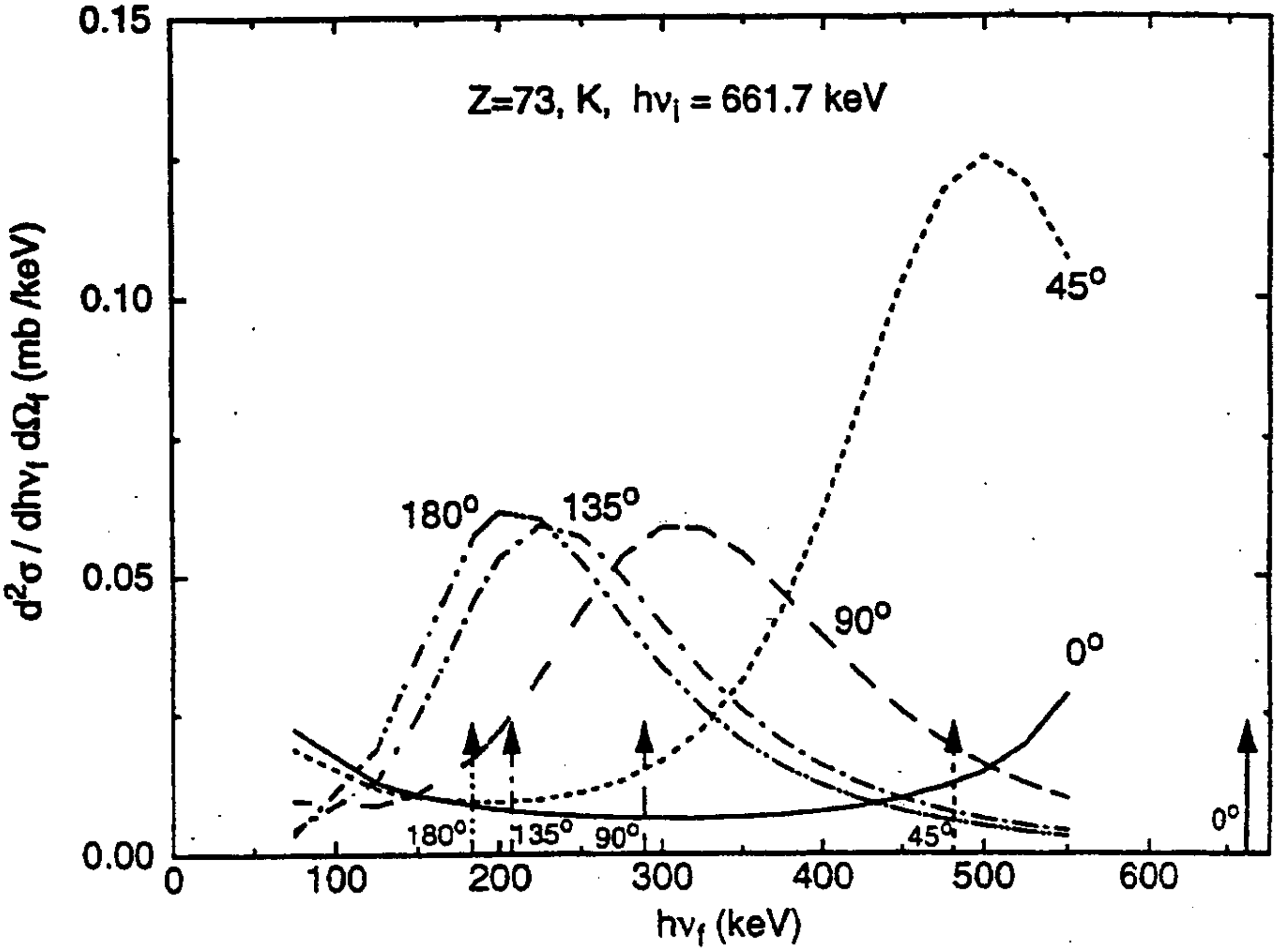
INCOHERENT SCATTERING



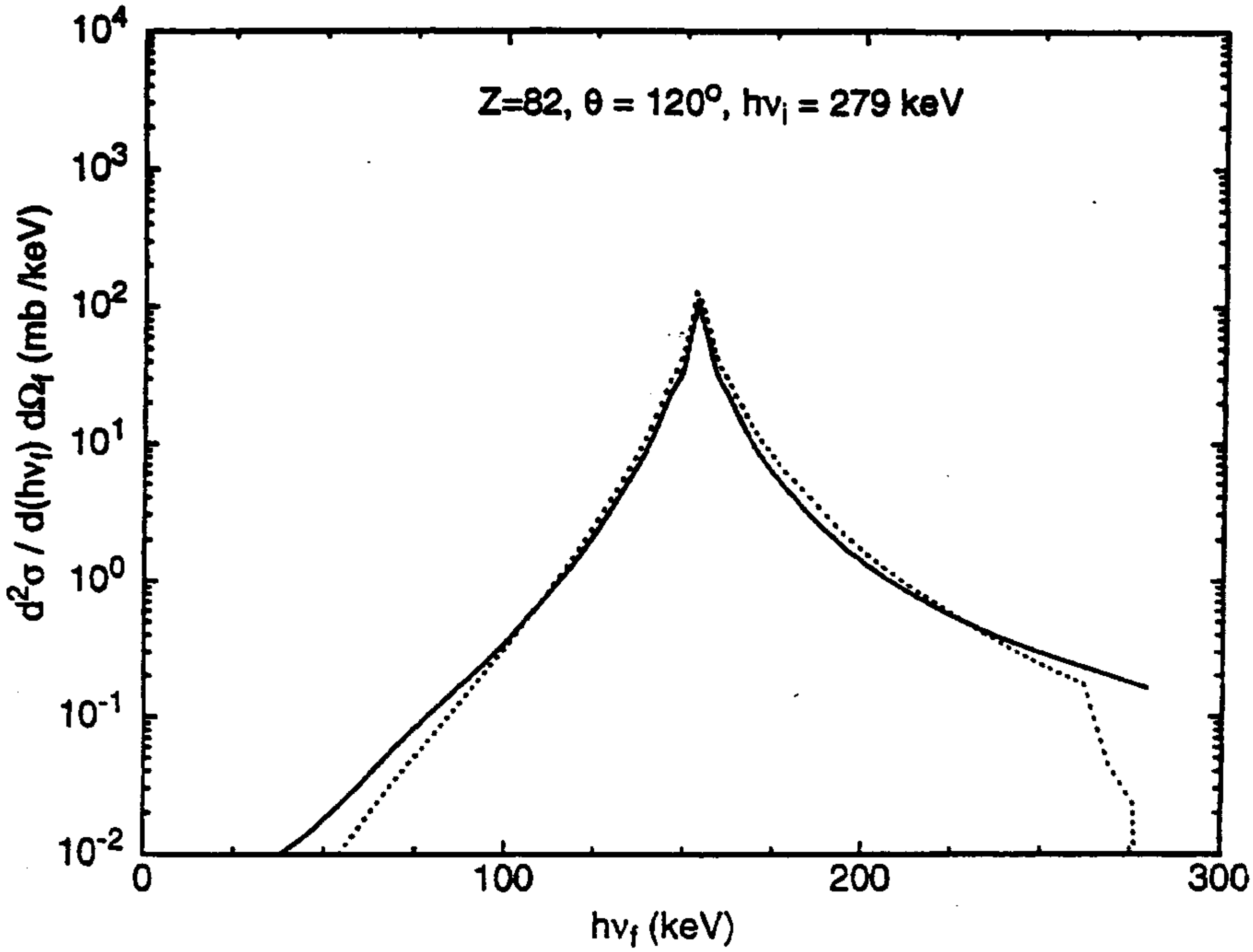


Inelastic (Incoherent) Compton Scattering

- **Form-factor approach: incoherent scattering function *S***
- **Impulse approximation: Compton profiles, Doppler broadening**
- **S-matrix calculations**



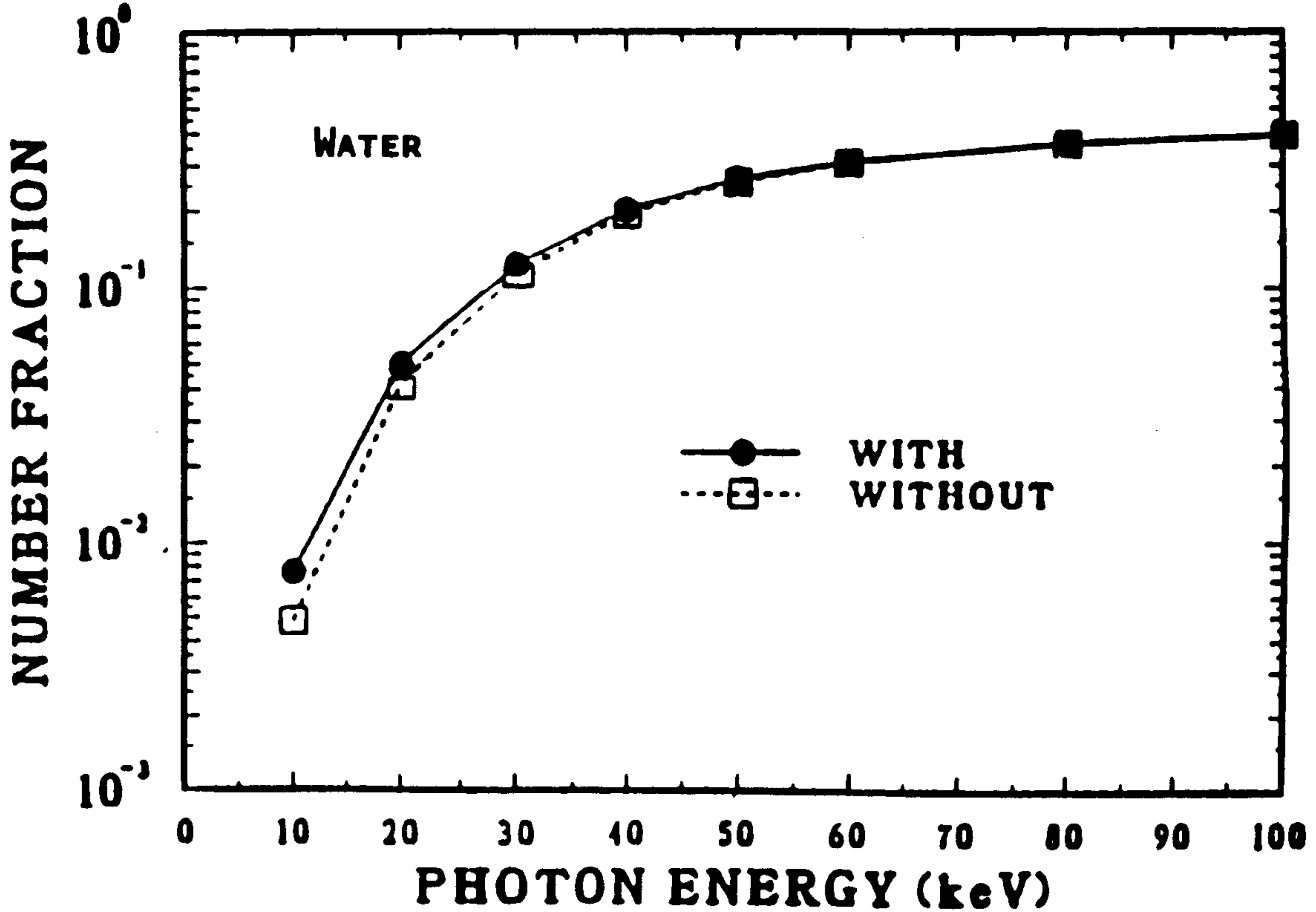
Cross section differential in scattered photon energy and angle for scattering of an unpolarized beam of 661.7 keV photons from a K-shell electron of tantalum. The photon scattering angles considered are 0° (solid curve), 45° (small dash), 90° (large dash), 135° (dot-dash), and 180° (dot-dash-dot). Results are from S-matrix calculations. Arrows show the predictions for the scattered photon energy given by Compton's formula. From Bergstrom and Pratt (1997).



Comparison of results for the scattering of 279 keV photons into 120° from *all* of the electrons in lead, from calculations in the impulse approximation: relativistic (dotted curve) and non-relativistic (solid curve). The tips of the sharp peaks are at the scattered photon energy given by Compton's formula, and the full-width-at-half-maximum of the distributions is about 5 keV. From Bergstrom and Pratt (1997).

Fraction of energy reflected for photons normally incident on water target

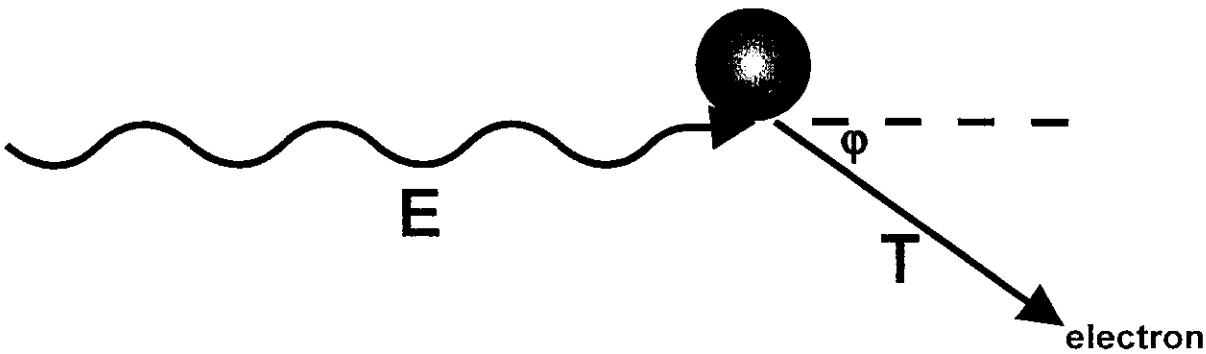
coherent scatter included	incoherent binding corrections	Incident photon energy, keV			
		100	50	20	10
no	no	0.202	0.194	0.0384	0.00457
yes	no	0.203	0.196	0.0444	0.00666
yes	yes	0.203	0.199	0.0475	0.00740



WATER
WITH
WITHOUT
10^{0}
10^{-1}
10^{-2}
10^{-3}
NUMBER FRACTION
0
10
20
30
40
50
60
70
80
90
100
PHOTON ENERGY (keV)

Photon Interactions

Photoelectric absorption



$$T \cong E-B$$

- Incident photon disappears
- Inner shell atomic electrons most probably ejected
- X-rays and Auger electrons emitted
- $\sigma \sim Z^{4-5}$

PHOTOEFFECT

σ_{pe} (or τ in much of the literature)

- Calculated by Scofield

- Solution of Dirac equation for the orbital electrons moving in a static Hartree-Slater potential

- For $2 \leq Z \leq 54$, Scofield gave renormalization factors to convert from Hartree-Slater to Hartree-Fock

- Comparisons with experiment favor the use of the unrenormalized Scofield results

Comparison of mass attenuation coefficients, in cm^2/g, for photoelectric absorption from various compilations.

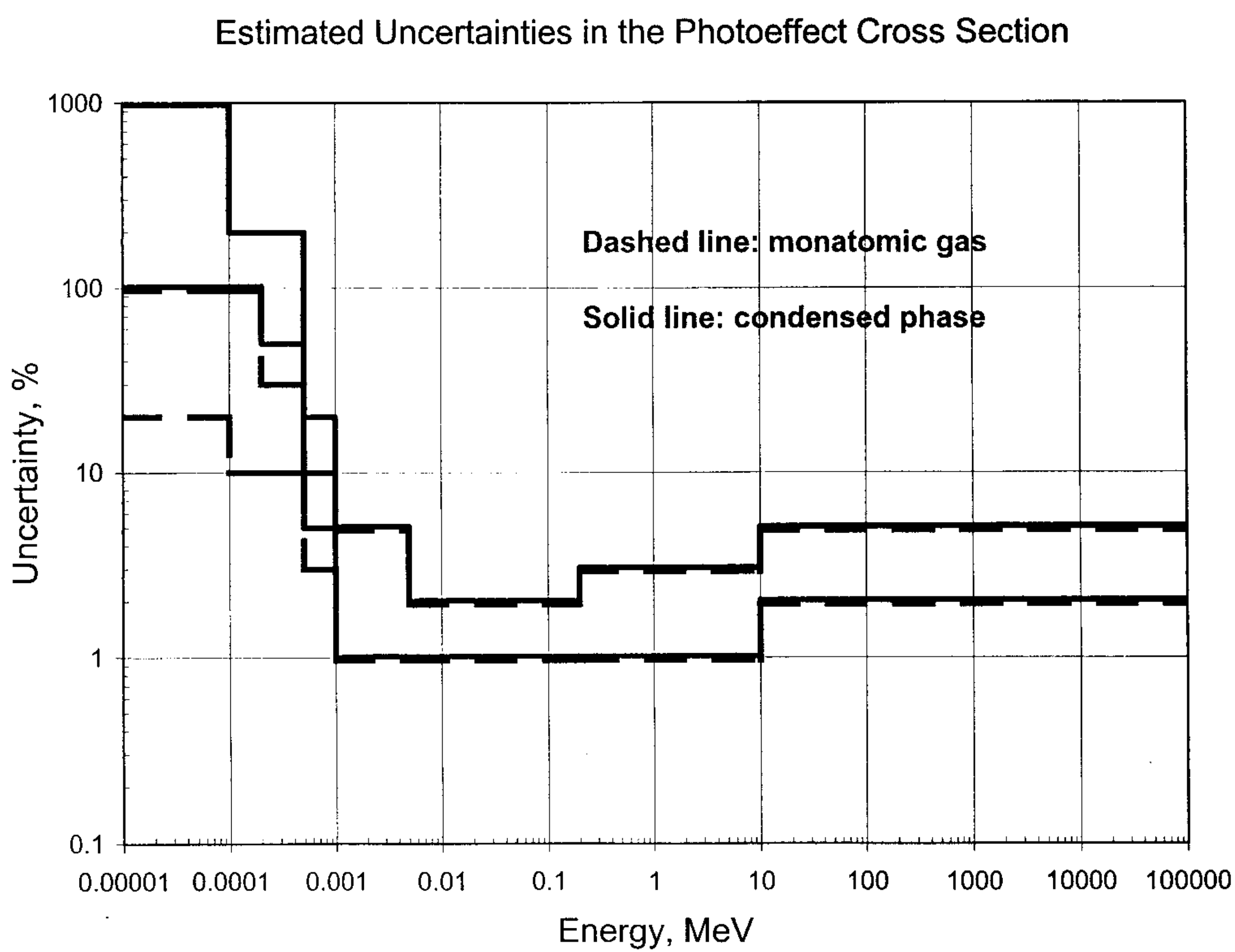
a. C (Z=6)

Photon Energy (MeV)	Hubbell (1969)	Storm-Israel (1970)	Biggs-Lighthill (1972)	Henke et al. (1982)	XCOM (1987)	EPDL89 (1989)	Henke et al. (1993)	Chantler (1995)	EPDL97 (1997)
0.0001	-	-	25950	23000	-	27445	24400	22541	27525
0.0002	-	-	5445	5180	-	5861	5070	4918	5791
0.0005	-	-	12470	14000	-	14085	13700	13257	14116
0.001	-	2180	2201	2220	2210	2197	2220	2072	2208
0.002	-	300	298.5	302	301.7	301.4	302	276.9	301.3
0.005	-	18.6	18.64	18.9	18.66	18.49	18.9	17.58	18.65
0.01	1.97	1.97	2.038	2.06	2.076	2.061	2.06	1.959	2.076
0.02	0.201	0.208	0.2018	-	0.2177	0.2166	0.202	0.2059	0.2174
0.05	0.00968	0.0101	0.00994	-	0.01042	0.01041	-	0.01009	0.01041
0.1	0.00108	0.00100	0.00102	-	0.00103	0.00104	-	0.00102	0.00103

Comparison of mass attenuation coefficients, in cm^2/g, for photoelectric absorption from various compilations.

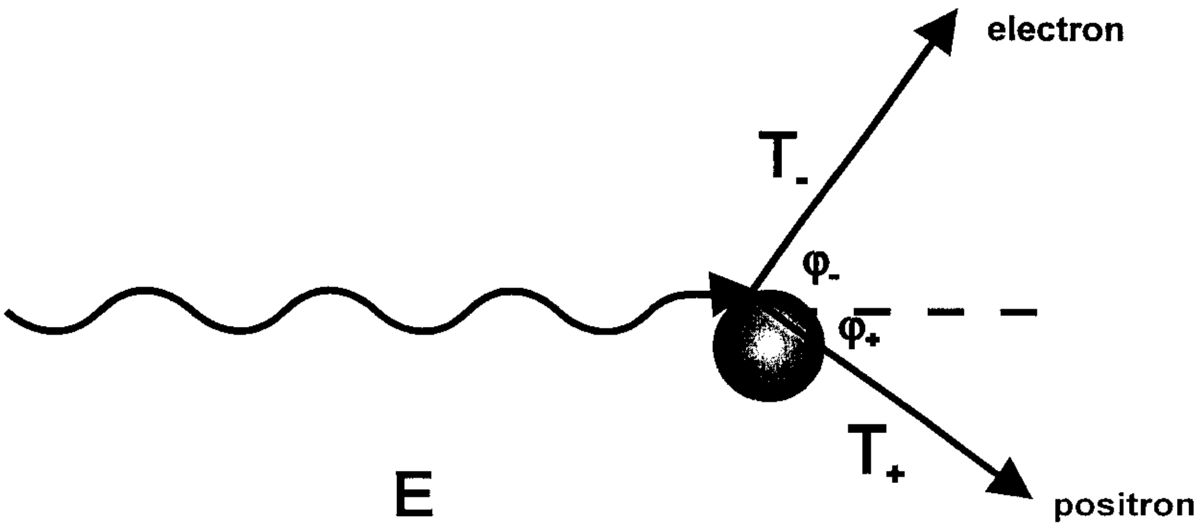
b. Pb (Z=82)

Photon Energy (MeV)	Hubbell (1969)	Storm-Israel (1970)	Biggs-Lighthill (1972)	Henke (1982)	XCOM (1987)	EPDL89 (1989)	Henke (1993)	Chantler (1995)	EPDL97 (1997)	Chantler (1999)
0.0001	-	-	23800	22700	-	13086	30200	12321	13089	12321
0.0002	-	-	11900	6620	-	3636	5060	5792	3625	5792
0.0005	-	-	16400	12900	-	13501	12300	12607	13501	12628
0.001	-	5120	5134	5680	5197	5191	5630	5003	5199	5004
0.002	-	1260	1265	1400	1274	1272	1350	1173	1274	1172
0.005	-	712	713.1	788	722.2	721.7	749	700.5	722.9	700.9
0.01	128	123	123.2	136	125.6	125.5	132	121.5	125.6	121.5
0.02	83.1	82.8	83.40	-	83.97	83.61	83.3	82.30	83.66	82.30
0.05	7.09	7.15	7.160	-	7.292	7.271	-	7.029	7.286	7.029
0.1	5.12	5.23	5.204	-	5.237	5.230	-	5.192	5.256	5.192
0.2	0.849	0.849	0.8497	-	0.8164	0.8463	-	0.8316	0.8471	0.8316
0.5	0.0849	0.0826	0.08259	-	0.08257	0.08261	-	-	0.08261	-
1	0.0186	0.0181	0.01791	-	0.01810	0.01810	-	-	0.01811	-
2	0.00515	0.00497	0.00494	-	0.00503	0.00504	-	-	0.00504	-


Estimated Uncertainties in the Photoeffect Cross Section
Dashed line: monatomic gas
Solid line: condensed phase
Uncertainty, %
1000
100
10
1
0.1
0.00001
0.0001
0.001
0.01
0.1
1
10
100
1000
10000
100000
Energy, MeV

Photon Interactions

Pair production (in field of nucleus)

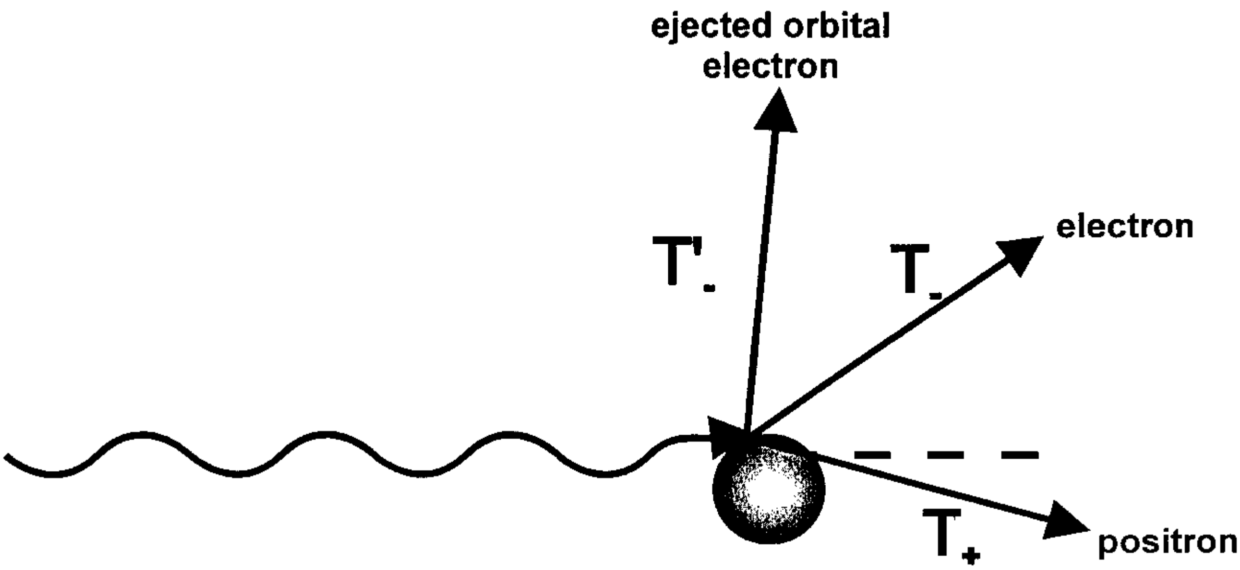


$$E=T_-+T_++2mc^2 \quad (E>2mc^2)$$

- Incident photon disappears
- $\sigma \sim Z^2$

Photon Interactions

Triplet production (in field of atomic electrons)



$$E=T_{-}+T'_{-}+T_{+}+B+4mc^2 \quad (E>4mc^2)$$

- Incident photon disappears
- Mostly outer shell electrons ejected ($B \approx 0$)
- $\sigma \sim Z$

PHOTON INTERACTIONS

- Mass attenuation coefficient

$$\frac{\mu}{\rho} = \frac{N_A}{A}\left[\sigma_{pe}+\sigma_{coh}+\sigma_{incoh}+\sigma_{pair}+\sigma_{trip}\right]$$

- Simple attenuation

$$\frac{I}{I_o} = e^{-\mu t}$$

PAIR AND TRIPLET PRODUCTION

σ_{pair} (or κ_n in much of the literature)

σ_{triplet} (or κ_e in much of the literature)

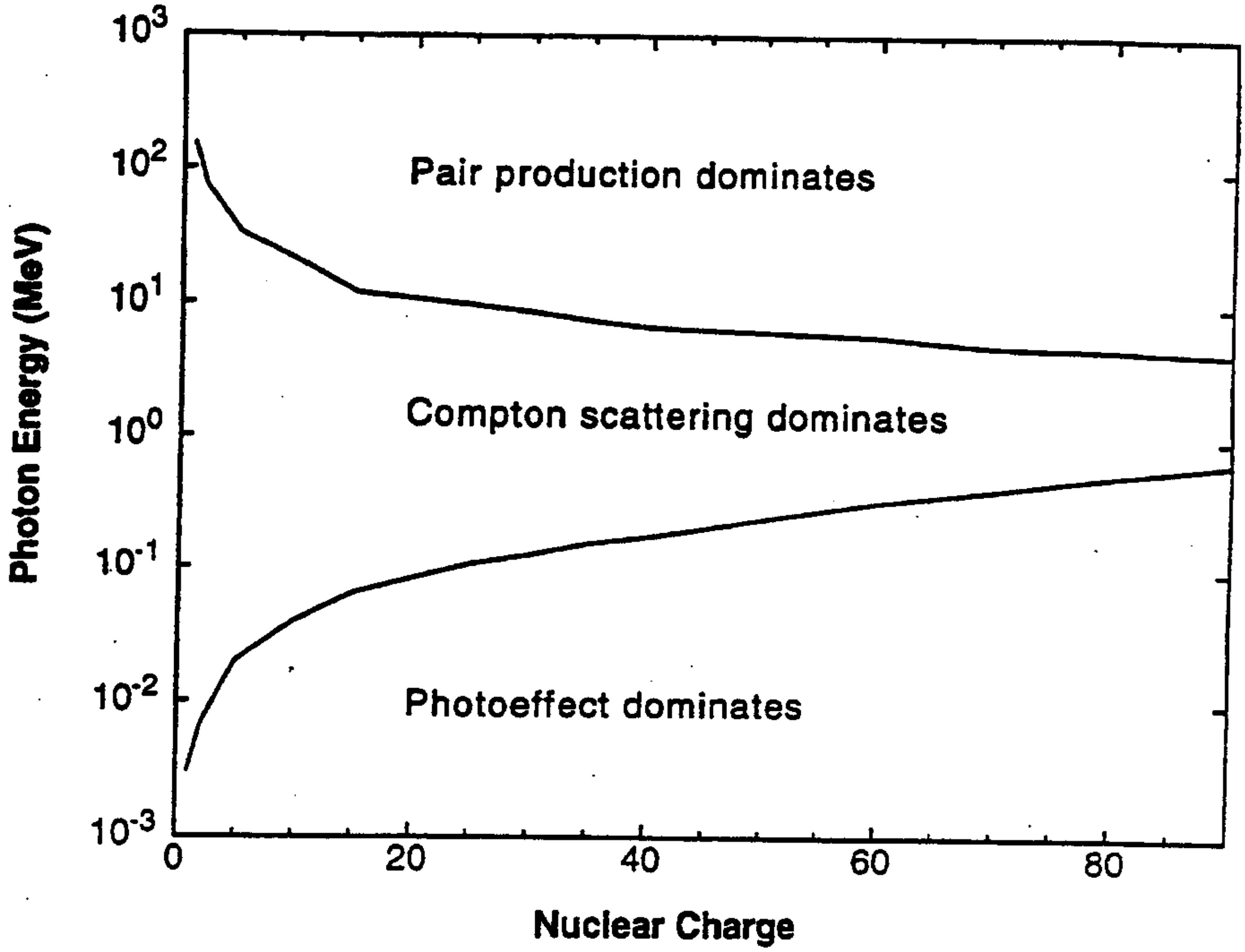
- Compiled through an elaborate synthesis of theoretical information

- Born-approximation without screening

- Coulomb corrections

- Screening corrections

- Radiative corrections



Regions of relative importance of photoeffect, Compton scattering, and pair production in total absorption cross sections, shown as a function of incident photon energy and atomic number (nuclear charge) Z. From Bergstrom and Pratt (1997).

PHOTON INTERACTIONS

- Mass attenuation coefficient

$$\frac{\mu}{\rho} = \frac{N_A}{A}\left[\sigma_{pe}+\sigma_{coh}+\sigma_{incoh}+\sigma_{pair}+\sigma_{trip}\right]$$

- Simple attenuation

$$\frac{I}{I_o} = e^{-\mu t}$$

Comparison of mass total attenuation coefficients, in cm^2/g, from various compilations.

a. C (Z=6)

Photon Energy (MeV)	Hubbell (1969)	Storm-Israel (1970)	Biggs-Lighthill (1972)	XCOM (1987)	EPDL89 (1989)	Chantler (1995)	EPDL97 (1997)
0.0001	-	-	25960	-	27446	22541	27526
0.0002	-	-	5445	-	5861	4919	5791
0.0005	-	-	13980	-	14086	13258	14117
0.001	-	2180	2202	2211	2198	2073	2209
0.002	-	301	299.3	302.6	302.6	277.8	302.2
0.005	-	19.1	19.11	19.12	18.96	18.07	19.12
0.01	2.28	2.27	2.337	2.373	2.362	2.271	2.376
0.02	0.429	0.432	0.4272	0.4420	0.4414	0.4359	0.4425
0.05	0.187	0.187	0.1870	0.1871	0.1869	0.1898	0.1871
0.1	0.152	0.151	0.1515	0.1514	0.1513	0.1506	0.1513
0.2	0.123	0.123	0.1231	0.1229	0.1226	0.1200	0.1229
0.5	0.0872	0.0867	0.08714	0.08715	0.08699	-	0.08699
1	0.0637	0.0637	0.06353	0.06362	0.06354	-	0.06354
2	0.0445	0.0443	0.04436	0.04443	0.04434	-	0.04434
5	0.0271	0.0271	0.02701	0.02708	0.02695	-	0.02695
10	0.0196	0.0196	0.01951	0.01959	0.01954	-	0.01954
20	0.0158	0.0157	0.01552	0.01575	0.01569	-	0.01569
50	0.0142	0.0142	0.01410	0.01426	0.01421	-	0.01421
100	0.0146	0.0145	0.01444	0.01457	0.01454	-	0.01454

Comparison of mass total attenuation coefficients, in cm^2/g, from various compilations.

b. Pb (*Z*=82)

Photon Energy (MeV)	Hubbell (1969)	Storm-Israel (1970)	Biggs-Lighthill (1972)	XCOM (1987)	EPDL89 (1989)	Chantler (1995)	EPDL97 (1997)	Chantler (1999)
0.0001	-	-	23810	-	13087	12321	13090	12321
0.0002	-	-	11910	-	3636	5792	3625	5792
0.0005	-	-	16410	-	13503	12608	13505	12634
0.001	-	5120	5145	5209	5194	5006	5206	5015
0.002	-	1260	1277	1285	1276	1201	1280	1184
0.005	-	721	721.2	730.5	728.3	709.1	730.6	709.5
0.01	133	128	128.2	130.6	128.7	126.5	129.9	126.5
0.02	85.7	85.2	85.81	86.37	86.06	84.71	86.01	84.71
0.05	7.81	7.88	7.863	8.042	7.983	7.759	8.001	7.759
0.1	5.40	5.52	5.503	5.549	5.540	5.489	5.545	5.489
0.2	0.991	0.994	0.9881	0.9986	1.003	0.9791	0.9958	0.9791
0.5	0.161	0.159	0.1583	0.1613	0.1607	-	0.1603	-
1	0.0708	0.0703	0.07000	0.07102	0.07062	-	0.07064	-
2	0.0455	0.0453	0.04538	0.04607	0.04594	-	0.04590	-
5	0.0424	0.0424	0.04221	0.04272	0.04259	-	0.04264	-
10	0.0484	0.0485	0.04901	0.04972	0.04967	-	0.04967	-
20	0.0606	0.0608	0.06073	0.06206	0.06200	-	0.06200	-
50	0.0804	0.0802	0.08052	0.08056	0.08053	-	0.08053	
100	0.0934	0.0933	0.09422	0.09310	0.09308	-	0.09308	-

Photon Interactions

- Mass energy-transfer coefficient

$$\frac{\mu_{tr}}{\rho} = \frac{N_A}{A}\left[f_{pe}\sigma_{pe} + f_{incoh}\sigma_{incoh} + f_{pair}\sigma_{pair} + f_{trip}\sigma_{trip}\right]$$

where the factors f are the average fractions of the photon energy transferred to kinetic energy of secondary charged particles.

- Mass energy-absorption coefficient

$$\frac{\mu_{en}}{\rho} = (1-g)\,\frac{\mu_{tr}}{\rho}$$

where g is the average fraction of the kinetic energy of secondary charged particles subsequently lost in radiative processes.

f_{pe}	accounts for:	complete cascade of fluorescence emission as atom relaxes following photoelectron ejection
f_{incoh}	accounts for:	energy of scattered photon
		cascade of fluorescence emission following Compton electron ejection
f_{pair}	accounts for:	energy converted to rest mass of electron-positron pair
f_{trip}	accounts for:	energy converted to rest mass of electron-positron pair
		fluorescence emission (cascade) following triplet electron ejection
g	accounts for:	*radiative losses of the secondary electrons and positrons as they slow down in the medium (and* ***includes effects of energy-loss straggling****)*
		bremsstrahlung production by electrons and positrons
		annihilation-in-flight of positrons
		fluorescence emission from electron- and positron-impact ionization

Photon Interactions

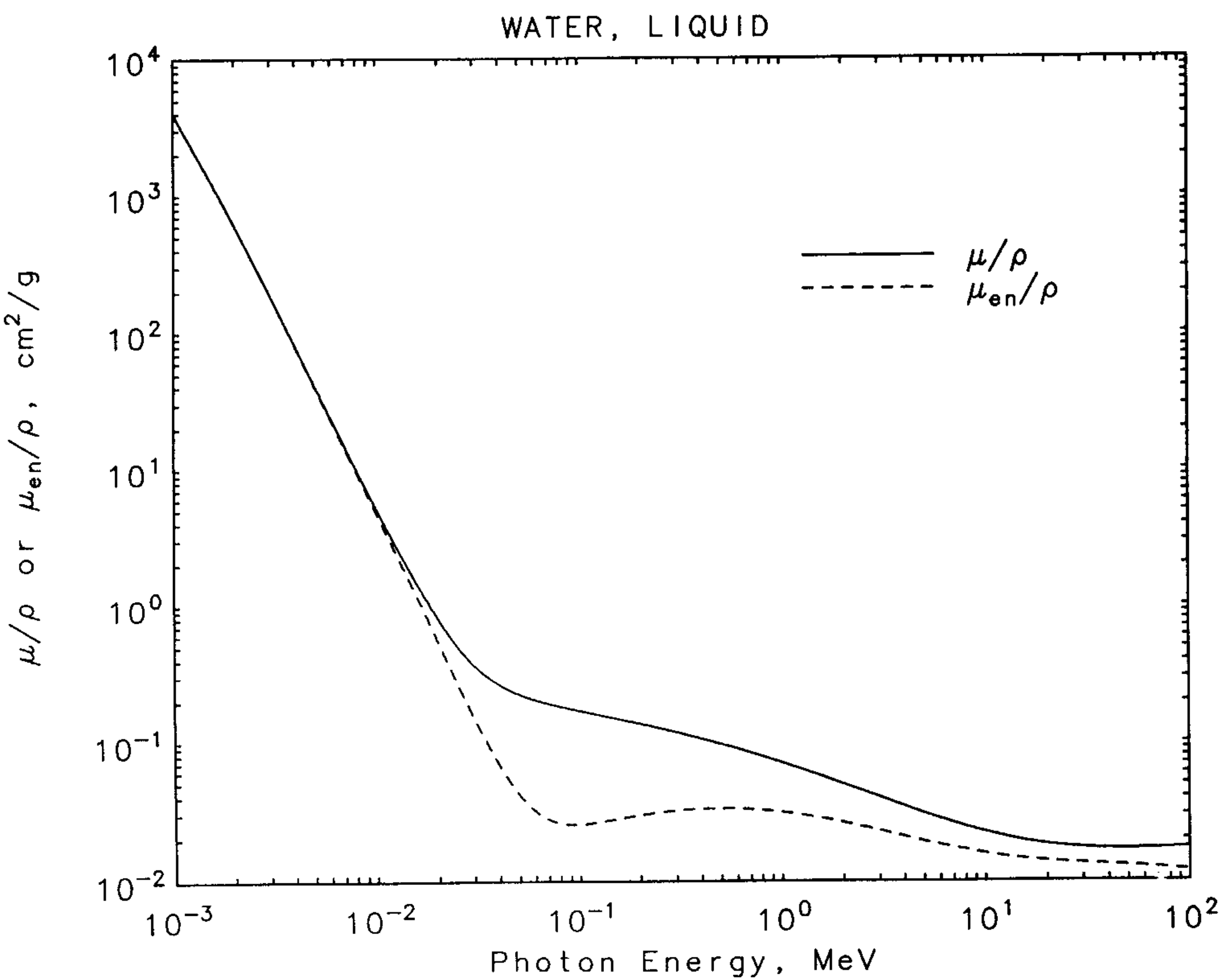
- *kerma*

$$K = \Phi E \frac{\mu_{tr}}{\rho}$$

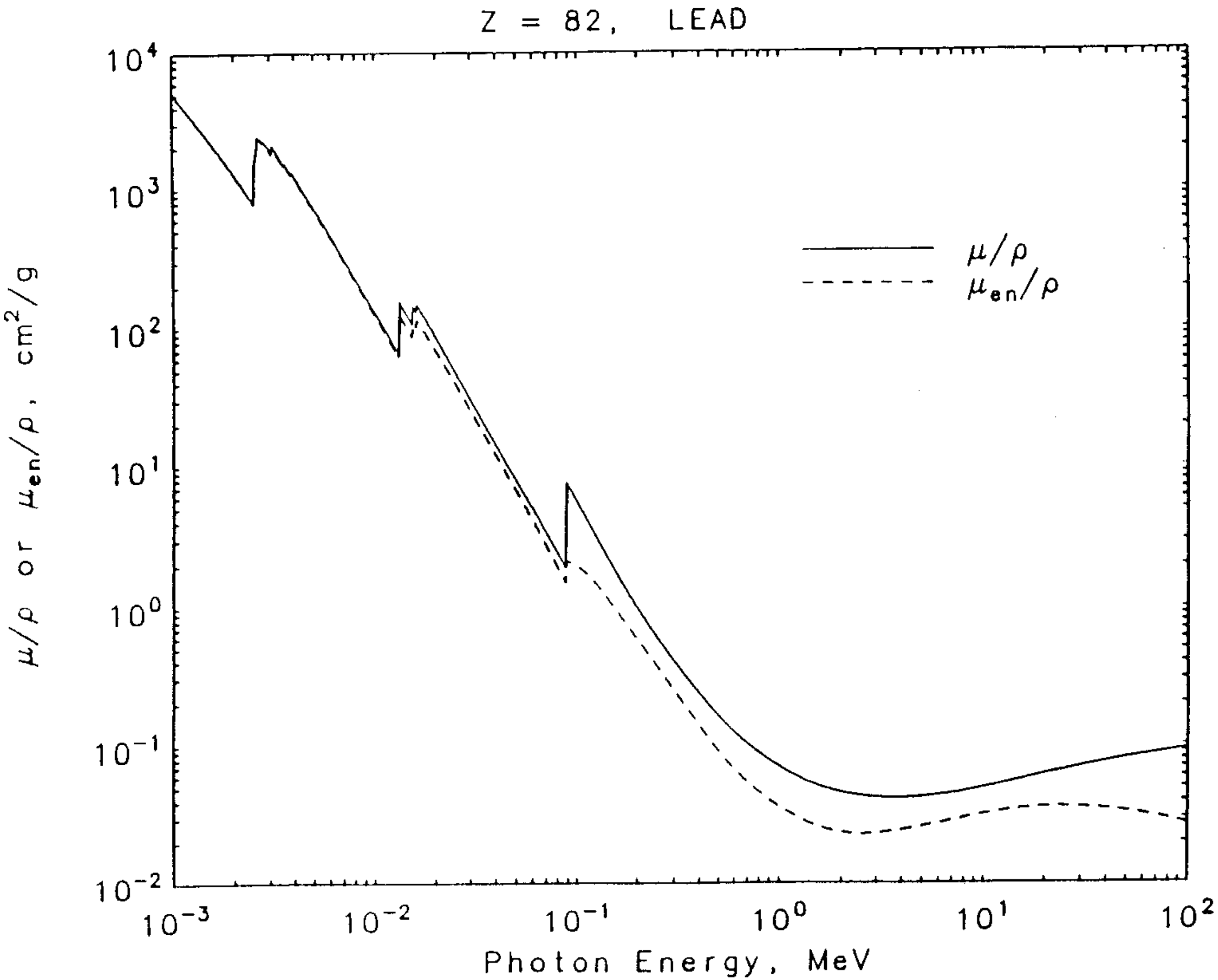
- Absorbed dose

$$D = \Phi E \frac{\mu_{en}}{\rho}$$

(under conditions of charged-particle equilibrium)



WATER, LIQUID
μ/ρ
μ_{en}/ρ
μ/ρ or μ_{en}/ρ, cm^2/g
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
Photon Energy, MeV



Z = 82, LEAD
μ/ρ
μ_{en}/ρ
μ/ρ or μ_{en}/ρ, cm^2/g
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Photon Energy, MeV

ICRU REPORT 60

Fundamental Quantities and Units for Ionizing Radiation

Issued: 30 December 1998

INTERNATIONAL COMMISSION ON RADIATION
UNITS AND MEASUREMENTS
7910 WOODMONT AVENUE
BETHESDA, MARYLAND 20814
U.S.A.

ELECTRON INTERACTIONS

MEAN ENERGY LOSS

STOPPING POWERS

ICRU REPORT 37

Stopping Powers for Electrons and Positrons

Issued: 1 October 1984

INTERNATIONAL COMMISSION ON RADIATION UNITS AND MEASUREMENTS
7910 WOODMONT AVENUE
BETHESDA, MD. 20814
U.S.A.

STOPPING POWER

$$\frac{1}{\rho} S = \frac{-1}{\rho} \frac{dE}{dx} = N_t \int W \frac{d\sigma}{dW} dW$$

$\frac{d\sigma}{dW}$ = cross section for scattering with energy loss W

N_t = number of scattering targets per unit mass

ρ = density of medium

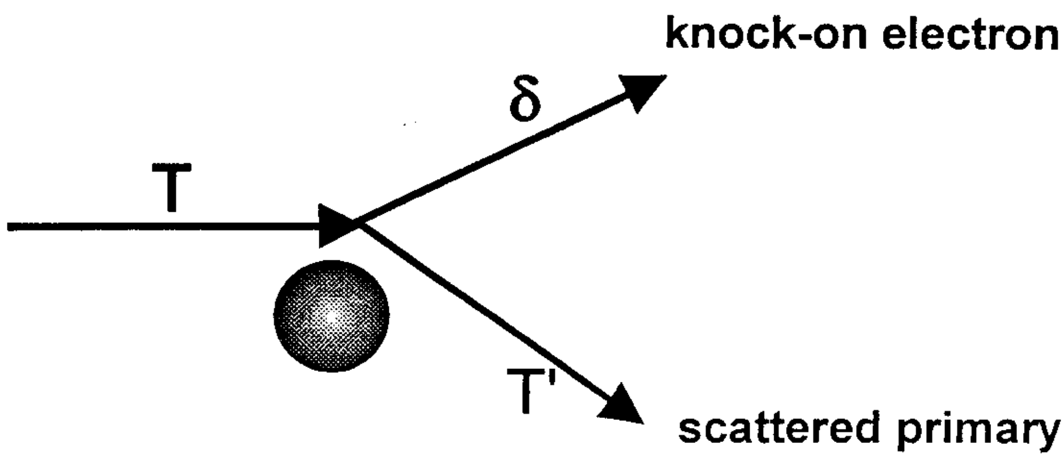
- *electronic* stopping power: inelastic scattering from bound atomic electrons, resulting in ionization and excitation (called *collision* stopping power by ICRU)

- *radiative* stopping power: inelastic scattering from atoms resulting in emission of bremsstrahlung photons (important for electrons and positrons)

- *nuclear* stopping power: elastic scattering from atoms with energy transfer to recoiling atom (important only for low-energy heavy charged particles)

ELECTRON INTERACTIONS

Ionization and excitation



- $\sigma \sim Z$
- Electronic stopping power-<u>average</u> energy loss by ionization and excitation per unit path length

$$\frac{1}{\rho} S_{el} \sim \frac{Z}{A}$$

- Mostly outer shell ionization
- X-ray and auger electron emission from inner shell vacancies

ELECTRONIC STOPPING POWER

$$\frac{1}{\rho} S_{col} = \frac{1}{\rho} S_{col}(W \le W_c) + \frac{1}{\rho} S_{col}(W > W_c)$$

$W_c \gg$ atomic electron binding energies

Distant Collisions

From Bethe theory

$$\frac{1}{\rho} S_{col}(W \le W_c) = \frac{2\pi r_e^2 mc^2 N_A}{\beta^2} \frac{Z}{A} z^2 \left\{ \ln\left[\frac{2mc^2 \beta^2 W_c}{(1-\beta^2) I^2} \right] - \beta^2 \right\}$$

with correction factors

$$\frac{1}{\rho}\, S_{col}(W \le W_c) = \frac{2\pi\, r_e^2\, mc^2 N_A}{\beta^2}\, \frac{Z}{A}\, z^2 \left\{ \ln\left[\frac{2mc^2 \beta^2 W_c}{(1-\beta^2) I^2}\right] - \beta^2 - \delta + 2\left[\frac{-C}{Z} + zL_1 + z^2 L_2\right] \right\}$$

where z = charge of projectile/charge of electron

non-trivial parameters:

I = mean excitation energy of medium

δ = density-effective correction

$$\left.\begin{array}{l} C/Z = \text{shell correction} \\ zL_1 = \text{Barkas correction} \\ z^2L_2 = \text{Block correction} \end{array}\right\} \begin{array}{l} \text{little or no information for} \\ \text{electrons and positrons} \end{array}$$

CLOSE COLLISIONS

$$\frac{1}{\rho}\, S_{col}(W > W_c) = N_A\, \frac{Z}{A} \int_{W_c}^{W_m} W\, \frac{d\sigma}{dW}\, dW$$

$\dfrac{d\sigma}{dW}$ = cross section for inelastic scattering from free electron

$$W_m = 2\tau(\tau+2)\, mc^2 \Big/ \left[1 + 2(\tau+1)\, \frac{m}{M} + \left(\frac{m}{M}\right)^2\right].$$

m = mass of target electron

M = mass of projectile

τ = projectile energy/rest energy

ELECTRONS:

W_m = T/2 (incident and target electrons are indistinguishable)

$$\frac{d\sigma}{dW} = \frac{2\pi r_e^2 mc^2}{\beta^2} \frac{1}{W^2}$$

$$\times \left[1 + \left(\frac{W}{T-W} \right)^2 + \frac{\tau^2}{(\tau+1)^2} \left(\frac{W}{T} \right)^2 - \frac{2\tau+1}{(\tau+1)^2} \left(\frac{W}{T-W} \right) \right]$$

Møller cross section (includes relativistic spin and exchange effects

POSITRONS:

$$W_m = T$$

$$\frac{d\sigma}{dW} = \frac{2\pi r_e^2 mc^2}{\beta^2} \frac{1}{W^2}$$

$$\times \left\{ 1 - a_B \left(\frac{W}{T}\right) + b_B \left(\frac{W}{T}\right)^2 - c_B \left(\frac{W}{T}\right)^3 \right.$$

$$\left. \times \left[1 - \frac{\tau}{2(\tau+1)} \left(\frac{W}{T}\right) \right] \right\}$$

Bhabha cross section (includes relativistic and spin effects)

$$a_B = \frac{\tau}{\tau+1} \left[\frac{\tau+2}{\tau+1} + \frac{\tau+3}{\tau+2} \right]$$

$$b_B = \left(\frac{\tau}{\tau+1}\right)^2 \left[3 + (\tau+2)^{-2}\right]$$

$$c_B = \frac{2\tau^3}{(\tau+1)(\tau+2)^2}$$

ELECTRONS AND POSITRONS

$$\frac{1}{\rho}\, S_{col} = \frac{2\pi\, r_e^2\, mc^2\, N_A}{\beta^2}\, \frac{Z}{A}$$

$$\times \left[\ln\left(\frac{T}{I^2}\right) + \ln\left(1 + \tau/2\right) - \delta + F^{\pm}(\tau) \right]$$

$$F^{-}(\tau) = \left(1 - \beta^2\right) \left[1 + \frac{\tau^2}{8} - (2\tau + 1)\,\ln 2 \right]$$

$$F^{+}(\tau) = 2\ln 2 - \frac{\beta^2}{12} \left[23 + \frac{14}{\tau + 2} + \frac{10}{(\tau + 2)^2} + \frac{4}{(\tau + 2)^3} \right]$$

MEAN EXCITATION ENERGY

- **Key, non-trivial, parameter I**

- **Sources of information on I**

Theoretical

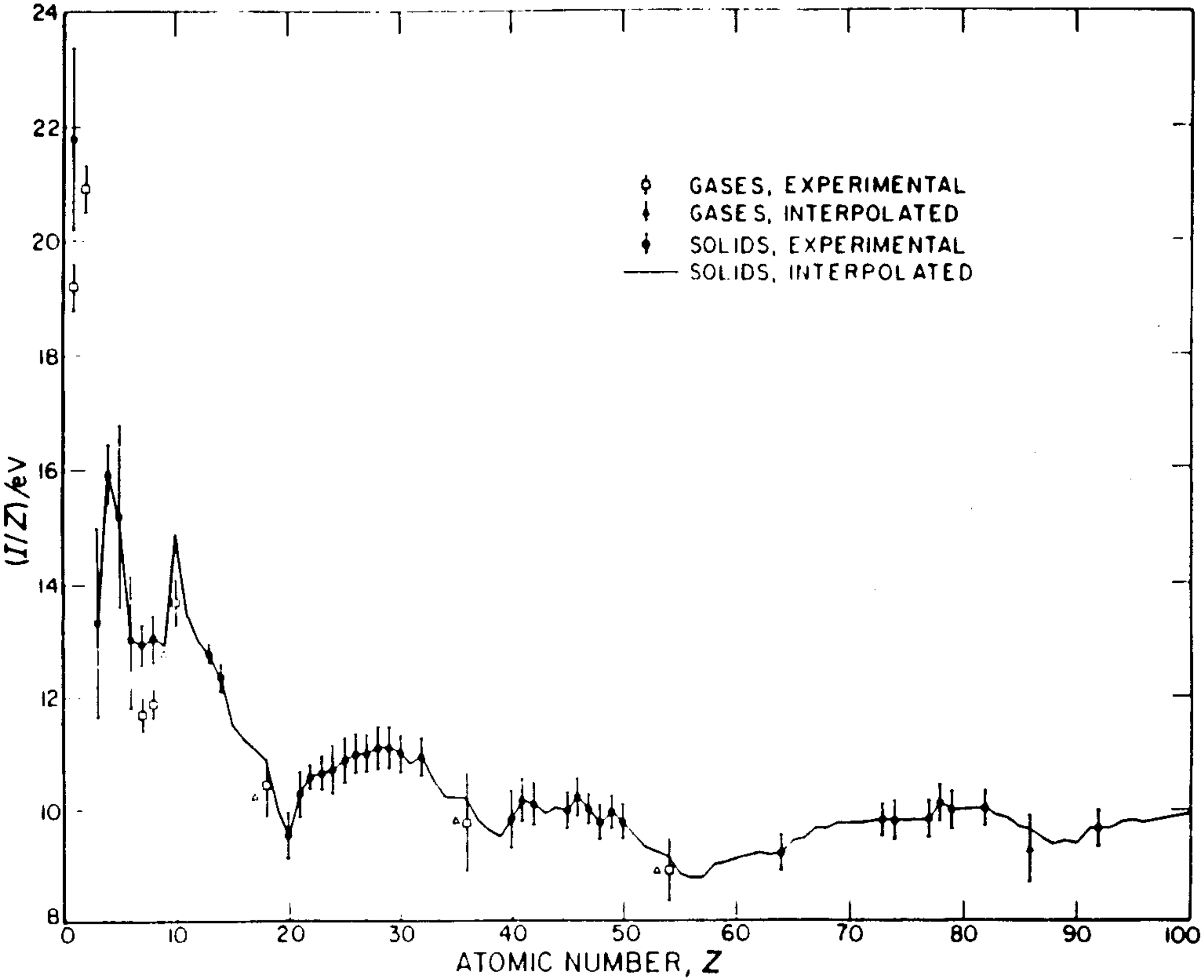
- Oscillator-strength data
- Dielectric-response function
- Local-plasma approximation

  (used here mainly for systematics)

Experimental

- Stopping-power and range data
- Mainly proton data; sensitive to estimates of

  shell corrections


GASES, EXPERIMENTAL
GASES, INTERPOLATED
SOLIDS, EXPERIMENTAL
SOLIDS, INTERPOLATED
(I/Z)/eV
ATOMIC NUMBER, Z

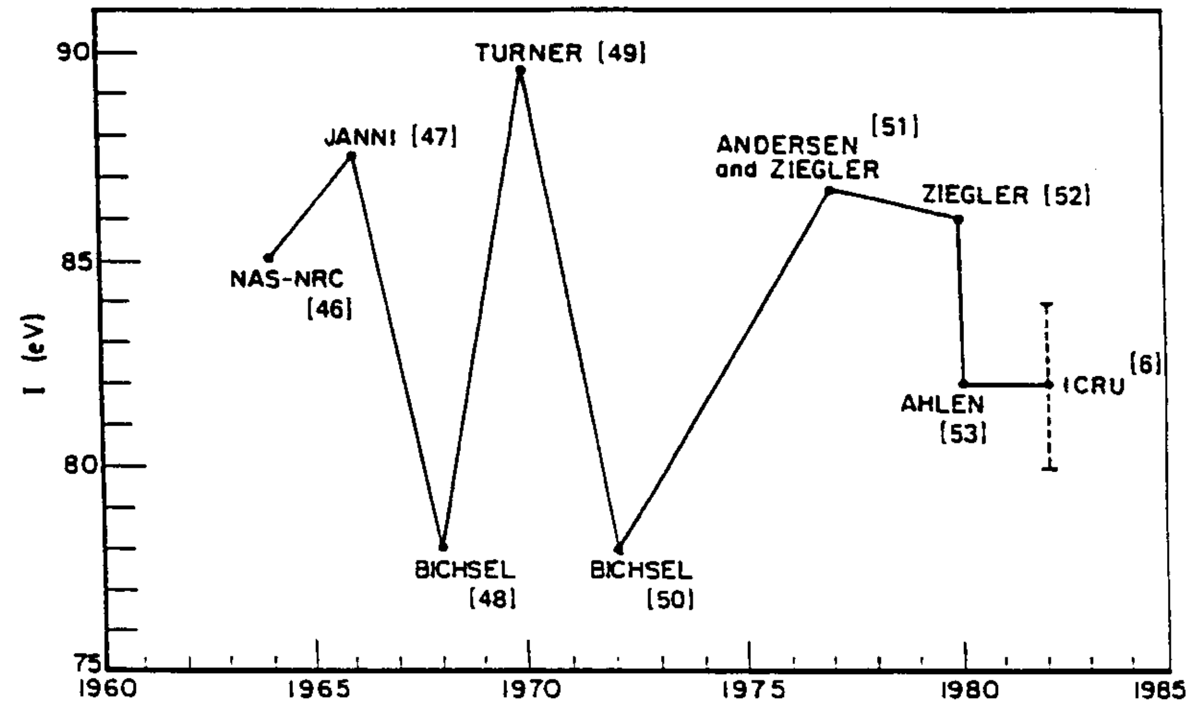


FIG. 3. Mean excitation energy for N_2 gas.

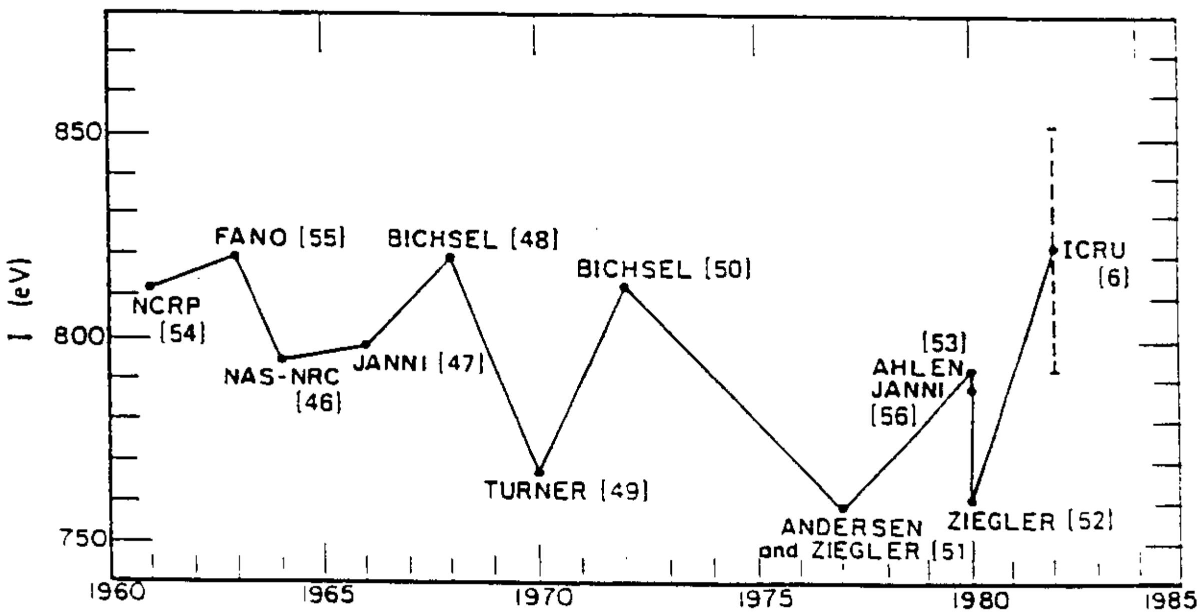


FIG. 4. Mean excitation energy for lead.

COMPOUNDS

Additivity Rule:

$$\langle Z/A \rangle = \sum_j w_j \frac{Z_j}{A_j}$$

$$\ln I = \langle Z/A \rangle^{-1} \sum w_j \frac{Z_j}{A_j} \ln I_j$$

$$w_j = \text{fraction by weight}$$

- **Accuracy improved by using I-values pertinent to compound**

TABLE 5.1—*Mean excitation energies adopted in the present work for atomic constituents of compounds*

Gases	
Constituent	I/eV
H	19.2
C	70
N	82
O	97

Liquids and Solids	
Constituent	I/eV
H	19.2
C	81
N	82
O	106
F	112
Cl	180
Others	$1.13 \times I^{a}$/eV

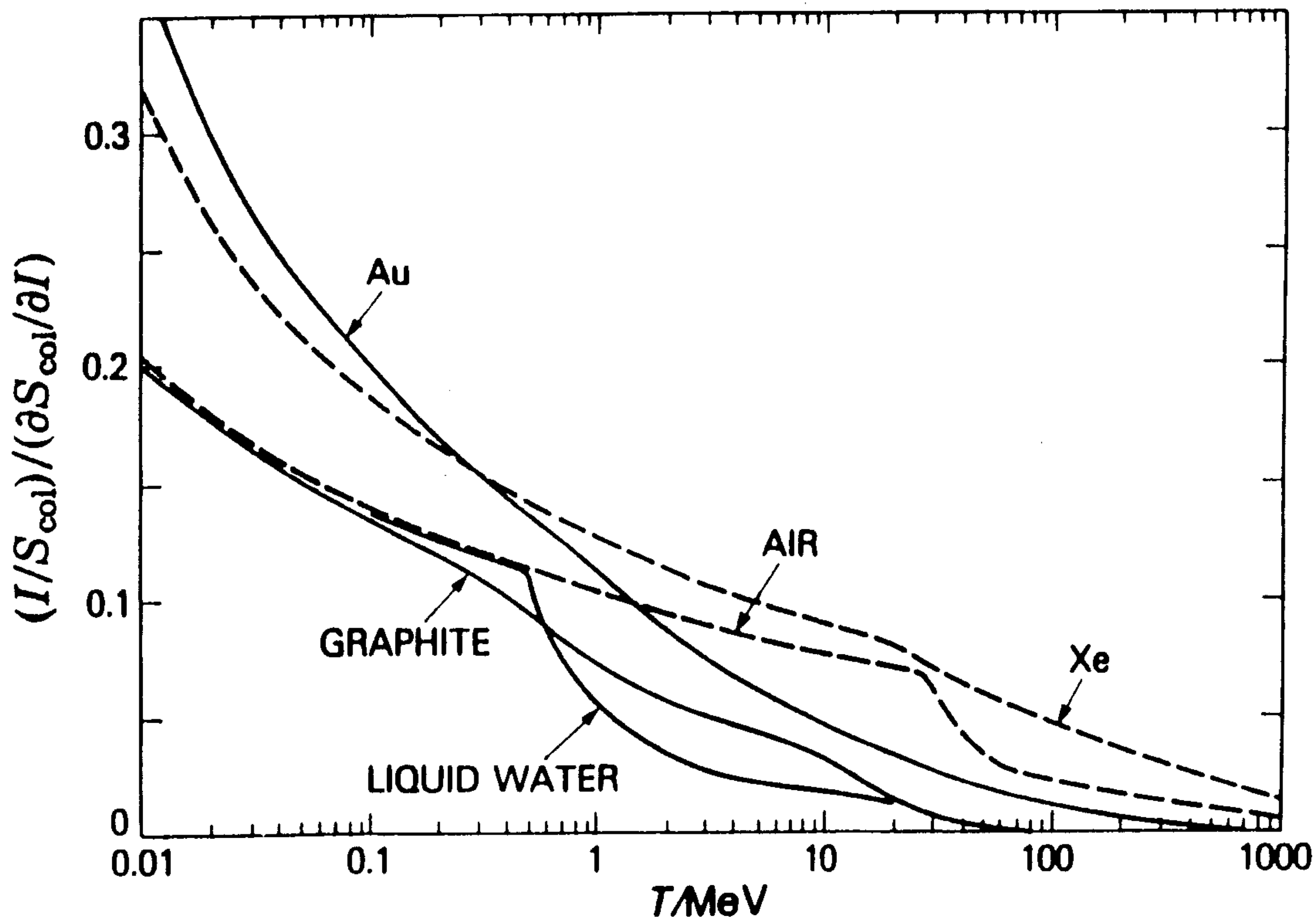
[a] I is the mean excitation energy for the element in the condensed phase given in Table 4.3.

Table 4
Values I (eV) derived from experimental measurements. The uncertainty ΔI (eV) is based on an uncertainty of $\pm 0.25\%$ of the experimental data. Also given are I-values from ref. [12] with their uncertainties.

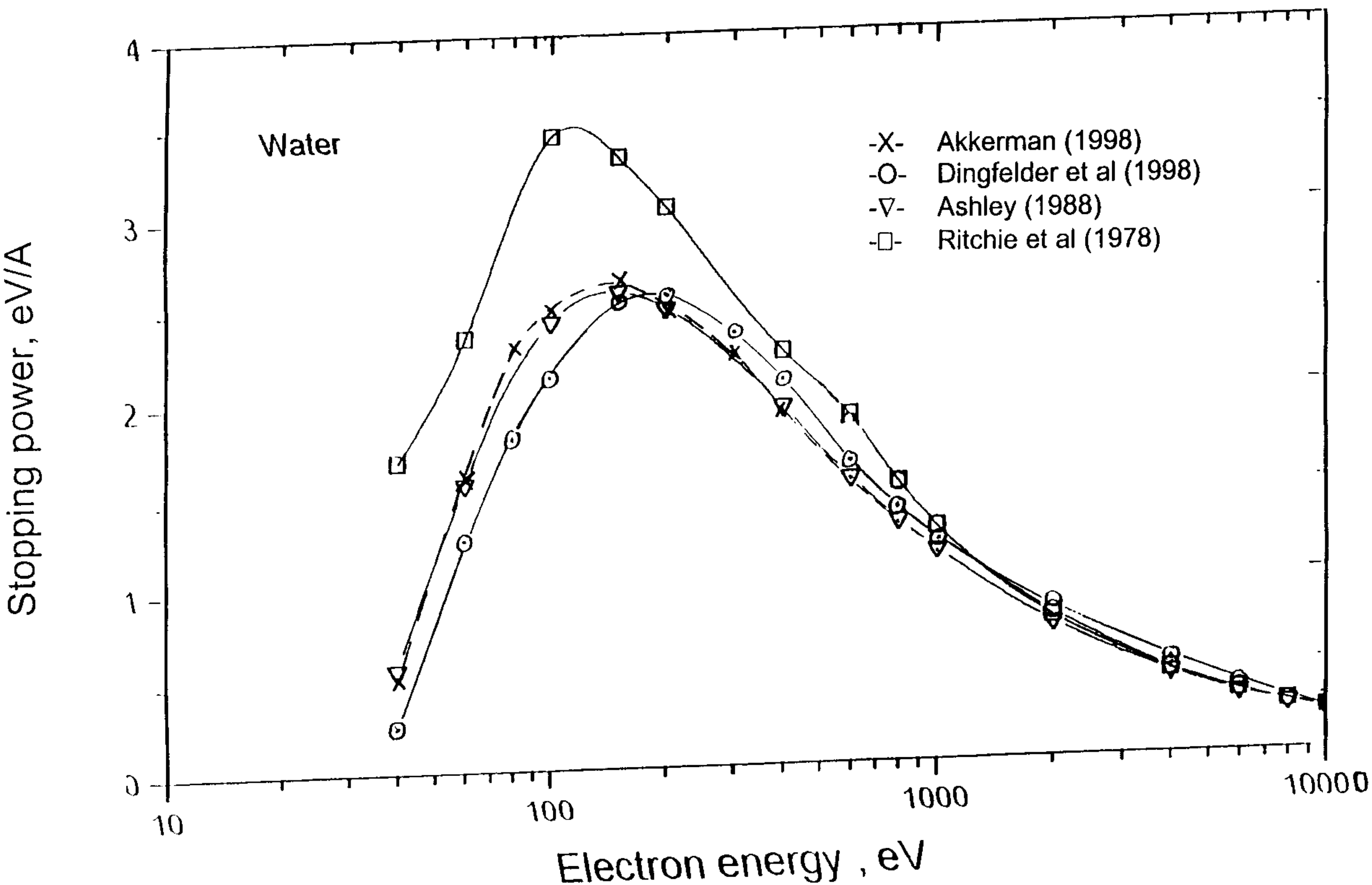
Z		I	ΔI	ref. [12]	ΔI
–	water	79.7	0.5	75	3
6	C	86.9	1.2	78	7
–	quartz	135.6	1.7	139.2	3
13	Al	166	–	166	2
14	Si	176	2	173	3
22	Ti	242	3	233	5
24	Cr	275	3	257	10
26	Fe	293	3	286	9
27	Co	311	3	297	9
28	Ni	322	3	311	10
29	Cu	339	4	322	10
30	Zn	336	4	330	10
40	Zr	417	4	393	15
41	Nb	431	4	417	15
42	Mo	440	4	424	15
47	Ag	489	5	470	10
48	Cd	518	5	469	20
49	In	503	5	488	20
50	Sn	508	5	488	15
73	Ta	736	6	718	30
74	W	766	7	727	30
82	Pb	816	7	823	30

Table 3. The mean excitation energy I for stopping power of water

Source, Year	Basis of determination	Value
	Bragg additivity	65 eV
Dalton and Turner, 1968 [73]	Stopping power of vapor	71.3 eV
Zeiss et al., 1977 [74]	From df/dE of vapor	71.6 eV
Ritchie et al. 1978 [75]	From $\eta(E)$ of liquid	75 eV
ICRU No. 37, 1984 [66]	Recommendation for vapor	71.6 eV
	Recommendation for liquid	75.0 eV
Bichsel and Hiraoka, 1992 [76]	Stopping power of liquid for protons of 30 - 70 MeV	79.8 eV
Dingfelder et al., 1998 [77]	From $\eta(E)$ of liquid	81.8 eV



0.3
0.2
0.1
0
0.01
0.1
1
10
100
1000
T/MeV
$(I/S_{col})/(\partial S_{col}/\partial I)$
Au
AIR
Xe
GRAPHITE
LIQUID WATER



Water
-X- Akkerman (1998)
-O- Dingfelder et al (1998)
-∇- Ashley (1988)
-□- Ritchie et al (1978)
Stopping power, eV/A
Electron energy , eV
0
1
2
3
4
10
100
1000
10000

DENSITY EFFECT

- **Passage of charged particle polarizes atoms in medium**

- **Polarization decreases EM field on particle**

- **Reduces electronic stopping power**

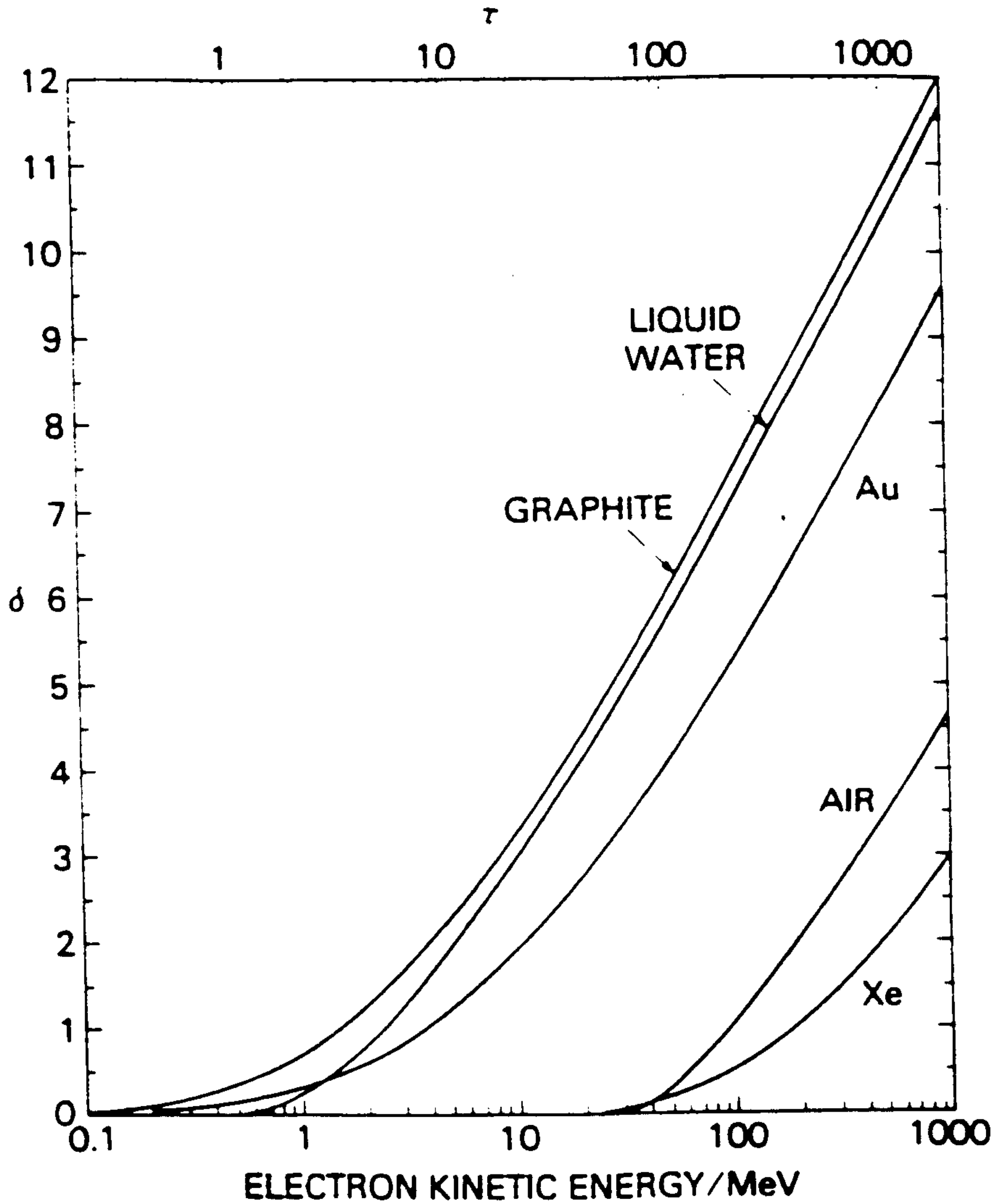
- **In limit of $\beta \to 1$**

$$\delta \to \ln \left[\frac{(\hbar\omega_p)^2}{(1-\beta^2)\, I^2} \right] - 1$$

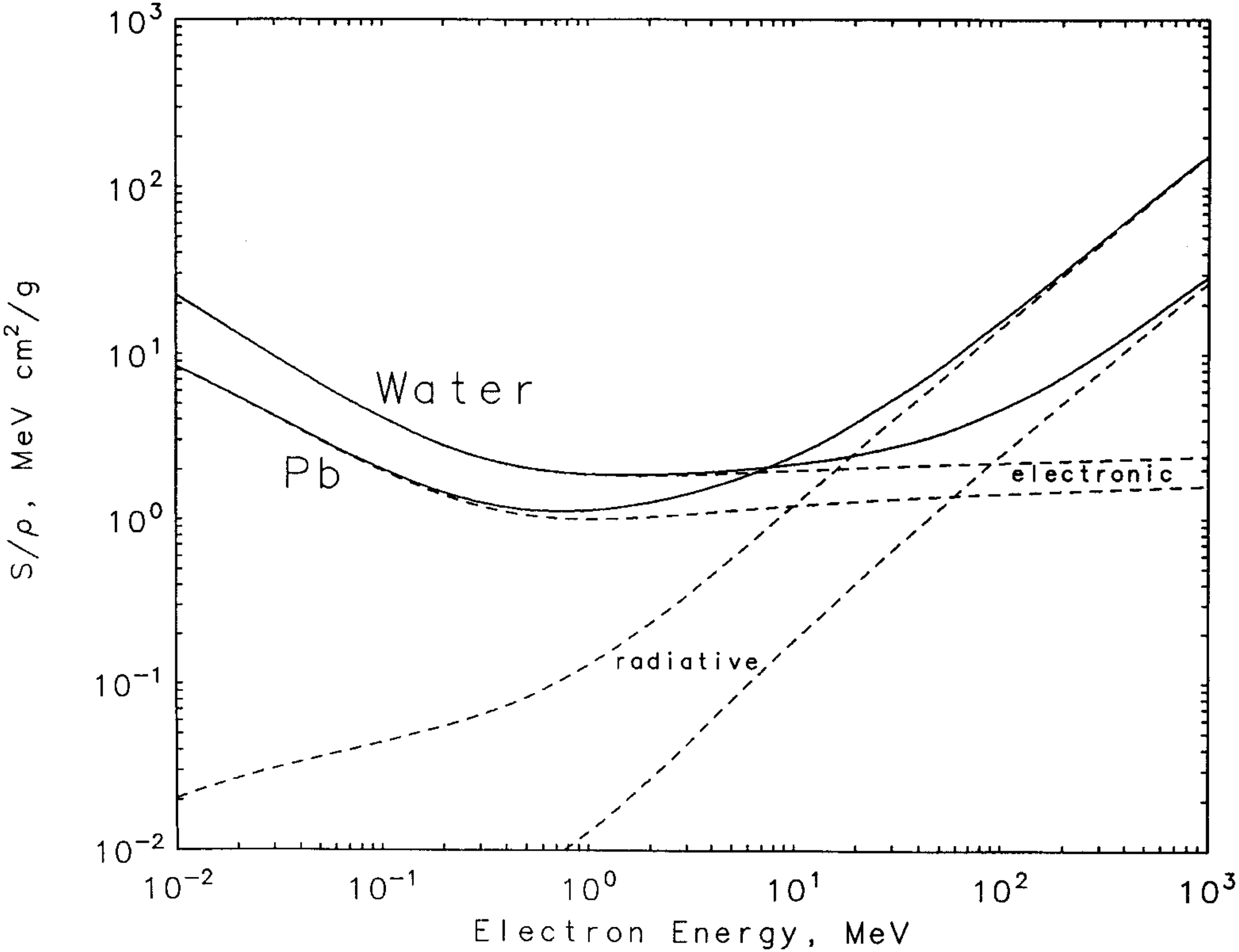
$$\hbar\,\omega_p = 28.8\ (\rho\, Z/A)^{1/2}\ \mathrm{eV} \qquad \text{plasma energy}$$

$$\frac{1}{\rho}\, S_{col} \to \frac{1}{\rho}\, S_{col}\ (\rho, Z/A)$$

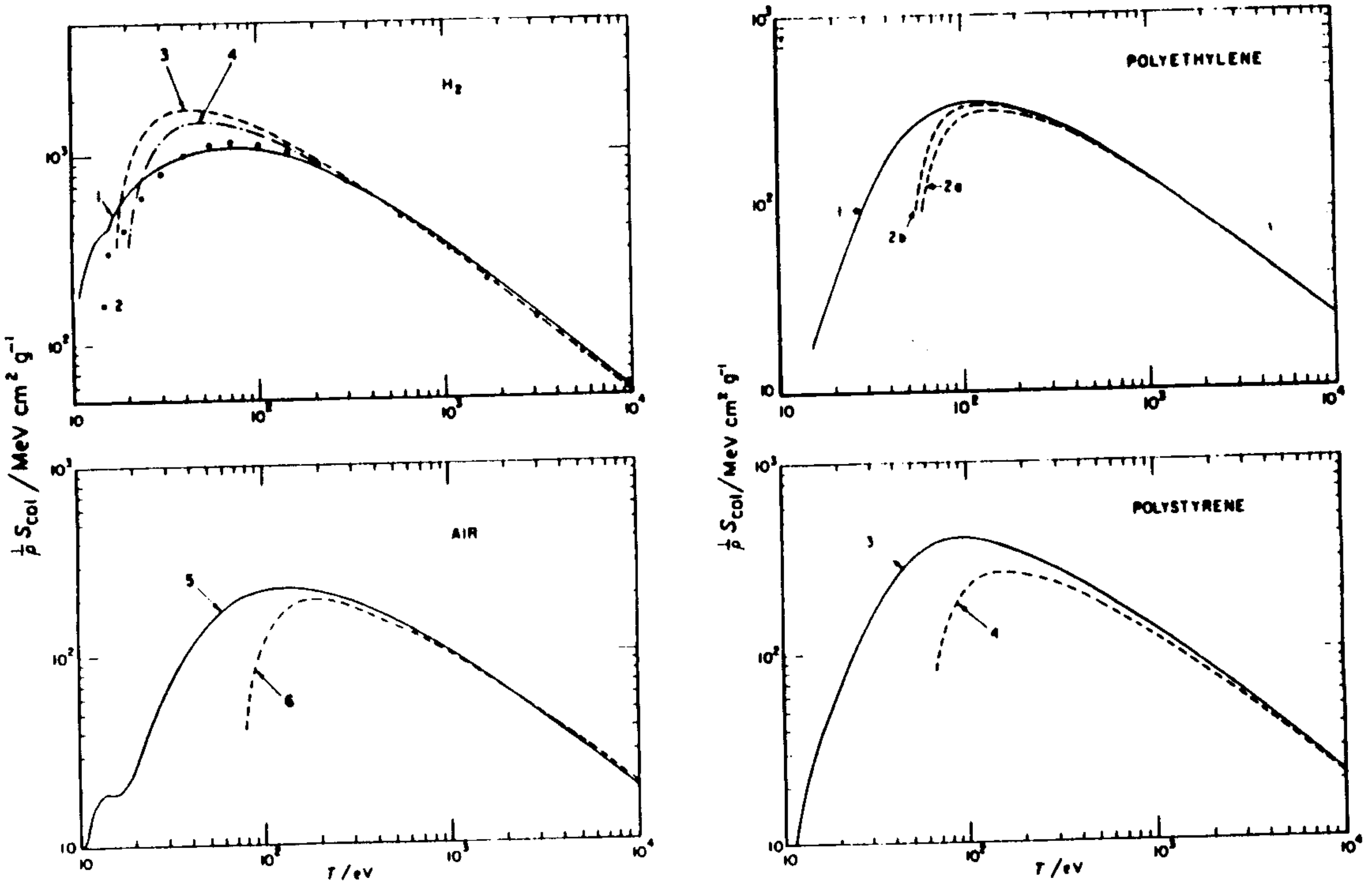
- **In general, calculate δ using Sternheimer's model**



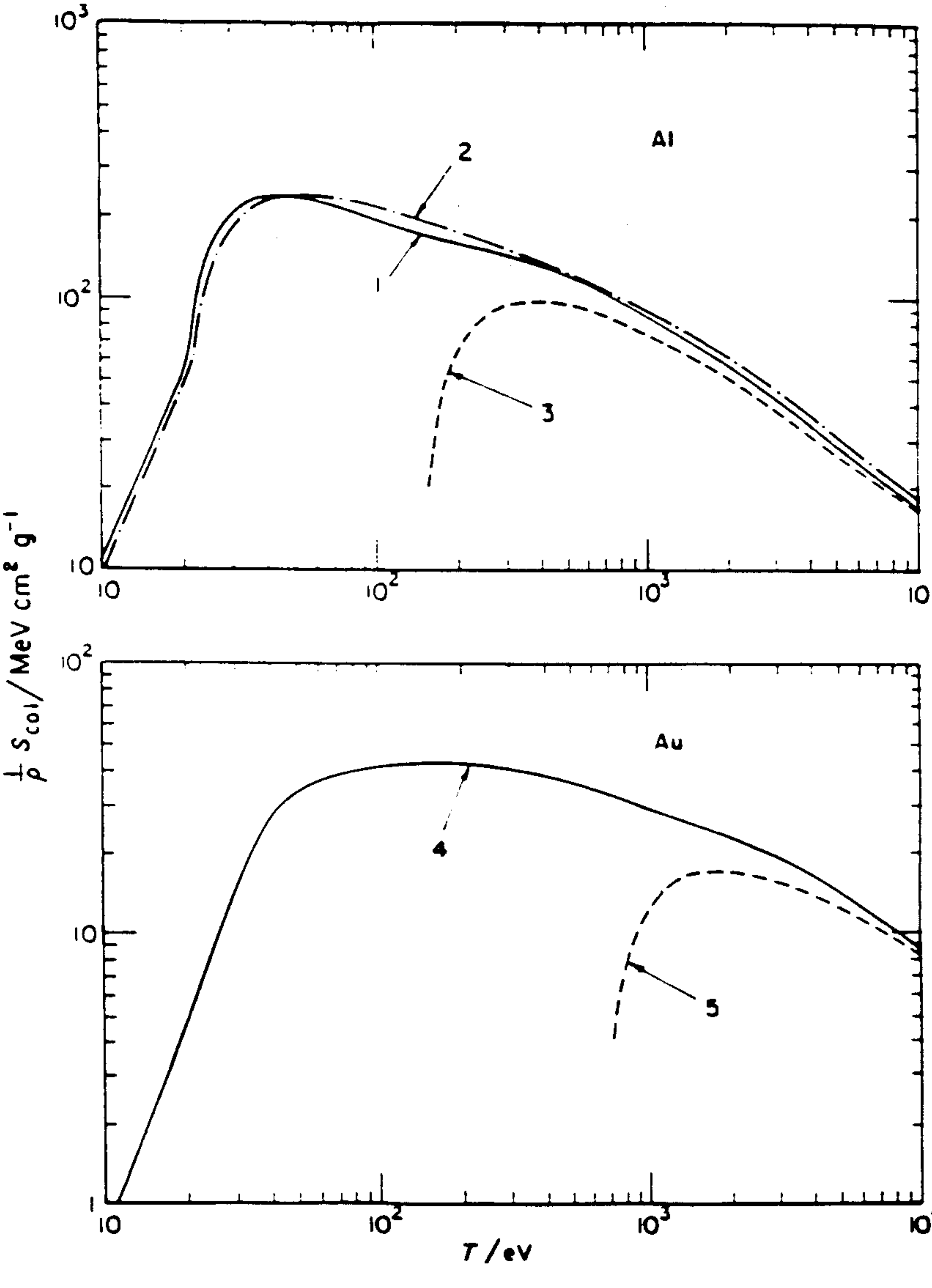
τ
1
10
100
1000
12
11
10
9
8
7
δ 6
5
4
3
2
1
0
LIQUID WATER
GRAPHITE
Au
AIR
Xe
0.1
1
10
100
1000
ELECTRON KINETIC ENERGY/MeV



Water
Pb
electronic
radiative
S/ρ, MeV cm^2/g
Electron Energy, MeV
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}



H$_2$
1
2
3
4
AIR
5
6
POLYETHYLENE
1
2a
2b
POLYSTYRENE
3
4
$\frac{1}{\rho} S_{col}$ / MeV cm^2 g^{-1}
T / eV
10
10^2
10^3
10^4



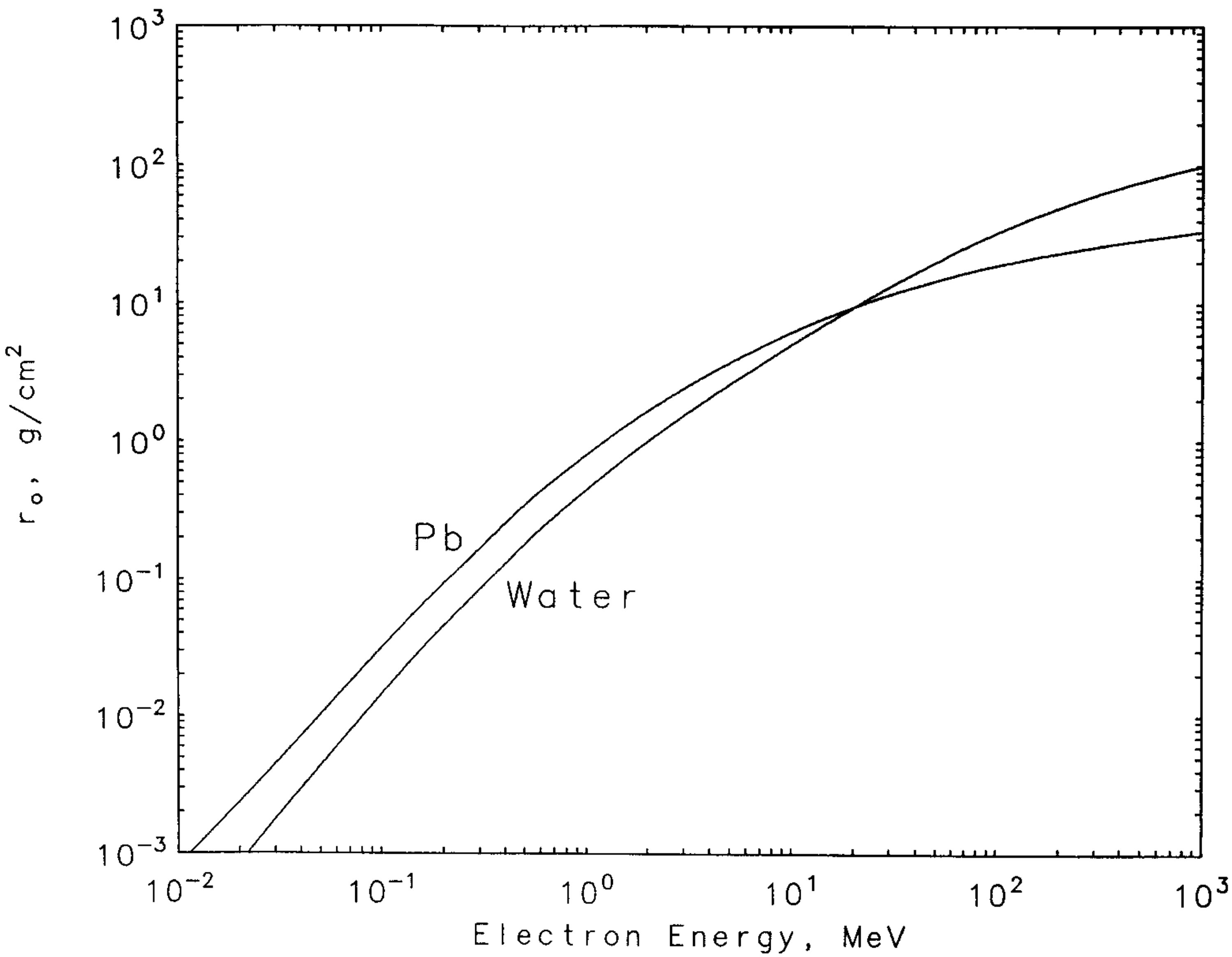
Al
Au
1
2
3
4
5
10
10²
10³
10⁴
1
T / eV
(1/ρ) S_col / MeV cm² g⁻¹

Electron Interactions

Range

$$r_0(T) = \int_0^T \frac{dT'}{S(T')}$$

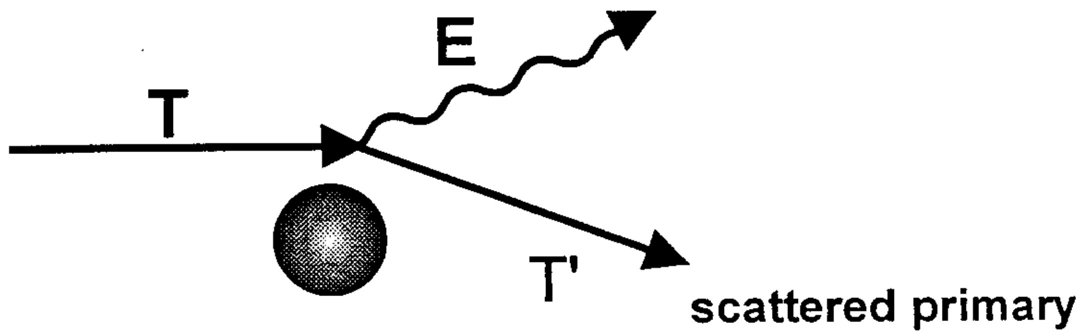
- $S = S_{el} + S_{rad}$
- Continuous-slowing-down approximation (csda)
- Total pathlength, not penetration distance



r_o, g/cm^2
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Pb
Water
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Electron Energy, MeV

Electron Interactions

Bremsstrahlung production



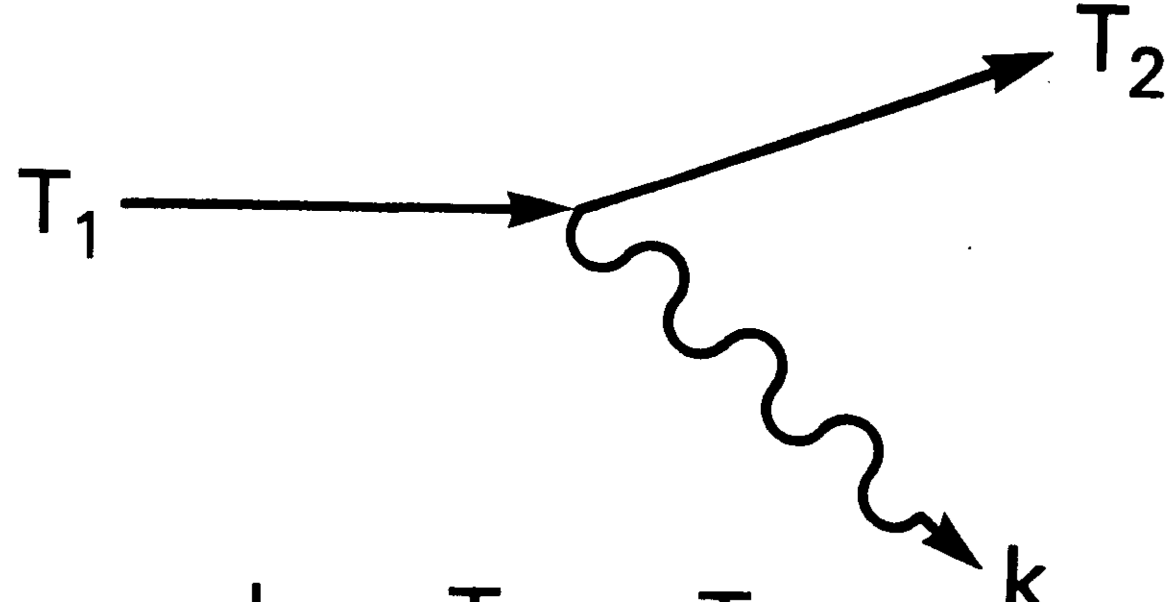
$$E = T - T'$$

- $\sigma_n \sim Z^2$ in field of nucleus
- $\sigma_e \sim Z$ in field of atomic electrons
- Radiative stopping power-<u>average</u> energy loss by bremsstrahlung per unit pathlength

$$\frac{1}{\rho} S_{rad} \sim \frac{Z(Z+1)}{A}$$

BREMSSTRAHLUNG CROSS SECTION

$$\frac{d\sigma}{dk}$$



$$k = T_1 - T_2$$

BREMSSTRAHLUNG PRODUCTION CROSS SECTIONS

(Electrons)

$$\frac{d\sigma}{dk} = \frac{d\sigma_n}{dk} + Z\,\frac{d\sigma_e}{dk} = \left[1 + \frac{\eta}{Z}\right]\frac{d\sigma_n}{dk}$$

$\frac{d\sigma_n}{dk}$: electron-nucleus cross section

$\frac{d\sigma_e}{dk}$: electron-electron cross section

$$\eta = \frac{d\sigma_e}{dk} \Big/ \left[\frac{1}{Z^2}\right]\frac{d\sigma_n}{dk}$$

(often assumed to be unity)

ELECTRON-NUCLEUS BREMSSTRAHLUNG

High energies (T_1 > 50 MeV)

$$\frac{d\sigma_n}{dk} = \frac{4\alpha r_e^2 Z^2}{k}\left[\chi_{\text{Born}}^{\text{unscr}} + \delta_{\text{screen}} + \delta_{\text{Coul}}\right]$$

$\chi_{\text{Born}}^{\text{unscr}}$ = Bethe-Heitler, Born-approximation result for unscreened nucleus (no energy approximations)

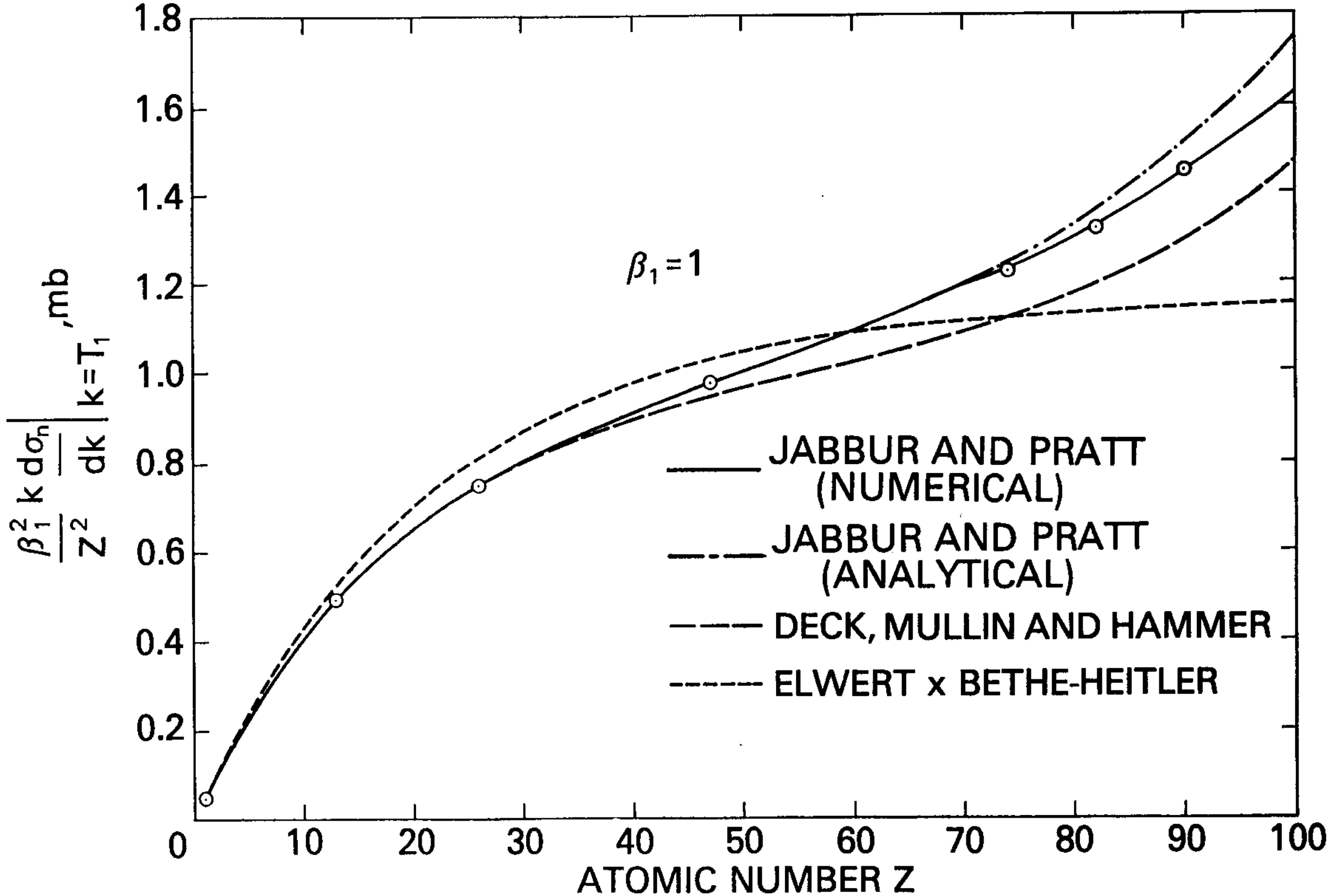
$\delta_{\text{screen}} = {}^{\text{HE}}\chi_{\text{Born}}^{\text{scr}} - {}^{\text{HE}}\chi_{\text{Born}}^{\text{unscr}}$ high-energy approx. using Hartree-Fock form factors in calculation of screening functions

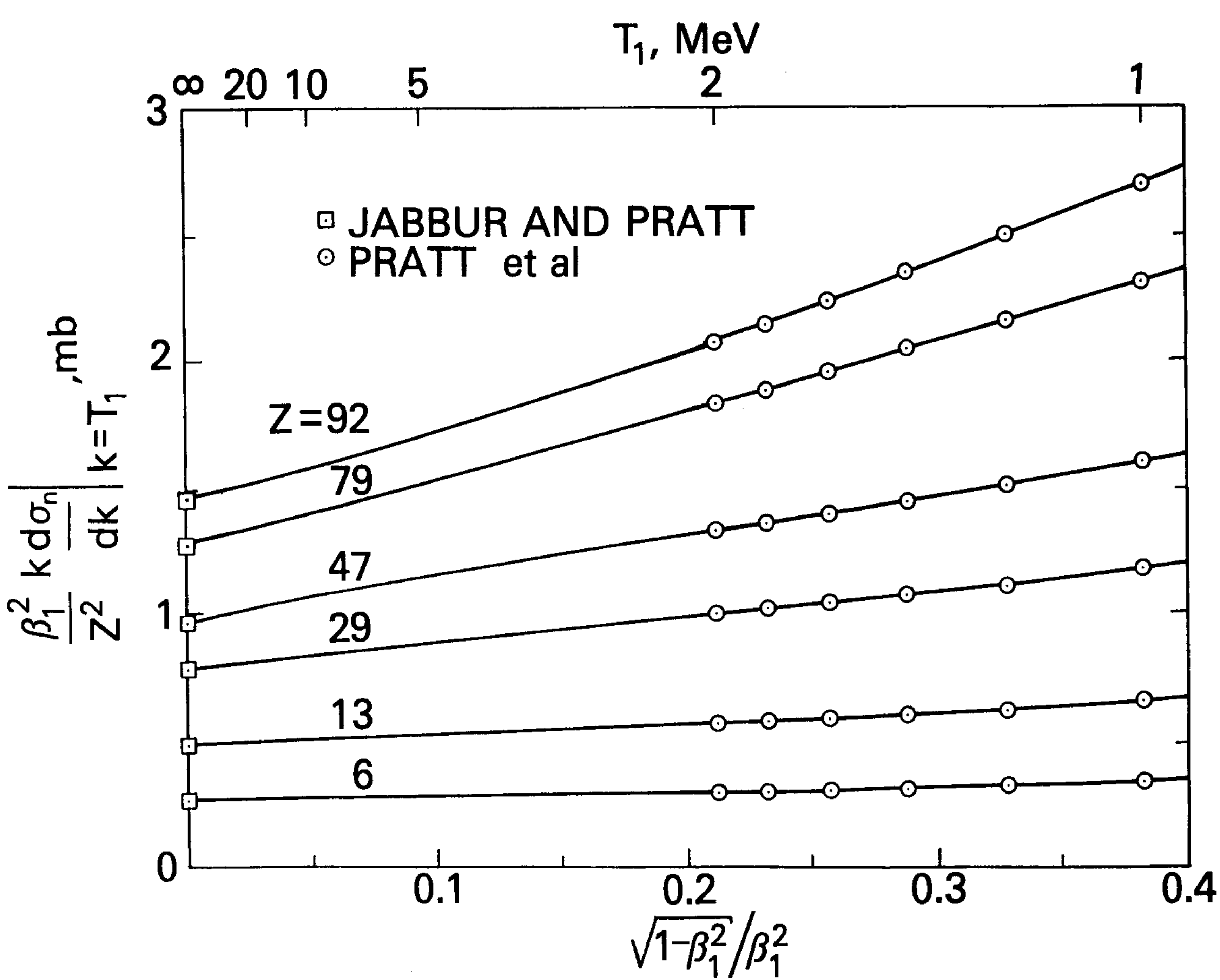
$$\delta_{\text{Coul}} = \left\{f_E \exp\left[\varsigma\left(1-\beta_2^2/\beta_1^2\right)\right] - 1\right\}\chi_{\text{Born}}^{\text{unscr}} + \omega(T_2)\,\delta_{\text{Coul}}^{\text{DBMO}}$$

f_E = Elwert factor

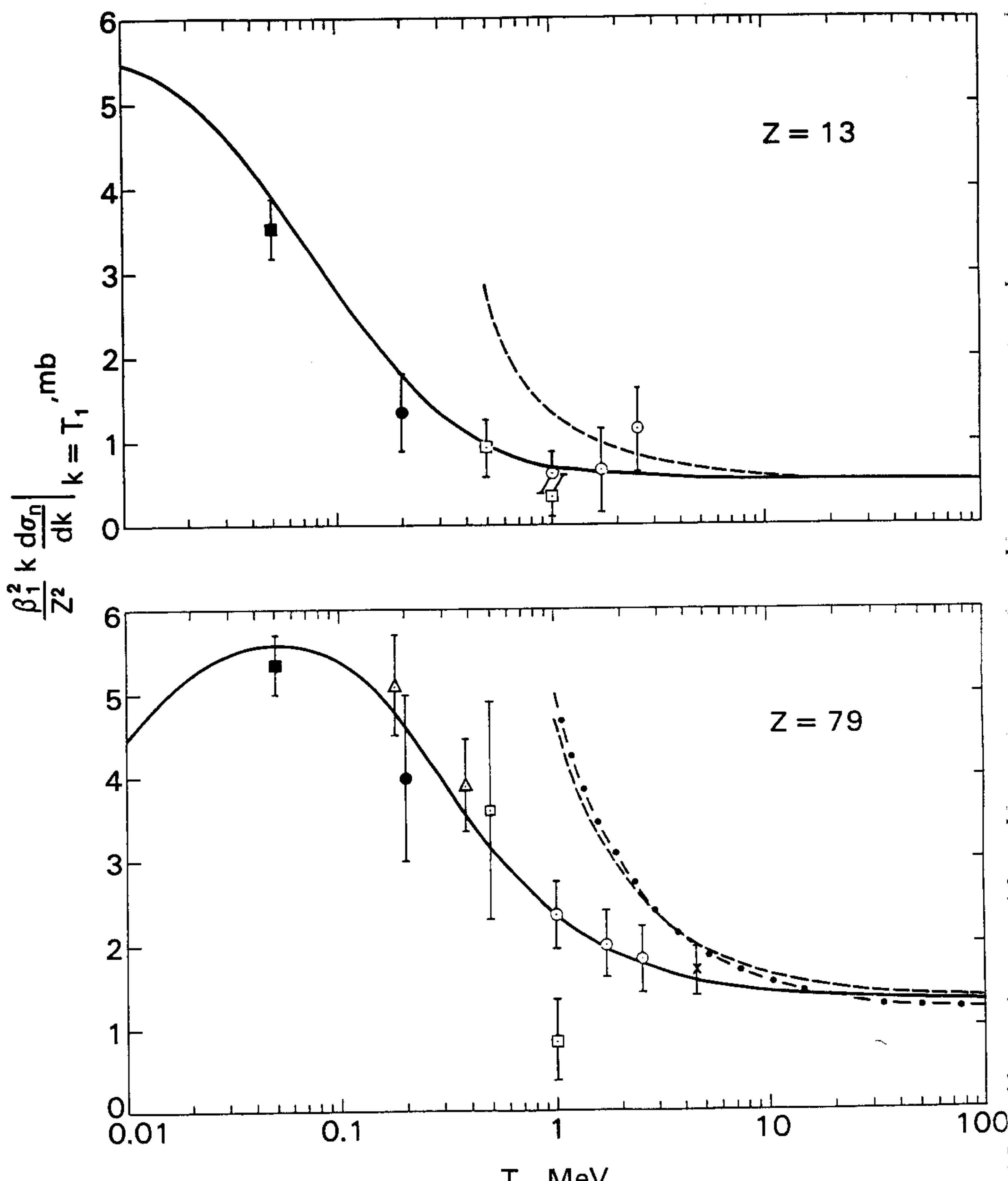
$\delta_{\text{Coul}}^{\text{DBMO}}$ = high-energy Coulomb correction

Two factors combined to give good tip values

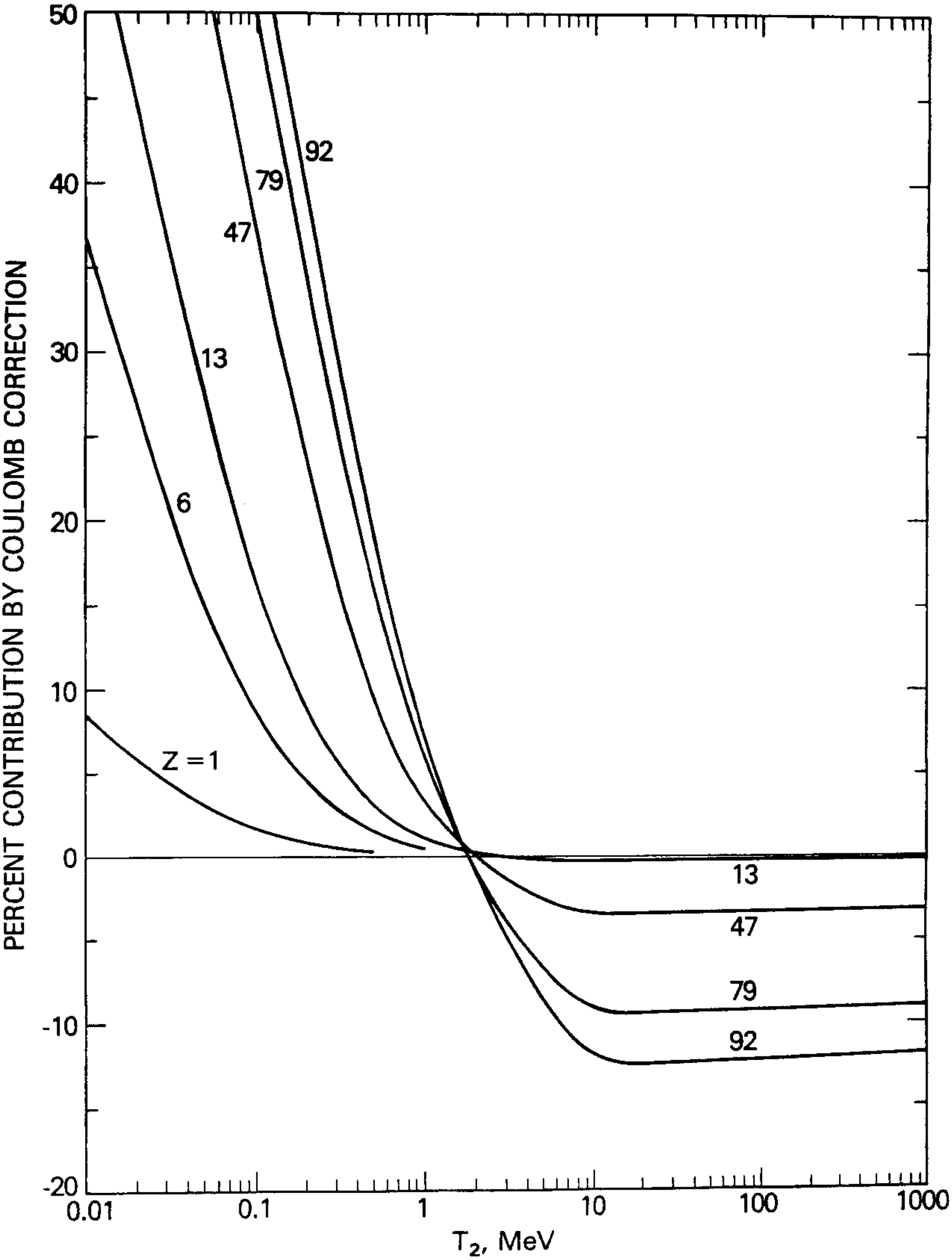

1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
10
20
30
40
50
60
70
80
90
100
ATOMIC NUMBER Z
$\frac{\beta_1^2}{Z^2} \frac{k\, d\sigma_n}{dk}\Big|_{k=T_1}$, mb
$\beta_1 = 1$
JABBUR AND PRATT (NUMERICAL)
JABBUR AND PRATT (ANALYTICAL)
DECK, MULLIN AND HAMMER
ELWERT x BETHE-HEITLER



T1, MeV
∞ 20 10 5 2 1
JABBUR AND PRATT
PRATT et al
Z=92
79
47
29
13
6
0 1 2 3
0.1 0.2 0.3 0.4
$\frac{\beta_1^2}{Z^2}\frac{k\,d\sigma_n}{dk}\Big|_{k=T_1}$, mb
$\sqrt{1-\beta_1^2}/\beta_1^2$



Z = 13
Z = 79
$\frac{\beta_1^2}{Z^2} k \frac{d\sigma_n}{dk}\Big|_{k=T_1}$, mb
T_1, MeV
0
1
2
3
4
5
6
0.01
0.1
1
10
100



PERCENT CONTRIBUTION BY COULOMB CORRECTION
50
40
30
20
10
0
-10
-20
0.01
0.1
1
10
100
1000
T_2, MeV
Z = 1
6
13
47
79
92
13
47
79
92

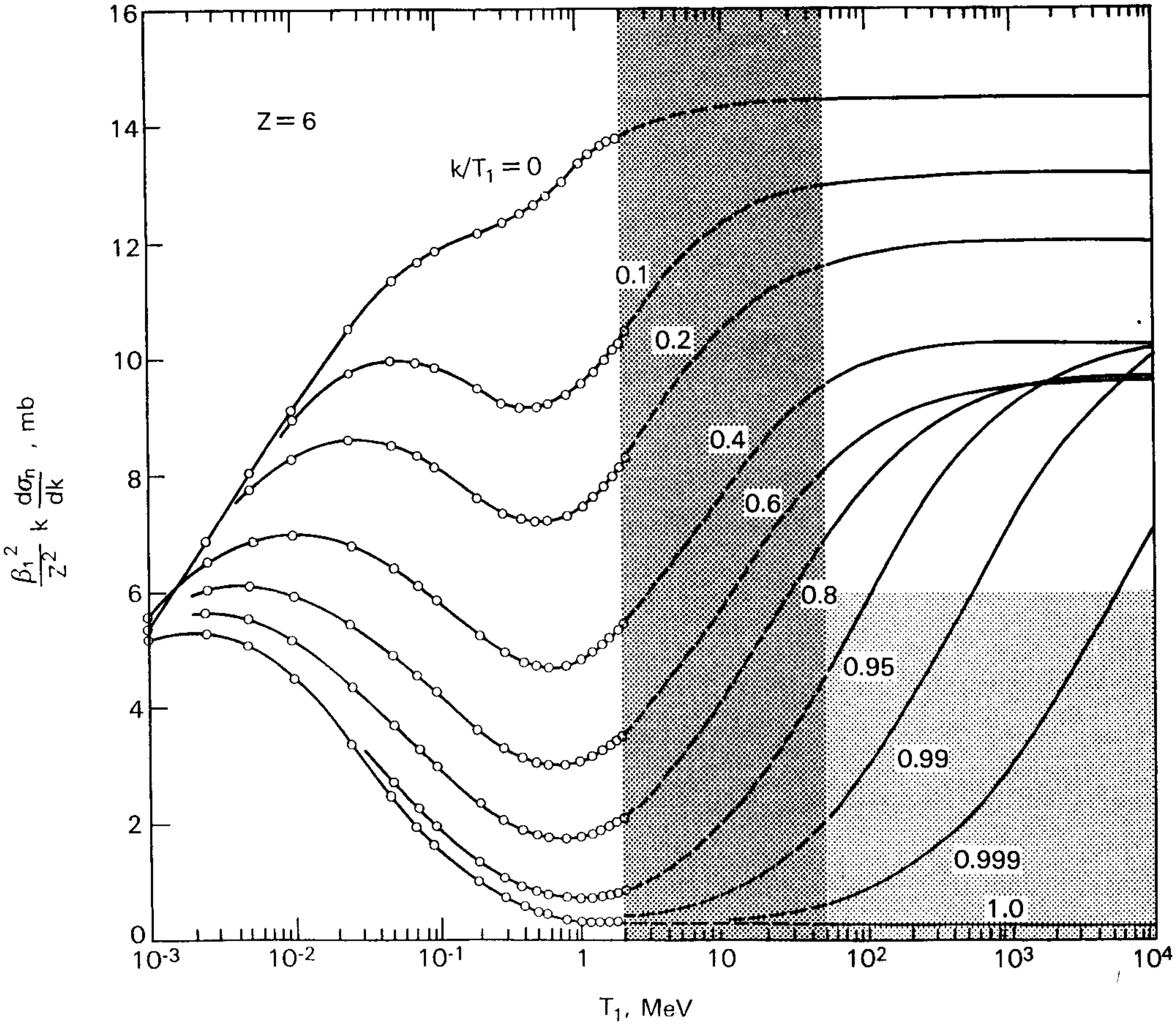
ELECTRON-NUCLEUS BREMSSTRAHLUNG

Lower energies

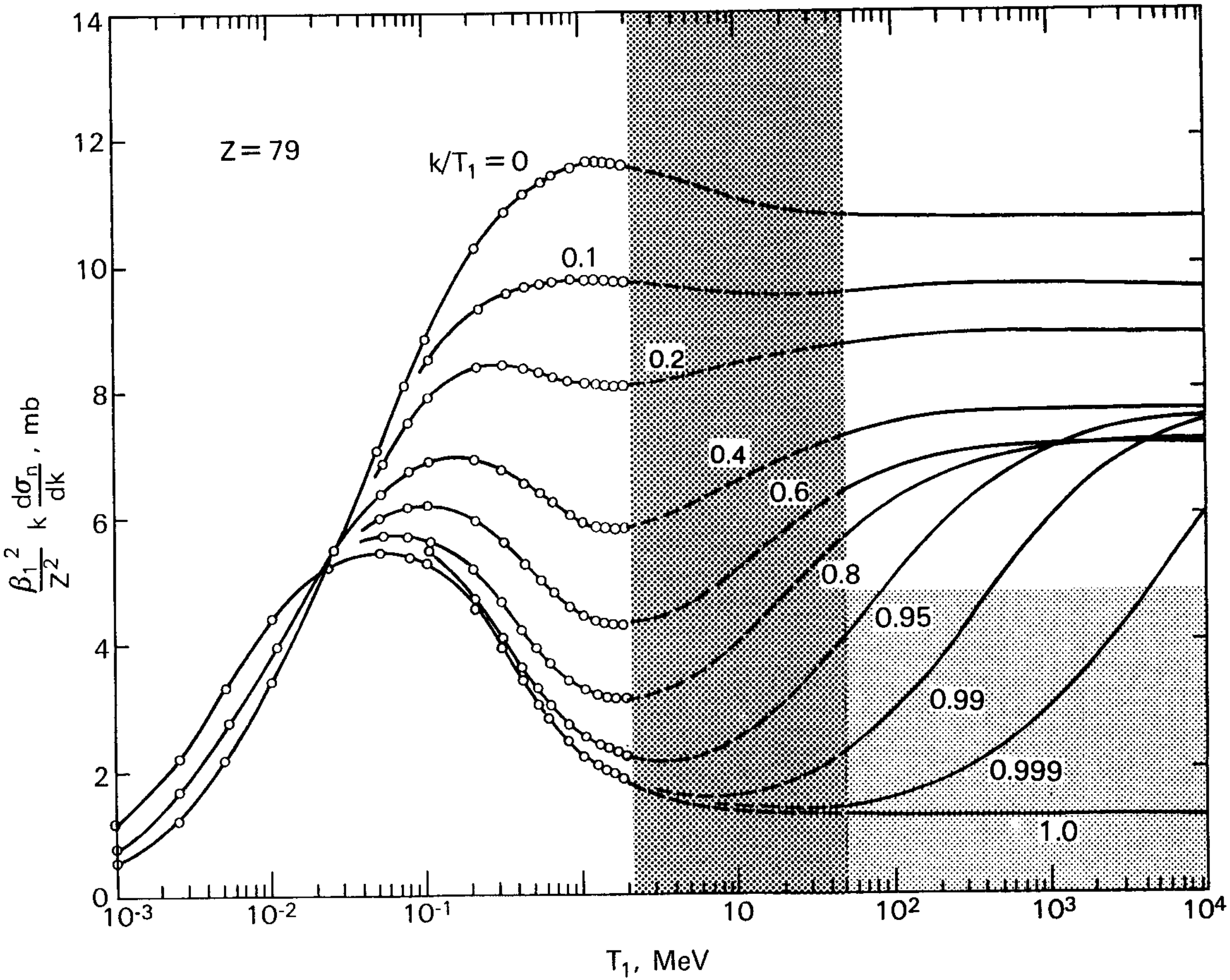
- Analytical results become unreliable

- Use results from Pratt, Tseng *et al.*, exact phase-shift calculations for
  $1 \text{ keV} \leq T_1 \leq 2 \text{ MeV}$

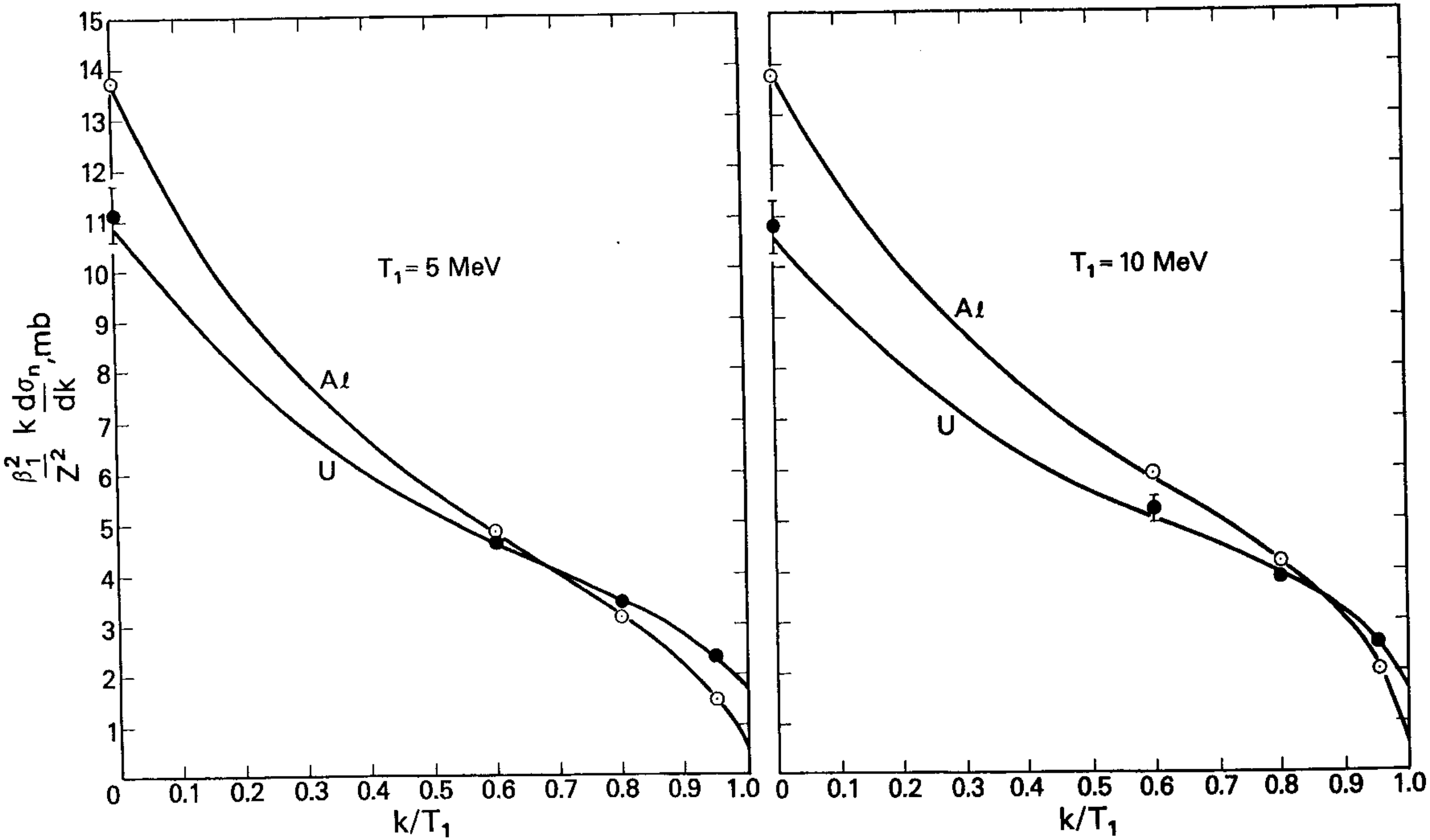
- Interpolate to fill in gap from 2 to 50 MeV



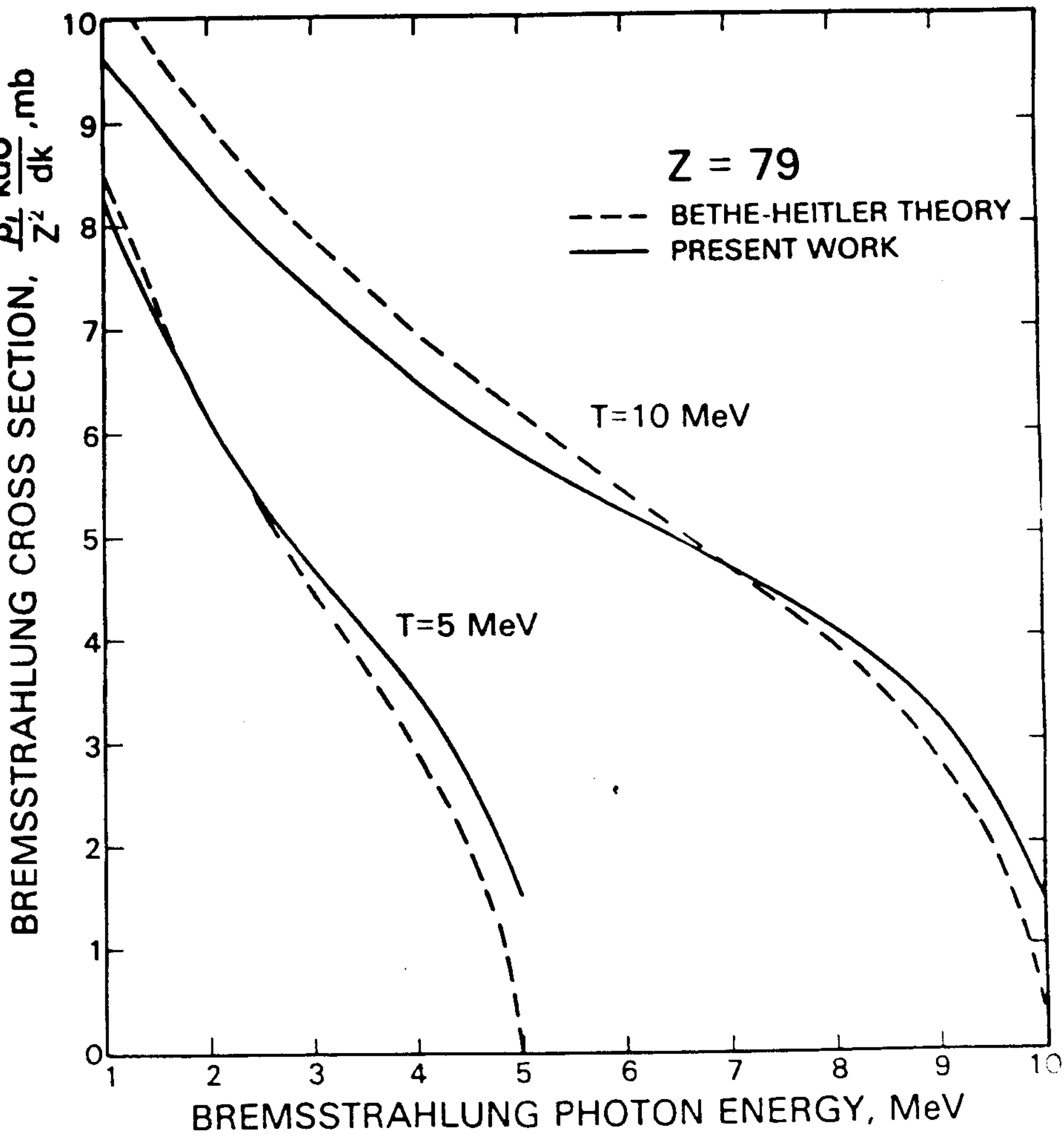
Z = 6
$k/T_1 = 0$
0.1
0.2
0.4
0.6
0.8
0.95
0.99
0.999
1.0
$\frac{\beta_1^2}{Z^2} k \frac{d\sigma_n}{dk}$, mb
0
2
4
6
8
10
12
14
16
10^{-3}
10^{-2}
10^{-1}
1
10
10^2
10^3
10^4
T_1, MeV



Z = 79
k/T1 = 0
0.1
0.2
0.4
0.6
0.8
0.95
0.99
0.999
1.0
β1² / Z² k dσn/dk , mb
T1, MeV



$\frac{\beta_1^2}{Z^2} \frac{k\,d\sigma_n}{dk}$, mb
$T_1 = 5$ MeV
$T_1 = 10$ MeV
Aℓ
U
k/T_1



Z = 79
BETHE-HEITLER THEORY
PRESENT WORK
T=10 MeV
T=5 MeV
BREMSSTRAHLUNG CROSS SECTION, $\frac{\beta_1^2}{Z^2}\frac{kd\sigma^{(m)}}{dk}$, mb
BREMSSTRAHLUNG PHOTON ENERGY, MeV
0
1
2
3
4
5
6
7
8
9
10

BREMSSTRAHLUNG PRODUCTION CROSS SECTIONS

(Electrons)

$$\frac{d\sigma}{dk} = \frac{d\sigma_n}{dk} + Z\,\frac{d\sigma_e}{dk} = \left[1 + \frac{\eta}{Z}\right]\frac{d\sigma_n}{dk}$$

$\dfrac{d\sigma_n}{dk}$: **electron-nucleus cross section**

$\dfrac{d\sigma_e}{dk}$: **electron-electron cross section**

$$\eta = \frac{d\sigma_e}{dk} \Big/ \left[\frac{1}{Z^2}\right]\frac{d\sigma_n}{dk}$$

(often assumed to be unity)

ELECTRON-ELECTRON BREMSSTRAHLUNG

$\eta \neq 1$

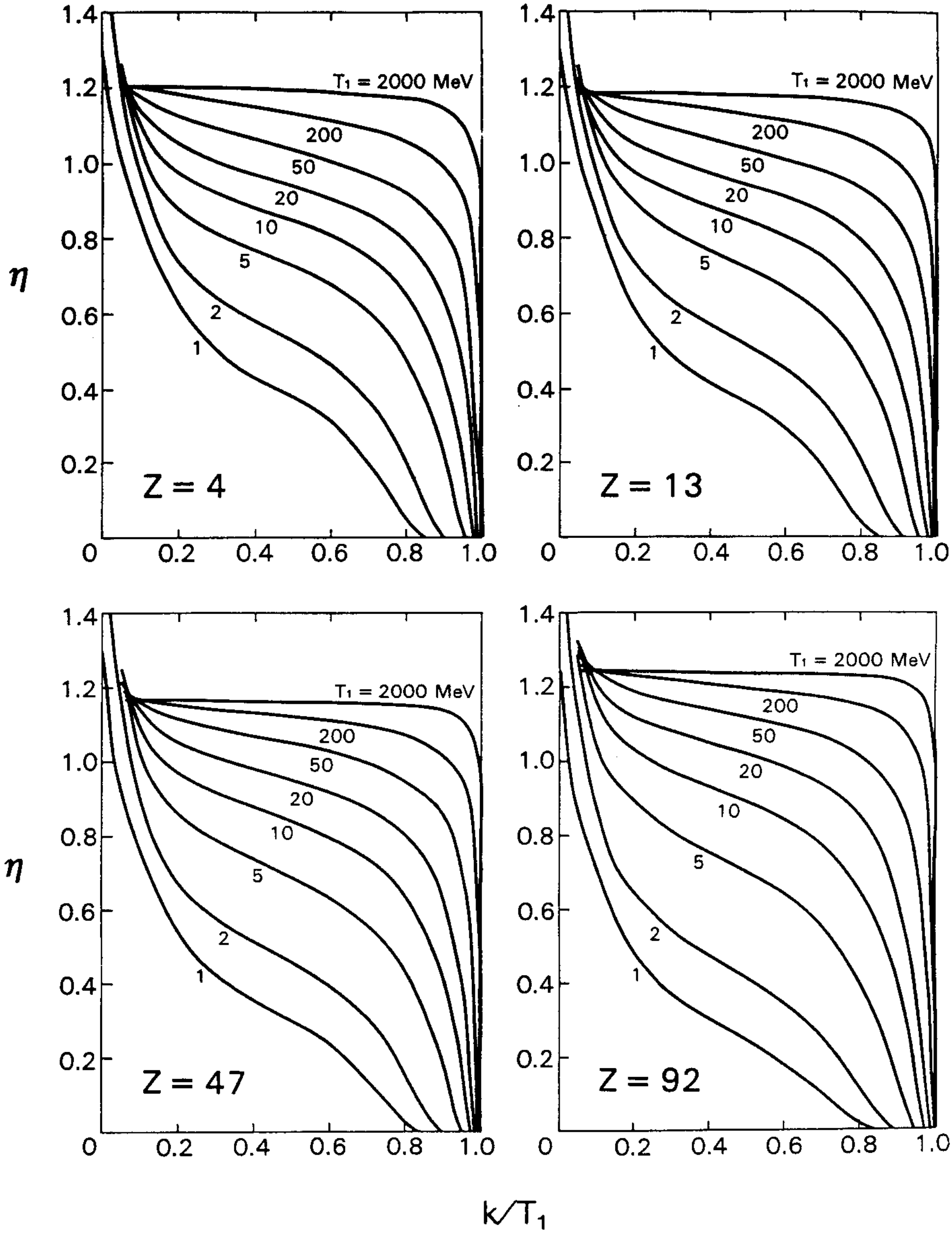
- Screening of atomic electrons different than e-n brems
- e-e brems goes to zero at high-frequency limit
- low-energy brems component due to recoiling target electron
- kinematic upper limit on emitted photon energy

$$T_1 < k_{max} < T_1/2$$

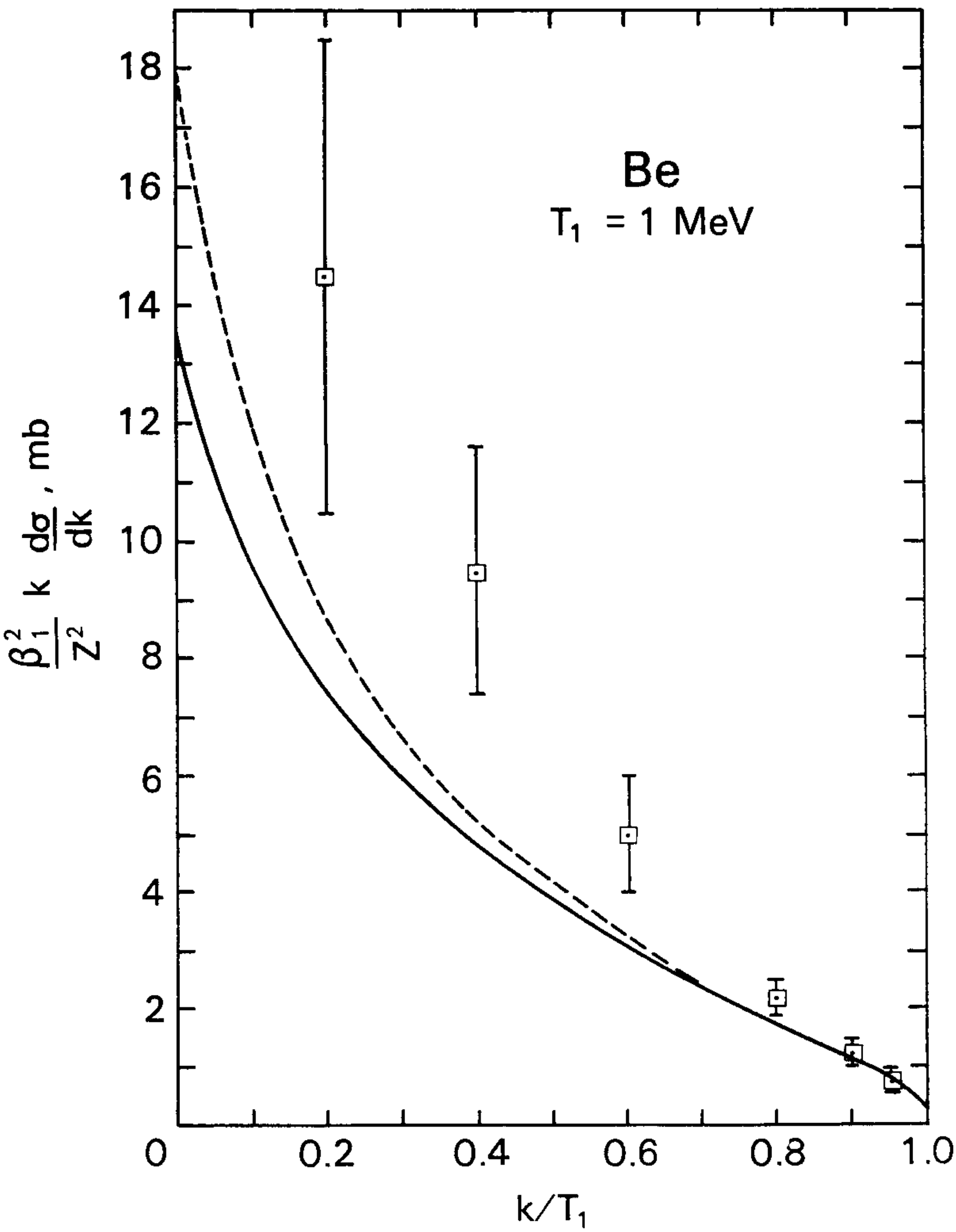
$$(\beta_1 = 1) \qquad\qquad (\beta_1 = 0)$$

- e-e brems cross section vanishes at low energies (no dipole moment)

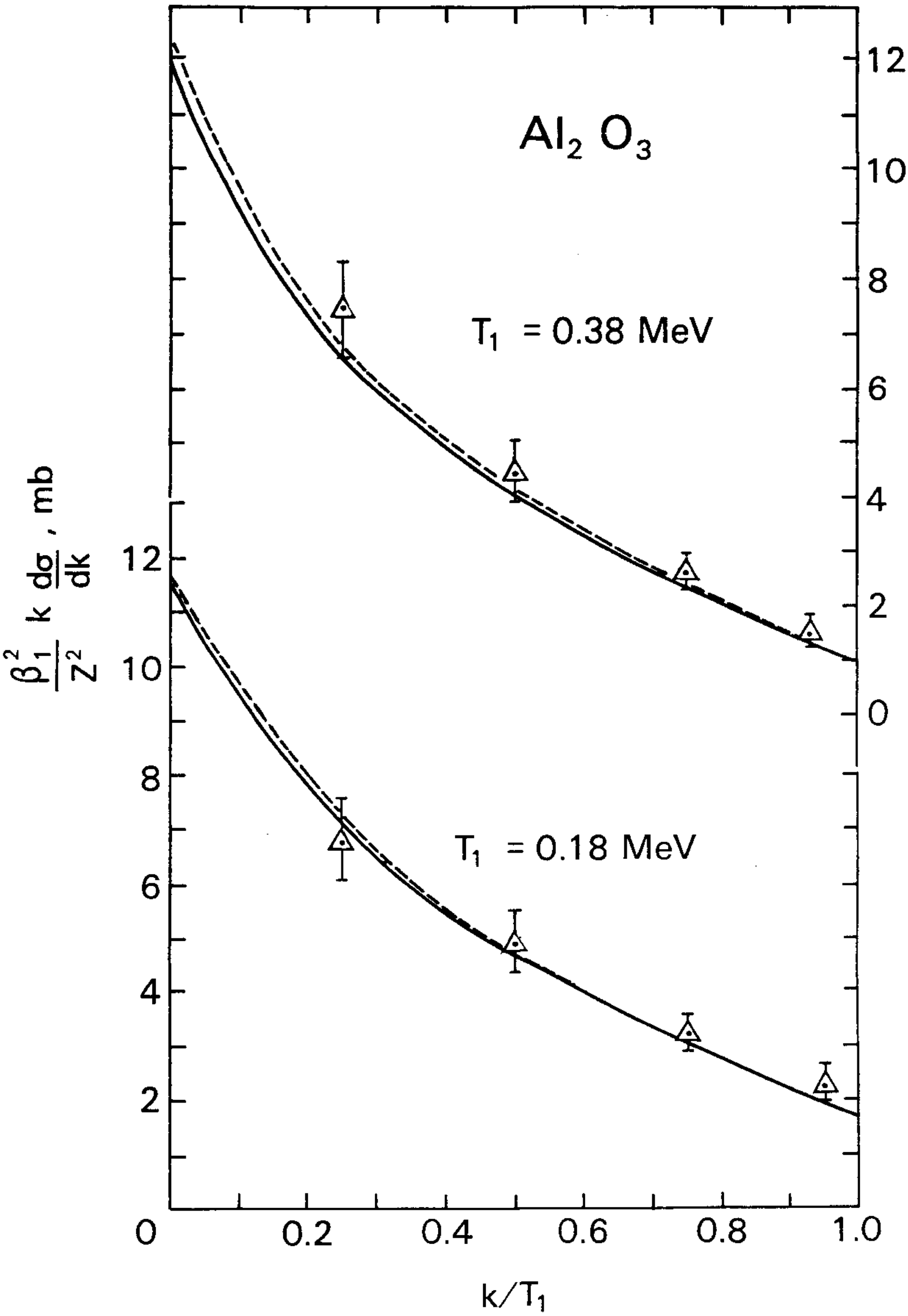
$$\frac{d\sigma_e}{dk} = \frac{4\alpha r_e^2}{k} \left\{ f_{e-e}\, \chi_{Haug} + \delta^e_{screen} \right\}$$



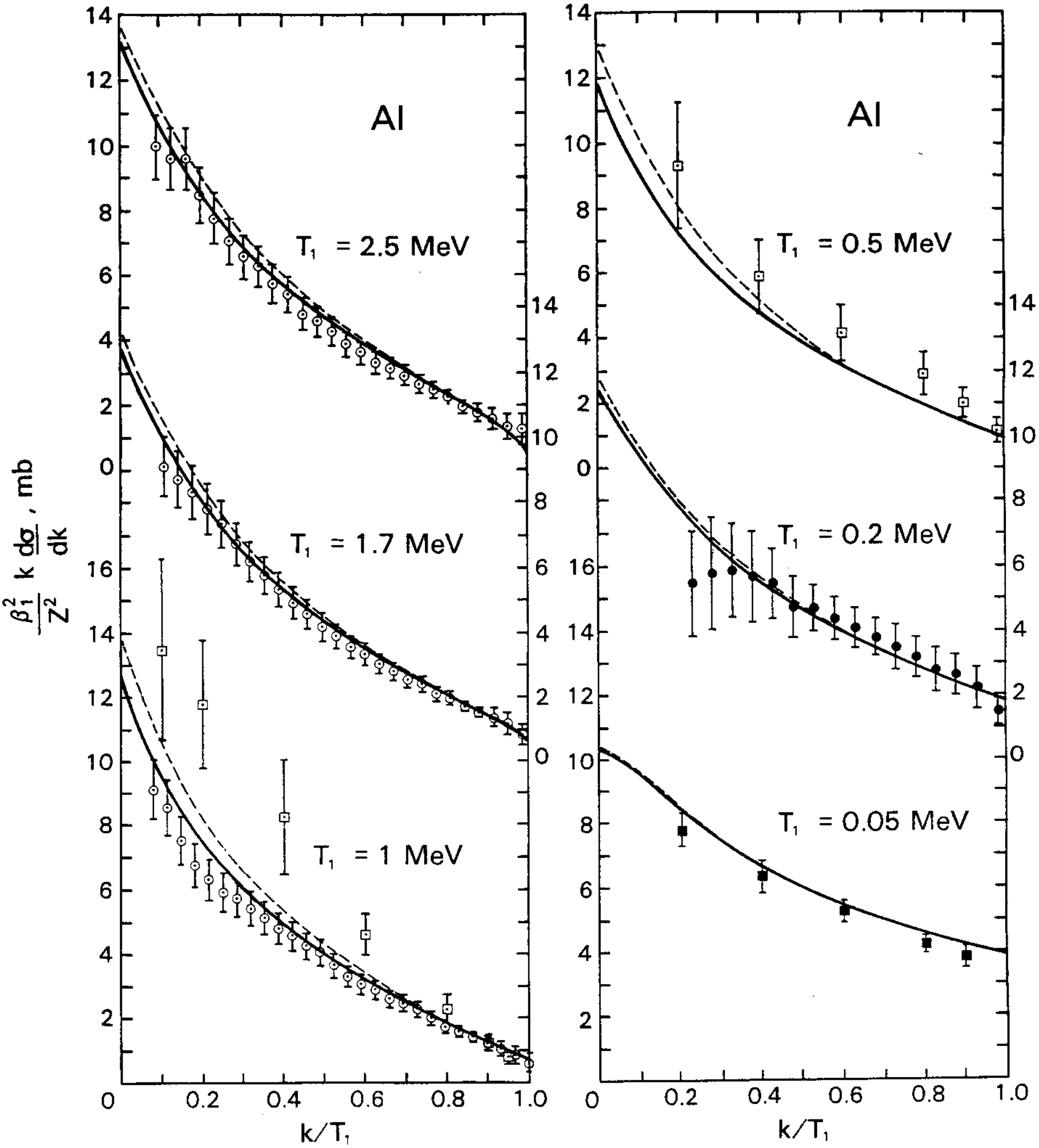
Z = 4
Z = 13
Z = 47
Z = 92
T1 = 2000 MeV
200
50
20
10
5
2
1
η
k/T1
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4



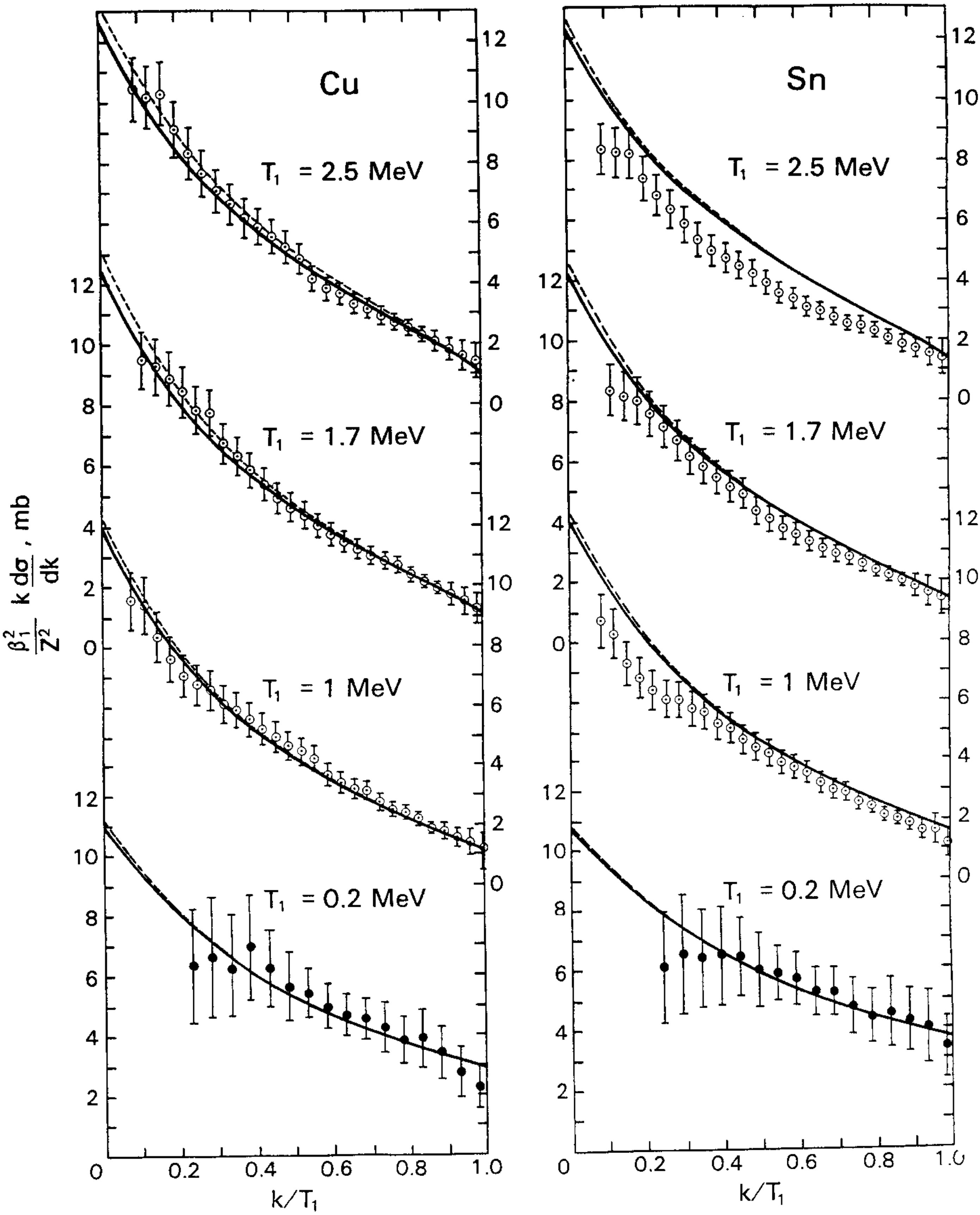
Be
T_1 = 1 MeV
$\frac{\beta_1^2}{Z^2}\, k\, \frac{d\sigma}{dk}$, mb
k/T_1
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
12
14
16
18



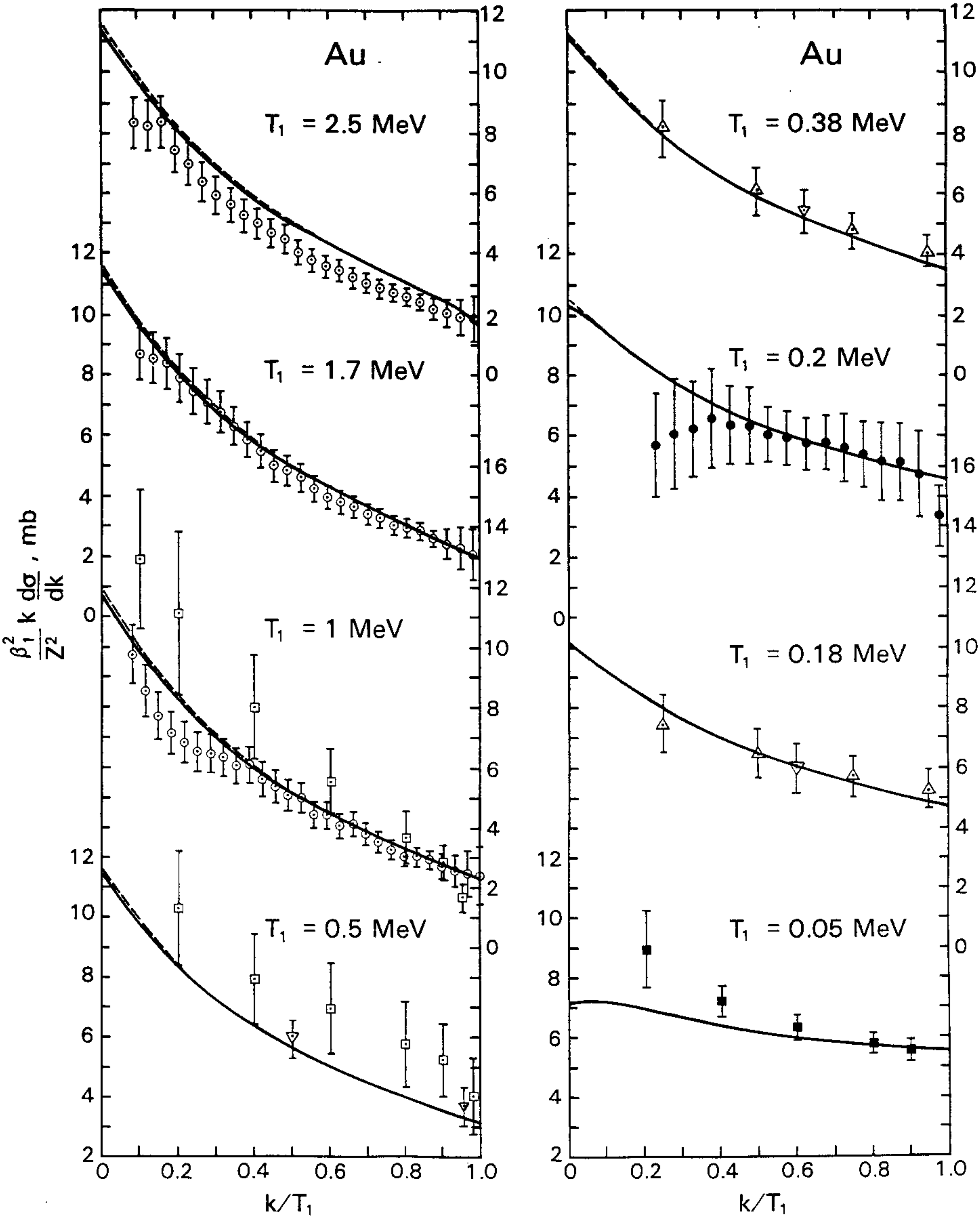
$Al_2\ O_3$
$T_1\ = 0.38$ MeV
$T_1\ = 0.18$ MeV
$\frac{\beta_1^2}{Z^2}\ k\ \frac{d\sigma}{dk}$, mb
k/T_1
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
12
0
2
4
6
8
10
12



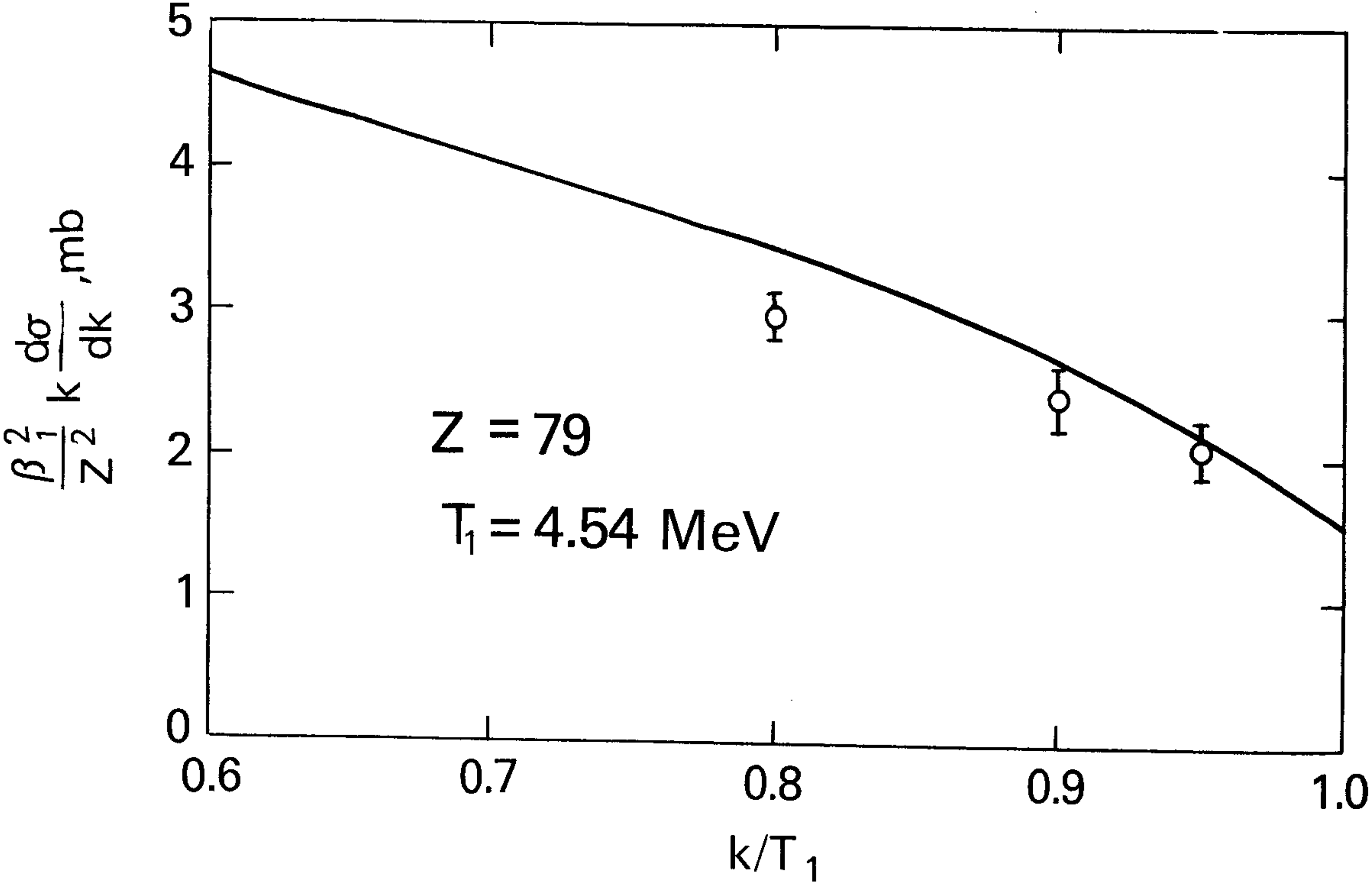
Al
T_1 = 2.5 MeV
T_1 = 1.7 MeV
T_1 = 1 MeV
T_1 = 0.5 MeV
T_1 = 0.2 MeV
T_1 = 0.05 MeV
$\frac{\beta_1^2}{Z^2} k \frac{d\sigma}{dk}$, mb
k/T_1



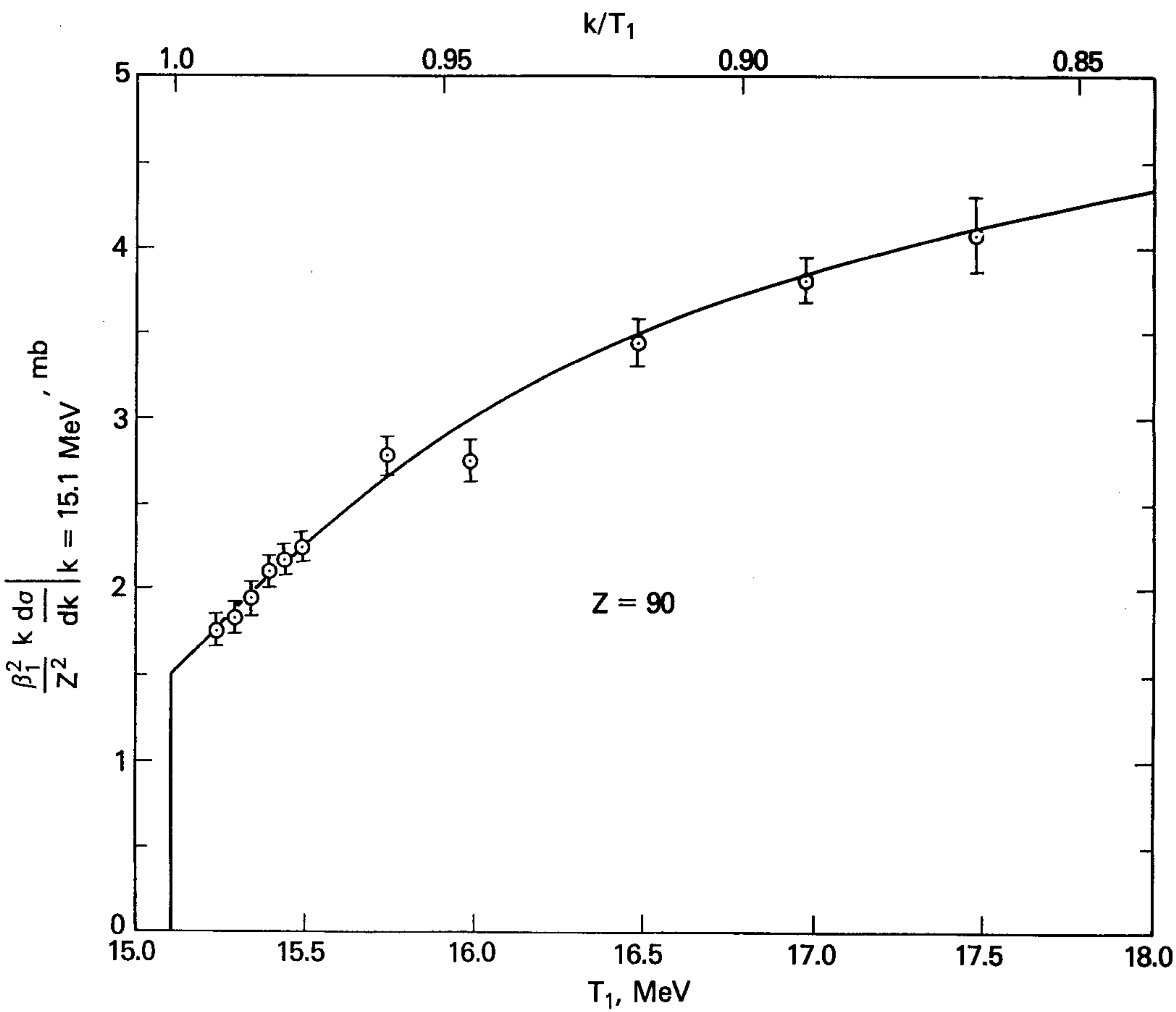
Cu
Sn
T_1 = 2.5 MeV
T_1 = 1.7 MeV
T_1 = 1 MeV
T_1 = 0.2 MeV
$\frac{\beta_1^2}{Z^2}\, k\frac{d\sigma}{dk}$, mb
k/T_1
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
12



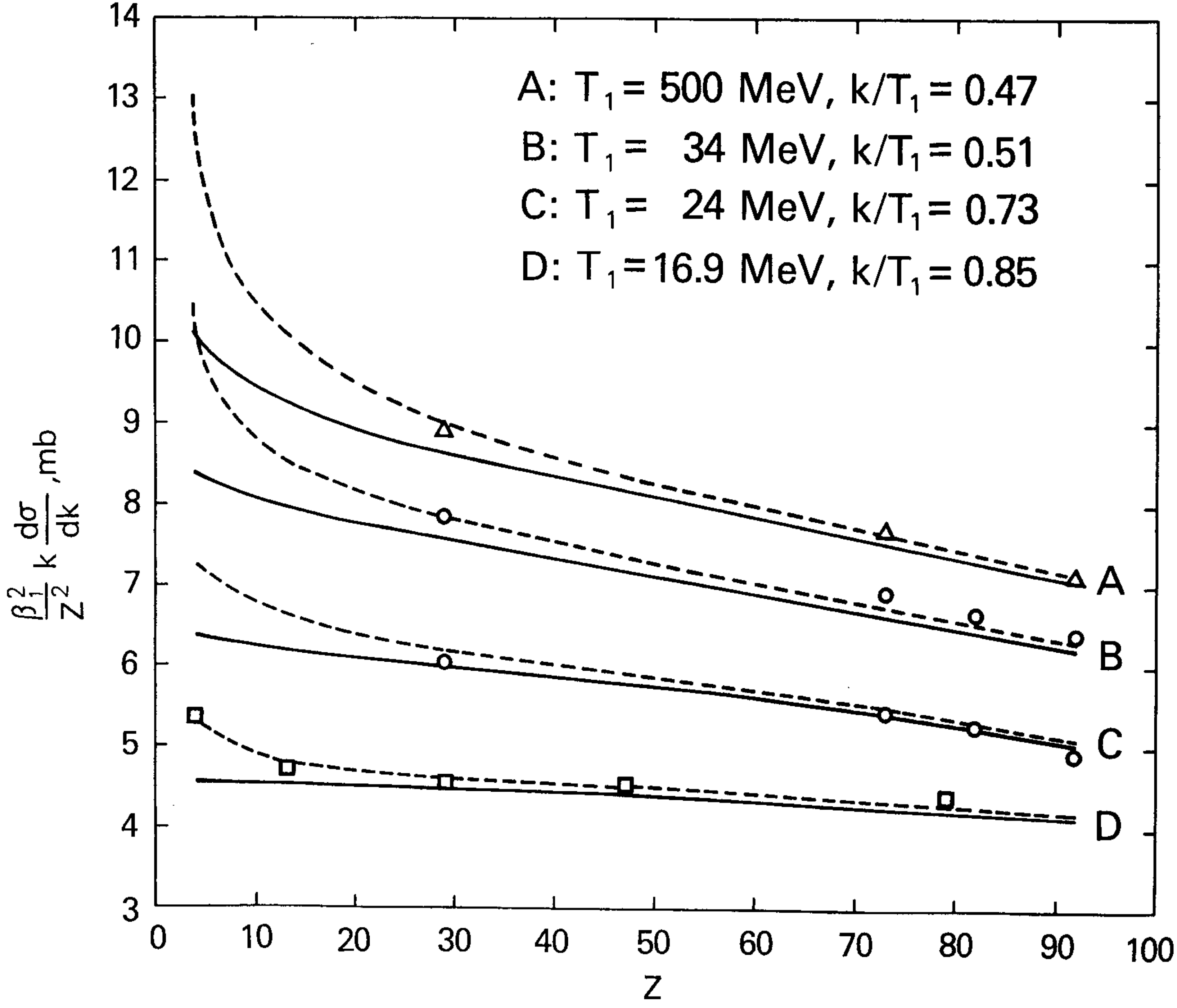
Au
T_1 = 2.5 MeV
T_1 = 1.7 MeV
T_1 = 1 MeV
T_1 = 0.5 MeV
Au
T_1 = 0.38 MeV
T_1 = 0.2 MeV
T_1 = 0.18 MeV
T_1 = 0.05 MeV
$\frac{\beta_1^2}{Z^2} k \frac{d\sigma}{dk}$, mb
k/T_1



$\frac{\beta_1^2}{Z^2} k \frac{d\sigma}{dk}$,mb
5
4
3
2
1
0
0.6
0.7
0.8
0.9
1.0
k/T_1
Z = 79
T_1 = 4.54 MeV



k/T_1
1.0
0.95
0.90
0.85
5
4
3
2
1
0
$\frac{\beta_1^2}{Z^2} \frac{k\, d\sigma}{dk}\Big|_{k = 15.1\ \mathrm{MeV}}$, mb
Z = 90
15.0
15.5
16.0
16.5
17.0
17.5
18.0
T_1, MeV



A: T_1 = 500 MeV, k/T_1 = 0.47
B: T_1 = 34 MeV, k/T_1 = 0.51
C: T_1 = 24 MeV, k/T_1 = 0.73
D: T_1 = 16.9 MeV, k/T_1 = 0.85
$\frac{\beta_1^2}{Z^2} k \frac{d\sigma}{dk}$,mb
Z
A
B
C
D
0
10
20
30
40
50
60
70
80
90
100
3
4
5
6
7
8
9
10
11
12
13
14

BREMSSTRAHLUNG PRODUCTION CROSS SECTIONS

(Electrons)

$$\frac{d\sigma}{dk} = \frac{d\sigma_n}{dk} + Z\,\frac{d\sigma_e}{dk} = \left[1 + \frac{\eta}{Z}\right]\frac{d\sigma_n}{dk}$$

$\dfrac{d\sigma_n}{dk}$: electron-nucleus cross section

$\dfrac{d\sigma_e}{dk}$: electron-electron cross section

$$\eta = \frac{d\sigma_e}{dk} \Big/ \left[\frac{1}{Z^2}\right]\frac{d\sigma_n}{dk}$$

(often assumed to be unity)

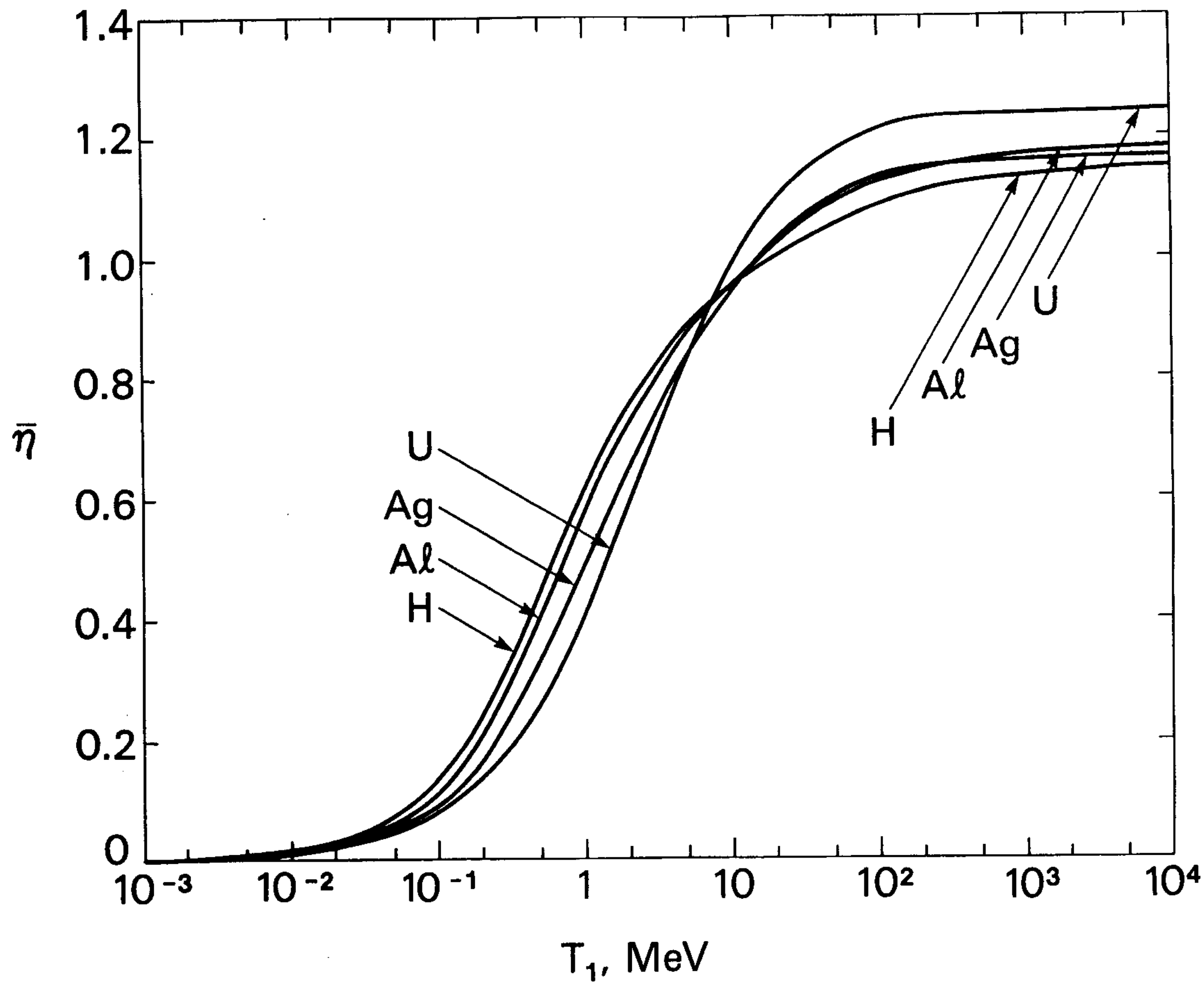
RADIATIVE STOPPING POWER

$$\frac{1}{\rho} S_{rad} = \alpha r_e^2 \frac{N_A}{A} Z^2(T_1 + mc^2) \Phi_{rad}$$

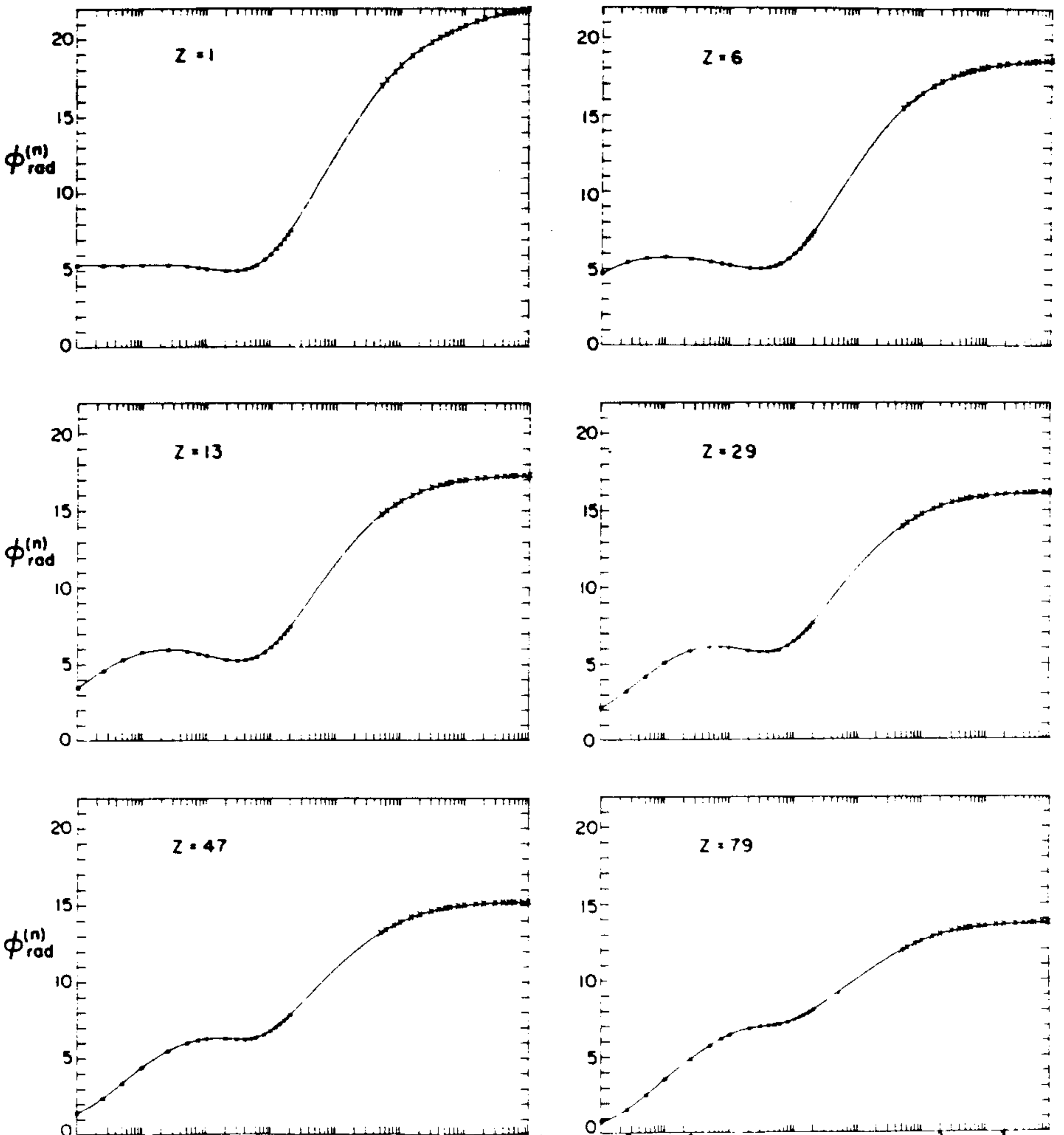
$$\Phi_{rad} = \left[\alpha r_e^2 Z^2 \left(T_1 + mc^2\right)\right]^{-1} \int_o^{T_1} k \frac{d\sigma}{dk} dk$$

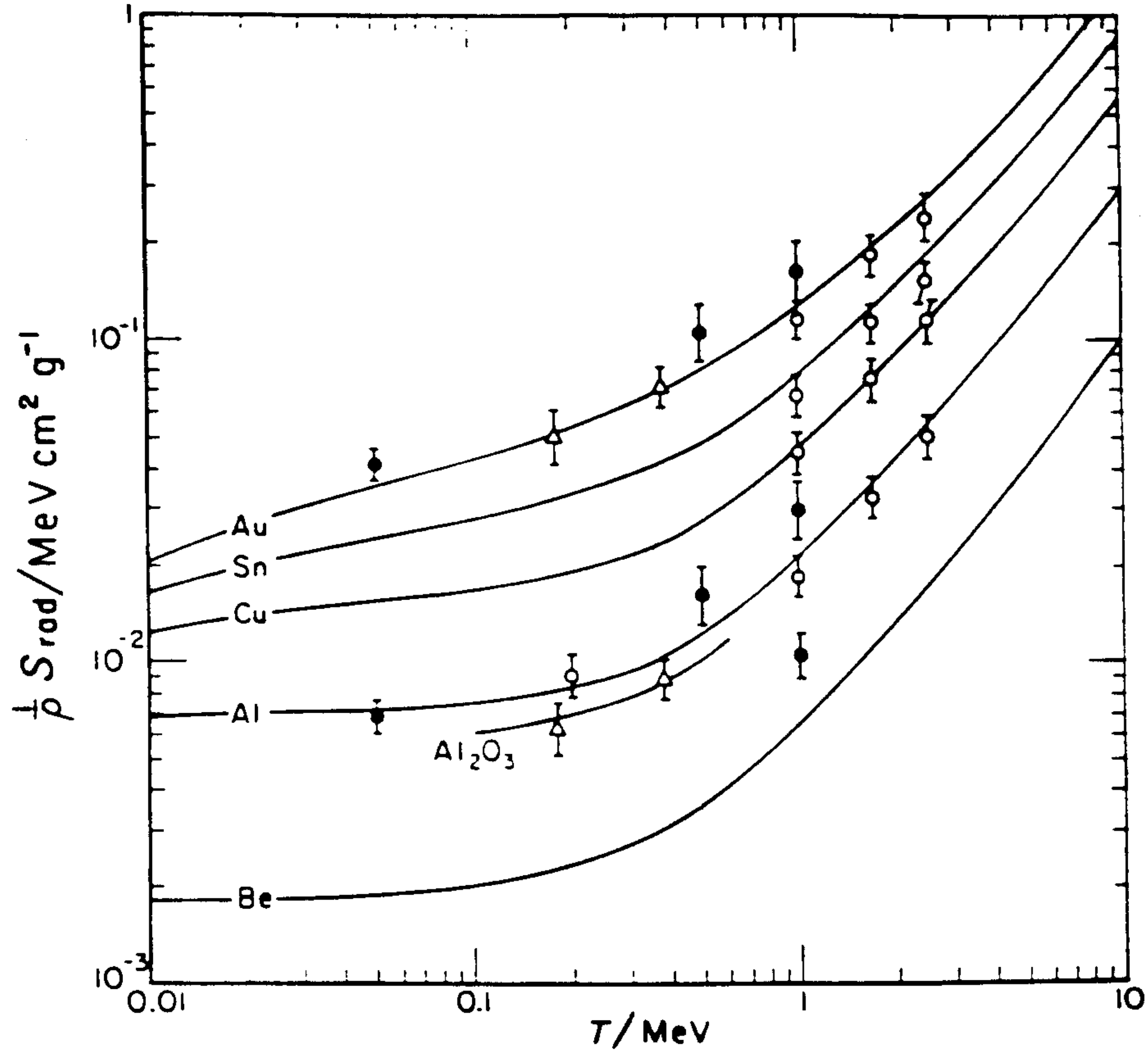
$$\begin{aligned} \Phi_{rad} &= \Phi_{rad}^{(n)} + Z \Phi_{rad}^{(e)} \\ &= (1 + \bar{\eta}/Z) \Phi_{rad}^{(n)} \end{aligned}$$

$$\bar{\eta} = \Phi_{rad}^{(e)} \Big/ \left[\frac{1}{Z^2} \Phi_{rad}^{(n)}\right]$$



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
$\bar{\eta}$
10^{-3}
10^{-2}
10^{-1}
1
10
10^2
10^3
10^4
T_1, MeV
U
Ag
Aℓ
H
H
Aℓ
Ag
U

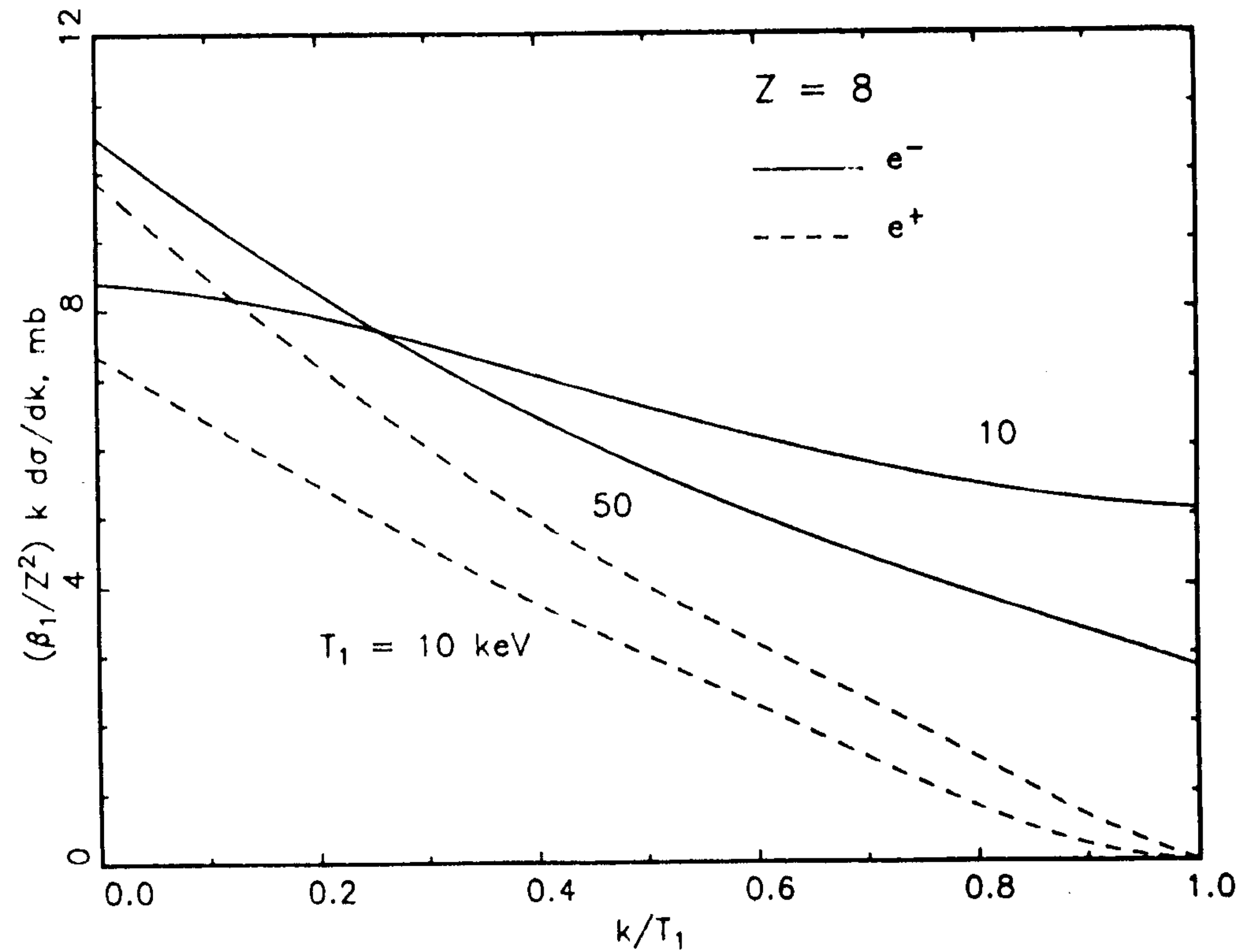

Z = 1
Z = 6
Z = 13
Z = 29
Z = 47
Z = 79
$\phi_{rad}^{(n)}$
T/MeV



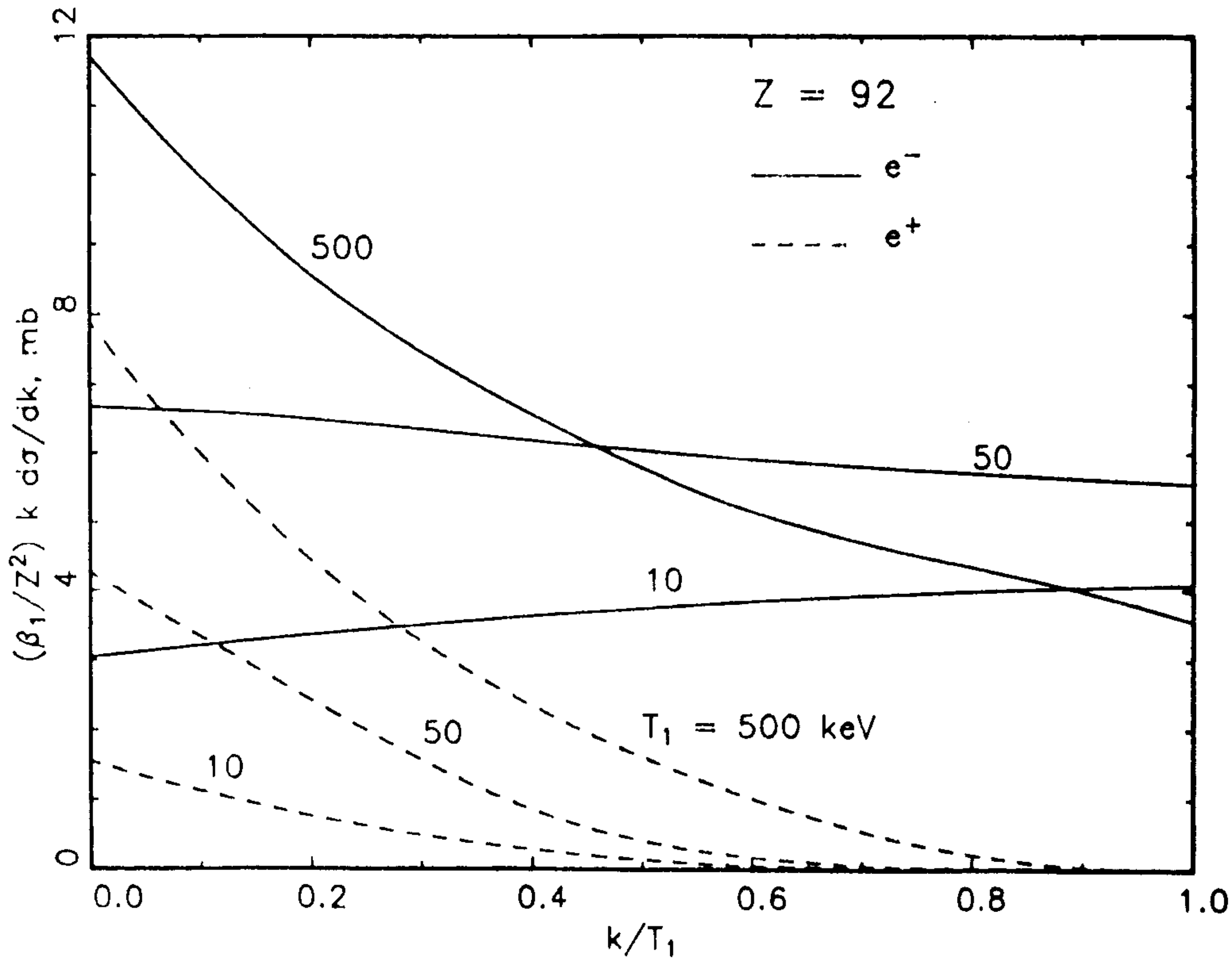
$\frac{1}{\rho} S_{rad}$/MeV cm² g⁻¹
T/MeV
1
10^{-1}
10^{-2}
10^{-3}
0.01
0.1
1
10
Au
Sn
Cu
Al
Al_2O_3
Be

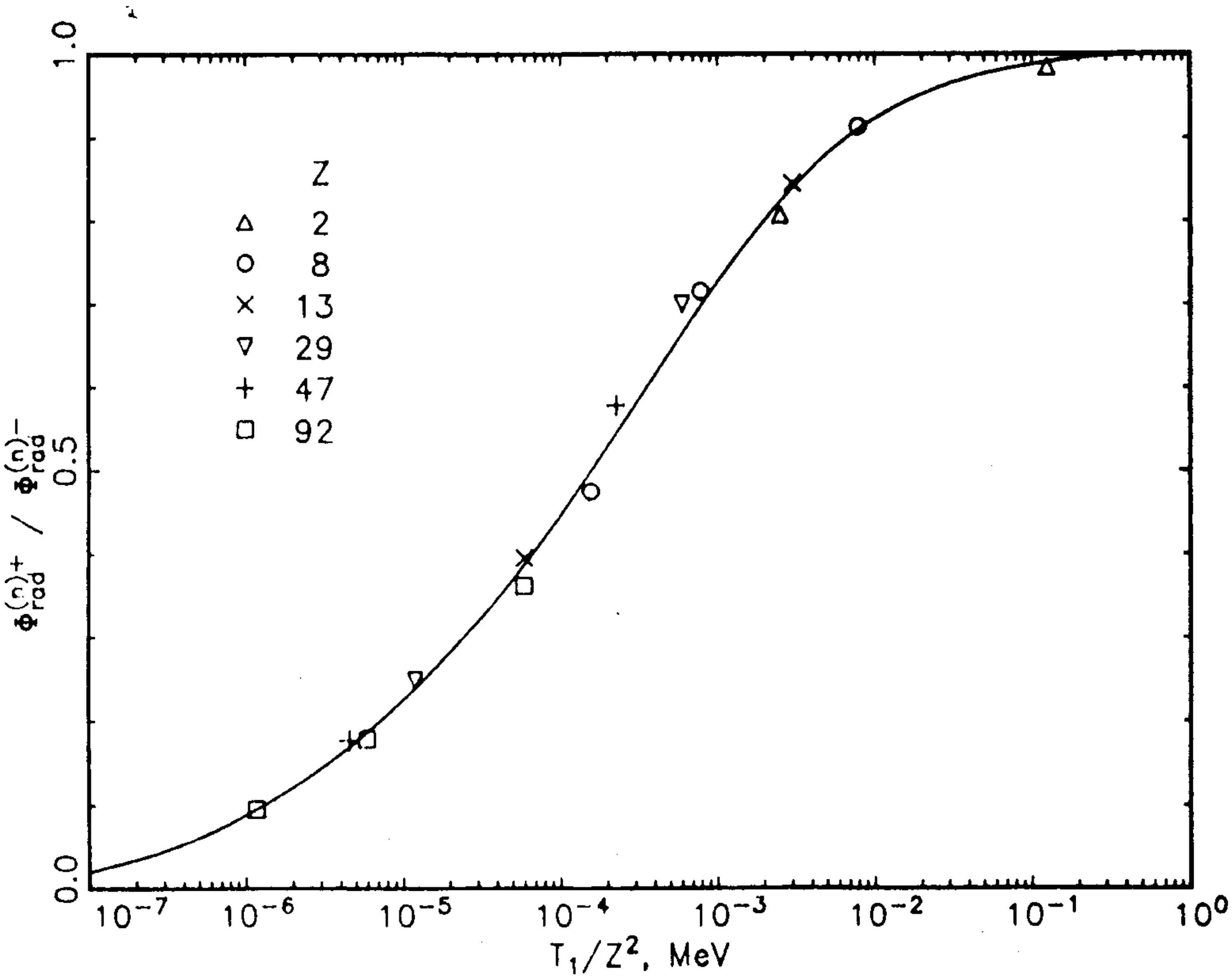
BREMSSTRAHLUNG PRODUCTION BY POSITRONS

- Positron-nucleus cross section vanishes at the high-frequency limit (tip)
- In high-energy approximation ($\gtrsim$ 50 MeV), positron-nucleus xsec $\approx$ electron-nucleus xsec
- At low energies, positron-nucleus xsec $\ll$ electron-nucleus xsec
- Positron-electron cross section should be finite at tip (as in the electron-proton system)
- At high energies, positron-electron xsec $\approx$ electron-electron xsec
- In the non-relativistc limit, positron-electron xsec $\approx$ ½ electron-proton xsec

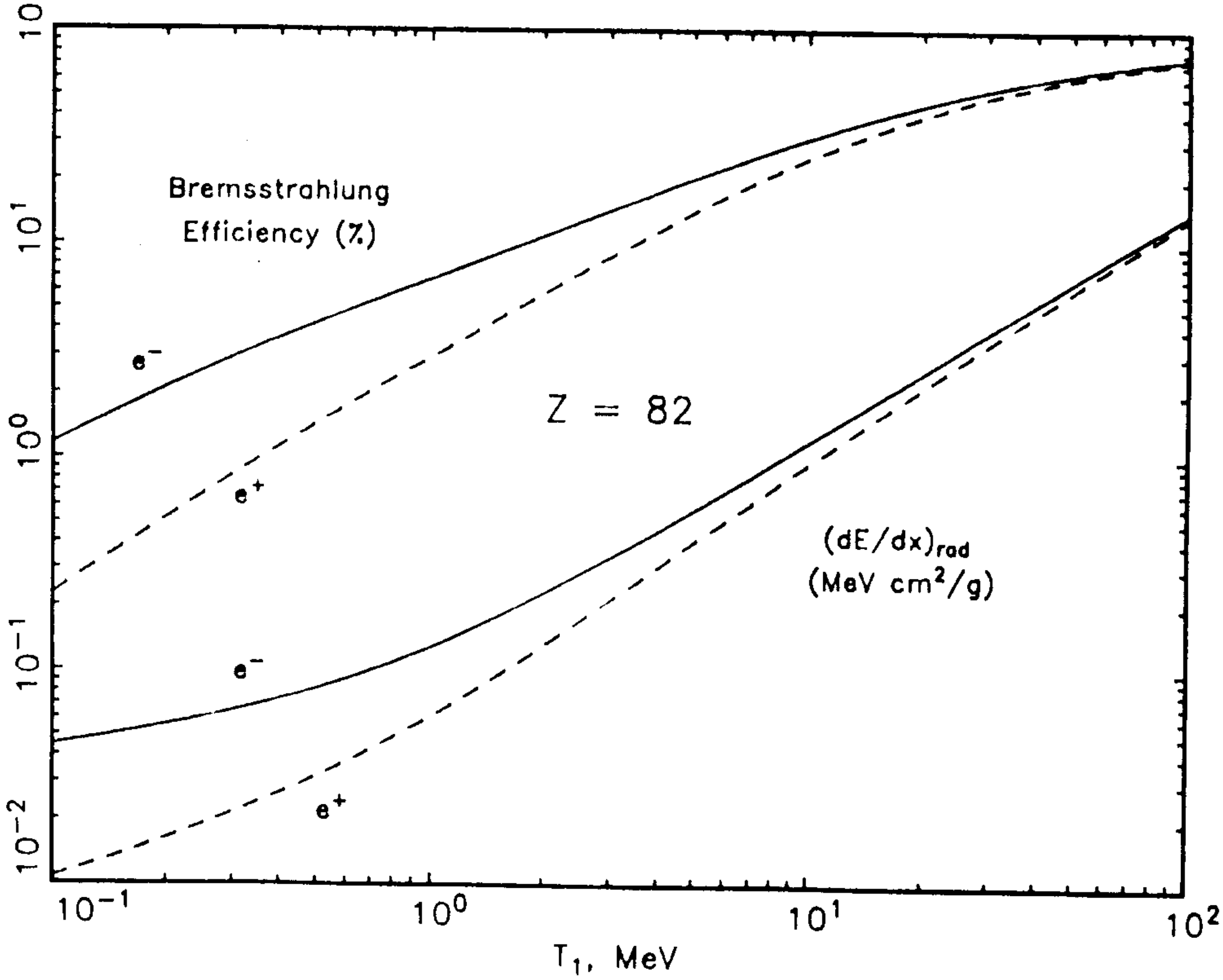


Z = 8
e⁻
e⁺
10
50
T₁ = 10 keV
(β₁/Z²) k dσ/dk, mb
k/T₁
0
4
8
12
0.0
0.2
0.4
0.6
0.8
1.0


Z = 92
e⁻
e⁺
500
50
10
T₁ = 500 keV
50
10
(β₁/Z²) k dσ/dk, mb
k/T₁
0.0
0.2
0.4
0.6
0.8
1.0
0
4
8
12



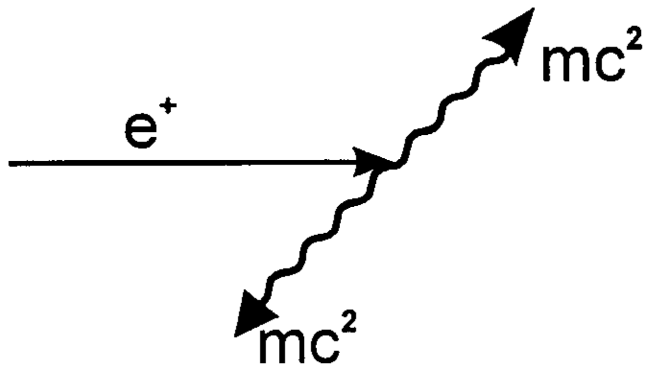
Z
2
8
13
29
47
92
1.0
0.5
0.0
$\Phi^{(n)+}_{rad} / \Phi^{(n)-}_{rad}$
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
T_1/Z^2, MeV



Bremsstrahlung
Efficiency (%)
e⁻
e⁺
Z = 82
e⁻
e⁺
(dE/dx)rad
(MeV cm²/g)
10⁻²
10⁻¹
10⁰
10¹
10²
10⁻¹
10⁰
10¹
10²
T1, MeV

Positron Interactions

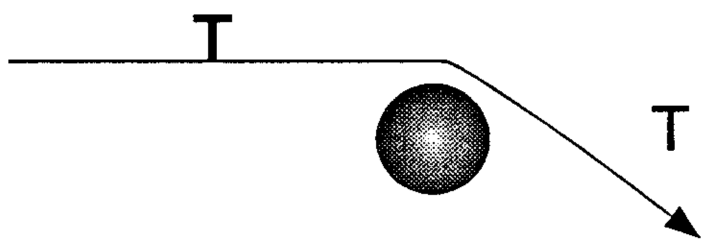
Annihilation



- Predominately for the positron <u>at</u> <u>rest</u>
- Two oppositely directed photons, each with an energy of 0.511 MeV

Electron Interactions

Elastic scattering



- Negligible energy loss
- $\sigma \sim Z^2$

ELASTIC-SCATTERING CROSS SECTION

Factored cross section

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{d\sigma_{Ruth}(\theta)}{d\Omega} K_{scr}(\theta)\, K_{rel}(\theta)$$

Spin-relativistic correction

$$K_{rel}(\theta) = \frac{d\sigma_{Mott}}{d\Omega} \Big/ \frac{d\sigma_{Ruth}}{d\Omega}$$

Screening correction

$$K_{scr}(\theta) = \left(\frac{1-\cos\theta}{1-\cos\theta+2\eta}\right)^2$$

Molière screening angle

$$\eta = \tfrac{1}{4}\left(\frac{\alpha\, mc}{0.885\, p}\right)^2 Z^{2/3}\,[1.13+3.76\,(\alpha Z/\beta)^2\,]$$

ELASTIC-SCATTERING CROSS SECTION

Zeitler & Olsen estimate relative error of factorization

$$\varepsilon \sim 2\alpha^2 Z^{4/3} \frac{\sqrt{1-\beta^2}}{\beta^2}$$

T(keV)	Z = 6	13	29	47	79	90
1028		0.002	0.004	0.007	0.01	0.02
512		0.003	0.006	0.01	0.02	0.03
256	0.001	0.005	0.01	0.02	0.04	0.05
128	0.003	0.01	0.02	0.04	0.08	0.10
64	0.005	0.02	0.04	0.08	0.15	0.18
32	0.01	0.04	0.08	0.15	0.30	0.35
16	0.02	0.07	0.15	0.29	0.59	0.70
8	0.04	0.14	0.31	0.58	1.16	1.38
4	0.07	0.28	0.61	1.16	2.32	2.75
2	0.15	0.55	1.21	2.31	4.62	5.50
1	0.30	1.10	2.43	4.62	9.23	11.0

ELASTIC SCATTERING OF ELECTRONS AND POSITRONS

- **Approximate, analytical theory becomes less reliable at lower energies**

- **Use Riley code for relativistic, partial-wave expansion calculations of differential elastic-scattering cross sections**

- **Treats scattering by a static, screened Coulomb potential**

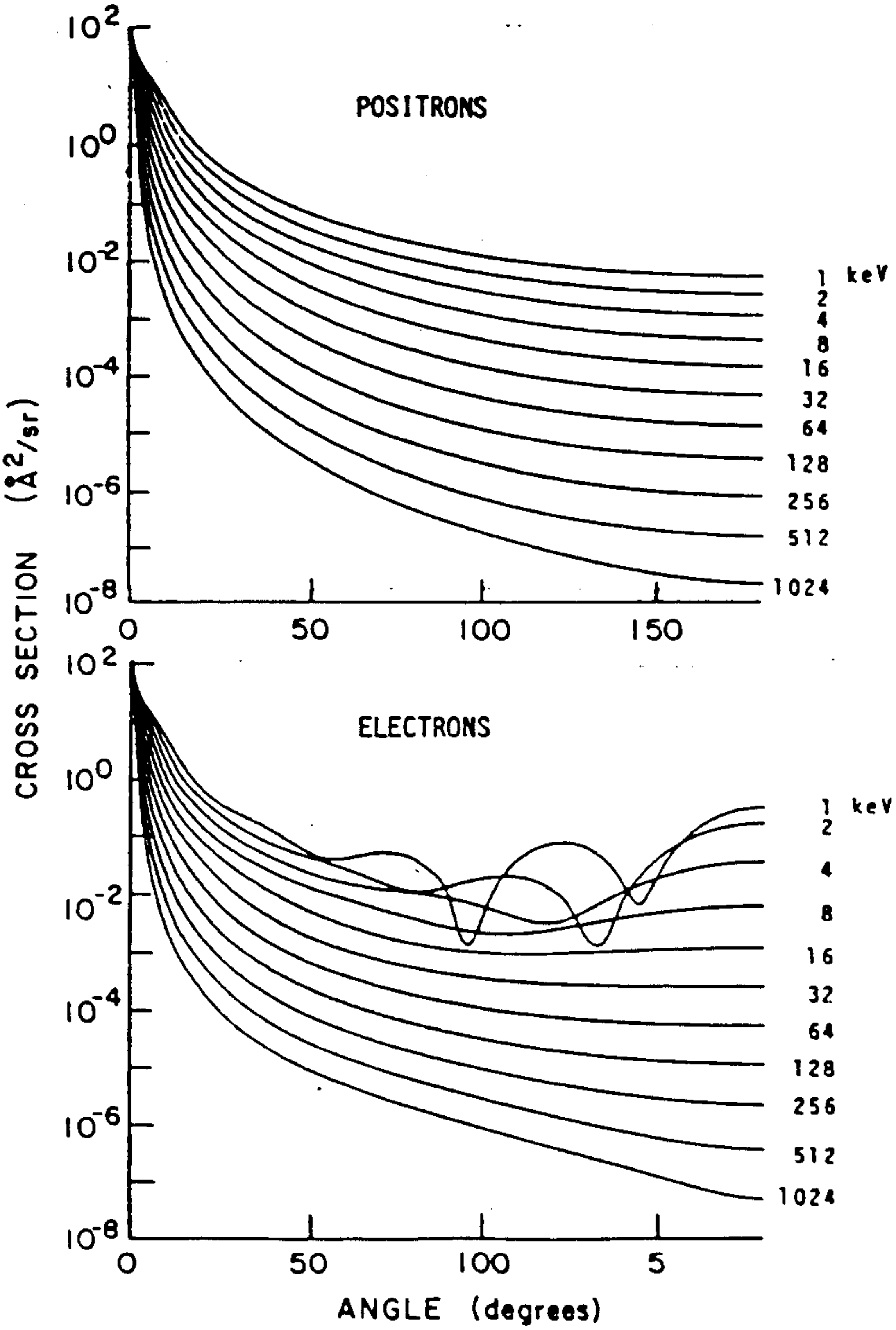
- **Screening: electron density distributions from relativistic Dirac-Fock wave-function program of Desclaux**

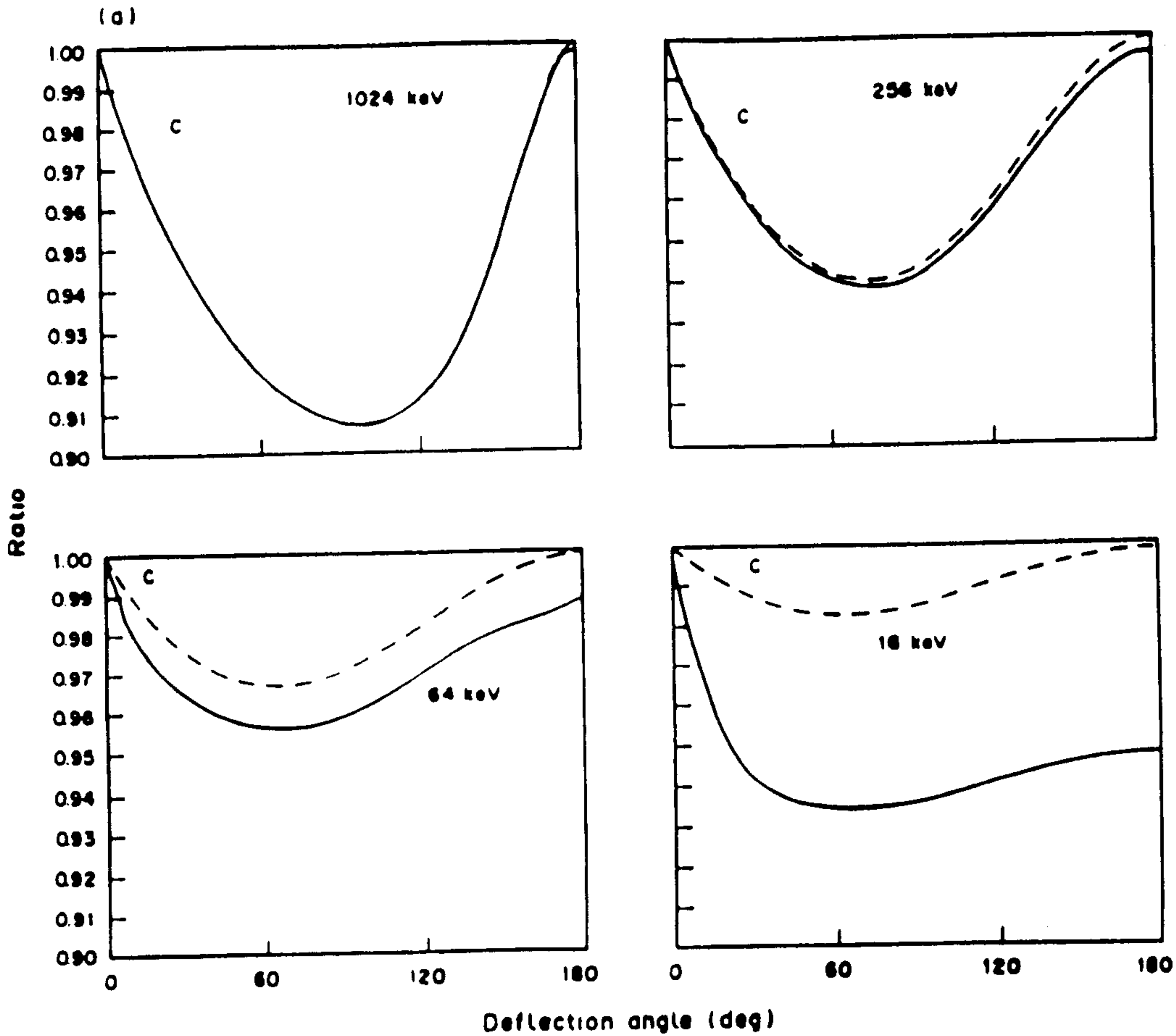
- **$1 \leq Z \leq 100$**

- **1 keV $\leq T \leq$ 1024 keV**

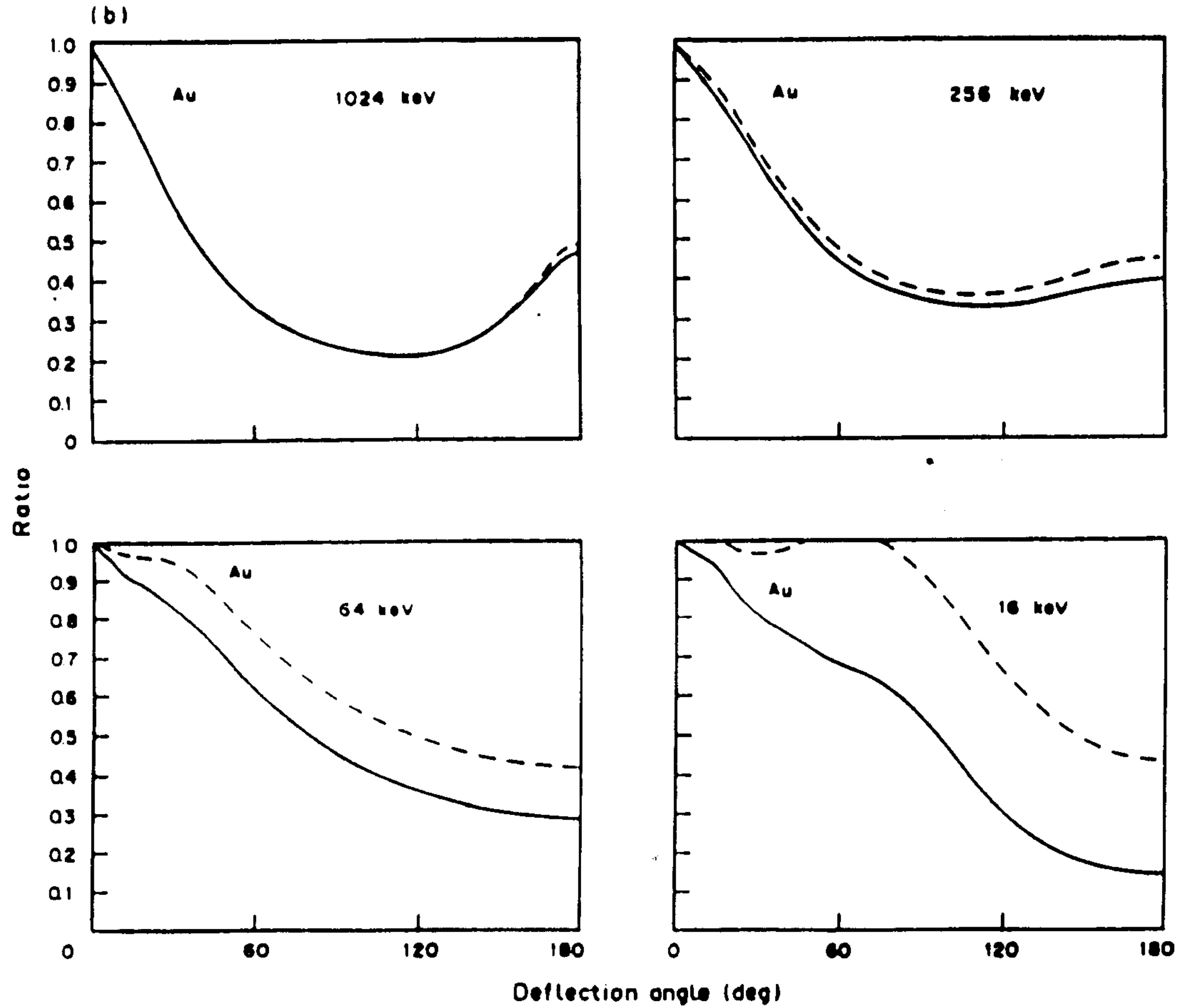
- **e^- and e^+**

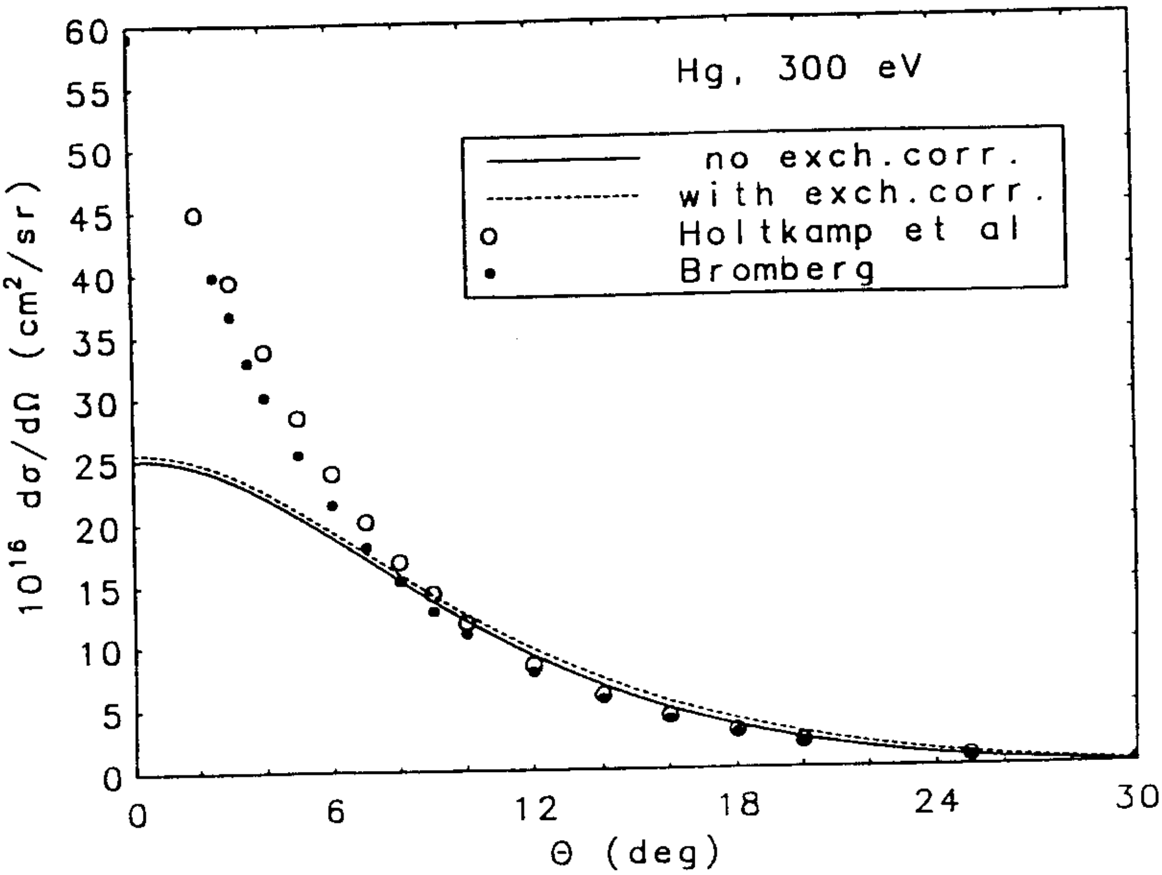
Elastic Scattering Cross Sections for Au

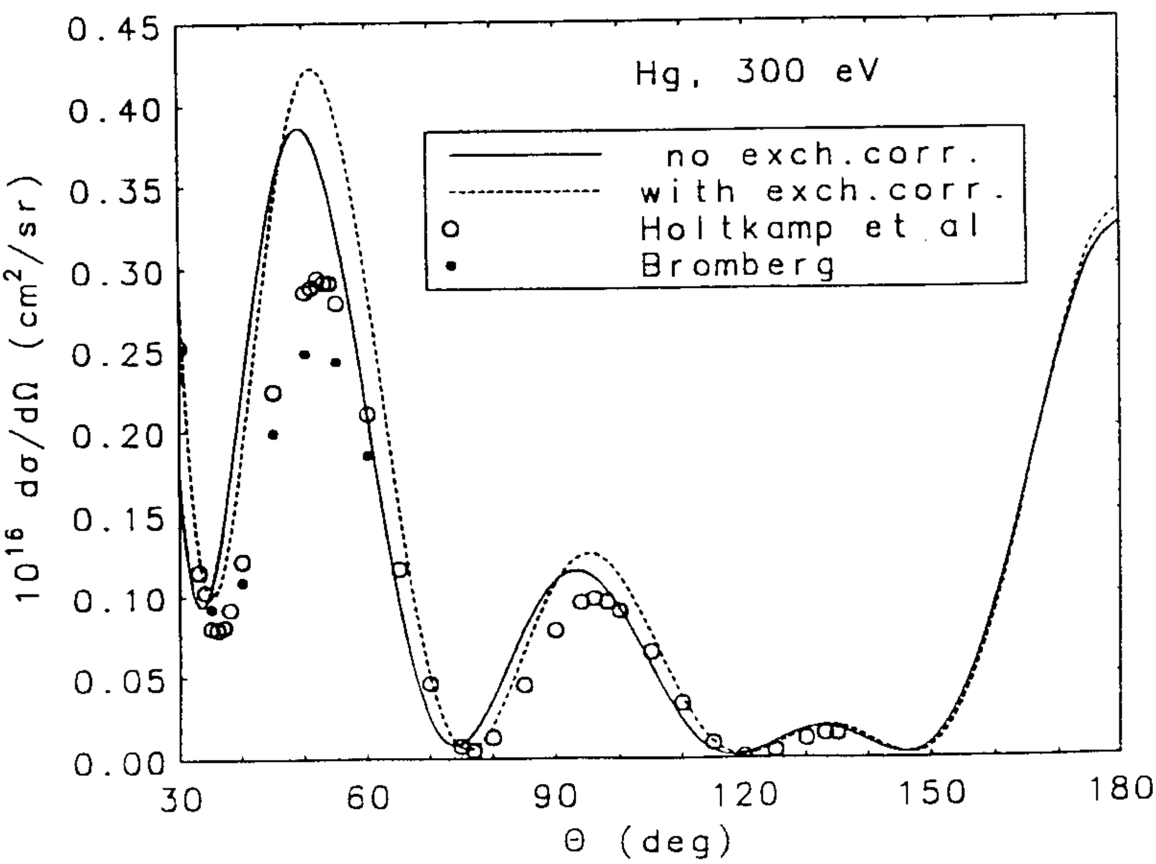


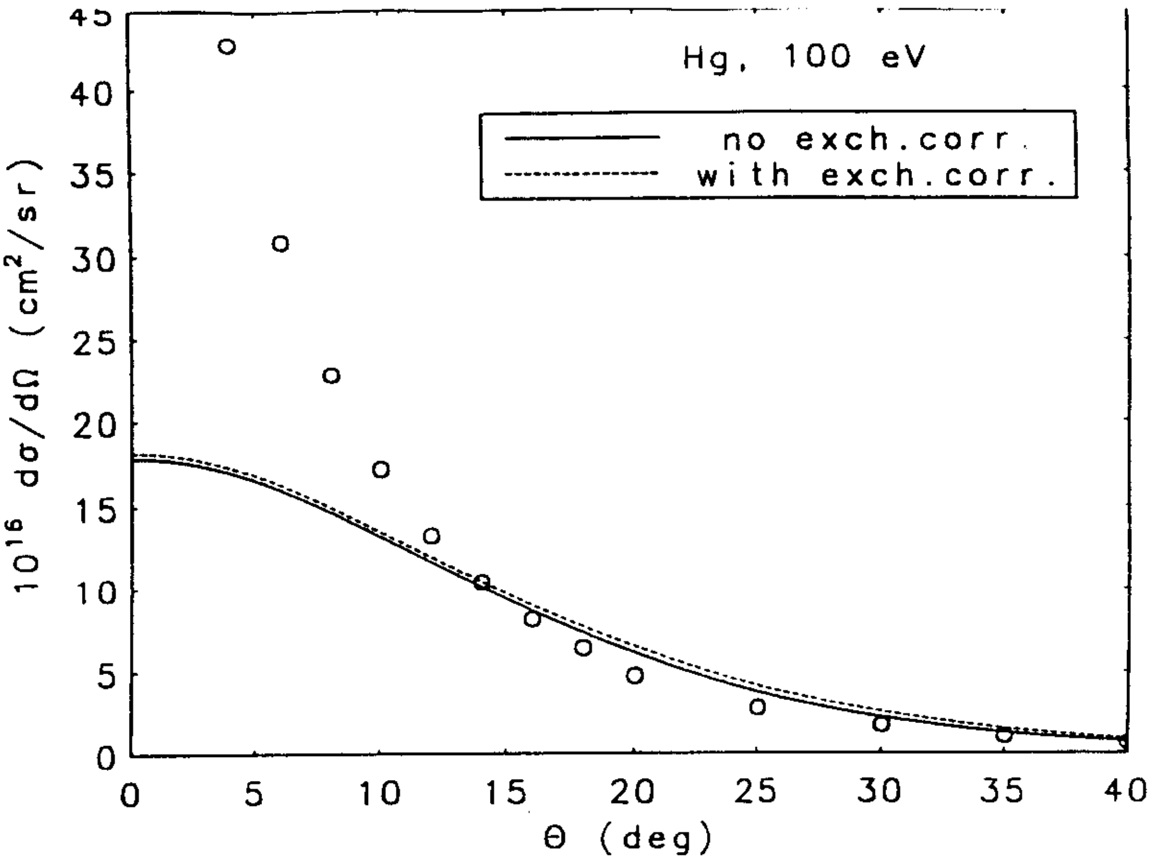


(a)
1024 keV
256 keV
64 keV
16 keV
C
Ratio
1.00
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.90
0
60
120
180
Deflection angle (deg)

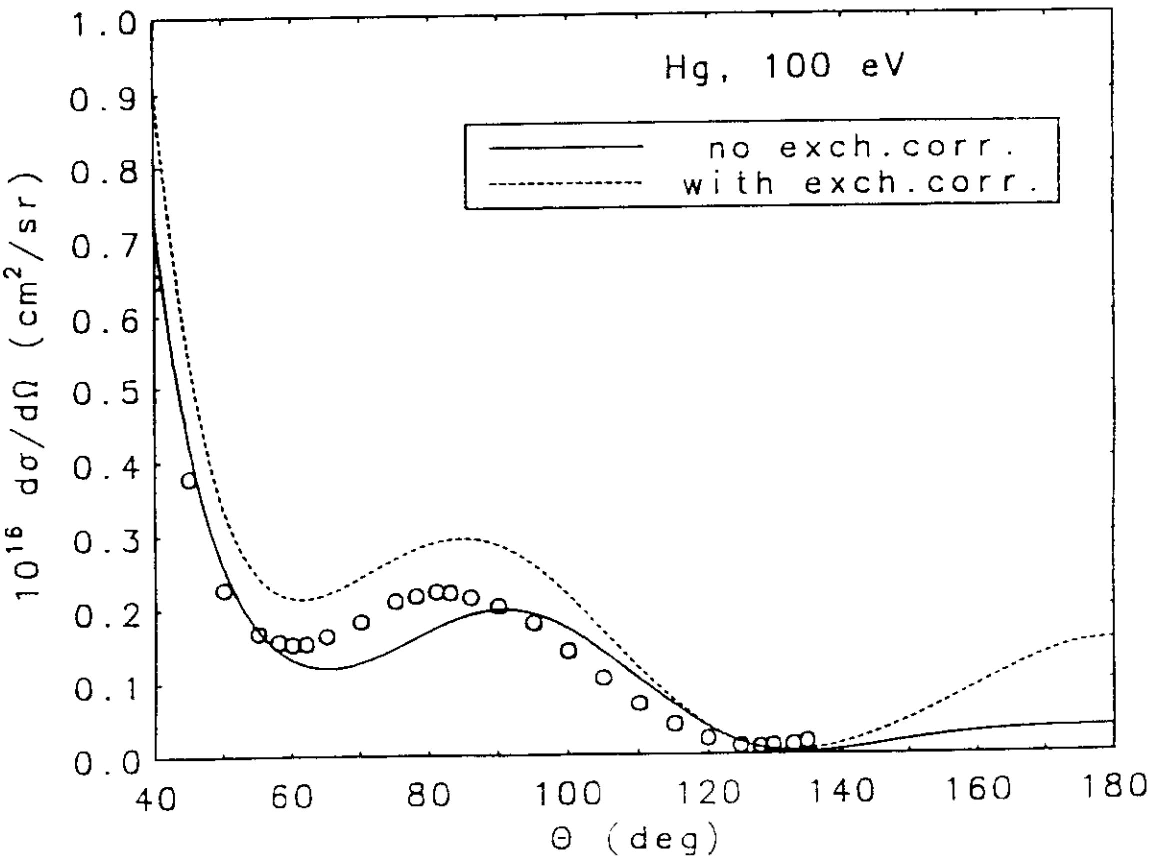

(b)
Au
1024 keV
Au
256 keV
Au
64 keV
Au
16 keV
Ratio
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
60
120
180
Deflection angle (deg)


Hg, 300 eV
no exch.corr.
with exch.corr.
Holtkamp et al
Bromberg
10^{16} dσ/dΩ (cm^2/sr)
Θ (deg)


Hg, 300 eV
no exch.corr.
with exch.corr.
Holtkamp et al
Bromberg
10^{16} dσ/dΩ (cm^2/sr)
Θ (deg)

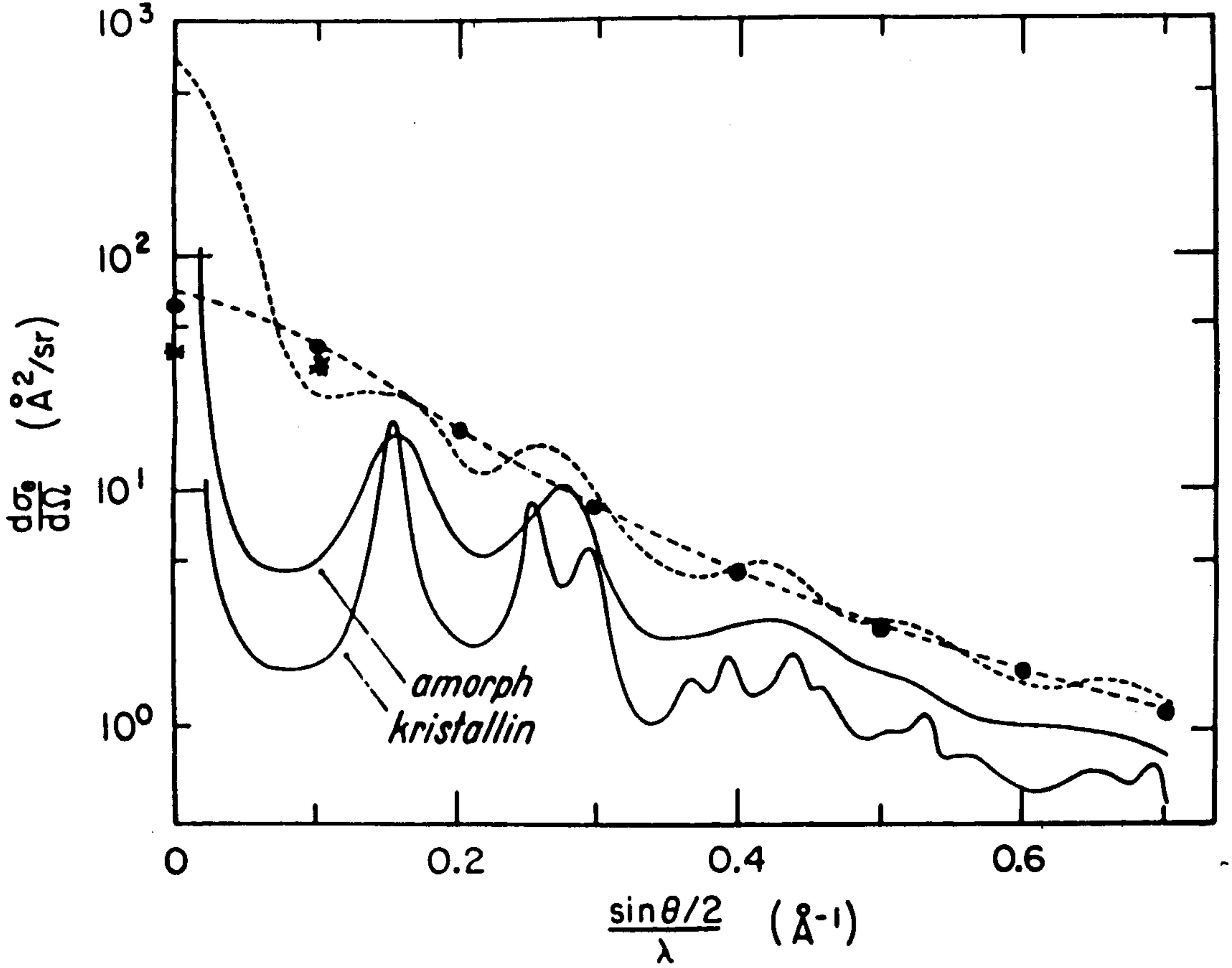


Hg, 100 eV
no exch.corr.
with exch.corr.
10^{16} dσ/dΩ (cm²/sr)
Θ (deg)



Hg, 100 eV
no exch.corr.
with exch.corr.
10^{16} dσ/dΩ (cm²/sr)
Θ (deg)

ELASTIC SCATTERING CROSS SECTIONS FOR 51.2-KEV ELECTRONS IN GE

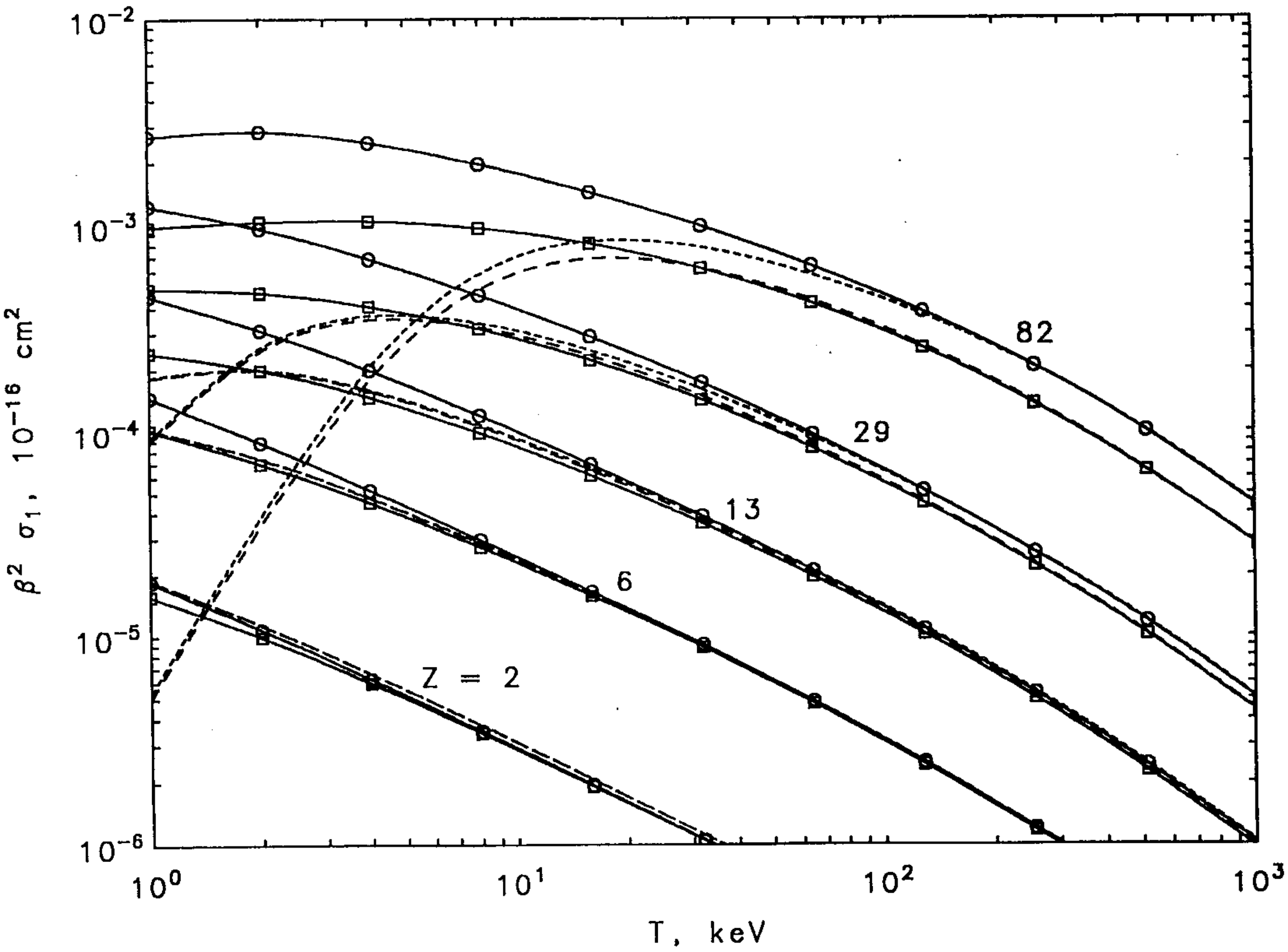


ELASTIC-SCATTERING CROSS SECTION

Transport (or momentum-transfer) cross section

$$\sigma_t = 2\pi \int_{-1}^{+1} \frac{d\sigma(\theta)}{d\Omega} (1 - \cos\theta)\, d(\cos\theta)$$

has dominant influence on multiple-scattering process



$\beta^2\ \sigma_1,\ 10^{-16}\ cm^2$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^0
10^1
10^2
10^3
T, keV
82
29
13
6
Z = 2

ELECTRON-IMPACT IONIZATION

ICRU REPORT 55

Secondary Electron Spectra from Charged Particle Interactions

Issued: 20 April 1996

INTERNATIONAL COMMISSION ON RADIATION
UNITS AND MEASUREMENTS
7910 WOODMONT AVENUE
BETHESDA, MARYLAND 20814
U.S.A.

ELECTRON-IMPACT IONIZATION CROSS SECTION

Goal: shell-by-shell cross section for all shells differential in ejected secondary electron energy

Approach: detailed implementation of the Weizsäcker-Williams method

$$\frac{d\sigma_{ion}}{dE} = \underbrace{\frac{d\sigma_c}{dE}}_{\substack{\text{close}\\\text{collisions}}} + \underbrace{\frac{d\sigma_d}{dE}}_{\substack{\text{distant}\\\text{collisions}}}$$

CLOSE COLLISIONS

Binary-encounter generalization of the Møller cross section

$$\frac{d\sigma_c^{(j)}}{dE} = \frac{2\pi r_e^2 mc^2}{\beta^2} n_j P_j \left\{ \frac{1}{(W+B_j)^2} + \frac{1}{(T-W)^2} + \frac{1}{T^2}\left[\frac{\tau}{\tau+1}\right]^2 - \frac{2\tau+1}{(\tau+1)^2}\frac{1}{(W+B_j)(T-W)} + G_j \right\}$$

$E = W + B_j$ energy transfer

W = KE of secondary electron, $0 \leq W \leq (T - B_j)/2$

B_j = binding energy of j^{th} orbital

n_j = number of orbital electrons

$P_j = T/(T + B_j + U_j)$

U_j = mean KE of orbital electron

$$G_j = \frac{8U_j}{3\pi}\left[\frac{1}{(W+B_j)^3} + \frac{1}{(T-W)^3}\right]\left[\arctan\sqrt{y} + \frac{\sqrt{y}(y-1)}{(y+1)^2}\right]$$

$y = W/U_j$

DISTANT COLLISIONS

Replace incident electron by equivalent field of virtual photons

$$\frac{d\sigma_d^{(j)}}{dE} = n_j \; I(W + B_j) \; \sigma_{PE}^{(j)}(W + B_j)$$

$\sigma_{PE}^{(j)}$ = **photoionization cross section, per orbital electron**

Virtual photon spectrum:

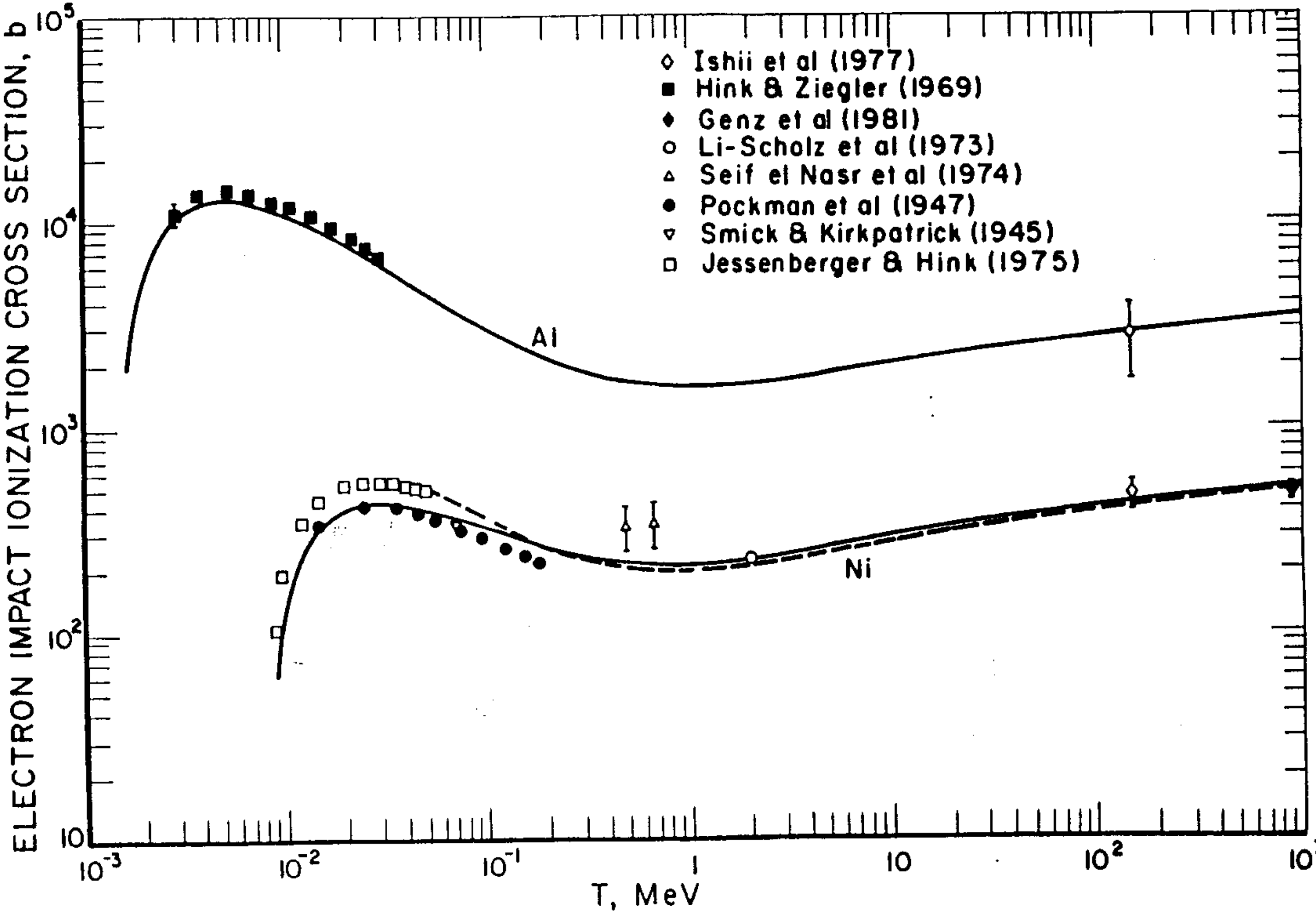
$$I(E) = \frac{2\alpha}{\pi\beta^2} \frac{1}{E} \left[F_t(x_{min}) - F_t(x_{max})\right]$$

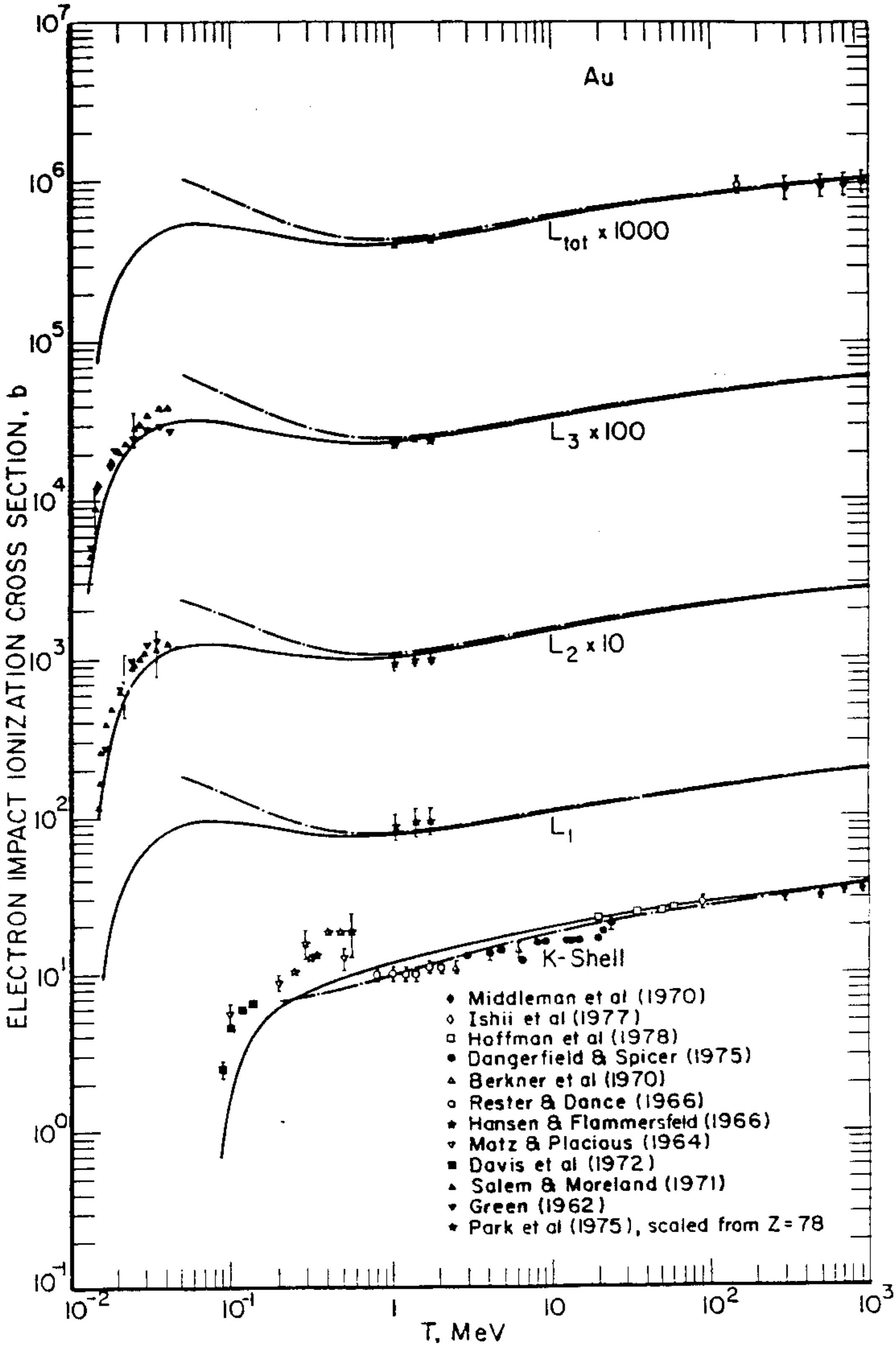
$$F_t(s) = x\,K_0(x)\,K_1(x) - \frac{x^2}{2}\left[K_1^2(x) - K_0^2(x)\right]$$

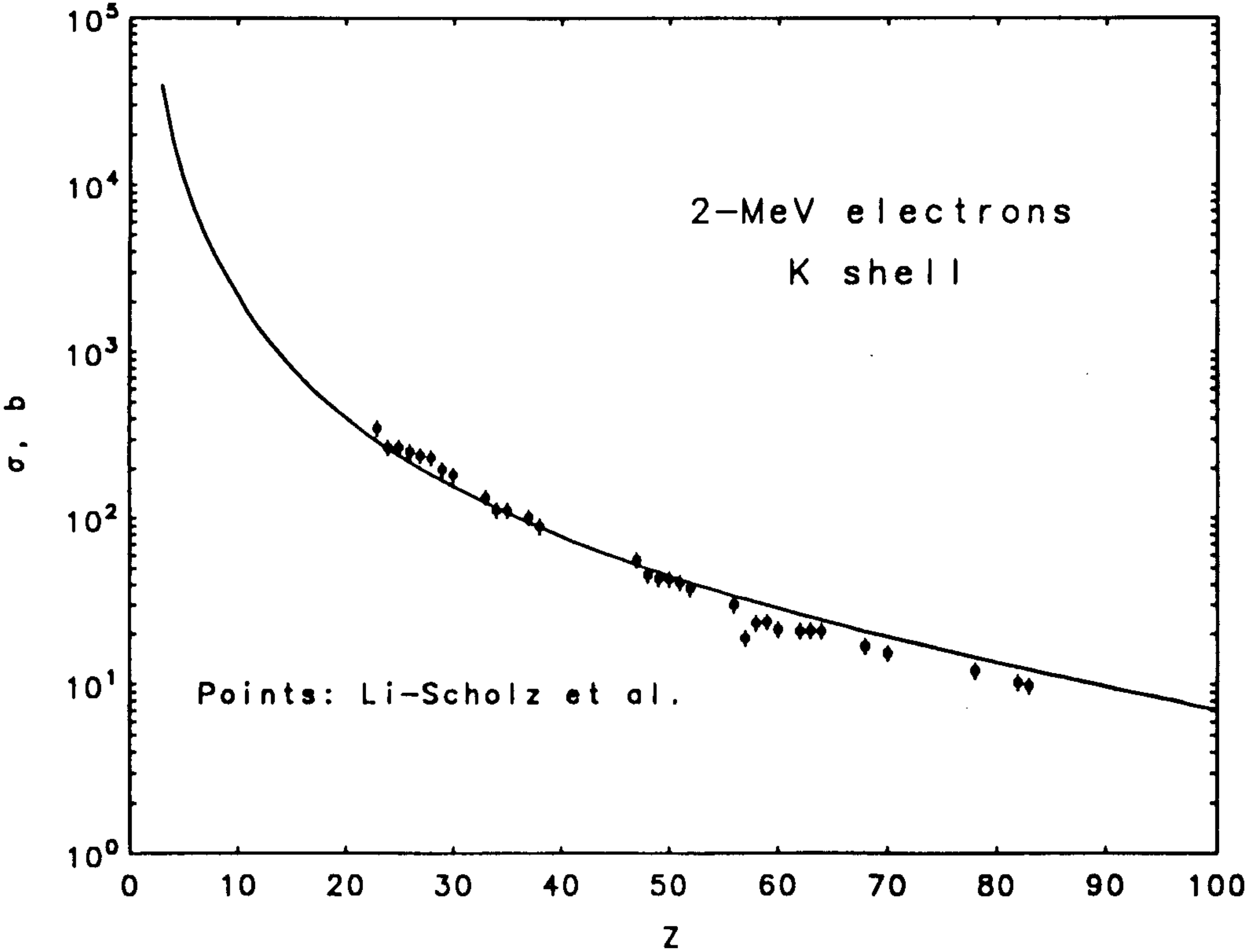
$$x = \frac{Eb}{\hbar c} \frac{\sqrt{1-\beta^2}}{\beta}$$

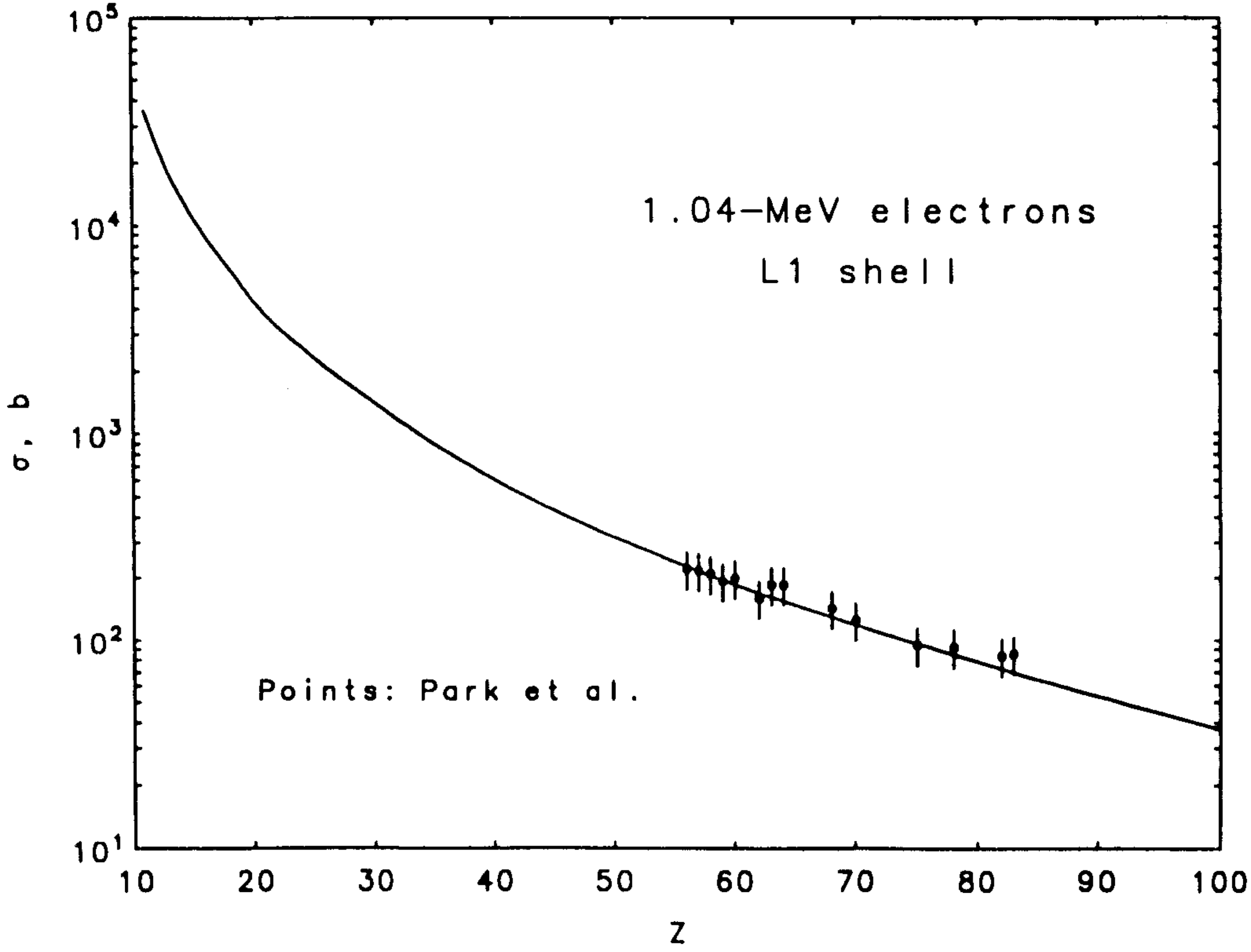
minimum impact parameter $b_{min} = \langle r \rangle_j$ **for** j^{th} **orbital**

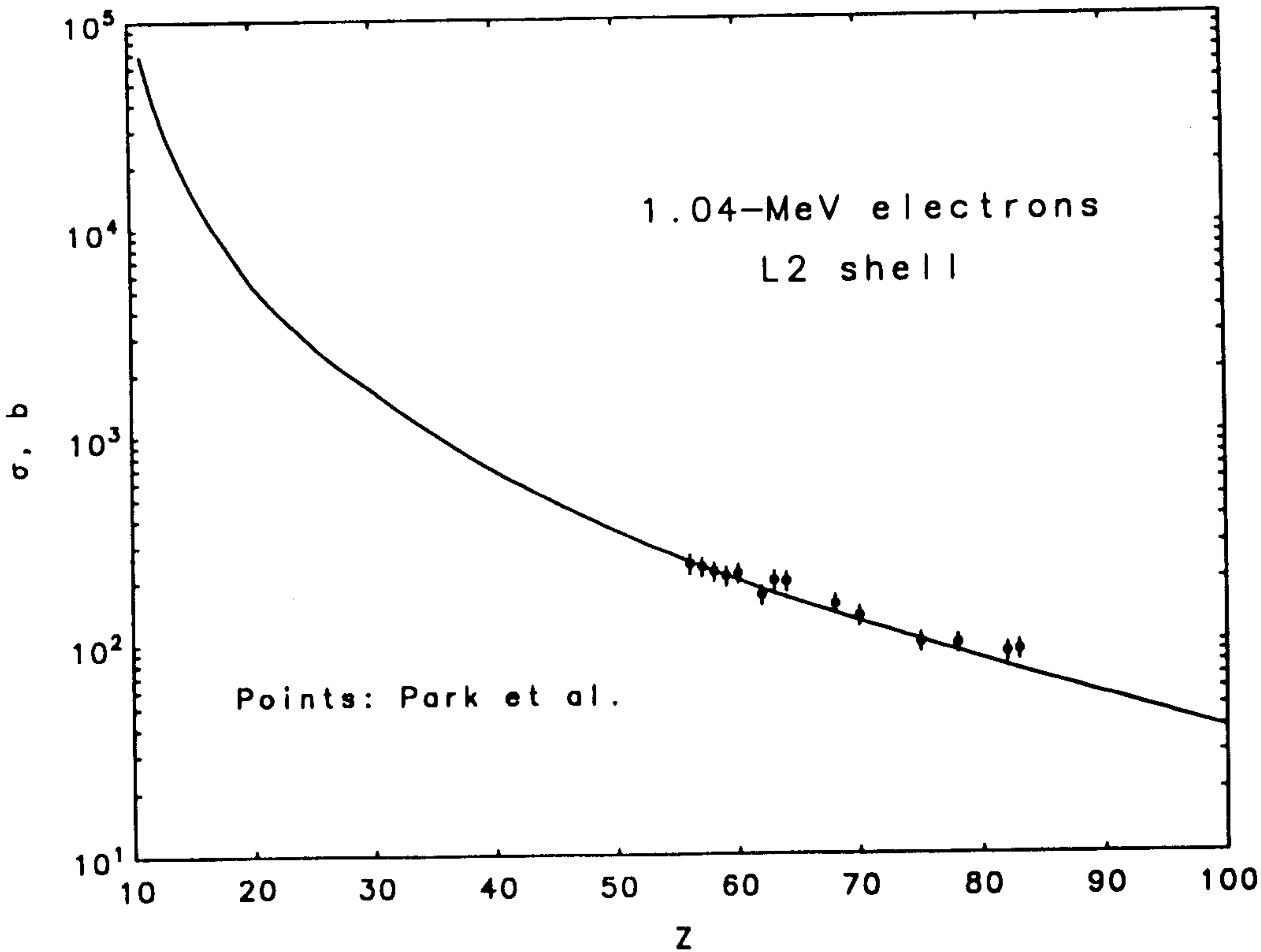
maximum impact parameter $b_{max} = \dfrac{1.123\ \hbar\, c\, \beta}{B_j \sqrt{1-\beta^2}}$

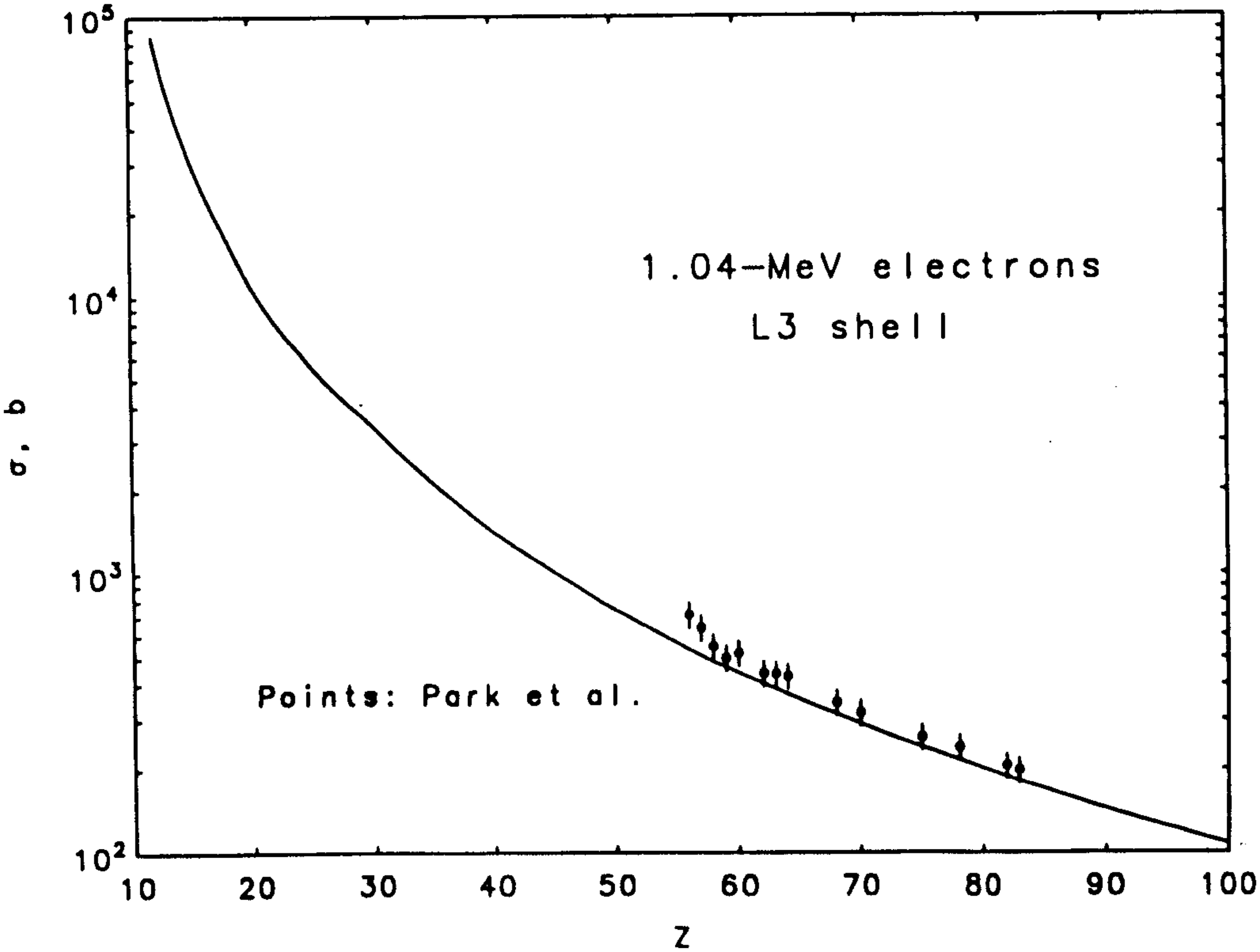

ELECTRON IMPACT IONIZATION CROSS SECTION, b
T, MeV
Ishii et al (1977)
Hink & Ziegler (1969)
Genz et al (1981)
Li-Scholz et al (1973)
Seif el Nasr et al (1974)
Pockman et al (1947)
Smick & Kirkpatrick (1945)
Jessenberger & Hink (1975)
Al
Ni


Au
$L_{tot} \times 1000$
$L_3 \times 100$
$L_2 \times 10$
L_1
K-Shell
Middleman et al (1970)
Ishii et al (1977)
Hoffman et al (1978)
Dangerfield & Spicer (1975)
Berkner et al (1970)
Rester & Dance (1966)
Hansen & Flammersfeld (1966)
Motz & Placious (1964)
Davis et al (1972)
Salem & Moreland (1971)
Green (1962)
Park et al (1975), scaled from Z = 78
ELECTRON IMPACT IONIZATION CROSS SECTION, b
T, MeV


2-MeV electrons
K shell
Points: Li-Scholz et al.
σ, b
10^0
10^1
10^2
10^3
10^4
10^5
0
10
20
30
40
50
60
70
80
90
100
Z


1.04–MeV electrons
L1 shell
Points: Park et al.
σ, b
10^5
10^4
10^3
10^2
10^1
10
20
30
40
50
60
70
80
90
100
Z


1.04–MeV electrons
L2 shell
Points: Park et al.
σ, b
Z
10^5
10^4
10^3
10^2
10^1
10
20
30
40
50
60
70
80
90
100


1.04–MeV electrons
L3 shell
Points: Park et al.
σ, b
Z
10
20
30
40
50
60
70
80
90
100
10^5
10^4
10^3
10^2

ELECTRON-IMPACT IONIZATION

Total sub-shell cross section

$$\sigma^{(j)} = \int_{o}^{(T-B_j)/2} \frac{d\sigma^{(j)}}{dE}\, dW$$

Spectrum of ejected electrons

$$\frac{d\sigma}{dW} = \sum_j \frac{d\sigma^{(j)}}{dW}$$

Ionization stopping power

$$\frac{1}{\rho}\, S_{ion} = \frac{N_A}{A} \sum_j \int_{o}^{(T-B_j)/2} (W + B_j)\, \frac{d\sigma^{(j)}}{dE}\, dW$$

Table 1. Spectrum of secondary electrons produced by 10-keV electrons in water vapor, as a function of the kinetic energy W of the secondary electron, given in terms of the ratio to the Moller cross section.

W(eV)	Ratio
0	0
10	0.427
20	0.896
50	1.33
100	1.12
200	1.00
500	0.962
1000	0.969
2000	0.978
3000	0.990
4000	1.01
4600	1.03
4800	0.815
4990	0.612
4992	0.408
4993	0.205
5000	0

LOW-ENERGY ELECTRON-IMPACT IONIZATION OF ATOMS AND MOLECULES

Binary-Encounter-Dipole Models of Y-K. Kim

BED Model for the differential cross section:

$$\frac{d\sigma_i(W,T)}{dW} = \frac{S}{B_i(t+u+1)}\left\{\frac{(N_i/N)-2}{t+1}\left(\frac{1}{w+1}+\frac{1}{t-w}\right)+\right.$$
$$\left.[2-(N_i/N)]\left[\frac{1}{(w+1)^2}+\frac{1}{(t-w)^2}\right]+\frac{\ln t}{N(w+1)}\frac{df_i(w)}{dw}\right\}$$

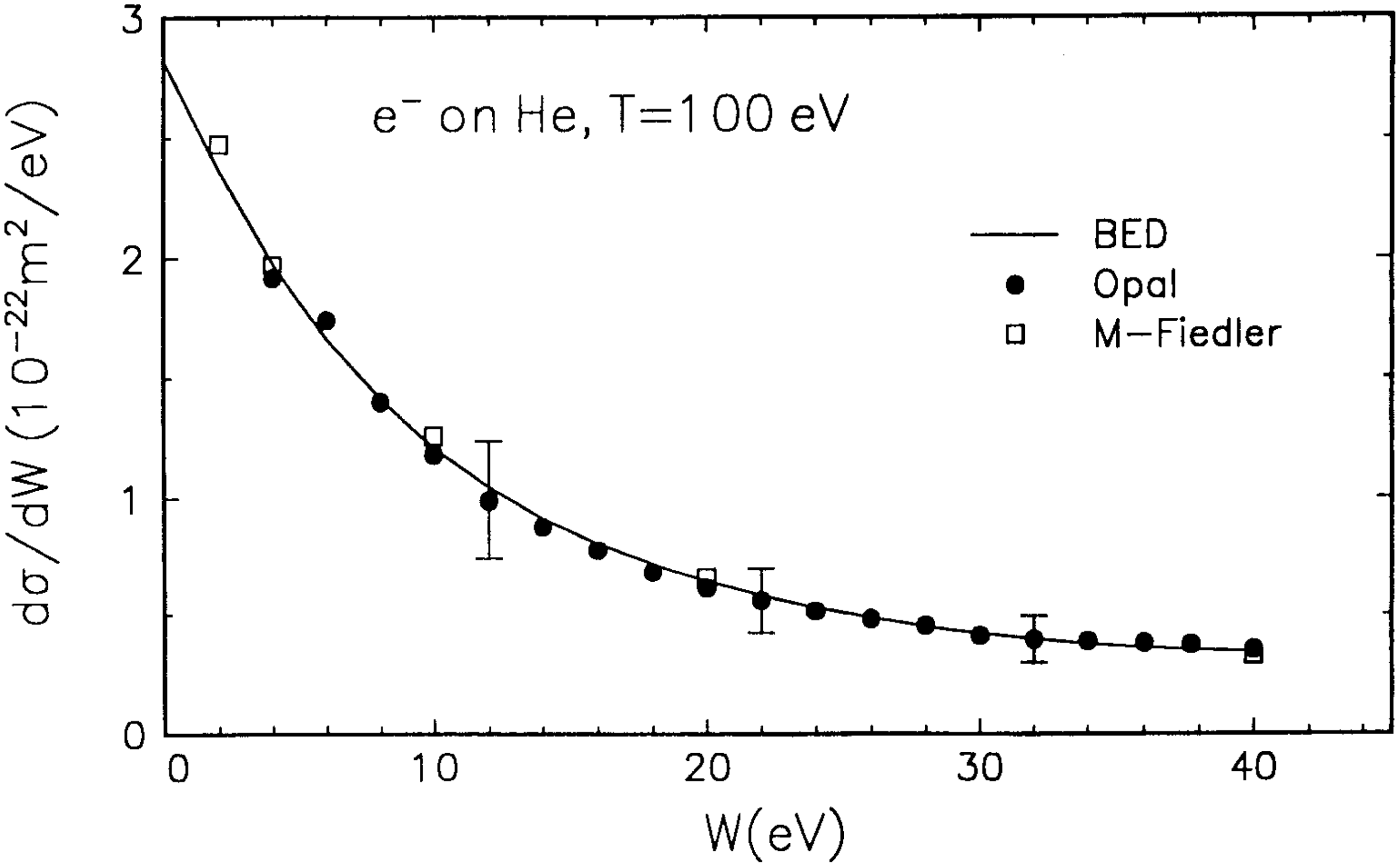
where $R = 13.6$ eV $t = T/B_i$

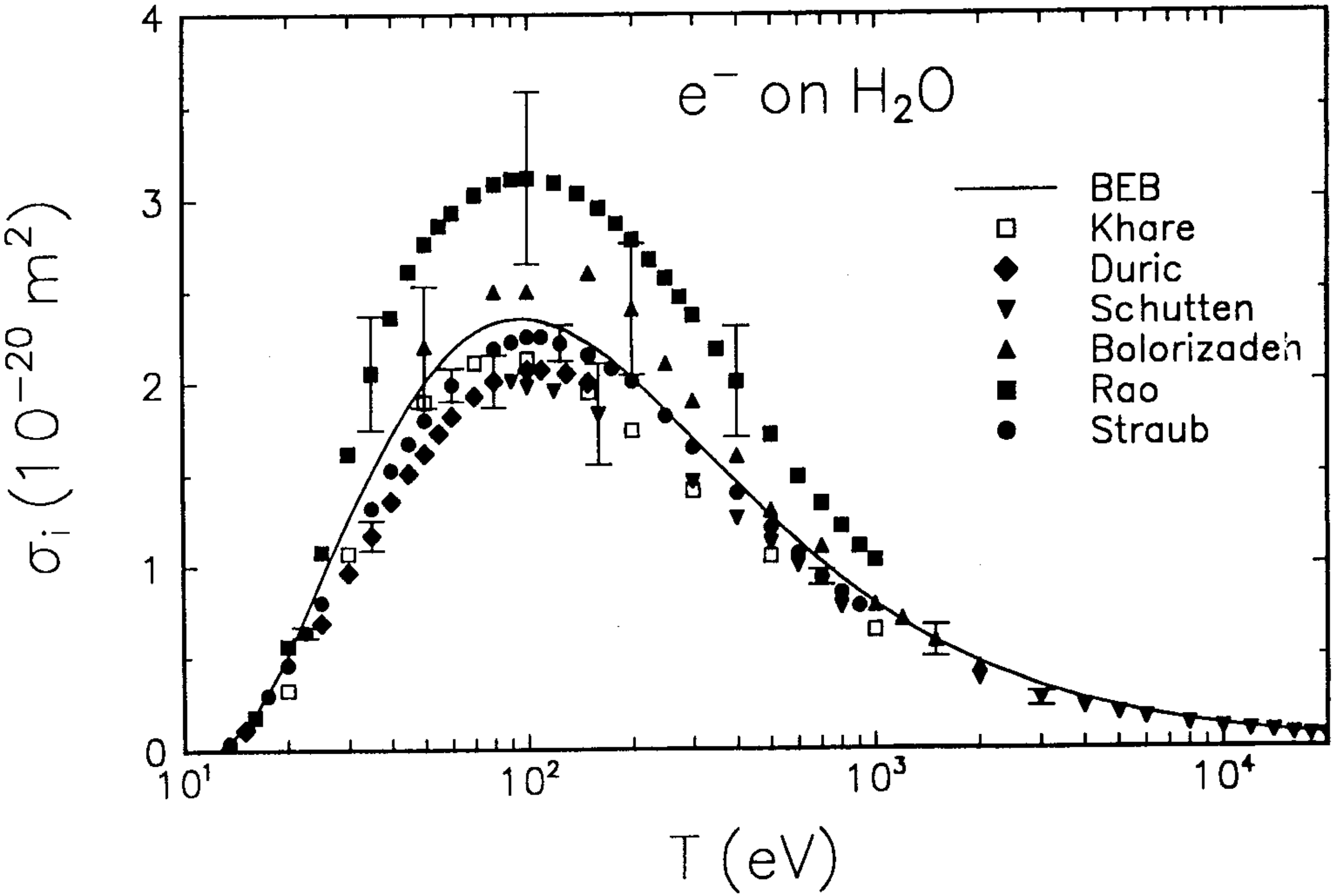
$S = 4\pi a_o^2 N(R/B_i)^2$ $u = U_i/B_i$

$a_o = 0.529$Å $w = W/B_i$

BEB Model for the total cross section:

$$\sigma_i = \frac{S}{t+u+1}\left[\frac{\ln t}{2}\left(1-\frac{1}{t^2}\right)+1-\frac{1}{t}-\frac{\ln t}{t+1}\right]$$


e⁻ on He, T=100 eV
BED
Opal
M−Fiedler
dσ/dW (10⁻²²m²/eV)
W(eV)
0
1
2
3
0
10
20
30
40



e⁻ on H₂O
BEB
Khare
Duric
Schutten
Bolorizadeh
Rao
Straub
σᵢ (10⁻²⁰ m²)
T (eV)
0
1
2
3
4
10¹
10²
10³
10⁴

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Photon, Electron, Proton and Neutron Interaction Data for Body Tissues

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