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TESTS OF FUNDAMENTALS OF QM WITH LASERS

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A CLASSICAL MODEL OF EPR EXPERIMENT WITH QUANTUM MECHANICAL CORRELATIONS AND BELL INEQUALITIES

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A simple model of a classical break-up process is given in which the correlation $E(a, b)$ of the components A and B of the spins of the two subsystems along directions a and b gives precisely the quantum mechanical result $-\cos(a \cdot b)$. The model is "local", but the normalization procedure of correlation functions in terms of "hidden variables" is different from that used in deriving Bell's inequalities. A discretization procedure of the classical spins is then given which reproduces fully the dichotomous quantum mechanical results both for probabilities and for correlation functions. This procedure illustrates particularly clearly the difference between quantum and classical spins and provides a possible intuitive picture for the notion of the "reduction of the wave function".

1. Introduction. Despite considerable amounts of literature in recent years, one of the difficulties with the interpretation of Bell's inequalities has been the lack of explicit models of hidden variables with which to compare classical and quantum correlations and to test the assumptions underlying their derivation. It is now generally believed that the so-called "local-realistic" hidden-variable (HV) theories are ruled out by experiment [1,2]. In this paper, we present a simple classical model of correlated events which can in principle be realized experimentally, and which exhibits the same correlations as those obtained in spin-1/2 correlation experiments. Our model is local in the sense of Bell, but is not in conflict with Bell's inequalities, because as we shall see the normalization of the correlation function is performed in different ways in our model and in the derivation of these inequalities.

We then use a technique formally similar to quantum projection operators to construct "joint probabilities" from the correlation function $E(a, b)$. Again, the observed probabilities are the same as in the quantum case, but the corresponding quantities at the "hidden variables" level are *not* positive definite, in contrast to Bell's requirement, and cannot be interpreted as probabilities. This situation can however be remedied by a discretization procedure leading to quantities that can be interpreted as probabilities also at the HV level. We then recover all results of quantum mechanics within a classical theory, but have to use a form of integration measure over the space of "hidden variables" different from that proposed by Bell. This simple example, and the three distinct ways to approach it, illustrate particularly simply the limitations of the hypotheses underlying the derivation of Bell's theorem.

2. Correlation functions. Consider a classical system initially at rest and which splits into two parts with opposite classical spins. The experiment consists in measuring the projections of the momenta (or equivalently of the angular momenta or spins) of the two separated subsystems along two directions a and b, respectively (fig. 1). The measured quantities are thus $p \cdot a$ and $-p \cdot b$. We indicate the relevant vectors in a 3-dimensional coordinate system shown in fig. 2.

We have

$$p \cdot a = pa \cos \mu , \quad -p \cdot b = -pb \cos \lambda , \tag{1}$$

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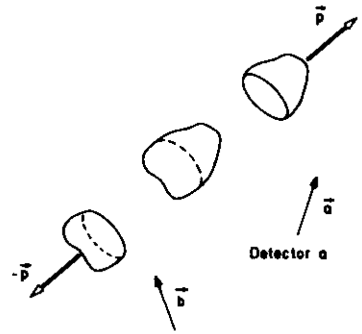

Fig. 1. The break-up process.

where p, a, and b are the magnitudes of the vectors p, a, and b. With respect to the axes as chosen in fig. 2,

$$A \equiv p \cdot a = pa \sin \theta \cos \varphi ,$$

$$B \equiv -p \cdot b = -pb(\cos \theta_0 \sin \theta \sin \varphi + \sin \theta_0 \sin \theta \sin \varphi) , \tag{2}$$

where θ_0 is the angle between a and b, $\cos \theta_0 = a \cdot b / ab$. We repeat the experiment a large number of times, and assume that the directions of the subsystems after splitting are randomly distributed in all directions. After averaging, we then obtain $\langle A \rangle = \langle B \rangle = 0$,

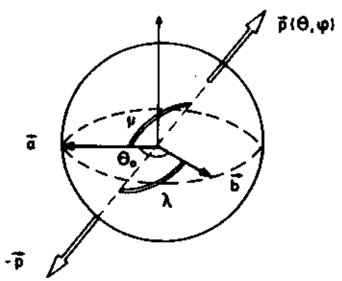

Fig. 2. The measured angles λ and μ of the projectiles relative to the detectors a and b.

$$\langle A^2 \rangle = \langle (p \cdot a)^2 \rangle = \frac{(pa)^2}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin \theta \cos^2 \mu = (pa)^2/3 , \tag{3}$$

and similarly

$$\langle B^2 \rangle = (pb)^2/3 . \tag{4}$$

The correlation $\langle AB \rangle = \langle (p \cdot a)(-p \cdot b) \rangle$ is found to be

$$\langle AB \rangle = -\frac{1}{4\pi} \iint d\theta \, d\varphi \, papb \sin \theta \cos \lambda \cos \mu = -\tfrac{1}{3} p^2 ab \cos \theta_0 . \tag{5}$$

The averages (3) to (5) still depend on the magnitudes of the various vectors. This is to be expected, since a classical spin can have any magnitude and therefore any value for a given component. If we are only interested in information about the direction of p, the relevant quantity is the normalized correlation function

$$E_c(a, b) = \frac{\langle AB \rangle}{\langle A^2 \rangle^{1/2} \langle B^2 \rangle^{1/2}} = \left\langle \frac{A}{\langle A^2 \rangle^{1/2}} \frac{B}{\langle B^2 \rangle^{1/2}} \right\rangle \equiv \widetilde{A}\widetilde{B} = -\cos \theta_0 . \tag{6}$$

This is identical with the correlation function obtained in the measurement of the spin components of a quantum mechanical spin-0 coherent state splitting into two spin-1/2 states:

$$E_Q(a, b) = \langle \psi | \tau_1 a \otimes \sigma_2 b | \psi \rangle , \tag{7}$$

where $|\psi\rangle = 2^{-1/2} [\uparrow\downarrow - \downarrow\uparrow]$.

In Bell's proof of the inequality [2]

$$-2 \leqslant E(a, b) - E(a, b') + E(a', b) + E(a', b') \leqslant 2 , \tag{8}$$

a crucial step is to write $E(a, b) = \int d\Lambda \, A_a(\Lambda) B_b(\Lambda)$, where Λ is a complete set of HV, and to assume that the expectation values of the unobserved $A_a(\Lambda)$ and $B_b(\Lambda)$ are bounded by one. This is an assumption, dictated by reasons to which we will return later on. In our case, we normalize the correlation function correctly by dividing it by the "magnitudes":

$$\widetilde{A}(\mu) = pa \cos \mu / \langle (p \cdot a)^2 \rangle^{1/2} = \sqrt{3} \cos \mu ,$$

$$\widetilde{B}(\lambda) = -pb \cos \lambda / \langle (p \cdot b)^2 \rangle^{1/2} = -\sqrt{3} \cos \lambda . \tag{9}$$

It is easily seen that in this case, Bell's proof does not follow. It is important to emphasize, however, that in our classical example, it is the procedure of normalization of the correlation function, rather than "nonlocality", that leads to a violation of Bell's inequalities.

We see, then, that the measured correlation functions alone do not tell too much about the details of the break-up process and can be reproduced by a simple classical picture. They merely describe the kinematics of the process, nothing more. In the next section, we show that joint probabilities already give a better indication of the difference between classical and quantum mechanics.

3. Probabilities. In experiments of the type performed e.g. by Aspect et al. [1], one does not measure directly the correlation function $E(a, b)$, but rather, the joint probabilities $P_{++}(a, b)$, $P_{+-}(a, b)$, etc. for the a- and b-component of the first and second spin, respectively, to be $+\hbar/2$ and $\pm\hbar/2$,... . These are obtained by decomposing the operator $\sigma_1 a \otimes \sigma_2 b$ in terms of projection operators as

$$4\sigma_1 a \otimes \sigma_2 b = (1 + \sigma_1 a)(1 + \sigma_2 b) + (1 - \sigma_1 a)(1 - \sigma_2 b) - (1 - \sigma_1 a)(1 + \sigma_2 b) - (1 + \sigma_1 a)(1 - \sigma_2 b) . \quad (10)$$

This yields

$$E_Q(a, b) = -\cos\theta_0 = \sin^2(\tfrac{1}{2}\theta_0) + \tfrac{1}{2}\sin^2(\tfrac{1}{2}\theta_0) - \tfrac{1}{2}\cos^2(\tfrac{1}{2}\theta_0) - \tfrac{1}{2}\cos^2(\tfrac{1}{2}\theta_0)$$
$$\equiv P_{++}(a, b) + P_{--}(a, b) - P_{-+}(a, b) - P_{+-}(a, b) . \quad (11)$$

In order to find a counterpart to this decomposition in our model, we use the identity

$$4\cos\lambda\cos\mu = (\alpha + \cos\lambda)(\beta + \cos\mu) + (\alpha - \cos\lambda)(\beta - \cos\mu) - (\alpha - \cos\lambda)(\beta + \cos\mu) - (\alpha + \cos\lambda)(\beta - \cos\mu) , \quad (12)$$

valid for arbitrary constants α and β.

By an appropriate choice of α and β we can find the observables whose correlation functions give the quantum mechanical ones $P_{ij}(a, b)$, $(i, j = \pm)$, namely

$$X_\pm(a, \Lambda) = \tfrac{1}{2}(1 \pm \widetilde{A}) , \quad Y_\pm(b, \Lambda) = \tfrac{1}{2}(1 \pm \widetilde{B}) . \quad (13)$$

In our case $\Lambda = [\lambda, \mu]$, and $X_\pm = \tfrac{1}{2}(1 \pm \sqrt{3}\cos\mu)$ and $Y_\pm = \tfrac{1}{2}(1 \mp \sqrt{3}\cos\lambda)$ are a measure of the correlation of the original observables A and B relative to their mean square deviations. Indeed, we find

$$P_{ij}(a, b) = \langle X_i Y_j \rangle = \tfrac{1}{4}(1 \pm \cos\theta_0) \quad (14)$$

(i.e., $\tfrac{1}{4}(1 - \cos\theta_0)$ for P_{++} and P_{--}, and $\tfrac{1}{4}(1 + \cos\theta_0)$ for P_{+-} and P_{-+}). We can now see why Bell's inequalities are inoperative here. If we write

$$P_{ij}(a, b) = \frac{1}{4\pi}\int d\Lambda\, P_{ij}(a, b, \Lambda) = \frac{1}{4\pi}\int d\Lambda\, P_i(a, \Lambda) P_j(b, \Lambda) , \quad (15)$$

then, our model is local in the sense of Bell, and all the final observed quantities (integrated over HV) are the same as quantum mechanically, but the densities $P_i(a, \Lambda) \equiv X_i(a, \Lambda) = \tfrac{1}{2}[1 \pm \widetilde{A}(a, \lambda)]$ are not positive definite. (Note that they take the same form in 3 dimensions as the "probabilities" obtained by Scully when describing the quantum mechanical problem with a Wigner distribution function [3].) Thus, they cannot be interpreted as probabilities.

In experiments aimed at testing Bell's theorem, the system may conceptually be regarded as a black-box connected to four lamps, two on each side, which always light up in pairs. One measures then the joint probability that a given pair of lamps light up by accumulating the results of single experiments [‡1]. Thus the experiment is reduced to counting. This is why one insists on having positive definite densities $P_i(a, \Lambda)$ and $P_j(b, \Lambda)$ for each single event. (It is a similar argumentation which leads to the requirement that $A_a(\Lambda)$ and $B_b(\Lambda)$ be bounded by 1.) We now show how this can be done in our model.

4. Discretization. Although all final correlations, probabilities, and joint probabilities are the same for both the classical and quantum mechanical cases, and apart from the difficulty with negative densities, there is clearly a fundamental difference between a quantized and a classical spin. In the quantum case, each event contributes to exactly one count in one of the $P_{ij}(a, b)$, say $P_{++}(a, b)$. In the classical case, each event contributes, in general, a certain amount to all of the $P_{ij}(a, b)$. This is of course related to another feature of the quantum theory of measurement, namely the collapse of the wave function. We do not wish to erase this difference. However, we can operationally discretize our continuous measurement so that the final recording of events will be exactly like the quantum mechanical "yes" or "no" experiment, with the same probabilities and correlation functions.

‡1 For a tutorial discussion of this point, see ref. [4].

For this purpose, we construct a detector that first measures both observables $X_+(a, \Lambda)$ and $X_-(a, \Lambda)$, and compares them. The system is then instructed to associate the region where $X_+(a, \Lambda) > X_-(a, \Lambda)$ with spin "up", or "+", relative to a. Assuming, without loss of generality, a and b to be in the equatorial plane, as shown in fig. 2, and generalizing slightly from our preceding discussion to label a by the angle φ_a (instead of zero), the region of angles for which $X_+ > X_-$ is given by

$$-\pi/2 \leqslant \varphi - \varphi_a \leqslant \pi/2 . \quad (16)$$

This is the upper hemisphere with the vector a as north pole. The lower hemisphere will then be spin "down". Similarly the region for which $Y_- > Y_+$ is given by

$$-\pi/2 \leqslant \varphi - \varphi_b \leqslant \pi/2 , \quad (17)$$

i.e. the upper hemisphere with vector b as north pole. (Note that the spin S_2 of the second particle is opposite to the spin S_1 of the first one, $S_2 = -S_1$, so that the component of S_2 in the upper half-plane is negative.)

The joint probability of, say, X_+ and Y_- occurs then in the intersection of the two regions (16) and (17). Similar considerations hold for X_+Y_+, X_-Y_- and X_-Y_+.

Recording each event at an angle φ with a weight factor $N|\cos(\varphi - \varphi_a)|$, respectively $N|\cos(\varphi - \varphi_b)|$, where $N = 1/4$ is a normalization constant yields for the single probabilities

$$P_+(a) = \frac{1}{4}\int_0^{2\pi} d\varphi |\cos(\varphi - \varphi_a)| \times [H(\varphi - (\varphi_a - \pi/2)) - H(\varphi - (\varphi_a + \pi/2))]$$
$$= \frac{1}{4}\int_{\varphi_a - \pi/2}^{\varphi_a + \pi/2} d\varphi |\cos(\varphi - \varphi_a)| = 1/2 , \quad (18a)$$

where $H(x)$ is the Heaviside function. This is just the sum of all spins in the upper hemisphere with respect to a, weighted by the factor

$$\Pi(\varphi, \varphi_a) = \tfrac{1}{4}|\cos(\varphi - \varphi_a)| \quad (19)$$

i.e., the sum of the projections of the spin components along the detector axis a. Similarly, $P_-(a) = 1/2$, and

$$P_-(b) = \frac{1}{4}\int_0^{2\pi} d\varphi |\cos(\varphi - \varphi_b)| \times [H(\varphi - (\varphi_b - \pi/2)) - H(\varphi - (\varphi_b + \pi/2))]$$
$$= \frac{1}{4}\int_{\varphi_b - \pi/2}^{\varphi_b + \pi/2} d\varphi |\cos(\varphi - \varphi_b)| = 1/2 . \quad (18b)$$

Note the difference between this and the averaging procedure used in the derivation of Bell's inequalities. In this last case, one uses the same measure to integrate $P_i(a, \Lambda)$, $P_j(b, \Lambda)$, and $P_{ij}(a, b, \Lambda)$ over the hidden-variables space:

$$P_i(a) = \int d\Lambda\, \rho(\Lambda) P_i(a, \Lambda) , \quad (20a)$$

$$P_j(b) = \int d\Lambda\, \rho(\Lambda) P_j(b, \Lambda) , \quad (20b)$$

$$P_{ij}(a, b) = \int d\Lambda\, \rho(\Lambda) P_i(a, \Lambda) P_j(b, \Lambda) , \quad (20c)$$

while in our case, the measure is replaced by a "projector" on the detectors axes a or b.

In our detection scheme, the joint probabilities are given by the intersection of the two relevant hemispheres, with a weight function that may be chosen as either that of the first detector, or of the second detector (or the average of both). For example:

$$P_{+-}(a, b) = \int_0^{2\pi} d\varphi\, \Pi(\varphi, \varphi_a) \times [H(\varphi - (\varphi_a - \pi/2)) - H(\varphi - (\varphi_a + \pi/2))] \times [H(\varphi - (\varphi_b - \pi/2)) - H(\varphi - (\varphi_b + \pi/2))]$$
$$= \int_0^{2\pi} d\varphi\, \Pi(\varphi, \varphi_b) \times [H(\varphi - (\varphi_a - \pi/2)) - H(\varphi - (\varphi_a + \pi/2))] \times [H(\varphi - (\varphi_b - \pi/2)) - H(\varphi - (\varphi_b + \pi/2))]$$
$$= \int_{\varphi_b - \pi/2}^{\varphi_a + \pi/2} d\varphi\, \Pi(\varphi, \varphi_a) = \tfrac{1}{4}(1 + \cos\theta_0) . \quad (18c)$$

This definition of the joint probability $P_{+-}(a, b)$, while quite different from the prescription of local-realistic HV theories, presents interesting features. Most notably, in the evaluation of joint probabilities one applies only one of the "projectors" $\Pi(\varphi, \varphi_a)$ or $\Pi(\varphi, \varphi_b)$. This is reminiscent of the quantum mechanical reduction of the wave packet: in that last case, once one of the spins is measured, then, the value of the other one is known with certainty. Our prescription does essentially the same thing: once one of the spins has been "projected" on its detection axis, no further projection is necessary. Thus, we can simulate the results of quantum mechanics by a completely classical descretization procedure.

Is this procedure local? The single probabilities (18a) and (18b) do not depend on the setting of the other detector. But the way $P_{ij}(a, b)$ is constructed, although independent of the relative setting of the two detectors, does depend on the fact that both detectors are present. This, of course, is also true in local-realistic HV theories. But the difference is that in that last case, one multiplies the single probabilities [eq. (20c)] while in our model $P_{ij}(a, b)$ is determined by the overlap region shown in fig. 3. In this region, the events at a are weighted by the "projector" (19), and the events at b with probability one (or vice versa) which introduces explicitly the fact that these are absolutely correlated events. (If the component of the first spin along a is known, we know for sure the value of the component of the second spin along b.) Whether our system can be called local, in some generalized sense, is then probably a question of taste.

5. Conclusions. We have compared the quantum mechanical spin correlation function (7) with the corresponding classical spin correlation (6) calculated completely classically, and found the same result in both cases. We also computed joint probabilities, and again found the same final observable results. Bell's inequalities are based on supplementary assumptions about the probability densities in the hidden variables space. There are no additional assumptions in our model.

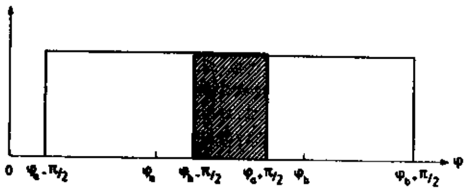

Fig. 3. Overlap region of spin up for the first particle and down for the second one.

Since quantum spins are discrete and the experiments yield discrete "yes" or "no" counts, but classical spins are continuous, we have introduced a natural discretization process counting all spins in the upper hemisphere relative to the observer as "up", and all spins in the lower hemisphere as down, together with a suitable measure. With this, our experiment also reproduces the discrete quantum mechanical correlations. We therefore conclude that a classical experiment can reproduce exactly observed quantum mechanical correlations, but that some of the assumptions underlying Bell's inequalities have to be removed. Our model further sheds some light on the differences and similarities between the classical and quantum mechanical spins and the corresponding spin correlation observables.

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IS REALITY REALLY REAL? - AN INTRODUCTION TO BELL'S INEQUALITIES

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1. INTRODUCTION

Ever since its development in the 1920's, quantum mechanics has been the object of numerous discussions, which are still going on, and will probably keep going on for some time. At the onset, one should agree on one point, namely, that quantum mechanics works extremely well, and allows us to predict the most minute aspects of, say, atomic spectra, with incredible accuracy. The problem is not there, but rather, lies in its interpretation. What is the meaning of the wave function, what is performed in a measurement, etc. ..., are questions which have fascinated, and still fascinate, many physicists. Some people make a living out of discussing these problems, but for most of us, this is a hobby, that we talk about during coffee breaks or in the evening, around a pitcher of beer. I am certainly not an expert on the foundations of quantum mechanics. But over the last few years, I have read a substantial amount of papers on this topic, and have realized that during my studies I had been "brain-washed" into accepting things which I should not have ... at least, not readily. I have come to understand that we live in a very strange world indeed, where the most trivial, self-evident truths don't apply. In this lecture, I would like to explain why it is so. What I will say is not new, it is just my way of understanding and summarizing the work of very clever people such as Einstein, Bohr, Bohm, Bell, Wigner, and many others. Of course, I may well misunderstand and misquote them at some point or the other, and apologize in advance for this. I hope nevertheless that these notes may be useful to some other "operational" physicists, who spend their life doing very concrete calculations or experiments, but ask themselves now and then "what on earth it all means".

In Section 2, I briefly review the Einstein-Podolski-Rosen "paradox". This is the starting point of most discussions of the foundations of quantum mechanics, puts the whole problem into a proper frame, and allows to introduce the central concepts of "locality" and "reality". In Section 3, I then open a parenthesis to discuss a variation on Bertlmann's socks adapted to the present summer school. This example, introduced by J.S. Bell to illustrate the strangeness of quantum mechanics, sparked my understanding of the problem, and maybe, it will help somebody else, too.

In Section 4, I then derive Bell's inequalities in a simple form, trying to explicitly show at which point each assumption enters -this is not necessarily obvious to see in the published literature. In Section 5, I review the most recent experiments, by Aspect and coworkers, using photon cascade in atomic transitions to test this inequality. In Section 6, I mention a loophole in Aspect's experiments, which leads to a brief discussion of delayed choice experiments. Finally, in Section 7, I ponder about the implications of these results. Is our world not local, or not real? ... I must say at the onset that any reader expecting to find an answer to these questions in this paper is going to be bitterly disappointed.

2. THE EINSTEIN-PODOLSKI-ROSEN "PARADOX"

It is well-known that Einstein, although he did not deny its operational success, never quite accepted quantum mechanics. In a famous paper[1], Einstein, Podolsky and Rosen (EPR) proposed a Gedankenexperiment aimed at proving that quantum mechanics is not a complete theory. We discuss here a variation of this experiment proposed by Bohm[2].

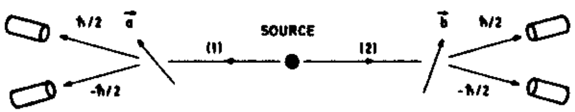

Fig. 1: Experimental set-up of the EPR-Bohm Gedanken experiment. The second analyzer $\vec{b}$ is not relevant here, but plays a central role in Bell's theorem (see Section 4).

Consider a source in which pairs of identical spin 1/2 particles are produced by, say, the decay of a diatomic molecule in the singlet state. Upon emerging from the source, these two particles fly towards two space-like separated analyzers and detectors, such as Stern-Gerlach magnets. (see Fig. 1).

Long after the particles are emitted, an observer sets the analyzer (magnet) 1, in order to measure the spin component $S_a = \vec{a} \cdot \vec{S}_1$ of particle 1 along $\vec{a}$. For a spin-1/2 particle, the result is $\pm \hbar/2$. Because the total spin of the system is zero, we know for sure, <u>without having to perform a measurement</u>, that the spin component S_a of the second particle along this same direction is then $\mp \hbar/2$.

At this point, EPR introduce the concept of <u>"reality"</u>[1]: "If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there is an element of physical reality corresponding to this physical quantity." EPR further require that "every element of the physical reality must have a counterpart in the physical theory".

According to this criterion, we can then attribute an element of physical reality to the spin component S_a. However, our observer might have chosen to set the detector 1 in direction $\vec{a}'$, thus measuring the spin component $S_{a'}$, and inferring, without in any way disturbing particle 2, its spin component $S_{a'}$. Thus, there is also an element of physical reality attached to $S_{a'}$.

According to quantum mechanics, however, one cannot predict precise values for non-commuting observables. If quantum mechanics is complete, the two observables S_a and $S_{a'}$ cannot have simultaneous reality (unless $\vec{a} = \vec{a}'$), in contradiction with the preceeding argument. Thus, EPR conclude that quantum mechanics is not complete[3].

Of course, many of the founders of quantum mechanics, and in particular Bohr[4], have refuted this argument, maintaining that the specification of the experimental procedure plays a central role in quantum mechanics, but for many years, no real progress was made. In fact, a majority of physicists believed that this whole discussion belonged more to the realm of philosophy that to physics, since it looked like no experiment was able to determine which was the correct attitude.

The situation was changed drastically since the early 1960's, due in particular to the work of J.S. Bell, who studied an extention of the original EPR experiment, where the analyzers 1 and 2 (see Fig. 1) are set at <u>different</u> angles $\vec{a}$ and $\vec{b}$. One then measures the joint probability of obtaining a given value (say, + ħ/2) for the spin components S_a and S_b.

Let us call $P_{++}(a,b)$ the joint probability of measuring S_a = ħ/2 and S_b = ħ/2, P_{+-} (a,b) the joint probability of measuring S_a = ħ/2, and S_b = -ħ/2, etc... According to quantum mechanics, we have

$$P_{++}(a,b) = P_{--}(a,b) = \frac{1}{2}\sin^2\theta/2,$$
$$P_{+-}(a,b) = P_{-+}(a,b) = \frac{1}{2}\cos^2\theta/2, \qquad (1)$$

where θ is the angle between $\vec{a}$ and $\vec{b}$. This indicates that quantum mechanics predicts strong correlations between the measurements at detectors 1 and 2. For instance, for θ = 0, we get $P_{++}(a,b)$ = $P_{--}(a,b)$ = 0, and $P_{+-}(a,b)$ = 1/2.

Of course, strong correlations are well-known in every-day life, too. Think for instance of two friends taking blindly one ball each out of a bag containing one white ball and one black ball, putting them into their pocket and then travelling one to the moon, and the other to Boulder. The two friends agree to look at the color of the balls at a given, prearranged time. When the traveler to the moon sees that he has, say, the black ball, he immediately knows that his friend has the white one, without need to even check. There is a strong correlation due to a common cause. (It is also legitimate for the moon traveler to attribute an element of physical reality to his friends's white ball - or is it? EPR would say yes.)

Are the correlations observed on the EPR-type experiments also due simply to common causes (the two spins are after all prepared by the decay of a single, common, molecule)? In the original EPR experiment, the answer could have been, yes! But the surprise is that in Bell's more elaborate version, the answer is, to a large degree of certainty, no (that is, provided that Aspect's results still hold in a "delayed choice" version of his experiment). And to understand better what this means, it is time to turn to Bertlmann's socks.

3. DR. BERTLMANN GOES TO BOULDER

In a recent paper [5], J.S. Bell discussed the case of Dr. Bertlmann's socks. I found this example, directly inspired by d'Espagnat[6], very illuminating, and tried to adapt it to the context of this school, which takes place in the foothills of the beautiful Rocky Mountains. In a series of experiments, I have asked participants to this Institute to perform two of three tests, con-

sisting of hikes on trails on flat ground, on slopes of 45°, and on vertical cliffs, respectively.

Now, for a fresh bunch of participants, one has that

(those who can hike at 0° but not at 45°)

\+

(those who can hike at 45° but not at 90°) (2)

is not less than

(those who can hike at 0° but not at 90°)

which is trivially correct, and not very deep, since members of the last group can either hike at 45°, and belong also to the second group, or not, in which case they belong to the first group. (Note that we do not assume that if somebody can hike at some angle, he can hike at lower angles, too.). However, it is hard to perform such an experiment, because if we bring somebody incompetent on a steep cliff, he may not be available for the next test! Also, after one test, a hiker is not fresh anymore, and may not pass another test!

But I have noticed that people always go in pairs of equivalently trained hikers, so that if one member passes a test, the other one would pass it, too.

Thus, I can rewrite relation (2) as

(the number of pairs where one member can hike at 0° and the other not at 45°)

\+

(the number of pairs where one member can hike at 45° and the other not at 90°) (3)

is not less than

(the number of pairs where one member can hike at 0° and the other not at 90°).

Assuming that the number of participants at the school is so large that one can go from single events to probabilities, this may be rewritten as

(the probability that one hiker can make it at 0° and the other not at 45°)

+

(the probability that one hiker can make it at 45° and the other not at 90°) (4)

is not less than

(the probability that one hiker can make it at 0° and the other not at 90°).

Now, spins in the EPR experiment are very much like hikers -or socks - except that they are anticorrelated: if one spin passes a test, the other one will not - if the spin component S_a is equal to $+\hbar/2$, then S_b is $-\hbar/2$. Thus, for spins, we must reexpress relation (4) as[24]

Prob (one spin having $\frac{\hbar}{2}$ at 0° and the other not $(-\frac{\hbar}{2})$ at 45°)

+

Prob (one spin having $\frac{\hbar}{2}$ at 45° and the other not $(-\frac{\hbar}{2})$ at 90°)

is not less than (5)

Prob (one spin having $\frac{\hbar}{2}$ at 0° and the other not $(-\frac{\hbar}{2})$ at 90°)

This is, in essence, Bell's inequality. From Eq. (1), we can also compute the predictions of quantum mechanics for such an experiment. Taking by convention that $+h/2$ corresponds to passing the test, we have

$$\frac{1}{2}\sin^2(22.5) + \frac{1}{2}\sin^2(22.5) = 0.1464,$$

and

$$\frac{1}{2}\sin^2(45) = 0.25.$$

Thus, quantum mechanics clearly violates Relation (5)! As we shall see later on, experiments up to now agree with quantum mechanics, and are in violation of Bell's inequalities. This indicates that at the microscopic level, things don't behave like socks or hikers anymore!

To understand better what is so peculiar about the microscopic world, let us now derive somewhat more formally Bell's inequalities, and try to isolate the hypotheses leading to them.

4. BELL'S INEQUALITIES

In 1964, Bell[8] showed that by performing correlation experiments of the type just discussed, one can distinguish between the predictions of quantum mechanics and those of so-called "local-realistic hidden-variable theories". Later on, his work was extended in particular by Clauser, Holt, Horne and Shimony [9]. Here, I limit my discussion to a simple case, using the derivation that I personnally like best [10,11].

The first point is that one should not be afraid by the term "local-realistic hidden-variable theories". What hides behind it is the idea that, following EPR, quantum mechanics is not complete. At some level, not yet understood, in an ultimate theory, there must be more, or different, variables providing a complete description of the system under study. Since these are not yet known, or measurable, they are hidden - thus hidden variables.

"Local-realistic" is a fancy word which, basically, means that one would like to have a "reasonable" ultimate theory. By reasonable, one means that three "self-evident truths" should hold, which d'Espagnat calls [6]

- realism,
- locality,
- free use of inductive inference.

I will try to explain these as we go along.

Consider an arrangement like that of Figure 1, with a source emitting two correlated particles (1) and (2). A property of these particles is measured by detectors 1 and 2, with settings $\vec{a}$, resp. $\vec{b}$, of the analyzers. (The analyzers could be Stern-Gerlach magnets for spins, polarizers for photons, etc ...) Using the principle of <u>inductive inference</u> - legitimate conclusions can be drawn from regularities in the results of experiments - we are entitled to speak about probabilities, rather than single events.

Let us denote $p_1(a)$ and $p_2(b)$ the probabilities of detecting particle (1), resp. (2) for settings $\vec{a}$ and $\vec{b}$ of the analyzers. (For instance, in the EPR-Bohm experiment, we could measure if a spin component is up along direction $\vec{a}$ and down along $\vec{b}$. If we had a complete theory at hand, $p_1(a)$ would be a function of all parameters $\{\lambda\}$ describing completely the emission process in the source. But at the present stage of physics (1983) we have no way to know, or measure, or even guess what these parameters might be. They are hidden, out or our control. What we detect in a series of measurements is some average over them:

$$p_1(a) = \int d\lambda \rho(\lambda) p_1(a,\lambda), \qquad (6)$$

where $d\lambda$ is a - unknown - measure over the space of hidden variables, and $\rho(\lambda)$ some weight function. For simplicity, we write λ instead of $\{\lambda\}$. Similarly,

$$p_2(b) = \int d\lambda \rho(\lambda) p_2(b,\lambda). \qquad (7)$$

We may, but don't have to require

$$\int d\lambda \rho(\lambda) = 1. \qquad (8)$$

Suppose for a moment that we actualy could control the hidden parameters $\{\lambda\}$, and know precisely what their value is. We could then ask the joint probability $p_{12}(a,b,\lambda)$ of detecting both particles for settings $\vec{a}$ and $\vec{b}$ of the analyzers. If the detectors are space-like separated, and the settings chosen long after the emission process, the result at one detector should be unaffected by the setting of the other. This is the principle of locality: no influence of any kind can travel faster than the speed of light -there can be no cross-talk between detectors 1 and 2. Thus, the counting-rates at detectors 1 and 2 should be uncorrelated, so that

$$p_{12}(a,b,\lambda) = p_1(a,\lambda) \cdot p_2(b,\lambda). \qquad (9)$$

Note, however, that the actually measured joint probability needs not be uncorrelated: Integrating over the hidden variables, we get with (9)

$$p_{12}(a,b) = \int d\lambda \rho(\lambda) p_1(a,\lambda) p_2(b,\lambda). \qquad (10)$$

The weight function $\rho(\lambda)$, which contains all informations about the correlations between hidden variables in the source, leads in general to a nonfactorizable joined probability distribution $p_{12}(a,b)$ (correlation through common cause). We present some examples of functions $\rho(\lambda)$ in Section 7.

A simple theorem [10] states that for any four numbers x,x',y, and y' between 0 and 1, the following inequalities hold:

$$-1 \leq xy - xy' + x'y + x'y' - x' - y \leq 0. \qquad (11)$$

Noting that probabilities lie between 0 and 1, and chosing two possible directions $\vec{a}$ and $\vec{a}'$, respectively $\vec{b}$ and $\vec{b}'$, for the analyzers 1 and 2, we obtain

$$\begin{aligned} -1 \leq\ & p_1(a,\lambda)p_2(b,\lambda) - p_1(a,\lambda)p_2(b',\lambda) \\ &+ p_1(a',\lambda)p_2(b,\lambda) + p_1(a',\lambda)p_2(b',\lambda) \\ &- p_1(a',\lambda) - p_2(b,\lambda) \leq 0, \end{aligned}$$

or, with Eq. (9)

$$\begin{aligned} -1 \leq\ & p_{12}(a,b,\lambda) - p_{12}(a,b',\lambda) + p_{12}(a',b,\lambda) \\ &+ p_{12}(a',b',\lambda) - p_1(a',\lambda) - p_2(b,\lambda) \leq 0. \end{aligned} \qquad (12)$$

Integrating Eq. (12) over the hidden variables yields finally

$$\begin{aligned} -\int d\lambda \rho(\lambda) \leq\ & p_{12}(a,b) - p_{12}(a,b') + p_{12}(a',b) \\ &+ p_{12}(a',b') - p_1(a') - p_2(b) \leq 0 \end{aligned} \qquad (13)$$

The left-hand side of this inequality is equal to -1 if condition (8) holds, but we actually don't need it. Keeping the right-hand side only yields:

$$\frac{p_{12}(a,b) - p_{12}(a,b') + p_{12}(a',b) + p_{12}(a',b')}{p_1(a') + p_2(b)} \leq 1, \qquad (14)$$

which is one form of Bell's inequalities, as derived by Clauser and Horne [10].

It is important to realize that when introducing two possible directions $\vec{a}$ and $\vec{a}'$, resp. $\vec{b}$ and $\vec{b}'$ for the analyzers, we implicitly assume that we can speak about the outset of measurements even if we don't actually perform them. (We cannot make simultaneous measurements of particle (1) along both $\vec{a}$ and $\vec{a}'$ directions!) This is the hypothesis of reality introduced by EPR [1], according to which there is an objective physical reality independent of whether we make an observation or not. Thus, three assumptions were indeed needed to derive Bell's inequalities: inductive inference, locality, and reality[6].

In practice, it is not the form (14) of Bell's theorem which is tested. Rather, one makes supplementary hypotheses, some of which can at least in principle be tested experimentally. In particular, if the joint probability $p_{12}(a,b)$ depends only on the angle θ between $\vec{a}$ and $\vec{b}$, and the probability $p_1(\vec{a})$ is independent of the direction of the analyzer, we can chose $\vec{a}$, $\vec{a}'$, $\vec{b}$, and $\vec{b}'$ as in Fig. 2.

Equation (14), becomes then

$$\frac{3p_{12}(\theta) - p_{12}(3\theta)}{p_1 + p_2} \leq 1. \qquad (15)$$

For the spin-1/2 case discussed in Section 1, quantum mechanics predicts $p_1 = p_2 = 1/2$, and with Eq. (1) and $\theta = 45°$, we obtain [12]

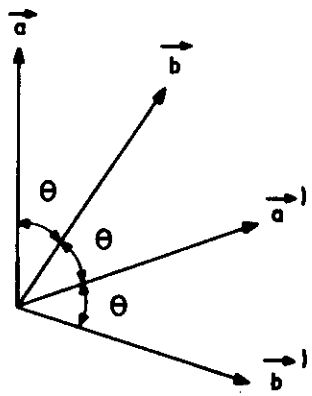

Fig. 2 Geometry used to derive the form (15) of Bell's theorem.

$$\frac{3p_{12}(\theta) - p_{12}(3\theta)}{p_1 + p_2} = 1.207,$$

in violation of Bell's theorem. Thus, quantum-mechanical correlations cannot be fully accounted for by any "local-realistic" hidden-variables theory. The question, then, is to determine which "self-evident truth" is violated by quantum mechanics.

Of course, the situation would be easiest if Bell's inequalities were not violated in experiments, in which case, our world would be "normal", after all. However, the best evidence today, although not definitive, clearly favors quantum mechanics.

5. EXPERIMENTS

The experimental efforts until 1978 to check Bell's theorem are summarized in the review by Clauser and Shimony[9]. Here, we briefly discuss the best experiments to-date, recently performed at Orsay by Aspect and coworkers [13,14]. Instead of spins, as in the original EPR-Bohm Gedankenexperiment, the system used here consists of pairs of optical photons emitted in an atomic radiative cascade (see Fig. 3) [15].

The $4p^2\ {}^1S_0$ level of calcium is populated by two-photon excitation, and decays back to the $4s^2\ {}^1S_0$ state over the $4s4p\,{}^1P_1$ level, emitting two photons of wavelengths λ_1 = 5513 Å and λ_2 = 4227 Å.

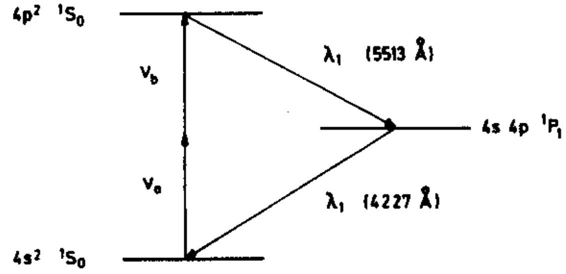

Fig. 3 Radiative cascade scheme used in the Aspect experimental test of Bell's theorem.

Because the change of angular momentum in the transition is $J = 0 \to J=1 \to J=0$, no net angular momentum is carried by the photons. For emitted photons counterpropagating in the ± z-directions, the state of polarization of the total system must therefore be of the form

$$\psi = \frac{1}{\sqrt{2}} \quad \{\binom{1}{0}\binom{1}{0} + \binom{0}{1}\binom{0}{1}\}, \qquad (16)$$

where $\binom{1}{0}$ represents the polarization along the $\hat{x}$-axis and $\binom{0}{1}$ along the $\hat{y}$-axis. The first column-vector describes the λ_1-photon and the second the λ_2-photon. Thus, in such a cascade, the polarization states are completely correlated, as the spins are completely anticorrelated in the Bohm set-up. But the difference between complete correlations and complete anticorrelation is irrelevant, as already discussed within the example of Section 3. Thus, photon cascades are appropriate systems to test Bell's inequalities.

The experimental difficulties are, however, considerable [9,16]. Most of the painstaking work consists in improving detectors, polarizers, sources, etc... The best experiment so far [14], however, has attained a high degree of perfection and is becoming agonizingly close to the idealized EPR-Bohm-Bell scheme. (There are, however, a couple of loopholes left. The most severe is discussed briefly in Section 6).

In photon-cascade experiments, one does not measure directly the quantitiy Eq. (14), but rather, another combination of correlation functions which satisfies the inequality [14]:

$$-2 \leq S \leq 2.$$

The detailed form of S in irrelevant here. It is sufficiant to note that this combination is more direct - or basic - in the sense that no single probabilities are used and both output channels of each analyzer are monitored. The experimental results, for an angle of 22.5° or 67.5° between the polarizers $\vec{a}$ and $\vec{b}$ is, however, S_{exp} = 2.697 ± 0.015, while quantum mechanics predicts $S_{QM} = 2.70 \pm 0.05$. (The theoretical uncertainty takes into account imperfections in the detection system). Thus, the experiment shows a spectacular violation of Bell's inequalities and is in excellent agreement with the predictions of quantum mechanics. Assuming that these results remain valid when the last two experimental loopholes are eliminated, which is most likely, we must therefore conclude that local-realistic hidden variable theories are wrong.

6. DELAYED-CHOICE EXPERIMENTS

As recognized already by Aspect et al., the experiments of Ref. 14 still have a couple of loopholes left. The first one, related to the low efficiencies of the detectors, does not appear to be very severe. The other is, however, of a conceptual nature and must be eliminated in future experiments. Specifically, the difficulty arises from the fact that the setting of the analyzers was fixed before the emission process has taken place. Thus, in principle, the analyzers could "tell" the source how to emit before hand, and the experiments do not actually test the locality or unlocality of quantum mechanics.

To eliminate this loophole, "delayed choice" experiments must be performed [16,17]. Aspect et al. [18] have recently gone one step in this direction by performing a series of experiments using variable polarizers which jump between two orientations in a time short compared to the photon transit time. The results are still in violation of Bell's inequalities. However, the experimental arrangement is not yet ideal: the change of analyzer direction is not random, but rather quasiperiodic, although the switches of the polarizers are driven by different generators at different frequencies. These recent experiments make "local-realistic" hidden-variable theories more and more improbable, but truly delayed-choice experiments, where the analyzers are set randomly long after the emission process, are nevertheless still absolutely required. Nobody really doubts that in this case, Bell's inequalities will still be violated. And nobody really knows what this means. We now know that the microscopic world is very strange, but what is strange in it is not yet clear.

7. INTERPRETATION ?

In Section 4, we derived Bell's inequalities using only three "self-evident truths": locality, reality, and the use of inductive inference. Their violation in photon cascade and other experiments indicates that at least one of these assumptions is wrong. Which one? Nobody actually knows, but many people seem to believe that locality is the bad guy. However, there is no proof of that.

I would like to suggest that, maybe, none of these "self-evident truths" is wrong. In the derivation of Bell's theorem, there was a fourth hypothesis, which one mostly doesn't pay much attention to, namely the fact that probabilities are positive and bounded by 1. However, if one abandons this requirement, one can easily build a local-realistic hidden variables model violating Bell's inequalities and reproducing the quantum mechanical results. In the following, I give a heuristic example of how to do this for the case of photon cascades. Using classical-looking representations such as the Wigner distributon, Scully has developed a systematic way to build such "hidden-variable theories"[19].

I start from Eqs. (6) and (10), and assume that the source is completely described by two sets of hidden variables α and β such that [20]

$$p_1(a) = \int_0 d\alpha \int_0 d\beta \rho(\alpha,\beta)\cos^2(a-\alpha), \qquad (17)$$

where a is the angle of the λ_1-photon polarizer with respect to the origin, and α varies between 0 and 2π. Similarly

$$p_2(b) = \int_0 d\alpha \int_0 d\beta \rho(\alpha,\beta)\cos^2(b-\beta) \qquad (18)$$

and

$$p_{12}(a,b) = \int_0 d\alpha \int_0 d\beta \rho(\alpha,\beta)\cos^2(a-\alpha)\cos^2(b-\beta). \qquad (19)$$

Belifante[20] has shown that the condition of "maximum source correlation",

$$\rho(\alpha,\beta) = \frac{1}{2\pi}\,\delta(\alpha-\beta) \qquad (20)$$

yields $p_1(a) = p_2(b) = 1/2$, as it should, but

$$p_{12}(a,b) = \frac{1}{4}\,[\cos^2(a-b) + \frac{1}{2}], \qquad (21)$$

which is within the bounds allowed by Bell's inequalities. The "problem" of quantum mechanics is that it produces more correlations than allowed by the common causes involved in local-realistic theories. How can one then produce more correlations in the source? For instance, by using a distribution sharper than a delta-function.

Taking as an example

$$\rho\ (\alpha,\beta) = \frac{1}{2\pi}\ \delta(\alpha-\beta)\ -\ \frac{1}{8\pi}\ \delta''(\alpha-\beta) \equiv \rho(\alpha,\beta)_{classical} + \rho(\alpha,\beta)_{QM} \qquad (20)$$

one finds readily

$$p_1\ (a) = p_2\ (b) = 1/2 \qquad (22)$$

and

$$p_{12}\ (a,b) = \frac{1}{2}\cos^2\ (ab), \qquad (23)$$

which is the correct (quantum mechanical) correlation function in the case of photon cascades [9]. (The half-angle appearing in Eq. (1) is characteristic of spin-1/2 particles). In such a model, one could interpret $\rho(\alpha,\beta)/2\pi$ as the "classical" source correlation, while $\rho''(\alpha-\beta)/8\pi$ appears as a singular, "quantum" correction.

The model described here is not unique. We have built a number of them [21], all doing the job. They are local, but involve negative probabilities. A fascinating point is that when trying to make quantum mechanics look classical, using e.g. Wigner distribution functions, it is usually not locality, but positive probability, which is sacrificed [22]. Could it be that our concept of probabilities is too naive, that they can, indeed, become negative, and that the quantum-mechanical wave-function is a way to handle them, very much like imaginary numbers are the way to handle the square-root of negative numbers? To stop these speculations, what is needed is somebody with a stroke of genius comparable to that of J.S. Bell, who comes up with a way to test separately the various "self-evident truths" used in local-realistic theories.

ACKNOWLEDGEMENTS

I have benefited from numerous discussions with A. Barut and M.O.Scully. I am greatful to A. Aspect for discussing his experiments in detail with me, for sharing his views of their implications, and for a careful reading of this manuscript. The fact that nonlocality can be avoided when introducting negative probabilities has been recognized by a number of people, including F. Laloe and Ph. Grangier[23]. That they did not publish their findings may be an indication of the general uneasiness about this concept. J.S. Bell has written [5], "... many physicists came to hold not only that it is difficult to find a coherent picture, but that it is wrong to look for one - if not actually immoral then certainly unprofessional." I am greatful to the Max-Planck Institute for Quantum Optics, and in particular to Prof. H. Walther, for allowing me to spend some of my time on such unprofessional ventures.

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