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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

(21 January - 22 March 1985)

THEORY OF OPTICAL BISTABILITY AND LASER INSTABILITIES (TRANSPARENCIES)

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THEORY of OPTICAL BISTABILITY

1) STEADY - STATE BEHAVIOUR
   (2 lectures)

2) SELF - PULSING and CHAOS

3) FLUCTUATIONS in OB

REVIEW ARTICLES ON OPTICAL BISTABILITY

INTRODUCTORY

GIBBS McCALL VENKATESAN, OPTICS NEWS 5, 6 (1979)

ABRAHAM SMITH, J. PHYS. E 15, 33 (1982)

MILLER, LASER FOCUS APRIL 82

ARECCHI SALIERI, PHYS. BULL. 33, 20 (1982)

LUGIATO, CONTEMP. PHYS. 24, 333 (1983)

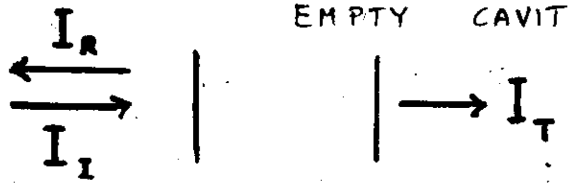
EXTENDED

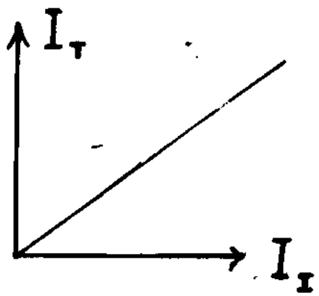
ABRAHAM SMITH, REP. PROGR. PHYS. 45, 815 (1982)

LUGIATO, PROGRESS IN OPTICS, VOL. XXI (1984)

WHAT IS OPTICAL BISTABILITY

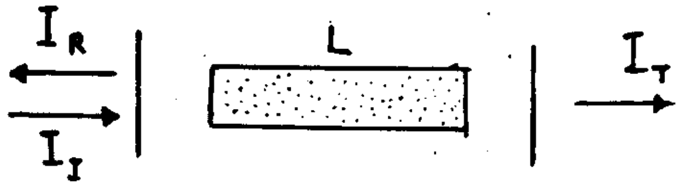
EMPTY CAVITY





$$I_T = \tau I_I \,, \quad \tau \leq 1$$

FILLED CAVITY

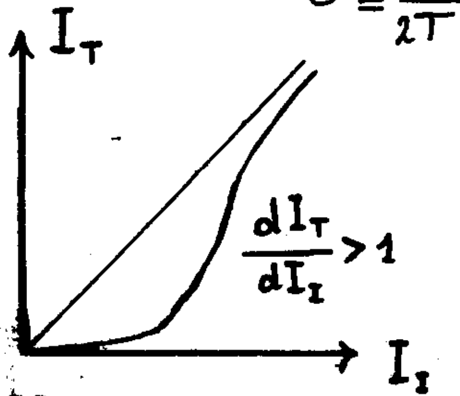


$$I_T = f(I_I) \,, \quad f \text{ NONLINEAR}$$

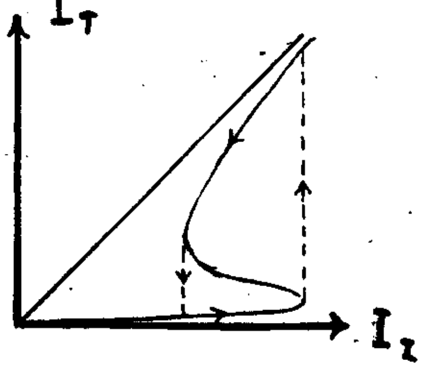
α = ABSORPTION COEFFICIENT PER UNIT LENGTH

T = TRANSMISSIVITY " OF THE MIRRORS

$$C \equiv \frac{\alpha L}{2T}$$

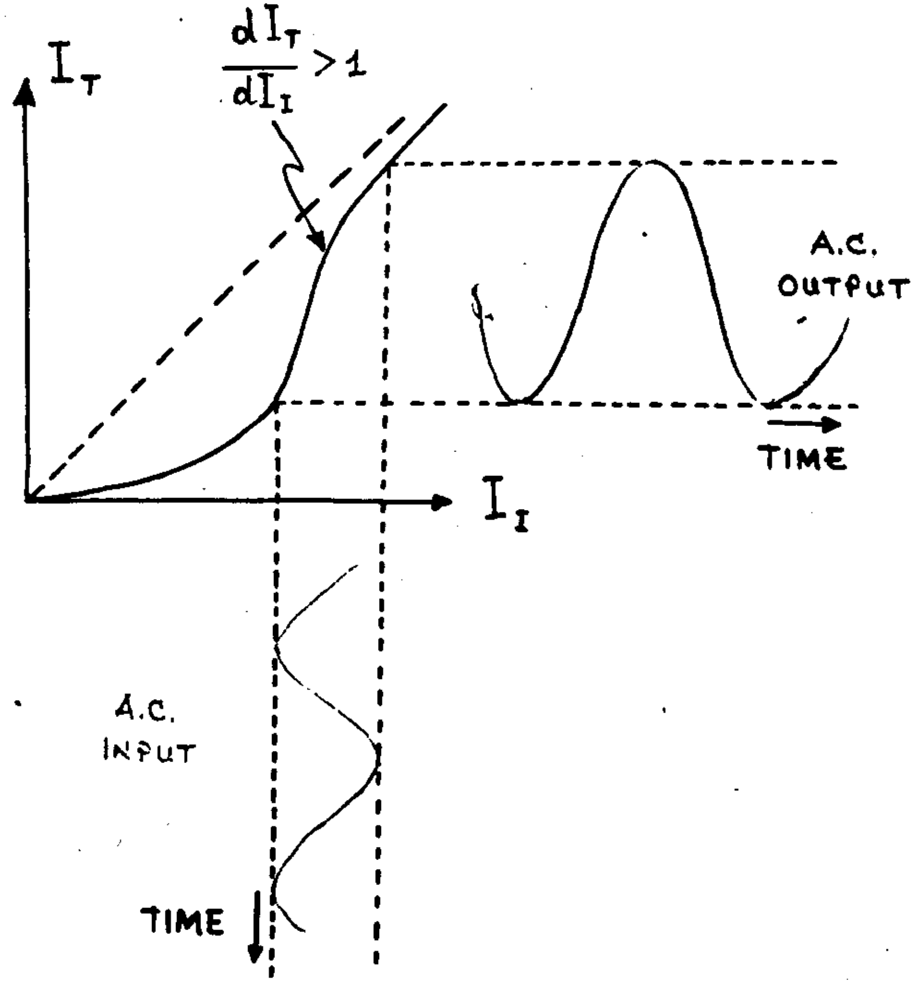


OPTICAL TRANSISTOR

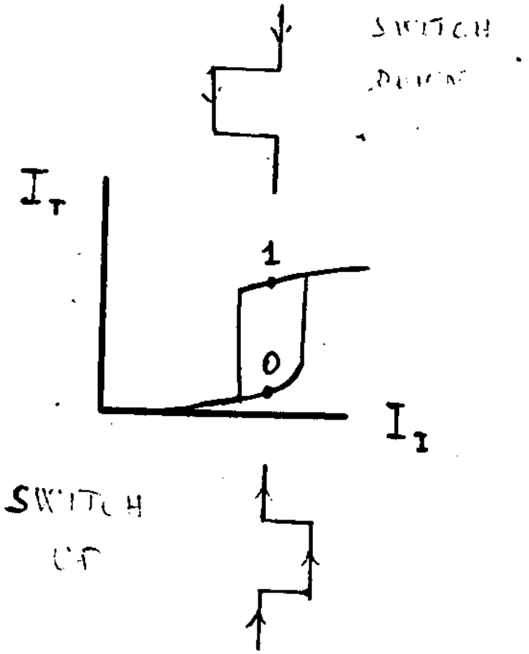


OPTICAL BISTABILITY

OPTICAL TRANSISTOR

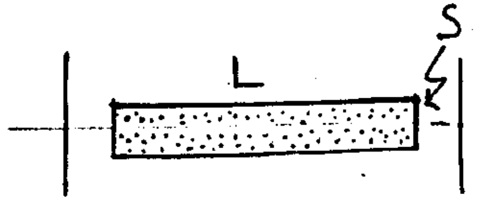


OPTICAL MEMORY



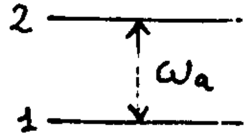
LUGIATO, PROGRESS IN OPTICS, VOL XXI

LASER CAVITY



$S \ll L^2$

$N \gg 1$ TWO-LEVEL ATOMS



ALL THE ATOMS ARE ASSUMED TO HAVE THE SAME TRANSITION FREQUENCY ω_a (HOMOGENEOUSLY BROADENED SYSTEM, i.e. NO DOPPLER BROADENING ETC)

$\mathcal{E}(z,t)$ ELECTRIC FIELD

$\mathcal{P}(z,t)$ MACROSCOPIC POLARIZATION

$\Delta(z,t)$ POPULATION DIFFERENCE

$$\Delta = \frac{1}{2}(N_1 - N_2)$$

THE POLARIZATION OF THE ELECTRIC FIELD IS ASSUMED RANDOM

- SLOWLY VARYING ENVELOPE APPROXIMATION

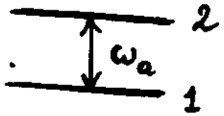
ω_i = CENTRAL FREQUENCY OF ELECTRIC FIELD

$k_i = \omega_i / c$

$$\mathcal{E}(z,t) = E(z,t)\, e^{i(k_i z - \omega_i t)} + E^*(z,t)\, e^{-i(k_i z - \omega_i t)}$$

$$\mathcal{P}(z,t) = -\mu \left\{ P(z,t)\, e^{i(k_i z - \omega_i t)} + P^*(z,t)\, e^{-i(k_i z - \omega_i t)} \right\}$$

$$\Delta(z,t) = (N_1 - N_2)\,\frac{1}{2}$$



μ = MODULUS OF THE ATOMIC DIPOLE MOMENT

THE ELECTRIC FIELD HAS RANDOM POLARIZATION

- DIPOLE APPROXIMATION
- ROTATING WAVE APPROXIMATION (no $e^{\pm 2i(k_i z - \omega_i t)}$)

$$H_{int} = -\mathcal{P} \cdot \mathcal{E}$$

$$H_{int} = \mu\left(E P^* + E^* P\right)$$

- SEMICLASSICAL APPROXIMATION

NEGLECT ALL CORRELATIONS BETWEEN ATOMIC SYSTEM AND ELECTRIC FIELD

MAXWELL - BLOCH EQUATIONS

HOMOGENEOUSLY BROADENED ATOMIC SYSTEM

EXACT RESONANCE $\omega_a = \omega_i \Rightarrow E = E^*,\ P = P^*$

$$\frac{\partial E}{\partial t} + c\,\frac{\partial E}{\partial z} = -g P$$

$\Omega = \frac{\mu E}{\hbar}$ REAL F.

$$R_1 \quad \frac{\partial P}{\partial t} = \frac{\mu}{\hbar} E \Delta - \gamma_\perp P$$

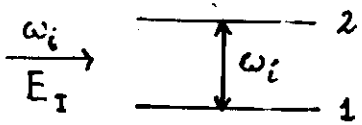
$\Delta = \frac{1}{2}(N_1 - N_2)$

$$-R_3 \quad \frac{\partial \Delta}{\partial t} = -\frac{\mu}{\hbar} E P - \gamma_\parallel \left(\Delta - \frac{N}{2}\right)$$

$\Delta \xrightarrow{t \to \infty} \frac{N}{2}$

$$\gamma_\parallel = T_1^{-1}, \quad \gamma_\perp = T_2^{-1}, \quad g = \frac{2\pi\omega_i}{V}\mu$$

ATOMIC LINEWIDTH



$$\Delta = \frac{N_1 - N_2}{2}$$

V = VOLUME OF ATOMIC SAMPLE

N = TOTAL NUMBER OF ATOMS

- RING CAVITY

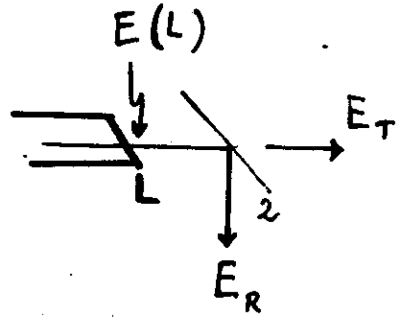
PLANE WAVE

HOMOGENEOUS BROADENING

ZERO ATOMIC AND CAVITY DETUNINGS

$\Longrightarrow$ PURELY ABSORPTIVE BISTABILITY

MIRRORS



$$I_T = E_T^2 \qquad I(L) = E^2(L)$$

$$I_T = T\, I(L)$$

$$E_T = \sqrt{T}\, E(L)$$

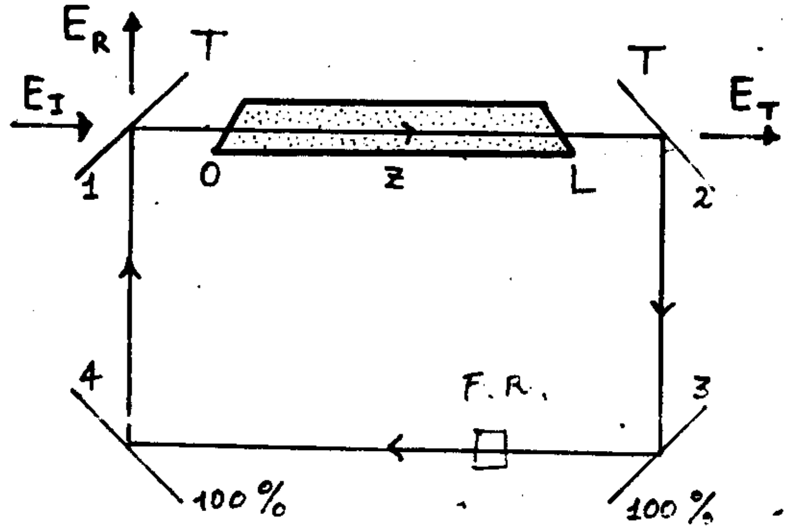
SIMILARLY

$$I_R = R\, I(L)$$

$$E_R = \sqrt{R}\, E(L)$$

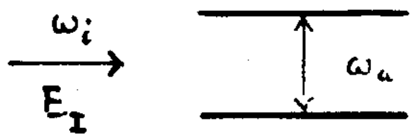
$$T + R = 1 \qquad 0 \leq T \leq 1$$

OB IN A RING CAVITY



BOUNDARY CONDITION

$$E(0) = \sqrt{T}\, E_I + \underbrace{\overset{\sqrt{R}\sqrt{R}}{R}\, E(L)}_{\text{FEEDBACK}}$$



PERFECT RESONANCE BETWEEN INCIDENT FIELD, ATOMS and CAVITY $\Longrightarrow$ PURELY ABSORPTIVE OPTICAL BISTABILITY

STATIONARY SOLUTION $\left(\frac{\partial P}{\partial t} = \frac{\partial \Delta}{\partial t} = \frac{\partial E}{\partial t} = 0\right)$

$$\Delta = \frac{N}{2}\frac{1}{1+\frac{\mu^2 E^2}{\hbar^2 \gamma_\perp \gamma_\parallel}} \quad , \quad P = \frac{N}{2}\sqrt{\frac{\gamma_\parallel}{\gamma_\perp}}\,\frac{\frac{\mu E}{\hbar\sqrt{\gamma_\perp\gamma_\parallel}}}{1+\frac{\mu^2 E^2}{\hbar^2\gamma_\perp\gamma_\parallel}}$$

$$F(z) = \frac{\mu E(z)}{\hbar\sqrt{\gamma_\perp \gamma_\parallel}}$$

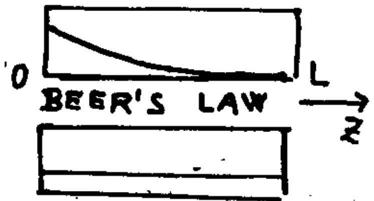
$$\Delta = \frac{N}{2}\frac{1}{1+F^2} \quad , \quad P = \frac{N}{2}\sqrt{\frac{\gamma_\parallel}{\gamma_\perp}}\frac{F}{1+F^2} = \chi F$$

χ = NONLINEAR DIELECTRIC SUSCEPTIBILITY

$$\longrightarrow \frac{dF}{dz} = -\alpha \frac{F}{1+F^2}$$

$\alpha = \frac{\mu g N}{\hbar c 2\gamma_\perp}$ LINEAR ABSORPTION COEFFICIENT

$F \ll 1 \qquad \frac{dF}{dz} = -\alpha F$



$F \gg 1 \qquad \frac{dF}{dz} = 0$ SATURATION

$$\ln F(z) + \frac{1}{2}F^2(z) = -\alpha z + C \Longrightarrow C = \ln F(0) + \frac{1}{2}F^2(0)$$

$$\ln\frac{F(0)}{F(z)} + \frac{1}{2}\left[F^2(0) - F^2(z)\right] = \alpha z$$

$z = L \qquad \ln\frac{F(0)}{F(L)} + \frac{1}{2}\left[F^2(0) - F^2(L)\right] = \alpha L$

$$y = \frac{\mu E_I}{\hbar\sqrt{\gamma_\perp\gamma_\parallel T}} \qquad x = \frac{\mu E_T}{\hbar\sqrt{\gamma_\perp\gamma_\parallel T}}.$$

SINCE $E_T = \sqrt{T}\, E(L)$ ONE HAS $x = F(L)$

$$\ln\frac{F(0)}{x} + \frac{x^2}{2}\left\{\left(\frac{F(0)}{x}\right)^2 - 1\right\} = \alpha L$$

$$F(0) = Ty + Rx \Longrightarrow \frac{F(0)}{x} = T\frac{y}{x} + R = 1 + T\left(\frac{y}{x} - 1\right)$$

$$\Rightarrow \ln\left[1 + T\left(\frac{y}{x} - 1\right)\right] + \frac{x^2}{2}\left\{\left[1 + T\left(\frac{y}{x} - 1\right)\right]^2 - 1\right\} = \alpha L$$

MEAN FIELD LIMIT

$\alpha L \longrightarrow 0 \;,\; T \longrightarrow 0 \;,\; C = \frac{\alpha L}{2T}$ CONSTANT

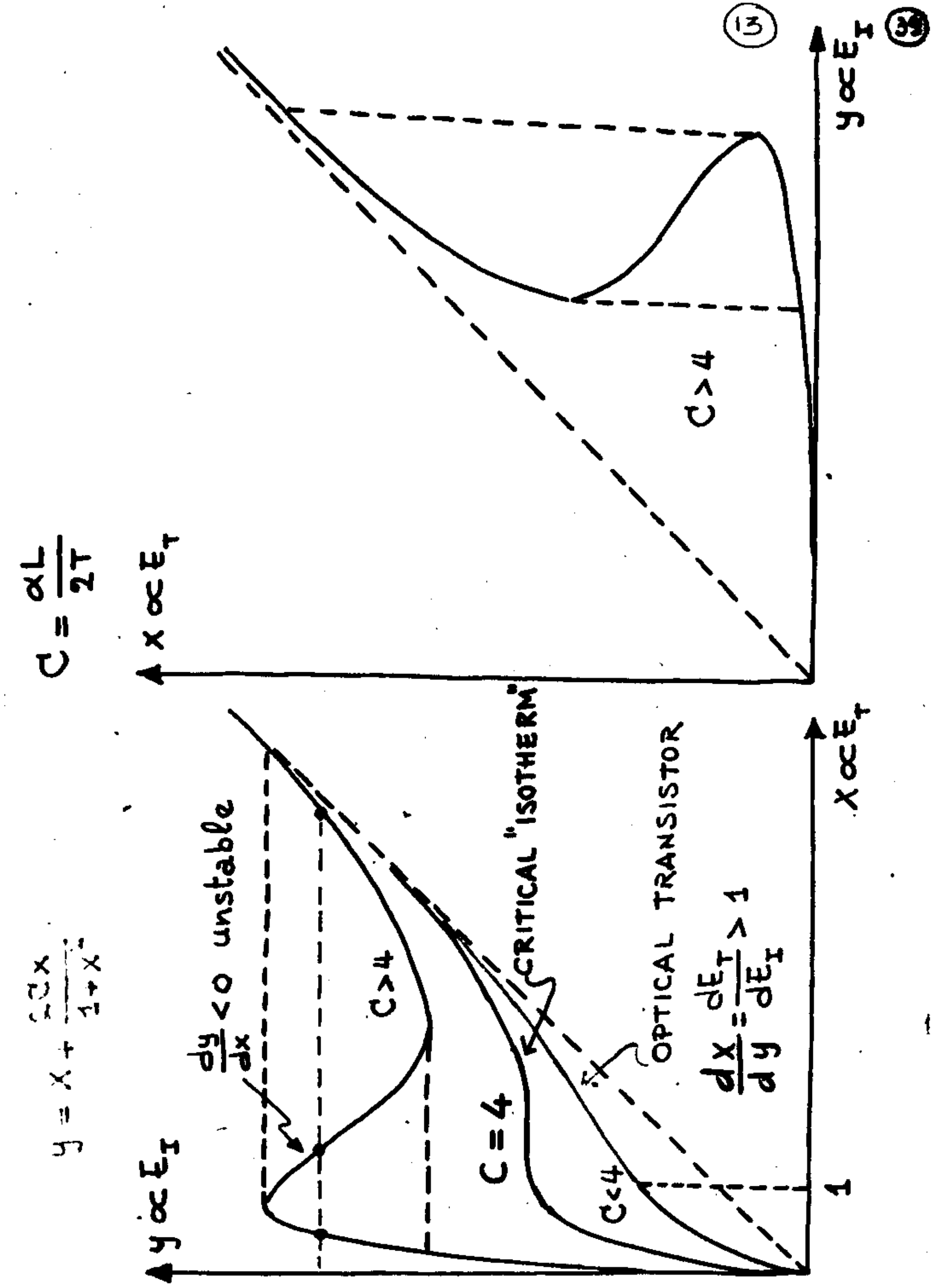
$$\boxed{y = x + \frac{2Cx}{1+x^2}}$$ MEAN FIELD STATE EQUATION

$\frac{dF}{dz} = -\alpha\frac{F}{1+F^2}$ FOR $\alpha L \to 0$ THE FIELD BECOMES UNIFORM

$$F(L) - F(0) = -\alpha\int_0^L dz\,\frac{F(z)}{1+F^2(z)} \simeq -\alpha L\,\frac{F(L)}{1+F^2(L)}$$

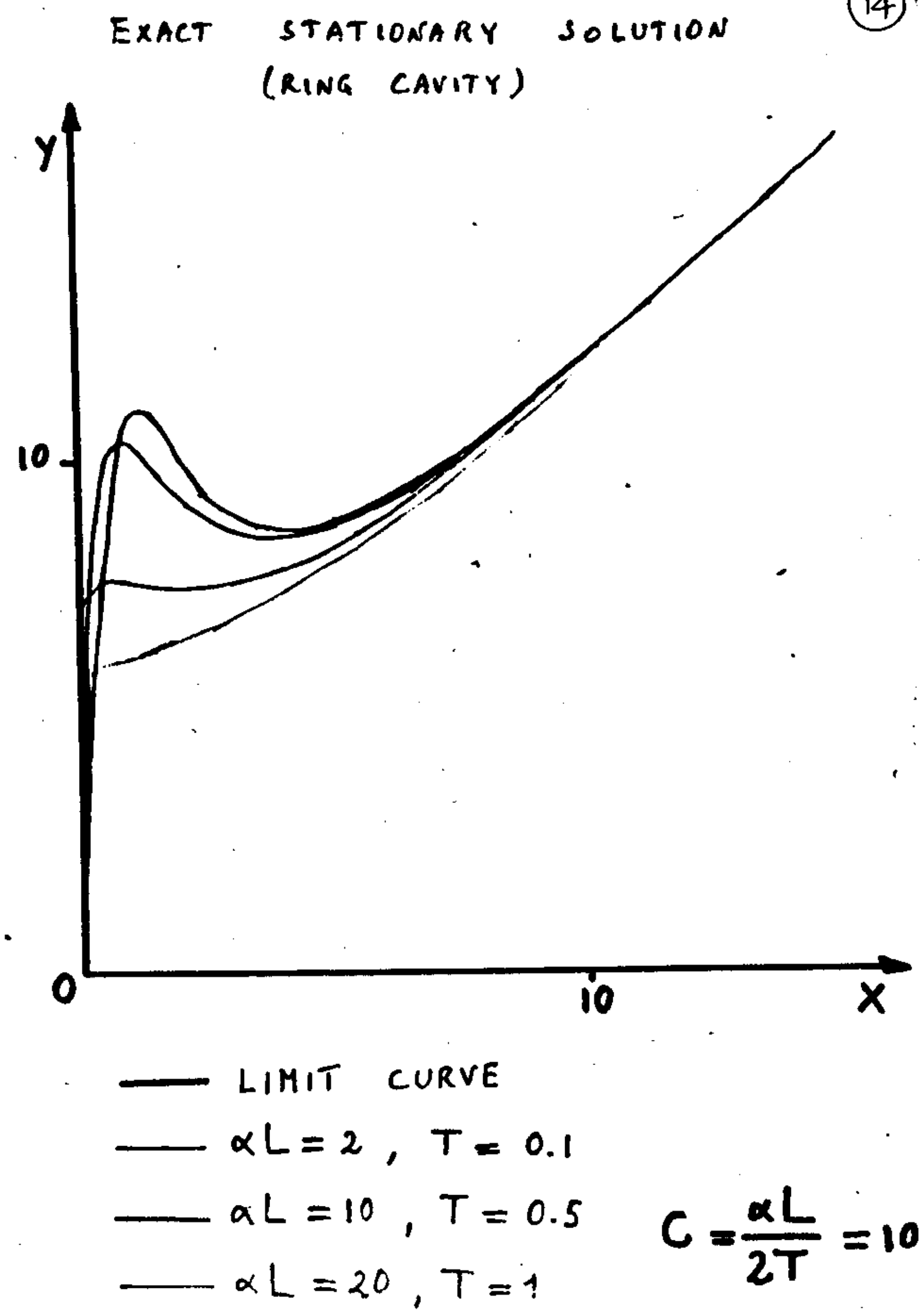
$$\left.\begin{aligned} x - F(0) &= -\alpha L\,\frac{x}{1+x^2} \\ F(0) &= Ty + Rx \end{aligned}\right\} \Longrightarrow y = x + \frac{2Cx}{1+x^2}$$

(13)



$y \propto E_I$
$x \propto E_T$
$C = \frac{\alpha L}{2T}$
C > 4
CRITICAL "ISOTHERM"
OPTICAL TRANSISTOR
$\frac{dx}{dy} = \frac{dE_T}{dE_I} > 1$
C > 4
C = 4
C < 4
1
$\frac{dy}{dx} < 0$ unstable
$y = x + \frac{2Cx}{1+x^2}$

(14)



EXACT STATIONARY SOLUTION
(RING CAVITY)
y
10
0
10
x
LIMIT CURVE
$\alpha L = 2$, $T = 0.1$
$\alpha L = 10$, $T = 0.5$
$\alpha L = 20$, $T = 1$
$C = \frac{\alpha L}{2T} = 10$

IN CONCLUSION MEAN FIELD MODEL

$$\dot{\bar{E}} = -k\left[\bar{E} - \frac{E_I}{\sqrt{T}}\right] - g\bar{P}$$

$$\dot{\bar{P}} = \frac{\mu}{\hbar}\bar{E}\bar{\Delta} - \gamma_\perp \bar{P}$$

$$\dot{\bar{\Delta}} = -\frac{\mu}{\hbar}\bar{E}\bar{P} - \gamma_\parallel\left(\bar{\Delta} - \frac{N}{2}\right)$$

$k = \dfrac{cT}{\mathcal{L}}$ CAVITY DAMPING CONSTANT

$\mathcal{L}$ = LENGTH OF THE CAVITY

BY SETTING

$$y = \frac{\mu E_I}{\hbar\sqrt{\gamma_\perp\gamma_\parallel T}} \quad , \quad x = \frac{\mu\bar{E}}{\hbar\sqrt{\gamma_\perp\gamma_\parallel}} = \frac{\mu E_T}{\hbar\sqrt{\gamma_\perp\gamma_\parallel T}}$$

ONE OBTAINS AT STEADY STATE

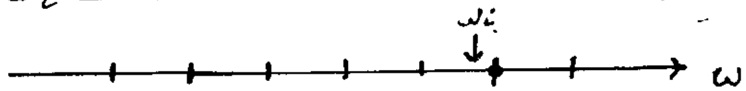
$$y = x + \frac{2Cx}{1+x^2}$$

$$C = \frac{\mu g N}{4\hbar\gamma_\perp k} = \frac{\alpha L}{2T}$$

ω_i = INCIDENT FIELD FREQUENCY

ω_a = ATOMIC TRANSITION FREQUENCY

ω_c = RESONANT CAVITY FREQUENCY



CAVITY FREQUENCIES $\dfrac{2\pi c}{\mathcal{L}} m$ $\quad m = 0, 1,$

UNTIL NOW, WE ASSUMED $\omega_i = \omega_a = \omega_c$

$\delta = \dfrac{\omega_a - \omega_i}{\gamma_\perp}$ ATOMIC DETUNING PARAMETER

$\theta = \dfrac{\omega_c - \omega_i}{k}$ CAVITY DETUNING PARAMETER

WHEN $\delta \neq 0$, NOT ONLY ABSORPTION BUT ALSO DISPERSION PLAYS A ROLE

MEAN FIELD MODEL FOR MIXED ABSORPTIVE + DISPERSIVE OB $(\alpha L \ll 1,\ T \ll 1)$

$$\dot{\bar{E}} = -k\left[\bar{E}(1 + i\theta) - \frac{E_I}{\sqrt{T}}\right] - g\bar{P}$$

$$\dot{\bar{P}} = \frac{\mu}{\hbar}\bar{E}\bar{\Delta} - \gamma_\perp(1 + i\delta)\bar{P}$$

$$\dot{\bar{\Delta}} = -\frac{\mu}{2\hbar}\left(\bar{E}^*\bar{P} + \bar{E}\bar{P}^*\right) - \gamma_\parallel\left(\bar{\Delta} - \frac{N}{2}\right)$$

5 DYNAMICAL EQUATIONS

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$$x = \frac{\mu \bar{E}}{\hbar\sqrt{\gamma_\perp \gamma_\parallel}} \qquad y = \frac{\mu E_I}{\hbar\sqrt{\gamma_\perp \gamma_\parallel T}}.$$

AT STEADY STATE

$$\bar{\Delta} = \frac{1+\delta^2}{1+\delta^2+|x|^2}\,\frac{N}{2}$$

$$\bar{P} = \frac{N}{2}\sqrt{\frac{\gamma_\parallel}{\gamma_\perp}}\,\frac{(1-i\delta)\,x}{1+\delta^2+|x|^2}.$$

$$y^2 = |x|^2\left\{\left[1+\frac{2C}{1+\delta^2+|x|^2}\right]^2+\left[\theta-\frac{2C\delta}{1+\delta^2+|x|^2}\right]^2\right\}$$

GENERAL MEAN FIELD STATE EQUATION

$$y^2 = |x|^2\left\{\left[1+\chi_1(|x|^2)\right]^2+\left[\theta-\chi_2(|x|^2)\right]^2\right\}$$

χ_1 = ABSORBTIVE PART OF DIELECTRIC SUSCEPTIBILITY

χ_2 = DISPERSIVE PART ...

$\propto \delta$

18

NORMALIZED VARIABLES

$$x = \frac{\mu \bar{E}}{\hbar\sqrt{\gamma_\perp \gamma_\parallel}}, \qquad y = \frac{\mu E_I}{\hbar\sqrt{\gamma_\perp \gamma_\parallel T}}$$

$$P = \left(\frac{N}{2}\sqrt{\frac{\gamma_\parallel}{\gamma_\perp}}\right)^{-1}\bar{P}, \qquad D = \left(\frac{N}{2}\right)^{-1}\bar{\Delta}$$

$$\frac{dx}{dt} = \kappa\left[\cancel{y} - x(1+i\cancel{\theta}) + 2CP\right]$$

$$\frac{dP}{dt} = \gamma_\perp\left[xD - (1+i\cancel{\delta})P\right]$$

$$\frac{dD}{dt} = -\gamma_\parallel\left\{\frac{1}{2}\left(xP^* + x^*P\right) + D - 1\right\}$$

SPECIAL CASE : (19)

ASSUME EXACT RESONANCE $\Longrightarrow \theta = 0, \ \delta = 0$

NO INCIDENT FIELD $\Longrightarrow y = 0$.

CONVERT ABSORBER INTO AMPLIFIER $\Rightarrow C \to -C$

$$\frac{dx}{dt} = \kappa\left(-x + 2CP\right)$$

$$\frac{dP}{dt} = \gamma_{\perp}\left(xD - P\right)$$

$$\frac{dD}{dt} = -\gamma_{\parallel}\left\{\frac{1}{2}\left(xP^{*} + x^{*}P\right) + D - 1\right\}$$

LASER MODEL !

NOW x and P CAN BE TAKEN REAL

$$\frac{dx}{dt} = -\kappa\left(x - 2CP\right)$$

$$\frac{dP}{dt} = \gamma_{\perp}\left(xD - P\right)$$

$$\frac{dD}{dt} = -\gamma_{\parallel}\left(xP + D - 1\right)$$

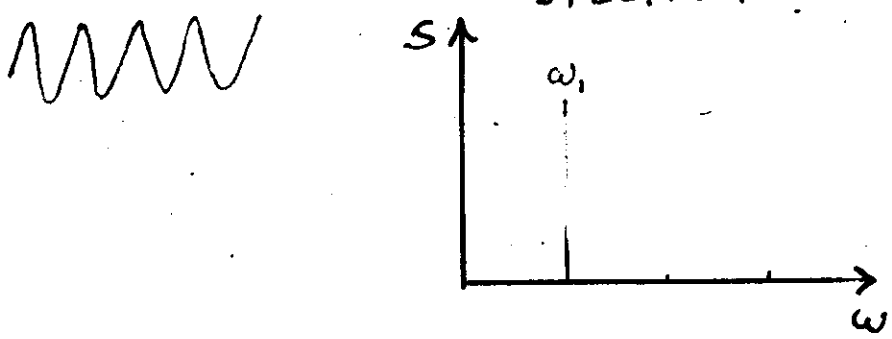
EQUIVALENT TO LORENZ MODEL !

$$\tilde{D} = D - 1$$

(20)

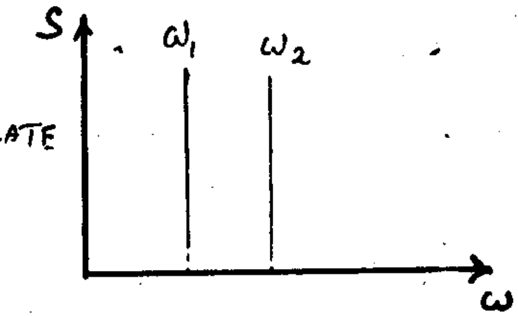
PERIODIC SOLUTION

SPECTRUM

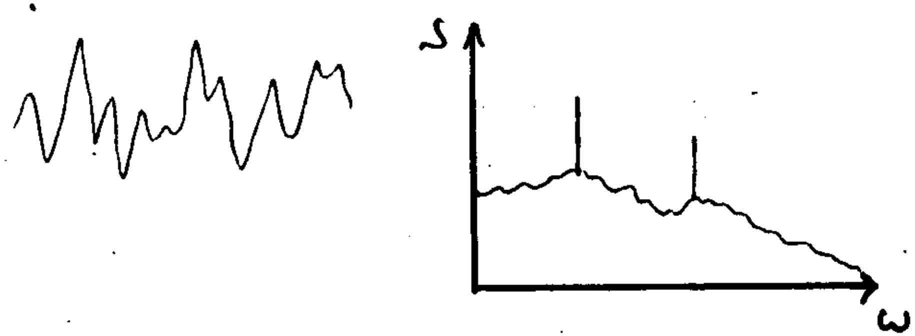


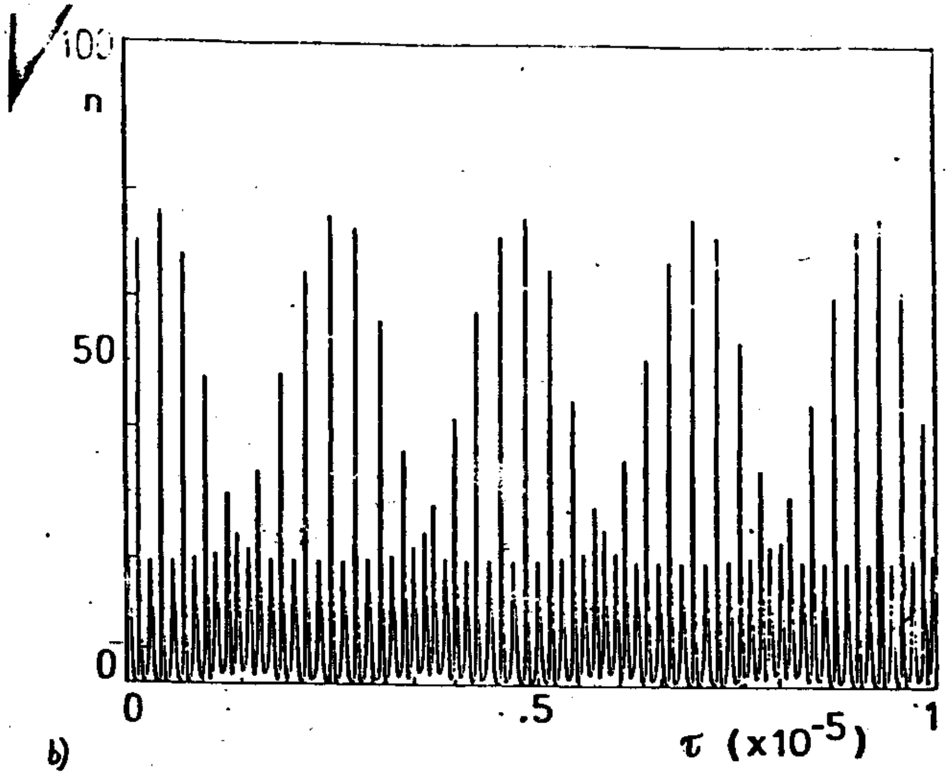
QUASIPERIODIC SOLUTION.

ω_1, ω_2 INCOMMENSURATE

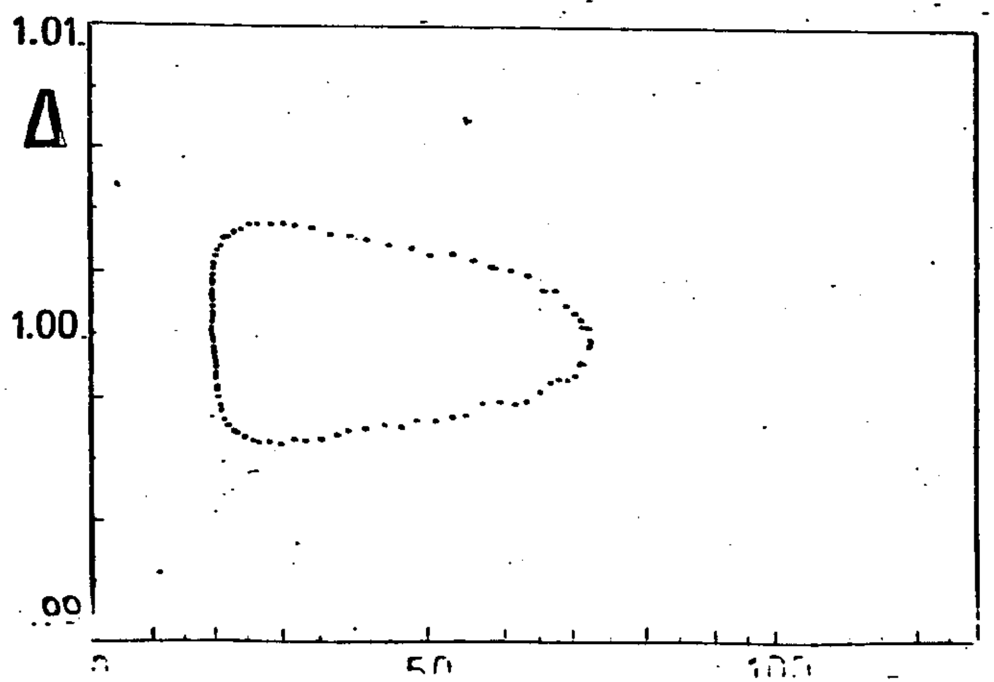


APERIODIC (CHAOTIC, TURBULENT) SOLUTION





b)



OPTICAL CLOCK

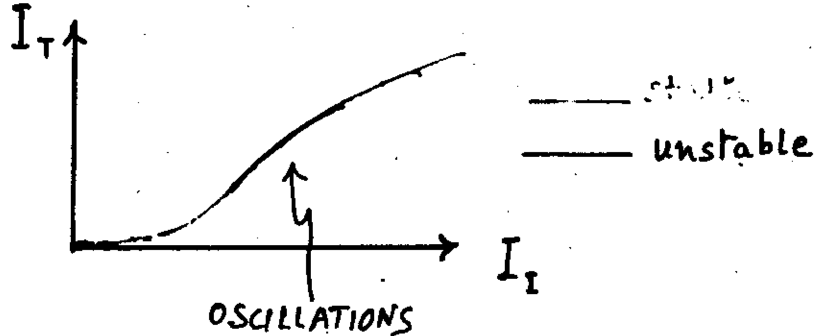
Fig. 20 Lugiato
Optical Bistability

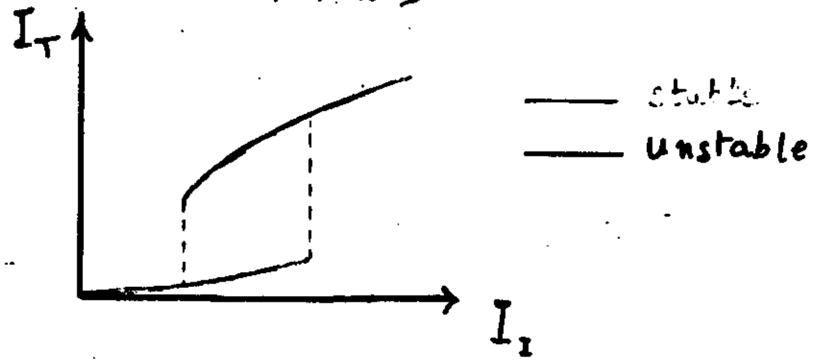
RESONANT MODE INSTABILITY IN OB

$\delta = \theta = 0 \Longrightarrow$ NO INSTABILITY

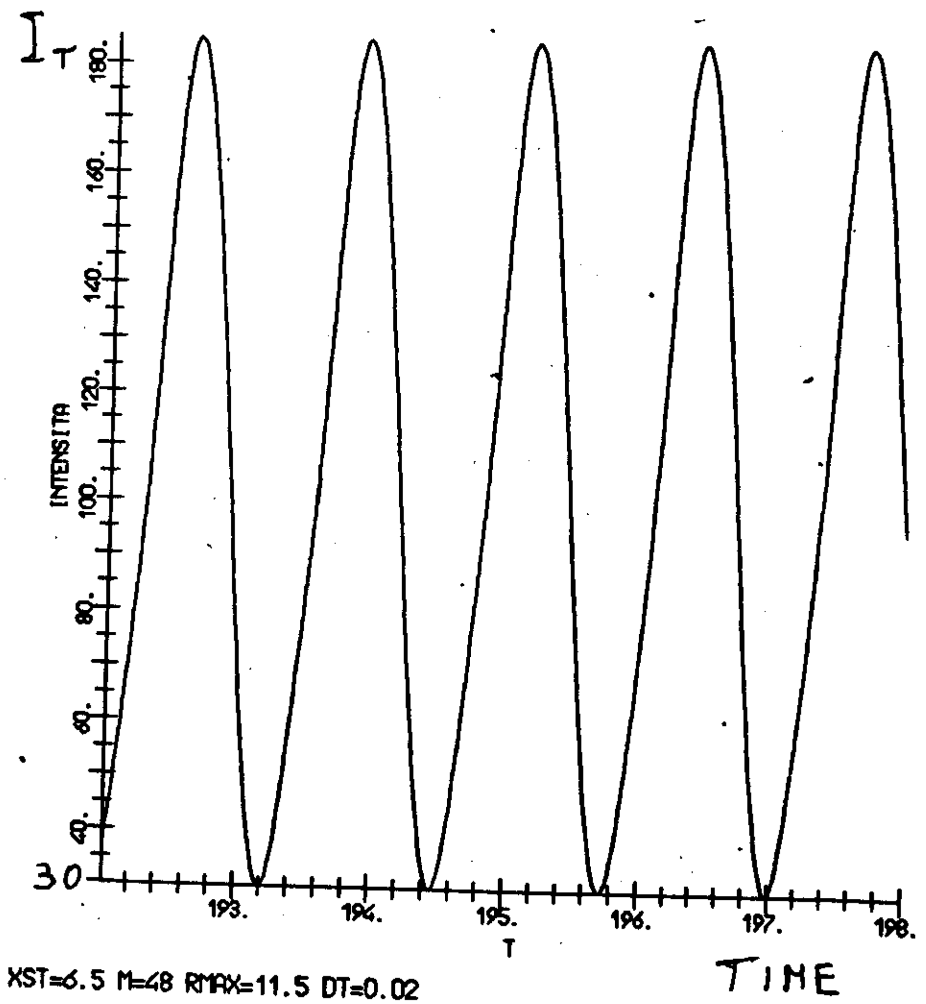
INCREASING THE DETUNING, ONE OBTAINS A HARD MODE INSTABILITY

$\Longrightarrow$ SPONTANEOUS OSCILLATIONS WITH FREQUENCY $\simeq$ K

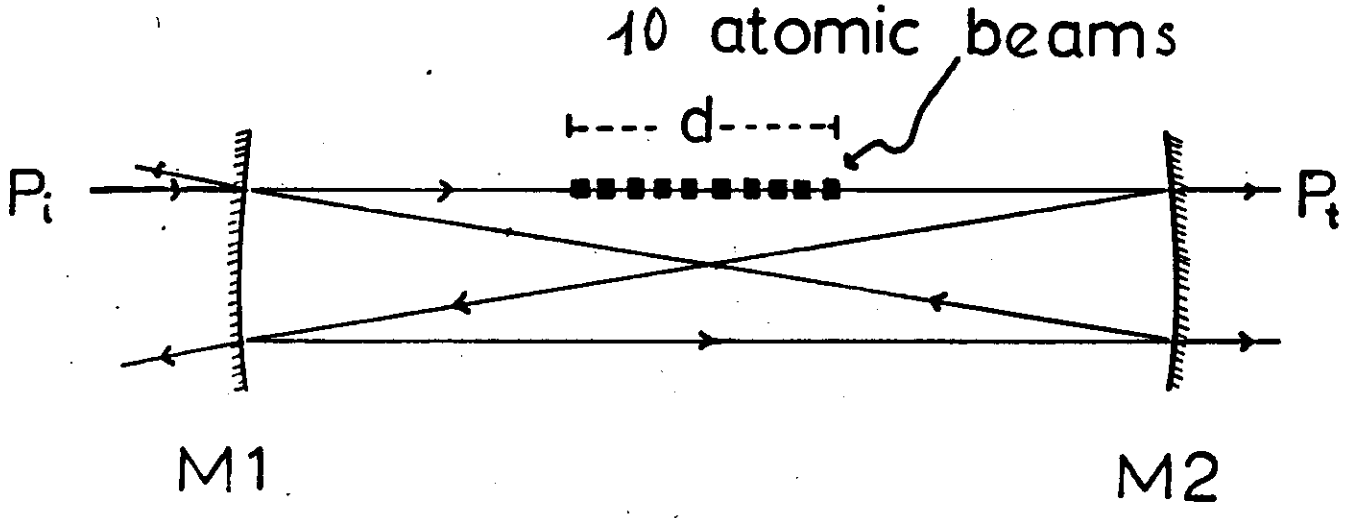




$C = 75 \,, \quad \theta = -3 \,, \quad \frac{\omega_a - \omega_c}{\gamma_\perp} = 1$

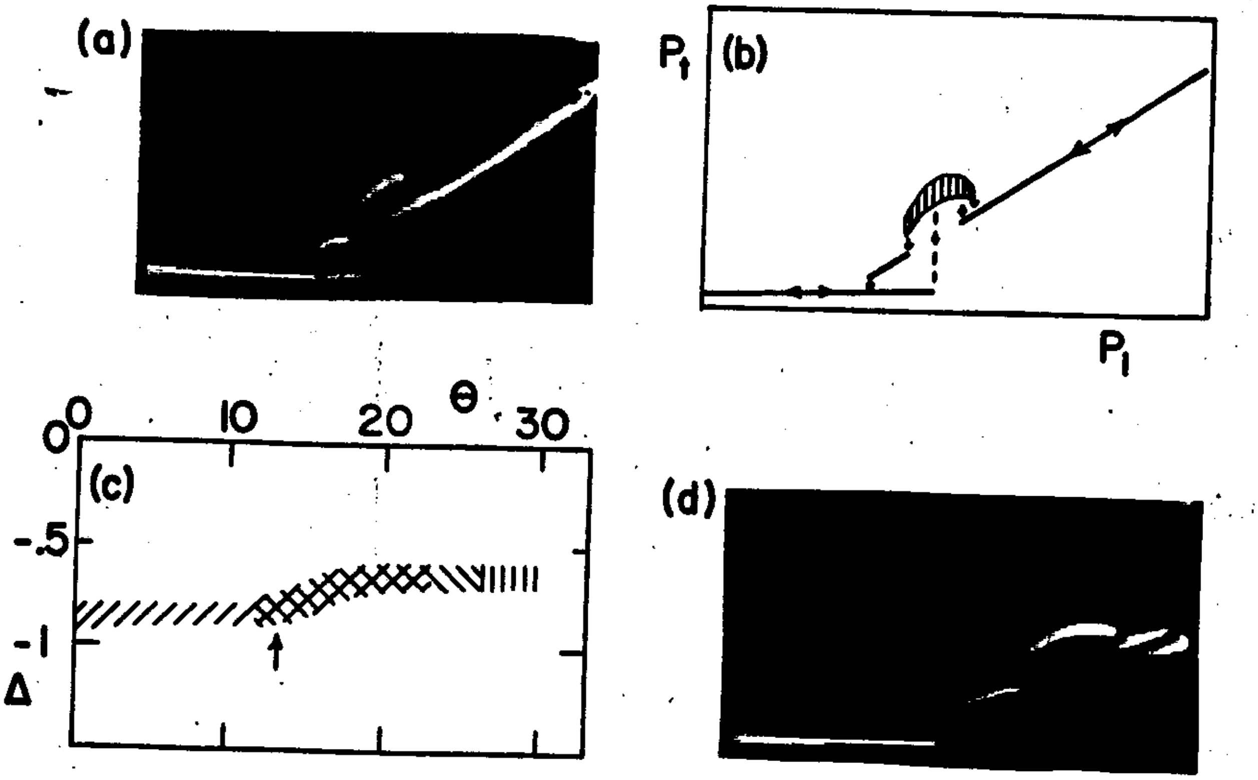


KIMBLE et al



(25)

KIMBLE OROZCO ROSEMBERGER



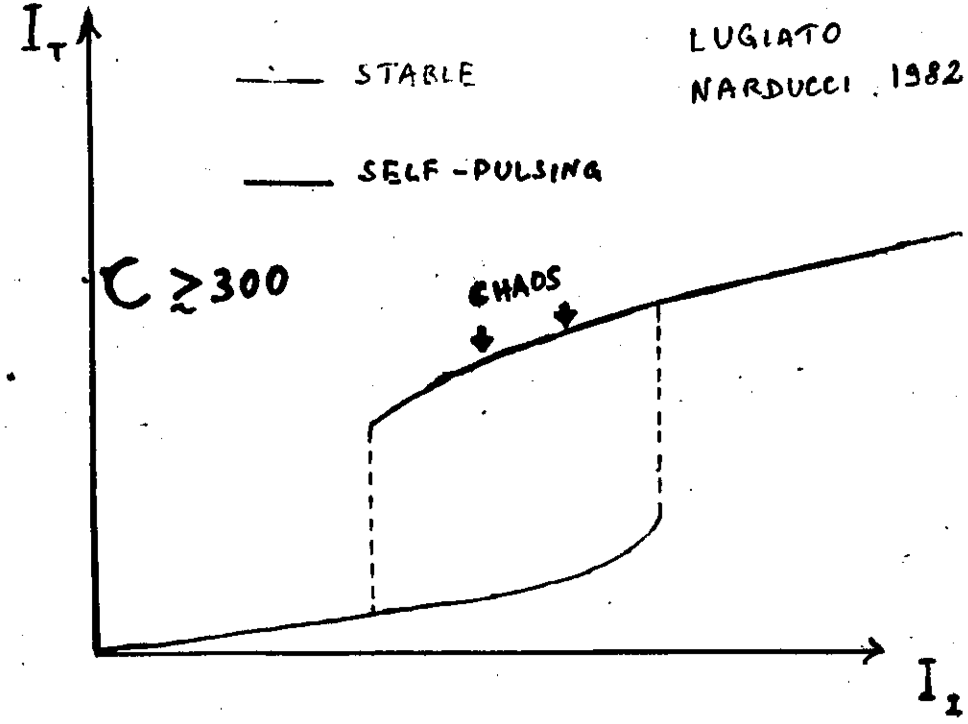
(26)

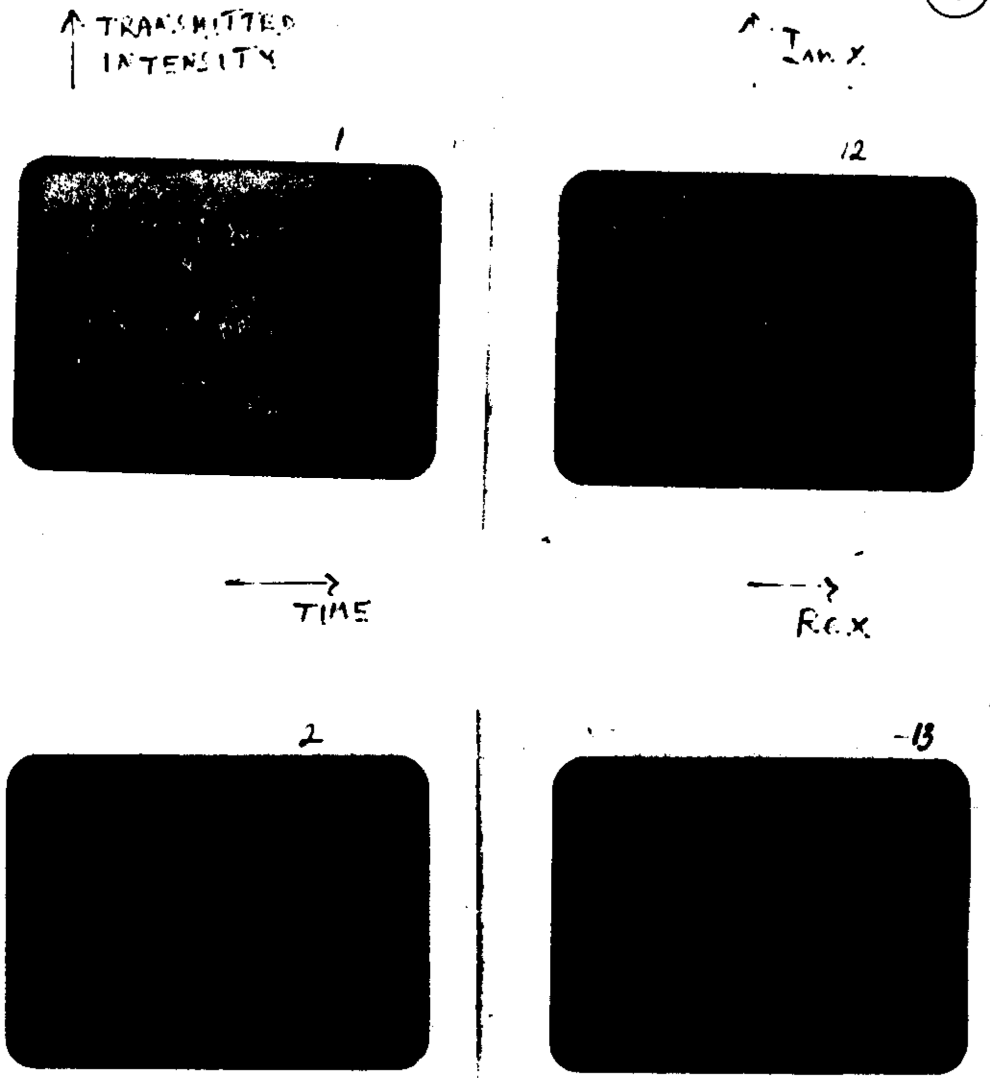
MEAN FIELD MODEL FOR ABSORPTIVE + DISPERSIVE OPTICAL BISTABILITY

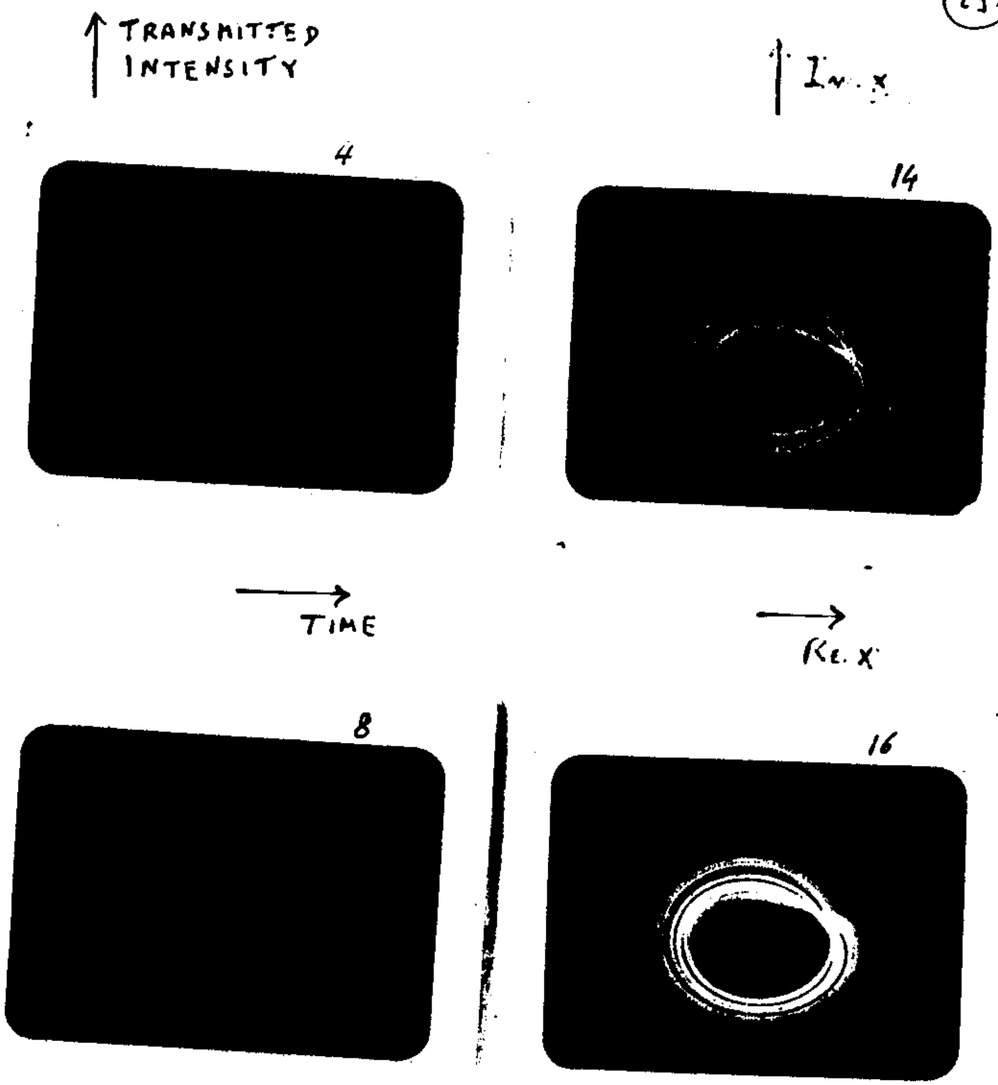
$$\kappa^{-1}\,\dot{x} = -i\theta x - (x-y) - 2Cp$$

$$\gamma_{\perp}^{-1}\,\dot{p} = xd - (1+i\Delta)\,p$$

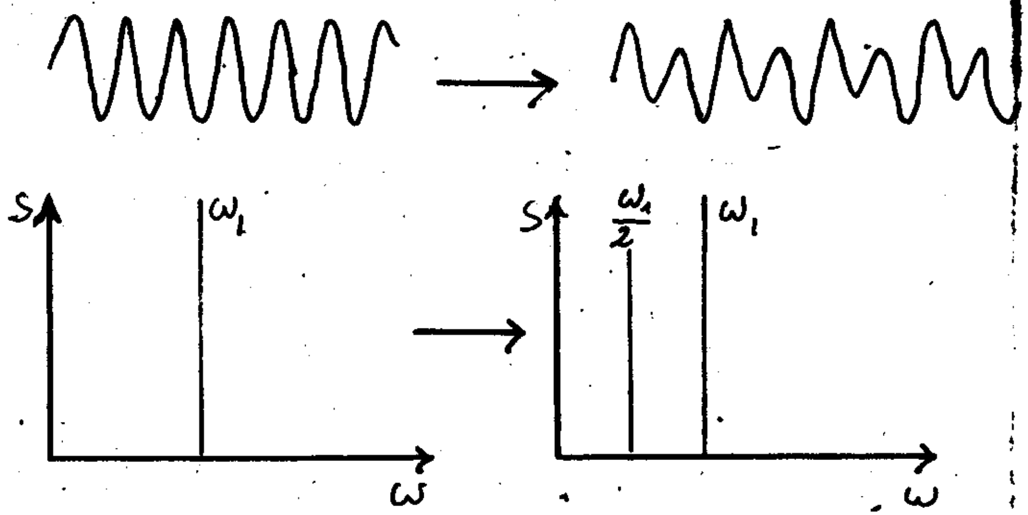
$$\gamma_{\parallel}^{-1}\,\dot{d} = -\frac{1}{2}\left(xp^{*} + x^{*}p\right) - d + 1$$







PERIOD DOUBLING



ΔY_m = RANGE OF Y IN CORRESPONDENCE OF WHICH THE SYSTEM EXHIBITS PERIOD-m BEHAVIOUR

$$\frac{\Delta y_m}{\Delta y_{m+1}} \xrightarrow{m \to \infty} 4.6692\ldots$$

MAGIC UNIVERSAL NUMBER

GROSSMANN and THOMAE 1977
FEIGENBAUM 1978
COULLET and TRESSER 1978

MIXED ABSORPTIVE + DISPERSIVE OB (31)

STABILITY ANALYSIS

$$\bar{E} = \bar{E}_{st} + \delta E$$
$$\bar{P} = \bar{P}_{st} + \delta P$$
$$\bar{\Delta} = \bar{\Delta}_{st} + \delta\Delta$$

$$\begin{cases} \delta\dot{E} = -\kappa\,\delta E(1+i\theta) - \bar{g}\,\delta P \\ \delta\dot{E}^* = -\kappa\,\delta E^*(1-i\theta) - \bar{g}\,\delta P^* \\ \delta\dot{P} = \frac{\mu}{\hbar}(\bar{E}_{st}\,\delta\Delta + \bar{\Delta}_{st}\,\delta E) - \gamma_\perp(1+i\delta)\,\delta P \\ \delta\dot{P}^* = \frac{\mu}{\hbar}(\bar{E}^*_{st}\,\delta\Delta + \bar{\Delta}_{st}\,\delta E^*) - \gamma_\perp(1-i\delta)\,\delta P^* \\ \delta\dot{\Delta} = -\frac{\mu}{2\hbar}(\bar{E}^*_{st}\,\delta P + \bar{P}_{st}\,\delta E^* + \bar{E}_{st}\,\delta P^* + \bar{P}^*_{st}\,\delta E) - \gamma_\parallel\,\delta\Delta \end{cases}$$

5. LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

$$\begin{pmatrix} \delta E \\ \delta E^* \\ \delta P \\ \delta P^* \\ \delta\Delta \end{pmatrix} = e^{\lambda t} \begin{pmatrix} \delta E^{(0)} \\ \delta E^{*(0)} \\ \delta P^{(0)} \\ \delta P^{*(0)} \\ \delta\Delta^{(0)} \end{pmatrix}$$

(32)

$$\dot{\bar{E}} = -\kappa\bar{E}(1+i\theta) - \bar{g}\,\bar{P}$$

$$\dot{\bar{E}}_{st} + \delta\dot{E} = -\kappa(\bar{E}_{st} + \delta E)(1+i\theta) - \bar{g}(\bar{P}_{st} + \delta P)$$

HENCE

$$\delta\dot{E} = -\kappa\,\delta E(1+i\theta) - \bar{g}\,\delta P$$

$$\dot{\bar{P}} = \frac{\mu}{\hbar}\bar{E}\bar{\Delta} - \gamma_\perp(1+i\delta)\bar{P}$$

$$\dot{\bar{P}}_{st} + \delta\dot{P} = \frac{\mu}{\hbar}(\bar{E}_{st}\bar{\Delta}_{st} + \bar{E}_{st}\,\delta\Delta + \bar{\Delta}_{st}\,\delta E + \delta E\cdot\delta\Delta) - \gamma_\perp(1+i\delta)(\bar{P}_{st} + \delta P)$$

HENCE

$$\delta\dot{P} = \frac{\mu}{\hbar}(\bar{E}_{st}\,\delta\Delta + \bar{\Delta}_{st}\,\delta E) - \gamma_\perp(1+i\delta)\,\delta P$$

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$\delta \dot{E} = \lambda \, \delta E$ ETC $\Longrightarrow$ LINEAR HOMOGENEOUS ALGEBRAIC SYSTEM OF EQUATIONS WITH COEFFICIENT MATRIX $M(\lambda)$

$\Longrightarrow$ EIGENVALUE EQUATION (5th ORDER)

$$\det M(\lambda) = 0$$

STABILITY CONDITION $Re \lambda_i \leq 0$

HURWITZ CRITERION

$$\lambda^5 + a_1 \lambda^4 + a_2 \lambda^3 + a_3 \lambda^2 + a_4 \lambda + a_5 = 0$$

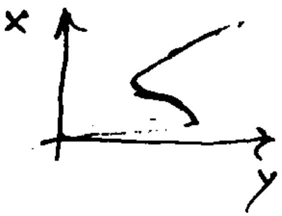
i) $a_1 > 0$

ii) $a_5 > 0$

iii) $a_1 a_2 - a_3 > 0$

.... > 0

$a_5 \propto \frac{dy}{dx}$ ONLY POSITIVE SLOPE BRANCHES ARE STABLE



WHEN a_5 CHANGES SIGN, SOFT MODE INSTABILITY

WHEN a_1 CHANGES SIGN, OR $a_1 a_2 - a_3$ CHANGES SIGN, OR ...... CHANGES SIGN, HARD MODE INSTABILITY

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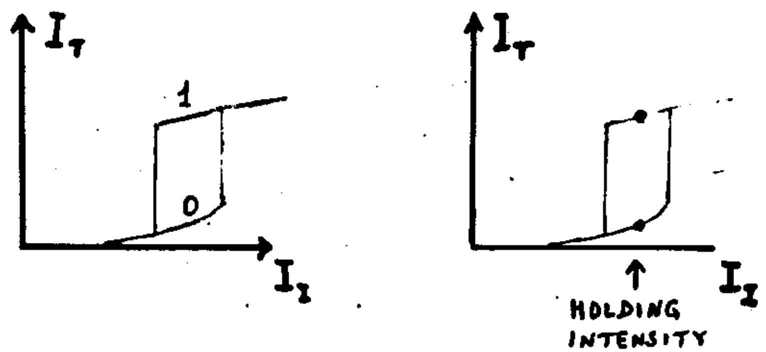
FLUCTUATIONS



EXTERNAL NOISE

THERMAL NOISE

INTRINSIC QUANTUM NOISE (SPONTANEOUS EMISSION)

STATISTICAL THEORY : NOT $I(t)$ BUT $P(I,t)$

FOKKER-PLANCK EQUATION FOR $P(I,t)$

FLUCTUATION PARAMETER $q \propto \Omega^{-1}$

STATIONARY SOLUTION $P_{st}(I) = e^{-\frac{V(I)}{q}}$

$V(I)$ = GENERALIZED FREE ENERGY

MEAN FIELD MODEL $(\delta = \theta = 0)$

$$\frac{dx}{dt} = k\,(y - x - 2CP)$$

$$\frac{dP}{dt} = \gamma_\perp\,(xD - P)$$

$$\frac{dD}{dt} = \gamma_\parallel\,(xP + D - 1)$$

ASSUME $k \ll \gamma_\perp, \gamma_\parallel \Longrightarrow$ ADIABATIC ELIMINATION OF P AND D

$$\left(\frac{dP}{dt} = \frac{dD}{dt} = 0 \right)$$

$$D = \frac{1}{1+x^2} \qquad P = \frac{x}{1+x^2}$$

CLOSED EQUATION FOR x

$$\frac{dx}{dt} = k\left(y - x - \frac{2Cx}{1+x^2} \right)$$

$$\tau = kt \qquad \dot{x} = \frac{dx}{d\tau}$$

$$\dot{x} = y - x - \frac{2Cx}{1+x^2} \qquad \dot{x} = -\frac{dV}{dx}$$

$$V = -\int \left(y - x - \frac{2Cx}{1+x^2} \right) dx$$

$$= -yx + \frac{x^2}{2} + C \ln (1+x^2)$$

SIMILAR TO THE OVERDAMPED MOTION OF A CLASSICAL PARTICLE IN A POTENTIAL

$$m\ddot{x} + \gamma\dot{x} = -\frac{dU}{dx} \Longrightarrow \dot{x} = -\frac{\partial V}{\partial x},$$

$$V = U/\gamma$$

FLUCTUATIONS IN THE AMPLITUDE OF THE INCIDENT FIELD.

$$y(t) = y_0 + \delta y(t)$$

$$\dot{x} = y_0 - x - \frac{2Cx}{1+x^2} + \delta y(t) \qquad \text{LANGEVIN EQUATION}$$

GAUSSIAN STATIONARY STOCHASTIC PROCESS

$$\langle \delta y(t) \rangle = 0 \quad , \quad \langle \delta y(t)\, \delta y(t') \rangle = 2q\, \delta(t-t')$$

$P(x,\tau)$ PROBABILITY DISTRIBUTION OF THE FIELD

FOKKER - PLANCK EQUATION

$$\frac{\partial P(x,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left\{\left(-\gamma_0 + x + \frac{2Cx}{1+x^2}\right)P(x,\tau)\right\}$$

DRIFT

$$+\, q\, \frac{\partial^2 P(x,\tau)}{\partial x^2}$$

DIFFUSION

$$\frac{\partial P(x,\tau)}{\partial \tau} = -\frac{\partial}{\partial x}\, j(x,\tau)$$

PROBABILITY CURRENT

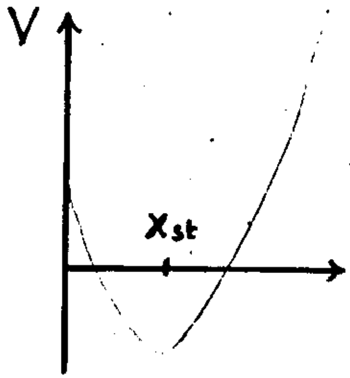
$$j(x,\tau) = \left(\gamma_0 - x - \frac{2Cx}{1+x^2}\right)P(x,\tau) - q\,\frac{\partial P(x,\tau)}{\partial x}$$

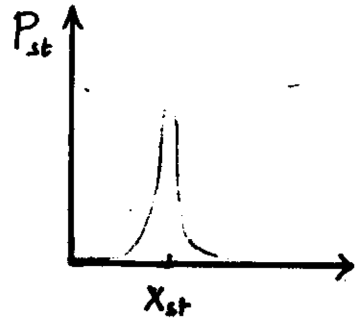
STATIONARY SOLUTION : $\frac{\partial P}{\partial \tau} = 0 \Longrightarrow j = 0$

$$\left(\gamma_0 - x - \frac{2Cx}{1+x^2}\right) P_{st} = q\,\frac{dP_{st}}{dx}$$

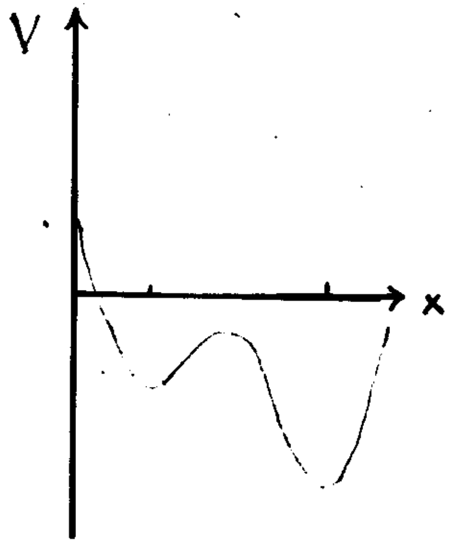
$$P_{st} = \frac{e^{-\frac{V(x)}{q}}}{\mathcal{N}}, \quad \mathcal{N} = \int dx\, e^{-\frac{V(x)}{q}}$$

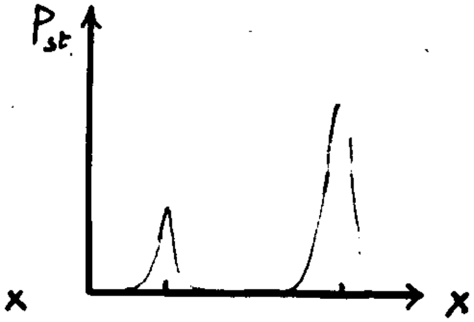
WHEN THE FLUCTUATIONLESS THEORY GIVES ONLY ONE STATIONARY SOLUTION X_{st}.



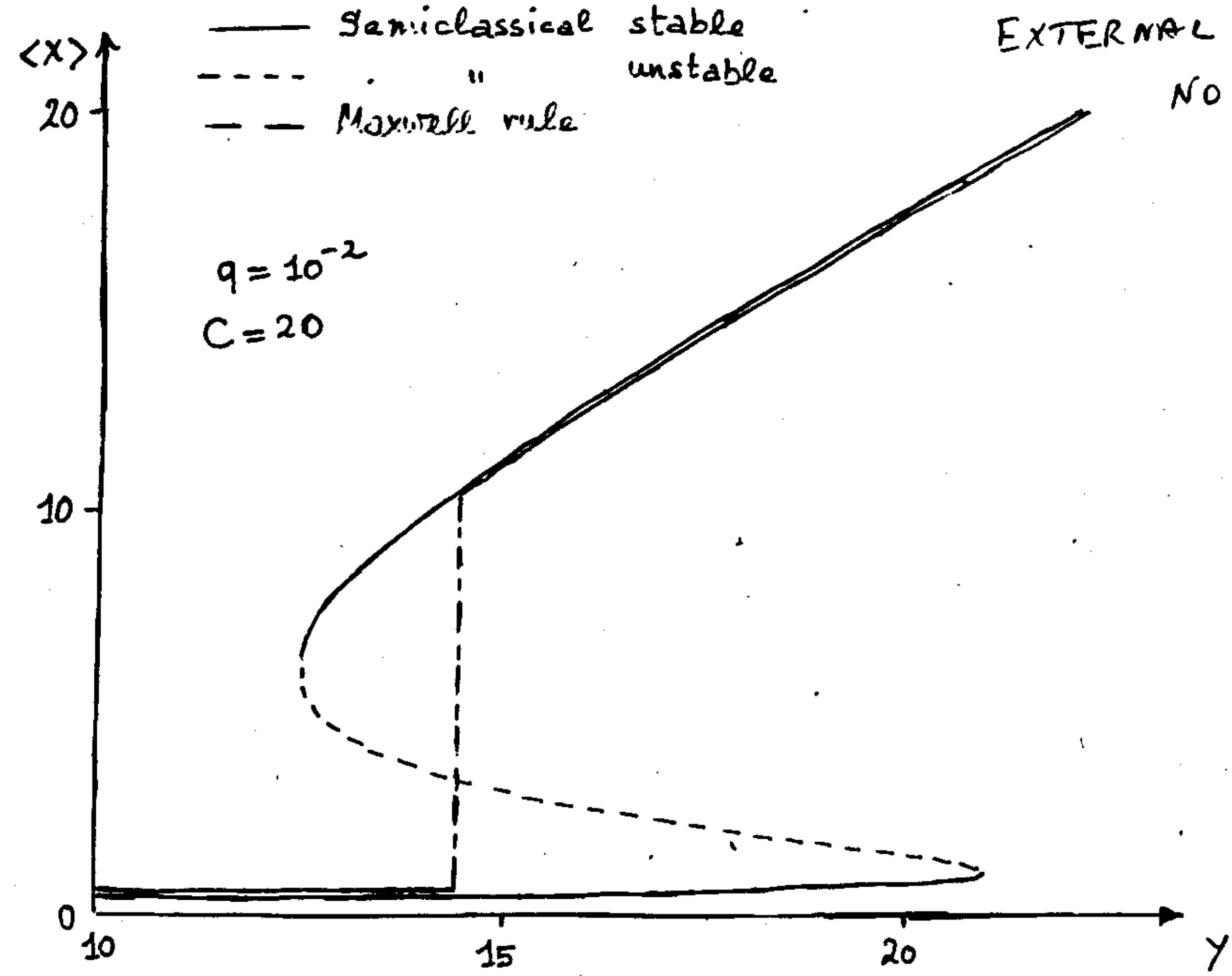


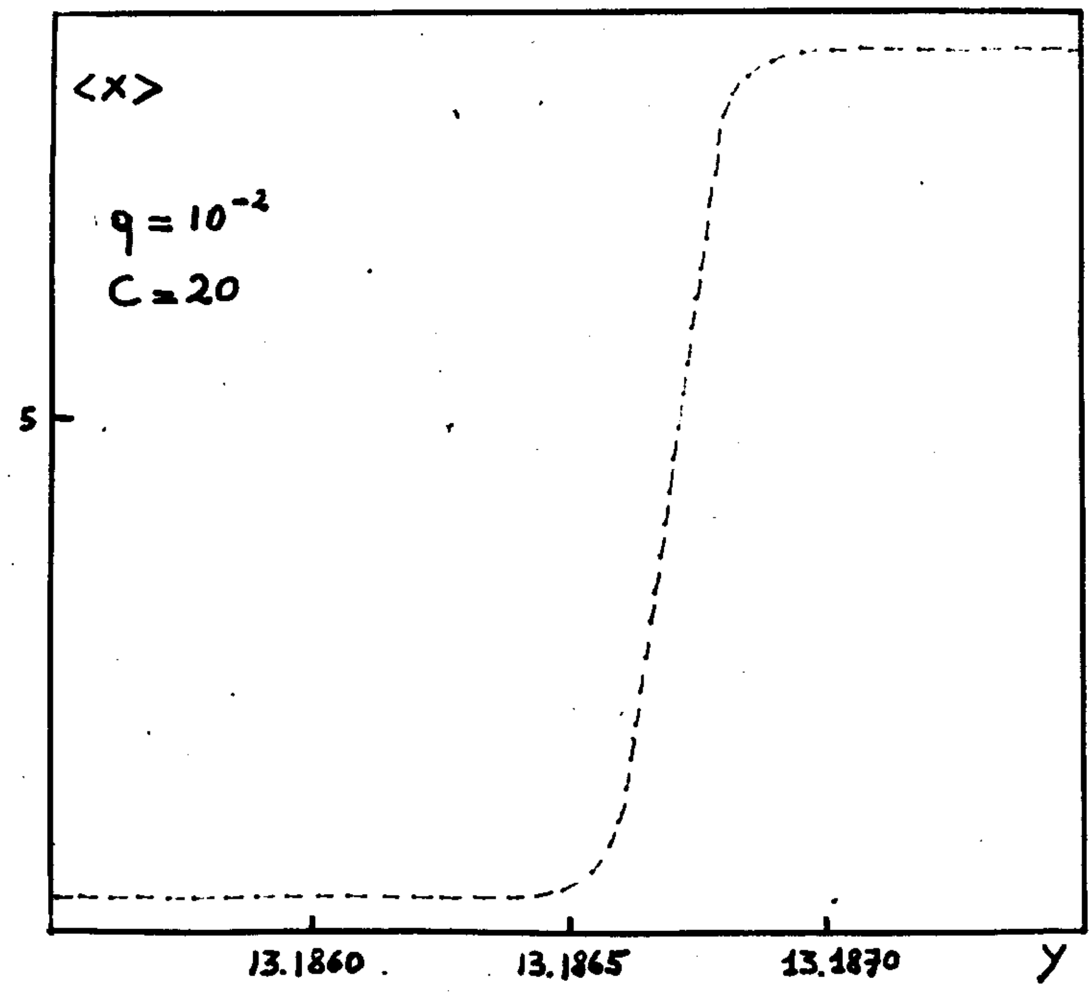
WHEN THE FLUCTUATIONLESS THEORY GIVES THREE STATIONARY SOLUTIONS



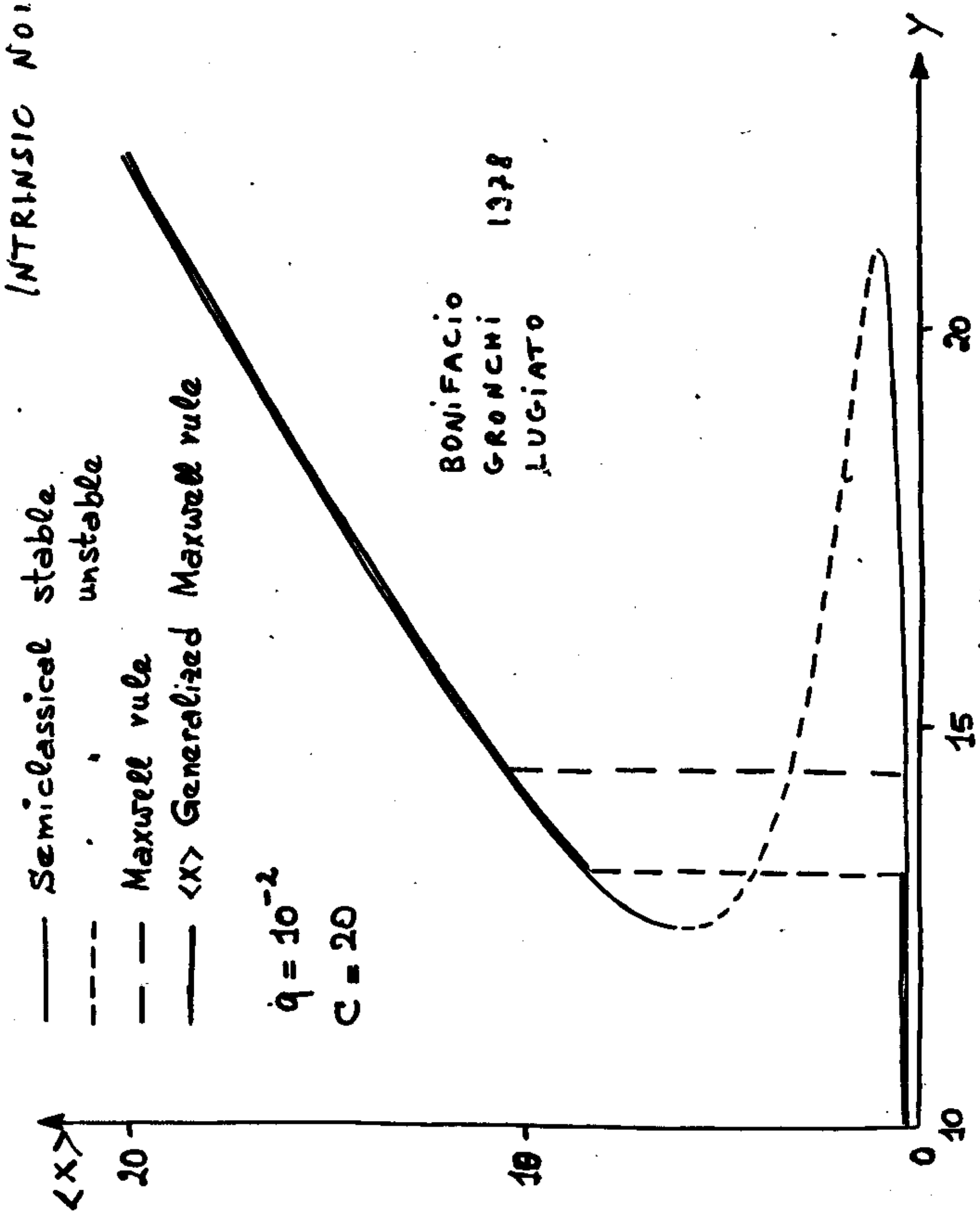


EXTERNAL NOISE

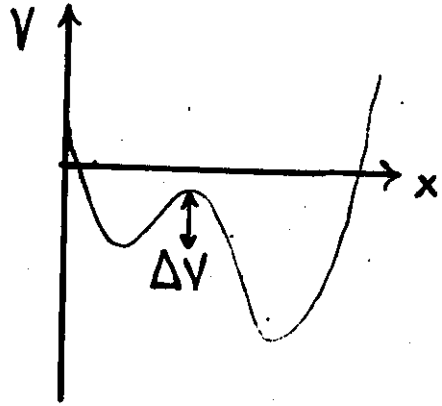




INTRINSIC NOISE



KRAMERS FORMULA FOR THE TRANSITION TIME



TRANSITION TIME

$$\propto e^{\frac{\Delta V}{q}}$$

INSTABILITIES IN NONLINEAR SYSTEMS

HAKEN'S SYNERGETICS

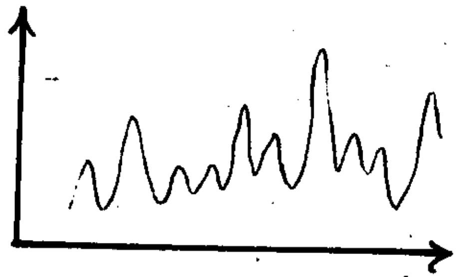
PRIGOGINE'S THEORY OF DISSIPATIVE STRUCTURES (NICOLIS and PRIGOGINE)

a = CONTROL PARAMETER

SELF-PULSING BEHAVIOUR

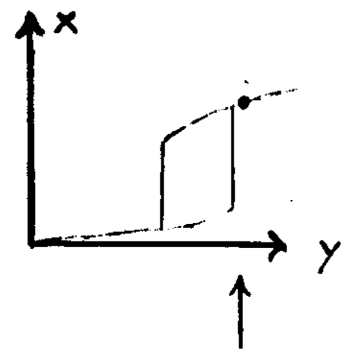
REGULAR (PERIODIC) SELF-PULSING

IRREGULAR (CHAOTIC) SELF-PULSING

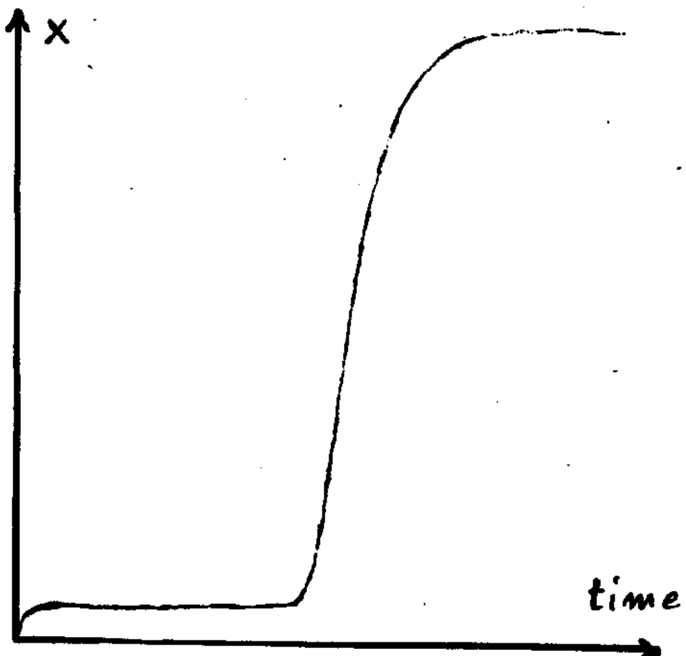


DETERMINISTIC CHAOS

TRANSIENT NOISE-INDUCED OPTICAL BISTABILITY (BROGGI LUGIATO)

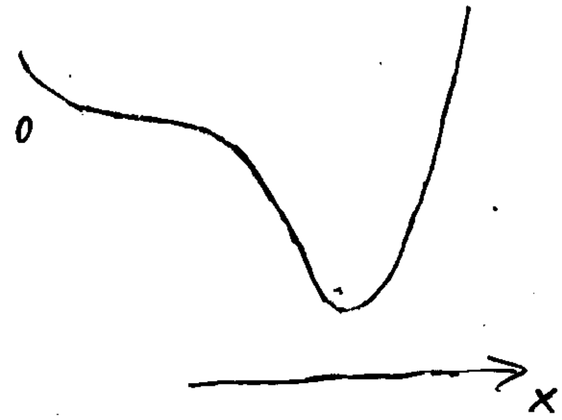


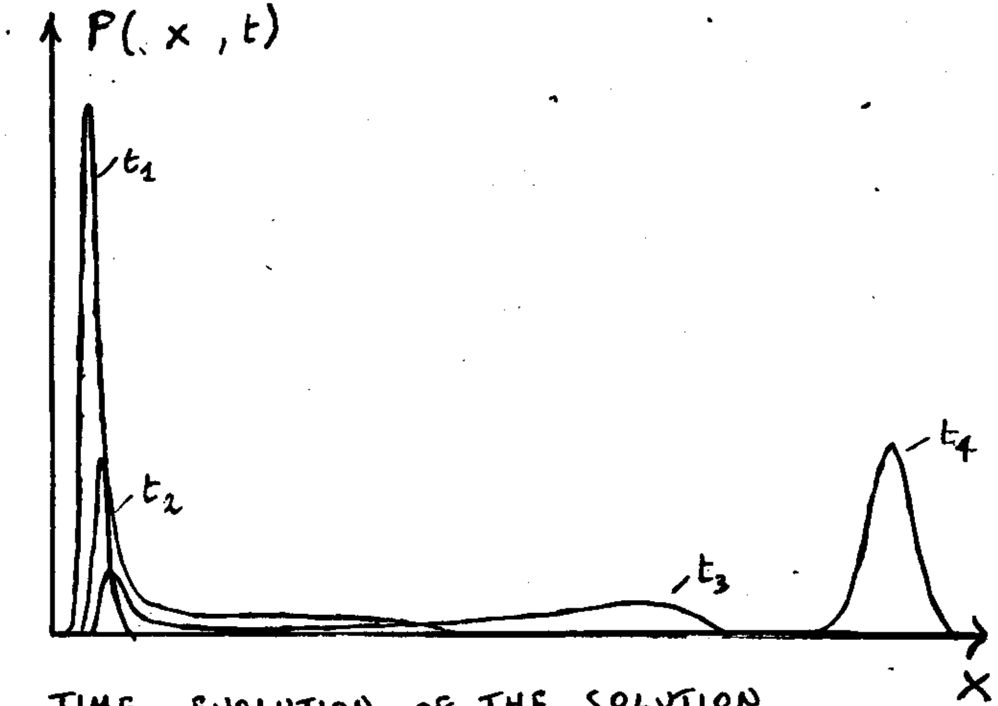
DETERMINISTIC EVOLUTION.: CRITICAL SLOWING DOWN



LANGE, MLYNEK DESERNO MISCHKE
EXPERIMENTAL OBSERVATION: GOZZINI
LONGO MACCARRONE (PISA); GRANT KIMBLE (AUSTIN)

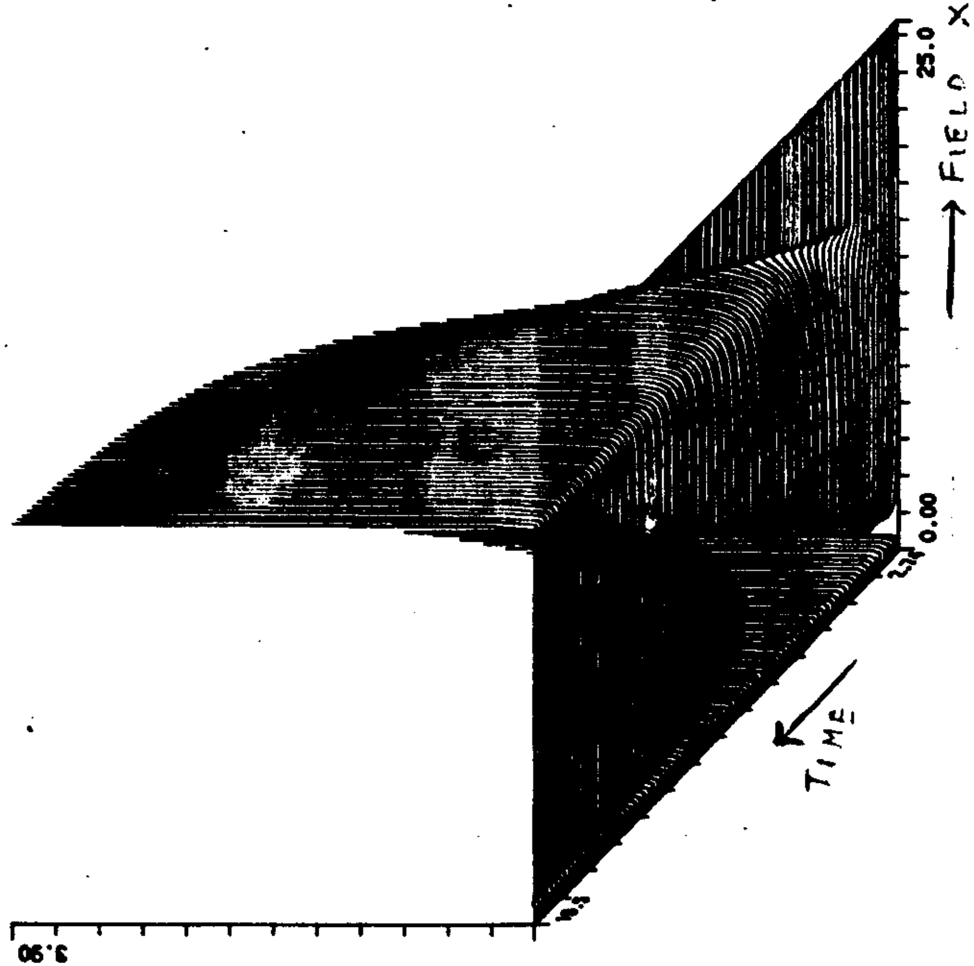
CRITICAL SLOWING DOWN.





TIME EVOLUTION OF THE SOLUTION

$t_1 < t_2 < t_3 < t_4$



TRANSIENT BIMODALITY