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MATRIX METHODS IN TREATING PHASE-CONJUGATE PHENOMENA

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Review *

Matrix methods in treating phase-conjugate phenomena

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Some sense in nonlinear optics (e.g. phase conjugation) could be treated by linear method (e.g. transfer matrix) as long as some new ideas for matrix may possibly be accepted.

Then, a series of complicated problems concerning phase conjugation are analysed and designed in such a simple way.

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1. Introduction

As we know that, phase conjugation it has potential applications in present and future laser systems.

In order to describe the operating characteristic of phase-conjugate mirror (PCM) in a simple way, there are two kinds of transfer matrix have been introduced.

One of them is <u>a ray transfer matrix</u> which was first defined by [1], and the other is <u>an equivalent transfer matrix</u> which was given latterly [2].

The both, either of them is useful for analysing different problems.

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[1] J. Au Yeung, D. Fekete, D.M. Pepper & A. Yariv, IEEE <u>QE-15</u> (1979) 1180.

[2] Wang Shaomin & H. Weber, Opt. Comm. <u>41</u> (1982) 360.

2. Two kinds of matrix

2.1 Ray transfer matrix M_I

Consider a Gaussian field E_i propagating along the axis z to be incident upon a <u>phase-conjugate mirror (PCM)</u>.

$$E_i = \mathcal{E}_i(\vec{r})\exp\left[i\left(\omega t - kz - \frac{kr^2}{2\rho}\right) - \frac{r^2}{W^2}\right] ; \qquad (1)$$

where, $\mathcal{E}_i(\vec{r})$ is the complex amplitude of E_i. This field can be also written as

$$E_i = \mathcal{E}_i(\vec{r})\exp\left[i\left(\omega t - kz - \frac{kr^2}{2q_i}\right)\right] , \qquad (2)$$

with the complex curvature

$$\frac{1}{q_i} = \frac{1}{\rho} - \frac{i\lambda}{\pi W^2} . \qquad (3)$$

If the PCM is formed by <u>degenerate four-wave mixing (DFWM)</u>, then the effect of the PCM is to «reflect» such an incident field as to yield its conjugate replica, leaving the wavefront and the spot size unchanged,

$$E_r \propto \mathcal{E}_i^*(\vec{r})\exp\left[i\left(\omega t + kz - \frac{kr^2}{2\rho}\right) - \frac{r^2}{W^2}\right] . \qquad (4)$$

As mentioned <u>the reflected axis is also defined positive</u>, Eq. (4) has to be rewritten as

$$E_r \propto \mathcal{E}_i^*(\vec{r})\exp\left[i\left(\omega t - kz + \frac{kr^2}{2\rho}\right) - \frac{r^2}{W^2}\right] , \qquad (5)$$

or

$$E_r \propto \xi_i^*(\vec{r})\exp[i(\omega t - \cdot z - \frac{K r^2}{2 q_r})] \quad (6)$$

The complex curvature o‚ reflected field subject to (6), (5), (3) and (2) is given by [1]

$$\boxed{\frac{1}{q_r} = -\frac{1}{\rho} - \frac{i\lambda}{\pi W^{\cdot}} - = -\frac{1}{q_i^*}} \quad ; \quad (7)$$

i.e., the wavefront is reverseo, or the phase is conjugated, that is why we call it <u>phase-conjug :tion</u>.

If one compares the r. flected property of PCM with that of flat reflector and cat's . ye reflector, <u>a ray transfer matrix</u> for describing the ray behavi :ur of PCM may be represented as follows [1]

$$\boxed{M_I = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} a_I & b_I \\ c_I & d_I \end{pmatrix}} \quad . \quad (8)$$

Some properties of PCM can be easily obtained by means of M_I. For example, <u>the ability c f PCM to compensate distortions of wavefront</u> may be analysed whe: the wave passes through an inhomogeneous medium there and bc :k. For a flat reflector,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} d & b \\ c & a \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ad+bc & 2bd \\ 2ac & ad+bc \end{pmatrix}, \quad (9)$$

it can not be compensated. F r a PCM,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} d & b \\ c & a \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (10)$$

it is compensated. By similar manner, the PCM has also the ability to compensate distortions even in misaligned systems [3].

However, the determinant of M_I which is not satisfied for that of linear optical system relation, it is

$$\boxed{\det(\vec{M}_I) = a_I d_I - b_I c_I = -1} \quad . \quad (11)$$

Even more, if we combine (7) with M_I, then get the reflected and the incident q parameters related by

$$\boxed{q_r = \frac{a_I q_i^* + b_I}{c_I q_i^* + d_I}} \quad . \quad (12)$$

Which is a ABCD law for M_I. <u>Note the conjugation operation upon q_i, as opposed to the conventional formation where the input field is not conjugated</u>. Therefore, it is not convenient to treat propagation, transformation and self-consistent solution of Gaussian beams.

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[3] Wang Shaomin, Opt. Quantum Electr. 16 (1984)

2·2 Transformation of transfer matrix

Some times, we would like to know what is the physical meaning by the transformation for an optical system, or to want calculating some problems in a simple way; thus can be done by making a transformation of ray transfer matrix through ABCD law[4].

For example,

$$q_2 = \frac{a_1\hat{q}_1 + b_1}{c_1\hat{q}_1 + d_1} = \frac{q_1}{\left(\frac{c_1\hat{q}_1 + d_1}{a_1\hat{q}_1 + b_1} - \frac{1}{q_1}\right)q_1 + 1} = \frac{a_2 q_1 + b_2}{c_2 q_1 + d_2}\ ; \qquad (13)$$

where, the caret means the original transformation. Then, an equivalent transfer matrix can be written as follows

$$\boxed{\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{c_1\hat{q}_1 + d_1}{a_1\hat{q}_1 + b_1} - \frac{1}{q_1} & 1 \end{pmatrix}} \quad , \qquad (14)$$

with $\det(\underset{\rightarrow}{M_2}) = a_2 d_2 - b_2 c_2 = 1$. (15)

It means, arbitrary optical element may thus be regarded as a thin lens with variable focal power if this element can be described by a ray transfer matrix.

[4] Wang Shaomin, J. Hangzhou University (in Chinese) 10 (1983) 476.

However, notice that the equivalent transfer matrix is applied to beams, not to rays.

2·3 Equivalent transfer matrix M_{II}

According to M_I, ABCD law for M_I and the equivalence transformation relation (14), we get

$$\boxed{\begin{pmatrix} a_{II} & b_{II} \\ c_{II} & d_{II} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{c_I q_i^* + d_I}{a_I q_i^* + b_I} - \frac{1}{q_i} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2/\rho & 1 \end{pmatrix} = M_{II}} \qquad (16)$$

with

$$\boxed{\det(\underset{\rightarrow}{M_{II}}) = a_{II} d_{II} - b_{II} c_{II} = 1} \quad , \qquad (17)$$

and

$$\boxed{q_r = \frac{a_{II} q_i + b_{II}}{c_{II} q_i + d_{II}}} \quad . \qquad (18)$$

Then, the operating property of PCM is getting clear. It is equivalent to a spherical reflector with variable curvature and which is identically equal to incident wavefront curvature $1/\rho$. This can be also seen by Fig. 1. In which, the PCM acts like a convergent lens for a divergent wave and like a divergent lens for a convergent wave. This physical situation is identical with that of [5].

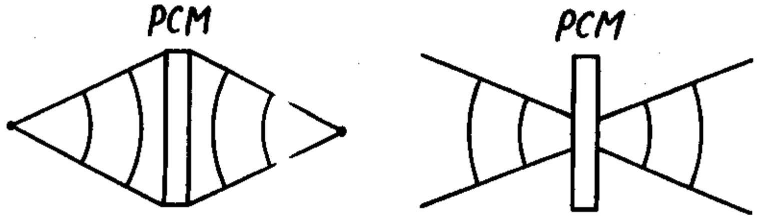


Fig. 1
Unfolded model of incident and reflected beams for a PCM

Or, in geometrical language, PCM makes the image distance is identically equal to the object distance; i.e.,

$$v \equiv -u \quad . \tag{19}$$

Then, the determinant of M_{II} equals unity, and the ABCD law for M_{II} is just like the conventional formalism. It is reasonable that all of the transformations (including conjugation operation) are embeded in the matrix elements and need not artificially added to the conjugate operation upon $\tilde{q}$. Consequently, it is convenient to treat propagation, transformation and self-consistent solution of optical beams.

However, remark that the M_{II} for PCM applies only to beams but not to rays.

[5] P. A. Bélanger, A. Hardy & A. E. Siegman, Appl. Opt. 19 (1980) 602.

As a matter of fact, the equivalent transfer matrix for PCM can be directly derived by means of the incident and reflected field relation and the ABCD law just like the derivation of transfer matrix for Gaussian mirror.

It is common knowledge that the matrix method used to analyse linear problems. Novelly, the matrix method can thus be expanded to analyse some nonlinear phenomena as long as the matrix elements are regarded as variables.

As a systematic example, we are going to discuss some main properties of a series of phase-conjugate resonators by means of these two matrices in next Section.

3. Phase-conjugate resonators (PCR)

3.1 PCR formed by degenerate four-wave mixing (DFWM)

There are two kinds of matrix representations for PCM have been given. Sometimes it is convenient to apply the equivalent transfer matrix M_{II} and sometimes we have to use the ray transfer matrix M_{I} if it is necessary to consider the characteristics of rays. We shall apply M_{I} in 3.6 and we are going to derive fundamental modes and some other properties of varied phase-conjugate resonators by M_{II} from 3·1 to 3·5.

Now, we consider the case of laser oscillation in a resonator in which one of the ordinary mirrors is repleced by a PCM and which is generated by DFWM as shown in Fig. 2. The R is the radius of curvature of real mirror (RM); and a, b, c and d are matrix elements of this optical system for starting from the RM and ending on the PCM but not including the RM and the PCM themselves.

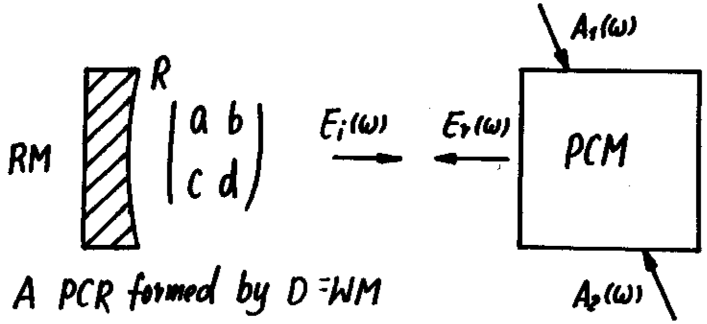


Fig. 2 A PCR formed by DFWM

Let us examine the PCR consisting of purely real optical elements at first. On the PCM surface, the transfer matrix for one round trip is

$$M=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}\begin{pmatrix} d & b \\ c & a \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2/\rho & 1 \end{pmatrix}=\begin{pmatrix} A_r & B_r \\ C_r & D_r \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2/\rho & 1 \end{pmatrix}=\begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (20)$$

with $A_rD_r-B_rC_r=1$ and $A_r=D_r$. (21)

Impose self-consistency

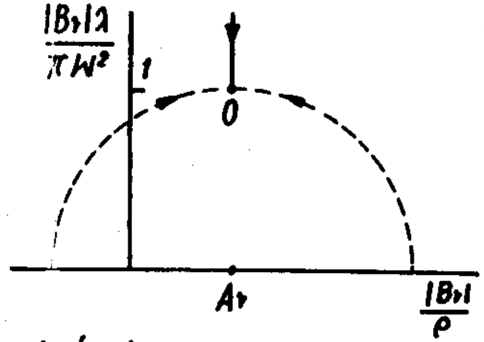
$$\frac{1}{q}=\frac{1}{\rho}-\frac{i\lambda}{\pi W^2}=\frac{C_r-2D_r/\rho+D_r/q}{A_r-2B_r/\rho+B_r/q}. \quad (22)$$

Compare the imaginary part of this equality, it is an identical equation of $0\equiv 0$; and compare the real part of it, get

$$\left(\frac{B_r\lambda}{\pi W^2}\right)^2+\left(\frac{B_r}{\rho}-A_r\right)^2=1. \quad (23)$$

It means that there is not any unique self-consistent transverse mode for the PCM cavity formed by DFWM with purely real optical elements which is shown in Fig. 3. This result is identical with that of given in [5] in which the conjugate relation (7) was directly used. In fact, the indefinity of modes is naturally understood when the PCM is regarded as a variable mirror.

Fig. 3 Modes on the PCM which is formed by DFWM
Dashed line – with purely real optical elements
Solid line – with complex optical elements



In view of the Gaussian reflectivity of PCM pumped by Gaussian beams [$A_1(\omega)$ and $A_2(\omega)$] [6], M_{II} (16) may be written as [7]

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[6] P. Trebino & A. E. Siegman, Opt. Comm. 32 (1980) 1.

[7] Wang Shaomin, Hong Xichun & Yu Jun, Acta Optica Sinica (in Chinese) 3 (1983) 41.

$$\boxed{M_{II} = \begin{pmatrix} 1 & 0 \\ -\left(\frac{2}{\rho} + \frac{i\lambda}{\pi\sigma^2}\right) & 1 \end{pmatrix}} \qquad (24)$$

Then, on the PCM surface, the transfer matrix for one round trip is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_r & B_r \\ C_r & D_r \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{\rho} + \frac{i\lambda}{\pi\sigma^2} & 1 \end{pmatrix}. \qquad (25)$$

Impose self-consistency,

$$\frac{1}{\rho} - \frac{i\lambda}{\pi W^2} = \frac{C_r - D_r\left(\frac{2}{\rho} + \frac{i\lambda}{\pi\sigma^2}\right) + D_r\left(\frac{1}{\rho} - \frac{i\lambda}{\pi W^2}\right)}{A_r - B_r\left(\frac{2}{\rho} + \frac{i\lambda}{\pi\sigma^2}\right) + B_r\left(\frac{1}{\rho} - \frac{i\lambda}{\pi W^2}\right)}. \qquad (26)$$

Compare the imaginary part and the real part of this equality, the fundamental mode will be fixed

$$\rho = \frac{B_r}{A_r}, \text{ and } \frac{|B_r|\lambda}{\pi W^2} = \frac{1}{2}\left\{-\left(\frac{|B_r|\lambda}{\pi\sigma^2}\right) \pm \left[\left(\frac{|B_r|\lambda}{\pi\sigma^2}\right)^2 + 4\right]^{1/2}\right\}. \qquad (27)$$

It has been shown in Fig. 3.

The point 0 in Fig. 3 is a critical state. On which, the spot size is the largest in the definite modes and is the smallest in the indefinite modes. Where

$$\rho = B_r/A_r, \text{ and } W^2 = \lambda|B_r|/\pi. \qquad (28)$$

If we define $\boxed{G = a - b/R}$, and $\boxed{G' = d - b/\rho}$; (29)

then we get $\boxed{\rho = \frac{2bG}{2dG-1}}$, and $\boxed{W^2 = \frac{\lambda}{\pi}2|bG|}$. (30)

By similar process, the fundamental mode on RM can be also obtained as follows $\boxed{\rho_{RM} = R}$, and $\boxed{W^2_{RM} = \frac{\lambda}{\pi}\left|\frac{b}{G}\right|}$. (31)

Combine (30) with (29), we find

$$\boxed{GG' \equiv 1/2}. \qquad (32)$$

Therefore, PCR formed by DFWM they are unconditionally confined. Perturbation stability of them is similar to that of conventional optical resonators. And they have the ability to compensate distortions in the cavity; however, by theoretical analysis [7] and experimental demonstration [8], the laser output must be extracted from the RM.

3.2 PCR formed by DFWM with apertures

It was difficult to treat PCR with finite-apertures except the «semiconfocal PCR» (R=L) which was discussed by means of integral equations [9].

[8] R.C. Lind & D.G. Steel, Opt. Lett. 6 (1981) 554.
[9] J.F. Lam & W.P. Brown, Opt. Lett. 5 (1980) 61.

However, as analysed before, the modes of PCR formed by DFWM are similar to that of conventional resonators as long as one considers PCM just like a zoom lens which all depends on incident beam. It offers possible to find out a spherical resonator equivalent to arbitrary PCR with apertures. Then, the diffraction losses and mode structures of any PCR could be solved by means of known numerical solutions of its equivalent spherical resonator.

For simplicity, an empty PCR with apertures is shown in Fig. 4. Then, the parameters (29)-(32) will be simplified to

$$g = 1 - L/R, \quad g' = 1 - L/\rho; \tag{33}$$

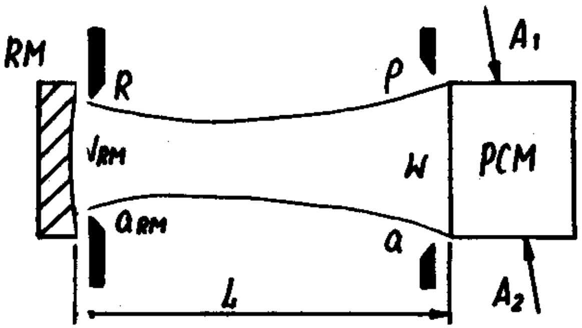
$$\rho = 2Lg/(2g-1), \quad W^2 = \frac{\lambda}{\pi} 2|Lg|; \tag{34}$$

$$\rho_{RM} = R, \quad W_{RM}^2 = \frac{\lambda}{\pi}\left|\frac{L}{g}\right|; \tag{35}$$

and

$$gg' \equiv 1/2. \tag{36}$$

Fig. 4



An empty PCR formed by DFWM with apertures

Generally speaking, resonator with apertures there are five parameters — a_1, a_2, R_1, R_2 and L to determine the beam properties.

By known transformations [10], only three independent parameters occur — G_1', G_2' and N:

$$N = a_1 a_2/\lambda L; \quad \begin{cases} G_1' = g_1(a_1/a_2), \\ G_2' = g_2(a_2/a_1). \end{cases} \tag{37}$$

It is reasonable that assume the ratio S between the radii of the apertures $a_{1,2}$ and the radii of the spot sizes $W_{1,2}$ to be identical for both mirrors, i.e.,

$$a_1/W_1 = a_2/W_2 = S. \tag{38}$$

Combining (38), (37) and

$$W_{1,2}^2 = \frac{\lambda L}{\pi}\left[\frac{g_{2,1}}{g_{1,2}(1-g_1 g_2)}\right]^{1/2}, \tag{39}$$

get [11]

$$G_1' = G_2' = \sqrt{g_1 g_2}, \quad N = S^2/\pi\sqrt{1-g_1 g_2}. \tag{40}$$

Then, a resonator with apertures is completely characterized by two parameters: S and $g_1 g_2$. Which means that for any asymmetric resonator (g_1, g_2) with adapted apertures there may be found a symmetric one with $\bar{g} = \sqrt{g_1 g_2}$.

Substituting (36) into (40) for PCR with apertures, get [2]

$$\boxed{\bar{g} = 1/\sqrt{2}, \quad N = \frac{\sqrt{2}}{\pi} S^2}. \tag{41}$$

[10] J.P. Gordon & H. Kogelnik, Bell Syst. Tech. J. 43 (1964) 2873.
[11] H.P. Kortz & H. Weber, Appl. Opt. 20 (1981) 1936.

An equivalence spherical symmetric resonator for any PCR with apertures then thus be found, and the number of independent characterizing parameters is reduced to only one: the factor S. The g and g' factors of PCR in the confinement diagram and its equivalence transformation are shown in Fig. 5. The equivalent relation between spherical resonators and PCR can also cf. [12].

For example, if we select $\bar{g} = 0.707$, a relation between the power loss for PCR with apertures and the factor S is obtained by means of [11] as shown in Fig. 6.

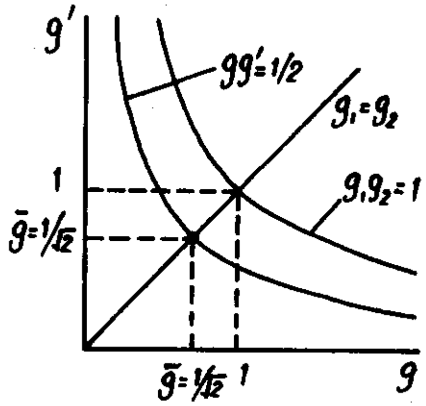


Fig. 5 gg' diagram and its equivalence transformation

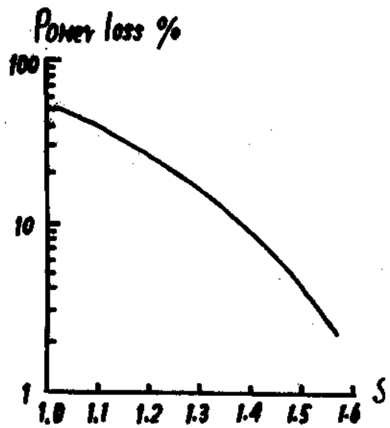


Fig. 6 Power loss per full round trip of the PCR vs. adapting factor S

[12] P.A. Bélanger, Opt. Eng. 21 (1982) 266.

3.3 PCR formed by nondegenerate FWM

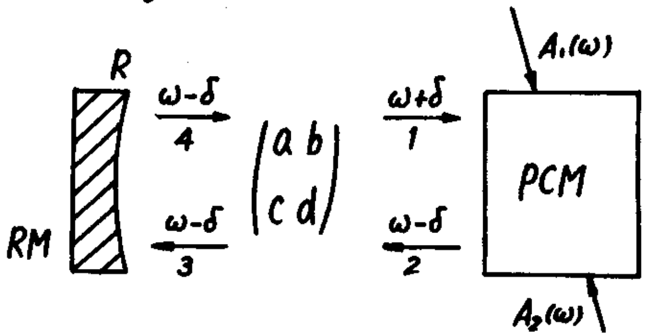


Fig. 7 A PCR formed by nondenerate FWM

If a PCR is formed by a nondegenerate FWM as shown in Fig. 7, due to the frequency flipping nature [13], the reflected complex radius of curvature ρ_r which is related to that of incident ρ_i by [1]

$$\boxed{\rho_r = -\rho_i^* K}, \quad K = \frac{1-\delta/\omega}{1+\delta/\omega}. \tag{42}$$

Then, the M_I is expanded to

$$\boxed{M_I^\delta = \begin{pmatrix} 1\mp\delta/\omega & 0 \\ 0 & -(1\pm\delta/\omega) \end{pmatrix}}, \tag{43}$$

with $\det(\underrightarrow{M}_I^\delta) = (\delta/\omega)^2 - 1$; (44)

and the M_{II} will be expanded to

$$\boxed{M_{II}^\delta = \begin{pmatrix} K^{\pm1} & 0 \\ -2/\rho_\pm & 1 \end{pmatrix}}, \tag{45}$$

with $\det(\underrightarrow{M}_{II}^\delta) = K^\pm$ and $\det[(\underrightarrow{M}_{II}^\delta)^2] = 1$ (46)

[13] D.M. Pepper & R.L. Abrams, Opt. Lett. 3 (1978) 212.

Both M_{I}^{δ} and M_{II}^{δ} they are equivalent transfer matrices. We are going to derive the fundamental modes of it by means of M_{II}^{δ} [14].

It is clear, there is not any self-consistent mode for one round trip for such a cavity. At the PCM surface, the transfer matrix for two round trips is

$$M_{II2r}^{\delta}=\begin{pmatrix} A_r & B_r \\ C_r & D_r \end{pmatrix}\begin{pmatrix} K^{-1} & 0 \\ -2/\rho_- & 1 \end{pmatrix}\begin{pmatrix} A_r & B_r \\ C_r & D_r \end{pmatrix}\begin{pmatrix} K^{+1} & 0 \\ -2/\rho_+ & 1 \end{pmatrix}=\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}. \qquad (47)$$

Substituting (47) into ABCD law and imposing self-consistency, get

$$\rho_{\pm}=\frac{A_r}{C_r}(1\pm\delta/\omega), \text{ and } W^2=\frac{c}{\pi\omega}\left[\frac{A_r^2 B_r}{C_r(1-A_r^2\delta^2/\omega^2)}\right]^{1/2}; \qquad (48)$$

the mode is fixed for the PCR with purely real optical elements. By the similar process, we get the beam parameters on RM

$$\rho_{RM}=R, \text{ and } W_{RM\pm}^2=\frac{c}{\pi(\omega\pm\delta)}\left[\frac{4b^2d^2}{B_rC_r}\left(\frac{1\pm A_r\delta/\omega}{1\mp A_r\delta/\omega}\right)\right]^{1/2}. \qquad (49)$$

According to (48), (49) and the determinant relation (46), a confinement condition for PCR formed by nondegenerate FWM is obtained

$$\boxed{|A_r| > 1}; \qquad (50)$$

or, for an empty PCR, we have

$$0>g, \; g>1, \text{ with } g=1-L/R. \qquad (51)$$

[14] Yu Jun, Dong Huiying & Wang Shaomin, J. Hangzhou University (in Chinese) 10 (1983) 8.

There is not any nondegenerate mode in such a PCR for one round trip, but the modes are there and fixed for two round trips even the PCR with purely real optical elements. However, they are not unconditionally confined, they are not satisfied the condition for dynamic stability and they are not sure whether satisfied perturbation stability. Therefore, it is not an ideal way to realize a PCR if which is formed by nondegenerate FWM besides the reflectivity is low.

Things are returned to PCR formed by DFWM. Unfortunately, if we would like get undistortion output, the pumping beams are required perfect in homogeneous or in Gaussian which is unrealistic. One of the approaches of surmount the difficulty is PCM formed still by DFWM but made passive (e.g. self-pumped) [15], and a PCM produced by stimulated scattering (SS) [16] is other way to overcome the difficulty.

3.4 PCR formed by stimulated scattering (SS)

[15] M. Cronin-Golomb, B. Fischer, J. Nilsen, J.O. White & A. Yariv, Appl. Phys. Lett. 41 (1982) 219.

[16] G. Giuliani, M.M. Denariez-Roberge & P.A. Bélanger, Appl. Opt. 21 (1982) 3719.

In order to analyse fundamental modes of SS-PCR, we had better offer a suitable transfer matrix to describe the operating property of SS-PCM at first. When a Gaussian beam (W_i, ρ_i) is incident on a SS medium, the scattered beam is still in Gaussian but with a beam spot size by [16]

$$W_s = \beta W_i \tag{52}$$

and a radius of curvature approximately by

$$\rho_s = -\rho_i \quad . \tag{53}$$

Where, the parameter β is a constant, which varies between 0 and 1 and which is related to the intensity of the field and to the gain of the process. Therefore, the relation between the reflected (scattered) beam and the incident beam can be written as

$$\boxed{\frac{1}{q_s} = \frac{1}{\rho_s} - \frac{i\lambda}{\pi W_s^2} = -\frac{1}{\rho_i} - \frac{i\lambda}{\pi \beta^2 W_i^2}} \tag{54}$$

It is difficult to find a ray transfer matrix for describing this phenomenon. However, an equivalent transfer matrix can be found by means of ABCD law as follows [17]

$$\boxed{\begin{pmatrix} a_I & b_I \\ c_I & d_I \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{2}{\rho_i} - \frac{i\lambda}{\pi}\left(\frac{1-\beta^2}{\beta^2 W_i^2}\right) & 1 \end{pmatrix}} \; , \tag{55}$$

[17] Wang Shaomin & H. Weber Optica Acta <u>31</u> (1984) 971.

with $\quad a_I d_I - b_I c_I = 1 \quad .$ (56)

<u>SS-PCM may thus be regarded as a Gaussian reflectivity mirror whose curvature is identically equal to that of incident beam and whose reflectivity profile depends on</u>

$$\boxed{R(r) = R(0) e^{-r^2(1-\beta^2)/\beta^2 W_i^2}} \tag{57}$$

Not only the real part of matrix element in (55) is a variable but also the imaginary part as well. If $\beta = 1$, the varied Gaussian reflectivity will be reduced to uniform reflectivity and the transfer matrix (55) will be reduced to (16).

Apply equivalent transfer matrix for SS-PCM (55), ABCD law and impose self-consistency; the incident and scattered beam parameters at SS-PCM surface are obtained as follows

$$\rho_i = B_r/A_r = 2bG/(2dG-1) \; , \tag{58}$$

$$\rho_s = -B_r/A_r = -2bG/(2dG-1) \; ; \tag{59}$$

$$W_i^2 = \lambda |B_r|/\pi\beta = \frac{\lambda}{\pi} 2|bG|/\beta \; , \tag{60}$$

and

$$W_s^2 = \lambda |B_r| \beta/\pi = \frac{\lambda}{\pi} 2|bG|\beta \; . \tag{61}$$

Notice that the spot size of scattered beam is different from that of incident beam, it is similar to (but not identical with) that of Gaussian reflectivity resonators. By the similar process, the incident and reflected beam parameters at RM are got as

$$\rho_{RMi} = b(1+\beta^2)/[a(1+\beta^2)-2G\beta^2] \quad , \qquad (62)$$

$$\rho_{RMr} = b(1+\beta^2)/[2G-a(1+\beta^2)] \quad ; \qquad (63)$$

and

$$W^2_{RMr} = W^2_{RMi} = \frac{\lambda}{\pi}[(1+\beta^2)/2\beta]\,|b/G| \; . \qquad (64)$$

The radius of reflected wavefront curvature is different from that of incident beam in value, and they are not coincident with that of RM.

According to (58), get

$$\boxed{GG' = 1/2} \qquad (65)$$

Things are getting clear, there are fixed modes in SS-PCR even the PCR with purely real optical elements. They are unconditionally confined, absolutely dynamically stable and stable against perturbation, as long as $\beta < 1$.

The fixed modes on the SS-PCM are illustrating in Fig. 8.

For the future, SS-PCR as practical devices, which could be recommended, provided to get higher reflectivity of SS-PCM.

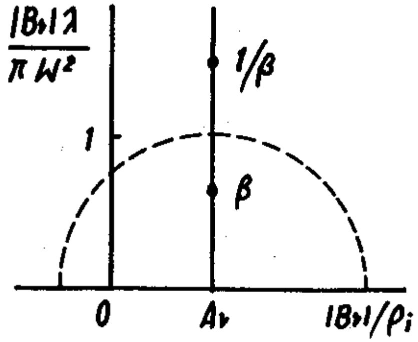


Fig. 8 Fixed modes at the SS-PCM in a SS-PCR

3.5 PCR formed by self-pumped FWM

Recent years, there were a lot of successful experimental works — PCR formed by self-pumped FWM which were presented [15], [18, 19].

It is important to define a suitable matrix and then to get its modes now.

In this case, the reflectivity profile of PCM depends on [6]

$$R(r) = R(0)\, e^{-r^2/W_i^2} \qquad (66)$$

Where, W_i is spot size of Gaussian beam inside the cavity and incident on the PCM itself. Then, the reflected spot size W_r is subject to

$$e^{-r^2/W_r^2} = e^{-r^2/W_i^2} \cdot e^{-r^2/W_i^2} \; ; \qquad (67)$$

or

$$W_r = W_i/\sqrt{2} \; . \qquad (68)$$

The total transformation could thus be got

$$\boxed{\frac{1}{q_r} = \frac{1}{\rho_r} - \frac{i\lambda}{\pi W_r^2} = -\frac{1}{\rho_i} - \frac{2i\lambda}{\pi W_i^2}} \; . \qquad (69)$$

[18] R.A. Mc Farlance & D.G. Steel, Opt. Lett. 8 (1983) 208.
[19] J. Feinberg & G. D. Backer, Opt. Lett. 9 (1984) 420.

According to ABCD law, an equivalent transfer matrix for describing the operation of self pumped PCM is given [20]

$$\boxed{\begin{pmatrix} a_{I} & b_{I} \\ c_{I} & d_{I} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{i\lambda}{\pi W_i^2} & 1 \end{pmatrix}} \tag{70}$$

with

$$a_{I} d_{I} - b_{I} c_{I} = 1 . \tag{71}$$

Then, the beam parameters at self-pumped PCM and at RM and the main property of self-pumped PCR are obtained

$$\rho_i = B_r/A_r = 2bG/(2dG-1) , \tag{72}$$

$$\rho_r = -B_r/A_r = -2bG/(2dG-1), \tag{73}$$

$$W_i^2 = \sqrt{2}\lambda |B_r|/\pi = \frac{\lambda}{\pi} 2\sqrt{2} |bG| , \tag{74}$$

$$W_r^2 = \lambda |B_r|/\sqrt{2}\pi = \frac{\lambda}{\pi} \sqrt{2} |bG| ; \tag{75}$$

$$\rho_{RMi} = 3b/(3a-2G) , \tag{76}$$

$$\rho_{RMr} = 3b/(4G-3a) , \tag{77}$$

$$W_{RMr}^2 = W_{RMi}^2 = \frac{\lambda}{\pi} \cdot \frac{\sqrt{2}}{1} |b/G| ; \tag{78}$$

and

$$\boxed{GG' \equiv 1/2} . \tag{79}$$

The properties of PCR formed by self-pumped FWM are very well just like the SS-PCR because it is a special case of SS-PCR in case of $\beta = 1/\sqrt{2}$.

[20] Wang Shaomin & Yu Guoan, Fundamental modes of self-pumped PCR, J. Hangzhou University (in Chinese and in Printing)

3.6 Misalignment sensitivity of PCR

Although the PCR have got a series of nice propertices, but the misalignment phenomenon has been observed [1].

Let us consider a PCR formed by DFWM in which the RM or/and the PCM is/are misaligned as shown in Fig. 9.

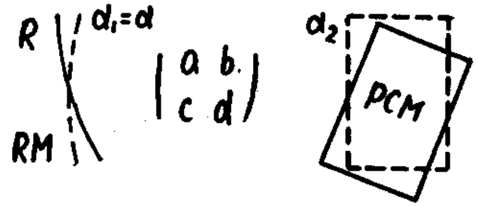


Fig. 9 A misaligned PCR

For a misaligned optical element which can be treated by an augmented 4×4 matrix. But note that an equivalent transfer matrix could not be augmented [3]. Therefore, we have to augment M_I to analyse this problem. For the misaligned PCM,

$$\boxed{M_I' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} . \tag{80}$$

It means that the PCM is a misalignment insensitive element. For the RM,

$$\boxed{M_{RM}' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2/R & 1 & 0 & -2d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} . \tag{81}$$

Then, the displacement of ray r_{21} and ray steering angle r_{21}' on PCM caused by tilted RM can be calculated by

$$\begin{pmatrix} r_{211} \\ r'_{211} \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2/R & 1 & 0 & -2\alpha \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d & b & 0 & 0 \\ c & a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{210} \\ r'_{210} \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} A & B & 0 & -2\alpha b \\ C & D & 0 & 2\alpha d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{210} \\ r'_{210} \\ 1 \\ 1 \end{pmatrix}, \qquad (82)$$

with

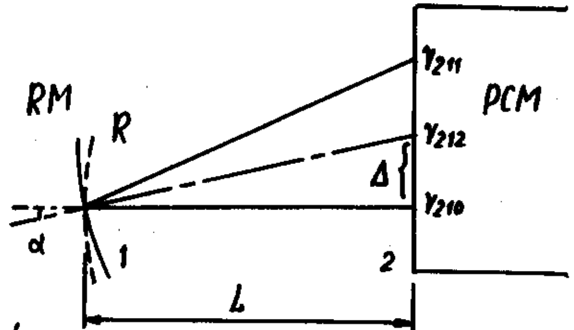
$$A = ab + bc - 2bd/R, \quad B = 2ab - 2b^2/R,$$
$$C = -(2cd - 2d^2/R), \quad D = -(ad + bc - 2bd/R). \qquad (83)$$

If the ray travels for two round trips in the PCR, we find

$$\begin{pmatrix} A & B & 0 & -2\alpha b \\ C & D & 0 & 2\alpha d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (84)$$

Which means that all of the rays are self-reproducing and auto-alignment after two round trips. For simplicity, this situation is illustrating in Fig. 10.

Fig. 10 Geometrical variations of rays caused by tilted RM in an empty PCR



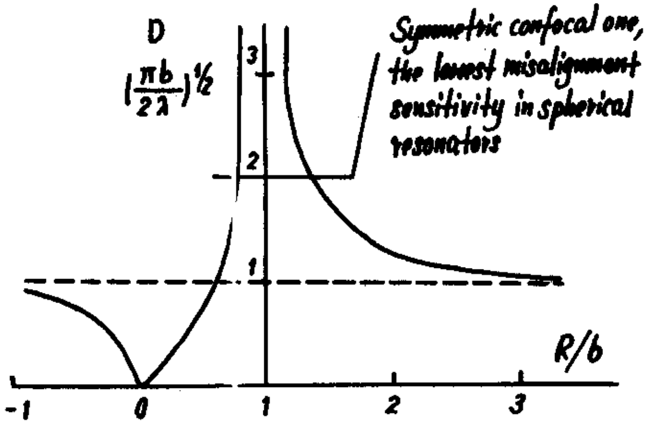
Substituting Δ into misalignment sensitivity formula [21], then we have [22]

[21] P. Hauck, H.P. Kortz & H. Weber, Appl. Opt. 19 (1980) 598.
[22] Wang Shaomin, Kexue Tongbao 28 (1983) 173.

$$\boxed{D = \left|\frac{\pi b}{2\lambda}\right|^{1/2} \left|\frac{1}{G}\right|^{1/2}} \qquad (85)$$

The physical meaning of D is: if a mirror is tilted by an angle α, and $\alpha = 1/D$; then an additional loss of ~10% is caused, in case of $\delta = a_1/w_1 = a_2/w_2 = 1.2$. A relation between D and R/b is shown in Fig. 11. It shows that the misalignment sensitivity of PCR is lower than that of spherical resonators except $R \simeq b$. It is convenient for adjusting.

Fig. 11 Misalignment sensitivity for PCR



As discussed before, FWM and SS they are perfect phase-conjugate phenomena. On the other hand, there are some physical phenomena which has ability to compensate distortions (e.g., corner cube array [23] and slab laser [24]), too. They were called pseudo-phase-conjugation, or called something as you please. They are useful and simple in practice.

[23] D. Mathieu & P.A. Bélanger, Appl. Opt. 19 (1980) 2262.
[24] J.M. Eggleston, T.J. Kane, J. Unternahrer & R.L. Byer, Opt. Lett. 7 (1982) 405.

4. Some pseudo-phase-conjugate phenomena

4.1 A polished rod solid state laser

A face pumped, face cooled, slab laser in which the optical path inside the gain medium undergoes total internal reflection, the effect of the thermal-optical distortions can be reduced [24, 25].

Then, a viewpoint on slab lasers having intrinsic pseudo-phase-conjugate properties was proposed and some demonstrations were given [26]. It may be useful to analyse some other slab lasers (e.g., slab dye laser [27]).

This concept is expanded to a polished rod laser [28]. The principle of ray and beam transformation for internal reflection in a polished rod is shown in Fig. 12, and some demonstrations are given in Fig. 13.

[25] T. J. Kane, R. C. Elkardt & R. L. Byer, IEEE QE-19 (1983) 1351.

[26] Wang Shaomin, Appl. Lasers (in Chinese) 4 (1984) 109.

[27] V. Rivano, P. Mazzinghi & P. Burlamacchi, Appl. Phys. B35 (1984) 71.

[28] Ying Chengren, Wang Shiaojing & Wang Shaomin, A tentative study of polished rod laser, to appear.

Fig. 12 A principle of transformation for internal reflection

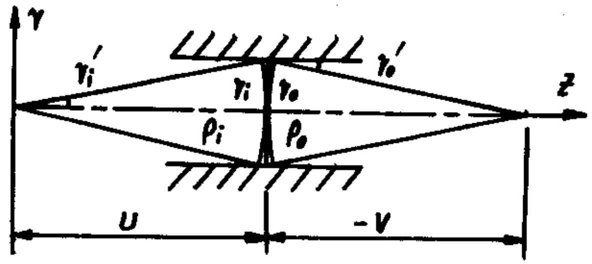


Fig. 13 Demonstrations for above principle

U = 190 -V = 120 -V = 190

As discussed in 2.1, backward-going conjugators have got ability to compensate distortions caused by inhomogeneous media. In this case, for a forward-going pseudo-conjugator,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} = \begin{pmatrix} a_1a_2 - b_1c_2 & a_1b_2 - b_1d_2 \\ c_1a_2 - d_1c_2 & c_1b_2 - d_1d_2 \end{pmatrix}. \quad (86)$$

Let

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

then the compensating conditions are obtained: A. the transfer matrix elements are identical for both sides; and B. $a = d$. If the rod with length l is pumped symmetrically, the index distribution in r direction generally expressed by

$$n(r) = n(0)\,(1 \pm \beta_0 r^2)\,. \qquad (87)$$

The corresponding ray transfer matries are

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} Ch(l\sqrt{2\beta_0}) & \frac{1}{\sqrt{2\beta_0}} Sh(l\sqrt{2\beta_0}) \\ -\sqrt{2\beta_0}\, Sh(l\sqrt{2\beta_0}) & Ch(l\sqrt{2\beta_0}) \end{pmatrix}, \quad \text{for } +\beta_0; \qquad (88)$$

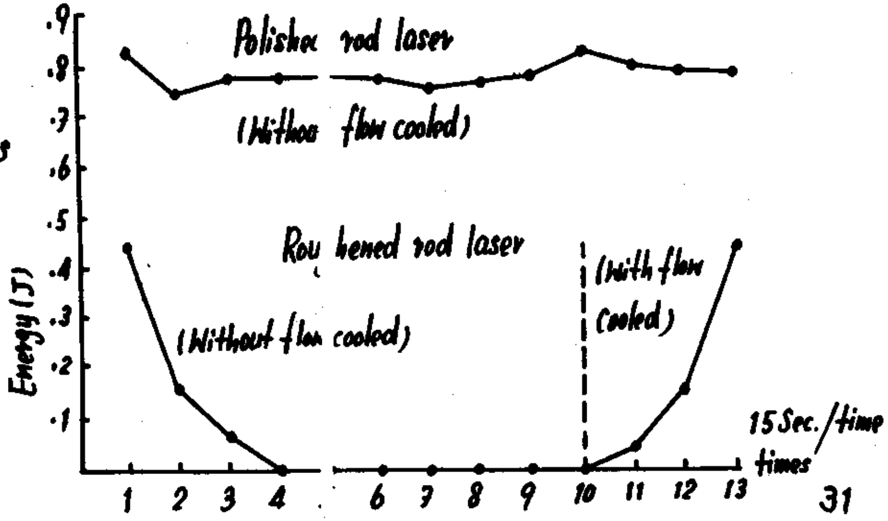
and

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} Cos(l\sqrt{2\beta_0}) & \frac{1}{\sqrt{2\beta_0}} Sin(l\sqrt{2\beta_0}) \\ -\sqrt{2\beta_0}\, Sin(l\sqrt{2\beta_0}) & Cos(l\sqrt{2\beta_0}) \end{pmatrix}, \quad \text{for } -\beta_0. \qquad (89)$$

The both, (88) and (89) the · are satisfied conditions A and B.

Then a set of polished rod laser and an another set of conventional roughened rod laser made of the same Nd glass for comparing the output properties are shown in Fig. 14.

Fig. 14 Properties of polished rod laser



It is clear, the output energy and dynamic stability of polished rod laser are better than that of conventional roughened rod laser. But, the mode structures are rather complicated. Even though so, it has some intrinsic pseudo-phase-conjugate properties and it is one of the simple ways to improve solid state laser in some industrial applications.

4.2 Specified arrays as pseudo-conjugators

On the other hand, some experimets were demonstrated which show how corner-cube arrays have been used to approximate phase conjugation, and used on laser amplifier or optical resonators compensated optical inhomogneities [29,30] [28]. Generally, the corner-cube arrays were the common plastic retroreflectors used on bicycles and highway signs.

It is a special problem of geometrical optics. But, it is difficult to analyse the novel phenomenon by means of classical optics because its imaging is not sure in Gaussian, i.e.,

$$\frac{1}{u} - \frac{1}{v} = \frac{1}{f} \qquad (90)$$

is not sure satisfied.

[29] V.K. Orlov, Ya. Z. Virnik, S.P. Vorotilin, V.B. Gerasimov, Yu. A. Kalinin & A. Ya. Sagalovich, Sov. J. Quantum Electr. 8 (1978) 799.

[30] H.H. Barrent & S.F. Jacobs, Opt. Lett. 4 (1979) 190.

If the individual optical element of array is arranged in regularity, it is easily treated by means of augmented 4x4 matrix [31, 32] [3]. Then, a conjugate distance equation for synthetical imaging of arrays, a ray transfer matrix for arrays and an equivalent transfer matrix for arrays can be derived as follows:

$$V = \frac{\left(\frac{R+b-l}{R}\right)u + b}{\left(\frac{d\mp 1}{R}\right)u + d} \quad ; \tag{91}$$

$$\begin{pmatrix} \underline{a}_I & \underline{b}_I \\ \underline{c}_I & \underline{d}_I \end{pmatrix} = \begin{pmatrix} (R+b-l)/R & b \\ (d\mp 1)/R & d \end{pmatrix}, \tag{92}$$

with $\det(\underline{\underline{M}}_I) = \underline{a}_I \underline{d}_I - \underline{b}_I \underline{c}_I = (dR \pm b - dl)/R$; (93)

and

$$\begin{pmatrix} \underline{a}_{II} & \underline{b}_{II} \\ \underline{c}_{II} & \underline{d}_{II} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{\underline{c}_I u + \underline{d}_I}{\underline{a}_I u + \underline{b}_I} - \frac{1}{u} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{(d\mp 1)/R}{\left(\frac{R+b-l}{R} + \frac{b}{u}\right)} + \left[\frac{d}{\left(\frac{R+b-l}{R} + \frac{b}{u}\right)} - 1\right] & 1 \end{pmatrix}, \tag{94}$$

with $\det(\underline{\underline{M}}_{II}) = \underline{a}_{II} \underline{d}_{II} - \underline{b}_{II} \underline{c}_{II} = 1$. (95)

Where, R is the radius of the array ; a, b, c, d and l are ray transfer matrix elements and geometrical thickness of individual element and the upper and lower of "$\mp$" and "$\pm$" they apply to forward-going and backward-going arrys, respectively.

[31] Wang Shaomin, J. Hangzhou University (in Chinese) 11 (1984) 79.

[32] Wang Shaomin, '83 ICL, China, Sep. (1983).

Let us select optical elements with

$$\boxed{b = 0 \ , \ d = -1} \tag{96}$$

to form a plane array ($R = \infty$), (91)-(95) will be reduced to

$$V = -u \quad ; \tag{97}$$

$$\begin{pmatrix} \underline{a}_I & \underline{b}_I \\ \underline{c}_I & \underline{d}_I \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \text{with } \det(\underline{\underline{M}}_I) = -1 \ ; \tag{98}$$

and

$$\begin{pmatrix} \underline{a}_{II} & \underline{b}_{II} \\ \underline{c}_{II} & \underline{d}_{II} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2/u & 1 \end{pmatrix}, \quad \text{with } \det(\underline{\underline{M}}_{II}) = 1 . \tag{99}$$

They are identical with that of DFWM. That is why corner-cube array has got pseudo-phase-conjugate property. According to condition (96), some new types of arrays who can perform approximate phase conjugation which have been offered. One of them is GRIN fiber array [33], and other is bead array [34] and a simple demostration was illustrated in [3]. Some pictures of these arrays are given in Fig. 15.

If we examine these phenomena in detail, the individual element sizes will introduce additional aberrations [35]

[33] Wang Shaomin, Zhou Guosheng, Wu Meiying, Peng lianhui & Tian Lijuan, Acta Physica Sinica (in Chinese) 32 (1983) 1357.

[34] Huang Weishai, Jian Xiuming, Chen Yingli, Zhao Jiaju, Zu Jinmin & Wang Shaomin, Appl. Lasers (in Chinese) 3 (1983) 5:27.

[35] Wang Shaomin, Topical Meeting on GRIN Systems, US. Apr. (1984).

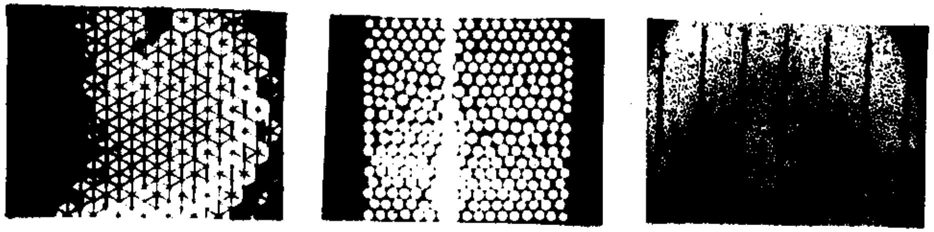


Fig. 15 Some arrays who can perform approximate phase conjugation

$$\Delta \underline{r}_{i2} = \mp\sigma\sqrt{1-\varepsilon_i'^2}\left\{(au+b)-\left[\left(\frac{R+b}{R}-\frac{l}{\ }\right)u+b\right](cu+d)\Big/\left[\left(\frac{d\mp1}{R}\right)u+d\right]\right\}\Big/u. \qquad (100)$$

In case of (96), it will be reduced to

$$\boxed{\Delta \underline{r}_2 \approx \mp\sigma(cu-2)} \qquad (101)$$

where, 2σ is individual element size. If σ is too small, diffraction effects will be produced. Which are shown in Fig. 16.

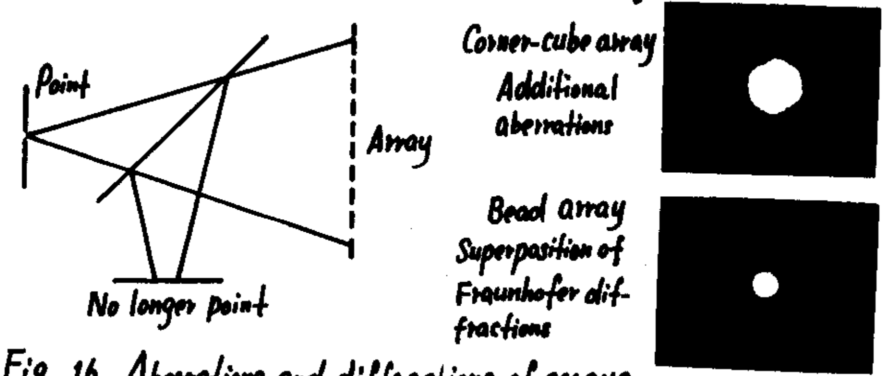


Fig. 16 Aberrations and diffractions of arrays

According to generalized Fresnel number [36]

$$N = \frac{\sigma^2}{\lambda}\left(\frac{A}{B}+\frac{1}{\rho}\right). \qquad (102)$$

If the individual element size is small, the number of the elements is large and an image system is followed the array; a clear imaging could be got, and the distortions could be compensated. An example is given in Fig. 17.

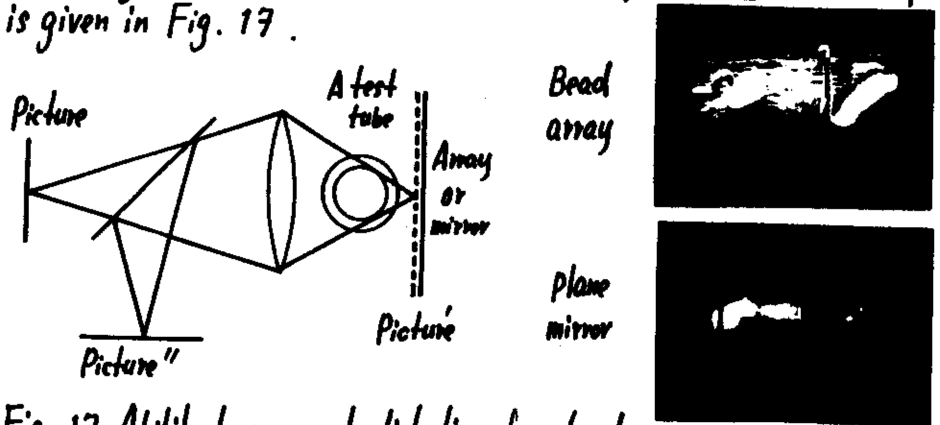


Fig. 17 Ability to compensate distortions for a bead array

As mentioned before, some specified arrays as pseudo-conjugators, they are not real phase conjugation. However, The arrays have significant potential advantages with respect to nonlinear phase conjugation, they are passive elements, the return frequency always equals the input frequency, they have larger areas and arbitrarily weak signals can be conjugated besides lower weight, lower cost

[36] Fan Dianyan, Acta Optica Sinica (in Chinese) 3 (1983) 319.

and simplicity.

Acknowledgment

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