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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

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EXPERIMENTAL FEATURES OF OPTICAL BISTABILITY

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Optical Bistability (OB) is a very young subject, the first experiment dating to 1976 (1). However it is a very lively subject, spread into different directions and it is very difficult to make a complete overview of the experimental developments. It should be noticed that the experiments of Optical Bistability have been reviewed in 1982 by Abraham and Smith (2), and in two Conferences devoted to this topic (3,4). In these lectures some experiments will be described to represent the most important features of this research. Of course the choice of the material here presented is very personal and several important experiments very likely will be not presented.

First of all some experimental effort has been devoted to the investigation of the phenomenon itself. In order to obtain optical bistability a threshold value for the cooperative parameter $C = \alpha LF/2\pi$ should be reached. Here α is the absorption coefficient of the medium under investigation, L the length of the absorbing sample and F the finesse of the cavity containing the medium under investigation. The first experiment of OB (1), as well a large majority of the following ones, have been performed on the resonance transitions of sodium that present an extremly large absorption coefficient. An important problem to be controlled in the experiments was the comparison with the theory for the threshold values of the cooperative C number and of the input power required to reach the bistability region. A careful comparison was performed by Kimble and coworkers (5) in an experiment involving the absorption oa a sodium sample composed by five primary beams and two secondary beams. Through optical pumping of the sodium beams with circularly polarized light the atoms were prepared in a single state and a two-level system was investigated. In this system the difficulties encountered in earlier works were to a large extent overcome, and a nearly homogeneously broadened absorber was interacting with the travelling wave laser beam. The sodium system had a maximum αL absorption equal to 1.5, and a finesse F=210 so that the cooperative parameter reached the value C=40. When a small inhomogeneous broadening due to the residual Doppler broadening was taken into the theoretical description, the agreement among the experimental data for the absorptive bistability and the theoretical predictions resulted very good.

A similar test of the threshold values for the input power and for the cooperative parameter C was performed in an absorptive bistability experiment on a microwave absorber by Gozzini, Maccarrone and Longo /6/. The ammonia inversion line at 23.9 GHz was employed in the experiment. Owing to the high finesse of the microwave cavity, the threshold value for the cooperative number C could be reached even if

the absorption of the sample is smaller than in the sodium experiment discussed above. In the comparison of the experimental observations with a theoretical model for the optical bistability in an homogeneous absorber, it turned out that a good theory-experiment agreement could not be obtained for the input power values of the swith-up and switch-down operations. On the contrary a good agreement could be obtanined for the ratio of these powers. A disagreement existed also for the measured minimum value of the cooperative parameter required to have a bistability diagram.

The microwave observations were determinant in showing the influence of the radial distribution of the electromagnetic field inside the cavity /7/. Because the saturation level of the absorber varies along the radius of the optical cavity, the overall optical bistability behaviour is modified: the threshold value for the input power required to reach the bistability region is increased by two or three times. As a function of the mixed Fresnel number defined as $\bar{F}=\pi w_o^2 / \lambda L$, the bistability region is greatly reduced at low Fresnel numbers /8/. The investigations on the sodium atoms /3/ and on the ammonia microwave transition /6/ presented above for the investigation of the threshold values, were performed inside a cavity with a Gaussian distribution of the field. For the sodium experiment in a ring cavity with a travelling wave laser the minimum C_{cri} value in order to have a bistability resulted C_{cri} =8, in good agreement with the theoretical predictions. For the experiments in sodium /10/ and in ammonia /6/ with a standing wave laser the measured C_{cri} was larger than the value predicted by the theory.

Another fundamental feature of the optical bistability phenomena is the time required to obtain a switch of the system from the lower to upper operation branch of the bistability diagram, or viceversa. Optical bistability is a phase transition phenomenon and near the transition points of the phase diagram a critical slowing down occurs, so that the response time of the optical system becomes very long. For instance for the operation of switching on the input power in a bistable system initially in the no transmission state, the theoretical analysis of Bonifacio and Lugiato has shown that a delay T_d exists between the time of switching on for the incident field and the time of rise for the output power. This delay time depends on the value of the incident power P_o , or more precisely on the distance $\Delta P = P_o - P_1$ between the incident power P_o and the minimum power P_1 required to make a transition to the upper branch. For large values of ΔP the delay time is equal to the transit time of the electromagnetic field inside the cavity. For small values of ΔP the delay time becomes very large. Such a

critical slowing down of the delay time has been observed in several experiments involving optical bistability in different systems (11-13,5). An important result from those experiments is the universality of the critical exponent relating the delay time to the parameter externally switched in the experiment. The delay time resulted proportional to $(\Delta P)^{-0.5}$ in an experiment where the input power was varied /5/, and resulted proportional to $(\Delta l)^{-0.5}$ in experiments where the length of the cavity was modified/13/.

The experiments where the critical slowing down in the delay time was measured, exhibited strong fluctuations in the output power of the bistable system. In effect fluctuations and instabilities represent another important feature of the phase transition phenomena, amd large attention is now devoted to these features also in the bistable devices. Quantum fluctuations produced by the presence of the spontaneous emission are a fundamental limitation in the operation of bistable systems. However their contributions may be neglected in most experiments, unless a specific experimental set-up is devised. On the contrary the thermal fluctuations due to the thermal photons, as well the amplitude and phase fluctuations of the incident field may play an important role. For instance the effect of amplitude noise in absorbitive optical bistability was desribed theoretically through a Fokker-Planck equation /14/. It resulted than the switching delay time undergoes considerable fluctuations and on the average is shorter than that predicted in absence of input noise. Furthermore in the critical slowing down regime, the probability distribution becomes two-peaked during the approach to the one-peaked steady state distribution in the upper branch of the phase diagram. This overall behaviour has been recently observed in experiments in sodium sample /15/.

A type of optical bistability based on self-focusing of counter-propagating laser beams has been also demonstrated /16/. Self-focusing occurs when a light beam having a nonuniform spatial profile (such as a Gaussian laser beam) and sufficient intensity propagates through a nonlinear medium having an intensity-dependent index of refraction. In the optical bistability experiments,laser radiation, tuned to the high-frequency side of the resonance transition , was focused in a sodium cell. A lens imaged the optical field on the exit face of the nonlinear sodium absorber onto a partially transmitting mirror. This mirror was aligned normal to the laser beam to provide optical feedback, and the self-focusing was produced by both the forward and backward optical waves. An aperture in front of the mirror caused the backward wave to depend on the laser beam spot size at the exit face of the cell. The power transmitted through the mirror was monitored. Experimental curves of

bistability were observed with input power in the 100 mW range; switching times in the 20 μsec range were obtained. Similar self-focusing experiments have been performed also on solid absorber samples/17/.

For what concern the optical bistability in solids, the first observations of optical bistability in a non-linear Fabry-Perot interferometer fashioned from parallel sided crystals have been made on InSb sample /18/ and GaAs films /19/. The required optical feedback is provided very simply by using the natural reflection of the crystal surface (for instance 0.36 for Insb) or by using reflection coatings. The experiments on InSb were first at cryogenic temperature, later at 77 °K, making use of CO laser lines. The nonlinear effects are due to band-gap resonant saturation of states immediately above the energy gap where the carrier excitation is presumed to be from band-tail absorption processes. The cw holding power in InSb samples ranges from 10 W/cm^2 to 300 W/cm^2 in crystal resonators with few hundred micrometers thickness. Room temperature InSb bistability has been also obtained making use of two-photon absorption of CO_2 lasers. The peak intensity power required in these experiments (~ 100kW/cm) was obtained from a TEA CO_2 laser so that only a transient optical bistability could be observed. Then the relation between instantaneous incident and transmitted intensities, showing the hysteresis cycle, could be reconstructed after the experimental observations were completed.

In GaAs etalons the optical bistability was observed in the 5 to 120 °K temperature range by using 819.9 nm radiation, 10-25 nm longer than the wavelength of the free-exciton peak /19/. The required cw holding-on intensity of order 20 kW/cm² was obtained by focusing 200 mW input radiation on a 10 μm diameter spot. The GaAs device was turned on or off in a 2 nsec time. The energy required to make the tramsition is extimated 4 nJ, the product of 200 mW power and 2 nsec switching time, but only a fraction of this energy was actually absorbed.

Progress towards practical all-optical switching and signal processing devices depends crucially on finding suitable non-linear materials. Considerable attention is now reserved to the so called quantum wells structure where GaAs carriers are confined in a 100 Å thick region by spacing with GaAlAs carriers/20/. The binding energy of the exciton confined in the 100 Å layer is increased and the room temperature spectrum of these quantum well structures presents the excitonic peaks. On the contrary in the GaAs crystals at room temperature the excitonic resonances are smeared out because of the thermal broadening of the resonances. Another benefit of the clear excitonic resonance in the multiple quantum well appears in the nonlinear

absorption behaviour: the non-linear absorption starts at powers at least ten times lower in the multiple quantum wells than in the GaAs crystals. Then holding and switching power are becoming very promising for the applications of these devices.

Finally, more for the investigation of the phenomenon itself than for the possible applications, it should be remembered that optical bistability has been investigated in great detail both from the theoretical and experimental points of view, in the laser with intracavity saturable absorber. Experimentally the phenomenon is observed very easily in infrared CO_2 lasers, making use of strong molecular absorbers /21/. The laser may operate in a cw regime, in a regime with output periodically modulated in time,and may present simultaneously bistability. Theoretically the laser with saturable absorber has been often used as a case where new analytical or numerical solutions of bistability features have been tested.

References

/1/ H.M. Gibbs, S.L. Mc Call and T.N.C. Venkatesan, Phys. Rev. Letts. 36, 1136 (1976)

/2/ E.Abraham and S.D. Smith, Rep. Progr. Phys. 45, 815 (1982)

/3/ "Optical Bistability", ed.by C.M. Bowden, M. Ciftan and H.R.Robl (Plenum Press, New York, 1981)

/4/ "Optical Bistability II ", ed. by C.M.Bowden, H.M. Gibbs and S.L.McCall (Plenum Press, New York, 1984)

/5/ A.T. Rosenberg, L.A. Orozco and H.J. Kimble, in" Fluctuations and Sensitivity in Non-equilibrium Sysyems", ed. by W. Horsthemke, D. Kondepudi, Lectures Notes in Physics, (Springer Verlag, Berlin, 1984)

/6/ A. Gozzini, F.Maccarrone and I. Longo, Nuovo Cimento D1, 489 (1982)

/7/ E. Arimondo, A. Gozzini, L. Lovitch and E. Pistelli, in ref. /3/ pag. 151

/8/ J.V. Moloney, M. Sargent III and H.M. Gibbs, Opt. Commun 44, 289 (1983)

/9/ R. Bonifacio and L. Lugiato, Opt. Commun. 19, 172 (1976)

/10/ D.E. Grant and H.J. Kimble, Opt. Commun. 44, 415 (1983)

/11/ S. Barbarino, A. Gozzini, F. Maccarrone, I. Longo

and R. Stampacchia, Nuovo Cimento B71, 183 (1982)

/12/ F. Mitschke, R. Deserno, J. Mlynek and W Lange, Opt. Commun. 46, 135 (1983)

/13/ S. Cribier, E. Giacobino and G. Grynberg, Opt. Commun. 47, 470 (1983)

/14/ G. Broggi and L.A. Lugiato, Phys. Rev. A29, 2949 (1984)

/15/ W.Lange, private communication (1984)

/16/ J.E. Bjorkholm, P.W. Smith, W.J. Tomlinson and A.E. Kaplan, Opt. Lett. 6, 345 (1981); J.E. Bjorkholm, P.W. Smith and W.J. Tomlinson,IEEE J. Quantum Electron. QE-18, 2016 (1982)

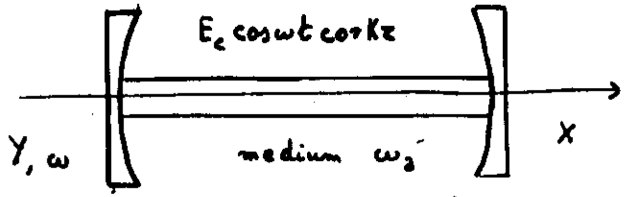
/17/ M. Dagenais and H. Winful, in ref. /4/, pag. 267

/18/ D.A.B. Miller, S.D. Smith an A.Johnston, Appl. Phys. Lett., 35, 658 (1981);S.D. Smith and F.A.P. Tooley, in ref. /4/, pag. 215

/19/ H.M. Gibbs, S.L.McCall, T.N.C. Venkatesan, A.C. Gossard, A. Passner and W. Wiegman, Appl. Phys. Lett. 35, 451 (1979); J.L. Jewell, S.T. Tarng, H.M. Gibbs, K. Tai, D. A. Weinberger, S. Ovadia, A.C. Gossard, S.L. McCall and A. Passner, in ref./4/, pag. 223

/20/ D.A.B. Miller, D.S.Chemla, A.C. Gossard and P.W. Smith, in ref. /4/ pag. 223

/21/ E. Arimondo, B.M. Dinelli and E. Menchi, in ref. /4/, pag. 317



In the saturable absorber the absorbed power is

$$P_a = \alpha_a \frac{c}{4\pi} E_c^2$$

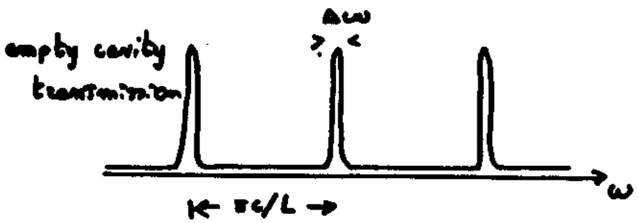
where

$$\alpha_a = \frac{\alpha_0}{1 + \left(\frac{\omega - \omega_a}{\gamma_\perp}\right)^2 + \frac{\mu^2 E_c^2}{\hbar^2 \gamma_\perp \gamma_\parallel}} = \frac{\alpha_0}{1 + \left(\frac{\omega - \omega_a}{\gamma_\perp}\right)^2 + \frac{I_c}{I_s}}$$

with $\alpha_0 = \frac{2\pi \omega_a \mu^2 N}{\hbar \gamma_\perp}$; $I_s = \frac{c}{4\pi} \frac{\hbar^2 \gamma_\perp \gamma_\parallel}{\mu^2}$

Threshold conditions on C, Y

$$C = \frac{\alpha_0 L}{2\pi} F > 4 \qquad \frac{F}{2\pi} = \frac{1}{T}$$



$$F = \frac{\pi c}{L \Delta\omega}$$

quality factor $Q_0 = \frac{\omega}{\Delta\omega}$

at resonance $\omega = \omega_a$

$$I_t = I_i \frac{T_0}{\left(1 + Q_0 \frac{\langle\alpha\rangle \lambda}{2\pi}\right)^2} \quad \leftarrow \text{transmission empty cavity}$$

$$= I_i \frac{T_0}{\left(1 + \frac{Q_0 \alpha_0 \lambda}{2\pi} \frac{1}{1 + \frac{\langle\mu\rangle^2}{3\hbar^2} \frac{E_c^2}{\gamma_\parallel \gamma_\perp}}\right)^2}$$

but $I_t = \frac{c}{4\pi} E_c^2 \frac{A}{Q_2}$ ← area; Q_2 ← output coupling factor

$$\Rightarrow T_0 I_i = I_t \left(1 + \frac{Q_0 \alpha_0 \lambda}{2\pi} \frac{1}{1 + \frac{4}{3} \frac{\langle\mu\rangle^2}{\hbar^2} \frac{\pi}{c} \frac{1}{\gamma_\perp \gamma_\parallel} \frac{Q_2}{A} I_t}\right)^2$$

state equation

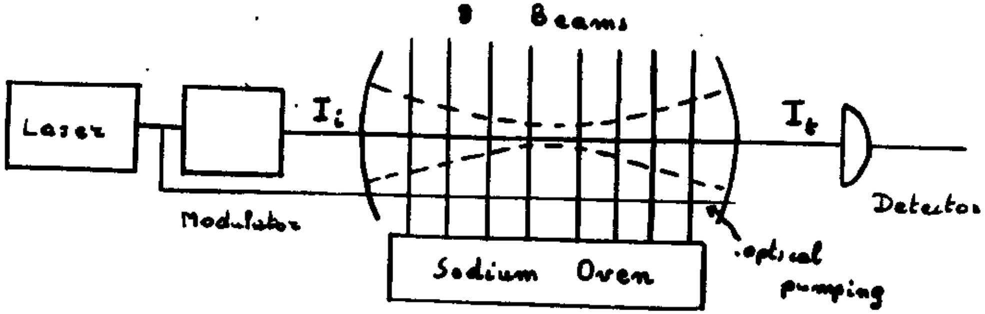
$$Y = X \left(1 + 2C \frac{1}{1 + aX}\right)^2$$

$Y = T_0 I_i$ $\qquad X = I_t$

$2C = \frac{Q_0 \alpha_0 \lambda}{2\pi}$ $\qquad a = \frac{4}{3} \frac{\langle\mu\rangle^2}{\hbar^2} \frac{\pi}{c} \frac{1}{\gamma_\perp \gamma_\parallel} \frac{Q_2}{A}$

Na absorptive bistability

(Grant & Kimble, Opt. Commun. 44 415, 1983)

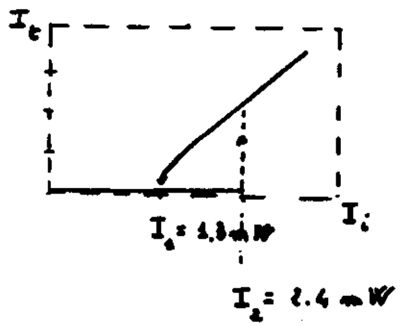


In the experiments $\alpha_0 L$ between 0 and 1.5

$F \simeq 400$ and C between 0 and 50

$L = 25$ cm

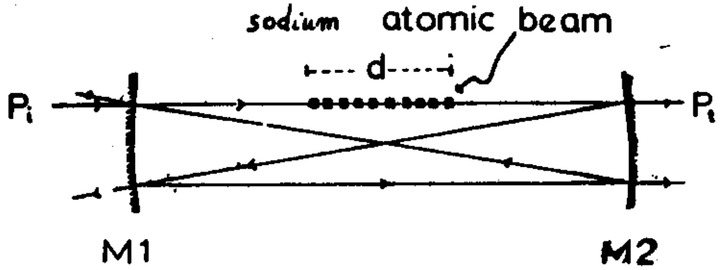
$\kappa = 4 \times 10^6$ sec^{-1} ; $\gamma_\parallel = 6 \times 10^7$ sec^{-1} ; $\gamma_\perp = 3 \times 10^7$ sec^{-1}

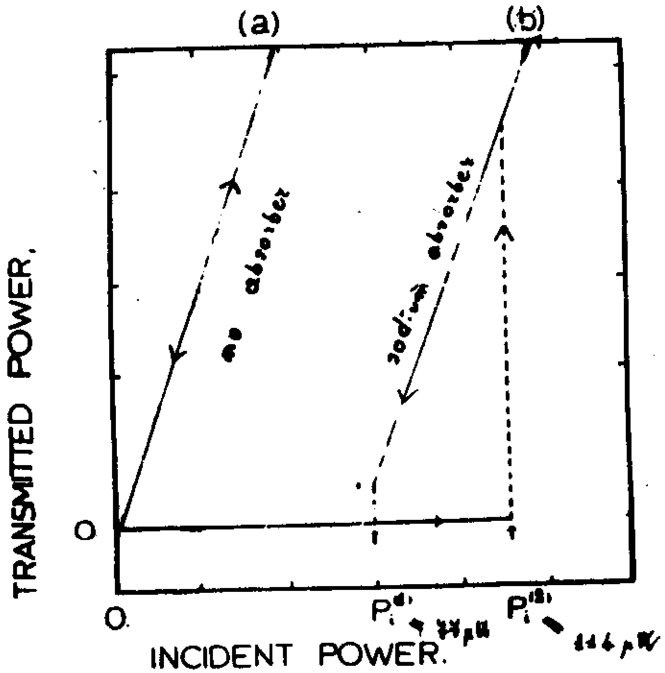


four times higher than predicted

& C_{eff} = 19 - 22 higher than predicted

Ring cavity experiment by Kimble et al. 1984



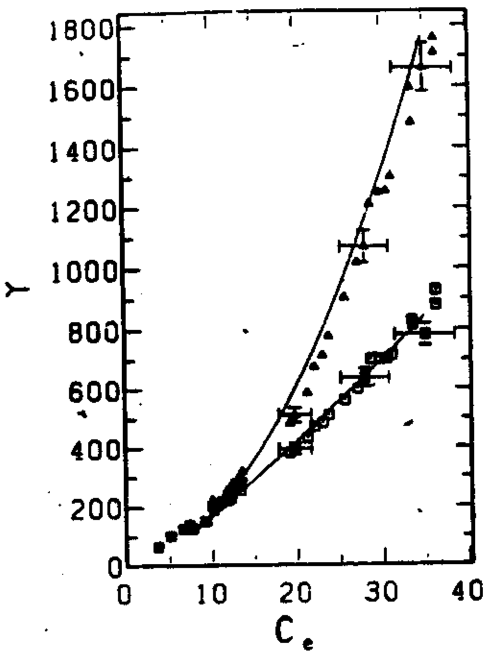


Microwave "optical bistability"

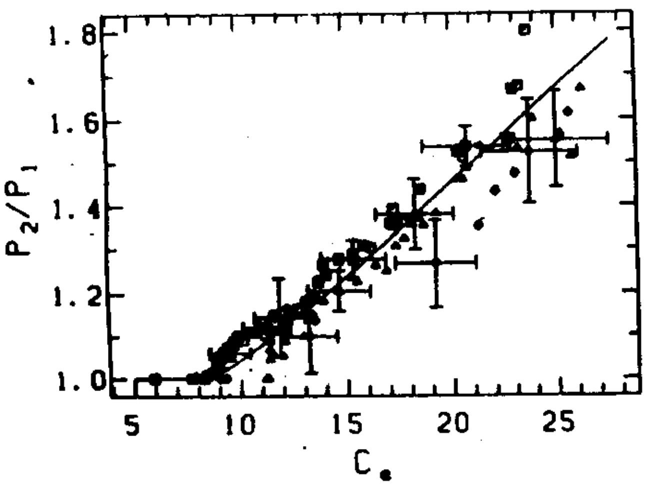
[Arimondo, Gozzini, Pistelli and Lovitch
Proc. Conf. Optical Bistability, 1981]

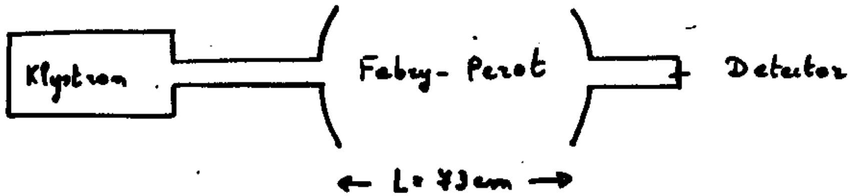
$Y = \frac{P_1}{I_s}, \frac{P_2}{I_s}$

scaled incident switching powers



C_e = effective cooperative number

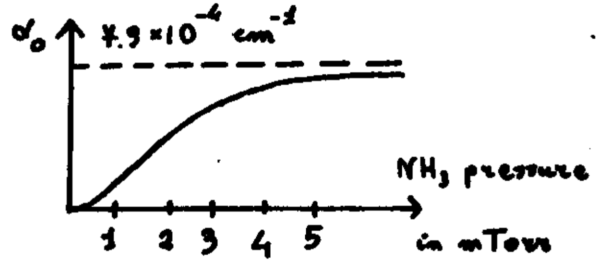




F ≈ 4300

(J = 3, K = 3) inversion line of NH_3 at 23.9 GHz

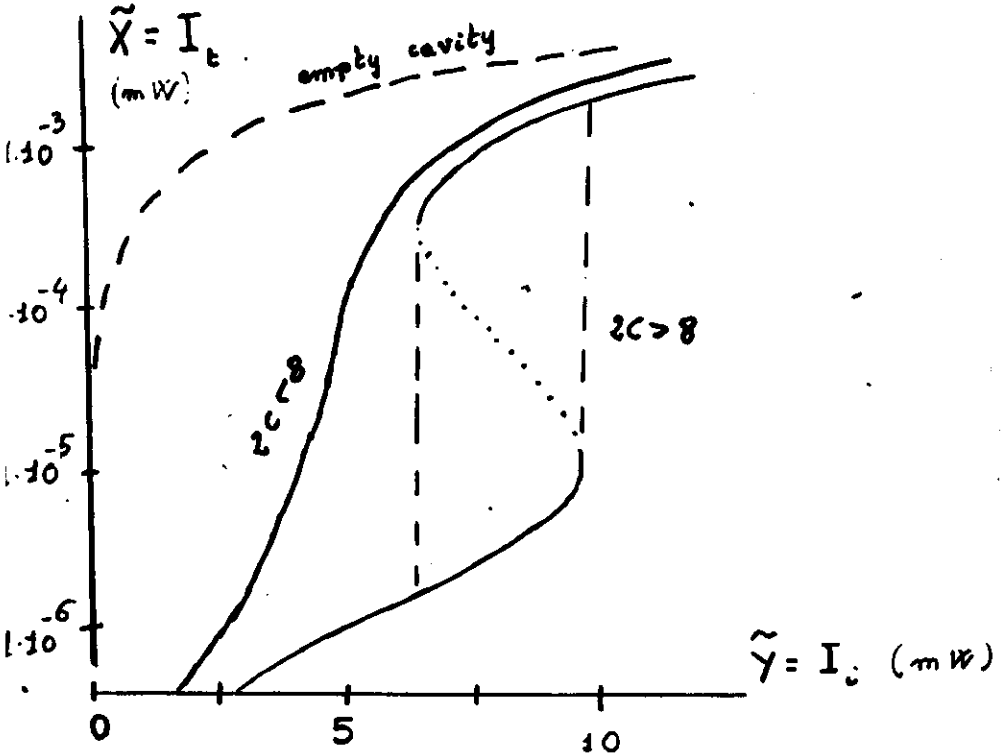
absorption coefficient



⟹ $\alpha_0 L$ between 0 and 0.06

C between 0 and 40

Absorptive bistability : $\omega = \omega_a$



threshold conditions: on $\tilde{Y} = I_i$

$$\text{on } 2C = \frac{\Omega_0 \alpha_0 \lambda}{2\pi} \gtrless 8$$

Absorptive optical bistability

(Gozzini et al., Nuovo Cimento 1D 489 (1982))

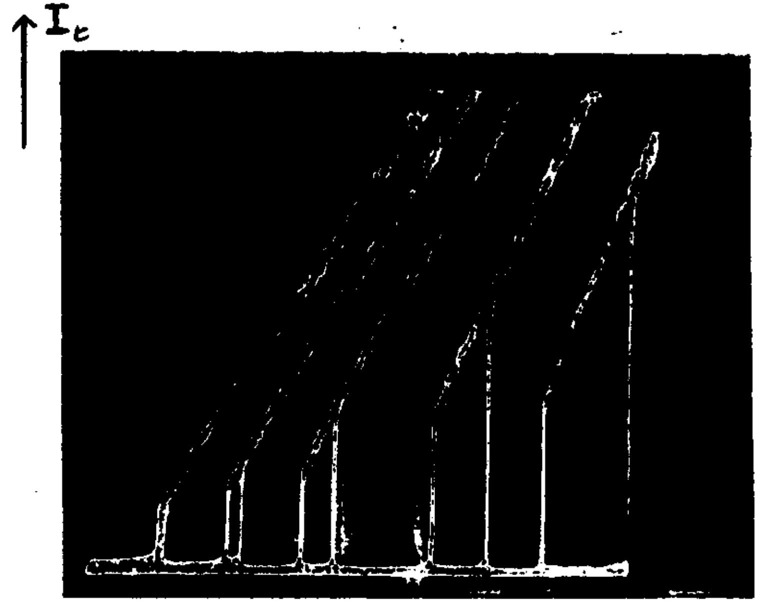


Fig. 6. – Cycles of P_t vs. P_i obtained at different gas pressures. The area enclosed into the cycles diminishes with the gas pressure.

I_1 I_2 larger gas pressure I_i

smaller gas pressure

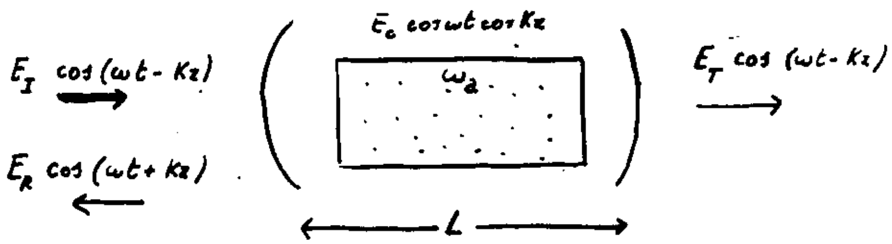
Threshold values for the input power I_1, I_2

1.5 times higher than predicted

$2C_{th} = 25 \pm 2$

$K = 3 \times 10^6 \text{ sec}^{-1}$ $\quad \gamma_\parallel = \gamma_\perp = 2 \times 10^6 \text{ sec}^{-1}$

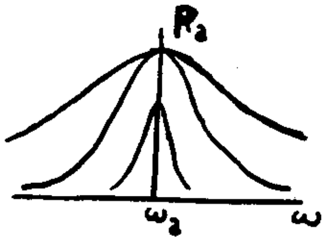
at 1 mTorr pressure



$$\frac{\lambda_R}{2} = \frac{L}{P} \qquad \text{modo } TEM_{oop}$$

$$P_a = \alpha_a \frac{c}{4\pi} E_c^2$$

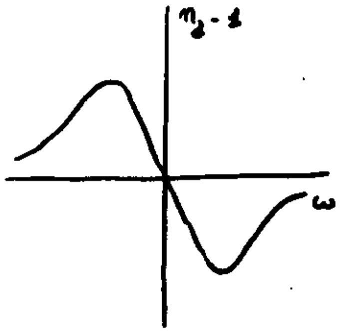
$$\alpha_a = \frac{\alpha_0}{1 + \left(\frac{\omega - \omega_a}{\gamma_\perp}\right)^2 + S}$$



$$S = \frac{\langle \mu^2 \rangle E_c^2}{3\hbar^2 \gamma_\parallel \gamma_\perp}$$

risonanza a $\frac{\omega_R}{n_a}$

$$n_a = 1 - \frac{\lambda}{4\pi} \frac{\omega - \omega_a}{\gamma_\perp} \alpha$$



Dispersive bistability

ω = e.m. field frequency

ω_0 = cavity resonance

ω_a = absorber resonance

$$\langle \alpha \rangle = \frac{\alpha_0}{1 + \left(\frac{\omega - \omega_a}{\gamma_\perp}\right)^2 + \frac{\langle \mu \rangle^2}{3\hbar^2} \frac{E_c^2}{\gamma_\perp \gamma_\parallel}}$$

$$\langle n \rangle = 1 - \frac{\langle \alpha \rangle \lambda}{4\pi} \frac{(\omega - \omega_a)}{\gamma_\perp} \quad \leftarrow \text{refractive index}$$

$$I_t = I_i \frac{T_0}{\left(1 + \frac{\Omega_0 \langle \alpha \rangle \lambda}{2\pi}\right)^2 + \left[2\Omega_0 \frac{(\omega - \omega_0/\langle n \rangle)}{\omega_0/\langle n \rangle}\right]^2}$$

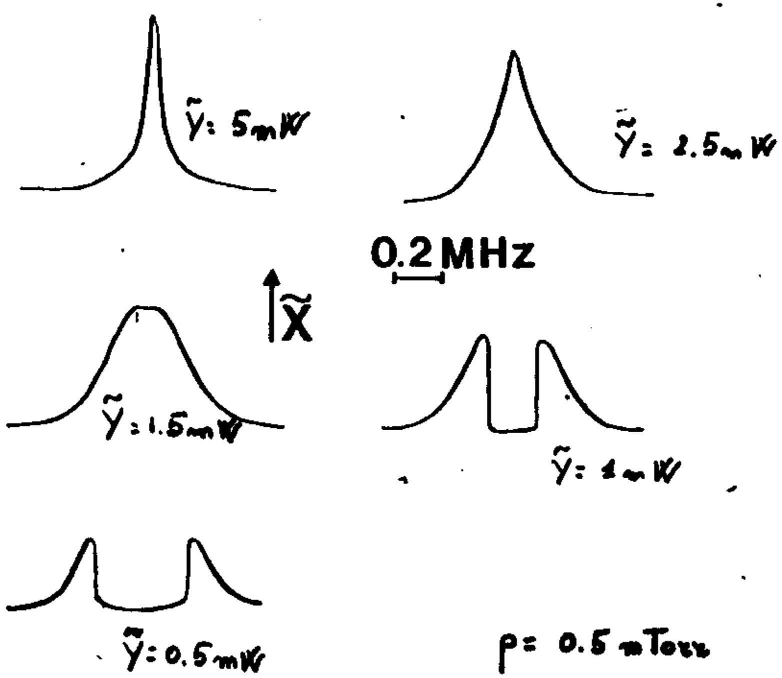
State equation

$$Y = X \left\{ \left(1 + \frac{2C}{1 + \Delta^2 + 2X}\right)^2 + \left(\Theta - 2C\Delta \frac{1}{1 + \Delta^2 + 2X}\right)^2 \right\}$$

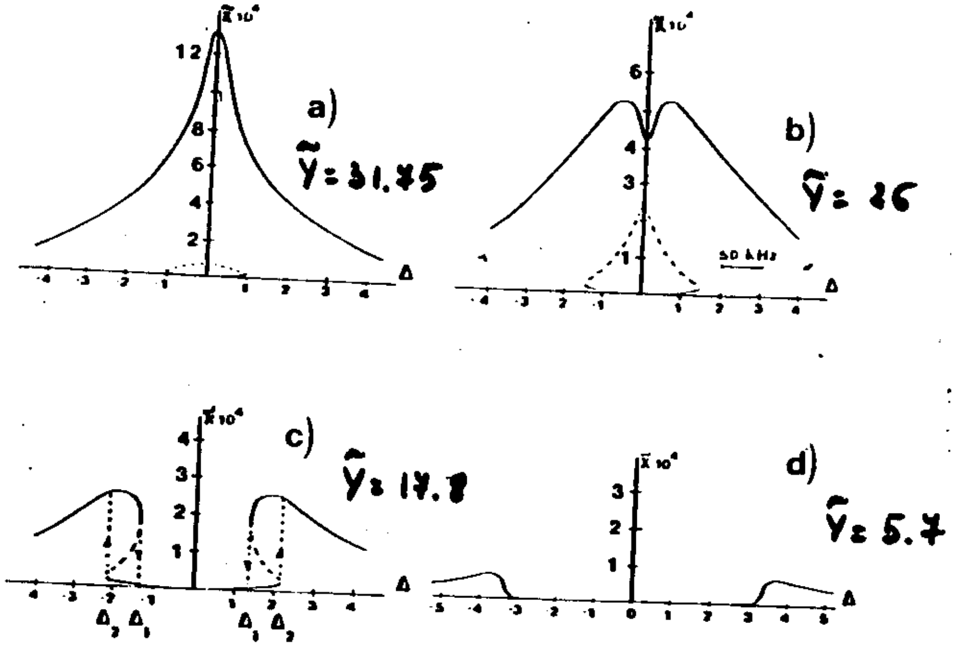
$$\Theta = \frac{2\Omega_0}{\omega_0}(\omega - \omega_0) \quad ; \quad \Delta = \frac{\omega - \omega_a}{\gamma_\perp}$$

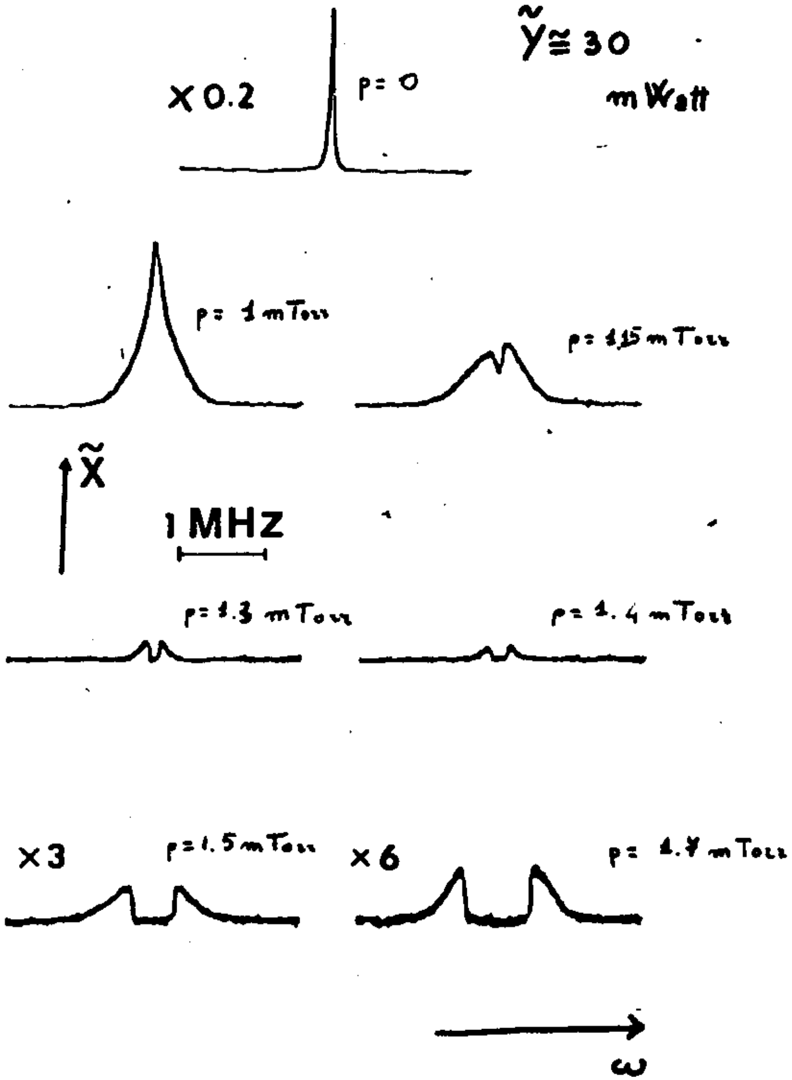
Experiments with

$\omega_0 = \omega_a$; ω varied


Ỹ: 5mW
Ỹ: 2.5mW
0.2MHz
X̃
Ỹ: 1.5mW
Ỹ: 1mW
Ỹ: 0.5mW
p= 0.5 mTorr

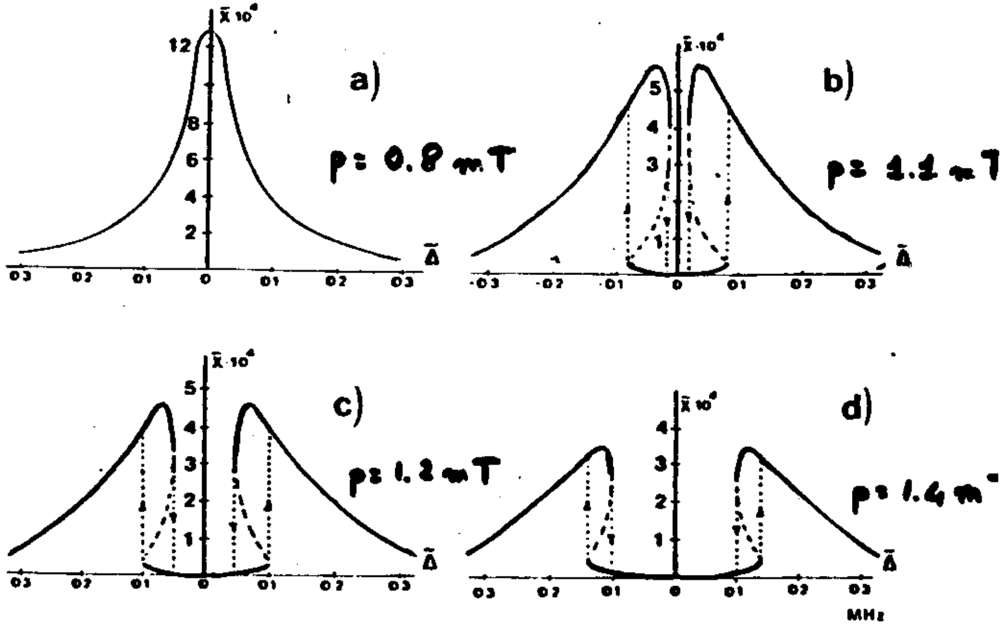
p: 1 mTorr


a)
Ỹ= 31.75
b)
Ỹ= 26
50 kHz
c)
Ỹ= 17.8
Δ₂ Δ₁
Δ₁ Δ₂
d)
Ỹ= 5.7



Dispersive bistability

$\tilde{Y}$ = 31.75 mW



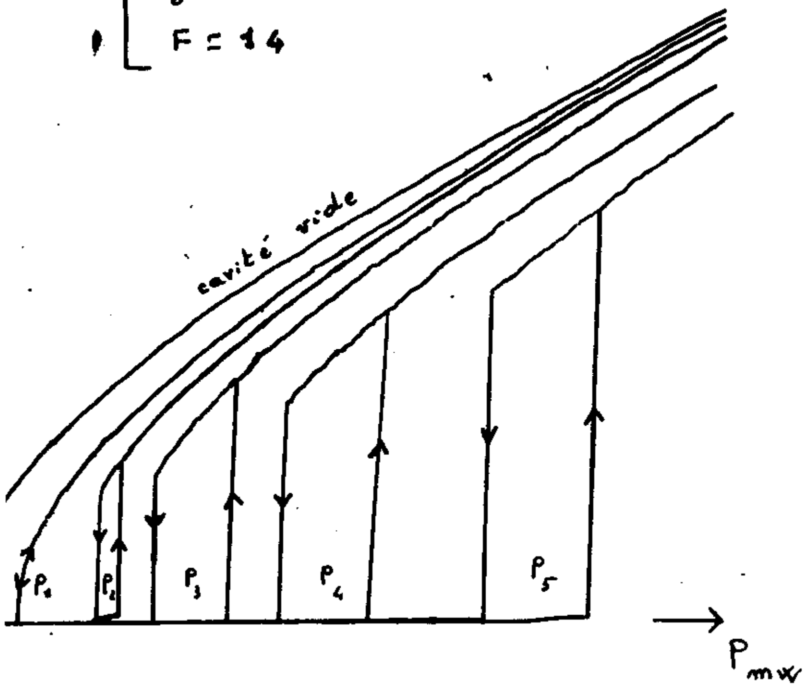
$\omega_c = \omega_a$

Bistabilité absorptive à microondes

dans $DC^{15}N$

$\omega_{J=0\to J=1} = 72\ \text{GHz}$ (Segard, Lille, 1984)

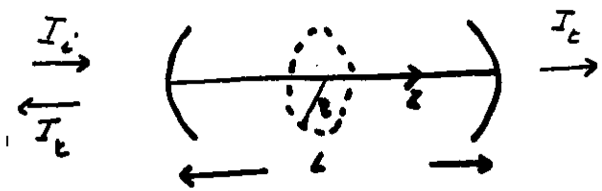
$$\begin{cases} \alpha_0 L = 720 \\ F = 14 \end{cases}$$



$P_1 < P_2 < P_3 < P_4 < P_5$

Gaussian distribution of the field

$$\langle \alpha \rangle = \int \alpha_0 \frac{E_c^2}{4\pi} dV$$



TEM$_{00p}$ mode

$$E^2(r,z) = \frac{E_0^2}{1+\xi^2} \exp\left(-\frac{A^2}{1+\xi^2} r^2\right) \sin^2 \varphi(r,z)$$

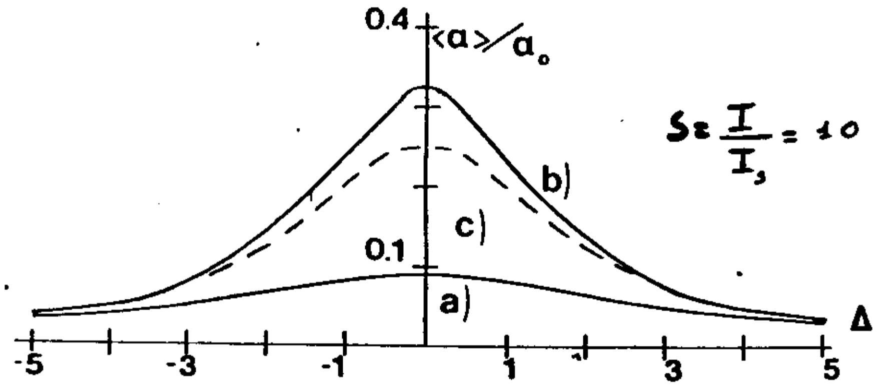
$$r = \sqrt{x^2+y^2} \qquad \xi = \frac{2z}{L} \qquad A^2 = \frac{4\pi}{L\lambda}$$

$$\langle \alpha \rangle = \alpha_0 \, \psi(\Delta, S)$$

$$\langle n \rangle = 1 - \frac{\alpha_0 \lambda \Delta}{4\pi} \psi(\Delta, S) = 1 - \frac{\lambda}{4\pi} \Delta \langle \alpha \rangle$$

$$\psi(\Delta, S) = \frac{16}{\pi S} \ln \frac{1}{2}\left(1 + \sqrt{1 + \frac{\pi}{4} \frac{S}{1+\Delta^2}}\right)$$

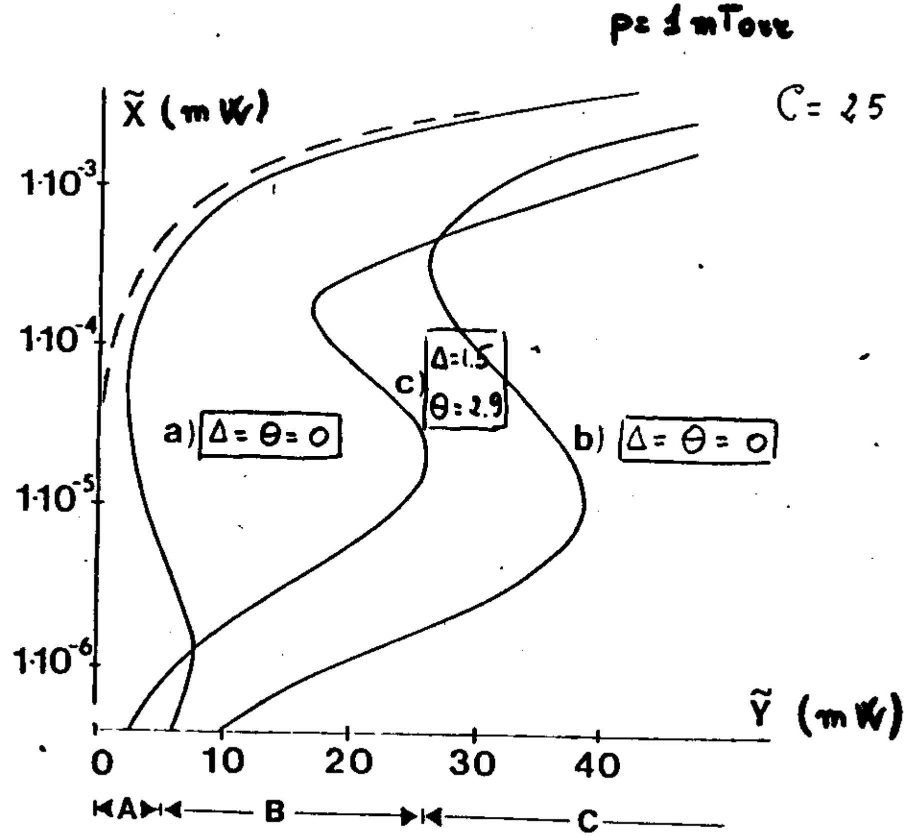
$$\Delta = \frac{\omega - \omega_0}{\gamma_\perp} \qquad S = \frac{\langle \mu \rangle^2}{3\hbar^2 \gamma_\parallel \gamma_\perp} \frac{16 Q_2}{L^2 c} I_0 = a X$$



Curva a) $\alpha = \dfrac{\alpha_0}{1+\Delta^2+S}$ } Bonifacio Lugiato

Curva b) $\langle\alpha\rangle = \alpha_0 \dfrac{16}{\pi S} \log \dfrac{1}{2}\left(1+\sqrt{1+\dfrac{\pi}{4}\dfrac{S}{1+\Delta^2}}\right)$

Curva c) $\langle\alpha\rangle = \dfrac{\alpha_0}{1+\Delta^2+\frac{3\pi}{32}S}$



$\Theta = \dfrac{2(\omega-\omega_c)}{k}$ $\Delta = \dfrac{\omega-\omega_a}{\gamma_\perp}$

Temps de transition

à la résonance, pour une cavité de haute qualité

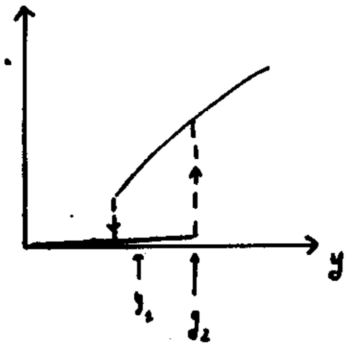
$$K << \gamma_\perp, \gamma_\parallel$$

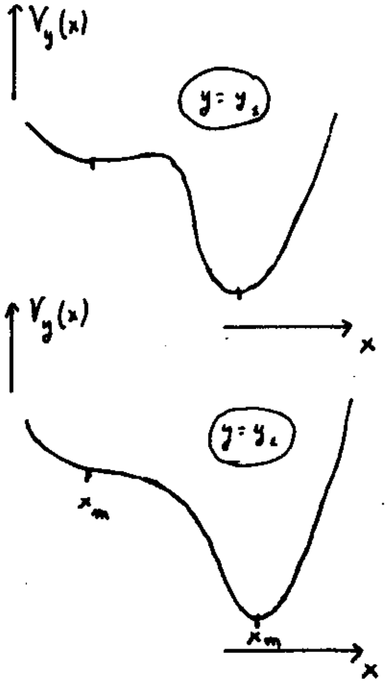
Dans les équations de Maxwell-Bloch on applique l'élimination adiabatique de la polarisation et des populations :

$$\dot{P} = \dot{D} = 0$$

On trouve une équation pour l'intensité du champ

$$\frac{1}{K}\frac{dx}{dt} = y - x - \frac{2Cx}{1+ax} = -\frac{\partial V_y(x)}{\partial x} \qquad \begin{cases} x \equiv I_t \\ y \equiv I_i \end{cases}$$





Ralentissement critique:

temps d'induction t_c

Transient and critical slowing down

in microwave experiments on ammonia

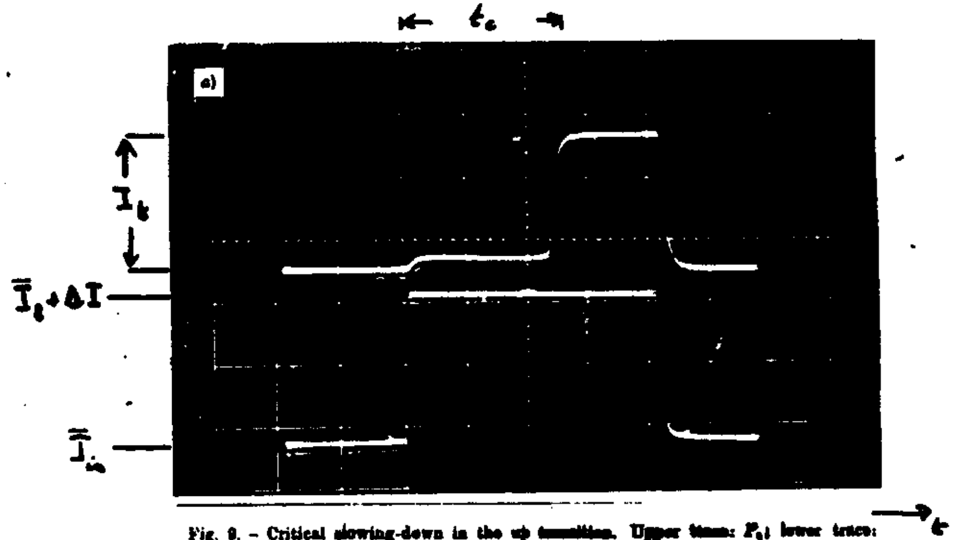
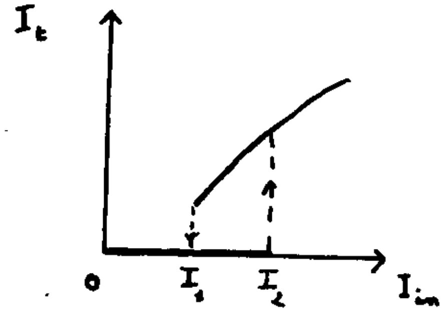
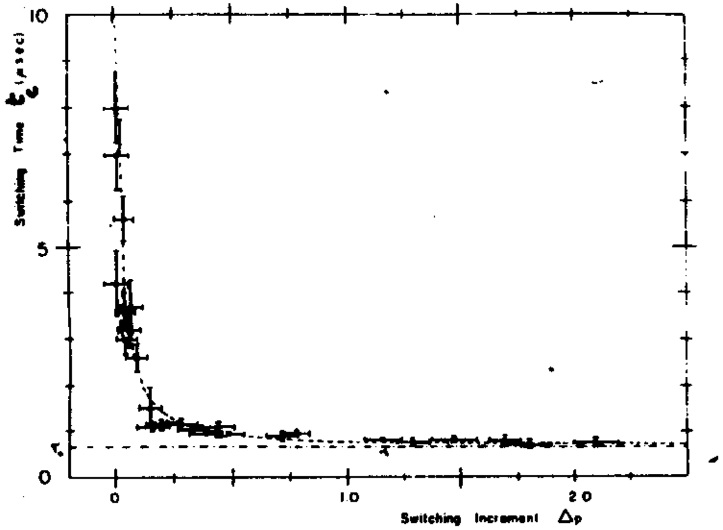


Fig. 9. – Critical slowing-down in the up transition. Upper trace: P_0; lower trace: P_1; a) time scale: 10 ms/cm, b) time scale: [illegible]



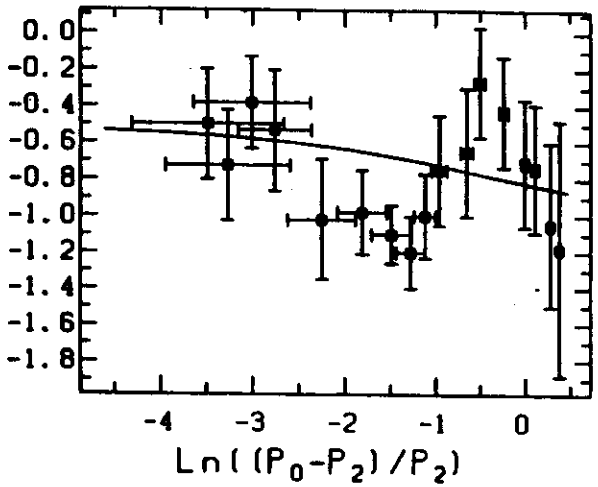
Critical slowing down in the sodium experiment by Kimble et al.



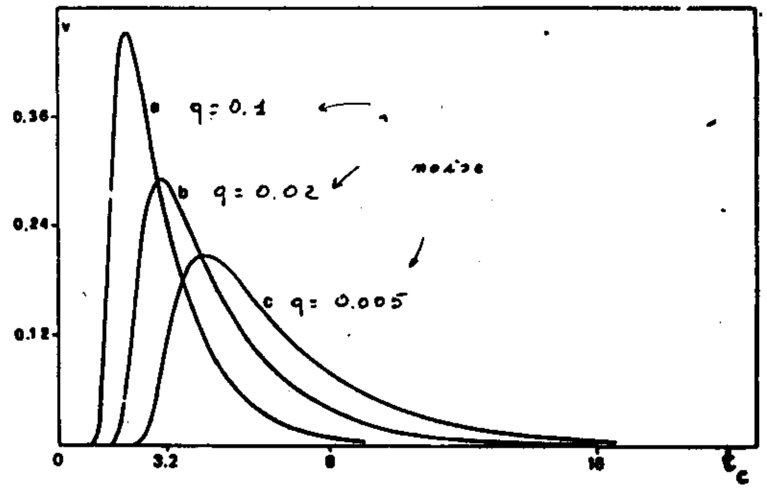
↑ input power

$$t_c \simeq \frac{1}{\Delta p^{+0.5}}$$

critical exponent ← F



Distribution of induction times



Broggi and Lugiato, Phys. Rev. A29 (1984)

Other features of
Optical Bistability observed
in sodium experiments.

1) bistability with optical pumping
= lower threshold power
by Arecchi and coworkers, Florence
by Sandle and coworkers, New Zealand.

2) tristability
by Arecchi and coworkers
by Lange and coworkers, Hannover

3) polarization switching
by Sandle and coworkers

double-peaked
probability distribution
from the Broggi and Lugiato theory

$P(x, \tau)$ ← observation times
↑
output power