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# THE RAMAN FEL

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## I. Introduction

The nonlinear physics of the FEL involves the several wave-like disturbances which are set up in a nearly cold electron beam when it passes through the undulator (or pump) field. In addition to the pump field, which is electromagnetic in the rest frame, there is the growing scattered wave which moves parallel to the electron direction. The nonlinear interaction of these two waves sets up a beat disturbance caused by the ponderomotive force, which exerts an axial force on the electron, reinforcing the growth of the scattered wave. Finally, the electron beam has itself a normal mode, the space-charge or plasma wave, which can interact with the ponderomotive wave in an important way. The type of FEL under consideration in this chapter, which we refer to as the "Raman FEL", involves the latter process. The name is drawn from Raman Lasers, where an intermediate energy-level is involved in a stimulated scattering interaction with a pump wave. This modification is not trivial, as the Raman device exhibits the desireable feature of exponential signal growth.

We can summarize the physics (Chapter II.9) appropriate to the small-signal growth problem, using the following dimensionless parameters. Defining T as $L/\gamma c\beta$, the time duration of the interaction observed in the rest frame, they are:

$$\Theta_n = [\gamma\Omega_\perp^2/2k_u c]\,T, \qquad (1)$$

which represents the strength of the pump wave [ $\Omega_\perp = eB/mc$ ];

$$\Theta_i = \omega_i T, \qquad (2)$$

which represents the idler or beat wave from the ponderomotive interaction [$\omega_i = \omega_0 - \omega$ ; $\omega_0 = \gamma k_u c$ in the rest frame]; and

$$\Theta_p = \omega_p T, \qquad (3)$$

which represents the space charge wave, or collective process [ $\omega_p^2 = 4\pi\rho e^2/\gamma m$ , the invariant plasma frequency].

If $\Theta_n >> \Theta_p$ but $\Theta_p^2\Theta_n << 1$, viz the pump wave is strong in a situation of high beam energy and low density, then we are dealing with the "two-wave" or Compton FEL. The physics can be modelled to good accuracy using a pendulum equation (see Chapter 9) and accounting for the energy converted from the beam motion into the EM fields; large-signal and saturation phenomena can be described by including the self-consistent EM wave equations. FEL gain optimises for a specific choice, $\Theta_i \simeq 2.6$ , implying a finite undulator length requirement for optimum net gain. The gain scales linearly in beam density.

In this chapter, we consider cases for which $\Theta_p \gg 1$: the undulator is therefore several plasma oscillations in length, and the beam mode has a well-defined meaning. Since $\rho \propto \gamma^3$, these effects are important at high charge density and comparatively

low energy. Dense beams imply large gain, even with the two-wave formula, although one must be careful not to use the simple pendulum model (Chapter II.5) beyond its limit. High gain suggests compact laser size, and it tends to have other favorable effects on the laser design. It turns out that there are two important categories of FEL in which dense beams figure importantly. First, if the ponderomotive wave resonates with the space charge wave, $\Theta_i \approx \Theta_p$, and if the pump wave is not too strong ( $\Theta_p > \Theta_n$ ), exponential signal growth will occur in what is called the "Raman" FEL. The growth coefficient for small signal at $\omega$ is of order

$$\left(eB_\perp/mc\right)\left(\omega_p \lambda_u f / 8\pi\gamma c^3\right)^{1/2} L, \tag{4}$$

where f is the beam filling factor; if $[\Theta_p\Theta_n/2]^{1/2} > 1$, the wave amplitude grows as

$$E_S(T) = E_S(0) \cosh \left[\Theta_p \Theta_n/2\right]^{1/2}, \tag{5}$$

where $E_S(0)$ is the amplitude of the growing mode at the undulator entry. [The actual input signal will be divided among the various modes at the FEL entrance; this is known as "coupling loss"]. Gain exceeding 100% per pass in a short (50–100cm ) undulator is not uncommon. Saturation will occur via several mechanisms, whereupon the efficiency is limited to $\lesssim \omega_p/\gamma K_u c$ for a simple undulator. Two-wave gain turns out to be almost negligible in this limit, although the Compton gain formula would



(erroneously) predict enourmous gain.

Conversely, if $\Theta_n >> \Theta_p$, but $\Theta_p^2 \Theta_n >> 1$, exponential growth still obtains with coefficient, $\frac{\sqrt{3}}{2}\left(\Theta_p^2\Theta_n\right)^{1/3}$ and $\Theta_i \approx \Theta_p$; this is called the strong pump regime or oscillating two-stream instability. Then the ponderomotive wave dominates the space-charge wave. The efficiency continues to increase in this limit, reaching a factor $\approx \frac{1}{2}\left(\Omega_\perp^2 \gamma/\omega_p K_u c\right)^{1/3}$ greater than the Raman result. However, this type of operation may be difficult to implement at low energy, owing to the velocity shear set up by the strong undulator field across a "big" electron beam ( $r_b \gtrsim 2\lambda$ ), required because of the "long" wavelength. On the other hand, it is becoming increasingly interesting in the area of X-Ray FEL research. There, one must use a very dense, energetic, but small beam moving through a strong pump field: the possibility of obtaining exponential growth is very attractive because of the difficulty with a suitable laser mirror. The point is to get a very high quality, high current density beam; current density in the range $10^5$ to $10^6$ A/cm$^2$ would open many possibilities.

What beam quality is essential for high gain and efficiency in a Raman FEL? This is an involved question, but two general conclusions have been reached. First, a cold beam is best, but since all beams have some temperature--partly due to the undulator--we must learn to live with some parallel electron velocity spread. The second conclusion is that if $\left(\frac{\delta\gamma}{\gamma}\right)_\parallel \lesssim N^{-1}$,

$N = L/\lambda_u$, then the gain of the FEL will not deteriorate significantly. This is essentially the same result as for the Compton FEL. Since N must be rather large to achieve adequate gain and coherence, a rather stringent limit is placed on the beam quality. The reult follows from the requirement that the space charge wave suffers no Landau damping, $\lambda_u/2\gamma \gtrsim 2\lambda_{Debye} = 2(V_{th}/\omega_p)$, or

$$\left(\frac{\delta\gamma}{\gamma}\right)_{\parallel} < \left(\frac{\lambda_u}{2\gamma}\right)\left(\frac{\omega_p}{2c}\right) \tag{6}$$

and since $\omega_p \cdot N\lambda_u/\gamma c \gg 1$, the $N^{-1}$ result follows. This leads to a lower bound on the wavelength of the collective FEL:

$$\lambda_s \gtrsim \left(\frac{2}{\gamma}\right)\left(\frac{c}{\omega_p}\right)\left(\frac{\delta\gamma}{\gamma}\right)_{\parallel} \approx \frac{2}{\delta N} \sim 10-100\mu, \tag{7}$$

since $c/\omega_p \gtrsim$ 1cm. In this chapter we shall therefore restrict our discussion to FELs operating in the 1cm to 0.1mm wavelength regime. A discussion of the various factors influencing beam quality will be included in section III, together with diagnostic methods for determining the beam quality of intense, relativistic

Saturation occurs in the Raman FEL when the vector potential of the scattered field is ~ $[1/2(\omega_p/\gamma k_u c)^3]^{1/2} mc^2/e$. However, one can improve the efficiency of the Raman FEL by programming the undulator field amplitude or period, using the "generalized" pendulum equation [Sprangle, 1981]. Unfortunately, there are as yet no experiments providing the wealth of definitive data on this matter, as is the case of the two-wave FEL at this time.



Using this technique, a strong EM wave is present at the FEL input. Space charge effects are important only if

$$\rho \gtrsim k_u^2 \gamma^3 A_0 A / 4\pi m c^2 = \frac{B E \gamma}{8\pi m c^2}, \tag{8}$$

where $A_0 = B/k_u$ and $A = cE/\omega$ are the vector potentials of the wave fields. At low $\gamma$, and for typical high power conditions, this critical density is of the same order as the density obtained from the requirement $\omega_p \gg 1$. Trapping of the electrons in the "buckets" of ponderomotive potential will be possible in a warm beam if

$$\left(\frac{\delta\gamma}{\gamma}\right)_{\parallel} \lesssim \frac{e}{mc^2}\sqrt{A_0 A} \tag{9}$$

which is compatible with (6) for signal strength > $10^6$ w/cm$^2$ using typical values of $k_u$, $\gamma$, and $A_0$. It is the trapping and deceleration of the bucket which permits extraction of appreciable energy from the beam: efficiency in the range of 20-50% is possible, according to numerical studies [Tang, 1981].

Early experimentation [Granatstein, 1977] found radiation at 400$\mu$ which was attributed to the stimulated backscattering of a powerful microwave from an intense, cold, relativistic beam. Theory, developed by Sprangle [1975] provided an interpretation of this result. Spectroscopic studies of superradiant power from a magnetostatic undulator were undertaken by Efthimion and Schlesinger [1977] in the microwave regime. This work was extended to millimeter wavelengths by Marshall [1977] and Gilgenbach [1979], obtaining considerable power, and identifying

the Raman and Cyclotron modes. The first proper collective FEL was operated by McDermott [1978] in an oscillator configuration which gave 1Mw at less than 1/2mm. Further spectroscopic studies by Birkett [1981], using the same configuration, showed the dispersion characteristics of the radiation obeyed the basic FEL wavelength relation. A redesign of the beam and undulator by a group at NRL demonstrated that high power and efficiency could be obtained [Parker, 1983]. Our approach in what follows will not be historical, as it has now been established what the basic physical process is. Rather, we shall provide examples which will show the direction of experimental research, and which serve as a guide to necessary design and construction procedure [Marshall, 1985].

## II. Dispersion Effects

We begin with a simple review of FEL dispersion theory. Imagine a simple case where an infinite, cold beam is guided along a magnetic field, through an undulator. The fast and slow beam space charge waves are modified by the undulator:

$$\omega_b = V_\parallel (k + k_u) \pm \omega_p/\gamma . \qquad (10)$$

The fast wave is stable, but the slow wave (negative sign) is unstable. These beam modes will intersect with the light wave, $\omega$ = kc, from which we obtain the simplified FEL equation:

$$\omega \approx 2\gamma_\parallel^2 (k_u V_\parallel - \omega_p/\gamma) \;, \quad [\text{lab frame}]. \qquad (11)$$



However, because the wavelengths in question can be rather long, it is necessary to contain them in an enclosed pipe, or drift tube, which also encloses the electron beam. Thus the dispersion of the light wave is better represented by

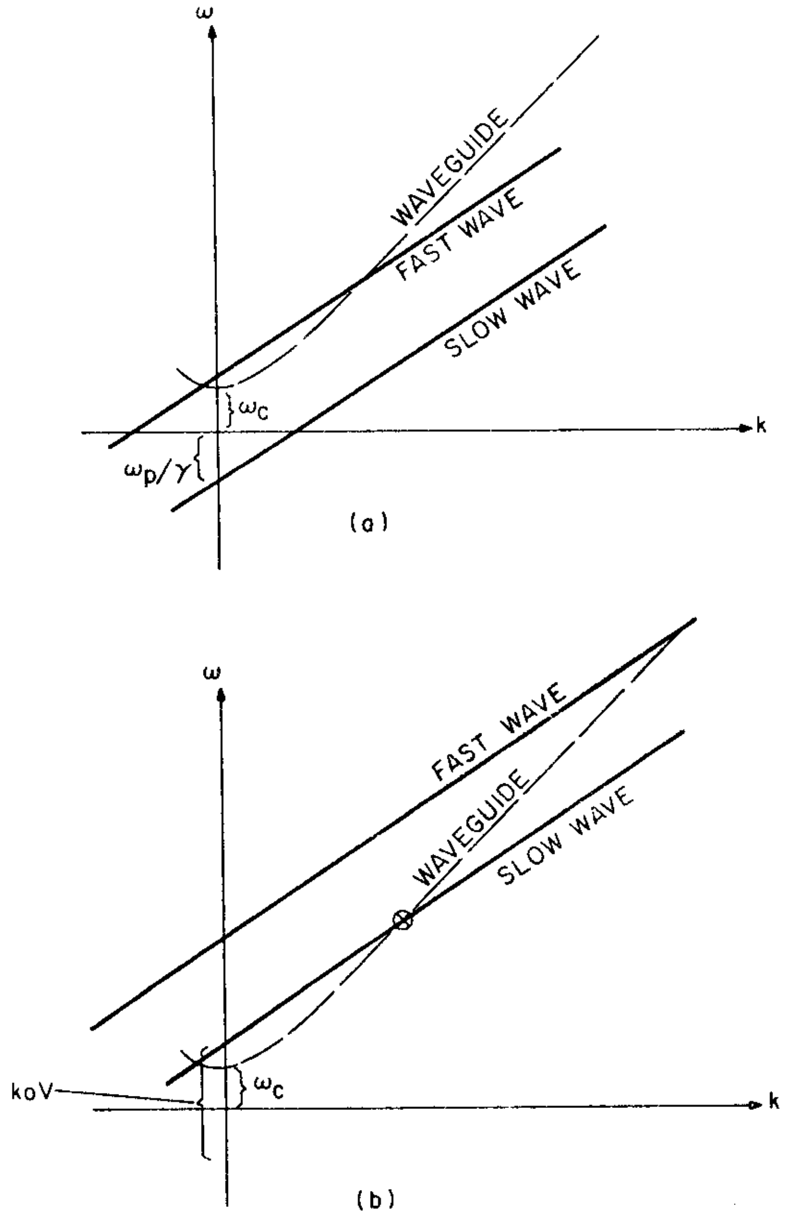
$$\omega^2 = k^2c^2 + \omega_c^2 + f\omega_p^2 , \qquad (12)$$

where $\omega_c$ is the cutoff frequency of the drift tube. The various EM modes have a spectrum of discrete cutoff frequencies, and therefore the axial wavenumber is different for each mode. If the FEL is to be a microwave device (eg a Ubitron), one or more waveguide modes could be excited. Solving eq 12 above simultaneously with the slow space charge wave in the limit of small plasma frequency gives ( $\omega_p/\omega_c \ll 1$ ):

$$\omega \approx k_u V_\parallel \gamma_\parallel^2 \{ 1 \pm \beta_\parallel^2 [1 - \omega_c^2/\gamma_\parallel^2 k_u^2 V_\parallel^2]^{1/2} \} , \qquad (13)$$

where $\gamma_\parallel^2 = (1-\beta_\parallel^2)^{-1}$. The intersection of the space charge wave with a waveguide mode is illustrated in Figure 1. The positive root in (13) corresponds to the high frequency backscattered mode, while the negative root is a microwave radiation that can propagate either with the electron flow ( if $\omega_c$ $k_u V_\parallel$ ) or against it. In the latter case, it is referred to as the absolute instability, and if $\omega_c \to 0$ , it has wavelength $\approx \lambda_u$ .

In the microwave region, one can introduce filters into the structure which will suppress all modes but the desired one. As the wavelength decreases, however, this becomes more difficult. The spectrum of oscillating modes becomes more






compressed, and it is characterized by a spread of axial wavenumbers:

$$\delta K_{\|}/K \approx \frac{1}{2}(\lambda/R)^2. \qquad (14)$$

To select the dominant mode of the system--avoiding unstable, off-axis FEL modes in the bargain-- one can "open" the Fabry-Perot resonator by separating the mirror from the drift tube with an open space. In this way, the dominant mode will have the lowest loss and will oscillate at the lowest pump threshold. In cases where the gain is not high, the reader is referred to Chapter 17 for a comprehensive discussion of resonator losses.

Interaction with the fast beam space charge mode is also possible, however no instability results. This is referred to as the Antistokes mode, whereas the slow, unstable space charge wave interaction, the Stokes mode, provides gain. If a strong signal wave is present, one can show that amplification by induced emission proceeds at the Stokes wavelength, but stimulated absorption occurs at the Antistokes frequency. An analogy with the two-wave FEL appears in the positive and negative peaks of the Compton gain. The frequency difference between the emission and absorption is

$$\Delta\omega/\omega \approx \frac{2\omega_p/\gamma}{k_u c}. \qquad (14)$$

If one considers the possibility of off-axis propagation, the scattered frequency is reduced as $\theta$, the angle relative to the backscattered direction, increases :

$$\omega \approx \frac{2\gamma_{\|}^2 k_u v_{\|}}{1+\gamma^2\theta^2}. \qquad (15)$$

The growing FEL radiation from the axially propagating Stokes mode may be absorbed by an off-axis propagating antistokes mode at the same frequency for angle $\theta_c$:

$$\theta_c \approx \frac{1}{\gamma}\sqrt{2\omega_p/\gamma k_u c} \,. \qquad (16)$$

This situation can be avoided if the diffraction angle of the rays, $\theta_d \approx \lambda/2R \approx 1/\gamma N^{1/4}$, is small enough: we find this to be the case if $\Theta_p \gg 1$. In a waveguide situation, correspondence of the Stokes and Antistokes frequencies could only occur for different modes with different cutoff frequencies: this is problematical, and the coupling would be imperfect at best.

An electron beam may be warm, or there may be a substantial component of transverse velocity ($V_{\perp 0}$) intentionally present, before entry into the undulator. In a guiding magnetic field ($B_c$), required for beam equilibrium, an intense beam can become unstable to cyclotron-type radiation given adequate $V_{\perp 0}/c$ ($\gtrsim$) . The frequency of the cyclotron waves on the beam, in the presence of an undulator, is given by

$$\omega_b = (k+k_u)v_\parallel \pm \Omega_0/\gamma \,, \qquad \Omega_c = eB_0/mc \,, \qquad (17)$$

and will interact with a light wave, $\omega = kc$, at

$$\omega \approx 2\gamma_\parallel^2 (k_u v_\parallel \pm \Omega_c/\gamma) \,. \qquad (18)$$

Radiation from the unstable slow cyclotron interaction with an undulator (minus sign) was observed by Gilgenbach [1979]in a moderately warm ($\frac{\delta\gamma}{\gamma} \approx 4\%$) beam, while an unstable, strong interaction was reported by Grossman [1983] using the fast cyclotron branch and a beam having a large $V_{\perp 0}/c$ ($\gtrsim 0.4$). The lower intersection of these waves with the waveguide dispersion curve can result in cyclotron type radiations: a lengthy series of reports, commencing with Granatstein and Herndon [1974], have documented this mechanism. The short wavelength radiation, which is of interest in this chapter, is Doppler-shifted cyclotron radiation modified by the undulator; the latter has the beneficial effect of lowering the threshold of instability as well as increasing the frequency of the radiation.

## III Electron Beam Considerations

As electrons stream in a relativistic beam, they experience forces from external fields (principally the external guiding field ) as well as the fields generated by the electron space charge and current. Under highly relativistic conditions, the latter two forces nearly balance. The search for various equilibria is an involved problem in the physics of non-neutral plasmas [Davidson, 1974]. Using no focussing magnets, a guiding field is necessary. One simple solution which provides some insight is the rigid rotor equilibrium--obtained by taking $B_0$ and $n$ constant across the beam radius. The radial force balance equation is

$$0 = \frac{\gamma m v_\theta^2}{r} + \frac{2\pi e^2 \rho\, r}{\gamma^2} - \frac{e v_\theta B_0}{c} \,, \qquad (19)$$

where the first term on the right is the centripetal effect of the azimuthal motion ($v_\theta$), the second is the combined effect of

the internal fields, and the last term is force exerted by the guiding field. I f the beam is not too dense, i.e. $\Omega_c^2 \gg \omega_p^2$ , then rotation occurs as a rigid body with two characteristic frequencies: a high frequency at $\approx \Omega_c/\gamma$ , and a low frequency at $\approx \omega_p^2/2\gamma\Omega_c$ which corresponds to the $\underset{\sim}{F_r} \times \underset{\sim}{B_c}$ drift of electrons in the radial force field. Thus, even before the beam enters the undulator, the motion is not simply parallel to the lines of $B_0$.

Injecting electrons into a cylindrical beam (radius $r_b$ ) contained in a metallic drift tube (R), will accumulate space charge near the axis, causing a build-up of electrostatic potential. This will cause a decrease in the forward motion of the electron, and for excessive current, electrons will be reflected. Assuming, however, we are content with smaller current, the space charge potential will not be important compared with $(\gamma-1)mc^2$ . Nevertheless, the space charge potential will vary across the beam radius and a shear or spread in electron parallel velocity will result. As $(\delta\gamma/\gamma)_{\parallel} = \gamma^2 v_{th}/c$ , where $\delta$ is a symbol representing the change in a quantity across the electron beam profile, it is easy to show

$$\left(\delta\gamma/\gamma\right)_{\parallel} \approx \omega_p^2 r_b^2/4c^2 \simeq \nu/\gamma , \qquad (20)$$

where $\nu$ is Budker's parameter, $\pi r_b^2 \rho(e^2/mc^2)$ , and "s.c." stands for the space charge contribution. The parameter $\nu/\gamma$ is a measure of what we shall term "beam intensity". Although it is possible to create beams having $\nu/\gamma \approx 1$, these are not suitable for FEL applications: taking the largest value of $\left(\frac{\delta\gamma}{\gamma}\right)_{\parallel}$ as $\approx$



1/N, we find $\nu/\gamma \approx .01$. In the future it may become possible to create equilibria which contain less velocity shear; indeed, the condition of Brillouin flow [$\omega_p^2 = \Omega_c^2/2$ ], if achieved, should substantially diminish the velocity shear represented by eq (20). This is a worthy but still unresolved objective. Energy spread due to finite axial bunch length is usually negligible unless a short pulse accelerator (eg linac or microtron) is used.

In an accelerator section, electrons may emerge with the same energy, but-- because of the configuration-- the momentum will be distributed between the parallel and transverse components. The average, $\langle v_\perp/v_\parallel \rangle$ , is referred to as the divergence, which is related to another parameter known as the transverse normalized emittance, $\varepsilon_n$ . We define $v_x/v \equiv (\gamma\beta)^{-1}\varepsilon_{nx}/X$, where x refers to that component of beam size. As $\frac{\delta v_\parallel}{v} \approx \delta v_x^2/2v^2$, we can calculate [Neil, 1979] the momentum spread which results from emittance. For a radially symmetric beam,

$$\left(\delta\gamma/\gamma\right)_{\parallel,\varepsilon} = \frac{1}{2}\left(\varepsilon_n/r_b\right)^2/(1+K^2) \qquad (21)$$

where $K = eB/k_u mc^2$ . [The quantity $(1 + K^2)$ in the denominator corrects the forward momentum of the electron for the transverse component of quiver motion induced by the undulator.] As the above is also a measure of our ability to focus the beam, there is defined a quantity known as the beam brightness, the electron flux through solid angle:

$$B_\varepsilon \equiv J_B / 2\pi \left(\frac{\delta\gamma}{\gamma}\right)_{\parallel,\varepsilon} . \qquad (22)$$

Careful design of the electron gun and focussing elements, using computational modelling of the trajectories, can improve the beam emittance, or aid in guiding it into the undulator. One

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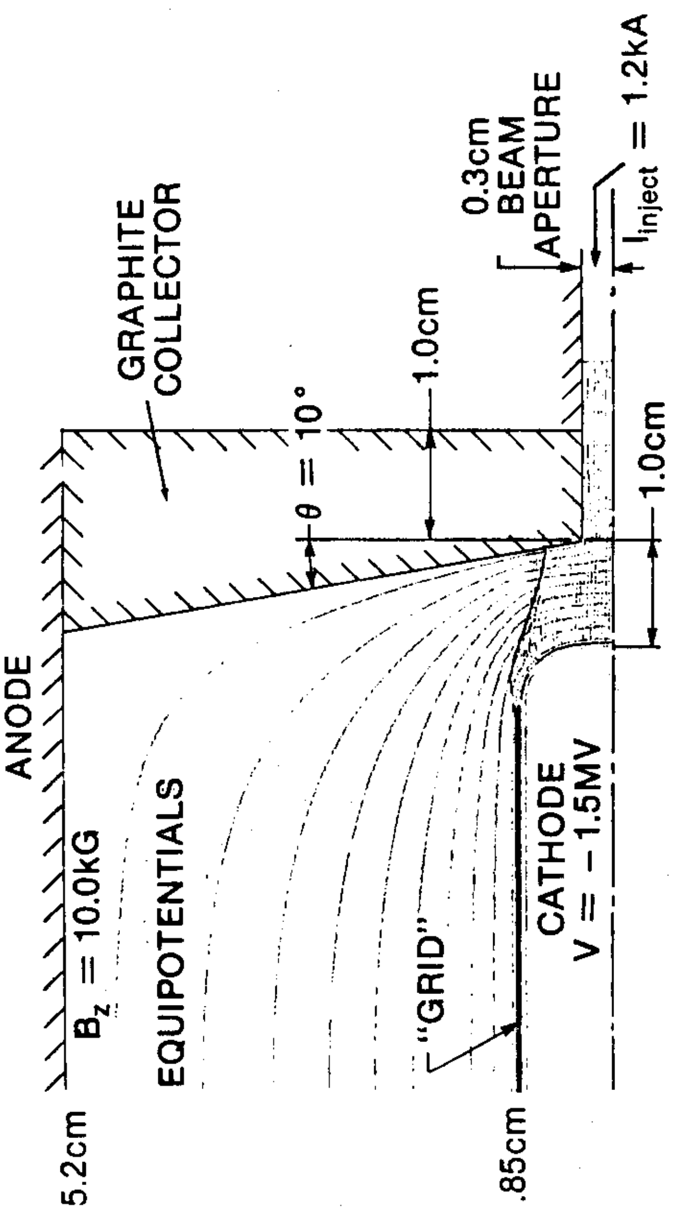


Figure III.11 Vacuum diode geometry (After Jackson et al., 1983)

Fig 2

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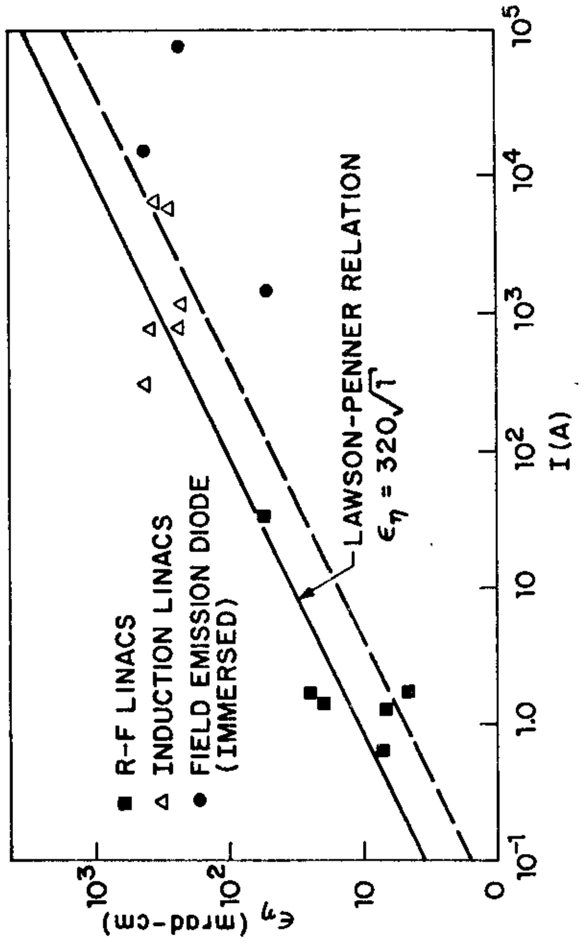
calculation [Jackson, 198 ] for the NRL VEBA diode, is shown in Figure 2. The electrode geometry was arranged to reduce the velocity spread of the beam, but an important aspect of the design-- common to many applications-- is the aperturing of the beam. Frequently, a strong magnetic field is imposed at the diode, or gun, as this restrains the gyromotion of electrons emitted from the cathode in the residual transverse electric field. On the other hand, injection of a beam from an unmagnetized electron gun into an increasing guide field is not advantageous as any initial transverse motion of the electrons is enhanced because of the adiabatic invariant $mv_{\perp 0}^2/B_z(z)$. Better results from an unmagnetized beam have been obtained with focussing elements [Pasour, 1983] only.

An empiracle relation between the current in the beam and the emittance is the Lawson-Penner relation:

$$\mathcal{E}_n\,(rad\text{-}cm) = S\sqrt{I\,(kA)}\ , \qquad (23)$$

where the factor S ~ 0.3 was initially determined from a study of rf linac performance. On the other hand, it also appears to represent data from other electron guns and sources (Fig 3, Roberson, 1983). [The dotted line is for a different choice of S, 0.15.] Field-immersed diodes perform somewhat better: it has been possible to obtain a divergence ≈ 30mrad even at very high current density. Simulations by Sloan and Thompson show the importance of a strong magnetic field to obtain low emittance at high current density; they find that the radial components of the






electric field in the gun should have an axial gradient long compared with ~ $c/\Omega_c$ .

Finally, the velocity shear of the beam will deteriorate further due to the undulator. The pump field varies radially:

$$\mathcal{B}_\perp(r) \approx \mathcal{B}_\perp(0)\left\{1 + \tfrac{1}{2}(k_u r)^2 + \cdots\right\}\left\{\begin{matrix}\sin\\ \cos\end{matrix}(\theta - k_u z)\right\}, \quad (24)$$

and hence the quiver velocity as well as the parallel velocity will depend on radius. Since

$$\delta v_\parallel / c \approx (\beta_\perp k_u r_b / 2)^2, \quad (25)$$

one finds

$$(\delta\gamma/\gamma)_{\parallel,und} \approx \frac{(k_u r_b K)^2/2}{(1+K^2)} \quad (26)$$

The electron motion in an actual helical undulator will approximate a helix only if $k_u r_o = K/\gamma \ll 1$ [Diament, 1981], where $r_o$ is the radius of the electron orbit helix. Thus a practical limit on the pump amplitude is $K \lesssim 1$. Since we should require $(\delta\gamma/\gamma)_{\parallel,und} < N^{-1}$ , we find $k_u r_b < N^{-1/2}$ ; taking $N \sim 100$ and $k_u \sim 1\text{cm}^{-1}$, then $r_b \approx 1\text{mm}$.

All the above contributions to $(\delta\gamma/\gamma)_\parallel$ are known as inhomogeneous broadenings. To avoid a decrease in FEL gain, it has been shown [Ibanez, 1983] that the inhomogeneous terms should not exceed* $(\delta\gamma/\gamma)_\parallel \approx 1/N$; or putting it somewhat differently, the length of the undulator may extend to $\lambda_u/(\frac{\delta\gamma}{\gamma})_\parallel$ (to obtain maximum gain) before warm beam effects will cause the gain to decrease appreciably.

* We can understand this result from the following dimensional argument. The Raman FEL fractional bandwidth, ie the homogeneous bandwidth, is $< \omega_p/\gamma k_u c$; this will accommodate a change in parallel momentum of the electron. However, $\omega_p > 2\pi$ and hence $\omega_p > 2\pi\gamma c/L$ : the 1/N result follows.

In an "ideal" helical undulator, where the fields derive from

$$A_{\perp}(z) = -B/k_u\{\hat{i}\cos k_u z + \hat{j}\sin k_u z\} \quad (27)$$

the transverse canonical momenta are constants:

$$P_\perp = \gamma m v_\perp - |e| A_\perp/c \, . \quad (28)$$

The electrons describe helical orbits with radius $r_o$ and constant axial velocity:

$$k_u r_o = v_\perp/v_\parallel = |e| B/\gamma m c k_u v_\parallel \, . \quad (29)$$

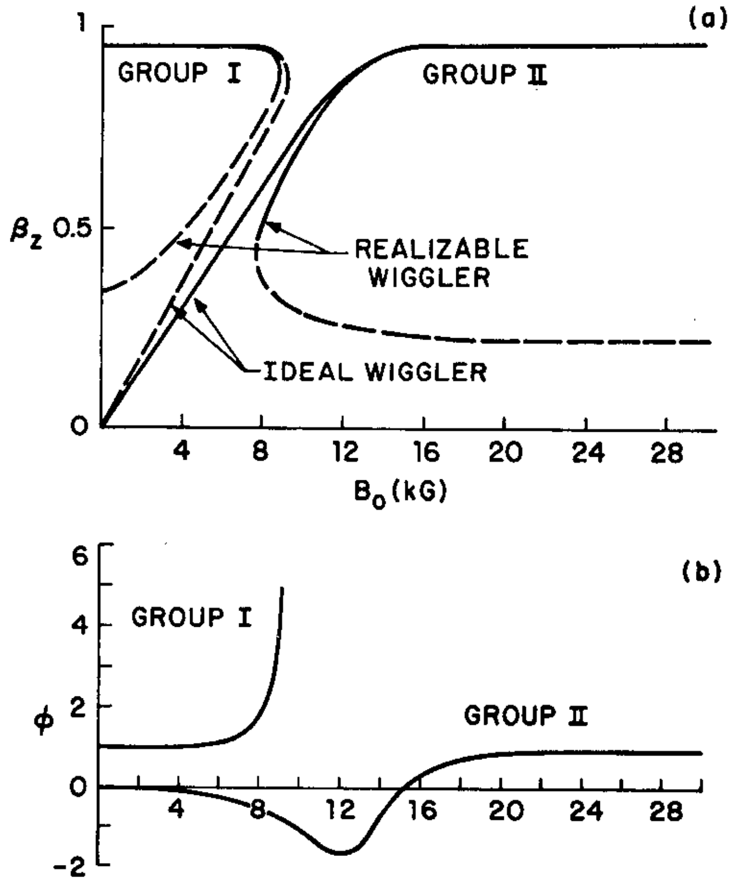
When a guiding field is superimposed, this simple description falls apart to some degree, although the approximation is fair if $k_u r_o$ remains small. The quiver velocity of the electron ($v_\perp$) becomes large near the "magnetoresonance", $\Omega_o \approx \gamma k_u v_\parallel$ :

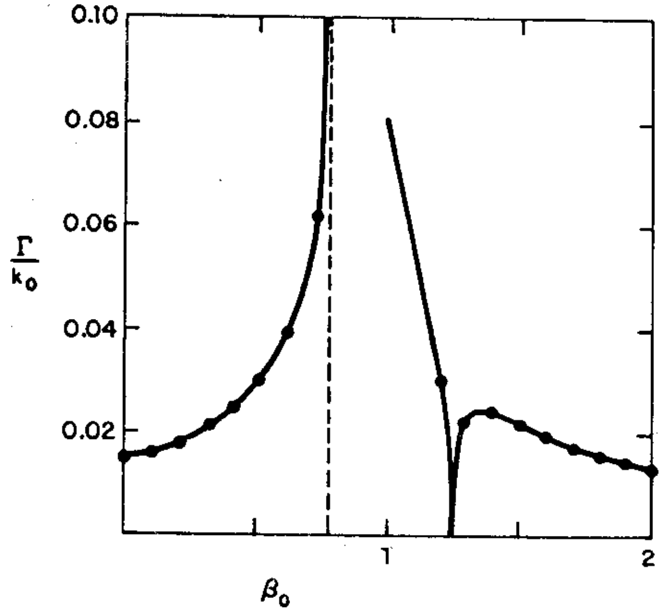
$$v_\perp = \frac{2\Omega_\perp v_\parallel [I_1(\zeta)/\zeta]}{\Omega_o - \gamma k_u v_\parallel \pm 2\Omega_\perp I_1(\zeta)} \, , \quad (30)$$

where $\zeta = k_u r_o$. High gain can therefore be obtained with only modest undulator B . Orbits are stable [Friedland, 1980] for $\Omega_o < \gamma k_u v_\parallel$ and $\Omega_o > \gamma k_u v_\parallel$ as shown in Fig 4, but the situation is more complicated near magnetoresonance. Freund [1983] has calculated the effect of guiding field upon the FEL gain, and finds an enhancement of the ponderomotive potential and gain in the vicinity of the resonance (Fig 5).

When injecting electrons into the undulator, it is necessary to "taper" the amplitude B over several periods so that the orbit becomes nearly helical. Nevertheless, simulations of orbital motion in a non-ideal undulator with a tapered B









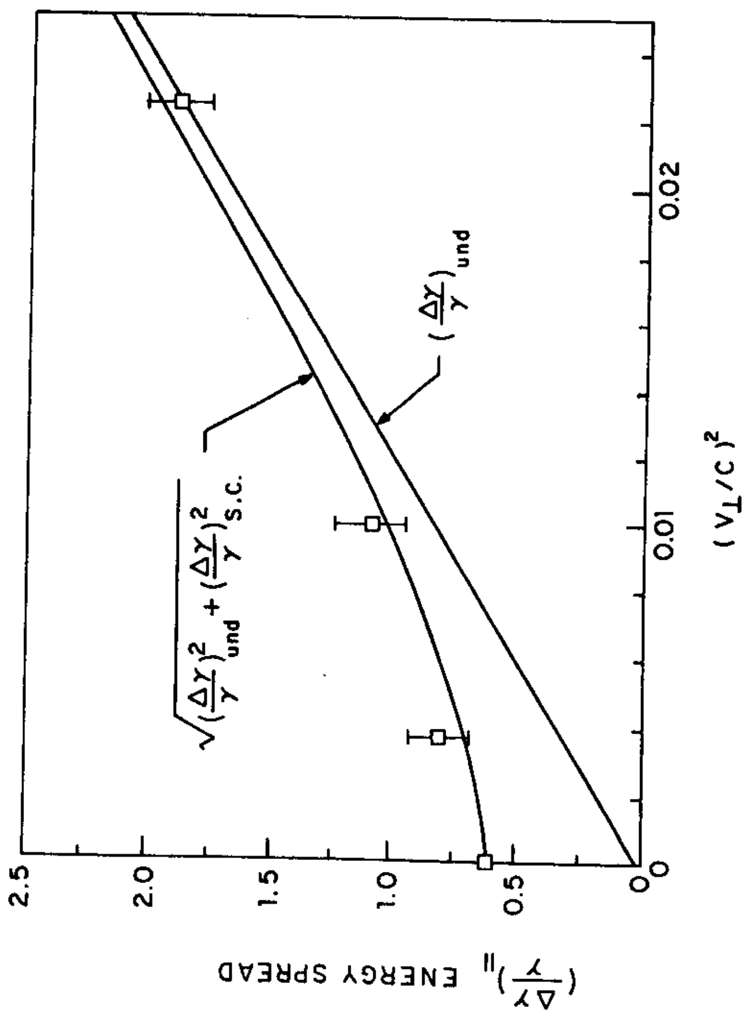

entry show an axial oscillation in $v_\perp$ is set up, which interacts with the undulator field gradients to cause a thermal spread in axial velocities [Jackson, 1982]. These effects impose an upper limit on the pump field or quiver velocity.

Since the helical undulator has "focussing" properties, that is, $B_\perp$ increases radially, it is a good choice for intense beams operating at low $\gamma$. The linearly polarized undulator has no such focussing in one plane, and indeed the electrons slowly drift off-axis owing to the $B_0 \times \nabla B_\perp$ drift. External focussing may be necessary for the other plane. Cusp field undulators provide $B_\perp$ only for cylindrical-shell beams (electrons located off-axis); on-axis, the periodic field has only a $B_z$ component. The latter configuration has been chosen for an FEL variation known as the "Lowbitron [Bekefi, 1982].

Many diagnostics have been developed to determine the average energy and momentum state of the beam (see for example Shefer [1983]). Some involve a perturbation in the beam (eg, pinhole, scattering foil, etc), while others analyze signals from externally located electrostatic or magnetic probes or loops. One technique which holds great promise for the accurate and non-interactive diagnosis of the electron beam is that of laser Thomson backscattering, developed at Columbia University [Chen, 1984]. A strong signal from a pulsed TEA CO2 laser at 9.6$\mu$ was backscattered to a wavelength of 0.5$\mu$ by interaction with a

1KA/cm$^2$ , $\gamma$ = 2.3 beam in an FEL configuration. When the incident photon is antiparallel to $v_\parallel$ and the scattered photon is parallel, the Doppler frequency upshift factor is $\approx 4\gamma^2$ and the differential photon scattering cross section is also enhanced by $\approx 4\gamma^2$. Both effects are essential to the success of the measurement. Measurement of the wavelength of the scattered light will determine the energy of the electron beam, with enough accuracy to permit an estimate for the space charge potential depression; the latter will give the beam density, which can be compared against a beam current measurement. More importantly, the width of the scattered spectrum can be measured by observing the transmitted light through a set of narrow-band, calibrated filters. Some interesting features of the measurement were that, with a resolution for $\left(\frac{\delta\gamma}{\gamma}\right)_\parallel \approx 0.1\%$, it was found that the space charge contribution to the inhomogeneous broadening dominated the emittance contribution using a diode similar to that shown in Fig 2: $\left(\epsilon\gamma/_{\gamma}\right)_\parallel$ = 0.6%. When the undulator was energized, the inhomogeneous broadening increased, the data showing that the space charge and undulator contributions should be quadratically combined to obtain the observed, total inhomogeneous width (Fig 6); this is a result of the total electron beam energy spread, $\sigma_\delta$.





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