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OPTICAL BEAM PHASE CONJUGATION: A REVIEW

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Lecture Notes on Optical Beam Phase Conjugation for the Winter College on Lasers, Atomic and Molecular Physics, at the International Centre for Theoretical Physics

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1. Optical Beam Phase Conjugation: A Review

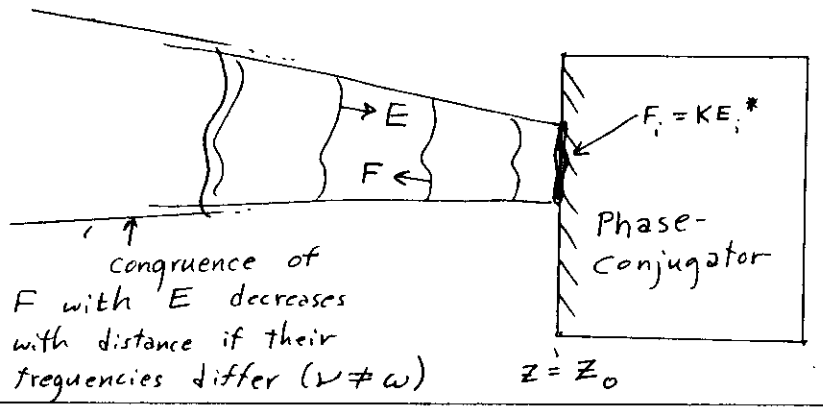
1.1. Definition

Suppose an optical beam at frequency ω impinges on the front plane ($z=z_o$) of a device where its electrical vector has complex amplitude $E_i(x,y,z_o)$. The subscript i labels spatial components. If this device causes a backscattered wave at frequency ν whose complex field amplitude in the same plane is $F_i(x,y,z_o)$, and

$$F_i = KE_i^* \text{ at } z=z_o, \tag{1.1}$$

where k is any complex constant, then the F beam is said to be a perfect (full-vector) "phase-conjugate" to the incident beam E. The device is then called a "phase-conjugator" with reflectivity $|K|^2$. When $\nu=\omega$ the device may be called a degenerate phase-conjugator. See Fig. 1.1.

Fig. 1.1. Schematic of conjugated wave.



In practice, devices called more loosely "phase-conjugators" produce a reflected field whose amplitude at ν is given by (repeated indices are to be summed)

$$F_i(x,y,z_o)=K_{ij}E_j^*(x,y,z_o)+C_i(x,y,z_o) \quad (1.2)$$

where the K_{ij} are complex constant matrix elements and

$$\int E_iC_i dxdy=0. \quad (1.3)$$

That is, there may be some degradation or selection of beam polarization through the matrix K_{ij} as well as a non-conjugate fraction f of the backscattered intensity given by

$$f = |\int F_iC_i dxdy|^2/\int |F_i|^2 dxdy \int |C_i|^2 dxdy \quad (1.4)$$

analogously as one would calculate the probability to be in state C if in state F in quantum mechanics.

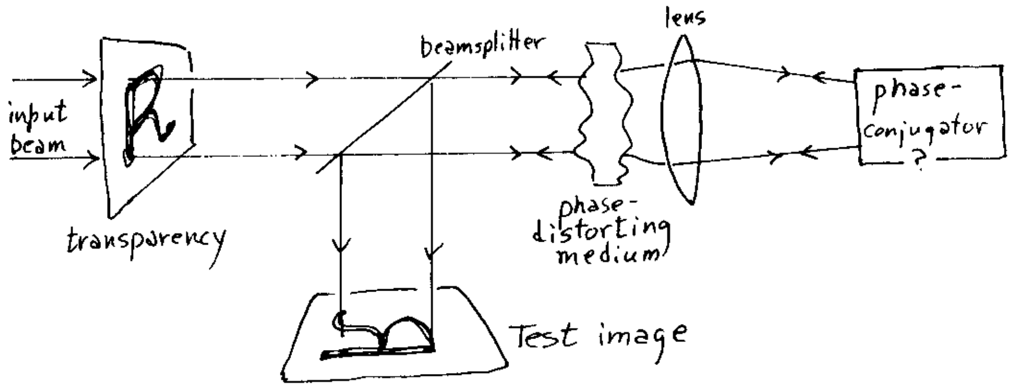
The calculation of f for many devices remains a challenge to theory.

1.2. Time-reversed Beam Replicas.

One of the main reasons for interest in phase-conjugation is the fact that, when $\nu=\omega$, the beam F will be essentially the time-reversed replica of the incident beam E in all

of space before the device, i.e., for $z<z_o$. This is provided that all of the beam E enters the phase-conjugator, and there are only non-dissipative, non-magnetic, transparent objects in this space. This time-reversed character allows the F-beam to retrace the path of the E-beam through a distorting medium, emerging as an undistorted replica of E. This distortion-correction property is the favorite way to verify that a device is functioning as a phase-conjugator. A typical experimental arrangement for this is shown in Fig. 1.2.

Fig. 1.2. Typical arrangement for verifying phase-conjugation.



Mathematically it is easily seen that, for $\nu=\omega$, the phase-conjugate wave is the time-reversed replica. If $\mathcal{F}_i$ and $\mathcal{E}_i$ are the real electric fields of the two waves

$$\mathcal{F}_i(\underset{\sim}{x},t)=\Re F_i e^{-i\omega t}=\Re KE_i^* e^{-i\omega t}=\Re K^* E_i e^{i\omega t}=|K|\mathcal{E}_i(\underset{\sim}{x},t_o-t) \quad (1.5)$$

where ωt_o is the phase angle of K. It is a property of the time-reversible form of Maxwell's equations that if equation 1.1 is true with $\nu=\omega$, and the beams have oppositely directed Poynting vectors, then $F_i=KE_i^*$ in all of space before the device (all $z<z_o$) as well as at $z=z_o$.

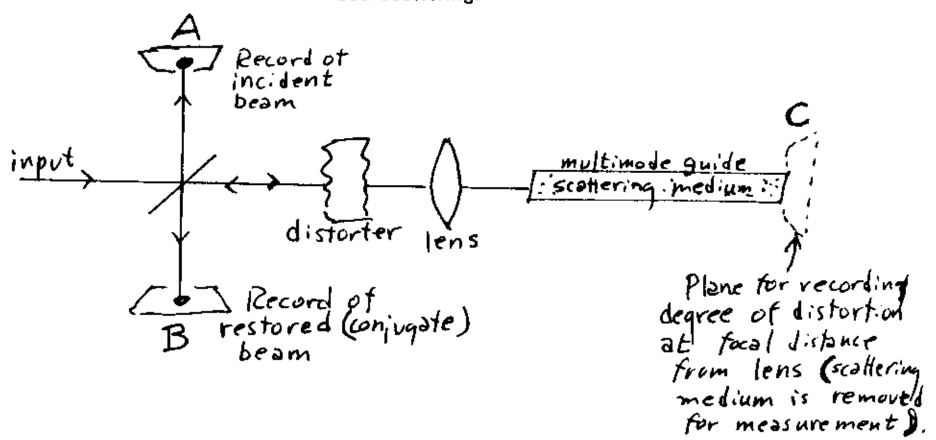
1.3. Nonlinear Optical Effects Capable of Phase-Conjugation.

Many nonlinear optical effects will produce a phase-conjugate wave (or a wave with an acceptable low value of f as defined in Eqn. 1.4). These are well-reviewed in Ref. 1, and include three-wave mixing, four-wave mixing, and all stimulated effects. By far the most effort has been expended on (1) stimulated scattering devices (especially stimulated Brillouin scattering), (2) on four-wave mixing via third-order nonlinear susceptibility, and (3) both four-wave mixing and two-beam coupling using the photorefractive effect. In this section we will describe all three briefly, and deal in detail with each in later sections. A final section will describe some measurements of material properties made easier or possible employing phase-conjugation.

1.3.1. Stimulated scattering.

The first report of optical beam phase-conjugation employed an arrangement that is a model of simplicity and has been used many times since. It is illustrated in Fig. 1.3.

Fig. 1.3. Schematic of phase conjugation by stimulated backscattering.



In this process the input beam is focused into a multi-mode guide, or sometimes into a semi-infinite container, containing a medium exhibiting strong stimulated

backscattering, usually Brillouin (i.e., from stimulated acoustic waves of half the optical wavelength). Once the required intensity for significant backscattering is reached, if this intensity is below the threshold for stimulated forward scattering or other nonlinear effects, then the backscattered light has "low-f" or small non-conjugate fraction and the input beam pattern (observed at A in Fig. 1.3) is seen to be reproduced (at B in Fig. 1.3). The degree of distortion which the conjugation is compensating is often measured by recording the far-field pattern of the distorter (at C in Fig. 1.3. with lens and guide removed) and comparing it with the far-field pattern of the input beam (seen at A in Fig. 1.3.) The medium can be in thermal equilibrium and no ancillary pump beams or other power supply is required. Threshold powers required are typically $\sim 10^5$W, easily obtained from a simple Q-switched laser. The coherence length of the input beam must be $\sim c/w$ or the effective interaction length (typically ~ 100 cm) whichever is shorter. Here $c \equiv$ velocity of light in vacuum, and $w \equiv$ spectral width (in rps) of the scattering excitation. In such conditions, 50 to 90 percent of the input beam is commonly observed to be reflected into the conjugate wave.[1,2] Of course, if conjugation occurs in a guide, the guide must support at least as many modes as there are modes or "elements" (or "pixels") in the distorted beam pattern (or in any undistorted image which is to be conjugated). Were this not the case it would be impossible to couple the entire incident beam into the guide (as is necessary). Also, the backscattered beam frequency ω equals that (ν) of the incident beam minus ω_e the frequency of the scattering excitation. If ω_e is too large, the incident and scattered wavefronts, which are conjugate at the entrance plane to the guide (or interaction region), may diverge too much to experience full correction of distortions. This seems never to be a problem with the small shifts (~ 1 cm^{-1}) accompanying Brillouin scattering, and was not even observed to be a problem when conjugating by scattering from the 656 cm^{-1} Raman excitation in liquid CS_2.[3]

Although stimulated backscattering provides the simplest experimental method for phase-conjugation, it is the most difficult to analyze and understand physically. It seems to work only for beams with a large number of modes, and never perfectly. A more detailed discussion of the theory will be given in the next Section 2.

1.3.2. Phase conjugation by four-wave mixing mediated by third-order nonlinear susceptibility $\chi^{(3)}$.

Here we discuss why four-wave mixing can be used to achieve phase-conjugation and review some experimental results. The nature of the degradation caused by finite pump beams will be discussed.

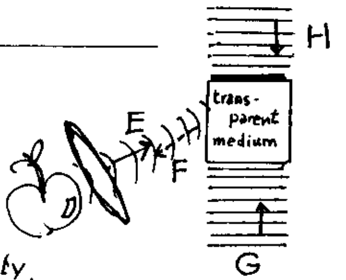
1.3.2.1. Theory, plane pump waves.

Consider the essentially instantaneous nonlinear polarization density of the form

$$\tfrac{1}{2}\,\sigma \underset{\sim}{E}_t(\underset{\sim}{r},t)\underset{\sim}{E}_t(\underset{\sim}{r},t)\cdot\underset{\sim}{E}_t(\underset{\sim}{r},t) \qquad (1.6)$$

where $\underset{\sim}{E}_t$ is the total electric field existing at $\underset{\sim}{r}$ and t, and σ is a constant. This describes accurately the electronic nonlinearity in fluids and glasses, which is the major portion of (pre-breakdown) nonlinearity in glasses and in fluids of isotropic molecules ($CC\ell_4$, CH_4, etc.). Suppose an optical field exists in such a medium which is the superposition of two counter-propagating pump waves G and H and an input image-bearing beam E as shown schematically in Fig. 1.4.

Fig. 1.4
Schematic of phase-conjugation by four-wave mixing using third-order nonlinear polarization density present in all transparent media.



This means that as a first approximation the total electric field may be written

$$\underset{\sim}{E}_t \approx \tfrac{1}{2}[\underset{\sim}{G}e^{ikz}+\underset{\sim}{H}e^{-ikz}+\underset{\sim}{E}(\underset{\sim}{r})]e^{-i\omega t} \qquad (1.7)$$

Then it is readily seen by substituting this in (1.6) that a nonlinear polarization density exists which will generate a fourth wave at frequency ω and whose complex amplitude is

$$\underset{\sim}{P}_4=\sigma\left(\underset{\sim}{G}\underset{\sim}{H}\cdot\underset{\sim}{E}^*+\underset{\sim}{H}\underset{\sim}{G}\cdot\underset{\sim}{E}^*+\underset{\sim}{E}^*\underset{\sim}{G}\cdot\underset{\sim}{H}\right)/4, \qquad (1.8)$$

in which the only spatial dependence is that of $\underset{\sim}{E}^*$. It is easy to imagine that if a polarization density exists proportional to $\underset{\sim}{E}^*$ everywhere in a slab orthogonal to this E beam, then it will radiate a time-reversed replica, or phase-conjugate, to this beam. The details of how this occurs are given in Ref. 4 which is attached as Appendix I.

In ordinary liquids and glasses, pump beam intensities $\sim 10^7$ to 10^{10} W/cm^2 are required to produce efficient phase conjugation in one centimeter of beam interaction length. This is typically done by focusing Q-switched pulses (~0.1 J) to beam diameters ~300 microns. Vapors (sodium, rubidium, etc.) which have an electronic resonance at the operating frequency may require three to four orders less pump beam intensity to achieve high efficiency.[1] Certain extraordinary semiconductors have orders-of-magnitude large nonlinear coefficients still.[1,6]

It is interesting to ask if there is any physical limit to how low an optical pulse energy would suffice as pump for phase-conjugation in the best possible medium. It is easy to show that in a medium of naturally-broadened two-level systems of optimum density, pulse energies approaching $\hbar\omega$ per picture element (or $\hbar\omega$ to conjugate a TEM_{00} gaussian beam) would be sufficient. This raises interesting questions about the kinds of photon states that could be produced by conjugation at single-quantum pump levels.

A less extreme limit exists for energy and power requirements to phase-conjugate in a transparent medium. Here the limit is imposed by the fact that any physical mechanism (electrostriction, molecular alignment, etc.) which produces the nonlinear refractive index also produces ordinary scattering (Brillouin, Raman, Rayleigh, etc.) which eventually attenuates all beams too much if the mechanism is enhanced by material design. The minimum pump-beam intensity in an optimal medium turns out to be $\sim k_B T \omega^3/c^2$ where k_B is Boltzmann's constant, T is the absolute temperature, and c is the velocity of light in vacuum. This is proved in Appendix II. It says that one will always need of-the-order of one Watt per cm^2 to conjugate visible beams in transparent media (no electronic excitations), but less at longer wavelengths by the cube of the wavelength ratio. This has interesting implications for microwave propagation. Media that are within some orders-of-magnitude of optimal may exist in nature.

That phase-conjugation by four-wave mixing might produce reflectivity $|K|$ greater than unity was predicted by Bloom, et al.,[6] from the theory of energy exchange among four coupled plane waves, neglecting self-focusing, and self-phase modulation, and pump-beam depletion. Even with these approximations, however, an analysis such as Appendix I of the image quality has bever been attempted in presence of gain, and experiments on this point are inconclusive.[1] Research along these lines was probably cut short by that discovery that high fidelity cw phase-conjugation with power gains $\sim 10^2$ and larger are easily obtainable with an argon laser beam in barium titanate crystals.[1,6] The photorefractive effect responsible for this method of phase-conjugation will be discussed in Sections 1.3.3 and 3.

If we designate the conjugate beam as the F-beam, and if the four beams are monochromatic, their frequencies are always related by

$$\omega_F = \omega_G + \omega_H - \omega_E. \tag{1.9}$$

The quantum process governing the wave mixing is a parametric process in which (when no energy is deposited in the medium) two pump photons, from G and H, are annihilated while a photon is created in both the E and F waves. This indicates the possibility of gain, or $|K|>1$ in ~~which~~ (1.5), which we will discuss below.

When the beams are not monochromatic, various frequency components of each beam must satisfy (1.9), suggesting the possibility of "filtering" which we also discuss below.

If all beams involved are nearly plane waves, their wavevectors obey

$$|\underset{\sim}{k}_G + \underset{\sim}{k}_H - \underset{\sim}{k}_E - \underset{\sim}{k}_F| < L^{-1} , \tag{1.10}$$

what is sometimes called the "phase-matching" condition because it allows the nonlinear polarization density of (1.8) to radiate constructively into the F-wave. The reason that phase-conjugation by degenerate four-wave mixing ($\omega_E = \omega_F = \omega_G = \omega_H$) in isotropic media is so convenient that it has essentially 4π acceptance solid angle. It is readily seen from (1.10) that when $\underset{\sim}{k}_G = -\underset{\sim}{k}_H$, phase-matching occurs for any direction of $\underset{\sim}{k}_E$ if the F-beam is conjugate so that $\underset{\sim}{k}_F = -\underset{\sim}{k}_E$. This simple argument is extended to complex image-bearing beams in Appendix I.

1.3.2.2. Four-wave mixing experiments.

Experimental verification of phase-conjugation using the test of Figure 1.2 and the device of Fig. 1.4 has been achieved with many media and at many wavelengths from infrared to ultraviolet.[1] Quantitative measurements of the non-conjugate fraction f of (1.4) are lacking however. In fact, convenient methods for measuring f are only now being developed.

Although perfect counter-propagating plane waves are required for pump beams in the "open" geometry of Fig. 1.4., it has been found,[7] as predicted,[8] that if the mixing occurs in a multi-mode optical guide, the pump beams can have any shape whatever without distorting the phase-conjugate reflection. In fact the input image can serve as its own pump. Guides of order 1 m long and 0.3 mm diameter have been used. (For details see Refs. 7 and 8 which are included as Appendices III and IV.)

If the pump beams are monochromatic (although perhaps of different frequencies) but the input image is broadband, the phase-conjugate is frequency-filtered. The frequency bandwidth of the filter is of order c÷[effective beam interaction length L], if the nonlinearity is not resonant. If a resonance in $\chi^{(3)}$ is used, and the width of this resonance is less than c/L, then the resonance width becomes the filter bandwidth. Details of this image-filtering are given in Appendix I for reflectivity less than unity with unguided waves, and in Appendix IV for the case of guided waves. Experimental demonstrations of this filtering are discussed in Refs. 9, 10, and 1.

Phase-conjugation by non-degenerate four-wave mixing can result in a large frequency shift in the phase-conjugate reflection. By this means, infrared-to-optical image conversion has been demonstrated. (See Ref. 11 which is attached as Appendix V.) Raman resonances can occur at certain frequency shifts and these have been used to make spectroscopic measurements, as we discuss in Section 4. Details are given in Ref. 12, attached as Appendix VI.

Most phase-conjugation configurations will conjugate only beams of a single specific state of polarization. However, by employing oppositely rotating circularly polarized pump beams, one can produce true vector phase conjugation in isotropic media. That is, the matrix K_{ij} in (1.2) is made to be $K\delta_{ij}$, and an image that has been distorted by a random birefringent medium is found to be restored when the phase-conjugate beam passes back through the distorter. Details are given in the reprint which is attached as Appendix VII.

1.3.2.3. Gaussian pump beams.

If the input image E in Fig. 1.4 impinges nearly parallel to one of the pump beams (G) and both pump beams have a constant congruent gaussian profile in the interaction region, then one can easily modify the analysis of Appendix I to show that the reflected beam F is the phase conjugate of E smeared over a diffraction spot size set by the pump beams. That

is, if the pump beam amplitude is proportional to $\exp[-kr^2/2q]$, then, in a plane at a distance z in free space in front of the conjugator

$$F_i(x)=K_{ij}\int d^2x' C(x-x')E_j^*(x') \qquad (1.11)$$

where x is the two dimensional vector in this plane (of magnitude r) and

$$C(x)=2\pi a^{-2}e^{-r^2/2a^2} \qquad (1.12)$$

where $a=(2z^2/kq)^{\frac{1}{2}}$. One can use this to calculate f or other desired aberration parameters. Other shape pump beams may be used in calculating the radiation pattern of P_4 of (1.8) to assess the non-conjugate fraction resulting.

1.3.3. Phase conjugation by photo-refractive effect.

In many insulating transparent materials there exist trapped electrons (or holes) which are essentially stationary unless excited by light to the conduction (or valence) band where they may migrate until recombining at another empty trap. When the light intensity is not uniform, the pattern of re-trapped electrons is also non-uniform and creates according to Poisson's equation a pattern of static electric field $\underset{\sim}{E}(x)$ in the material. This pattern may persist in the dark for seconds to months, depending on the material. If, in addition, the material lacks a center of inversion symmetry, then the electric field E creates a proportional pattern of refractive index change everywhere via the electro-optic (Pockel's) effect. Such a pattern is often called an index "grating". Such index change was called "optical damage" by its discovers, Ashkin, et al.,[13] who found it troublesome while attempting to phase-match optical beams to enhance harmonic generation. The potential of such index gratings produced by light beams for applications such as real-time holography[14] and optical memories[15,16] was soon recognized, as were the essential physical processes involved.[17-20] The effect became known as the "photorefractive effect".[19-22]

It is now widely employed for phase-conjugation. Because it is essentially an effect that is nonlinear in beam energies rather than beam powers, it can phase-conjugate low-power (even microwatt) beams, but taking longer to respond to the lower powers employed. Barium titanate, for example, responds typically in tenths of seconds at one Watt cm^{-2} levels, depending on beam angles, applied fields, and other factors. This crystal also yields large conjugate-wave gains.

A remarkable property of the photorefractive effect is that it can produce phase-conjugation in either the manner and geometry of stimulated backscattering, as depicted in Fig. 1.3., or in the manner and geometry of four-wave mixing, as depicted in Fig. 1.4. A more detailed description of the optical nonlinearities of photorefractive materials is given in Section 3.

2. Phase-conjugation By Stimulated Scattering

The remarkably simple device of Fig. 1.3 is perhaps the most difficult method of phase-conjugation to understand. We will first review the basic nature of stimulated scattering and then show how the theory of the phase conjugator proceeds without going through the details, many of which are best handled by numerical computation.

In its simplest form, stimulated scattering is the exponential power gain e^G experienced by a weak monochromatic probe wave at ω when it traverses a strong monochromatic pump wave at a higher frequency ν in a transparent medium having an (Raman-active) excitation whose frequency is $\nu-\omega$.

When this excitation is acoustic the process is called "stimulated Brillouin scattering"; when vibrational it is called "stimulated Raman scattering," etc., for a number of different physical processes. The rate of growth of the gain factor G with distance ℓ along the probe beam is given by

$$dG/d\ell = gI \qquad (2.1)$$