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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

(21 January - 22 March 1985)

Topical Meeting on the Free Electron Laser

# RAMAN FREE ELECTRON LASERS

Lecture I: Theory
Lecture II: Experiment

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## (I) Categories of FEL

1) "Compton" or "single-particle":
- gain scales as $n$, $\omega_p^2 (= 4\pi n e^2/m)$ or current ($I$).
- gain is "low".

2) "Collective":
- gain does not scale as $n$;
- exponentially-growing instability convecting with electron flow.
- gain is "large".

Raman category: dense, cold beam with weak pump (undulator).

- if beam is warm, exponential growth persists, but scattering from single electrons causes recovery of Compton process.

Go to electron rest frame :

principle classes of waves (cold electrons):

EM $\quad \omega'^2 = k'^2 c^2 + \omega_p^2$

ES $\quad \omega' = \pm \omega_p$

$\omega_p = (4\pi n e^2/\gamma m)$ = invariant plasma frequency.

... for "warm" electrons

ES $\quad \omega'^2 = \omega_p^2 + 3k'^2 V_T^2$

$V_T$ = thermal velocity

$\lambda_D = V_T/\omega_p$ = characteristic distance (Debye length)

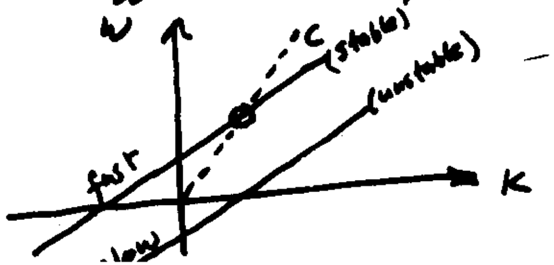
criterion for ES wave : $\lambda' = 2\pi/k' \gtrsim 2\lambda_D$

Now transform these to the Lab. frame :

use relativistic Doppler effect

$$\omega' = \gamma(\omega - kv) \equiv \pm\omega_p$$

find $\quad \omega = kv \pm \omega_p/\gamma \quad$ (lab. frame)






– if the EM wave is slowed (e.g. by a helix), the slow space charge wave will amplify a signal.

– to make an FEL, we undulate the e-beam using a magnetostatic "wiggler" or "undulator"

period (lab) = $l_0$

wavenumber = $2\pi/l_0 = k_0$

rest frame $\quad l_0' = l_0/\gamma$

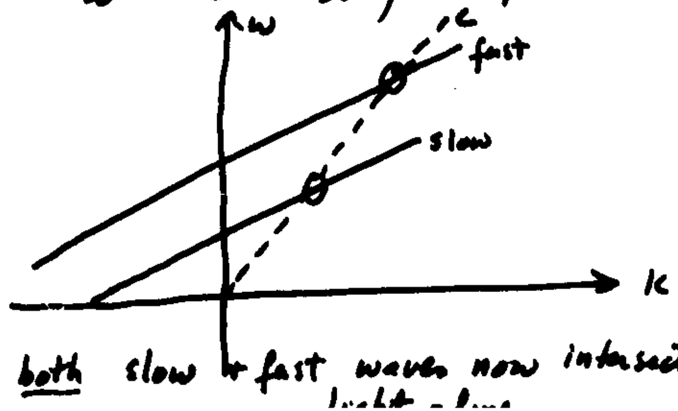
$k_0' = \gamma k_0$

In the rest frame, the magnetostatic lab disturbance appears at frequency

$\sim k_0' v$

$$\therefore \quad \omega' = k_0' v \pm \omega_p$$

or $\quad \omega = (k + k_0) c\beta \pm \omega_p/\gamma$




Note both slow & fast waves now intersect the light-line for EM wave, $\omega \approx kc$ ; equate freq. of EM + ES waves: find

$$\omega = 2\gamma_\parallel^2 (k_0 c - \omega_p/\gamma)$$

for the unstable (growing) interaction.

$$\left[\; 2\gamma_\parallel^2 \approx \frac{1}{1-\beta_\parallel} \;\right]$$

At small $k$, the EM and ES wave dispersion is modified by transverse structure.

<u>Criteria</u>

1.) Pump (undulator) wave couples EM and ES modes. It must be "weak" so it doesn't "destroy" the ES wave.

2.) The beam must "sense" its collective response: number of plasma oscillations must be large:

$$\omega_p \tau \gg 1$$

or

$$\boxed{\omega_p \cdot L/\gamma c \gg 1}$$



[redacted]

is sustained <u>sans</u> Landau Damping:

$$\ell_0/2\gamma \gtrsim 2\lambda_D = 2\, v_T/\omega_p$$

since $(v_T/c)_{R.F.} \equiv (\delta\gamma/\gamma)_\parallel$

Then $\left(\frac{\delta\gamma}{\gamma}\right)_\parallel < \frac{\ell_0}{2\gamma} \cdot \frac{\omega_p}{2c} \sim \frac{1}{N}$

where $N = L/\ell_0$ ; <u>(follows from $\omega_p L/\gamma c \gg 1$.</u>

This defines a <u>restriction</u> on $\gamma$, $\ell_0$, or the FEL wavelength (for Raman operation)

$$\lambda_s \gtrsim \frac{2}{\gamma}\left(\frac{c}{\omega_p}\right)\left(\frac{\delta\gamma}{\gamma}\right)_\parallel \sim 10\text{-}100\,\mu,$$

since $c/\omega_p$ = EM skin depth $\sim 1$ cm.

4) <u>High gain</u>:

The ponderomotive (beat wave) $\left[\text{due to } \frac{\tilde{v}_\perp}{c} \times \tilde{B}_w\right]$ resonates with plasma wave. The space charge fluctuation <u>enhances</u> the scattering by dielectric resonance.

However, if the ponderomotive (beat) wave does not resonate with $\omega_p$ wave, the dense electron gas (dielectrically) shields the response, which is much reduced.

... Can one obtain high gain "off-resonance"?

... yes!! Just "destroy" the plasma wave by pumping "very hard" (more on this later).

Raman FEL is a 3-wave system:

$$\left\{\begin{array}{lll} \text{idler (plasma, sp. chg.) wave:} & E_i,\ \omega_i,\ K_i & \longleftarrow K_i \\ \text{signal (em) wave:} & E_s,\ \omega_s,\ K_s & \longrightarrow K_s \\ \text{pump (undulator) wave:} & E_0,\ \omega_0,\ K_0 & \longleftarrow K_0 \end{array}\right.$$

growing, upshifted, scattered wave:

resonance if $\omega_i = \omega_p$.
$$\left\{\begin{array}{l} \omega_s = \omega_0 - \omega_i \\ K_s = K_0 - K_i \end{array}\right. \qquad (\omega_i \approx \omega_p)$$

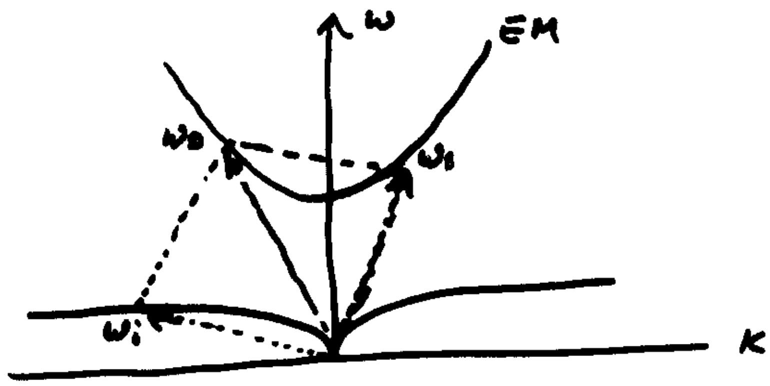


"Stokes" wave is "unstable" (growing)

"Anti-stokes" wave is "stable" (absorbing)

... corresponds to stimulated emission/absorption.

Stokes Diagram (rest frame)

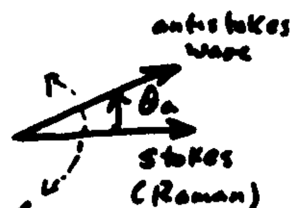



stim Raman Absorption occurs at

$$\omega_A \approx 2\gamma_\parallel^2 (K_0 c + \omega_p/\gamma)$$

$$\Delta\omega_a/\omega = \frac{2\omega_p/\gamma}{K_0 c} = 2(\Delta\gamma_a/\gamma)$$

$$\theta_a = \frac{1}{\gamma}\sqrt{\frac{2\omega_p}{\gamma K_0 c}}$$




freq. decreases as θ increases

stokes wave grows if

$$\theta_a > \theta_{diff} = \lambda_s/2R$$

$\left(\text{use } z_{Ray} \approx L = \frac{\pi R^2}{\lambda_s}\right)$

How does the Raman FEL interaction Develope?

I. (Simple) 3-wave parametric Amplifier model

... take pump wave constant amplitude

EM wave $c^2 \nabla\times(\nabla\times\underline{E}) + \ddot{E} = -4\pi\, \partial \underline{j}/\partial t$

.... wave equations are fourier-analyzed spatially, lead to eqs. for amplitudes:

$$\begin{cases} \left(c^2 k_s^2 - \omega_s^2 + 2i\omega_s \frac{d}{dt} + \frac{d^2}{dt^2}\right) E_s = -4\pi i\omega_s J_x \\ \left(-\omega_i^2 + 2i\omega_i \frac{d}{dt} + \frac{d^2}{dt^2}\right) E_i = -4\pi i\omega_i J_z \end{cases}$$

NB -- no $\nabla\times(\nabla\times E)$ term for ES mode.

J = [linear current] + [non-linear current]

gives linear Dispersion Relation — causes growth.

$$\begin{cases} (\omega_s^2 - \omega_p^2 - c^2k_s^2) E_s = 2i\omega_s[\dot{E}_s + 2\pi J_{x\,NL}] \\ (\omega_i^2 - \omega_p^2) E_i = 2i\omega_i[\dot{E}_i + 2\pi J_{z\,NL}] \end{cases}$$

Left-hand side of these equations $\approx 0$ near 3-wave resonance:

$$\therefore \boxed{\frac{dE}{dt} \approx -2\pi J_{NL}}$$

NL currents result from mixing: i.e.,

$$J_{x\,NL}(\omega_s) = -|e|\,\underbrace{\tilde{n}(\omega_i)}_{\text{s.c. wave}}\,\underbrace{V_x(\omega_0)}_{\text{pump-induced quiver motion}}$$

we can use $\begin{cases} \partial n/\partial t + \nabla\cdot(n\underline{v}) = 0 \\ \underline{v} = \frac{ie}{m\omega}\underline{E}(\omega) \end{cases}$

to write $J_{NL}$ in terms of $\underline{E}$. Then:

$$\begin{cases} \ddot{E}_s(\omega_s, t) = g^2 E_s(\omega_i, t) \\ \ddot{E}_i(\omega_i, t) = g^2 E_i(\omega_i, t) \end{cases}$$

where $g^2 = \frac{\omega_p^4 \omega_0}{\omega_i^3}\left(\frac{e}{mc\omega_0}\right)^2 |E_0(\omega_0)|^2$,

where $\omega_0 = \gamma k_0 c\beta$.

Initial conditions $\begin{cases} E_s(t=0) = E_0 \\ E_i(t=0) = 0 \end{cases}$

give solutions $E_s(t) = E_0 \cosh gt$

$E_i(t) = E_0 \sinh gt$

where $t = L/\gamma c$.

$g \approx \frac{eB_\perp}{mc}\left(\omega_p/4\gamma k_0 c\right)^{1/2}$ ~~note~~

Exponentially-growing signal:

$G$ = Power gain/pass = $\cosh^2\left(\frac{gL}{\gamma c}\right)$

$gL/\gamma c$ is typically "large", viz, $\sim 1$.

In an <u>oscillator</u>, signal regenerates after $M$ passes to $P(M)/P_0 \propto M + e^{GM}\left(\frac{GM-1+}{(1+G)^M}\right)$;

<u>Threshold</u> for oscillation depend on EM and ES losses (rate $\nu_L, \nu_i$ respectively):

$$\frac{g}{2\gamma c} \sim \sqrt{\nu_L \nu_i} \ .$$

<u>Saturation of small-signal growth</u>:

Since frequencies have Re and Im parts,

$$\omega_0 - \omega_s - \omega_i \neq 0 \ ; \text{ in fact}$$

$$\gamma k_0 c\beta - \omega_s - \omega_p \approx g \ .$$

as signal saturates, $g \to 0$, and $\beta \downarrow$ to $\beta_{sat}$.

Then $\gamma k_0 c\beta_{sat} - \omega_s - \omega_p \approx 0$

$$\therefore \quad (\beta - \beta_{sat}) \equiv \Delta\beta \approx \frac{g}{\gamma k_0 c} = \frac{g}{\omega_s}$$

efficiency $= \eta = |\Delta\beta|_{\text{rest-frame}}$



$$\therefore \eta_{Raman} \sim \frac{g}{\gamma k_0 c} \lesssim \frac{\omega_p}{\gamma k_0 c} \ .$$

since, if $g \gtrsim \omega_p$, we have growth on the time scale of the plasma oscillation. This defines a transition to the "strong-pump" regime.

It's easy to show

$$\eta_{Raman}/\eta_{Compton} \sim \omega_p \tau \gg 1 .$$

Now suppose the beat (Ponderomotive) Disturbance does <u>not</u> resonate with the space-charge wave (non-resonant excitation). Can we recover the various limiting cases?

- Calculate <u>temporal</u> behaviour of scattered wave using a <u>travelling-wave model</u> (in <u>rest frame</u>) on a <u>cold electron fluid</u>; solve, using LaPlace transforms.

Fourier transform the spatial variable & get:

$$\left(c^2 k_s^2 + \omega_p^2 + \frac{d^2}{dt^2}\right) E_s(t) = 2\pi e n \left\{ \Omega_\perp u(t)\, e^{i\omega_0 t} - \frac{i\alpha\Omega_\perp}{k_0} \frac{d}{dt}\left[ e^{i\omega_0 t} \frac{n(t)}{n} \right] \right\} \qquad (\{\ \} = j_{NL})$$

where $\frac{\Omega_\perp}{\omega_0} = eB_\perp / k_0 m c^2$, $\alpha_\perp = [1 + \Omega_\perp^2/\omega_0^2]^{-1/2}$

$$B_\perp = \hat{l}\, \frac{B_\perp}{2} \left[ e^{i(\omega_0 t + k_0 z)} + c.c. \right]$$

This eqn for the scattered wave amplitude involves $n(t)$, $u(t)$:

$$\frac{d}{dt} u(t) = -\frac{e}{m} \mathcal{E}(t) - \frac{1+\alpha_\perp}{2} \Omega_\perp V_s(t)\, e^{-i\omega_0 t},$$

$$\frac{d}{dt} V_s(t) = -\frac{e}{m} E_s(t),$$

$$\frac{d}{dt}\left(\frac{n(t)}{n}\right) = i(k_0 + k_s)\, u(t),$$

and the ES wave amplitude is given by

$$\left(\omega_p^2 + \overbrace{\frac{d^2}{dt^2}}^{\sim \omega_i^2}\right) \mathcal{E}(t) = -2\pi e n (1+\alpha_\perp)\, \Omega_\perp V_s(t)\, e^{-i\omega_0 t}$$

↑ plasma dielectric response* ↑ ponderomotive excitation

write solution for $E_s$ in terms of residues:

*"plasma" effect dominates if $\omega_p \gg \omega_i \sim 2.6/T$ ......

Hence "Raman" is $\boxed{\omega_p T \gg 1}$

$$\frac{E_s(t)}{E_s(0)\, e^{i\omega_0 t}} = -\theta_p^2 \theta_n \left\{ \frac{e^{-i\theta_1}}{\theta_1(\theta_1-\theta_2)(\theta_1-\theta_3)} + \frac{e^{-i\theta_2}}{\theta_2(\theta_2-\theta_1)(\theta_2-\theta_3)} + \frac{e^{-i\theta_3}}{\theta_3(\theta_3-\theta_1)(\theta_3-\theta_2)} \right\}$$

where $\theta_1, \theta_2, \theta_3$ are roots of the cubic:

$$-\theta_p^2 \theta_n = \theta\left(\theta + [\theta_i + \theta_p]\right)\left(\theta + [\theta_i - \theta_p]\right),$$

where:

$\theta_p = \omega_p T$; $\theta_i = \omega_i T$; $\theta_n = (\Omega_\perp^2 \gamma / 2k_0 c)\, T$ and $T = L/v_g c$.

If we examine the case $\omega_i \approx \omega_p$

$$-\theta_p^2 \theta_n \approx \theta^2 (\theta + 2\theta_p).$$

<u>if</u>: $\theta_n \ll \theta_p$, pump & growth are "small"

so growth parameter $\sim \sqrt{\theta_p \theta_n h}$

and we obtain the "Raman" case.

A more detailed calculation for a typical Raman situation is shown:

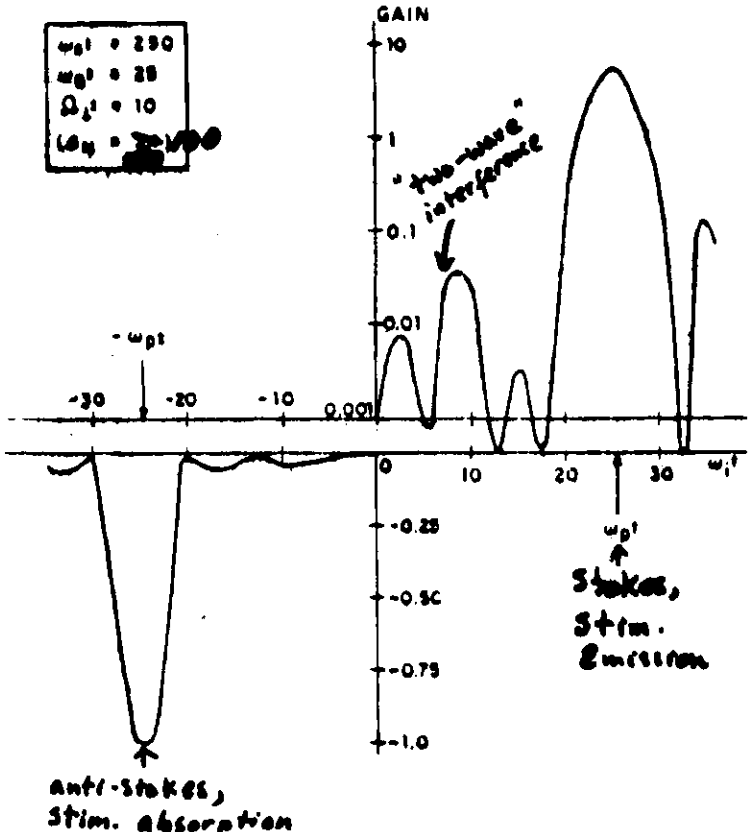



Figure IV-6. The dependence of total gain upon the pump-signal mismatch for the experimental conditions of the Columbia/NRL Collective Free Electron Laser.

Note -- "dielectric shielding" reduces 2-wave gain by a factor of $10^4$ here!

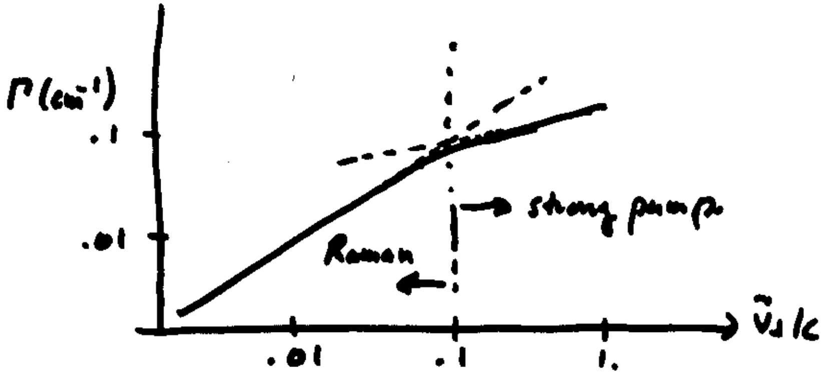


If $\theta_n \gg \theta_p$, we have two cases.

a) If $\theta_p^2 \theta_n \gg 1$, exponential growth for $\theta_i \sim \theta_p$ is still dominant; then

growth parameter $\sim (\theta_p^2 \theta_n)^{1/3}$;

this is the strong-pump case.




N.B.: No "disaster"... strong pump $\eta$ increases over Raman case. This may be useful for short-$\lambda$ lasers.

b) If $\theta_p^2 \theta_n \ll 1$, exponential growth is unimportant, gain is dominated by interference effects (2-wave Compton case).

2 roots lie near $-\theta_i$, + residues are combined in form of a derivative:

$$G \simeq -4\theta_p^2\theta_n \frac{\partial}{\partial\theta}\left(\frac{\sin\frac{1}{2}\theta_i}{\theta_i}\right)^2$$

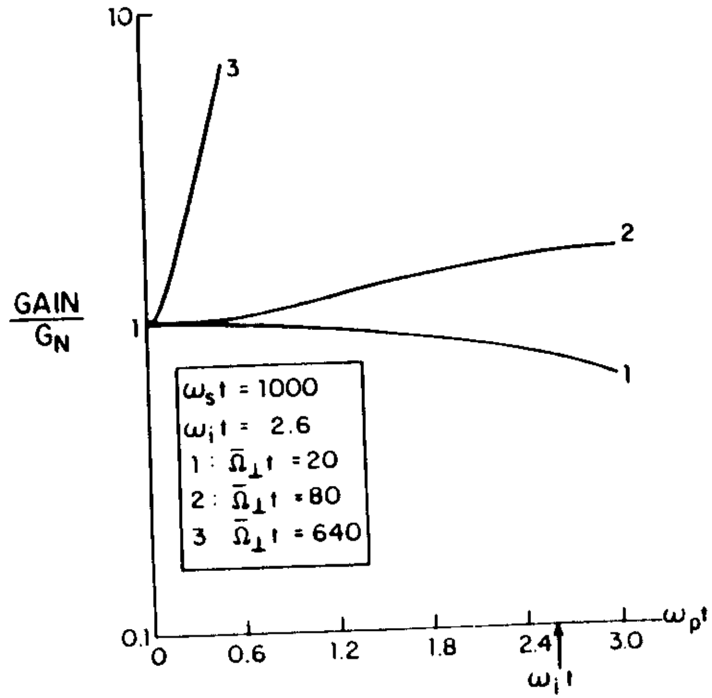
a max for $\theta_i = 2.6$ and

$$G_m = 0{,}27\ \theta_p^2\theta_n .$$

What happens if we take a 2-wave case, $\theta_i = 2.6$, and increase $\omega_p$?

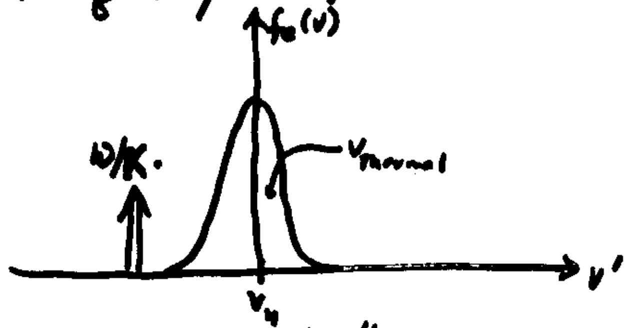
(1) If pump is weak, $G \downarrow$ below $G_m$ due to dielectric shielding.

(2) If pump is strong, $G \uparrow$ above $G_m$. "strong-pump" limit where wiggle "disrupts" the dielectric shielding of the space-charge.

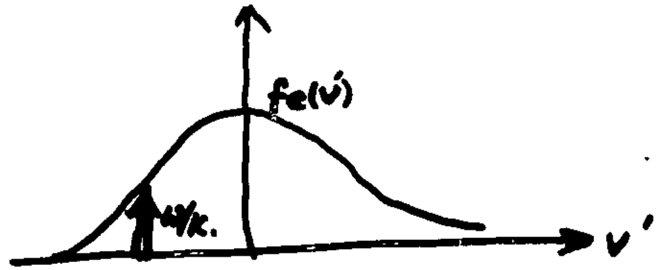



$\approx 10^3 A$ ($10^5 A/cm^2$) if $\gamma = 80$, $\lambda_s = 3\mu$

Adequately "cold" beam :




Beam is "hot" :




1. Use a Kinetic Equation approach with a realistic model for $f_e(v)$, i.e., a Maxwellian in the rest-frame.

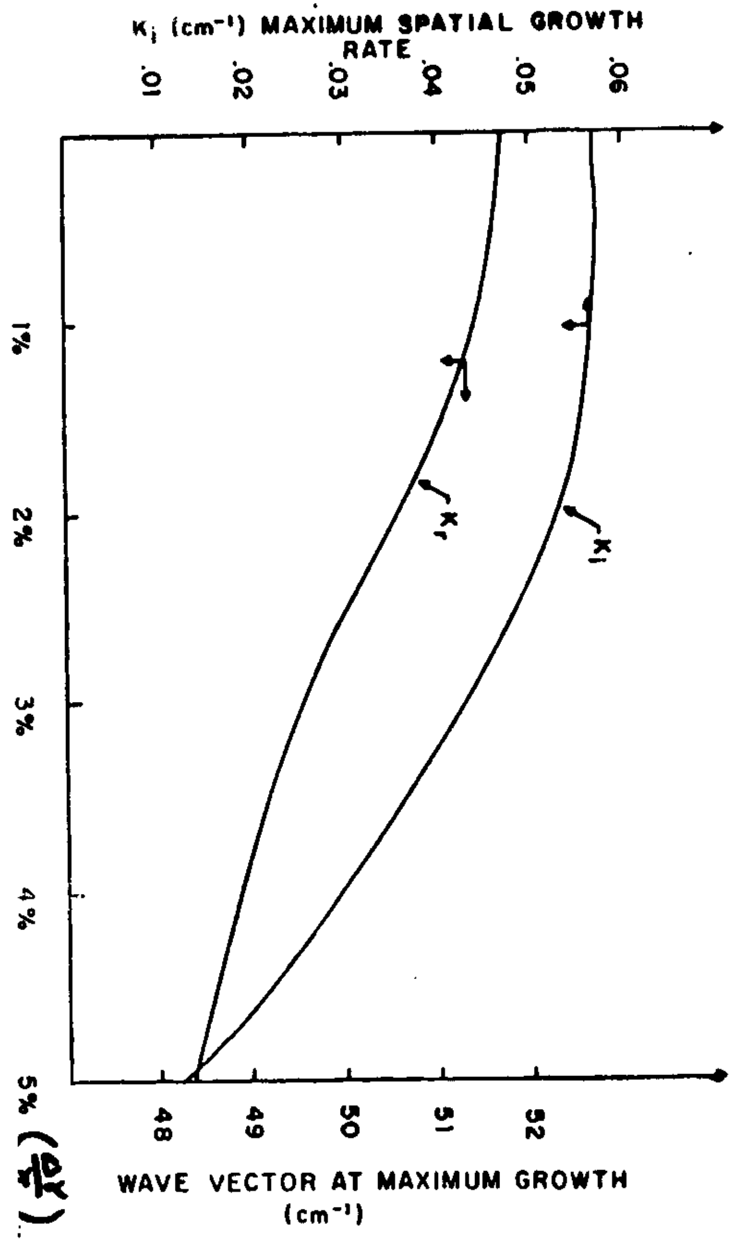
2. Find: growth parameter

$$\sim 0.2 \left(\frac{\omega_p^2}{\gamma K_0 c}\right)\left(v_\perp'/v_T\right)^2 .$$

3. growth is small (although still exponential). scattering due to electrons having $v \sim \omega_i/(K_0+K_i)$

Note : scales as $\omega_p^2$, $\therefore$ is Compton .






Raman Amplifier (Ebrahim + Johnston)
$\lambda = 1.2$ mm.

## Demands on Beam Quality

For most FELs, to keep gain constant (with constant current), $L$ must be increased as $\lambda_s$ decreases:

$$L \sim \lambda_s^{-1/2}.$$

Since requirement on inhomogeneous broadening is

$$\left(\frac{\delta\gamma}{\gamma}\right)_{\parallel} < \frac{1}{N} = \frac{\ell_0}{L}$$

Then $(\delta\gamma/\gamma)_{\parallel}$ must scale as $\lambda_s^{1/2}$.

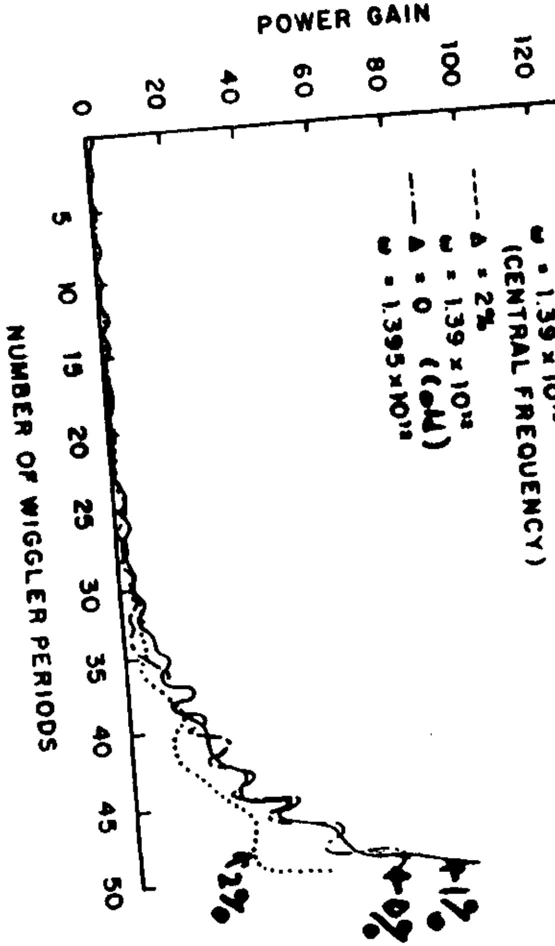
Short Wave-length limitation of Raman FEL ---

Emittance is $(\delta\gamma/\gamma)_{\parallel,\varepsilon} \gtrsim 0.1\%$ at present;

$(\delta\gamma/\gamma)_{\parallel,\text{space charge}} \propto 1/\gamma$ and scales favorably.

$\Theta_p \gg 1$ requirement scales as $L\gamma^{-3/2}$

and $L$ scales as $\lambda_s^{-1/2} \propto \gamma$; so one can maintain Raman condition, at present, for $\lambda_s$ as low as $10\mu$.




Raman Amplifier, $\lambda = 1.2$ mm (Jbones + Johnston

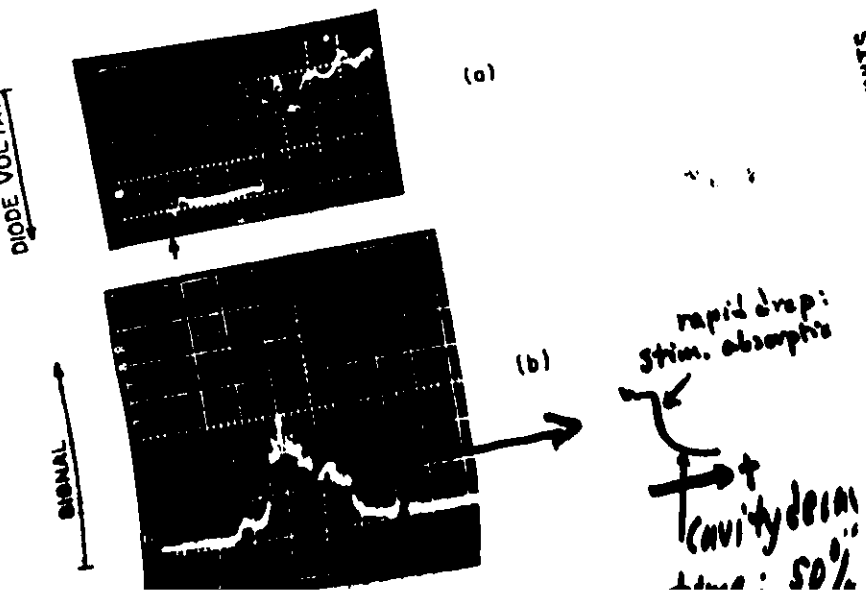
characteristic length where thermal effect on beam matters is $\sim \frac{\gamma\ell_0}{\left(\frac{\Delta\gamma}{\gamma}\right)}$ = so $\frac{\Delta\gamma}{\gamma} \simeq \frac{1}{N}$

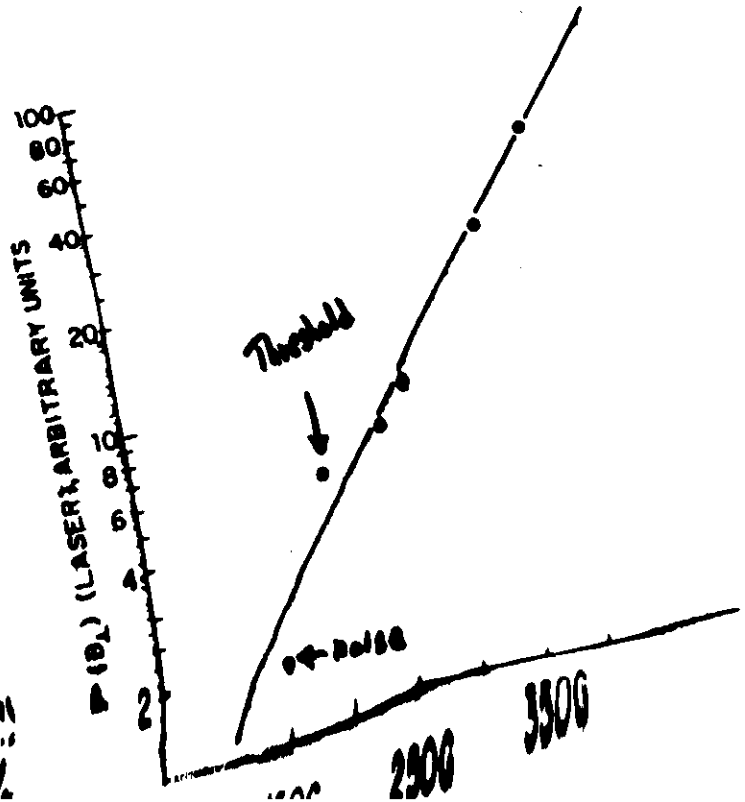
II

22

27

26


DIODE VOLTAGE
(a)
SIGNAL
(b)
rapid drop:
Stim. absorptio
cavity decay
time: 50%


$(I_L)$ (LASER) ARBITRARY UNITS
100
80
60
40
20
10
8
6
4
2
Threshold
noise
2500
3500

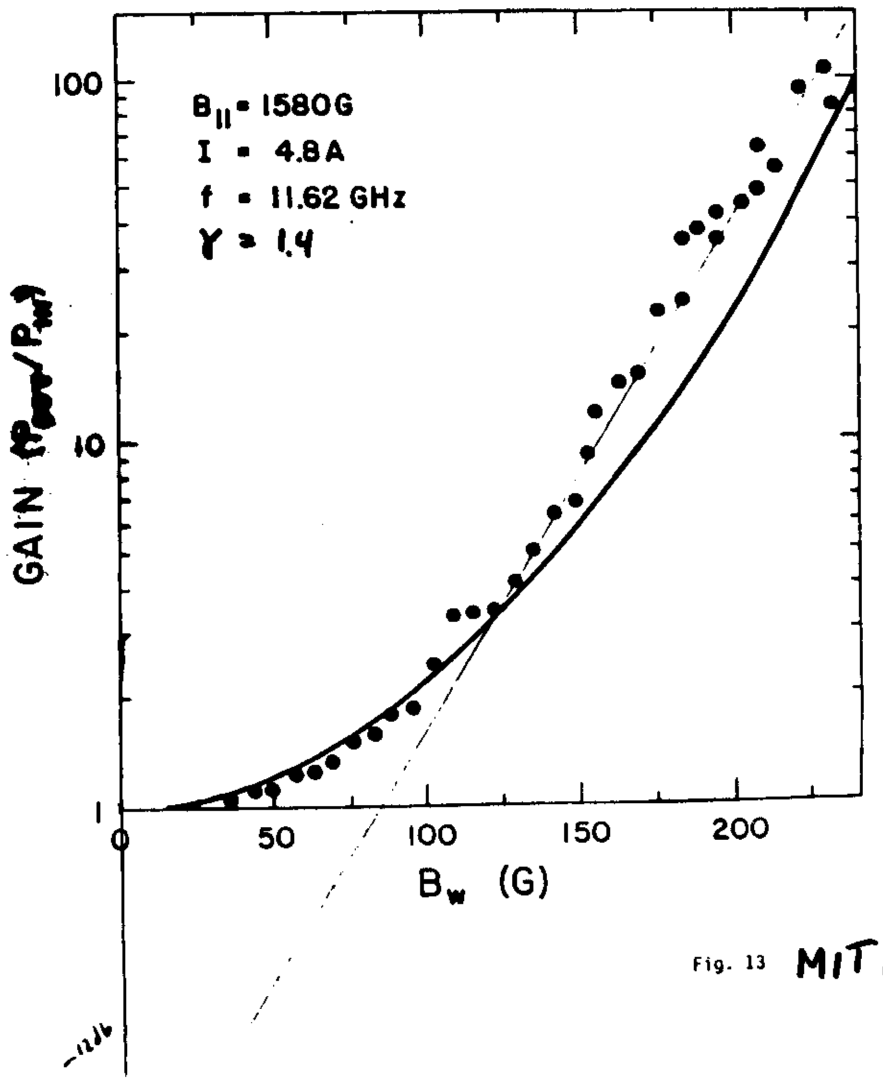
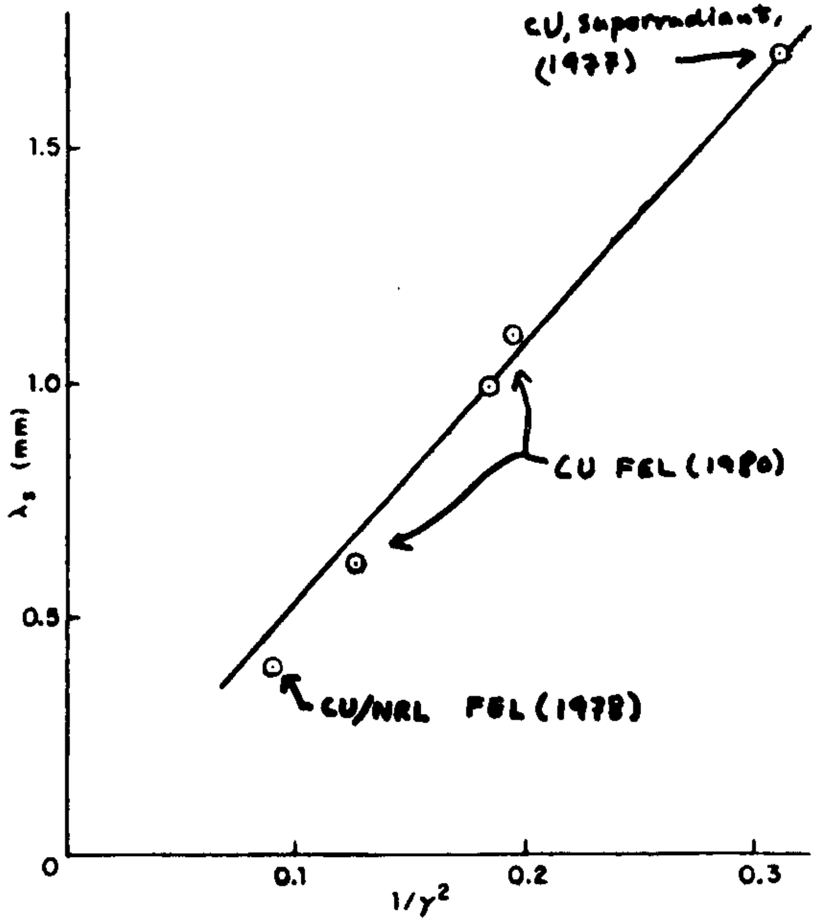


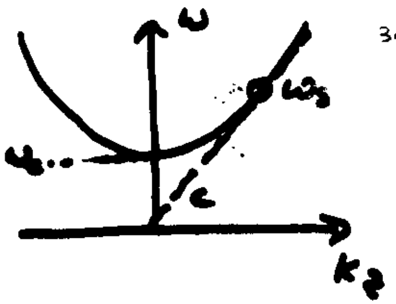

Fig. 13 MIT (1984)

$$\lambda_s \approx \frac{l}{2\gamma^2} : \; l = 8.3 \text{ mm}$$




Columbia (1980)

## Dispersion




Waves in Drift-tube pipe are slightly dispersive - due to finite radius, a:

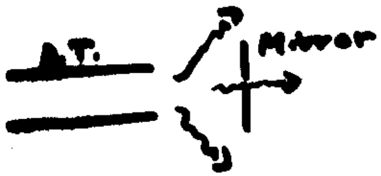
$$\Delta K_z/K_z \sim \tfrac{1}{4}(\lambda/a)^2 \sim 1/1000.$$

∴ spread of EM wave phases along wiggler is

$$\Delta K_z L = K_0 L/1000 = \frac{2\pi}{1000}\frac{L}{\lambda} \sim 2\pi$$

→ Situation can be described by the Fresnel Number $= a^2/L\lambda \sim 1$, which is a borderline case for assuming all modes travel at c.

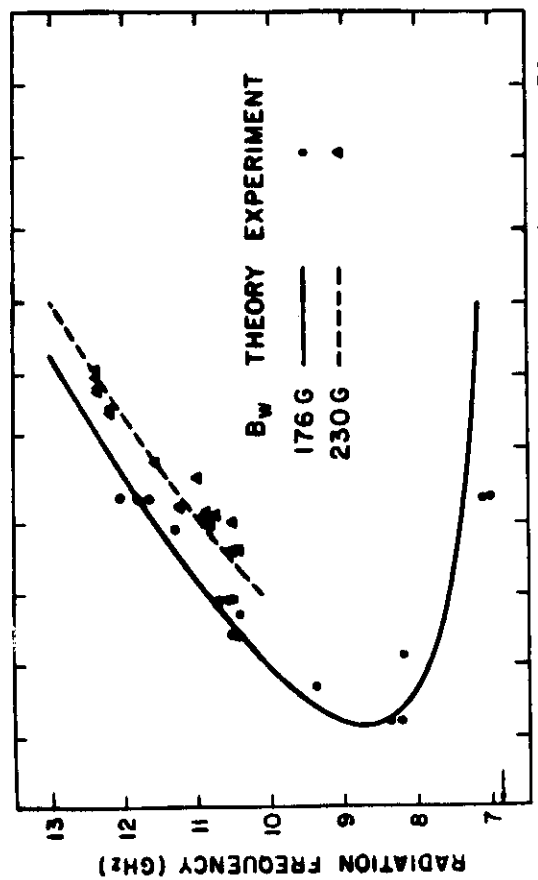
### Cavity losses




"open resonator" permits loss of radiation with appreciable $K_\perp$. Fastest-growing mode is the dominant mode, smallest $K_\perp$.



Effect of waveguide.

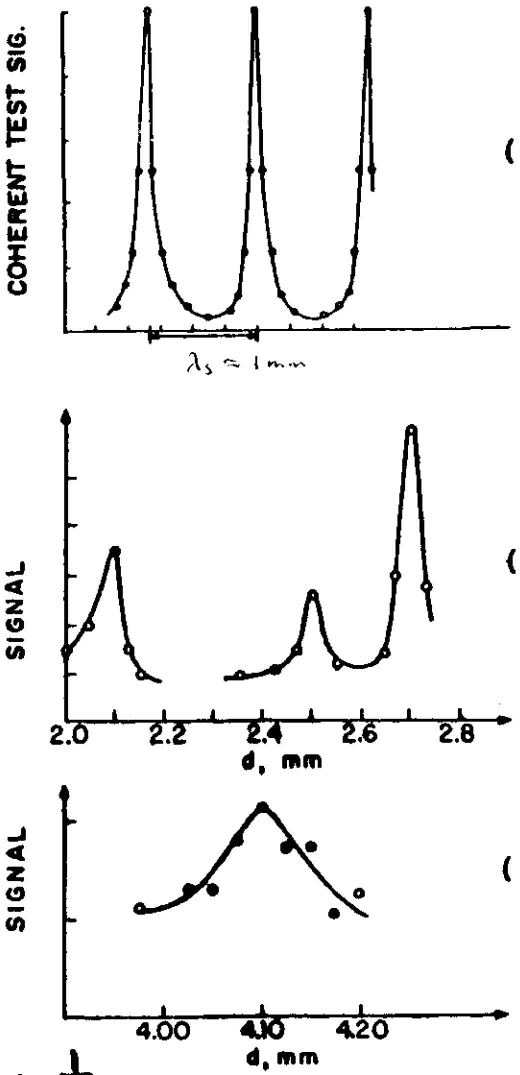



$\gamma = 1 + \frac{eV}{m_0c^2}$

Fig. 9 MIT (1984)

$$\omega_s \approx K_0 c \gamma_z^2 \left\{ 1 \pm \beta_z \left[ 1 - (\omega_c/K_0 \gamma_z c)^2 \right]^{1/2} \right\}$$

Columbia (1978)

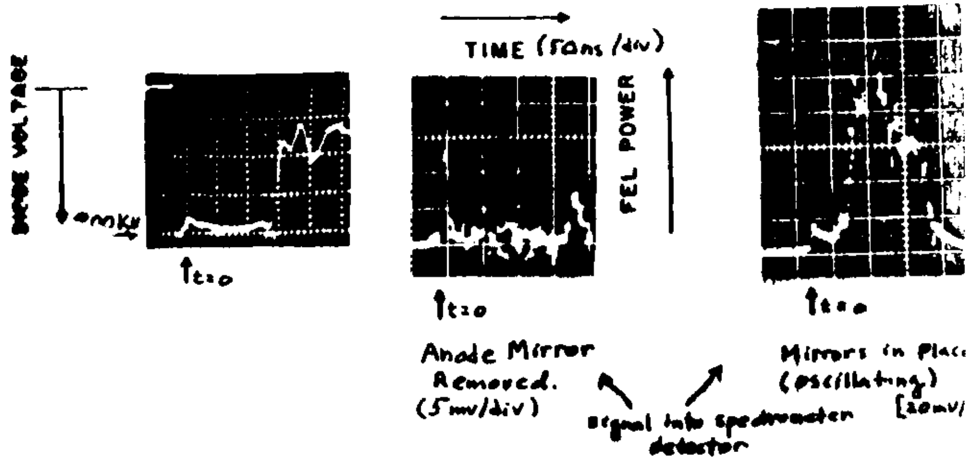


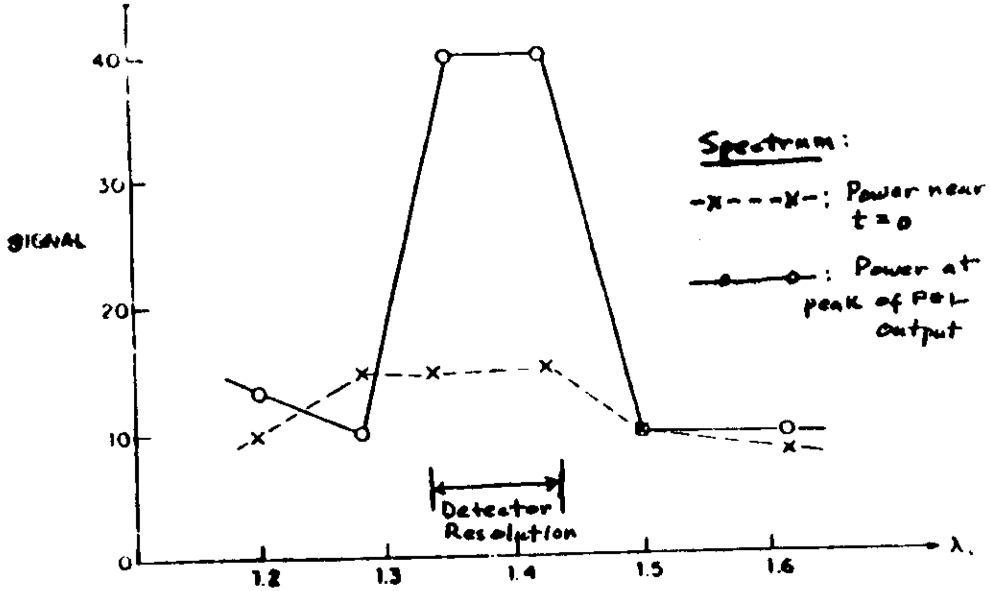

$$\frac{\Delta\lambda}{\lambda} \approx 2\% \sim \frac{1}{N}$$

(holds for resonator with high losses)

Fabry-Perot Spectral Analysis ($\lambda_s = 0.4$ mm)

Columbia (1981)



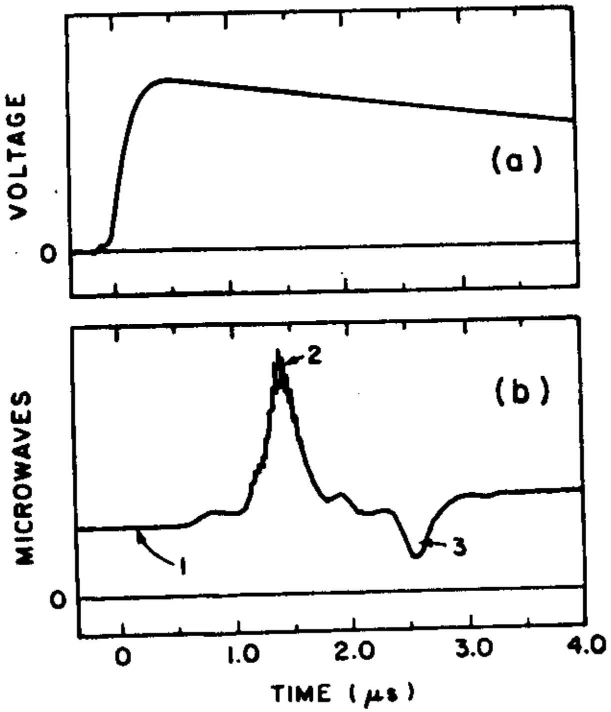



Fig. 10 MIT (1984)



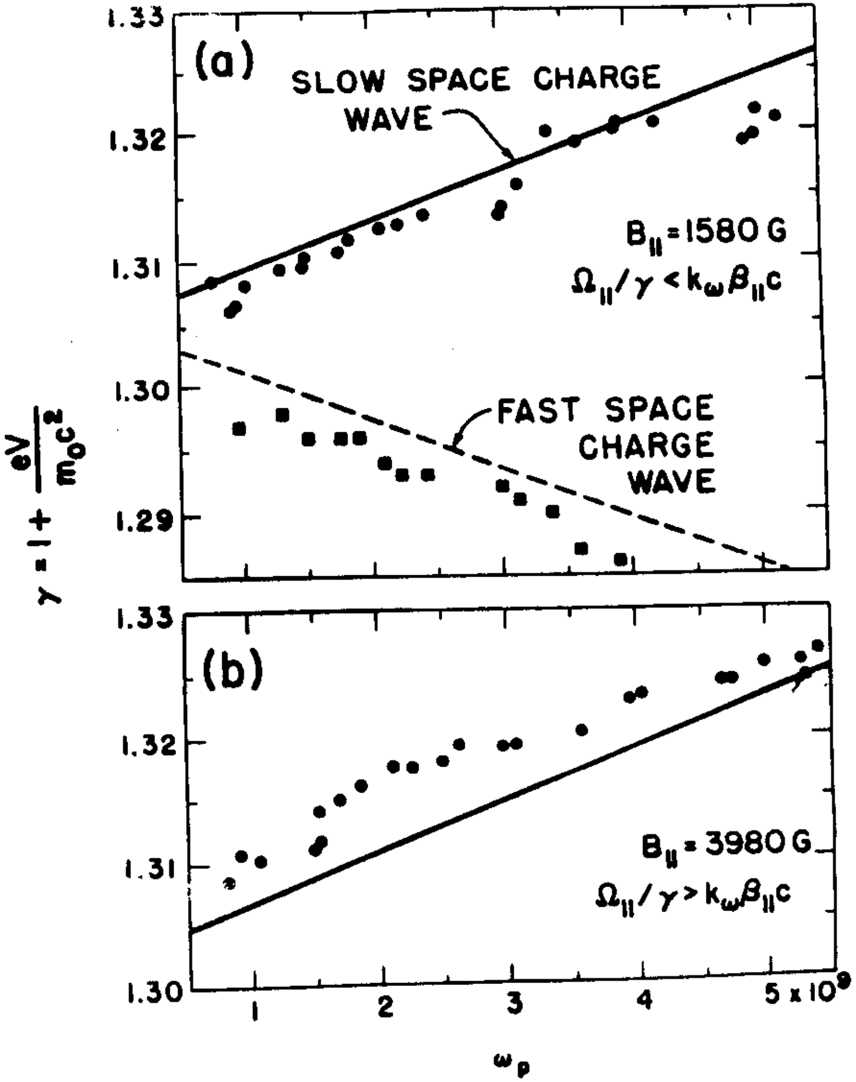
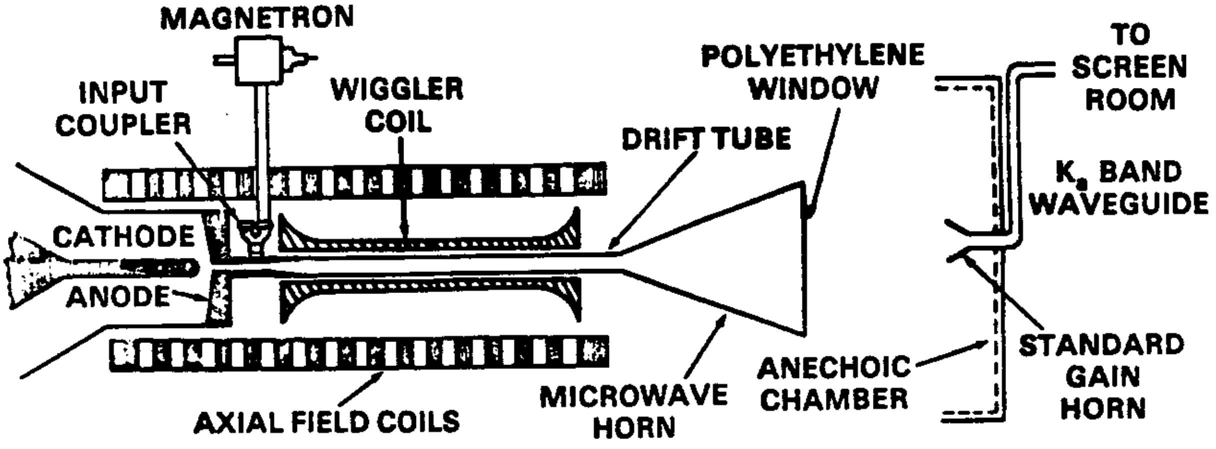



Fig. 15 MIT (1984)




FEL AMPLIFIER CONFIGURATION
[NRL]

$P \sim 80$ MW at 8 mm $= \lambda$
$\eta \approx 7\%$. (1982)

Figure 1.

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[S. Gold]
NRL
FEL:
Amplification of 35 GHz coherent signal
$\gamma = 2.8$
$\ell \sim 3$ cm
(1983)

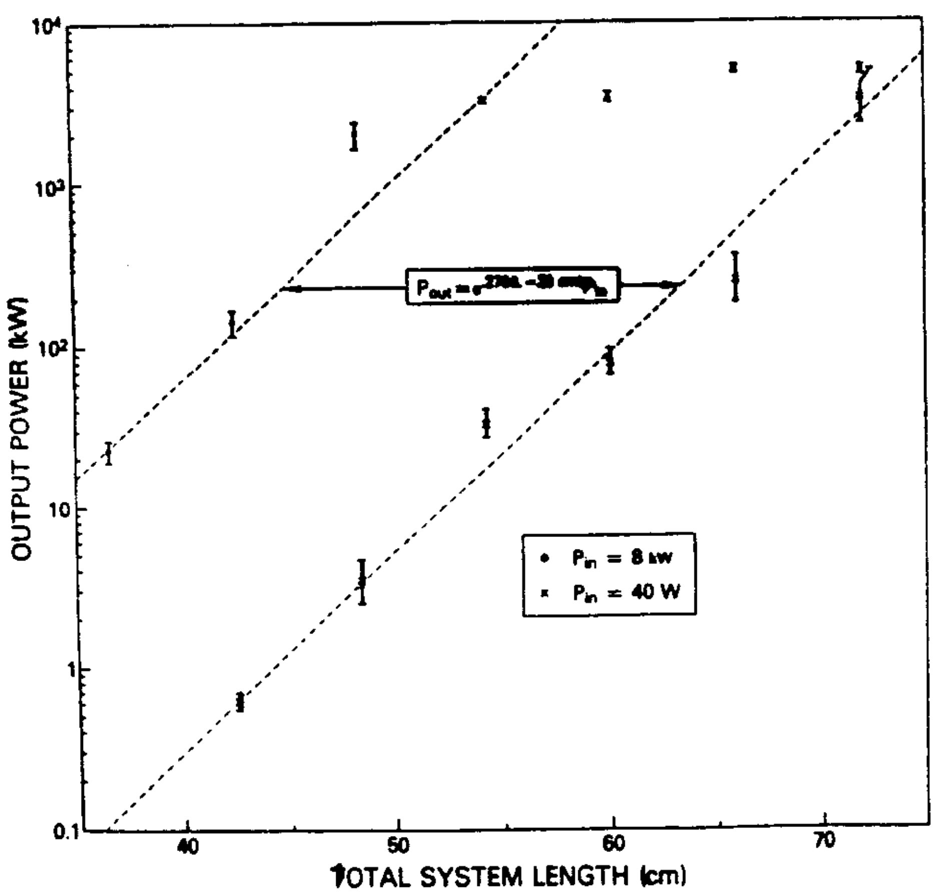



Figure 2

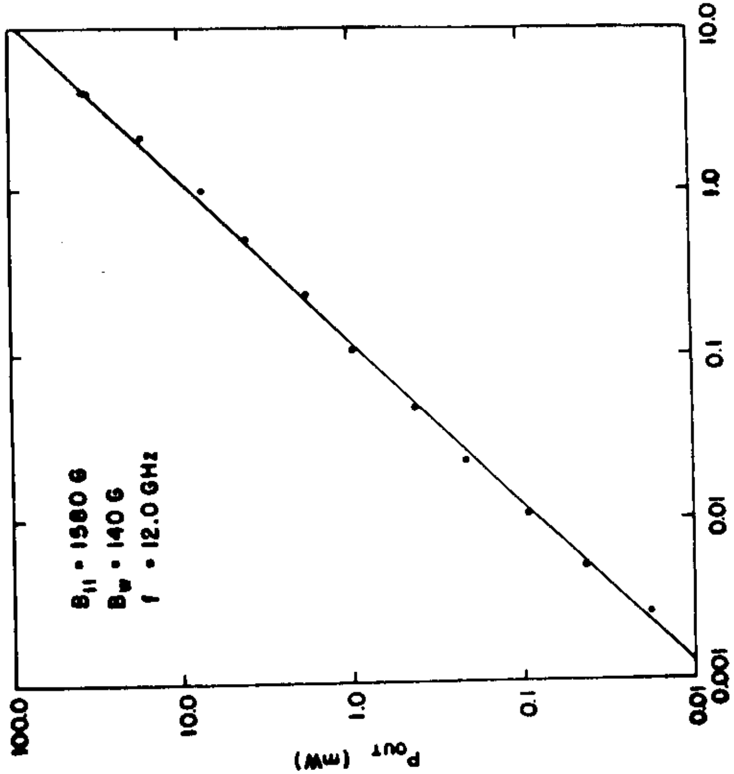



Fig. 11 MIT (1984)



To preserve Raman FEL gain, the inhomogeneous line-broadening should be $(\delta\gamma/\gamma)_{\parallel, inh.} \lesssim 1/N \sim \left(\frac{\delta\gamma}{\gamma}\right)_{\parallel, ho}$

N = # of undulator periods.

[Why? Homog. bandwidth of Raman FEL < $\overset{\text{growth rate}}{\omega_p/\gamma k_0 c}$ ; however - for Raman FEL, $\omega_p \cdot \frac{L}{\gamma c} \gtrsim 2\pi$ ...... $1/N$ follows]

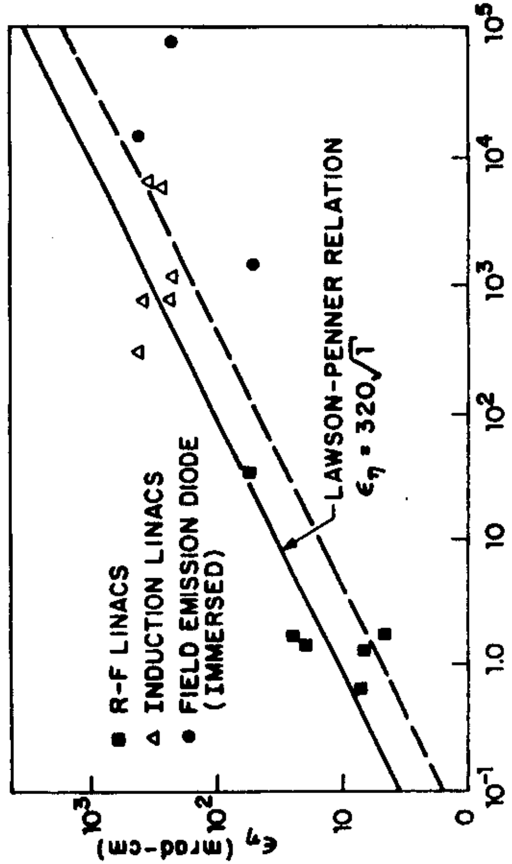
Contributions to inhomog. broadening:

1. Space-charge: $\left(\frac{\delta\gamma}{\gamma}\right)_{\parallel, s.c} \sim \frac{\omega_p^2}{4\gamma c^2} r_b^2 \sim \nu/\gamma \ll 1$.

2. Emittance (or divergence) of diode:
$(\delta\gamma/\gamma)_{\parallel, \varepsilon} \sim \frac{1}{2}(\varepsilon_N/r_b)^2/(1+a_w^2)$ ; $v_\perp/v \sim \frac{\varepsilon_N}{\gamma r_b}$

3. Undulator: $(\delta\gamma/\gamma)_{\parallel, un} \sim \frac{k_0^2 a_w^2 \, r_b^2}{1+a_w^2}$ $(B_0 =$

since $B_\perp(r) = B_\perp(0)\{1 + \frac{1}{4}(k_0 r_b)^2 + \cdots\}$
$\cdot \, {}^{\sin}_{\cos}\,(\theta - k_0 z)$




[Roberson 1982]



Diagnostic for measuring $(\delta\gamma/\gamma)_\parallel$ --
intense $e^-$ beam, $\gamma \lesssim 10$, $j > 10^3\ A/cm^2$:

**Thomson Backscattering of Coherent Light.**

Intense laser beam scatters from electron beam, spectral analysis gives $\delta\beta_\parallel$.

Electron Kinetic energy $= (\gamma - 1) mc^2$

In $e^-$ rest frame, there is a parallel velocity spread (or "Temperature"):

$$\left(\frac{\delta v_\parallel}{c}\right)_0 = \left(\frac{\delta\gamma}{\gamma}\right)_\parallel$$

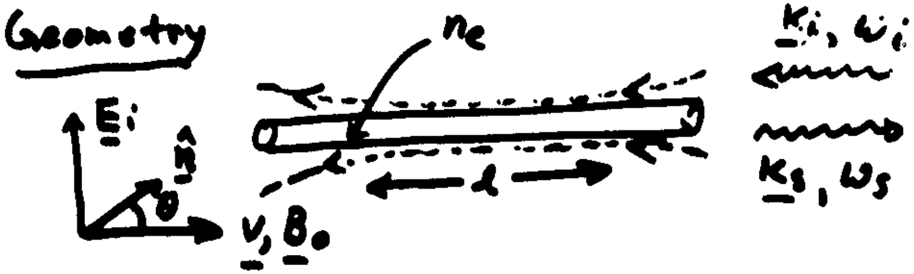
$$\left(\frac{\delta v_\parallel}{c}\right)_{lab} = \frac{1}{\gamma^2}\left(\frac{\delta v_\parallel}{c}\right)_0 = \frac{1}{\gamma^2}\left(\frac{\delta\gamma}{\gamma}\right)_\parallel$$

Photon scattering cross section is enhanced if scattered radiation is $\parallel$ to $\underline{v}_\parallel$:

$$\left(\frac{d\sigma}{d\Omega}\right)_{photon} \propto 4\gamma^2 r_0^2 ;$$

Energy scattered is upshifted by another factor $\sim 4\gamma^2$.

Geometry




$$\frac{\omega_s}{\omega_i} = \frac{1+\beta\cos\theta}{1-\beta\cos\theta} \simeq \frac{1+\beta}{1-\beta\cos\theta} \simeq \frac{4\gamma_{\parallel}^2}{1+\gamma^2\theta^2}.$$

scattering into solid angle $d\Omega$,

$$d\Omega \sim 2\pi\theta\, d\theta$$

causes a finite spread in $\omega_s$, $\delta\omega_s$,

$$\frac{\delta\omega_s}{\omega_s} \simeq \frac{\theta\,\delta\theta}{1-\beta} \simeq \frac{\gamma^2}{\pi} d\Omega$$

This should be small compared with

$$\left(\frac{\Delta\omega_s}{\omega_s}\right)_{\text{Doppler}} = 2\left(\frac{\Delta\gamma}{\gamma}\right)_{\parallel}.$$

If we take $\delta\omega_s/\omega_s = \frac{1}{2}\%$,

$d\Omega = 2\times10^{-3}$, $\theta \sim \delta\theta \sim 2\times10^{-2}$,

$\therefore$ f-20 optics



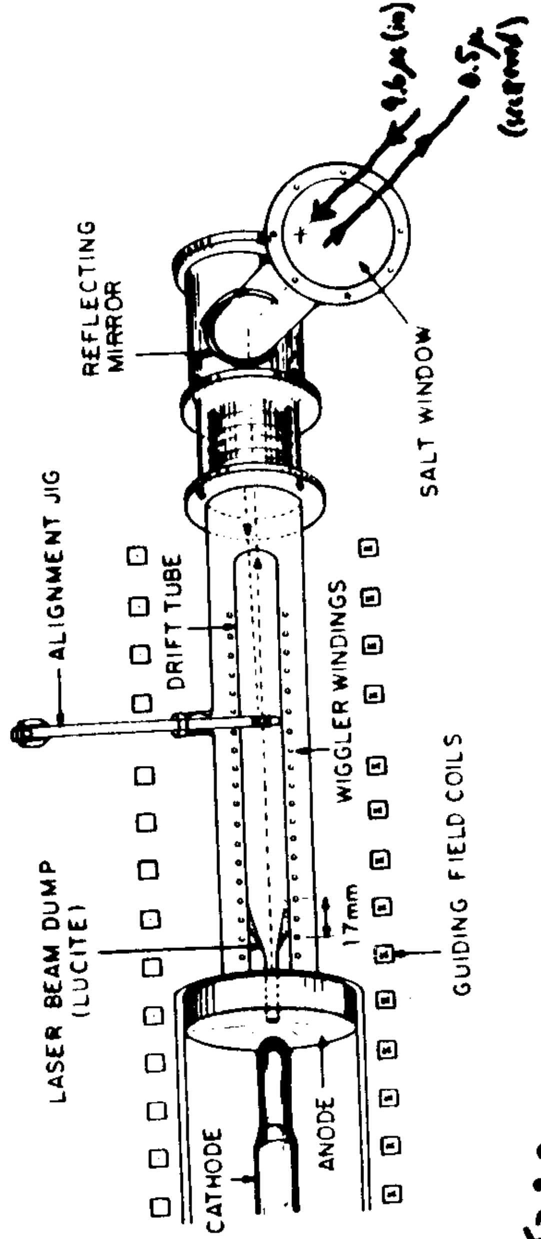



Figure III.1? Diagram of scattering region

$\gamma = 2.3$

$V_g = 700$ KV

$E = 670$ KV

$\Delta\phi = 30$ KV ($n = 3\times10^{11}$/cc)

or $j \sim 1$ KA/cm$^2$.

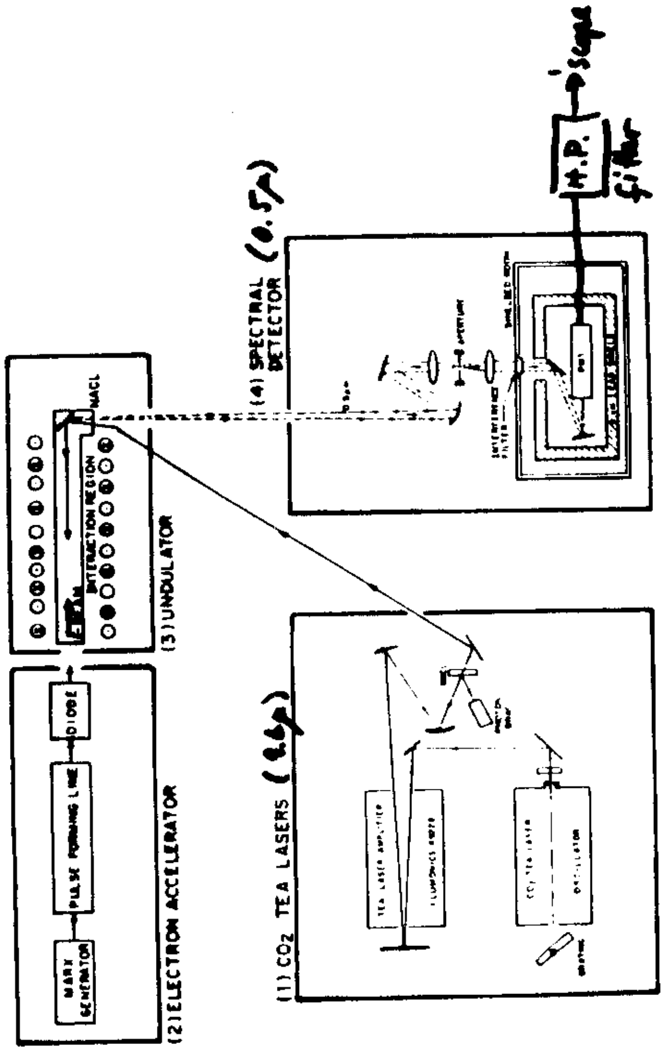
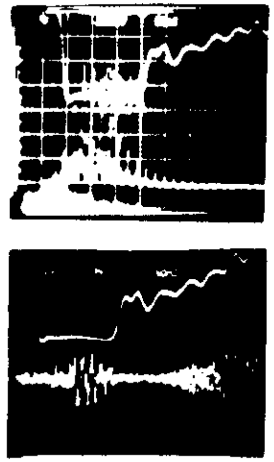
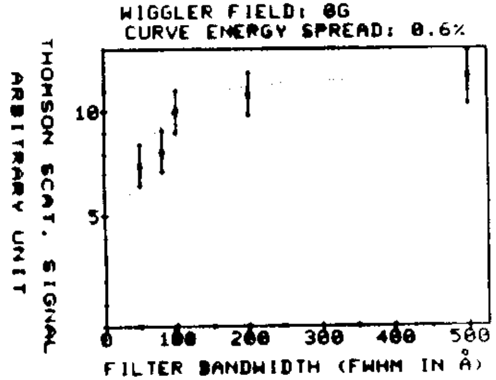


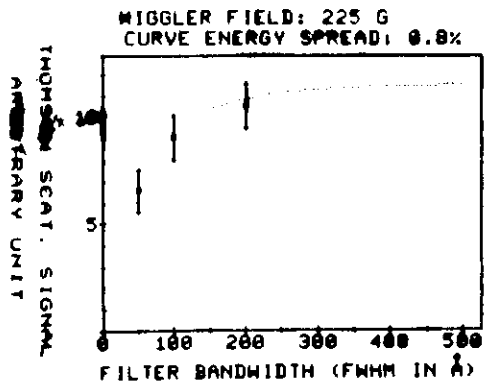

Figure III.1 Four major components of the experiment

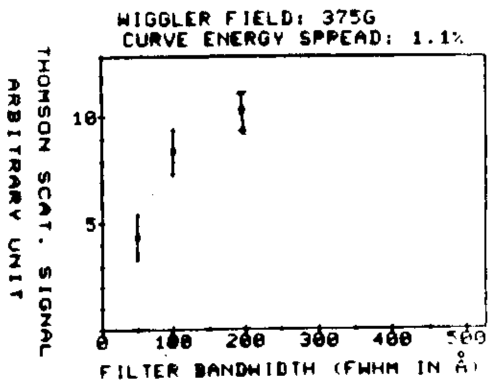


# RESULTS (1983)



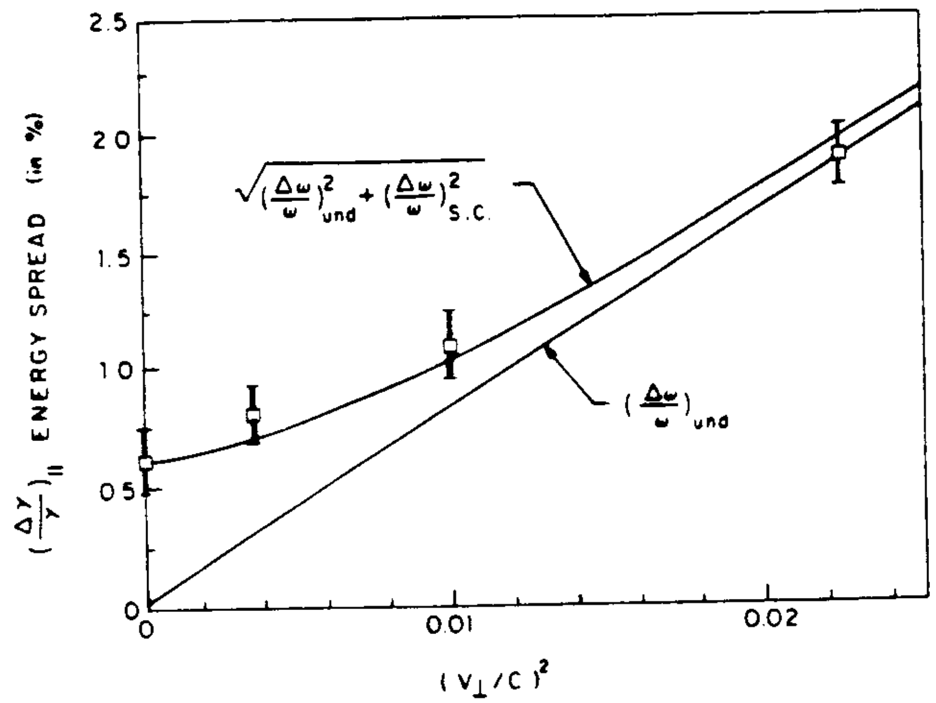



Figure ... Dependence of beam parallel energy spreads on the transverse velocity

Columbia (1984)

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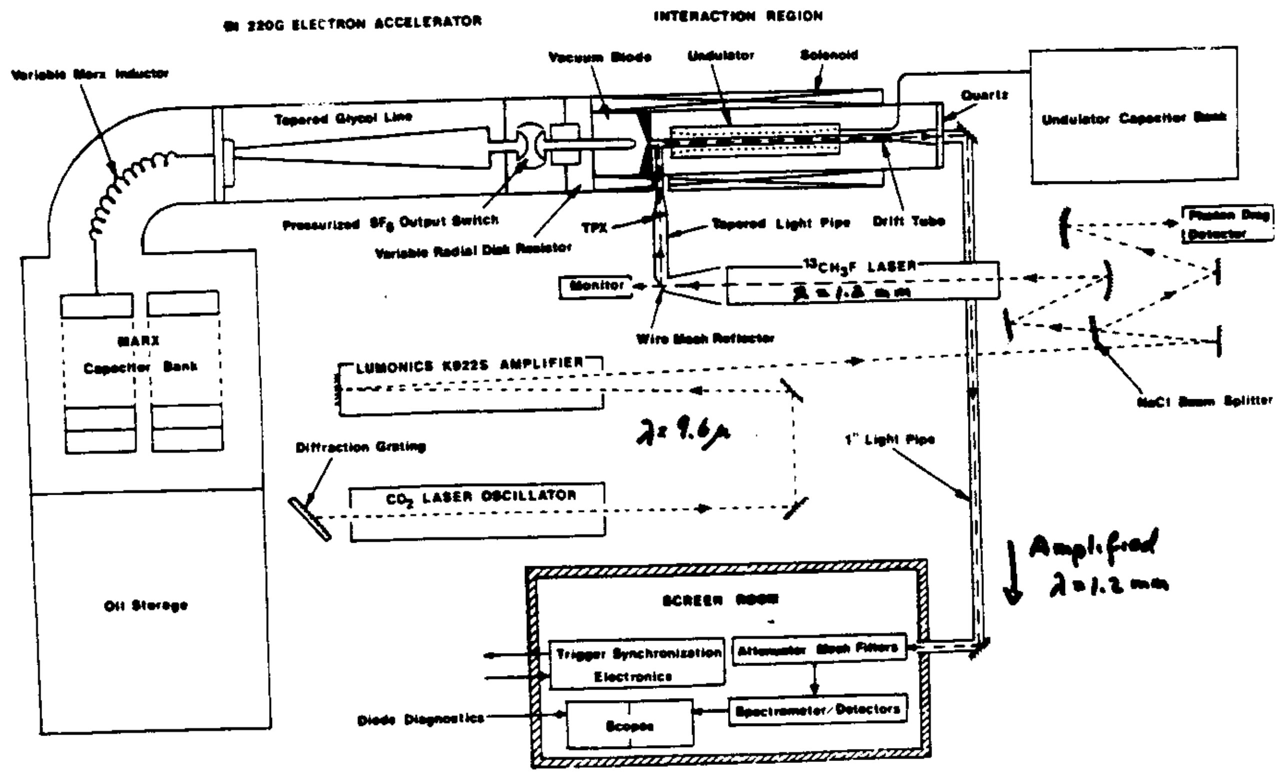
Columbia Raman FEL Amplifier (1989)

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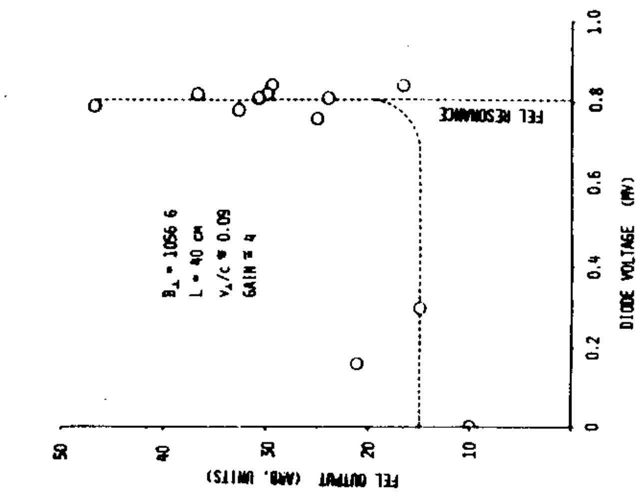
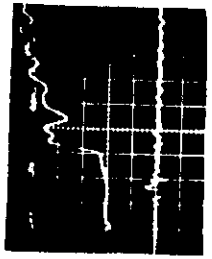
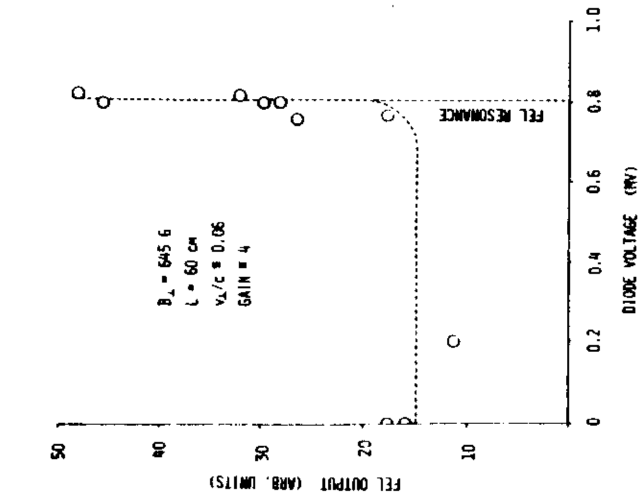
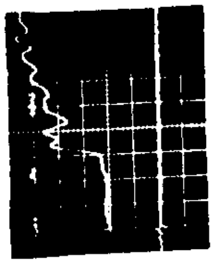




MEASURED FEL OUTPUT AT DIFFERENT TRANSVERSE FIELD VALUES VS. ELECTRON ENERGY FOR INTERACTION LENGTH, L = 60 AND 40 CM. THE SIGNAL IS NORMALIZED WITH RESPECT TO THE PHOTON DRAG TO PARTIALLY COMPENSATE FOR INPUT LEVEL FLUCTUATIONS.



## What about efficiency enhancement?

- use a generalized pendulum eqn

$$\psi'' = \varphi'' + \frac{dk_0}{dz} - \frac{\omega}{4c}\left(\frac{e}{mc^2}\right)^2 \frac{\partial^2 A_w}{\partial z^2}$$

$$- \frac{\omega}{2}\left(\frac{e}{\gamma mc^2}\right)^2 k_0 A_w A_s \cos\psi$$ ← pendulum term

$$+ \frac{2\omega_p^2}{c^2\gamma_\parallel^2}\left[\langle\cos\psi\rangle \sin\psi - \langle\sin\psi\rangle \cos\psi\right]$$

↖ space charge term.

s.c. can be "ignored" here if

$$n \ll k_0^2\gamma^3 A_w A_s / 4\pi mc^2 \sim B_\perp E_s \gamma / 8\pi mc^2$$

[at low $\gamma$, critical $n$ is same order as requirement $\theta_p \gg 1$]

Wave grows "slowly"

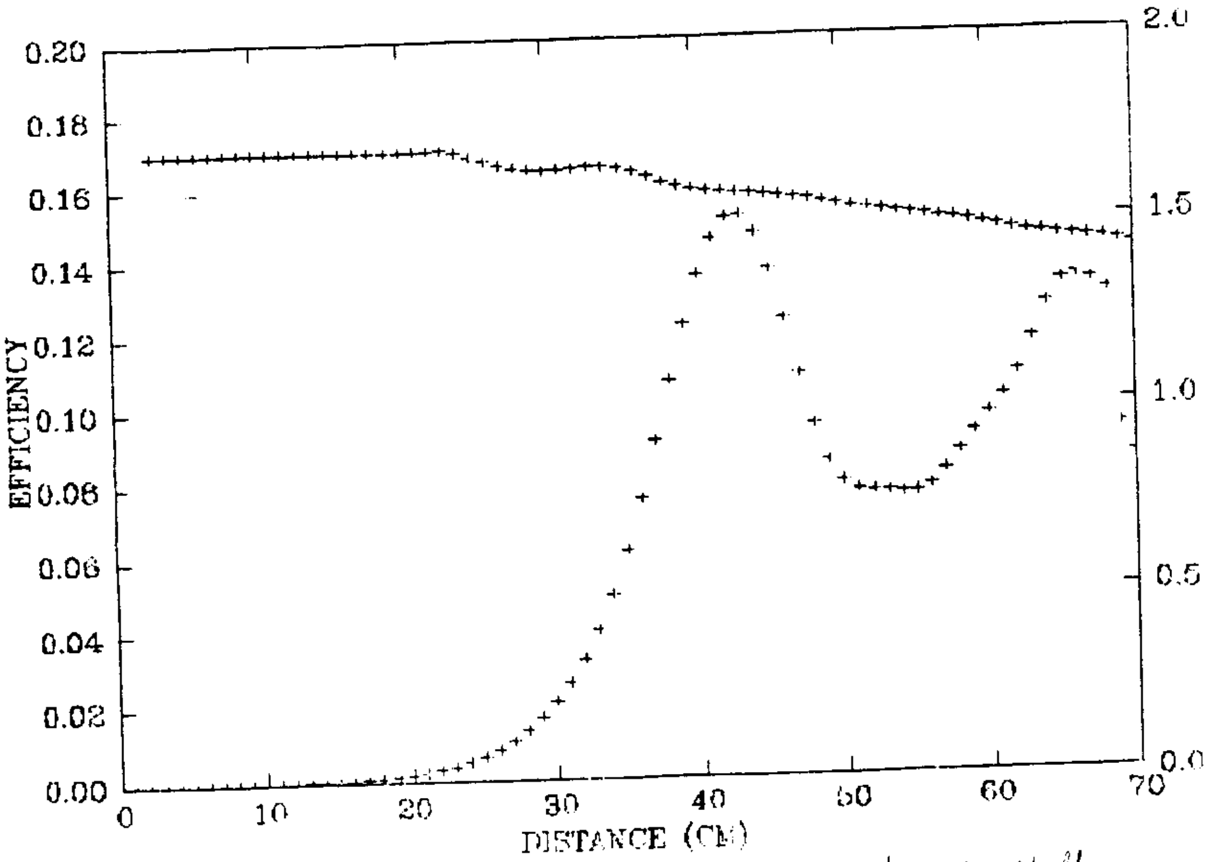
$$\left(\frac{\omega}{c} - k\right) A_s = \frac{\omega_p^2}{c^2}\left(\frac{c}{2\omega}\right)\left\langle \frac{\sin\psi}{\gamma}\right\rangle A_w$$

$$k^{1/2}\frac{d}{dz}\left(A_s k^{1/2}\right) = \frac{1}{2}\frac{\omega_p^2}{c^2} A_w \left\langle\frac{\cos\psi}{\gamma}\right\rangle$$

Re and Im part of refract. index of e-beam.

$\eta$ can be increased by changing wiggler period, amplitude, or both, so electron stays in resonance with the signal wave.

# FEL EFFICIENCY




Yee + Marshall

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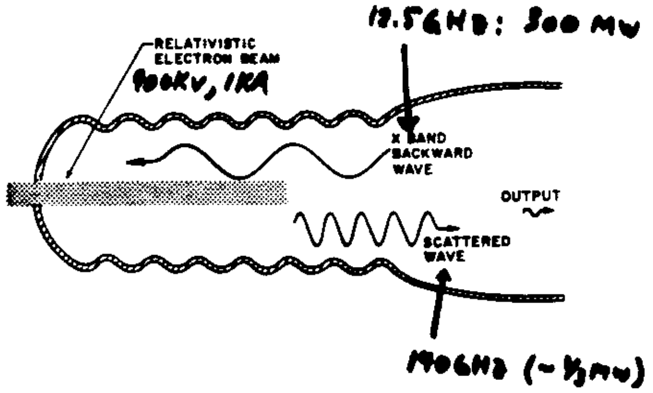


EM Undulator - FEL

[Carmel]
NRL (1983)

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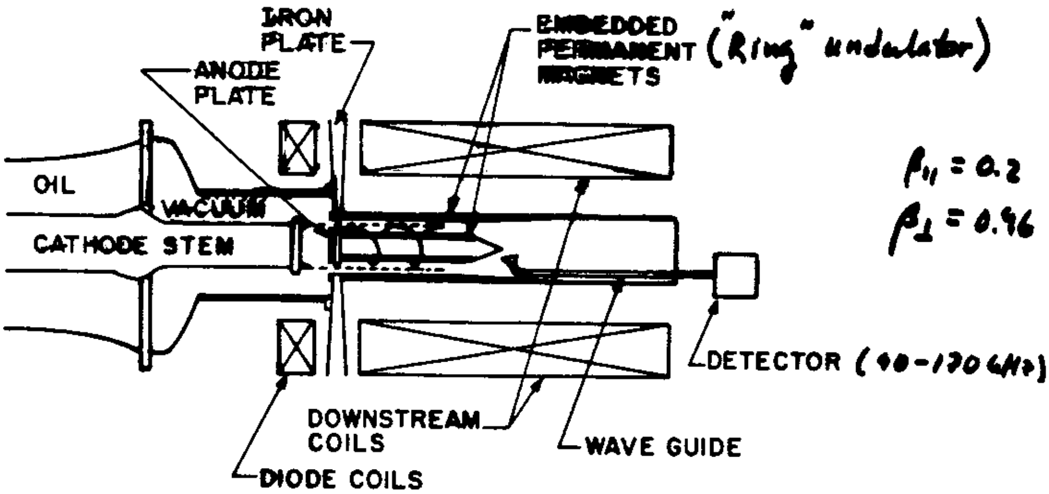
RING FEL (MIT - U Md.)
[Bekefi, Destler] (1983)

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