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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

(21 January - 22 March 1985)

Topical Meeting on the Free Electron Laser

STORAGE RINGS FOR FEL APPLICATIONS

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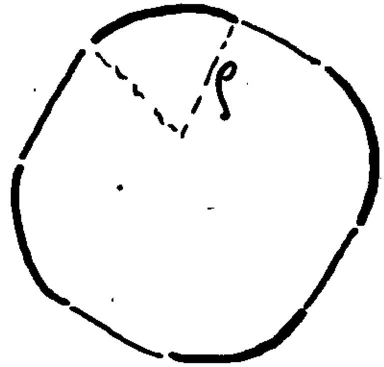
STORAGE RINGS FOR FEL APPLICATIONS

- High peak e- current
- Small beam sizes
- Short damping times
- Small momentum spread

- Basic physics of S.R.
- Limits to performance
- Examples

STORAGE RING



A string of bending magnets in a closed loop defines a reference orbit.
(Arcs of circle connected by straight sections)

E_0 : electron energy
B_0 : magnetic field (bending)
ρ_0 : bending radius

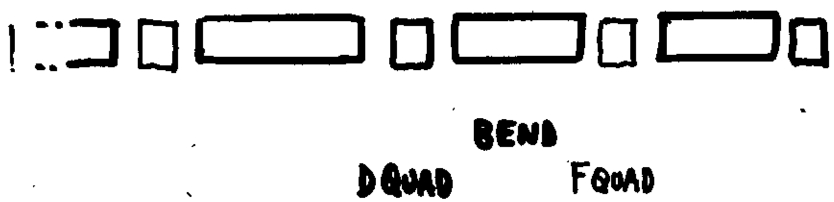
$$E_{0\,(GeV)} = .3\, B_{0\,(T)} \cdot \rho_{0\,(m)}$$

$$2\pi\rho_0 + \Sigma_u l_i = 2\pi R = C$$

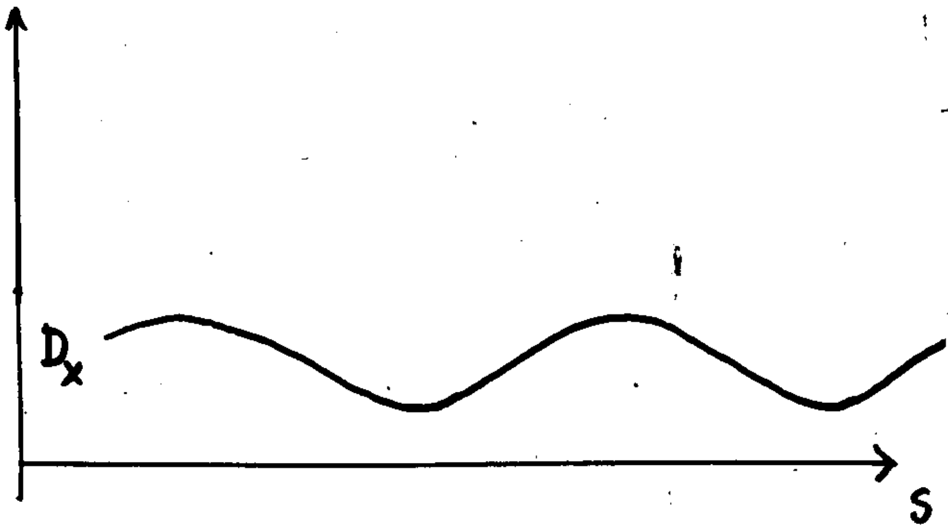
R : average radius

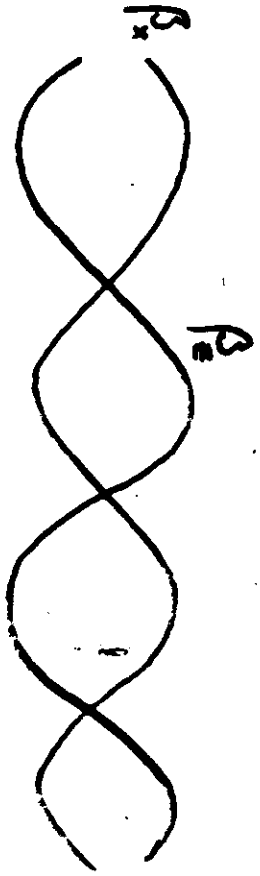
Define a linear coordinate, s, along the reference orbit

A well known ('strong focusing') lattice called 'FODO'.



Others are possible!





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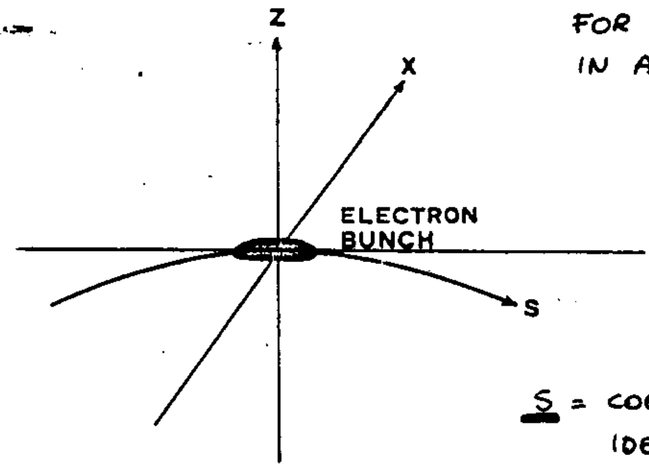
DEFINE A REFERENCE SYSTEM <u>WITH RESPECT TO</u>
<u>THE REFERENCE ORBIT</u>

REFERENCE SYSTEM
FOR AN ELECTRON
IN A STORAGE RING



$\underline{S}$ = COORDINATE ALONG THE IDEAL ORBIT, **<u>LONGITUDINAL</u>**

$\underline{X, Z}$ = <u>TRANSVERSE</u> COORDINATES FOR BETATRON AND SYNCHROTRON OSCILLATIONS

$x', z' = \frac{\partial x}{\partial s}, \frac{\partial z}{\partial s}$

TRANSVERSE EQUILIBRIUM ORBITS

If bending only is provided

- A particle started in the vicinity of the reference orbit, with initial conditions (x, x', z, z', E) different from those of the reference particle would be unstable: i.e. its trajectory would drift away from the reference one until the particle is "lost".

- In order to have stable orbits suitable linear restoring forces have to be provided, e.g. by means of quadrupolar fields interspersed between the bending magnets (or even incorporated in them).

Analogue of an optical system:

Bending magnets : prisms
F, D Quadrupoles : F, D lenses.

Note: because of Maxwell a magnetic lens that is focusing in one plane is defocusing in the other!

When the proper focusing is provided a set of periodic (closed) orbits is generated, defined by

$$\begin{pmatrix} x(s) \\ z(s) \end{pmatrix} = \begin{pmatrix} D_x(s) \\ D_z(s) \end{pmatrix} \cdot \frac{\Delta E}{E_0} \qquad (1)$$

$\left.\begin{matrix} D_x \\ D_z \end{matrix}\right\}$ are the "dispersion" functions.

Their shape (as funct. of s) is determined by the properties (strength, location..) of the magnetic elements.

The "sources" of dispersion are the bending fields.

Usually, therefore, $D_z = 0$.
(there are no vertical bends)

$$\boxed{\text{We will assume : } D_z \equiv 0}$$

Note:

The new trajectory does not have the same length as the reference one.

By simple geometrical arguments it can be shown that (to first order)

$$\frac{\Delta C}{C} = \alpha_c \frac{\Delta E}{E} \qquad (2)$$

with

$$\alpha_c = \frac{1}{C} \oint \frac{D_x(s)}{\rho(s)} ds \qquad (3)$$

(• If $D_x(s)$ is small in the bending magnets (where $\rho \neq \infty$) then α_c is small)

α_c is the "MOMENTUM COMPACTION" factor

LONGITUDINAL EQUILIBRIUM

• Electrons radiate energy in the bending fields (and may exchange energy with the EM field of an FEL).

• Unless the average energy radiated in one turn is restored the electron will drift away from its orbit (according to (1)).

Let U_0 be the average energy radiated per turn:

$$\boxed{U_0 = 88.5 \frac{E_{GeV}^4}{\rho(m)} \quad keV} \qquad (4)$$

• If, as it is usually, $U_0 \ll E$ a single RF station can be used to restore the energy by selecting the frequency, f_{RF}, voltage, V_{RF}, and phase, φ, with respect to the particle (bunch) so that:

$$f_{RF} = k \frac{c}{2\pi R} = k f_0 \qquad (5)$$

k: harmonic n°

and

$$e V_{RF}(\varphi) = U_0 \qquad (6)$$

NOTE: (5) can only be met exactly at one energy (see (2)), e.g. E_0.
It is the "synchronism" condition

OSCILLATIONS

A) TRANSVERSE (Betatron)
B) LONGITUDINAL (Synchrotron)

A) Transverse oscillations

- A particle not traveling exactly along the reference orbit corresponding to its energy will start oscillating around it, because of the restoring forces provided along the channel.
- The periodic envelopes of the oscillations are determined by two periodic functions: $\beta_x(s)$, $\beta_z(s)$

$$\boxed{E_{x,z} = A_{x,z}\sqrt{\beta_{x,z}}} \qquad (7)$$

- But: the oscillation of single particles is not periodic with the lattice period. The particle oscillates with a frequency:

$$f_{\beta_{x,z}} = Q_{x,z}\, f_0 \qquad Q_{x,z} = INT_{x,z} + \delta_{x,z}$$

$Q_{x,z}$ are the betatron wavenumbers (or tunes)
They must not be integer (certain fractions are also to be avoided) because of resonances.

K(s) = LINEAR RESTORING FORCE COEFFICIENT (Q-POLES)

$$\frac{d^2x(s)}{ds^2} = K_x(s)\,x(s)$$

K_x, K_z PERIODIC

$$\frac{d^2z(s)}{ds^2} = K_z(s)\,z(s)$$

BETATRON OSCILLATIONS

$$\begin{cases} x(s) = A_x\sqrt{\beta_x(s)}\cos(\varphi_x(s) - \theta_x) \\ z(s) = A_z\sqrt{\beta_z(s)}\cos(\varphi_z(s) - \theta_z) \end{cases}$$

$\beta_{x,z}(s)$ = BETATRON FUNCTIONS

$\varphi_{x,z}(s)$ = BETATRON PHASES $\qquad \varphi_{x,z}(s) = \int_0^s \frac{ds}{\beta_{x,z}(s)}$

BETATRON WAVENUMBERS (TUNES)

$$2\pi Q_x = \int_0^L \frac{ds}{\beta_x(s)}\ ;\ 2\pi Q_z = \int_0^L \frac{ds}{\beta_z(s)}$$

B. LONGITUDINAL OSCILLATIONS

- In the longitudinal direction a particle with energy different from the 'synchronous' one (that for which the trajectory is exactly $\frac{k\,c}{f_{RF}}$ long) will not be in phase with the accelerating field.
- It will therefore oscillate around the equilibrium phase, the restoring force being provided by the derivative of the accelerating field.

(Phase stability principle)

- Longitudinal oscillations are of course associated with energy oscillations.

'Longitudinal', 'Synchrotron' or 'Energy' oscillations are synonimous.

The oscillation frequency is

$$f_s = \nu_s f_0$$

ν_s (or Q_s) is the synchrotron tune (Wavenumber)

USUALLY: $\nu_s \ll 1$

ELECTRONS RADIATE PER TURN AN ENERGY

$$U_0 = 8.85 \times 10^{-5} \frac{E^4}{\rho} \qquad (U_0 \text{ IN GeV})$$

WITH THE SPECTRAL DISTRIBUTION $F(\xi)$ $\quad \xi = \lambda/\lambda_c$

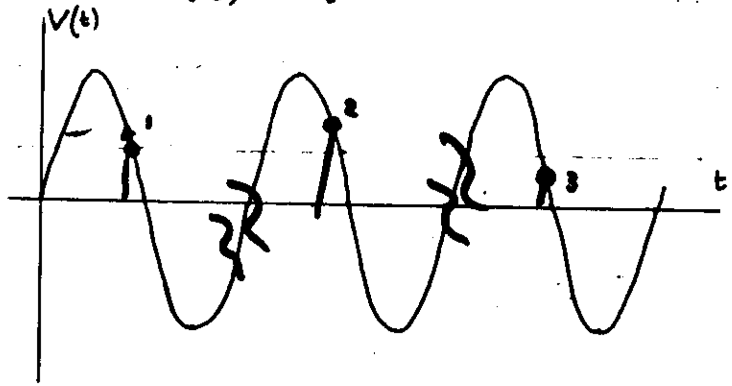
$$\lambda_c(\text{Å}) = \frac{18.6}{B(T)\, E^2(\text{GeV})}$$

THE ENERGY IS RESTORED BY A SYSTEM OF R.F. CAVITIES

$$V = V_0 \cos(h\omega_0 t + \chi)$$

FOR THE SYNCHRONOUS PARTICLE

$$eV(t_s) = U_0$$



$$\alpha_c = \frac{\delta L/L}{\delta E/E} > 0$$

SMALL AMPLITUDE OSCILLATIONS

$$\nu_s = \frac{1}{2\pi}\sqrt{\frac{\alpha_c\, e\, \dot{V}_0}{T_0\, E}} \qquad T_0 = \frac{L}{c}$$

NOTE: Usually

$Q_{x,z} \gg 1 \quad , \; Q_s \ll 1$

Synchrotron oscillations are very slow compared to betatron ones -

i.e.: betatron motion can be treated without taking synchrotron oscillations into account - (They are then considered as a slow perturbation)

THE ROLE OF RADIATION

- Transverse and longitudinal oscillations are excited by the random quantum emission of radiation. (Think of emission of a quantum where $D_x \neq 0$) (They can also be excited by other causes: interaction with EM fields, intrabeam scattering, ...).
- Radiation also provides a damping mechanism:
  - on average only transverse momentum is radiated because the RF restores the average longitudinal one
  - the amount of radiated energy depends on energy: particles with higher than average E radiate more -

The interplay of excitation and damping leads to finite equilibrium oscillation amplitude distributions for the particles of a beam (bunch): Beam Sizes

The distributions are gaussian with s.d. $\sigma_x, \sigma_z, \sigma_s$

NOTE : The longitudinal amplitude distribution s.d. σ_s corresponds to an energy distribution with s.d.

σ_E (or σ_p)

BETATRON AND SYNCHROTRON OSCILLATIONS ARE DAMPED

$$\tau_E(s) \approx \frac{T_0 E}{U_0} = 4.8\times 10^{-4}\frac{R\rho}{E^3\left(2+\frac{\alpha_c R}{\rho}\right)} \qquad R=\frac{L}{2\pi}$$

$$\tau_x(s) = 4.8\times 10^{-4}\frac{R\rho}{E^3\left(1-\frac{\alpha_c R}{\rho}\right)} \approx \frac{2T_0 E}{U_0}$$

$$\tau_z(s) = 4.8\times 10^{-4}\frac{R\rho}{E^3} \approx \frac{2T_0 E}{U_0}$$

STOCHASTIC EMISSION OF SYNCHROTRON RADIATION PHOTONS EXCITES SYNCHROTRON AND BETATRON OSCILLATIONS : THEY ARE COUNTERACTED BY DAMPING $\Rightarrow$ GAUSSIAN EQUILIBRIUM DISTRIBUTIONS

"NATURAL" OR "RADIATION" VALUES

$$\sigma_p = \frac{\sigma_E}{E} = 8.6\times 10^{-4}\frac{E(\text{GeV})}{\sqrt{\rho(\text{m})}}$$ ENERGY SPREAD

$$\sigma_s = \frac{c\,\sigma_p\,\alpha_c}{2\pi\nu_s}$$ BUNCH LENGTH

$$\sigma_x = \sigma_p\sqrt{\frac{2M\beta_x(s)}{1+\chi^2}+\eta^2(s)}$$ BUNCH WIDTH

$$\sigma_z = \sigma_p\sqrt{\frac{2M\beta_z(s)\chi^2}{1+\chi^2}}$$ BUNCH HEIGHT

χ IS THE COUPLING FACTOR BETWEEN VERTICAL AND HORIZONTAL BETATRON OSCILLATIONS

$$\varepsilon_{MIN} < \varepsilon < 1$$

NOTE: The damping can be increased by increasing the energy radiated per turn.

e.g.: by adding extra bends (wigglers)

If proper precautions are taken damping can be increased more than excitation and smaller (but not <u>much</u> smaller) natural beam dimensions can be obtained.

BEAM SIZES AND EMITTANCES

- Transverse envelopes are proportional to $\sqrt{\beta}$ (see (7)).

It is therefore obvious that beam sizes are proportional to the same quantity:

$$\sigma_\beta \cong \sqrt{\mathcal{E}_\beta} \cdot \sqrt{\beta}$$

$\mathcal{E}_\beta$'s are called the beam EMITTANCES (betatron)

they are 'invariants':

$$\mathcal{E}_\beta \neq \mathcal{E}_\beta(s)$$

In reality the horizontal beam size is also affected by the beam energy spread:

$$\sigma_x^2 = \mathcal{E}_{\beta_x}^2 \beta_x + S^2(s)$$

$$\sigma_z^2 = \mathcal{E}_z^2 \beta_z$$

NOTE:
- wherever $D_x(s) = 0$ also $S(s) = 0$
- $\mathcal{E}_z$ would be neglibly small if no coupling existed between the x and the z planes

THE "TWISS FUNCTIONS"

$$\alpha_{x,z}(s) = -\frac{1}{2}\,\frac{d\beta_{x,z}(s)}{ds}$$

$$\gamma_{x,z}(s) = \frac{1+\alpha^2_{x,z}(s)}{\beta_{x,z}(s)}$$

$$\eta \equiv D_x$$

$$\boxed{M = \frac{1}{2\pi\rho}\int_{\text{MAGNETS}} \left(\gamma_x \eta^2 + 2\alpha_x \eta\eta' + \beta_x \eta'^2\right) ds}$$

THE "INVARIANT"

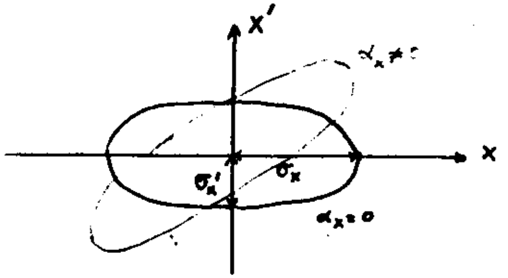
THE BEAM DIVERGENCES (CONNECTED WITH THE INHOMOGENEOUS BROADENING OF THE LINEWIDTH)

$$\sigma_{x'} = \sigma_p \sqrt{\frac{2M\gamma_x(s)}{1+\varepsilon^2} + \eta'^2(s)}$$

$$\sigma_{z'} = \sigma_p \sqrt{\frac{2M\gamma_z(s)\,\varepsilon^2}{1+\varepsilon^2}}$$

THE "NATURAL EMITTANCE"

$$\boxed{\boxed{\varepsilon_0 = 2M\sigma_p^2}} = \sigma_x\,\sigma_{x'} \quad \text{FOR } \varepsilon = 0,\ \alpha_x = 0$$



$\pi\varepsilon_0 = \pi\sigma_x\sigma_{x'}$ IS THE AREA OF THE ELLIPSE

IN SUMMARY:

In an e- storage ring

ε_x is determined by the lattice (FOR ANY GIVEN ENERGY)

ε_z by ε_x and the minimum obtainable coupling (errors, misalig.,.. mult. Touschek..)

$\sigma_x(s)$, $\sigma_z(s)$ determined by $\varepsilon_{x,z}$ and lattice:

$$\boxed{\begin{aligned}\sigma^2_{x}(s) &= \beta_x(s)\,\varepsilon_x\,\frac{1}{1+\chi^2}\\ \sigma^2_{z}(s) &= \beta_z(s)\,\varepsilon_z\,\frac{1}{1+\chi^2}\end{aligned}}$$

where $D_x = 0$

e.g. some straight sections

$$\boxed{\varepsilon_z = \chi^2\,\varepsilon_x}$$

χ : coupling coeff : $\begin{cases} 0 & \text{no coupling} \\ 1 & \text{full coupling}\end{cases}$

— *** —

For a very well corrected lattice

$\chi^2 \approx .01$ (and without other problems!)

Conservatively

$\chi^2 \approx .1$

AT PLACES WHERE $D_x \neq 0$

$$\boxed{\sigma_x^2 = \varepsilon_x \beta_x \left(\frac{1}{1+k^2}\right) + \sigma_p^2 D^2}$$

LAST :

$$\boxed{\begin{aligned} \sigma_x'^2 &= \frac{\varepsilon_x}{\beta_x(s)} \\ \sigma_z'^2 &= \frac{\varepsilon_z}{\beta_z(s)} \end{aligned}}$$

$$\varepsilon_x \cong \sigma_x \cdot \sigma_x'$$

- The unperturbed longitudinal beam size is determined by the energy spread, σ_p :

$$\boxed{\sigma_s = \sigma_p \frac{\alpha_c}{Q_s} \frac{c}{2\pi}}$$

The longitudinal beam size determines the peak current.

Assume we have stored n_b electrons in a bunch. The average circulating current is

$$\boxed{i = e\, n_b f_0}$$

and the peak current

$$\boxed{i_p = i \frac{2\pi R}{\sqrt{2\pi}\, \sigma_s}}$$

CONCLUSIONS

To achieve high peak currents and small beam dimensions (at the place where we have one insertion device: usually a straight):

- small σ_e
- small emittances
- $D_x = 0$ in the straight

SMALL EMITTANCE LATTICES

- For all usual lattices it can be shown analytically that

$$\boxed{\varepsilon_x = K_\varepsilon \, \theta_B^3 \, \gamma^2} \quad ; \quad \gamma = \frac{E_0}{m_0 c^2}$$

where K_ε is a constant for any given lattice configuration, and θ_B is the bend angle

$$\theta_B = \frac{l_{bend}}{\rho_0}$$

- To make K_ε small one has to make D_x small in the bending magnets

- Making D_x small makes α_c small (see (3)) and this helps in having a small (unperturbed) σ_e.

WARNING: The dependence on γ^2 is to be considered with care: it is much easier to make θ_B small at high energy than at low energy.

A special, easy to understand, type of lattice is the so called Chasman-Green lattice.

It has, at best:

$$K_\varepsilon = 2.5\ 10^{-14}\ m \cdot rad^{-2}$$

Also, it is designed to naturally give $D_x = 0$ in the straights where one would put an insertion device.

NOTE : • In practice the above value of K_ε is <u>difficult</u> to achieve

$$K_\varepsilon \simeq 5\ 10^{-14}\ m \cdot rad^{-2}$$

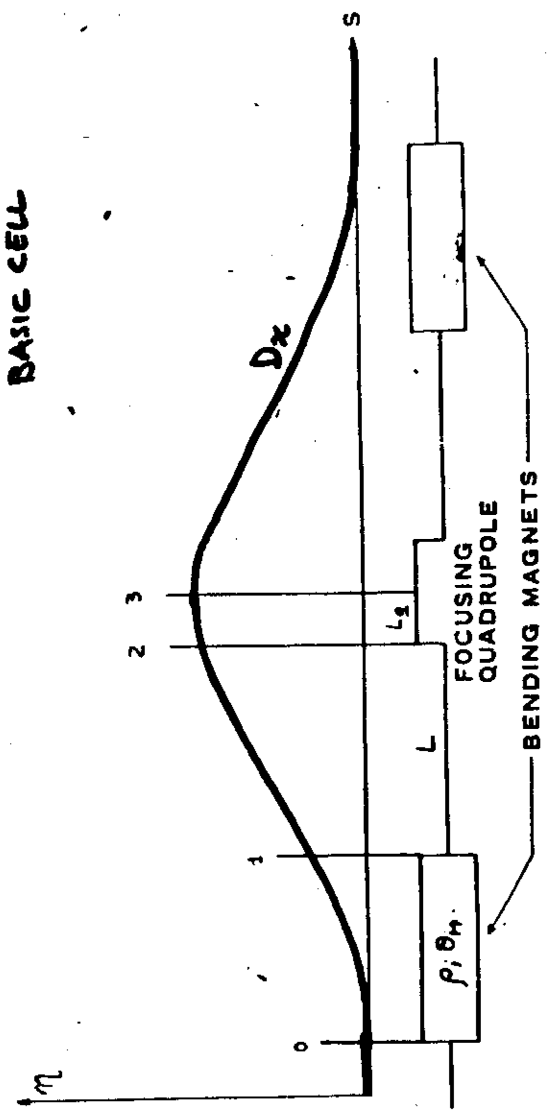
<u>is more usual</u>.

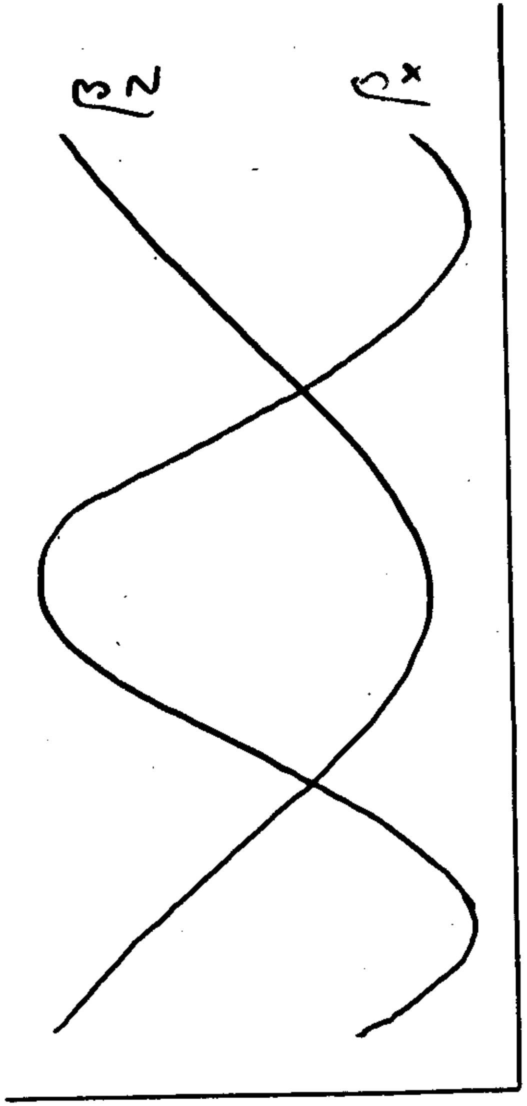
IN GENERAL:

$\varepsilon_x \ll 10^{-8}\ \pi\, m \cdot rad$ is <u>difficult</u> to achieve

> NOTE : For photon beams things are much simpler:
>
> $$\varepsilon_x = \varepsilon_z = \lambda \quad !$$

CHASMAN-GREEN LATTICE
BASIC CELL





THERE ARE MANY PROBLEMS

to actually build a machine with small beam sizes, high peak currents, long lifetime and a comfortable operating range of tunes.

We will glance at some:

- Nonlinearities (aberrations)
- Lifetimes
- Trapped ions
- Longitudinal instabilities leading to lengthening of the bunch

SMALL BEAM SIZES

SMALL EMITTANCES

STRONG FOCUSSING !

STRONG ABERRATIONS

- CHROMATIC
- OSC. AMPL. DEPENDENT
- ...

STRONG CORRECTIONS

VERY NONLINEAR DYNAMICS

EFFECT OF ERRORS
TOLERANCES

CHROMATICITY:

The natural linear chromaticity is:

$$\boxed{\frac{\Delta Q}{\Delta p/p} = \xi_T = \frac{1}{4\pi}\int_0^C \beta(s)\, K(s)\, ds} \qquad 1)$$

(derived from the equation of motion
$x'' + \left(\frac{1}{\rho^2} + K(s)\right)x = \frac{\Delta p/p}{\rho}$)

- Eq. 1 can be written in both planes (x, z).

$K(s) = \frac{G(s)}{B\rho}$ is the strength of the q-pole field at s, $G = B/a$

- ξ_T is large when strong focusing elements are placed where the β is high.

If typically the beam energy spread is $\sim \pm .02$ $(10\,\sigma_p)$ then

$$\Delta Q > .5 \text{ if } \xi_T \gtrsim 25$$

Physically: a particle with higher/lower energy is less/more focused

In the low emittance lattices we discussed above we have chromaticities of ~ 100.

They have to be corrected by the addition of sextupoles producing:

$$\left[\Delta Q / \Delta p/p\right]^{(S)} = \xi_S = -\frac{1}{4\pi}\int_0^C \beta(s)\cdot\eta(s)\,K'(s)\,ds \quad 2)$$

$K'(s)$ is the sextupole field strength at S

$$\left(K'(s) = \frac{G^S(s)}{B\rho_0} \quad , \quad G^S = \frac{1}{2}\frac{B''}{a^2} \right)$$

- Eq. has to be applied in both planes. Remember: Sx-pole effect opposite in the two planes!

- ξ_S' has to be large, to compensate large ξ_T.
- $\eta(s)$ is small: $K'(s)$ tends to be larger than for more usual lattices.

$\boxed{\text{i.e.: STRONG SEXTUPOLES}}$

Non linear effects

Strong sextupoles $\longrightarrow$ strong nonlinear terms

(Eq. of motion now is

$$x''(s) + \left(\frac{1}{\rho^2}+K\right)x = \frac{1}{\rho}\frac{\Delta p}{p} - \frac{1}{\rho}\frac{\Delta B}{B}$$

where ΔB is the diff. with respect to q-pole)

- To second order in $\Delta p/p \equiv \delta$ find several other effects: (chromatic)
  - β variation with δ
  - η " " "

  :

They are essentially related to integrals of lattice functions such as: ..

- $\beta^2(s)\,\eta(s)\,K'(s)$
- $\eta^2\,K'(s)$
- $\beta^{3/2}\,\eta(s)\,K(s)$

- Non-chromatic higher order effects are also found: e.g. Q-shift on β-tron amplitude

- Strong nonlinearities also excite resonances

  e.g. 3rd order resonances excited by sextupolar fields.

  — DYNAMIC APERTURES!!

All these effects have to be minimized to have good dyn. apertures.

- Complicated sextupole arrangements (e.g. 6 families instead of the 2 fam. necessary to correct the linear part, special locations) are studied by means of tracking programs —

This is one of the most interesting problems in lattice design.

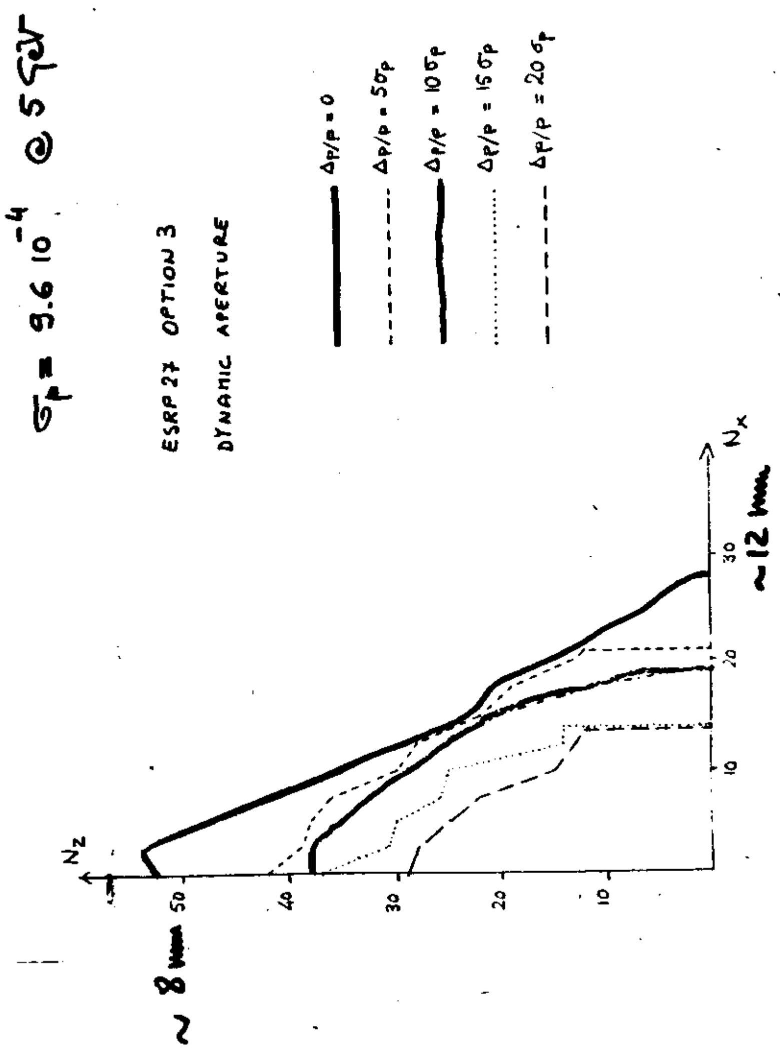


FIG. 12 - DYNAMIC APERTURE FOR DIFFERENT SYNCHROTRON OSCILLATION AMPLITUDES

ION TRAPPING

(Baconnier, Brianti)

Single beam (e^- only)

- Mass A is trapped if

$$\boxed{A \geq A_{c_{x,z}} = 2\left(\frac{\pi R}{b}\right)^2\left(\frac{m_e}{m_p}\right)\frac{I}{I_0}\,\frac{1}{\sigma_{x,z}(\sigma_x+\sigma_z)}}$$

b: number of evenly spaced bunches

I: total current, $I_0 = 17045$ A

- The ion density in the beam cannot exceed

$$\boxed{d_{i_\ell} = A_c\,\frac{b^2(1+\sigma_z/\sigma_x)}{k}}, \quad k = 5.33\,10^{-14}\ m^3$$

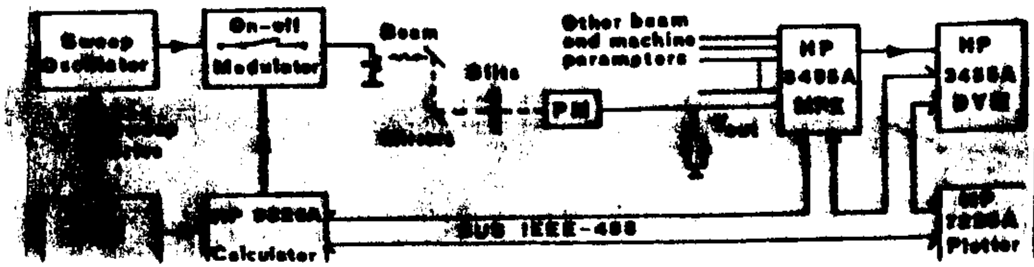
- A lower limit, d_m exists if only singly ionized ions are trapped

$$\boxed{d_m = 3.2\,10^{13}\cdot P_{(\text{ntorr})}}\ (m^{-3})\ @\ 300^\circ K$$

- An ion density d_i produces a Q spread (in z), with a maximum Q-shift given by

$$\boxed{\Delta Q_z = \frac{r_e}{\gamma}\int\frac{d_i\,\beta_z(s)}{1+\frac{\sigma_z}{\sigma_x}(s)}\,ds}$$

Measurement of Q_z



'Microwave' instability

(limits achievable peak current)

- Is due to self generated EM fields in the beam surroundings acting back on the same bunch.

- Is characterised by a 'broad band' normalised impedance

  $\left|\frac{Z_n}{n}\right|$: a few Ohms for smooth vacuum chambers.

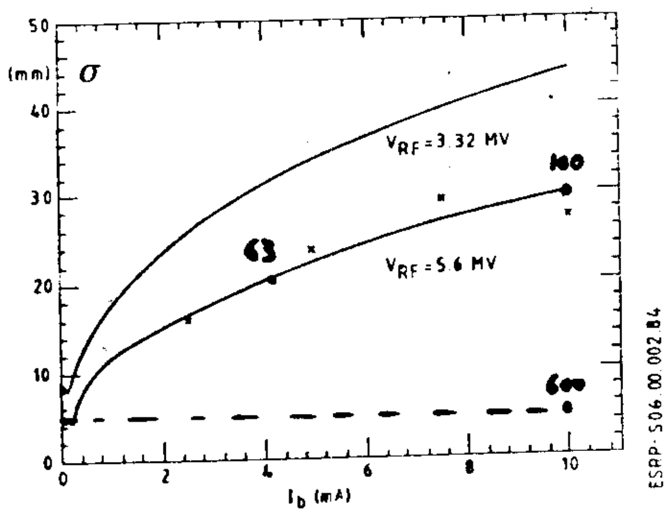
- The threshold current is

$$\boxed{i_{p_{th}} = \frac{2\pi E_0\,\alpha_c}{e\,|Z_n/n|}\,\sigma_p^2}$$

- If the threshold is exceeded the bunch starts lengthening and peak current increases very slowly when increasing the average current.

- Worse: σ_p increases (with σ_e)

I_p (A)


σ
(mm)
50
40
30
20
10
0
0
2
4
6
8
10
I_b (mA)
V_{RF} = 3.32 MV
V_{RF} = 5.6 MV
160
63
600
ESRP-S06.00.002.84

ESRF