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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

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Topical Meeting on the Free Electron Laser

COLLECTIVE INSTABILITY OF AN FEL

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COLLECTIVE INSTABILITY of an FEL

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nsls

VUV

X Ray

Basics of Free Electron Devi[ces]

Kinetic Energy $(\gamma-1)mc^2$ Free Energy Source Active Medium

← Electron Beam

How do I convince the electrons to give up their energy?

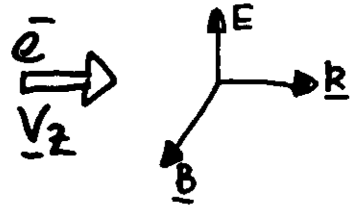
1.) <u>spontaneous</u> <u>emission</u>: accelerate them!

incoherent, low power

2.) <u>stimulated</u> <u>emission</u>: $mc^2\dot{\gamma} = -e\underline{v}\cdot\underline{E}$

To have a net energy exchange you must have $\underline{v}$ along E!

i) Free electron & free EM wave



$\underline{v}\cdot\underline{E} = 0$

No E_z!

ii.) Waveguide EM wave

TM wave: E_x, E_z, B_y (no B_z, TM)

Now $\underline{v}\cdot\underline{E} \neq 0$! BUT

$E = E_0(x,y)\cos(kz-\omega t)$

$\dot{\gamma} \sim v_z E_0 \cos(kz-\omega t)$; $z = v_z t$

$v_z - c = v_z - v_\phi$ $\quad \cos[k(v_z-c)t]$; $c = v_\phi$

averages to zero because it is rapidly oscillating.

<u>Synchronism</u> ($v_z = v_\phi$)

$\dot{\gamma} \sim \cos\phi \Rightarrow \dot{\phi} = 0$ <u>Stationary Phase</u>

Since we can't make $v_z = c = \omega/k = v_\phi$, we must find a way to make

$v_z = v_\phi < c$

iii.) Slow wave Structures

a.) dielectric loaded waveguide: $v_\phi = \frac{c}{\sqrt{\varepsilon}}$
b.) disk loaded structures for acc.
$v_\phi < c$

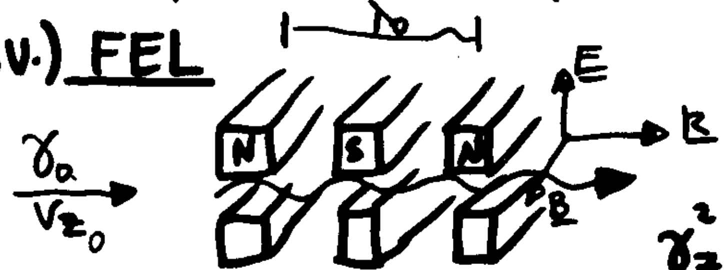
Couple v_z to E_z!

Problem: E_z exists only near the surfaces of the structure — "near" field devices.

$$E_z \sim e^{-\frac{2\pi x}{\lambda\beta\gamma}}$$

Can't fabricate devices small enough; ($a \sim \lambda$)
also power dissipation problems.

iv.) FEL



$$\gamma_z^2 = \frac{\gamma_0^2}{(1+K^2)}$$

In the undulator: $\frac{v_z}{c} = \left(1 - \frac{(1+K^2)}{2\gamma_0^2}\right)$

Field: $E = E_0 e^{i(kz-\omega t)}$

$$v_\perp = \frac{cK}{\gamma_0} e^{i(2\pi/\lambda_0)z}$$

Look at energy change of electron,

$$\dot{\gamma} \sim \underline{v} \cdot \underline{E} = \frac{KE_0}{\gamma} \cos[(k_0+k)z - \omega t]$$

Phase velocity, $v_\phi = \frac{\omega}{(k_0+k)} < c$

Couple $v_\perp$ to $E_\perp$ through the undulator.

Synchronism: $v_\phi = v_z$

Set $v_z = \frac{\omega}{(k_0+k)}$ use $\omega = ck$

Solve for $\omega = \omega(v_z, k_0)$

$$\boxed{\omega = 2ck_0\gamma_z^2 = \frac{2ck_0\gamma_0^2}{(1+K^2)}}$$

Resonance or Synchronism Condition

Absolute Phase: Accelerator or FEL (IFEL)

Collective Instability

Question: If I keep adding electrons will I reach a point after which the electrons no longer behave as individual particles, but instead exhibit collective behavior?

Will the system organize itself?

Ans. Perhaps!

Depends on
i) number of emitters & their proximity ⇒ n_e
ii) strength of communication b/wn electrons
iii) time given to organize itself ⇒ L_W

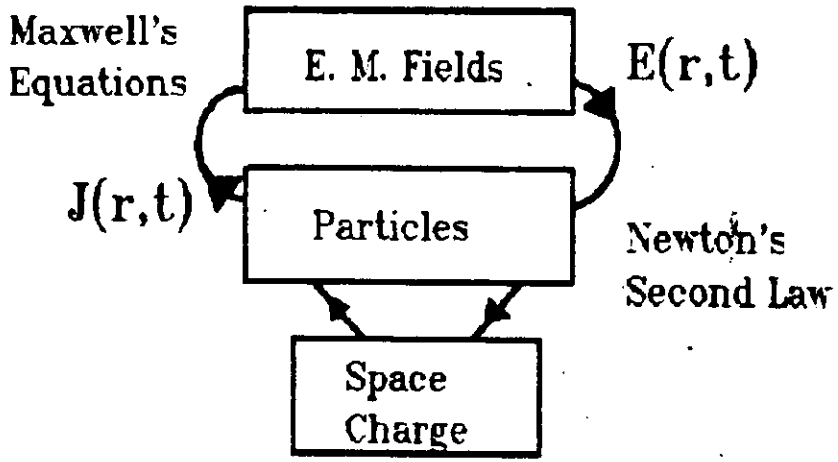
How do the electrons communicate?

i) radiation field: "Mean" field ⇒ E
ii) space charge
a.) bad if it doesn't aid organization
b.) good if it does (plasma wave) ⇒ Raman

Communication Breakdown

i) energy spread ii) space charge iii) etc.

Self Consistent Eqns.



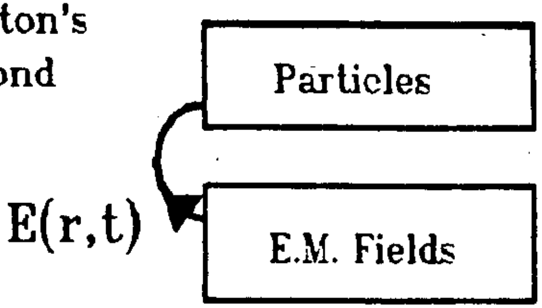
Particles: (φ,γ)

$\varphi = (k_0 + k)z - \omega t$
= phase of e^- relative to pond. wave
γ = energy of electron
(ω, k) = frequency & wave number of the radiation field
λ_0 = period of undulator
$k_0 = 2\pi/\lambda_0$

Field: $E = E(z,t)\exp(i[kz - \omega t])$

Small Signal Gain

Newton's Second Law



Assume changes in amplitude & phase of $\underline{E}$ are negligible!

$$\text{Gain} = (E_f^2 - E_i^2)/E_i^2$$

$E_f^2/8\pi$ = change in kinetic energy density of the electron beam in one pass through the undulator

Compute ΔK.E. to second order in E_i from

$$\dot{\gamma} = \frac{eKE_i}{\gamma mc}\cos(\varphi[z] + \varphi_0)$$

Gain < 1 & $d\alpha = 0$ for this analysis to be valid!

Equations of Motion

Particles: $\dot{\varphi}_j = (k+k_o) - \omega$

$$\dot{\gamma}_j = \frac{-e\underline{v}\cdot\underline{E}}{mc^2}$$

In an undulator: $\underline{v}_\perp = \frac{cK}{\gamma}\exp(ik_o z)$

$$v_z = c(1 - [1+K^2]/2\gamma^2)$$

Substituting in the above gives,

$$\dot{\varphi}_j = ck_o(1 - \gamma_r^2/\gamma_j^2)$$

$$\dot{\gamma}_j = \frac{-eK}{2\gamma mc}\left[E_o\exp(i\varphi_j) + \text{c.c.}\right]$$

Fields: Maxwell's Wave eqn (1-Dim.)

$$\nabla_T^2 E + \left[\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]E = -\frac{4\pi}{c^2}\frac{\partial J}{\partial t}$$

transverse effects (Dr. Sessler)

FEL Equations

$$\dot{\theta}_j = \frac{1}{2\rho}\left[1 - 1/(\rho\Gamma_j)^2\right]$$

$$\dot{\Gamma}_j = -\frac{1}{\rho}\left[\frac{A}{\Gamma_j}e^{i\theta_j} + c.c.\right]$$

$$\dot{A} = i\delta A + \left\langle \frac{e^{-i\theta_j}}{\rho\Gamma_j} \right\rangle \qquad A = \epsilon e^{i\alpha}$$

$\rho \equiv \left[\frac{K}{4}\left(\frac{\gamma_0}{\gamma_R}\right)^2 \frac{\Omega_p}{\omega_0}\right]^{2/3}$ = coupling parameter depends on both n & K!

$\Omega_p^2 = \frac{4\pi e^2 n_e}{m\gamma_0^3}$ = plasma freq., depends on both n_e and γ_0

$\theta_j = (k+k_0)z_j - \omega t - \dot{\phi}_0 t$ = phase

$\Gamma_j = \frac{1}{\rho}\frac{\gamma_j}{\gamma_0}$ = normalized energy of e^-

$|A|^2 = \frac{|E_0|^2}{[4\pi\rho m_0 c^2 \gamma_0 n_e]}$ = field energy density

Set of $2N_\lambda+2$ Nonlinear Coupled O.D.E.

$$\langle \quad \rangle = \frac{1}{N_\lambda}\sum_{j=1}^{N_\lambda}$$

$$\cdot = \frac{d}{d\tau}$$

$$\tau = 2\omega_0\rho\left(\frac{\gamma_R}{\gamma_0}\right)^2 t$$

$$\delta = \frac{1}{\rho}\left[\frac{\gamma_0^2-\gamma_R^2}{2\gamma_R^2}\right] = \text{detuning}$$

Conservation Law

$$\langle\Gamma\rangle + |A|^2 = \text{const} = \langle\Gamma_0\rangle + |A_0|^2$$

The energy that leaves the electrons goes into the radiation field.

Recalling that $\Gamma = \frac{\gamma}{\rho\gamma_0}$ we have

$$\frac{\langle\gamma_j-\gamma_0\rangle}{\gamma_0} = \text{efficiency} = \rho\left[|A_f|^2 - |A_0|^2\right]$$

Linear Theory

$$\theta_j = \theta_{oj} + \psi_j$$
$$\rho \Gamma_j = 1 + \eta_j$$
$$A = 0 + A$$

Substitute into eqns. of motion,

$$\dot{\psi}_j = \eta/\rho$$
$$\dot{\eta}_j = -\rho[A\exp(i\theta_j) + c.c.]$$
$$\dot{A} = i\delta A - i\langle e^{-i\theta_{oj}}\psi_j\rangle - \langle e^{-i\theta_{oj}}\eta_j\rangle$$

Assume a soln of the form: $\exp[i\lambda\tau]$

Form macroscopic variables,

$$X = \langle e^{-i\theta_{oj}}\psi_j\rangle \qquad Y = \langle e^{-i\theta_{oj}}\eta_j/\rho\rangle$$

Eqns. are reduced to algebraic form,

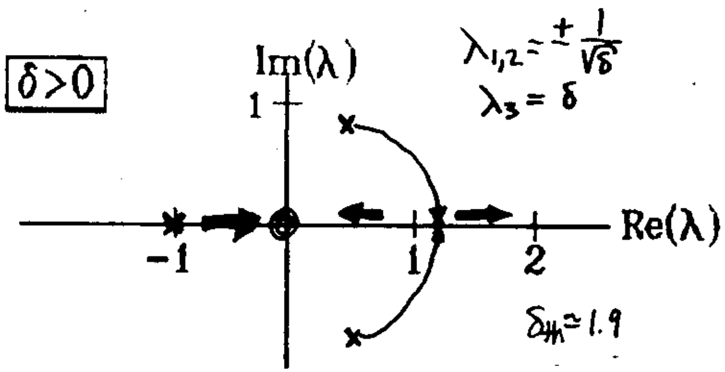
$$i\lambda X = Y$$
$$i\lambda Y = -A$$
$$i\lambda A = i\delta A - iX - \rho Y$$

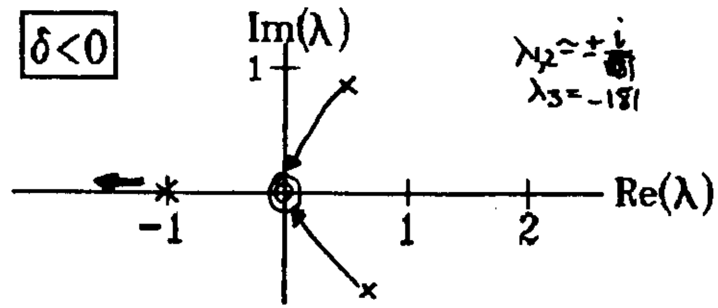
Analysis of Cubic

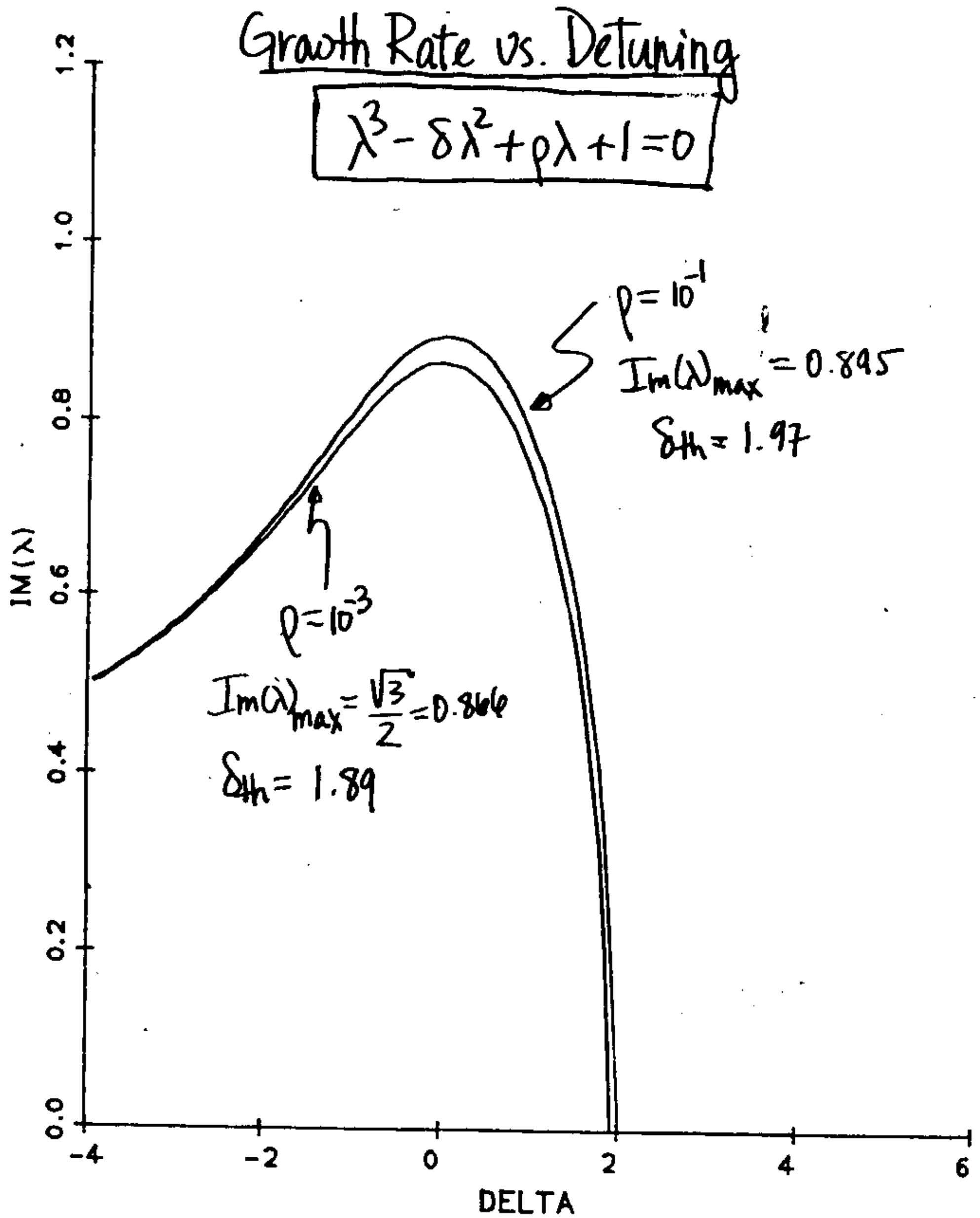
$$\boxed{\lambda^3 - \delta\lambda^2 + \rho\lambda + 1 = 0}$$

Dispersion relation is real!

- *three real roots
- *1 real root, c.c. pair







Phase Space Dynamics

Consider the particle eqns.,

$$\dot\theta_j = \frac{1}{2\rho}\left[1 - \frac{1}{(\rho\Gamma_j)^2}\right]$$

$$\dot\Gamma_j = -\frac{1}{\rho}\left[\frac{a e^{i\theta_j}}{\Gamma_j} + c.c.\right]$$

Take $\gamma_j = \gamma_0(1+\eta_j) \Rightarrow \rho\Gamma_j = (1+\eta_j)$, the above pair becomes

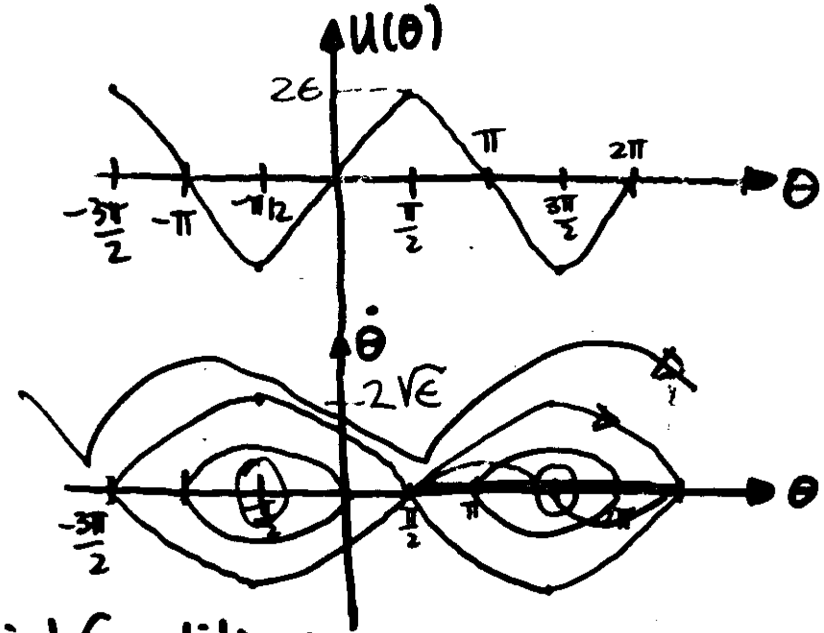
$$\dot\theta_j = \frac{\eta}{\rho} \qquad \frac{\dot\eta}{\rho} = -(a e^{i\theta_j} + c.c.)$$

Taking $a = \epsilon e^{i\alpha}$ and combining gives,

$$\ddot\theta_j + 2\epsilon\cos(\theta_j + \alpha) = 0$$

Define: $U(\theta) = 2\epsilon\sin(\theta+\alpha)$

Then, $\ddot\theta = -\nabla_\theta U(\theta)$



Initial Conditions:

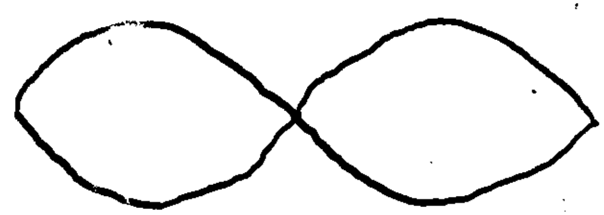
$\theta_j = [0, 2\pi)$ uniformly/randomly

$\Downarrow$ No initial bunching

$\dot{\theta}_j = \frac{\eta_j}{\rho} = 0$ cold electron beam, $\delta = 0$

Small signal gain limit ($\delta=0$) $\epsilon, \alpha = \text{const}$

The picture is symmetric, there is NO net gain because the same number of particles move up as down.



High Gain or Collective Instability Regime

Recall $\epsilon \equiv$ electric field amplitude
$\alpha \equiv$ slowly varying phase of field

They are NOT constants in a self consistent theory,

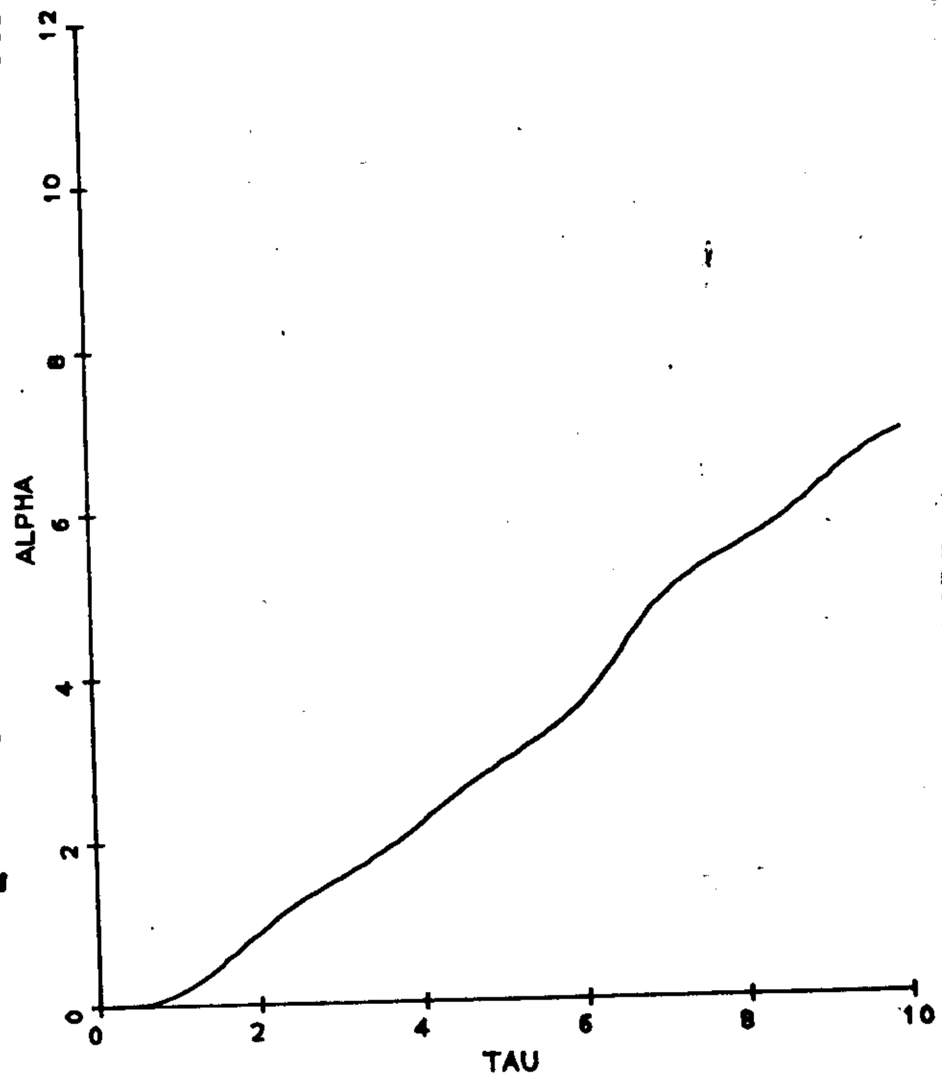
$$\dot{\epsilon} \simeq \left\langle \frac{\cos(\theta_j + \alpha)}{\rho \Gamma_j} \right\rangle \qquad (1)$$

$$\dot{\alpha} = i\delta - \frac{1}{\epsilon} \left\langle \frac{\sin(\theta_j + \alpha)}{\rho \Gamma_j} \right\rangle \qquad (2)$$

(1) ➡ Height of bucket changes

(2) ➡ Bucket slips w.r.t. the particles

For $\alpha > 0$ the bucket slips to the left, the picture is no longer symmetric!
(Plot of α vs. τ)



Movie/Slides

Back of the Envelope Explanation

The field eqn is roughly

$\dot{A} \sim J$ "field driven by current"

Linear theory: $J = \sigma A$

Replace e^- beam by a complex conductivity

$\sigma \equiv$ complex "conductivity"
$= \sigma_r + i\sigma_I$
$=$ (resistance) $+ i$ (dielectric const., ●)

For $A = \epsilon(\tau)\exp(i\alpha(\tau))$, we have

$\dot{\epsilon} \simeq \sigma_r \epsilon$; σ_r drives amp. (gain)

$\dot{\alpha} \sim \sigma_I$; σ_I changes phase velocity of wave

Complete phase of pond. wave is

$\phi = (k_0+k)z - \omega t + \alpha \Rightarrow \boxed{V_\phi = \frac{\omega - \dot{\alpha}}{(k_0+k)}}$

if $\alpha > 0$, wave slows down

Summary of Slides

1.) Phase space dynamics is highly nonlinear but "explainable".

2.) Electrons undergo a net shift in energy, which appears as radiation.

3.) Electrons acquire a net energy spread

$\sigma_{(\gamma/\gamma_0)} \simeq \rho$.

→ trouble for a recirculating e^- beam.
→ must damp σ_γ before the beam can be used again.

4.) Field grows to about $|A| \simeq 1$

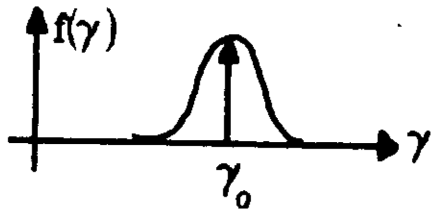
→ efficiency $= \left\langle \frac{\gamma - \gamma_0}{\gamma_0} \right\rangle = \rho |A|^2 \sim \rho$

5.) Field grows exponentially at first, traps particles in the bucket & saturates,

$|\dot{A}|^2 = 2\lambda_I |A|^2 - \beta |A|^4$

Van der Pol eqn.

Warm Beam Analysis



Cold beam: $f(\gamma) = \delta(\gamma - \gamma_o)$

Warm beam: $f(\gamma) = \dfrac{1}{\pi} \dfrac{\Delta_\gamma}{([\gamma-\gamma_o]^2 + \Delta_\gamma^2)}$

In normalized variables, $\mu = \eta/\rho$,

$$f_o(\mu) = \frac{1}{\pi} \frac{\Delta}{(\mu^2 + \Delta^2)}$$

Note: $\Delta = \Delta_\gamma/(\rho\gamma_o)$

Use Vlasov eqn & single particle eqns to obtain,

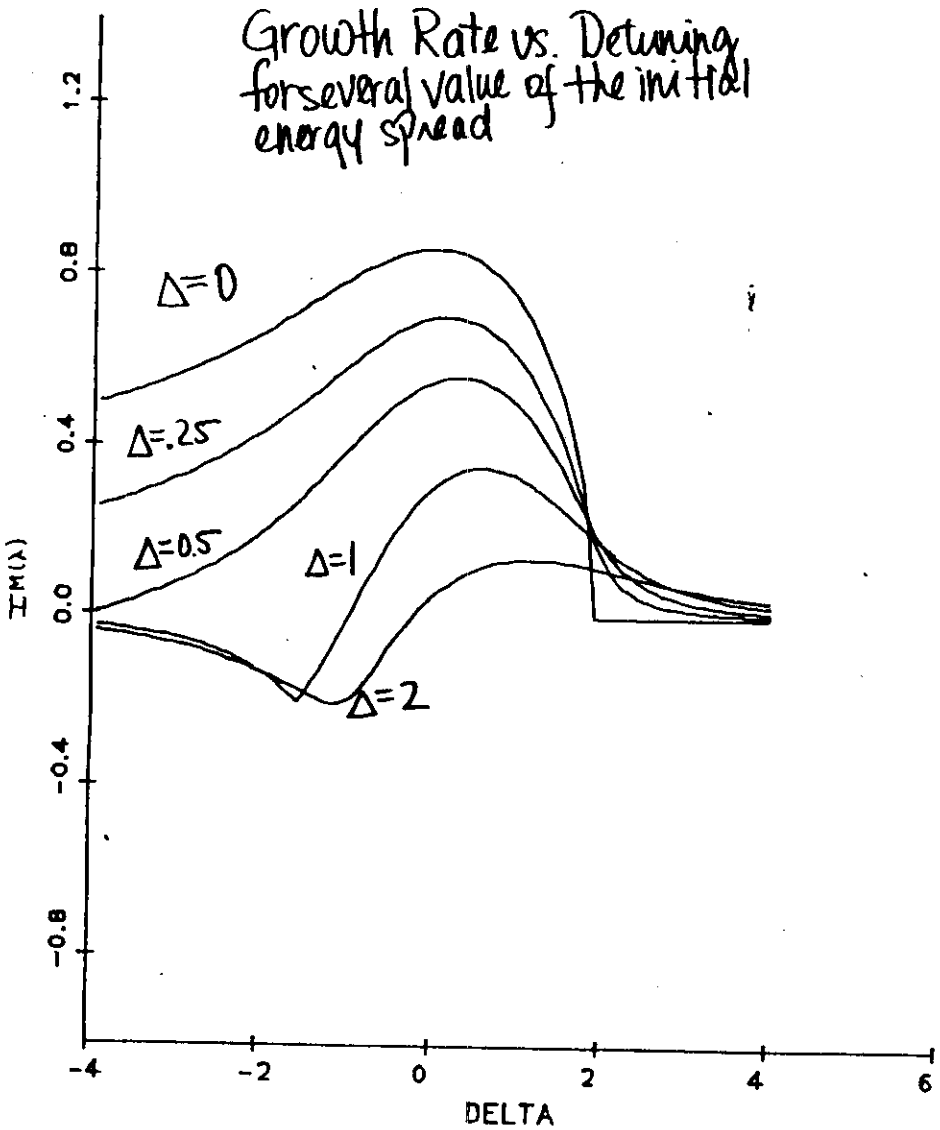
$$(\lambda - \delta) = \int d\mu \, \frac{\partial f_o(\mu)/\partial\mu}{(\lambda - \mu)}$$

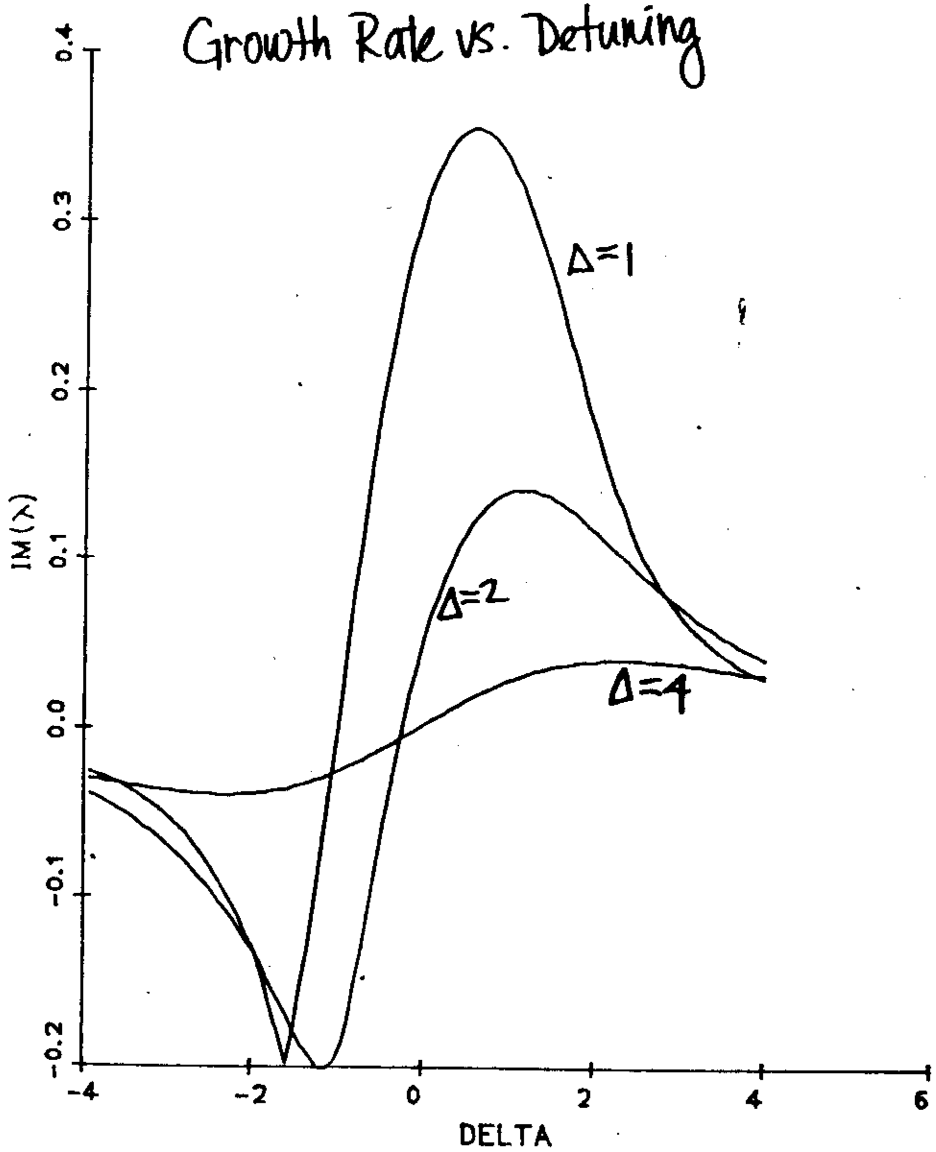
For a Lorentzian the dispersion relation is,

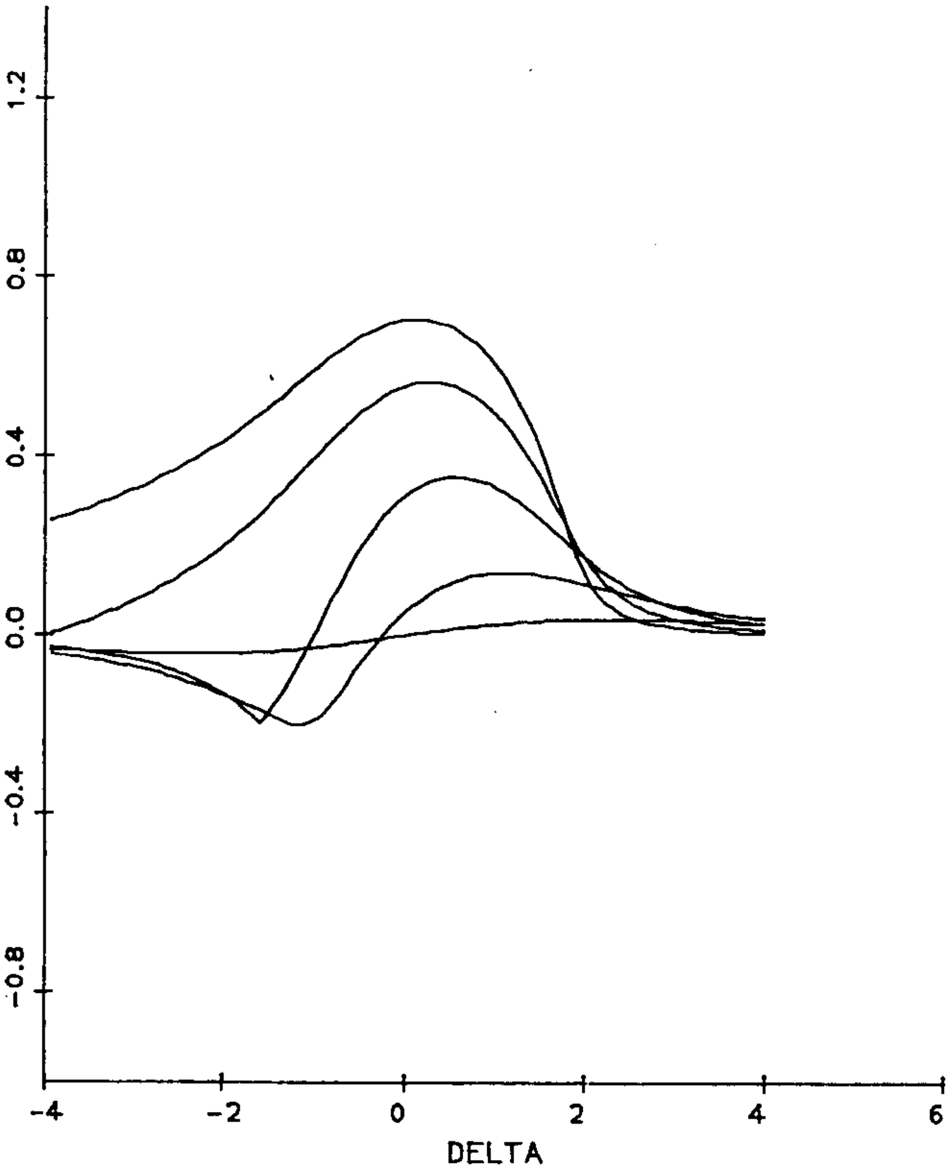
$$\boxed{(\lambda-\delta)(\lambda+i\Delta)^2 + 1 = 0}$$

Character of dispersion relation has changed: i appears explicitly!

$$\int d\mu \, \frac{\partial f_o(\mu)/\partial\mu}{(\lambda-\mu)} = PV\!\int d\mu \, \frac{\partial f_o(\mu)/\partial\mu}{(\lambda-\mu)} - i\pi \frac{\partial f_o}{\partial\mu}\bigg|_{\lambda=\mu}$$


Growth Rate vs. Detuning
for several value of the initial energy spread
Δ=0
Δ=.25
Δ=0.5
Δ=1
Δ=2
IM(λ)
DELTA


Growth Rate vs. Detuning
Δ=1
Δ=2
Δ=4
IM(λ)
DELTA


IM(λ)
1.2
0.8
0.4
0.0
-0.4
-0.8
-4
-2
0
2
4
6
DELTA

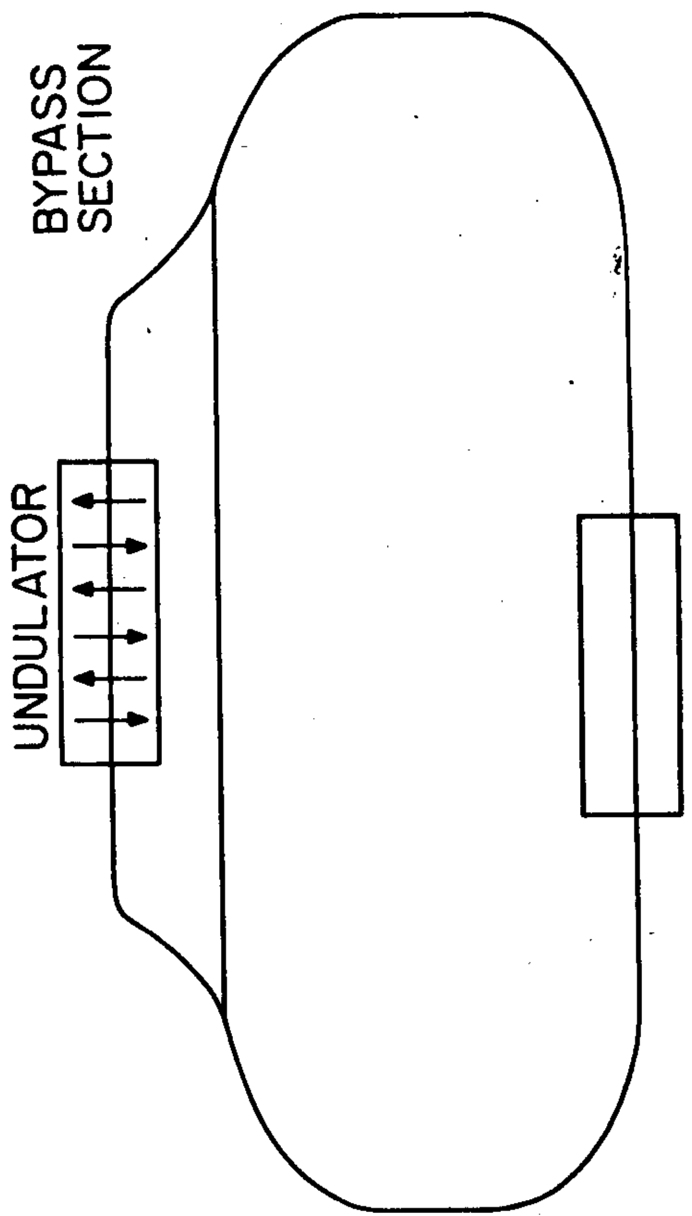

BYPASS
SECTION
UNDULATOR
RF SYSTEM

Fig. 3.