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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS
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SPIN-POLARIZED HYDROGEN

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lecture #1

SPIN-POLARIZED HYDROGEN

(by D. Kleppner - M.I.T.)

- or, a study in laser-free atomic physics
(and, hopefully, molecule-free atomic physics)

"To understand hydrogen
is to understand everything"
attributed to Victor F. Weisskopf
by Gerhardt Herzberg

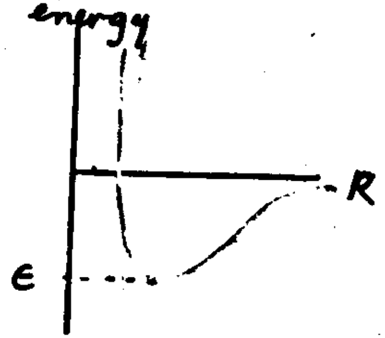
"I never said it -
but I wish I had!"
- V. F. Weisskopff

A "Thumbnail" History of Hydrogen

Paracelsus ~ 1510 probably saw H_2.
Cavendish ~ 1774 isolated & identified H_2.
Lavoisier ~ 1776 $H_2 + O \to H_2O$: gave name "hydrogen"
Fraunhofer ~ 1818 solar absorption - C line = H_α.
Angstrom ~ 1860's lab. measurement of H spectrum
Huggins ~ 1870's - astron. " " " "
Balmer ~ 1883 $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ $n = 3, 4, 5$
Bohr ~ 1912 H "theory" $\Longrightarrow$ birth of quant. mech.
Dirac ~ 1926 relativistic e theory
Rabi ~ 1946 hyperfine structure - problems ?!
Lamb ~ 1948 Lamb shift $\Longrightarrow$ Q.E.D.
Purcell & Field, von Hulst ~1950 21cm line in space.
Metzger & Hoaglund ~1965 recomb. lines $n = 101 \to$ ...
1970's & 80's: Lamb shift measured - proton structure limit; 1s-2s 2-photon spectroscopy, saturation spectroscopy of Balmer series, QED of high-Z "hydrogen", etc...
1980 - Spin-polarized H created by Silvera & Walraven, & at M.I.T. **WHY?**

Solid, Liquid or Gas?

The state of matter is determined by a competition between energy order due to attractive potential ϵ, and disorder due to kinetic energy $E \sim kT$.



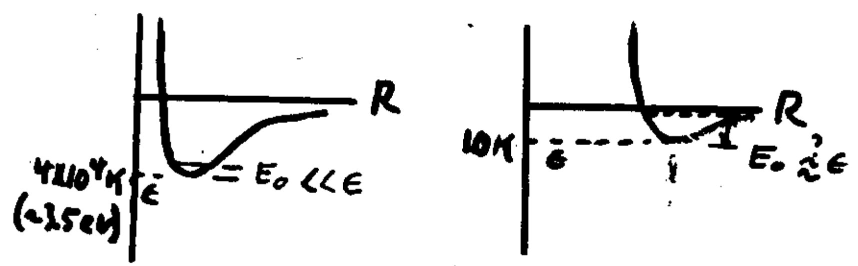
$E \gg \epsilon$ kinetic energy wins: gas

$E \ll \epsilon$ potential energy wins: solid

$E \approx \epsilon$ intermediate state: liquid

$E \sim kT$. As $T \to 0$, all matter becomes solid, with one (or two) exceptions.

Exception – He remains liquid as $T \to 0$.



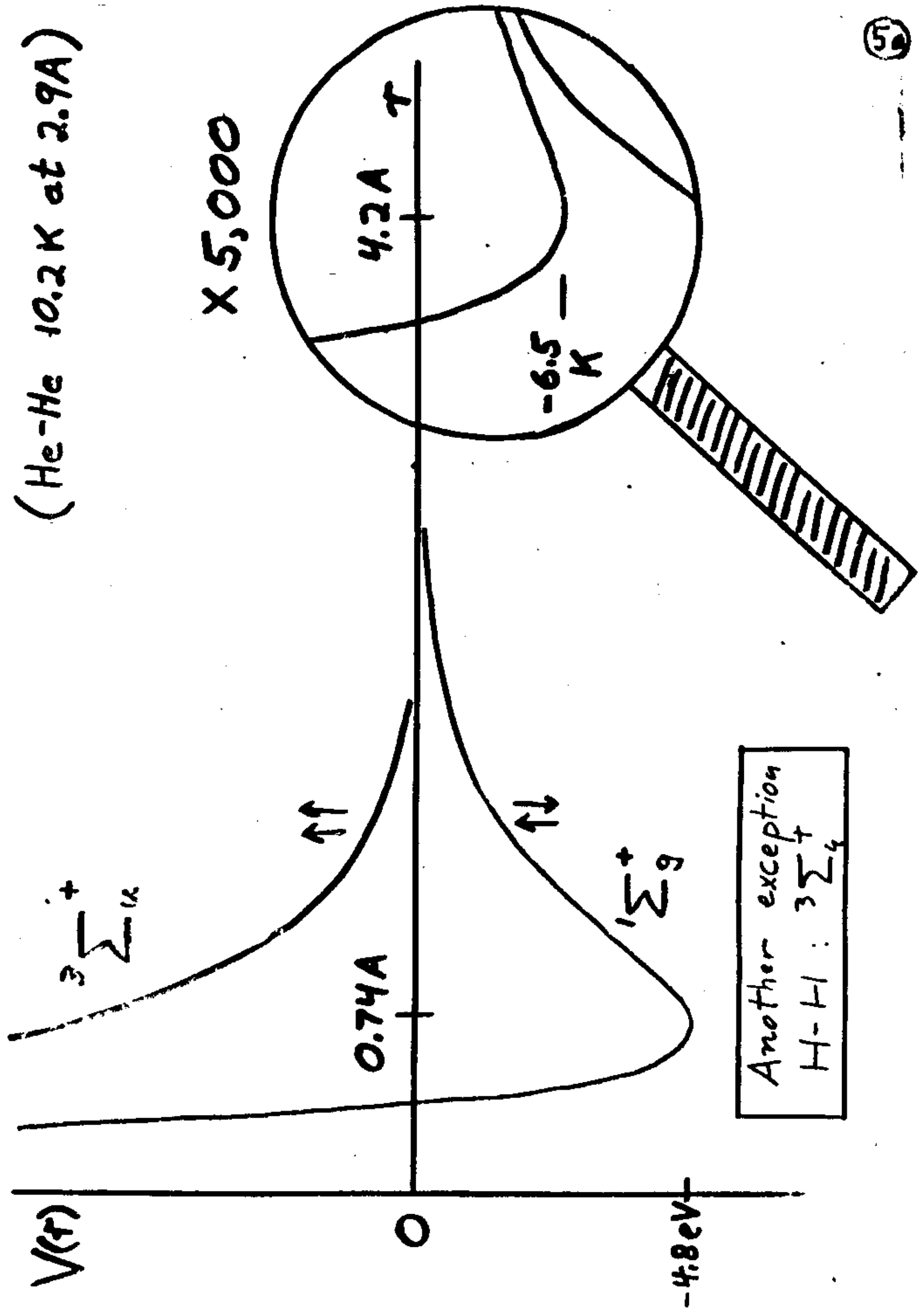
typical atom-atom potential

He-He potential

In He, the 0-point energy E_0 is so large that as $T \to 0$, $E \to E_0 \sim \epsilon$. System forms a "loose" liquid, but not a solid.

E_0 neglected
no K.E.
$\rho = \rho_{max}$

E_0 included
much K.E.
$\rho(\text{actual}) \sim \frac{1}{4}\rho_{max}$



Another exception
H-H : $^3\Sigma_u^+$

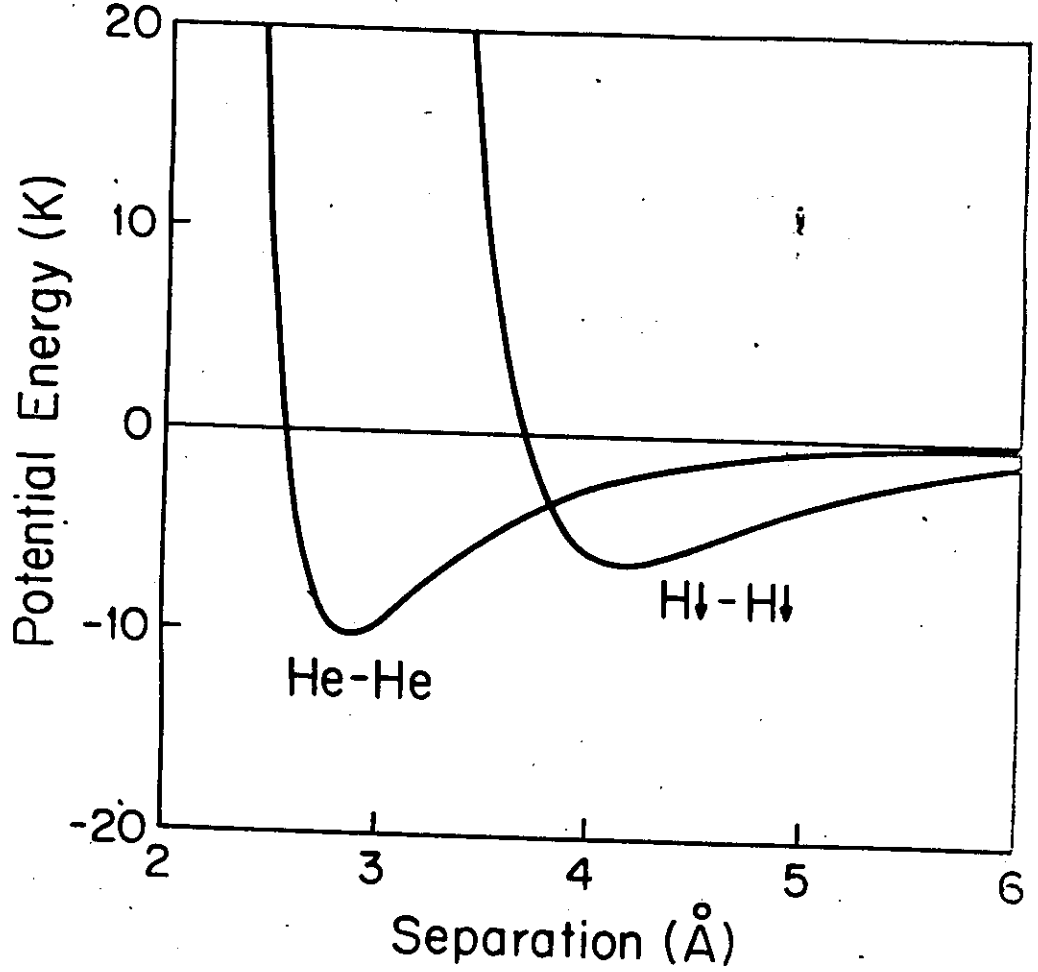
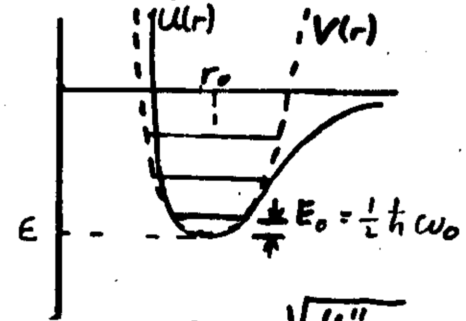


Fig. 5.1

Effect of zero-point energy on equation of state

- can be predicted with a universal scaling law.



$V = -\epsilon + \frac{1}{2}U''\big|_{r_0}(r-r_0)^2$

$U''\big|_{r_0} = \text{const}' \times \frac{\epsilon}{r_0^2}$ ← depends on form of U.

$$\omega_0 = \sqrt{\frac{U''}{M}} = \sqrt{\frac{\text{const}'\,\epsilon}{M r_0^2}}$$

$$\frac{\text{0-point energy}}{\text{Binding energy}} = \frac{\frac{1}{2}\hbar\omega_0}{\epsilon} = \text{const}\sqrt{\frac{\hbar^2}{M\epsilon r_0^2}}$$

$\eta = \frac{\hbar^2}{M\epsilon r_0^2}$ = "quantum parameter"

Law of corresponding states

reduced variables: $E^* = \frac{\text{energy}}{\epsilon}$; $T^* = \frac{kT}{\epsilon}$; $r^* = \frac{\text{distance}}{r_0}$; $M^* = \frac{\text{mass}}{M}$.

> All systems having same form for potential have same thermodynamic behavior in terms of reduced variables. Behavior depends only on η.

Why is spin-polarized hydrog (H↓) interesting?

- because it is predicted to remain a gas as $T \to 0$. This prediction is very firm, based on the law of corresponding states and other studies. So far, it has been confirmed to 60 mK.

- because it offers the opportunity to study matter in a new quantum regime, the regime where the translational wave packets overlap ($\Lambda_{deBroglie}$ > distance between atoms)

- because it has created a new technology for producing H at high density and low temperature with many potential applications to spectroscopy, atomic physics, particle & nuclear physics, etc.

Plan of lectures

I. Introduction -

H↓ and Bose condensation

II } strategies for making H↓
III } what we have discovered
how we fooled Mother Nature
how Mother Nature fooled us
prospects

IV applications - including spectroscopy

Reference:

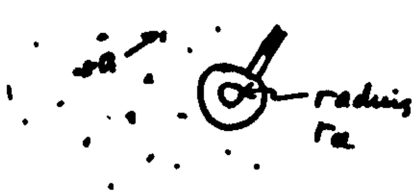
Lectures on Spin-Polarized Hydrogen
T. J. Greytak & D. Kleppner
in New Trends in Atomic Physics, Vol II
G. Grynberg & R. Stora, ed.
North Holland, 1984

Copies will be distributed in a week or two

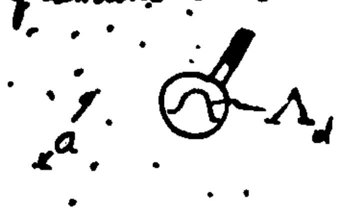
H↓ and the Bose-Einstein transition

A real gas, at low density, is accurately described by classical statistical mechanics. For instance, momentum described by Maxwell-Boltzmann statistics.

classical view



quantum view



low density: $r_a \ll a$

low density: $\Lambda_d \ll a$

For most atoms, $\Lambda_d = \frac{h}{M V_{thermal}} \ll r_a$.

So - if gas is at low density classically it is also at low density quantum-mechanically.

Quant. mech. effects important when

$$a \sim \Lambda = \frac{h}{p(\text{thermal})} = \frac{h}{\sqrt{2\pi k T M}}$$

density $n = \frac{1}{a^3}$ atoms cm^{-3}.

$$n_0 = (\text{const}) \times \left(\frac{2\pi k T M}{h^2}\right)^{3/2}$$

For Bose particles, phase transition occurs when

$$n_0 = (2.612\ldots)\left(\frac{2\pi k T M}{h^2}\right)^{3/2}.$$

A very quick summary of Bose-Einstein transition.

Note: H obeys Bose statistics. To interchange 2 H atoms, first interchange electrons, then protons. Each interchange reverses sign of wavefunction. $(-1)(-1) = +1$. This is true so long as the atoms are not electronically excited. (In such a case, one would have 4 fermions, instead of 2 bosons.) The situation is analogous to the deuteron, which is composed of 2 fermions - a proton and a neutron - but behaves like a simple boson at low energy.

(summary of Bose transition - continued).

Mean occupation number, $\bar{N}_p$
- non interacting particles
- momentum $\vec{p}$
- energy ε_p.

$$\bar{N}_p = \begin{cases} \dfrac{1}{\exp\beta(\varepsilon_p-\mu)-1} & \text{Bose particles} \quad \left(\beta = \frac{1}{kT}\right) \\ \dfrac{1}{\exp\beta(\varepsilon_p-\mu)+1} & \text{Fermi particles} \\ \dfrac{1}{\exp\beta(\varepsilon_p-\mu)} & \text{Boltzmann law - classical} \end{cases}$$

$\sum \bar{N}_p = N$ (total # particles)

# of states: box quantization $V = L^3$

$p_x = \frac{2\pi\hbar}{L} j_x$; $p_y = \frac{2\pi\hbar}{L} j_y$; $p_z = \frac{2\pi\hbar}{L} j_z$

j_x, j_y, j_z integers

$$\sum_{j_x, j_y, j_z} \longrightarrow \left(\frac{L}{2\pi\hbar}\right)^3 \iiint dp_x\, dp_y\, dp_z$$

Change from integral over momenta to integral over energy.

$\varepsilon_p = \frac{p^2}{2M}$. $\iiint dp_x\, dp_y\, dp_z \rightarrow 4\pi \int p^2 dp$
$\rightarrow 4\pi \sqrt{2M}^3 \int \sqrt{\varepsilon}\, d\varepsilon$.

summarizing:

$$\sum_{j_x, j_y, j_z} \longrightarrow \frac{V}{(2\pi)^2}\left(\frac{2M}{\hbar^2}\right)^{3/2} \int \sqrt{\varepsilon}\, d\varepsilon = \int D(\varepsilon)\, d\varepsilon.$$

$$D(\varepsilon) = \frac{V}{(2\pi)^2}\left(\frac{2M}{\hbar^2}\right)^{3/2} \sqrt{\varepsilon} = \text{density of states in energy.}$$

Require:

$$\int \bar{N}(\varepsilon)\, D(\varepsilon)\, d\varepsilon = N.$$

$$\bar{N}(\varepsilon) = \frac{1}{\exp\beta(\varepsilon-\mu)-1}$$

. μ is adjustable parameter (chemical potential).

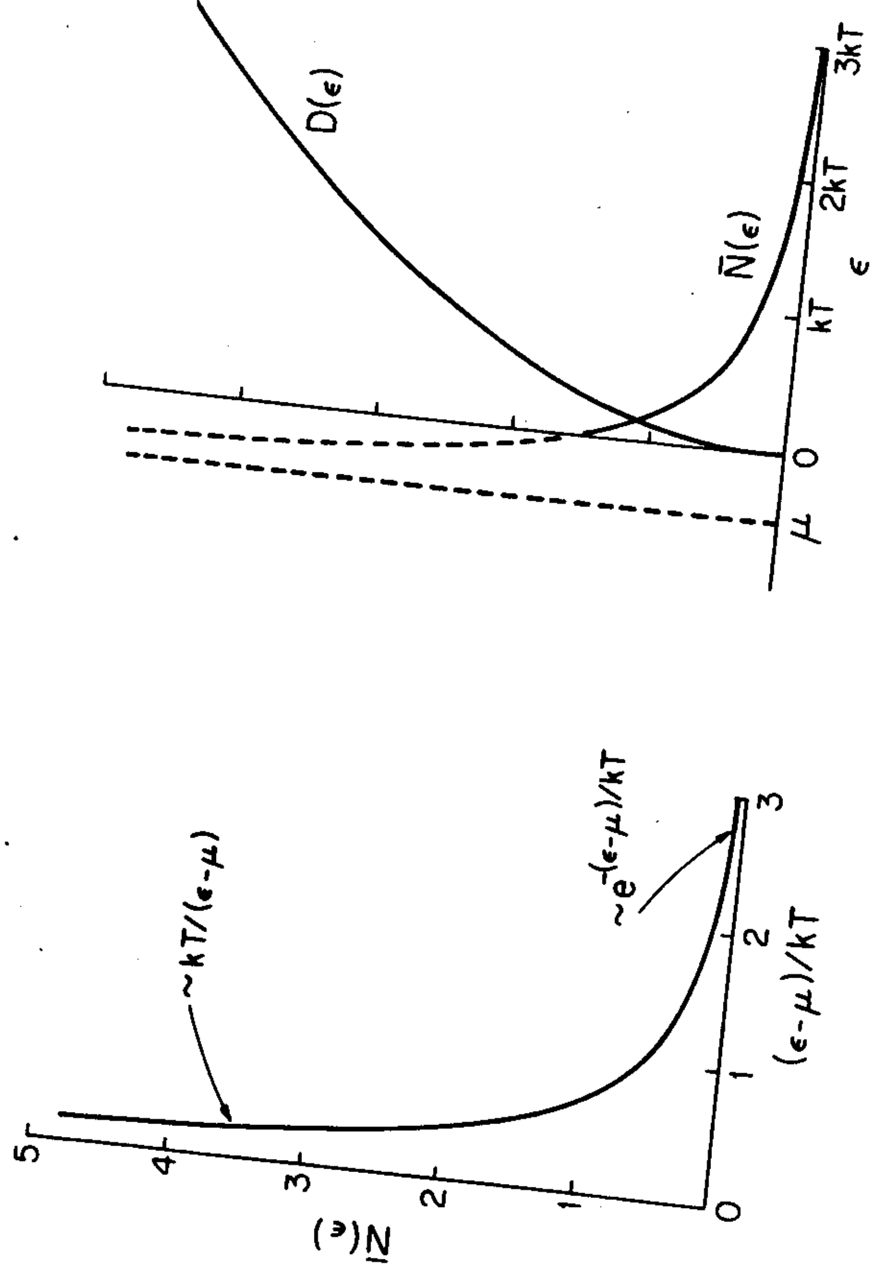


Fig. 1.1

As T decreases, μ increases toward 0 so that $N = \int \bar{N}(\epsilon) D(\epsilon)\, d\epsilon$ is satisfied.

Maximum temperature: $\mu = 0$, $T = T_0$

$$N = V \left(\frac{M k T_0}{2\pi \hbar^2}\right) \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sqrt{x}\, dx}{e^x - 1}$$

$$= V \frac{1}{\Lambda(T_0)^3} \times 2.612\ldots$$

$$n = \left(\frac{M k T_0}{2\pi \hbar^2}\right)^{3/2} \times 2.612\ldots$$

What happens if $T < T_0$?

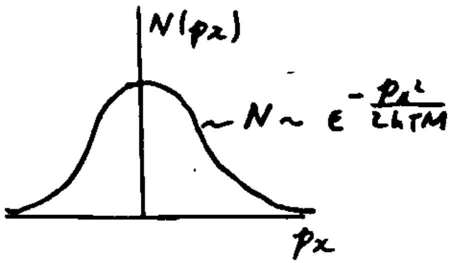
Finite number of particles drop into the 0-energy state.

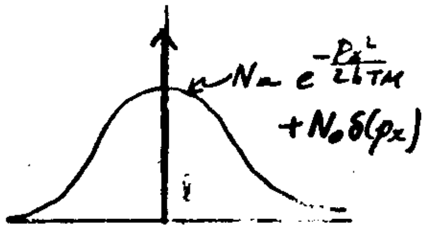
(This state was discarded in converting sum over states into integral over energy.)

Can show that the number of particles in 0-energy state is

$$\bar{N}_0 = \begin{cases} N(1 - T/T_0)^{3/2} & T \le T_0 \\ 0 & T > T_0 \end{cases}$$

Evidence for Bose-Einstein condensation





Most systems: solid before T_0 is reached.

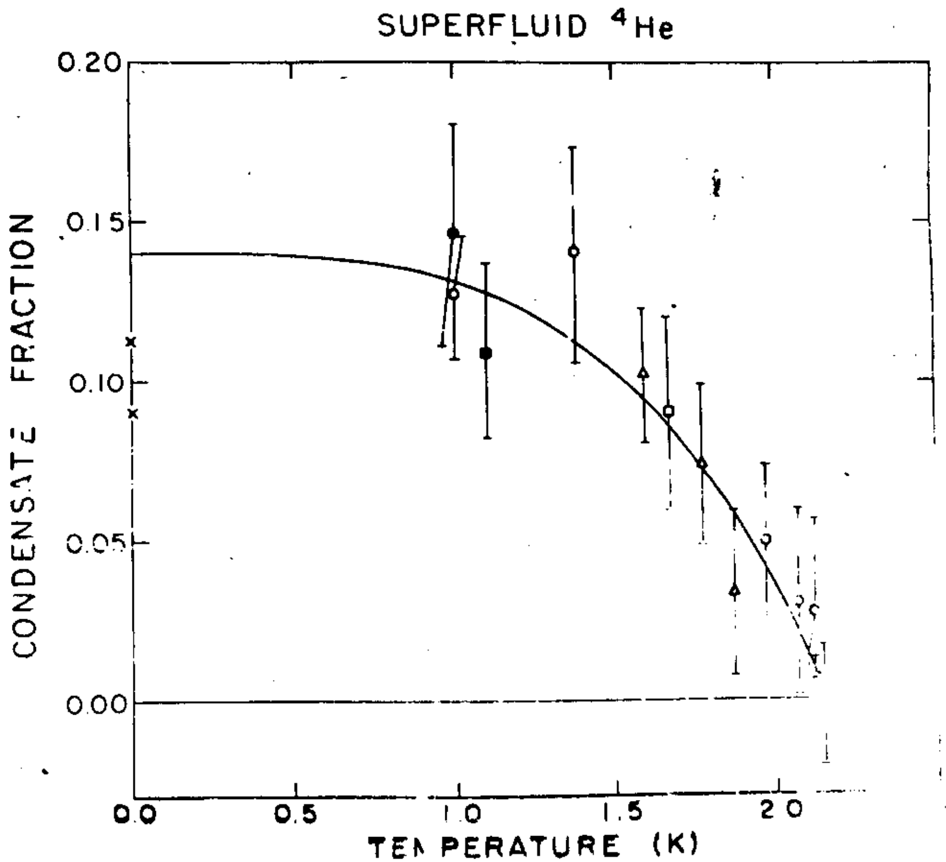
Liquid ^4He: $n = 2.20 \times 10^{22}$ atoms cm^{-3}
(liquid density)
$T_0 = 3.15 K$
Close to superfluid temp., 2.24
Is superfluid ^4He a Bose condensate?

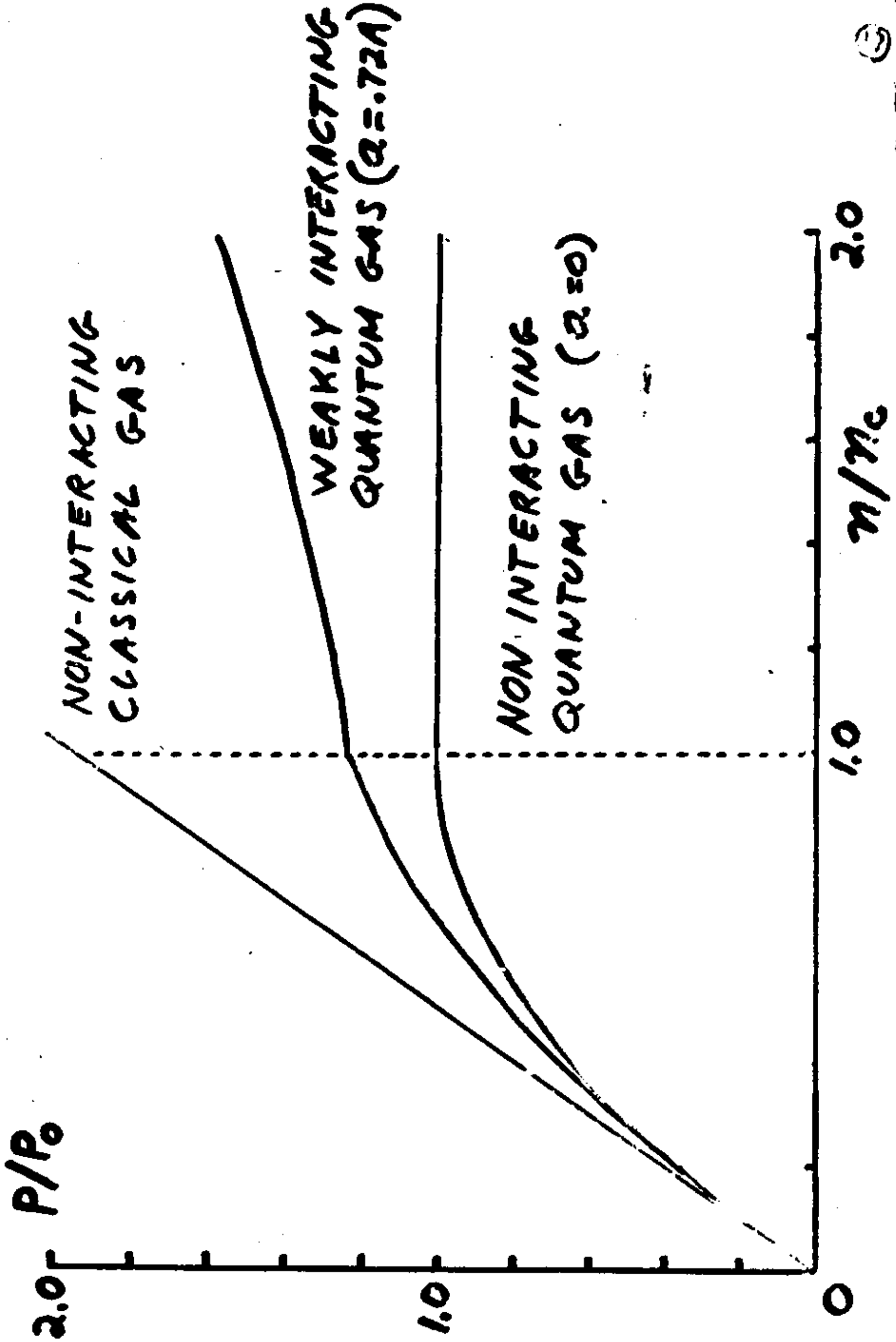
$$T_0 = \frac{2\pi\hbar^2}{kM}\left(\frac{n}{2.612}\right)^{2/3}$$

require minimum M!

Hydrogen: for $T_0 = 0.3K$, $n = 6 \times 10^{19}\ cm^{-3}$

<u>Can B-E condensation be achieved?</u>

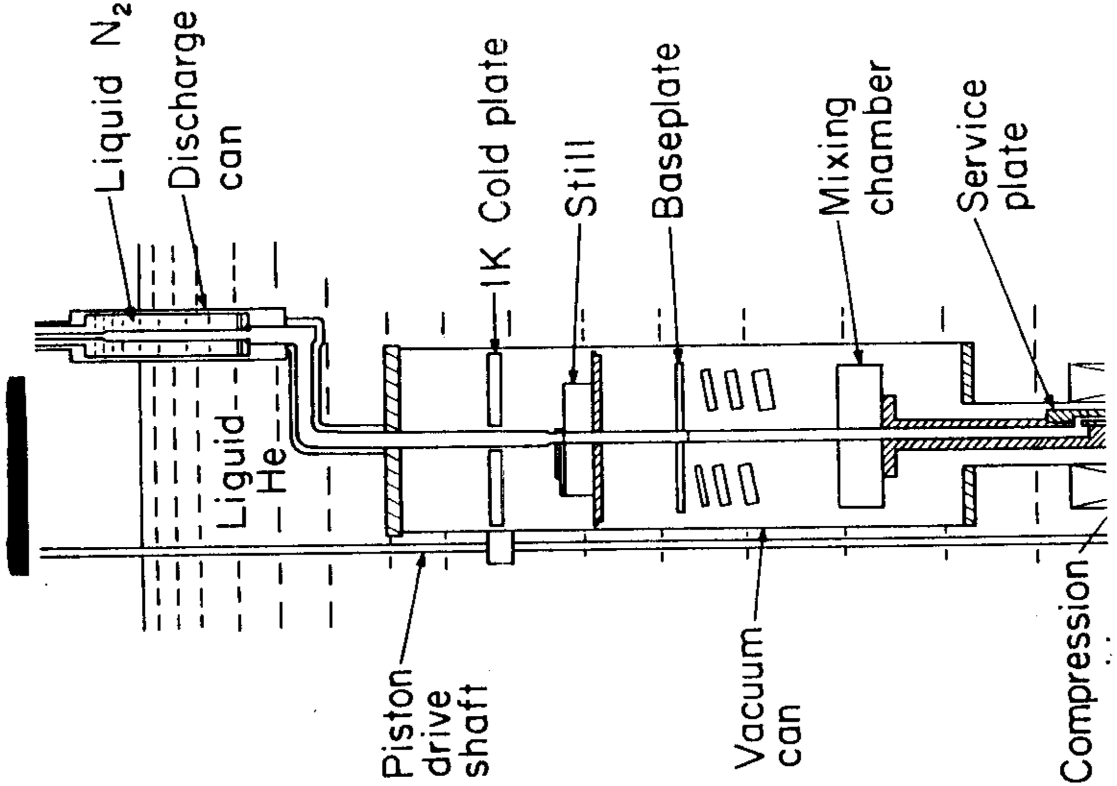


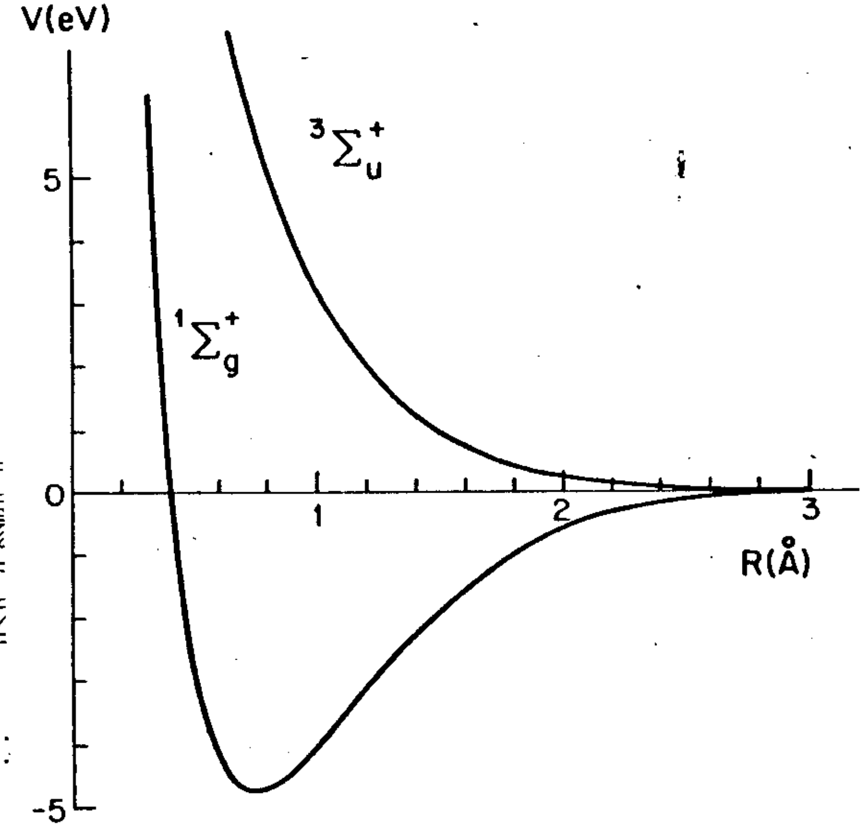


Quest to observe B-E transition in H↓

Required: atomic hydrogen

- at temperature < 1K
  (previous lowest temperature, 78K)
- at density $> 10^{14}$ cm^{-3}
  (previous maximum density for H (without H_2) ~ 10^{12} cm^{-3})
- highly spin polarized
  (a few percent of wrong spin state would rapidly destroy system)
- stabilized against any sort of spin relaxation





H_2 POTENTIALS