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Modelling of Block Structure Dynamics

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Introduction

The study of seismicity by statistical and phenomenological analysis of real earthquake catalogs has this disadvantage that the data usually sample time intervals of about one hundred years or even less. This time interval is very short in comparison with the duration of tectonic processes responsible for seismic activity, therefore the patterns of earthquake occurrence identifiable in a real catalog may be only apparent and not recur in the future. On the other hand, a synthetic catalog obtained by numerical modelling of the seismogenic process can cover very long time intervals, thus allowing more reliable estimation of seismicity parameters.

Mathematical models of lithosphere dynamics are tools for the study of the earthquake preparation process. These models are also useful in earthquake prediction studies. An adequate model should indicate the physical basis of premonitory patterns determined empirically before large events. Note that the available data often do not constrain the statistical significance of the premonitory patterns. The model can be used also to suggest new premonitory patterns that might exist in real catalogs.

A block model simulates the dynamics of the block structure and the tectonic movements of a real seismic region and is used to produce a synthetic catalog of earthquakes. The basic principles of the model are developed in Gabrielov et al. (1990).

Although there is no adequate theory of the seismo-tectonic process, various properties of the lithosphere, such as spatial heterogeneity, hierarchical block structure, different types of non-linear rheology, gravitational and thermodynamic processes, physico-chemical and phase transitions, fluid migration and stress corrosion, are probably relevant to the properties of earthquake sequences. The qualitative stability of these properties in different seismic regions suggests that the lithosphere can be modeled as a large dissipative system that does not essentially depend on the particular details of the specific processes active in a geological system.

The model exploits the hierarchical block structure of the lithosphere proposed by Alekseevskaya et al. (1977). According to this model, the blocks of the lithosphere are separated by comparatively thin, weak, less consolidated fault zones, such as lineaments and tectonic faults. In the seismotectonic process major deformation and most earthquakes occur in such fault zones.

A seismic region is modelled by a system of absolutely rigid blocks divided by infinitely thin plane faults. Relative displacement of all blocks is supposed to be infinitely small relative to their geometric size. Blocks interact between themselves and with the underlying medium. The system of blocks moves as a consequence of prescribe motion of boundary blocks and of the underlying medium.

As the blocks are absolutely rigid, all deformation takes place in the fault zones and at the block base in contact with the underlying medium. Relative block displacements take place along the fault planes. This assumption is justified by the fact that for the lithosphere the effective elastic moduli in the fault zones are significantly smaller than those within the blocks.

The blocks are in viscous-elastic interaction with the underlying medium. The corresponding stresses depend on the value of relative displacement. This dependence is assumed to be linear elastic. The motion of the medium underlying different blocks may be different.

Block motion is defined so that the system is in a quasi-static state of equilibrium.

The interaction of blocks along fault planes is viscous-elastic ("normal state") so far as the ratio of the stress to the pressure remains below a certain strength level. When the critical level is exceeded in some part of a fault plane, a stress-drop ("failure") occurs (in accordance with the dry friction model), possibly causing failure in other parts of the fault planes. These failures produce earthquakes. Immediately after the earthquake and for some time after, the affected parts of the fault planes are in a state of creep. This state differs from the normal state because of a faster growth of inelastic displacements, lasting until the ratio of the stress to the pressure falls below some other level.

This numerical simulation gives rise to a synthetic earthquake catalog.

Block Structure Geometry

A layer, *d*, with thickness *H* limited by two horizontal planes is considered (Fig.1), and a block structure is defined as a limited and simply connected part of this layer. Each lateral boundary of the block structure is defined by portions of the parts of planes intersecting the layer *d*. The subdivision of the structure into blocks is performed by planes intersecting the layer. The parts of these planes, which are inside the block structure and its lateral faces, are called "fault planes".

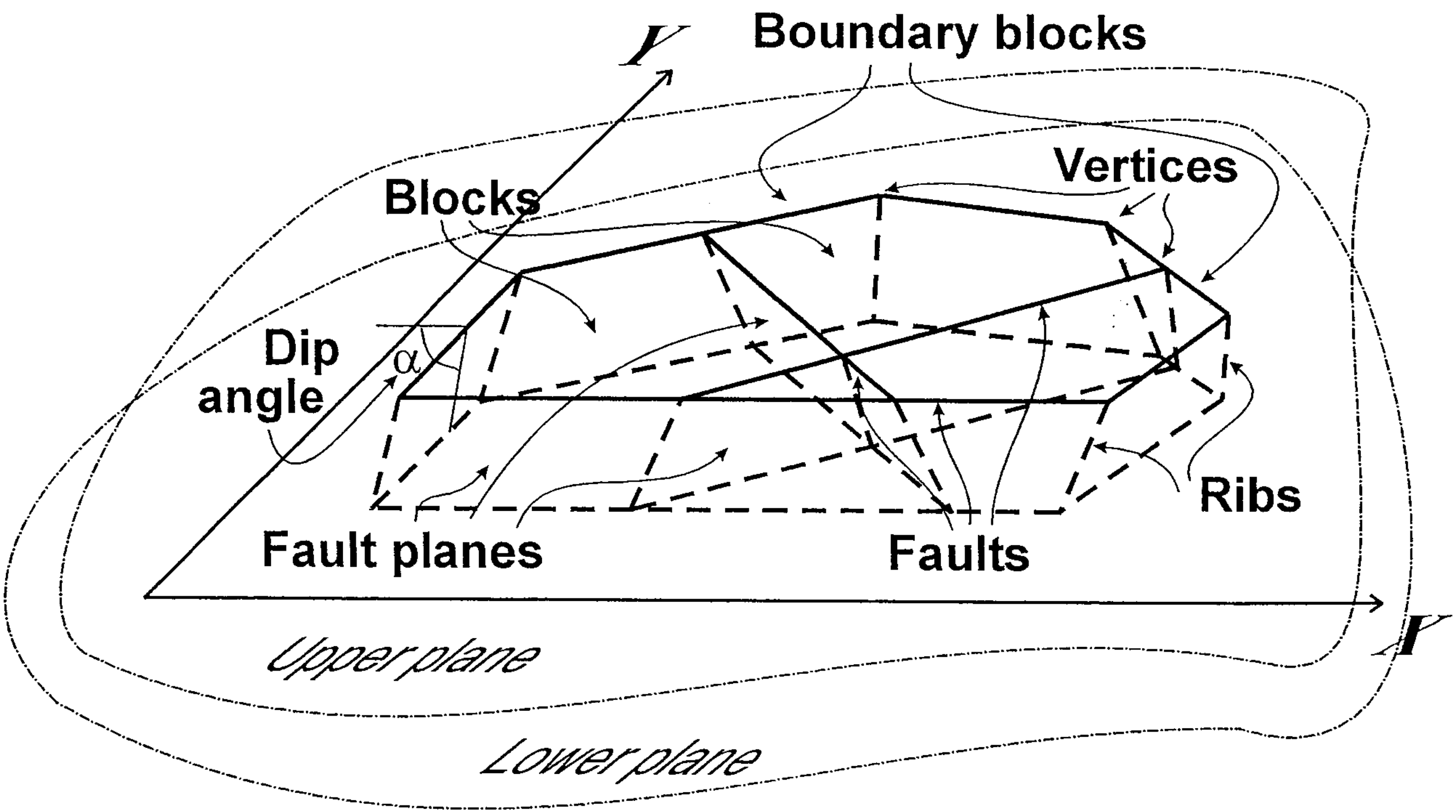

FIGURE 1 On definitions used in the block structure model.

The geometry of a block structure is defined by the lines of intersection between the fault planes and the upper plane which bounds the layer *d* (these lines are called "faults"), and by the angles of dip of each fault plane. Three or more faults cannot have a common point on the upper plane, a common point of two faults being called a "vertex". The direction is specified for each fault and the angle of dip of the fault plane is measured on the left of the fault. The fault planes can have arbitrary dip angles, which are specified on the basis of information on the deep structure of the region under consideration. The positions of a vertex on the upper and the lower plane by which the layer *d* is bounded are connected by a segment ("rib") of the line of intersection of the corresponding fault planes. The part of a fault plane between two ribs corresponding to successive vertices on the fault is called a "segment". The shape of a segment is a trapezium. The common parts of a block with the upper and lower planes are polygons, and the common part of a block with the lower plane is called "bottom".

We assume that the block structure is within a confining medium, whose motion is prescribed on its continuous parts contained between two ribs of the block structure boundary. These parts of the confining medium are called "boundary blocks".

Block Movement

The blocks are assumed to be rigid and all their relative displacements take place along the bounding fault planes. The interaction of the blocks with the underlying medium takes place along the lower plane, any kind of slip being possible.

The movements of the boundaries of the block structure (the boundary blocks) and of the medium underlying the blocks are assumed to be an external force on the structure. The rates of these movements are considered to be horizontal and known.

Non-dimensional time is used in the model, therefore all quantities that contain time in their dimensions are referred to one unit of the non-dimensional time, so that their dimensions do not contain time. For example, velocities in the model are measured in units of length, so a velocity of 5 cm means 5 cm per one unit of the non-dimensional time. When interpreting the results a realistic value is given to the unit. For example, if it is one year, then the velocity of 5 cm prescribed for the model means 5 cm/year.

At each time the displacements of the blocks are defined so that the structure is in a quasi-static equilibrium, and all displacements are supposed to be infinitely small compared with the block size. Therefore the geometry of the block structure does not change during the simulation and the structure does not move as a whole.

Interaction between the Blocks and the Underlying Medium

The elastic force which is due to the relative displacement of a block and the underlying medium, at some point of the block bottom, is assumed to be proportional to the difference between the total relative displacement vector and the vector of slippage (inelastic displacement) at the point.

The elastic force per unit area $\mathbf{f}^{u} = (f_x^{u}, f_y^{u})$ applied to the point with coordinates (X,Y), at some time t, is given by

$$f_x^{u} = K_u(x - x_u - (Y - Y_c)(\varphi - \varphi_u) - x_a),$$
$$f_y^{u} = K_u(y - y_u + (X - X_c)(\varphi - \varphi_u) - y_a). \qquad (1)$$

where X_c, Y_c are the coordinates of the geometrical center of the block bottom; (x_u, y_u) and φ_u are the translation vector and the angle of rotation (following the general convention, the positive direction of rotation is anticlockwise) around the geometrical center of the block bottom, for the underlying medium at time t; (x,y) and φ are the translation vector of the block and the angle of its rotation around the geometrical center of its bottom at time t; (x_a, y_a) is the inelastic displacement vector at the point (X,Y) at time t.

The evolution of inelastic displacement at (X,Y) is described by the equations

$$\frac{dx_a}{dt} = W_u f_x^{u}, \quad \frac{dy_a}{dt} = W_u f_y^{u}. \qquad (2)$$

The coefficients K_u and W_u in (1) and (2) may be different for different blocks.

Interaction between Blocks along Fault Planes

At time t and some point (X,Y) of a fault plane separating the blocks numbered i and j (the block numbered i is on the left and that numbered j on the right of the fault), the components Δx, Δy of the relative block displacement are given by

$$\Delta x = x_i - x_j - (Y - Y_c^i)\varphi_i + (Y - Y_c^j)\varphi_j,$$

$$\Delta y = y_i - y_j + (X - X_c^i)\varphi_i - (X - X_c^j)\varphi_j. \qquad (3)$$

where X_c^i, Y_c^i, X_c^j, Y_c^j are the coordinates of the geometrical centers of the block bottoms, (x_i, y_i), and (x_j, y_j) are the translation vectors of the blocks, and φ_i, φ_j are the angles of rotation of the blocks around the geometrical centers of their bottoms at time t.

In accordance with the assumption that relative block displacements take place only along fault planes, the displacements along a fault plane are related to the horizontal relative displacement by

$$\Delta_t = e_x\Delta x + e_y\Delta y,$$

$$\Delta_l = \Delta_n / \cos\alpha, \quad \text{where} \quad \Delta_n = e_x\Delta y - e_y\Delta x. \qquad (4)$$

That is, the displacements along a fault plane are projected onto the horizontal plane (Fig.2A). Here Δ_t, Δ_l are the displacements along the fault plane parallel (Δ_t) and normal (Δ_l) to the fault line on the upper plane, (e_x, e_y) is the unit vector along the fault line on the upper plane, α is the dip angle of the fault plane, and Δ_n is the horizontal displacement normal to the fault line on the upper plane.

The elastic force per unit area $\mathbf{f} = (f_t, f_l)$ acting along the fault plane at the point (X,Y) is given by

$$f_t = K(\Delta_t - \delta_t),$$

$$f_l = K(\Delta_l - \delta_l). \qquad (5)$$

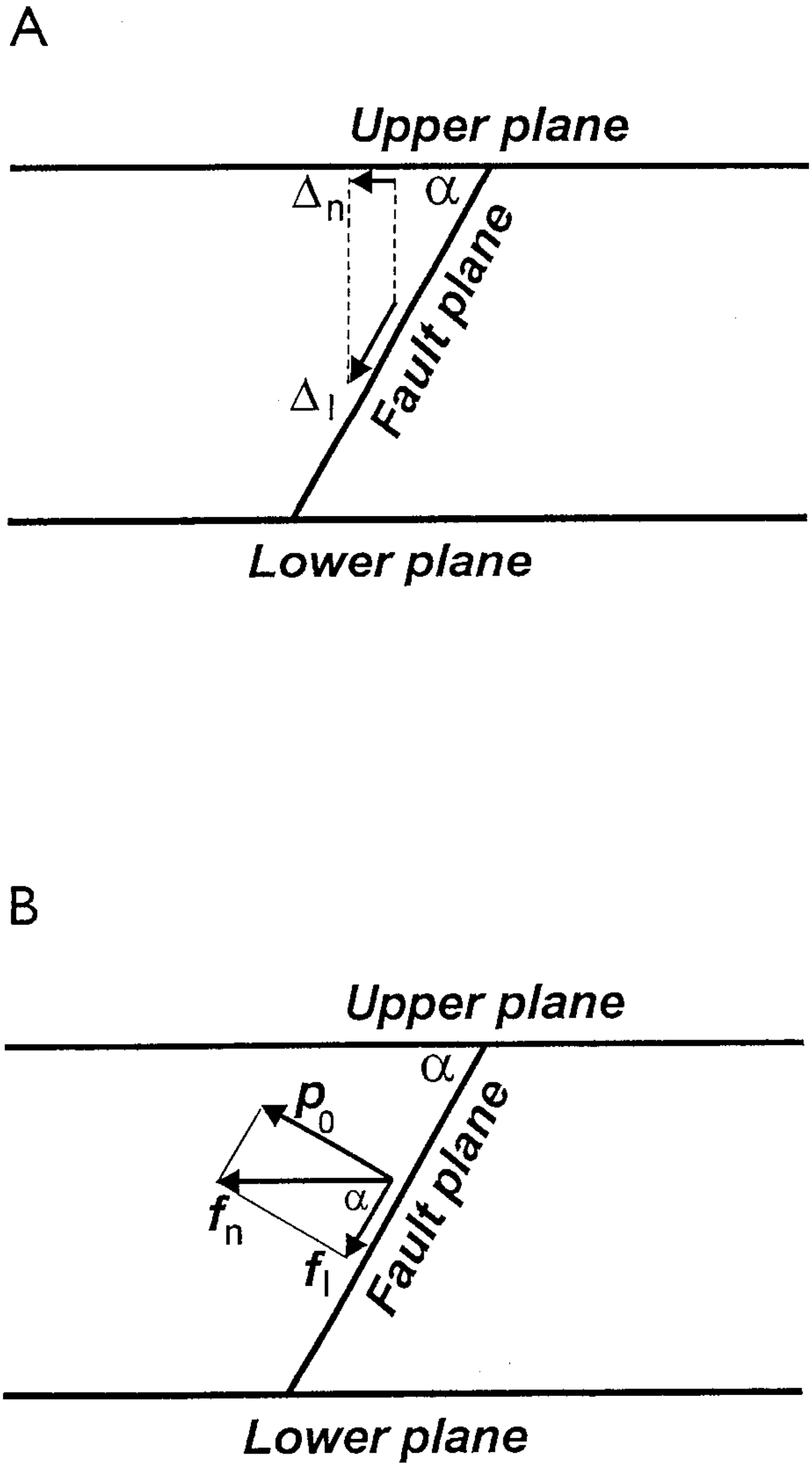

FIGURE 2 Vertical section of the block structure orthogonal to a fault. Relative displacements of blocks (A) Δ_n and Δ_l and forces (B) p_0, f_l, and f_n are shown.

Here δ_t, δ_l are inelastic displacements along the fault plane at the point (X,Y) at time t, parallel (δ_t) and normal (δ_l) to the fault line on the upper plane.

The evolution of the inelastic displacement at the point (X,Y) is described by the equations

$$\frac{d\delta_t}{dt} = Wf_t, \quad \frac{d\delta_l}{dt} = Wf_l. \tag{6}$$

The coefficients K and W in (5) and (6) may be different for different faults. The coefficient K can be considered as the shear modulus in the fault plane.

In addition to the elastic force, there is a reaction force which is normal to the fault plane; the work done by this force is zero, because all relative movements are tangent to the fault plane. The elastic energy per unit area at (X,Y) is equal to

$$e = (f_t(\Delta_t - \delta_t) + f_l(\Delta_l - \delta_l))/2. \tag{7}$$

From (4) and (7) the horizontal component of the elastic force per unit area normal to the fault line on the upper plane, f_n, can be written as:

$$f_n = \frac{\partial e}{\partial \Delta_n} = \frac{f_l}{\cos\alpha}. \tag{8}$$

It follows from (8) that the total force acting at a point of the fault plane is horizontal if there is a reaction force, which is normal to the fault plane (Fig.2B). The reaction force per unit area is equal to

$$p_0 = f_l \operatorname{tg} \alpha. \tag{9}$$

Since we have introduced the reaction force (9), there are no vertical components of forces acting on the blocks and no vertical block displacements.

Formulas (4), (5) and (8) lead to the following formulas for the horizontal components of the vector (f_x, f_y) of the elastic force per unit area at the point.

$$f_x = K\left(\left(e_x^2 + \frac{e_y^2}{\cos^2\alpha}\right)\Delta x + e_x e_y\left(1 - \frac{1}{\cos^2\alpha}\right)\Delta y - \delta_t e_x + \frac{\delta_l e_y}{\cos\alpha}\right),$$

$$f_y = K\left(\left(e_y^2 + \frac{e_x^2}{\cos^2\alpha}\right)\Delta y + e_x e_y\left(1 - \frac{1}{\cos^2\alpha}\right)\Delta x - \delta_t e_y - \frac{\delta_l e_x}{\cos\alpha}\right).$$

The formulas given above are valid for the boundary faults too. In this case one of the blocks separated by a fault is a boundary block. The movement of such a block is described by its translation and rotation around the origin of coordinates. Therefore the coordinates of the geometrical center of the block bottom in (3) are zero for a boundary block. For example, if the block numbered j is a boundary block, then $X_c^j = Y_c^j = 0$ in (3).

Equations of Equilibrium

The components of block translation vectors and the angles of rotation around the geometrical centers of the bottoms are found from the condition that the total force and the total moment of forces acting on each block are both zero. This is the condition of quasi-static equilibrium of the system, and at the same time the condition of minimum energy. The forces arising from prescribed movements of the underlying medium and of the boundaries of a block structure are considered only in the equations of equilibrium. In fact it is assumed that the action of all other forces (gravity, etc.) on a block structure is balanced and does not cause displacements.

According to (1), (3-5), (8), and (9), forces acting on the blocks are linear functions of translation vectors and the angles of rotation. Therefore the system of equations which describes the equilibrium is linear and has the following form

$$A\mathbf{z} = \mathbf{b} \qquad (10)$$

where the components of the unknown vector $\mathbf{z} = (z_1, z_2, ..., z_{3n})$ are components of the translation vectors and angles of rotation around the geometrical centers of the bottoms (n is the number of blocks), i.e. $z_{3m-2} = x_m$, $z_{3m-1} = y_m$, $z_{3m} = \varphi_m$ (m is block number, $m = 1, 2, ..., n$).

The matrix A ($3n$x$3n$) does not depend on time and its elements are given by (1), (3-5), (8), and (9). The moment of the forces acting on a block is calculated relative to the geometrical center of its bottom. The elements of A can be calculated by the formulas

$$a_{3m+1,3m+1} = S_u^m K_u^m c_m + \sum_{p=1}^{r_m} S_p^m K^{mp} C_1^{mp},$$

$$a_{3m+1,3m+2} = a_{3m+2,3m+1} = \sum_{p=1}^{r_m} S_p^m K^{mp} C_2^{mp},$$

$$a_{3m+1,3m+3} = a_{3m+3,3m+1} = \sum_{p=1}^{r_m} S_p^m K^{mp} (C_2^{mp} (X_c^{mp} - X_c^m) - C_1^{mp} (Y_c^{mp} - Y_c^m)),$$

$$a_{3m+2,3m+2} = S_u^m K_u^m c_m + \sum_{p=1}^{r_m} S_p^m K^{mp} C_3^{mp},$$

$$a_{3m+2,3m+3} = a_{3m+3,3m+2} = \sum_{p=1}^{r_m} S_p^m K^{mp} (C_3^{mp} (X_c^{mp} - X_c^m) - C_2^{mp} (Y_c^{mp} - Y_c^m)),$$

$$a_{3m+3,3m+3} = K_u^m c_m \left(\int_S (X^2 + Y^2)\, dS - S_u^m \left((X_c^m)^2 + (Y_c^m)^2 \right) \right) +$$

$$+ \sum_{p=1}^{r_m} K^{mp} \left(\int_S (C_3^{mp} X^2 + C_1^{mp} Y^2 - 2 C_2^{mp} XY)\, dS - S_p^m \left(C_3^{mp} \left(2 X_c^{mp} X_c^m - (X_c^m)^2 \right) + C_1^{mp} \left(2 Y_c^{mp} Y_c^m - (Y_c^m)^2 \right) + 2 C_2^{mp} (X_c^m Y_c^m - X_c^{mp} Y_c^m - Y_c^{mp} X_c^m) \right) \right).$$

Here S_u^m, X_c^m, Y_c^m are the square and the coordinates of the geometrical center of the bottom of a block numbered m; K_u^m is the coefficient K_u in (1) for a block numbered m; r_m is the number of vertices of a block numbered m; S_p^m, X_c^{mp}, Y_c^{mp} are the square and coordinates of the geometrical center of the fault segment between block vertices numbered p and $p + 1$ (if $p < r_m$) or r_m and 1 (if $p = r_m$); K^{mp} is the coefficient K in (5) and (10) for the fault to which the segment belongs. As defined above a fault segment means a part of the fault plane limited by the upper and lower planes and lines which connect positions on the upper and lower planes of two sequential vertices of the fault.

The coefficients c_m, C_1^{mp}, C_2^{mp}, C_3^{mp} are calculated by the formulas

$$c_m = \min_{1 \le p \le r_m} \cos^2\alpha_{mp}, \quad C_1^{mp} = (e_x^{mp})^2 c_m + \frac{(e_y^{mp})^2 c_m}{\cos^2\alpha_{mp}},$$

$$C_2^{mp} = e_x^{mp} e_y^{mp}\left(c_m + \frac{c_m}{\cos^2\alpha_{mp}}\right), \quad C_3^{mp} = (e_y^{mp})^2 c_m + \frac{(e_x^{mp})^2 c_m}{\cos^2\alpha_{mp}},$$

where α_{mp}, e_x^{mp}, e_y^{mp} are the values of α, e_x, and e_y for the fault to which the segment belongs.

Let $m \neq k$. If blocks numbered m and k have no common segments, the elements $a_{3m+i,\,3k+j}$ $(i, j = 1, 2, 3)$ of the matrix A are equal to zero. Otherwise

$$a_{3m+1,\,3k+1} = -\sum_p{}' S_p^m K^{mp} C_1^{mp},$$

$$a_{3m+1,\,3k+2} = a_{3m+2,\,3k+1} = -\sum_p{}' S_p^m K^{mp} C_2^{mp},$$

$$a_{3m+1,\,3k+3} = \sum_p{}' S_p^m K^{mp}\left(C_1^{mp}(Y_c^{mp} - Y_c^k) - C_2^{mp}(X_c^{mp} - X_c^k)\right),$$

$$a_{3m+2,\,3k+2} = -\sum_p{}' S_p^m K^{mp} C_3^{mp},$$

$$a_{3m+2,\,3k+3} = \sum_p{}' S_p^m K^{mp}\left(C_2^{mp}(Y_c^{mp} - Y_c^k) - C_3^{mp}(X_c^{mp} - X_c^k)\right),$$

$$a_{3m+3,\,3k+1} = \sum_p{}' S_p^m K^{mp}\left(C_1^{mp}(Y_c^{mp} - Y_c^m) - C_2^{mp}(X_c^{mp} - X_c^m)\right),$$

$$a_{3m+3,\,3k+2} = \sum_p{}' S_p^m K^{mp}\left(C_2^{mp}(Y_c^{mp} - Y_c^m) - C_3^{mp}(X_c^{mp} - X_c^m)\right),$$

$$a_{3m+3,\,3k+3} = \sum_p{}' K^{mp}\Bigg(\int_S (2C_2^{mp}XY - C_3^{mp}X^2 - C_1^{mp}Y^2)\,dS +$$

$$+ S_p^m\left(C_3^{mp}\left(X_c^{mp}(X_c^m + X_c^k) - X_c^m X_c^k\right) + C_1^{mp}\left(Y_c^{mp}(Y_c^m + Y_c^k) - Y_c^m Y_c^k\right) + \right.$$

$$\left. + C_2^{mp}\left(X_c^m Y_c^k + X_c^k Y_c^m - X_c^{mp}(Y_c^m + Y_c^k) - Y_c^{mp}(X_c^m + X_c^k)\right)\right)\Bigg).$$

In these formulas summarizing over common segments of blocks numbered m and k is only made.

The components of the vector $\mathbf{b} = (b_1, b_2, \ldots, b_{3n})$ are defined from (1), (3-5), (8), and (9) as well. They depend on time explicitly because of the movements of the underlying medium and of block structure boundaries and, implicitly, because of the inelastic displacements and are calculated by the formulas

$$b_{3m+1} = c_m\left(K_u^m\left(S_u^m x_u^m + \int_{S_u^m} x_a\, dS\right) + \sum_{p=1}^{r_m} K^{mp}\int_{S_p^m}\left(\delta_t e_x^{mp} - \frac{\delta_1 e_y^{mp}}{\cos\alpha_{mp}}\right)dS\right) + d_{3m+1},$$

$$b_{3m+2} = c_m\left(K_u^m\left(S_u^m y_u^m + \int_{S_u^m} y_a\, dS\right) + \sum_{p=1}^{r_m} K^{mp}\int_{S_p^m}\left(\delta_t e_y^{mp} + \frac{\delta_1 e_x^{mp}}{\cos\alpha_{mp}}\right)dS\right) + d_{3m+2},$$

$$b_{3m+3} = c_m\left(K_u^m\left(\int_{S_u^m}\left(y_a(X - X_c^m) - x_a(Y - Y_c^m) + \varphi_u^m(X^2 + Y^2)\right)dS -\right.\right.$$

$$\left. - S_u^m\varphi_u^m\left((X_c^m)^2 + (Y_c^m)^2\right)\right) + \sum_{p=1}^{r_m} K^{mp}\int_{S_p^m}\left(\delta_t\left(e_y^{mp}(X - X_c^m) - e_x^{mp}(Y - Y_c^m)\right) + \right.$$

$$\left.\left. + \delta_1\frac{e_x^{mp}(X - X_c^m) + e_y^{mp}(Y - Y_c^m)}{\cos\alpha_{mp}}\right)dS\right) + d_{3m+3}.$$

Here x_u^m, y_u^m, and φ_u^m are the components of the translation vector and the angle of the rotation, around the geometrical center of the block bottom for the medium underlying the block numbered m.

If a block numbered m has no common segments with the boundary blocks, the items d_{3m+i} ($i = 1, 2, 3$) are equal to zero. Otherwise

$$d_{3m+1} = \sum_p{}' S_p^m K^{mp}\left(C_1^{mp}(x_{mp} - \varphi_{mp}Y_c^{mp}) + C_2^{mp}(y_{mp} + \varphi_{mp}X_c^{mp})\right),$$

$$d_{3m+2} = \sum_p{}' S_p^m K^{mp}\left(C_2^{mp}(x_{mp} - \varphi_{mp}Y_c^{mp}) + C_3^{mp}(y_{mp} + \varphi_{mp}X_c^{mp})\right),$$

$$d_{3m+3} = \sum_p{}' K^{mp}\left(S_p^m\left(x_{mp}\left(C_2^{mp}(X_c^{mp} - X_c^m) - C_1^{mp}(Y_c^{mp} - Y_c^m)\right) + \right.\right.$$

$$+ y_{mp}\left(C_3^{mp}(X_c^{mp} - X_c^m) - C_2^{mp}(Y_c^{mp} - Y_c^m)\right) + \varphi_{mp}\left(X_c^{mp}(C_2^{mp}Y_c^m - C_3^{mp}X_c^m) + \right.$$

$$\left.\left.\left. + Y_c^{mp}(C_2^{mp}X_c^m - C_1^{mp}Y_c^m)\right)\right) + \varphi_{mp}\int_{S_p^m}(C_3^{mp}X^2 + C_1^{mp}Y^2 - 2C_2^{mp}XY)\, dS\right).$$

Discretization

Time discretization is performed by introducing a time step Δt. The state of a block structure is considered at discrete values of time $t_i = t_0 + i\Delta t$ (i = 1, 2, ...), where t_0 is the initial time. The transition from the state at t_i to the state at t_{i+1} is made as follows: (i) new values of inelastic displacements x_a, y_a, δ_t, δ_l are calculated from equations (2) and (6); (ii) the translation vectors and rotation angles at t_{i+1} are calculated for the boundary blocks and the underlying medium; (iii) the components of **b** in equations (10) are calculated, and these equations are used to determine the translation vectors and angles of rotation. Since the elements of A in (10) are not functions of time, it is sufficient to find A and the associated inverse matrix only once, at the beginning of the calculation.

Formulas (1-9) describe the forces, the relative displacements, and the inelastic displacements at points of fault segments and block bottoms. Therefore some discretization of these surfaces is required for numerical simulation. The space discretization is defined by a parameter ε, and is applied to the surfaces of fault segments and to block bottoms. The integrals over the surfaces of fault segments and block bottoms in formulas for elements of the matrix A and for the components of the vector **b** are replaced by finite sums in accordance with the discretization.

The discretization of a fault segment is done as follows. Each fault segment is a trapezium with bases a and b and height $h = H / \sin \alpha$, where H is the thickness of the layer d, and α is the dip angle of the fault plane. If we define

$n_1 = \text{ENTIRE}(h/\varepsilon) + 1$, and $n_2 = \text{ENTIRE}(\max(a,b)/\varepsilon) + 1$,

the trapezium is divided into $n_1 n_2$ small trapeziums by two groups of segments inside it: n_1-1 segments parallel to the trapezium bases and spaced at intervals h/n_1, and n_2-1 segments connecting the points spaced at intervals of a/n_2 and b/n_2, respectively, on the two bases (Fig. 3). The small trapeziums obtained in such a way are called "cells". The coordinates X, Y in (3) and the inelastic displacements δ_t, δ_l in (5) are supposed to be the same for all points of a cell. These values of the coordinates and inelastic displacements are considered as the average values for a cell. When substituted in (3-5), (8), and (9), they yield the average (over the cell) elastic and reaction forces per unit area. The forces

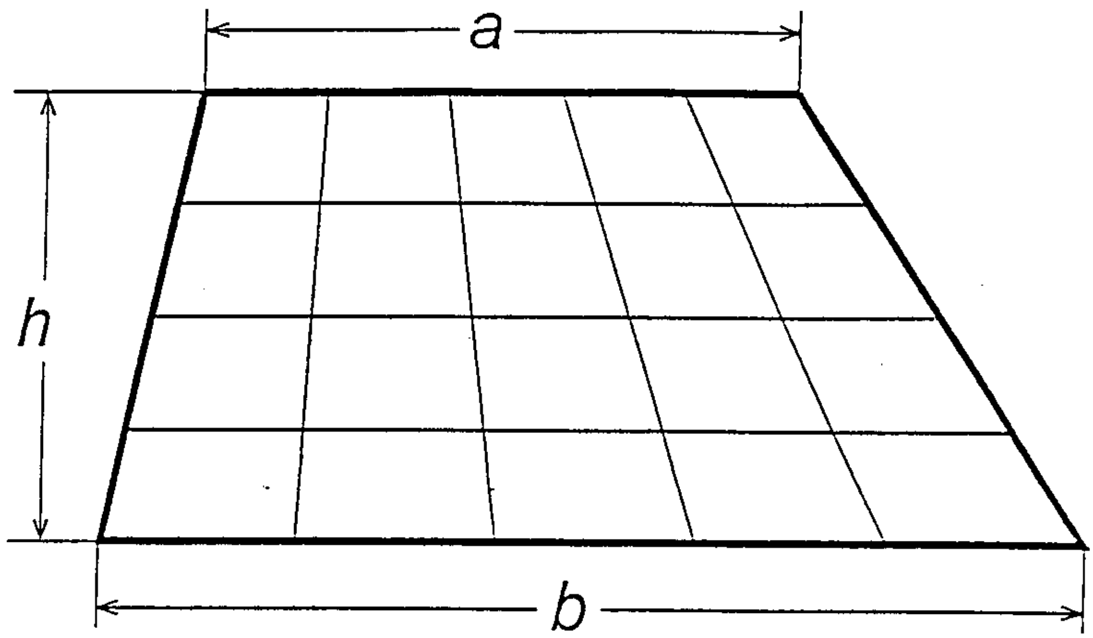

FIGURE 3 Discretization of the fault segment ($n_1 = 4$, $n_2 = 5$).

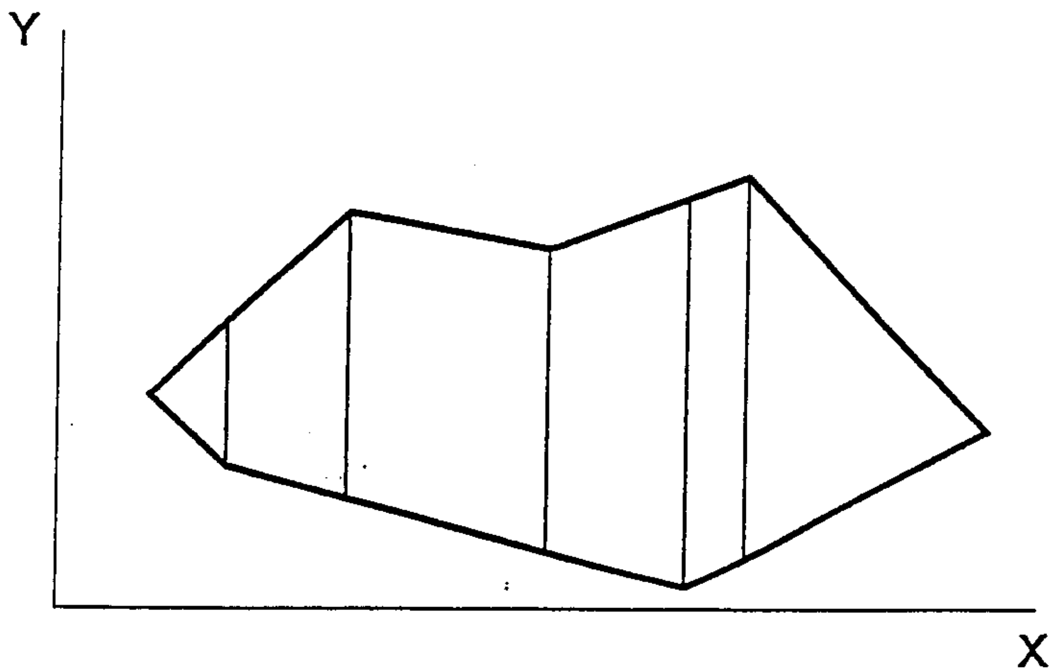

FIGURE 4 Division of the block bottom into trapeziums and triangles.

acting on the cell are obtained by multiplying the average forces per unit area by the area of the cell.

The bottom of a block is a polygon. Before discretization it is divided into trapeziums (triangles) by segments passing through its vertices and parallel to the Y axis (Fig. 4). The discretization of these trapeziums (triangles) is performed in the same way as in the case of fault segments. The small trapeziums (triangles) are also called "cells". For all the points of a cell the coordinates X, Y and the inelastic displacements x_a, y_a in (1) are assumed to be the same.

Earthquake and Creep

Let us introduce the quantity

$$\kappa = \frac{|\mathbf{f}|}{P - p_0} \quad (11)$$

where **f** is the elastic stress given by (5), P is a parameter of the model which is assumed to be equal for all the faults and can be interpreted as the difference between the lithostatic (due to gravity) and the hydrostatic pressure, and p_0 is the reaction force per unit area given by (9+). The value of P reflects the average effective pressure in fault planes, and the difference $P - p_0$ is the actual pressure for each cell.

For each fault the following three values of κ are considered

$B > H_f \geq H_s$.

Let us assume that the initial conditions for the numerical simulation of block structure dynamics satisfy the inequality $\kappa < B$ for all the cells of the fault segments. If, at some time t_i, the value of κ in any cell of a fault segment reaches the level B, a failure ("earthquake") occurs. By failure we mean slippage during which the inelastic displacements δ_t, δ_l in the cell change abruptly to reduce the value of κ to the level H_f. Thus, the earthquakes occur in accordance with the dry friction model.

The new values of the inelastic displacements in the cell are calculated from

$$\delta_t^e = \delta_t + \gamma f_t, \quad \delta_l^e = \delta_l + \gamma f_l \quad (12)$$

where δ_t, δ_l, f_t, f_l are the inelastic displacements and the components of elastic force vector per unit area just before the failure. The coefficient γ is given by

$$\gamma = 1/K - PH_f/(K(|\mathbf{f}| + H_f f_l \operatorname{tg} \alpha)) \quad (13)$$

It follows from (5), (9), (11-13) that after the calculation of new values of the inelastic displacements the value of κ in the cell is equal to H_f.

After calculating the new inelastic displacements for all cells that have failed, the new components of the vector **b** are calculated and, from equations (10), the translation vectors and the angles of rotation for the blocks are found. If $\kappa > B$ for some cell(s) of fault segments, the procedure given above is repeated for this cell (or cells). Otherwise the state of the block structure at the time t_{i+1} is determined as follows: the translation vectors, the rotation angles (at t_{i+1}) for the boundary blocks and for the underlying medium, and the components of **b** in equations (10) are calculated, and then equations (10) are solved.

Different times could be attributed to the failures occurring at different steps of the procedure: if the procedure consists of p steps, the time $t_i + (j - 1)\delta t$ can be attributed to the failures occurring at the jth step, and a value of δt is chosen to satisfy the condition $p\delta t < \Delta t$.

The cells of the same fault plane in which failure occurs at one and the same time form a single earthquake.

The parameters of the earthquake are defined as follows.

The origin time is $t_i + (j - 1)\delta t$.

The epicentral coordinates and the source depth are the weighted sums of the coordinates and depths of the cells included in the earthquake (the weight of a cell is given by its area divided by the sum S of areas of all the cells included in the earthquake).

The magnitude, M, of earthquakes can be defined by using the difference between the energy of the system before and after an earthquake, which can be considered as the strain energy E released through an earthquake. As shown in Keilis-Borok et al. (1997) in the block-structure model there is the linear dependence between E and S, that can be explained by the fact that the energy is distributed along planes, and the energy released through an earthquake depends mainly on the total square of the fault plane involved in the earthquake. Therefore the use of

$$M = 0.98 \lg S + 3.93 \qquad (14)$$

proposed by Utsu and Seki, (1954) where S is the total area of the failured cells, measured in km^2, seems to be reasonable.

Immediately after the earthquake, it is assumed that the cells in which failure has occurred are in a state of creep. It means that, for these cells, in equations (6) which describe the evolution of inelastic displacement, the parameter W_s ($W_s > W$) is used instead of W, and W_s may be different for different faults. After each earthquake a cell is in a state of creep as long as $\kappa > H_s$, while when $\kappa \leq H_s$, the cell returns to the normal state and henceforth the parameter W is used in (6) for this cell.

Hierarchy of Faults

Fault features can be taken into consideration through the values of the constants K, W, W_s and the levels B, H_f, H_s.

The hierarchy of faults is controlled by the hierarchy of structures separated by them. Larger faults separate larger structures. Note that accordingly to the fault definition the lager fault does not mean the longer fault.

It seems natural that the same value of elastic displacement leads to a smaller elastic force for the larger fault than for a smaller one. Thus the value of K has to be smaller for a larger fault.

Larger faults separating larger structures are usually the more strongly fractured and less consolidated zones than smaller faults, and the same force can lead to larger slippage (inelastic displacement) for a larger fault than for a smaller one. Thus the values of W and W_s have to be larger for larger faults than for smaller ones.

The more strongly fracturing of the larger faults can be a cause that earthquakes occur in the larger faults for smaller values of the parameter κ than in the smaller ones. This can be reflected in smaller values of the levels B, H_f, H_s for the larger faults than for the smaller ones.

The qualitative arguments given above can be used as some indications for selecting the values of constants K, W, W_s and levels B, H_f, H_s if the fault hierarchy is known.

REFERENCES

Alekseevskaya,M.A., Gabrielov,A.M., Gvishiani,A.D., Gelfand,I.M. and Ranzman,E.Ya., 1977. Formal morphostructural zoning of mountain territories. *J. Geophys. Res.*, **43**: 227-233.

Gabrielov,A.M., Levshina,T.A. and Rotwain,I.M. 1990. Block model of earthquake sequence. *Phys. Earth and Planet. Inter.*, **61**: 18-28.

Keilis-Borok,V.I., Rotwain,I.M. and Soloviev,A.A., 1997. Numerical modeling of block structure dynamics: Dependence of a synthetic earthquake flow on the structure separateness and boundary movements. *Journal of Seismology*, **1**, 2, 151-160.

Utsu,T. and Seki,A., 1954. A relation between the area of aftershock region and the energy of main shock. *J. Seism. Soc. Japan*, 7: 233-240.