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H4.SMR/1150 - 12

Fifth Workshop on Non-Linear Dynamics and Earthquake Prediction

4 - 22 October 1999

The Simplicity of Generating Complexity

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Lecture I: The Simplicity of Generating Complexity

- **Generating Power Laws**
- **Simple Examples of Complex Patterns**
- **Self-Organized Criticality (SOC)**
- **Earthquake Models of Complexity**
- **A CA Model Using Permeability as a Toggle Switch**
- **Application of the CA Model to Dehydration Reactions**

Lecture II: Contemporary Models of the Earthquake Process

- Dislocations and Brittle Faulting
- Stress Transfer Models (e.g. NAFZ in Turkey)
- Coupling Fluid Flow Model to Large Scale Tectonics
- The Behavior of a 3-D Fluid-Controlled Earthquake Model

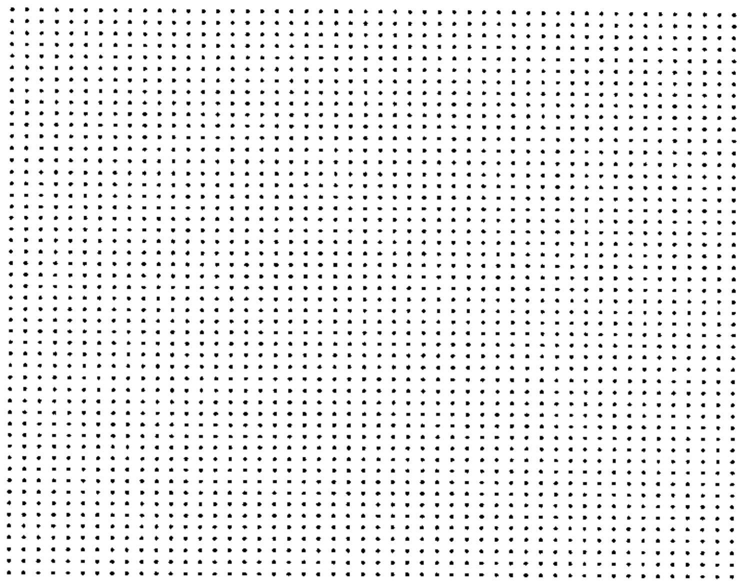
Lecture III: Properties of Large Ruptures and Surface Deformation Field

- Review of Some Rupture Models
- Complex Slip of Earthquakes
- General Strain Fields Around Elastic Dislocations
- The Properties of Large Ruptures (e.g. slip, rise time, moment release)
- Comparisons with Earhtquake Catalogs
- The Surface Deformation Field

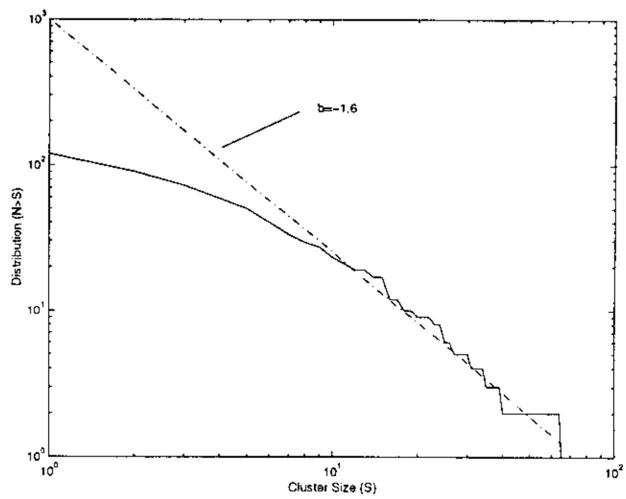
Objective: Build a model that includes:

- Complexity (Implied by Gutenberg-Richter)
  1. How to generate complexity
  2. Self-organized criticality (SOC)
  3. Molecular dynamics models
- Long-range stress interactions (Elasticity Theory)
  1. Brittle faulting as elastic dislocations
  2. Stress transfer models (earthquake triggering)
- Observation that faults are weak (In General)
  1. Stress measurements
  2. Heat flow anomalies
  3. Role of high pore pressures

Simple Complexity: The Dot Game

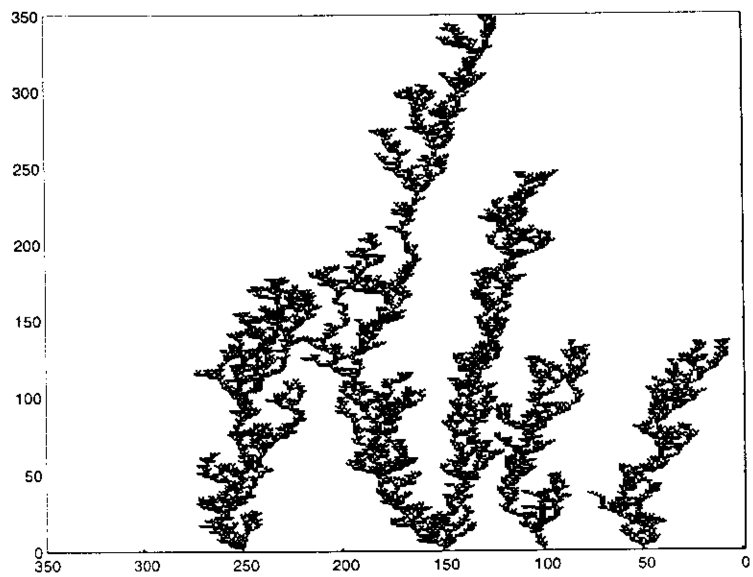


Connect the dots randomly until a box is created. If a box in enclosed, draw another line. Continue turn until no more boxes can be enclosed. At the end of the game, count the size of each box cluster created during one turn



Cluster size statistics. At the end of the game, count the size of each box cluster created during one turn

Growth in an A. oryzae colony in a nutrient poor medium. From Matsuura and Miyazima (1993).



Simple Matlab program of positive feedback

```
colormap('gray')
V=[-1 0];
N=500;  matrix size
x=rand(N);  generate random number
y=x;
y(1:50:N)=-1;   perturb the boundary with a negative number
for k=1:30;  loop to search for connectivity to -1.
for j=2:N-1;
for i=2:N-1;
while y(i,j)>=0.95  & (y(i,j-1)==-1 |  y(i+1,j)==-1 | y(i-1,j)==-1)
y(i,j)=-1;
y(i+1,j+1)=-1;
y(i-1,j+1)=-1;
y(i,j+1)=y(i,j+1)+.1;
y(i+1,j)=y(i+1,j)+.1;
y(i-1,j)=y(i-1,j)+.1;
end;
end;
end;
step forward in time
y(y>=0)=y(y>=0)+.01; step forward in time
h=pcolor(y);caxis(V);shading flat;pause(.5); plot results
end;
```

Self-Organized Criticality (SOC)

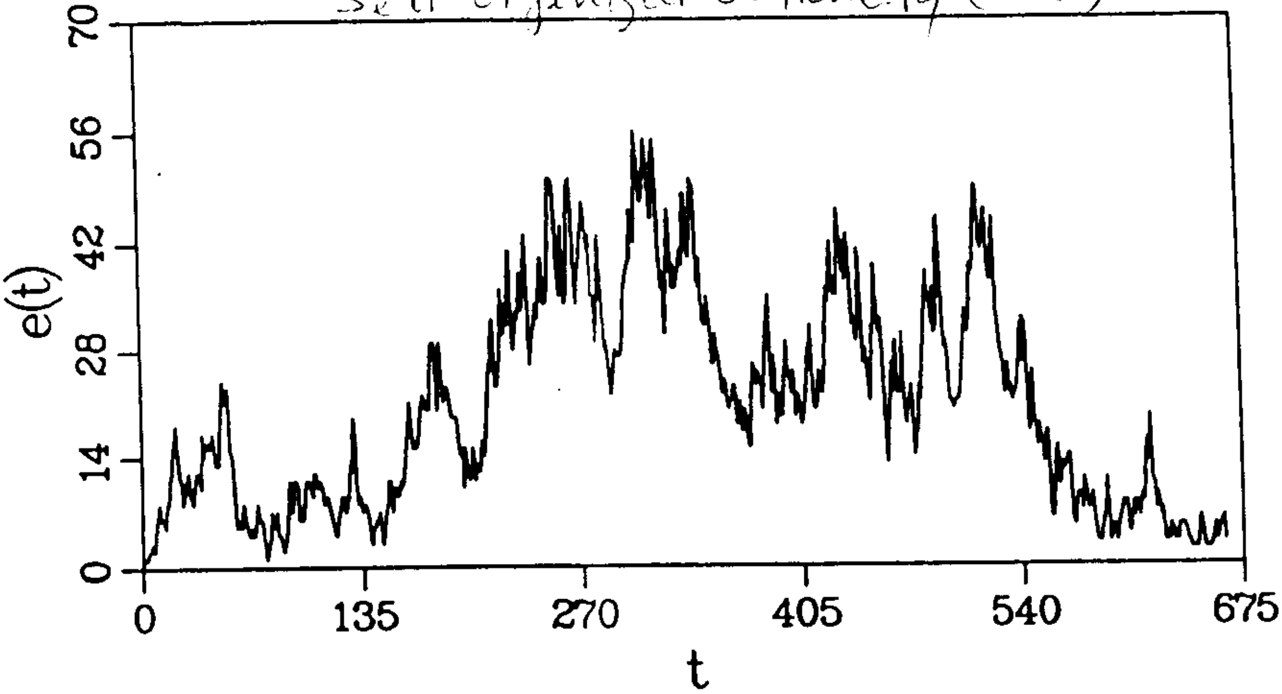
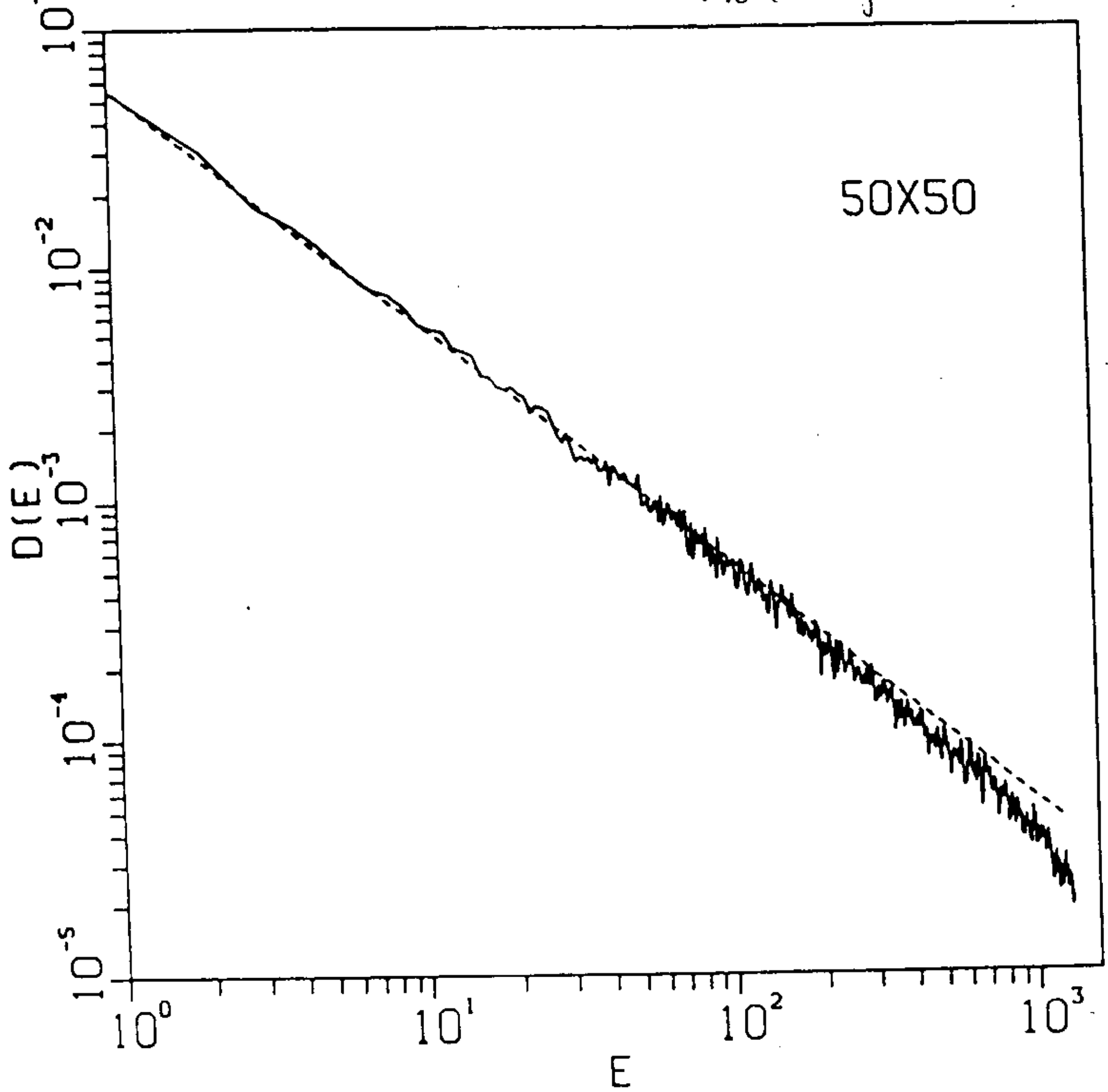
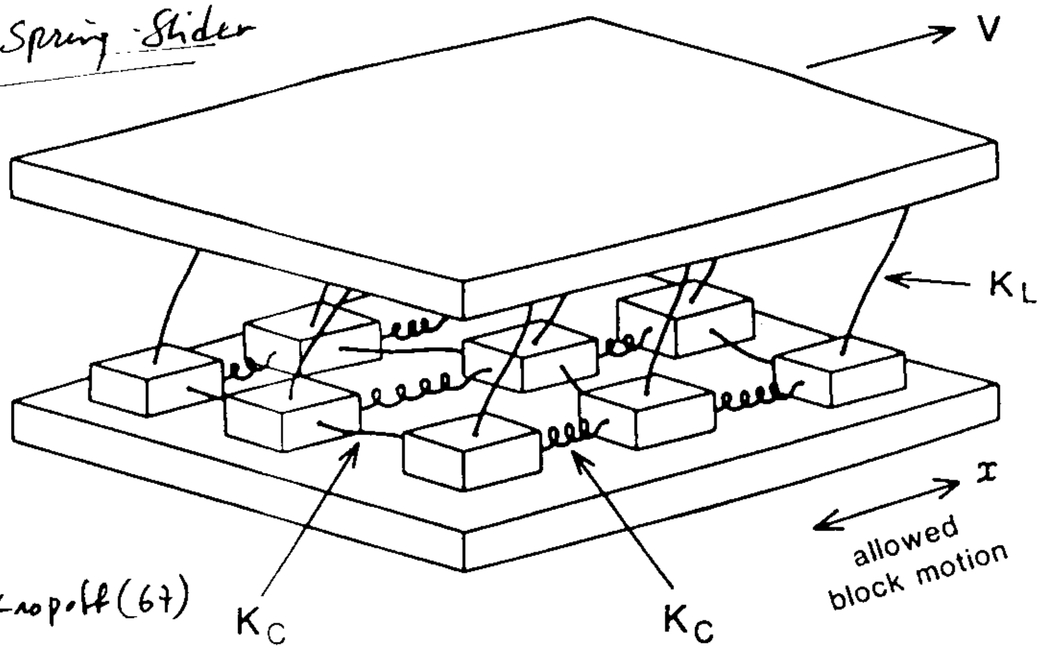


Fig. 1. Energy release versus time during a typical earthquake.

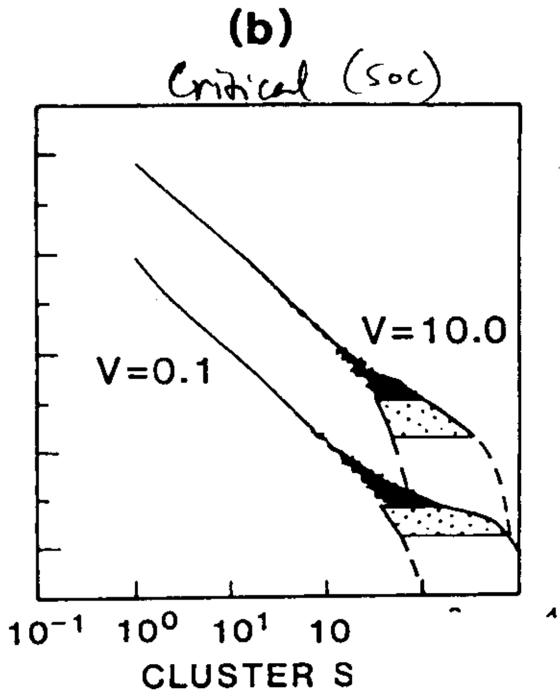
Bak & Tang (1989)

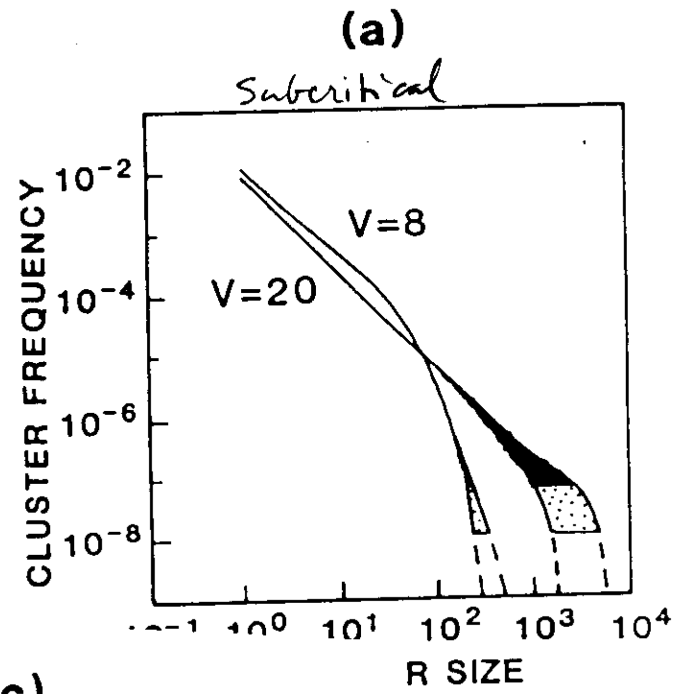


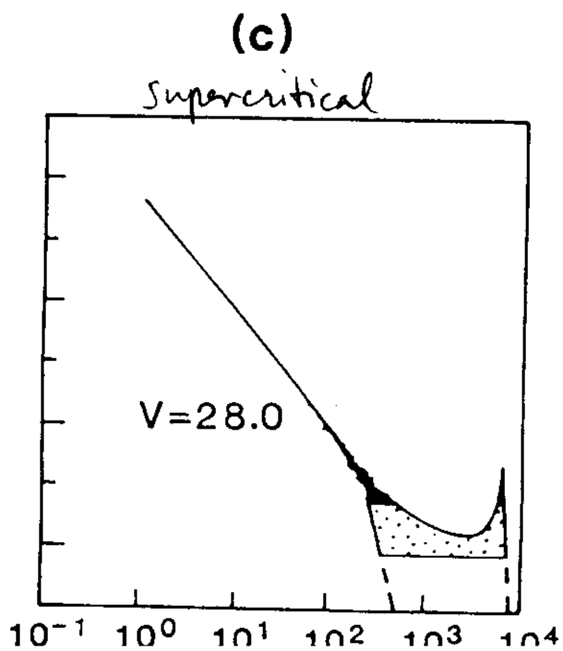
Complexity of Spring-Slider



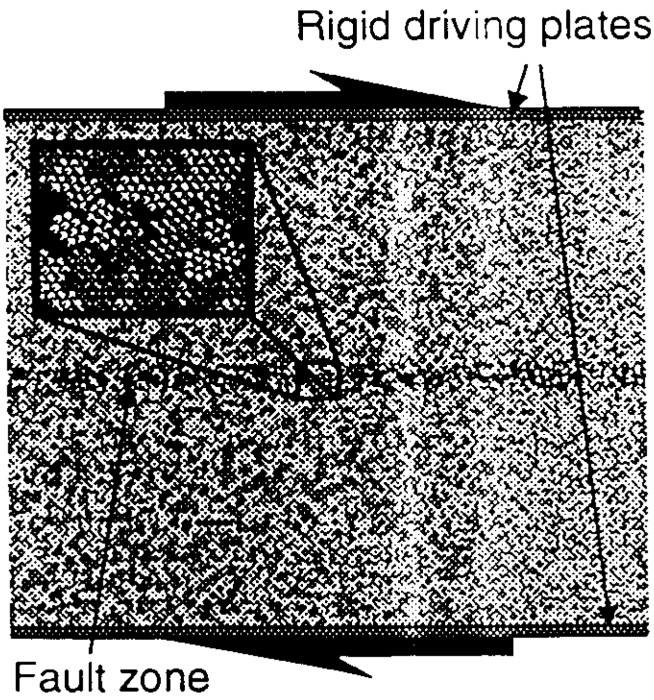
Main (1996)
after Burridge-Knopoff (67)



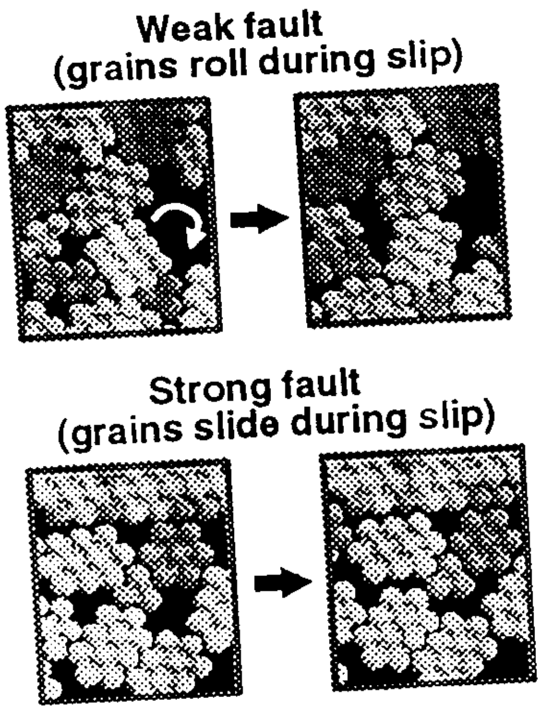




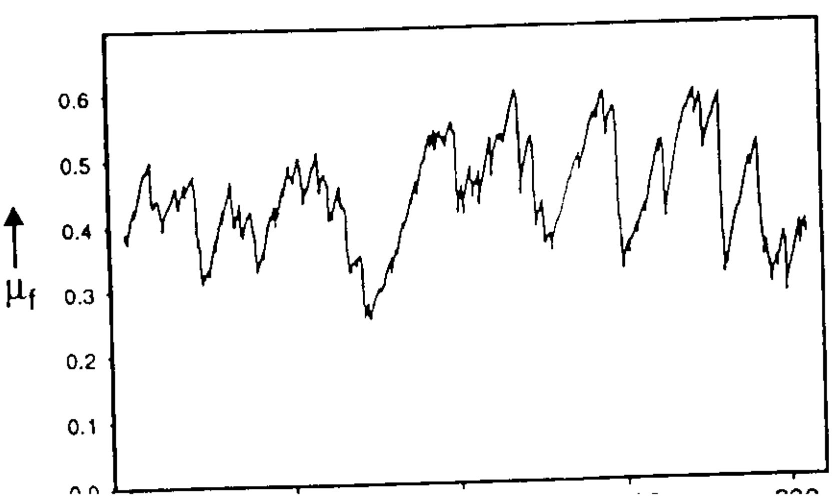
Main (96)
from Rundle & Klein (93)



Lattice-Solid Model of Fault Zones



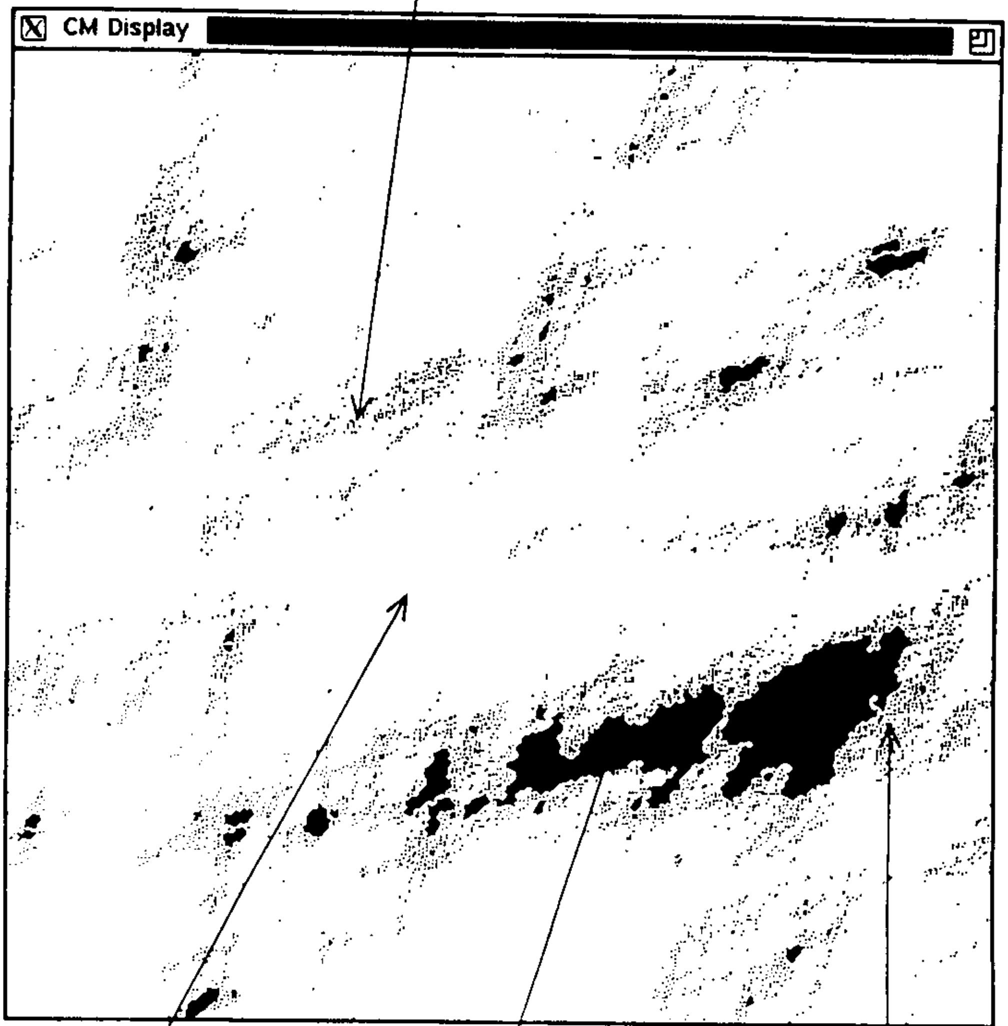
Mora & Place (98)



Cellular Automata of Damage

Henderson et. al (94)

Zone of Damage



Intact Rock

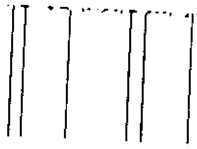
Fracture

'Process Zone'

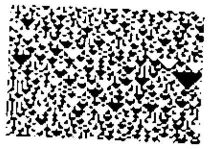
wolfram(86)

rule 90 (01011010)

rule 94 (01011110)

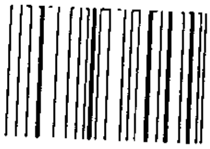


rule 104 (01101000)

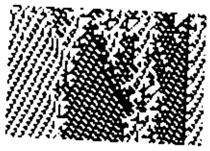


rule 105 (01101001)

rule 106 (01101010)



rule 108 (01101100)

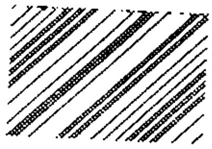


rule 110 (01101110)

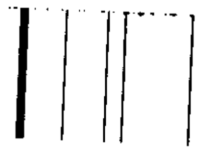
rule 122 (01111010)

rule 126 (01111110)

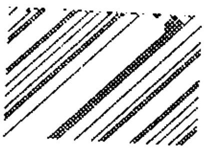
rule 128 (10000000)



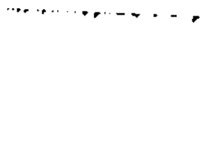
rule 130 (10000010)



rule 132 (10000100)

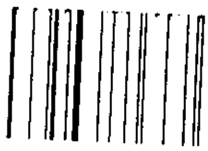


rule 134 (10000110)



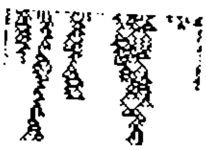
rule 136 (10001000)

rule 138 (10001010)



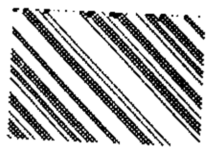
rule 140 (10001100)

rule 142 (10001110)

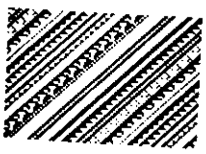


rule 146 (10010010)

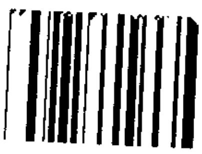
rule 150 (10010110)



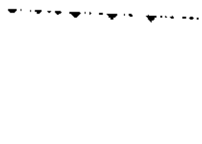
rule 152 (10011000)



rule 154 (10011010)

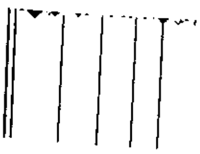


rule 156 (10011100)

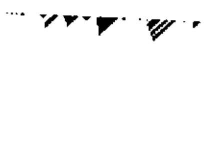


rule 160 (10100000)

rule 162 (10100010)

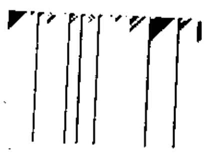


rule 164 (10100100)



rule 168 (10101000)

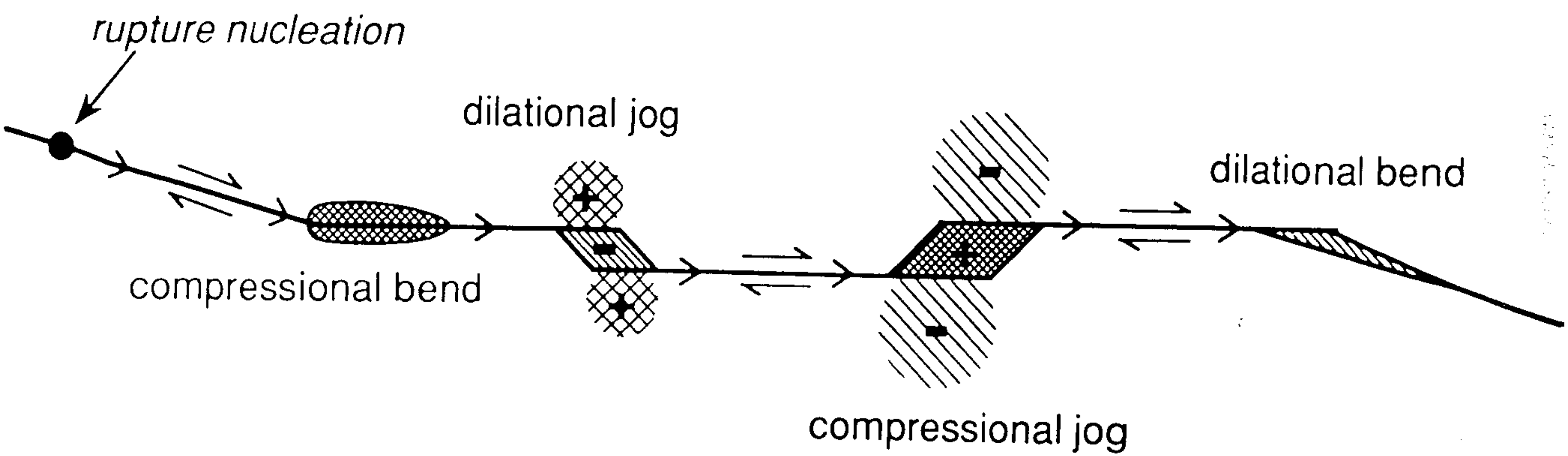
rule 170 (10101010)



rule 172 (10101100)

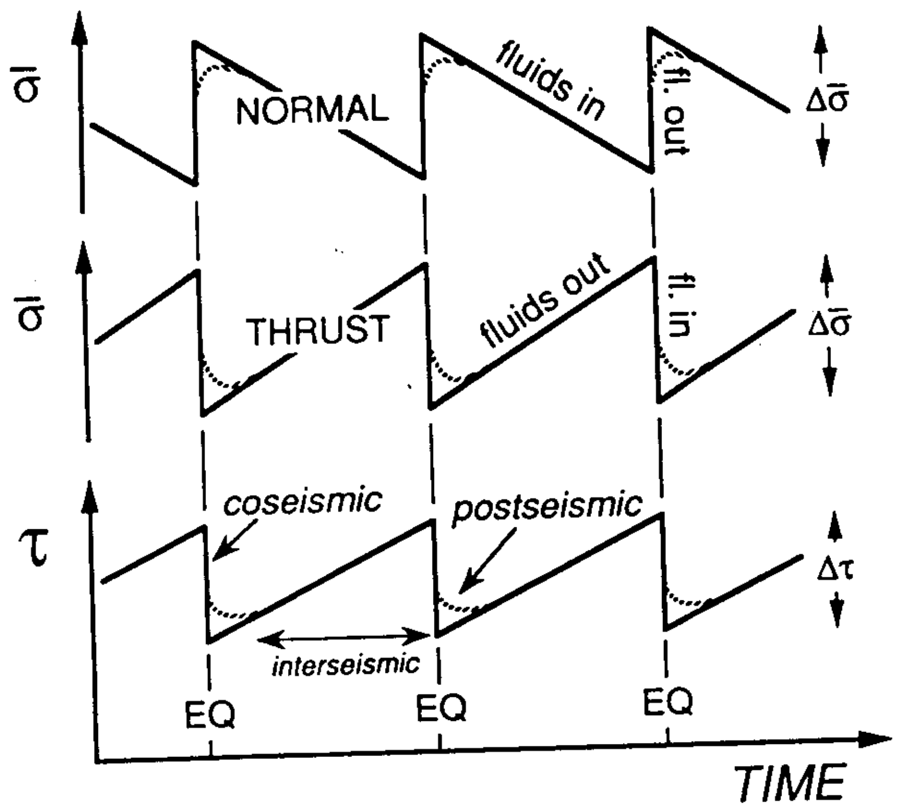
FAULT VALVE BEHAVIOR

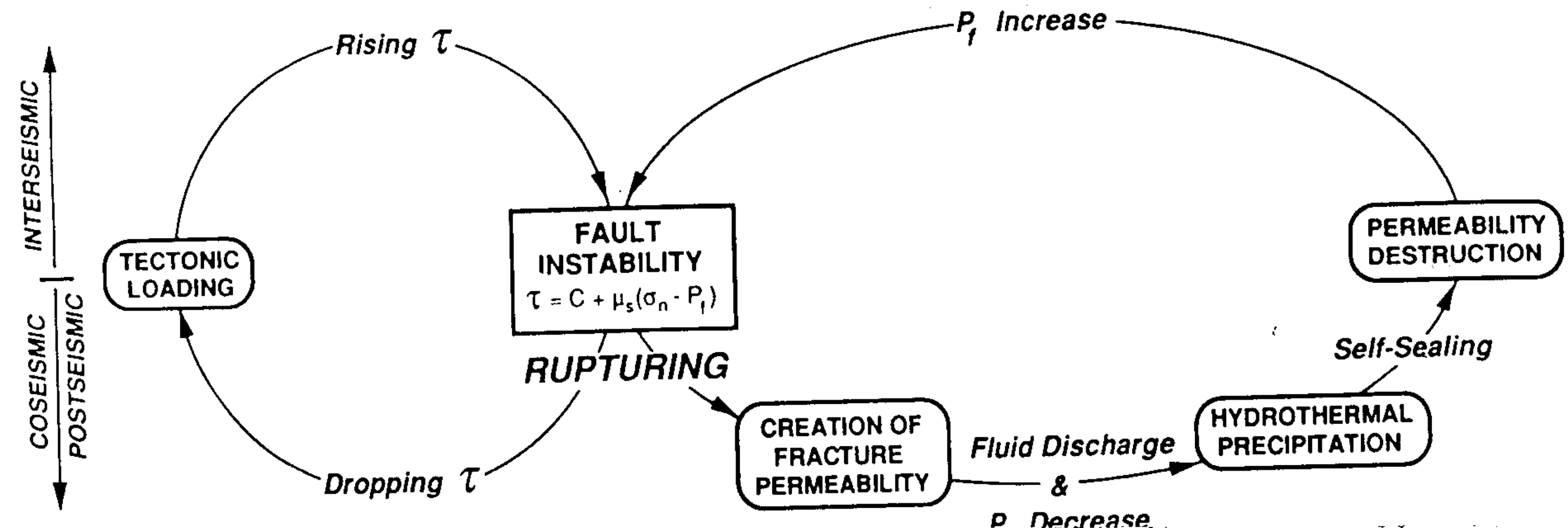
SIBSON ('73, '92)



ig. 5. Seismotectonic carbon of an irregular rupture trace (not to scale), showing areas of enhanced (+ and ross-hatched) and reduced (− and diagonal hachures) mean stress arising from a rupture propagating from left o right. Note that the response of isolated fault bends depends on the direction of rupture propagation. iagram represents a map view of a strike-slip fault, or a cross-section through a dip-slip fault.

FAULT VALVE BEHAVIOR
SIBSON ('92)





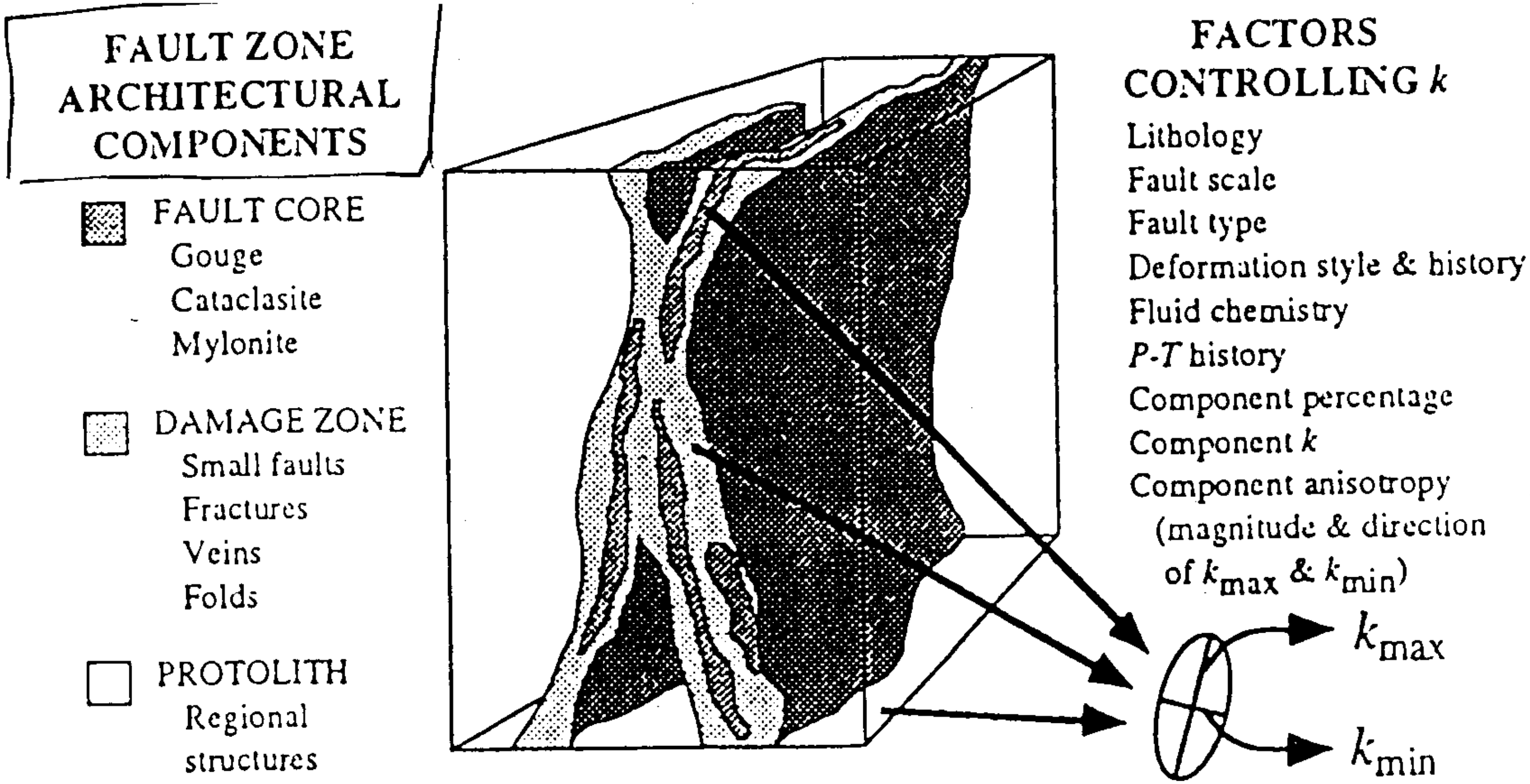


Figure 1. Conceptual model of fault zone with protolith removed (after Chester and Logan, 1986; Smith et al., 1990). Ellipse represents relative magnitude and orientation of the bulk two-dimensional permeability (k) tensor that might be associated with each distinct architectural component of fault zone.

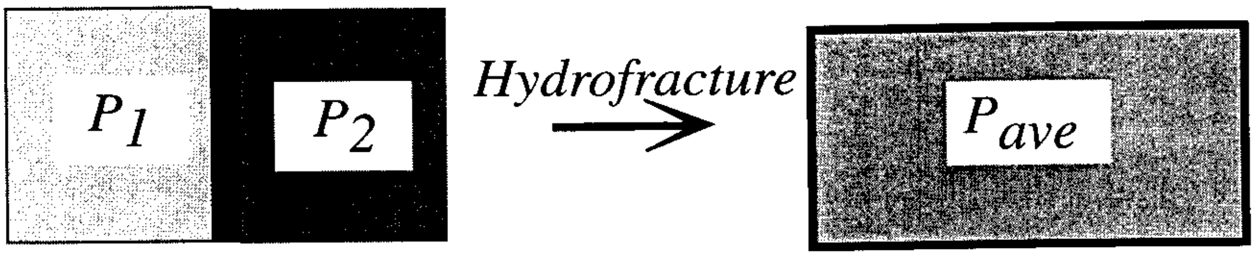
CAINE et. al ('96)

CELLULAR AUTOMATA of CRUSTAL FLUID FLOW

Objective:

Answer the simple question:

"What happens if two neighboring cells at different pore pressures suddenly communicate?"



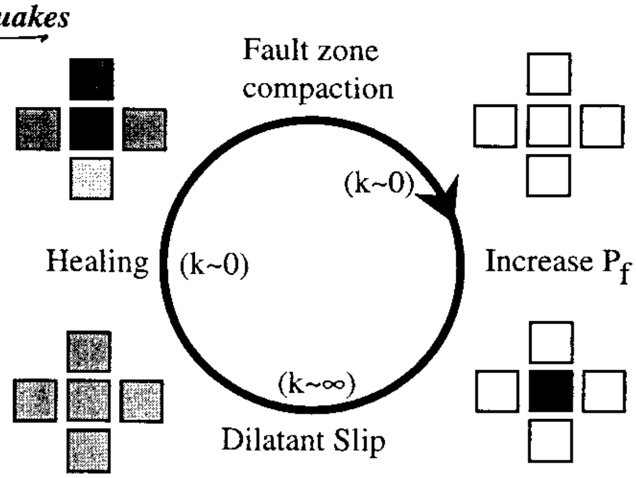
Approach:

- **Investigate a system where permeability is treated as a *toggle-switch***
- **Apply the model to approximate fluid flow in fault zones, couple to *elastic dislocation* theory, and build a fluid-controlled *earthquake model***
- **Apply the model to *dehydration reactions* and compare with laboratory experiments**

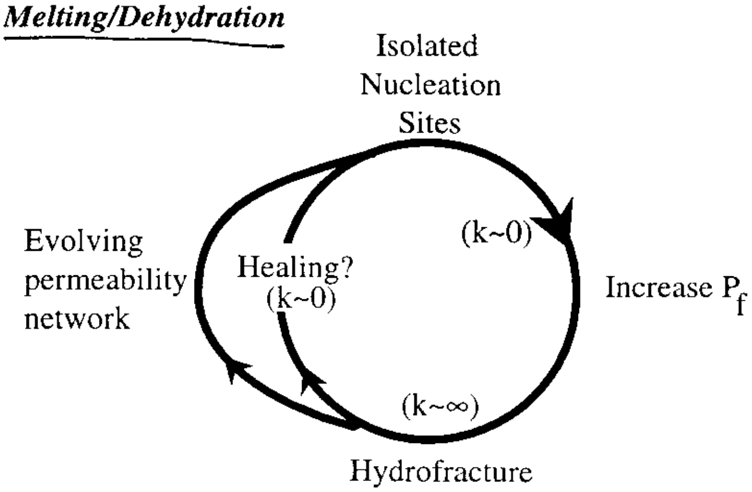
INTERACTION RULE

Permeability as a Toggle Switch

a) Earthquakes



b) Melting/Dehydration



Diffusion Equation with Source Term

$$\frac{\partial P_f}{\partial t} = \frac{1}{\phi(\beta_\phi + \beta_f)} \left[\frac{k}{v}\nabla^2 P_f - (\dot{\phi}_{plastic} - \dot{\Gamma})\right] \qquad (1)$$

where

- ϕ is porosity
- β_ϕ and β_f are the pore and fluid compressibility
- ν is the viscosity
- ρ is the fluid density
- k is the intrinsic permeability of the matrix.
- $\dot{\phi}_{plastic} - \dot{\Gamma}$ is a source term

For an impermeable matrix ($k \sim 0$)

$$\frac{\partial P_f}{\partial t}\Big|_{noflow} = \frac{(\dot{\Gamma} - \dot{\phi})_i}{\phi_i \beta_i} \qquad (2)$$

At failure (hydrofracture), fluid pressures equilibrate with nearest neighbor cells. The equilibrium pressure (by conserving fluid mass and ignoring gravity, is:

$$\overline{P} = \frac{\Sigma_{i=1}^{m}(\phi\beta)_i P_i}{\Sigma_{i=1}^{m}(\phi\beta)_i} \qquad (3)$$

where $\overline{P}$ is the average pressures of affected cells.

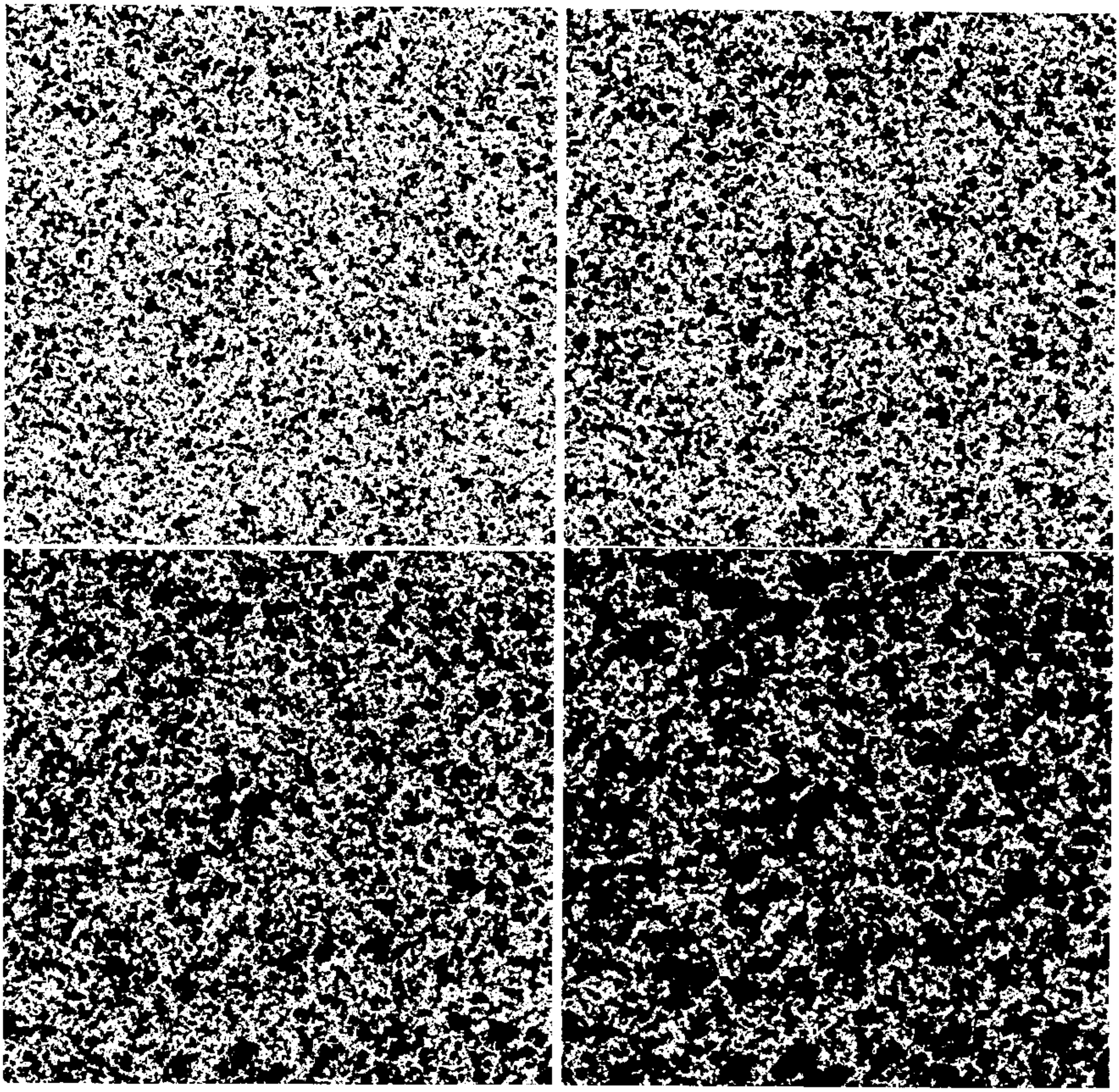
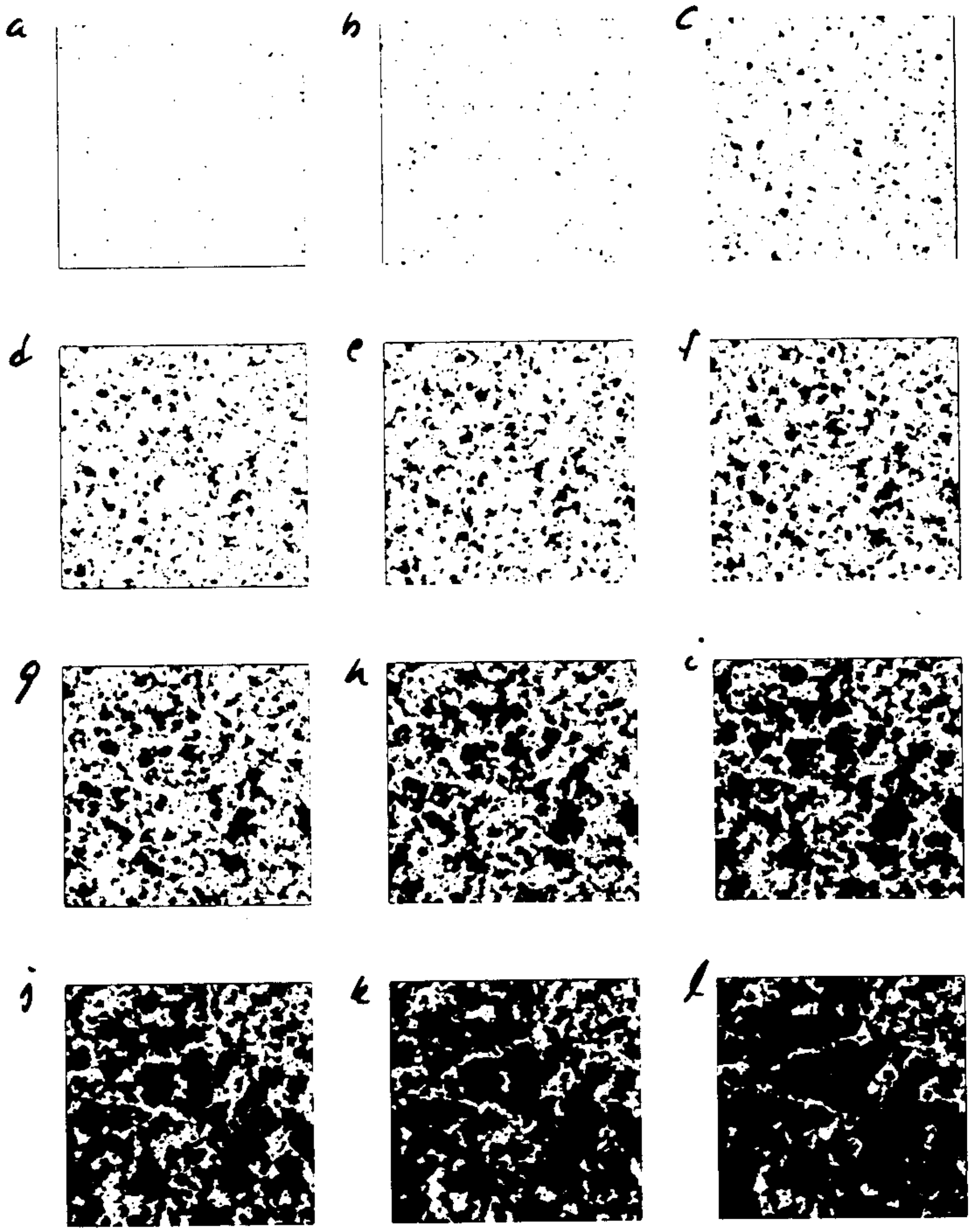
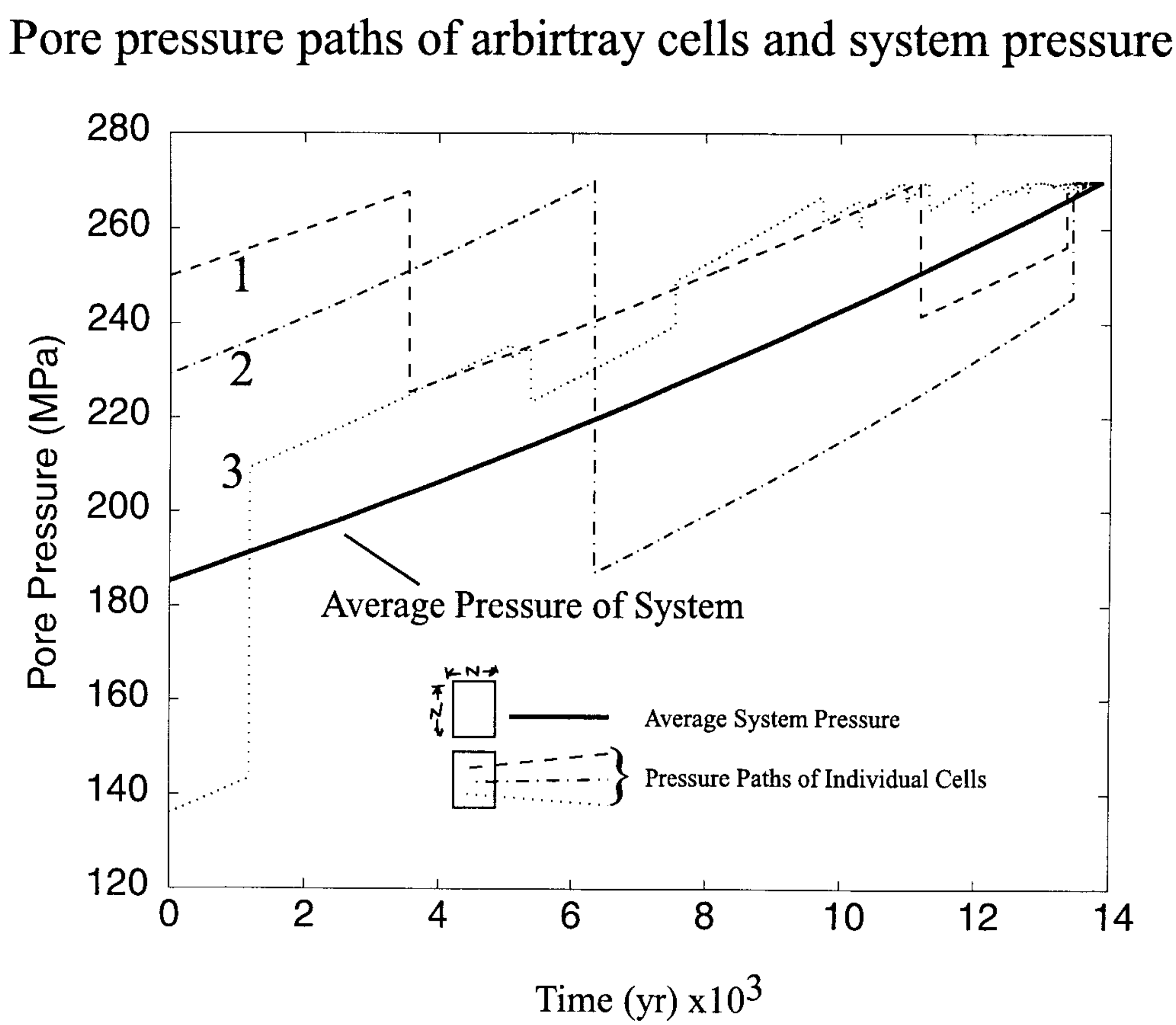


Figure 5: Evolution of high pore pressure connectivity from cellular automaton model

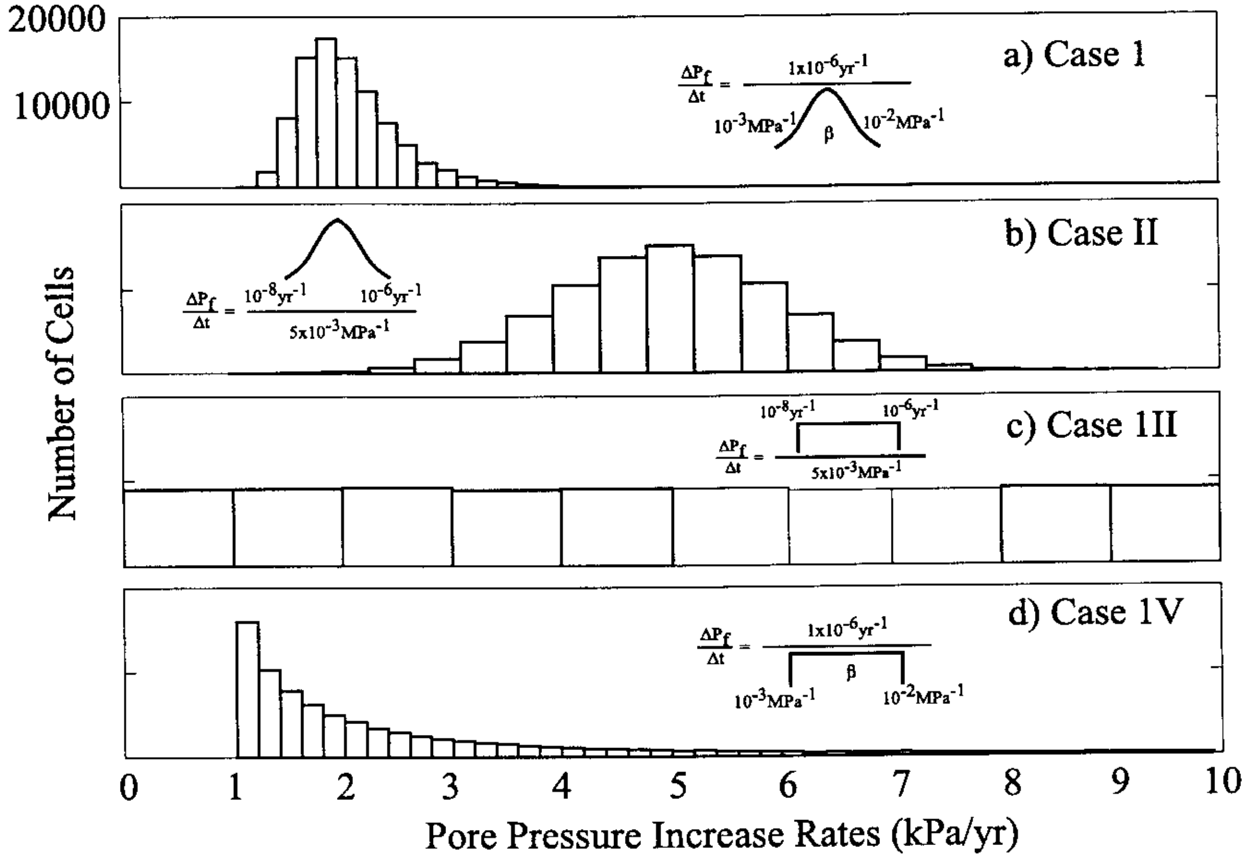
EVOLUTION of INCIPIENT FAILURE



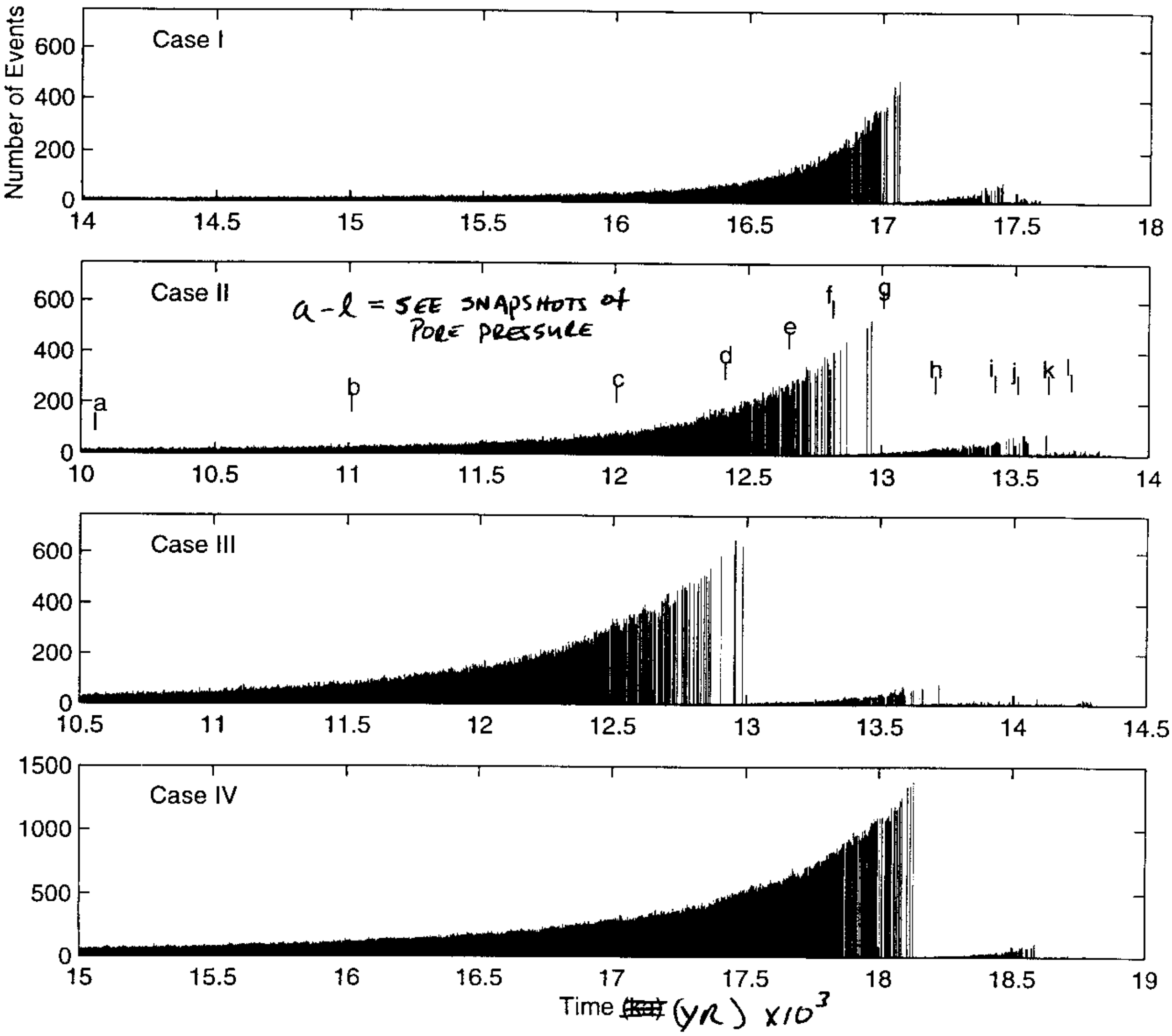
Black = Incipient Failure


Pore pressure paths of arbirtray cells and system pressure
Pore Pressure (MPa)
280
260
240
220
200
180
160
140
120
0
2
4
6
8
10
12
14
Time (yr) x10^3
1
2
3
Average Pressure of System
Average System Pressure
Pressure Paths of Individual Cells
N
Z

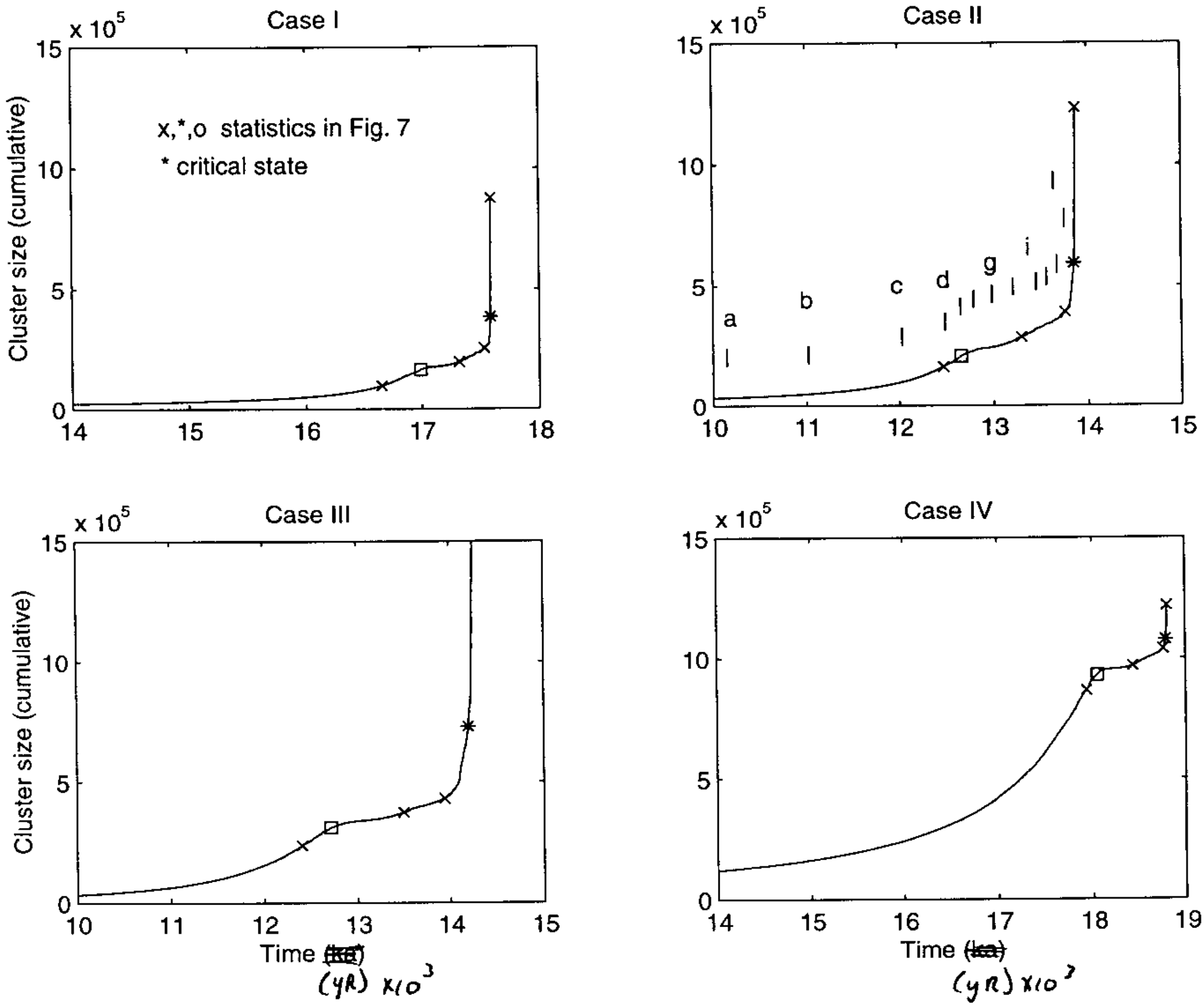
INPUT TO CA MODEL; FOUR CASES of PORE PRESSURE INCREASE RATE DISTRIBUTIONS

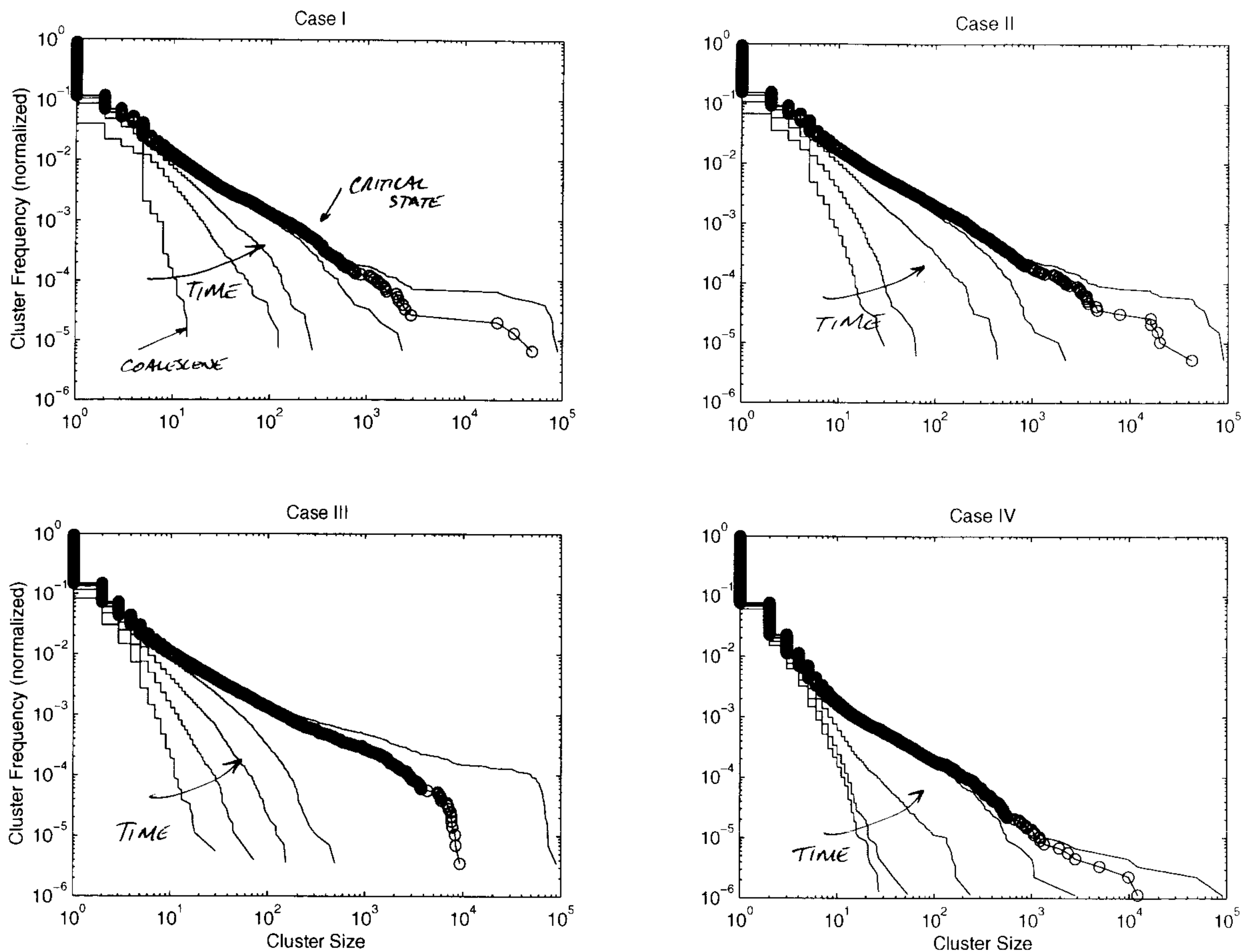


TIMELINE of NUMBER of EVENTS

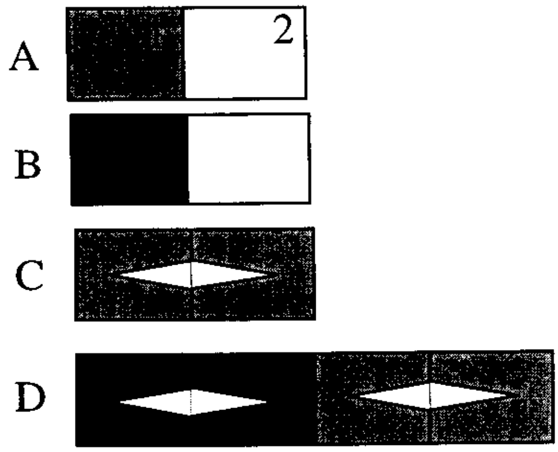


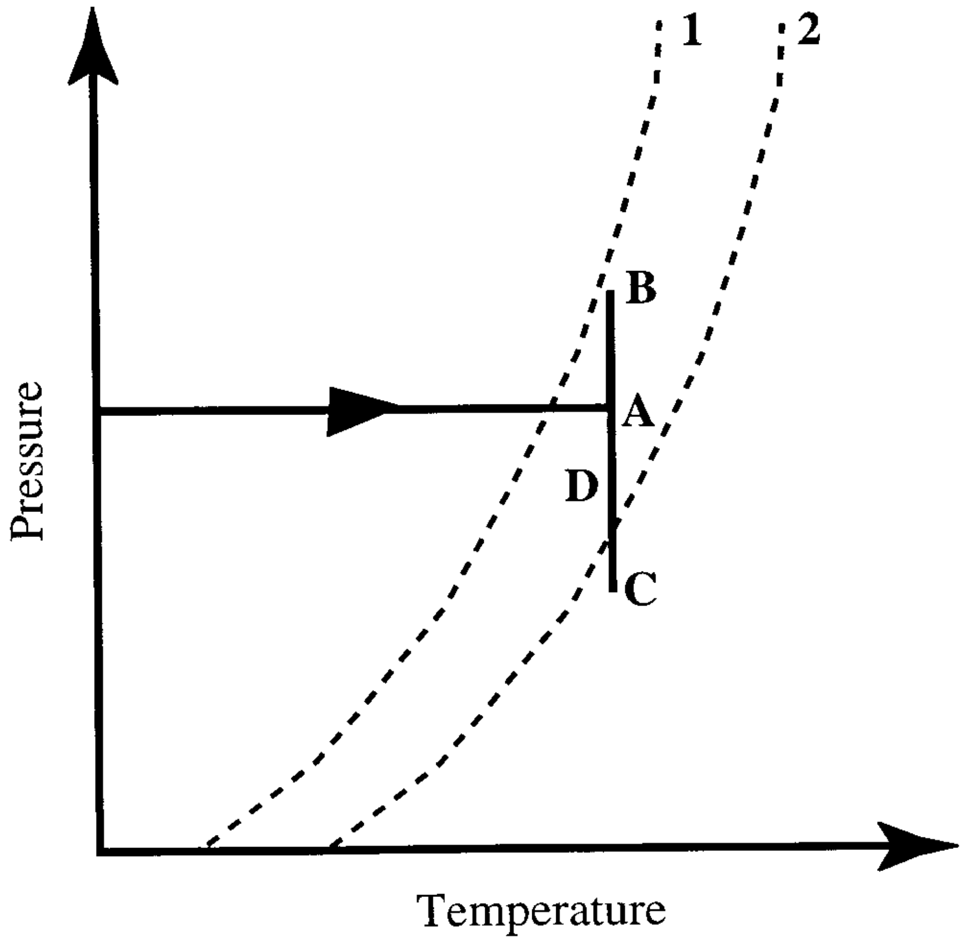
EVOLUTION OF CLUSTER SIZES




Case I
Case II
Case III
Case IV
Cluster Frequency (normalized)
Cluster Size
CRITICAL STATE
TIME
COALESCENCE

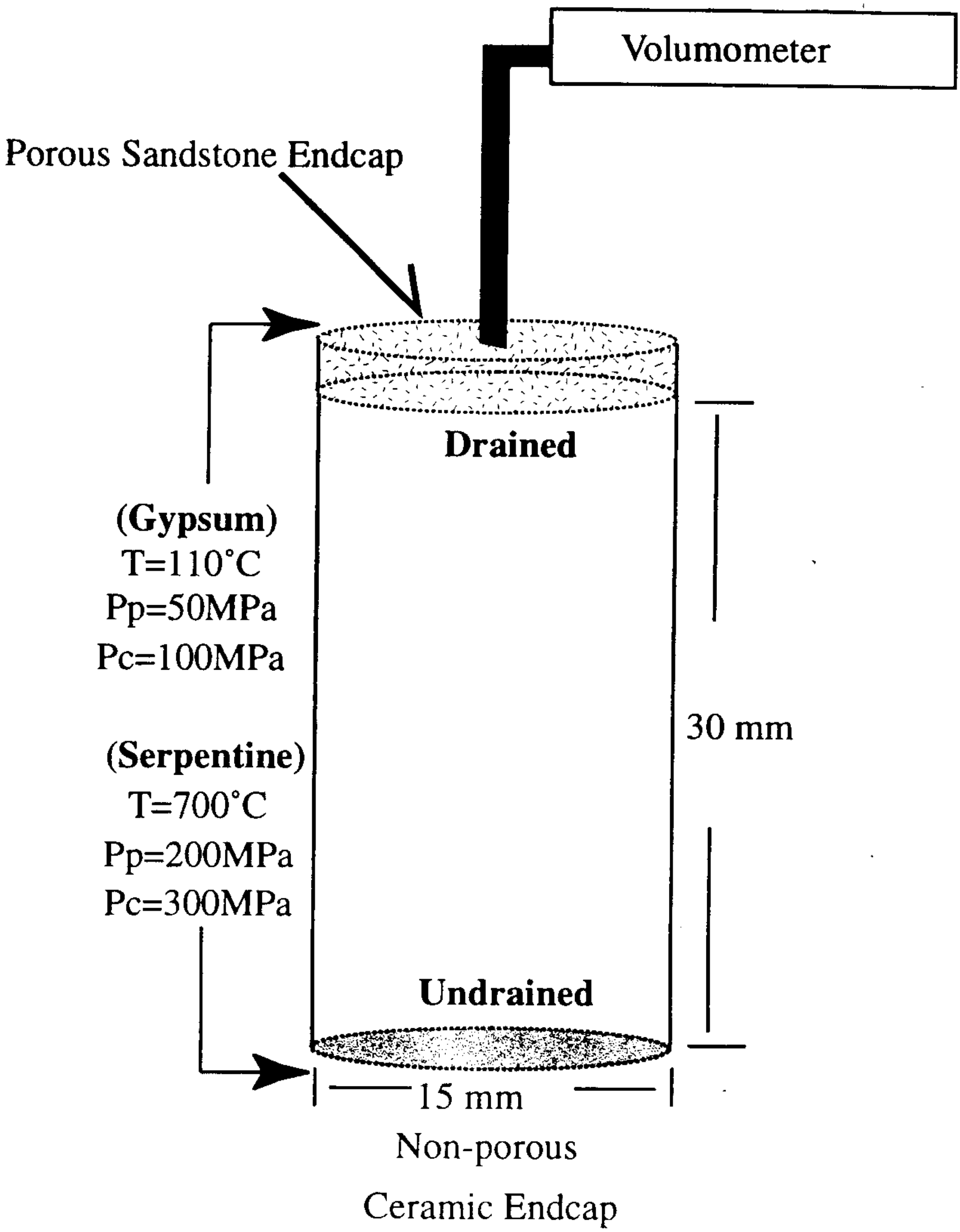
CONCEPTUAL MODEL of DEHYRATION REACTIONS USING TOGGLE-SWITCH PERMEABILITY ASSUMPTION



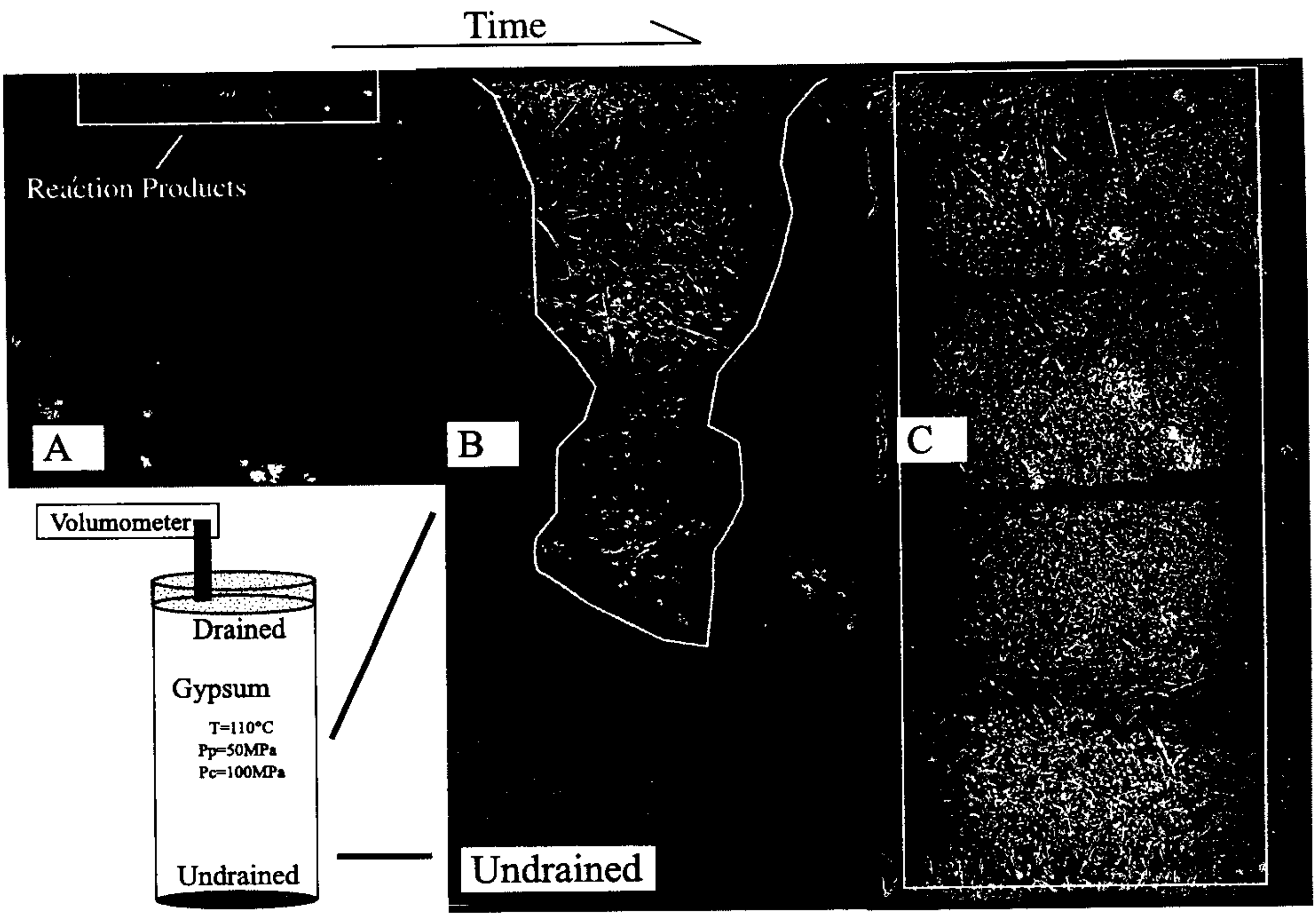


Miller, vanderZee, Olgaard, Connolly ('99, submitted)

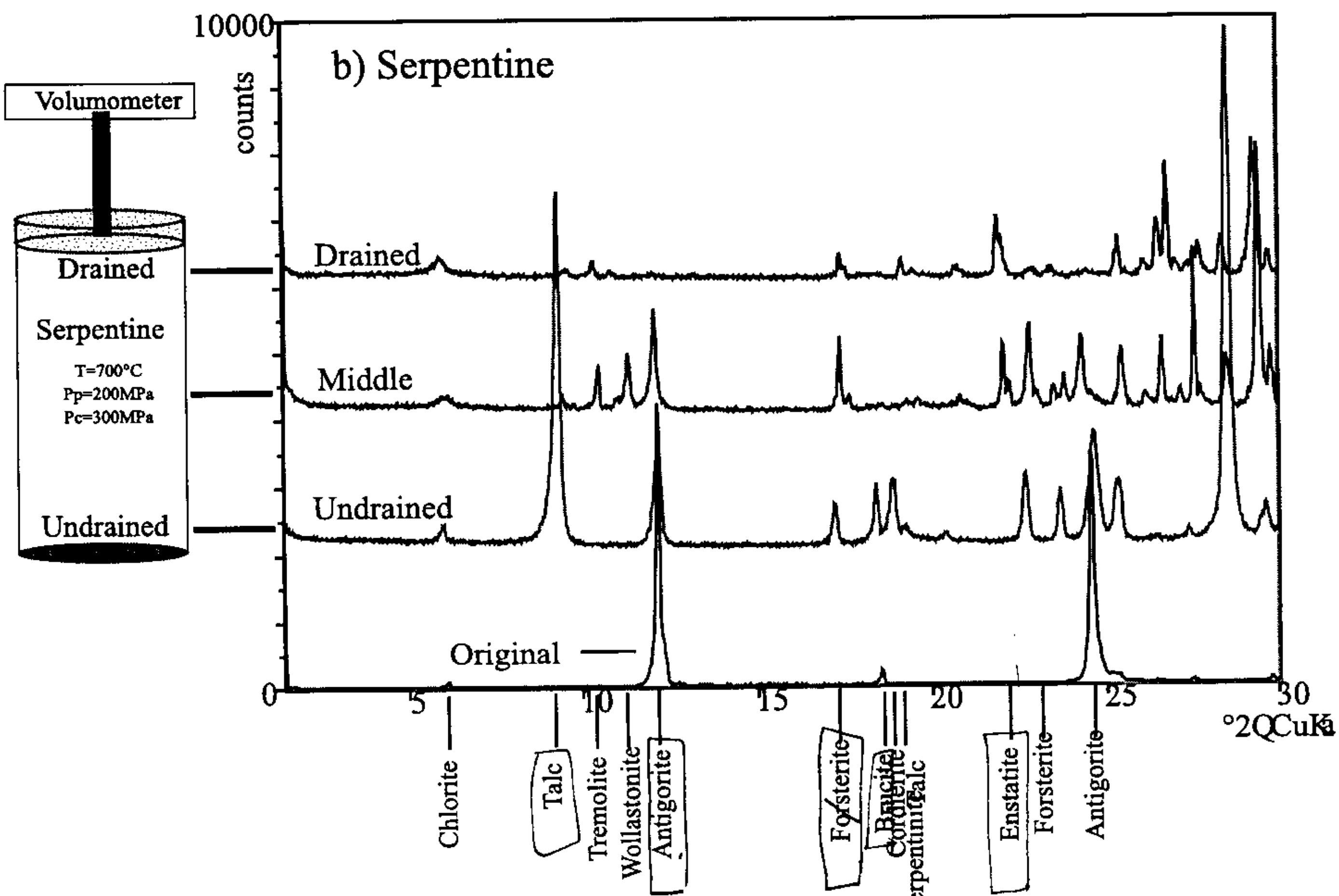
Experimental Setup



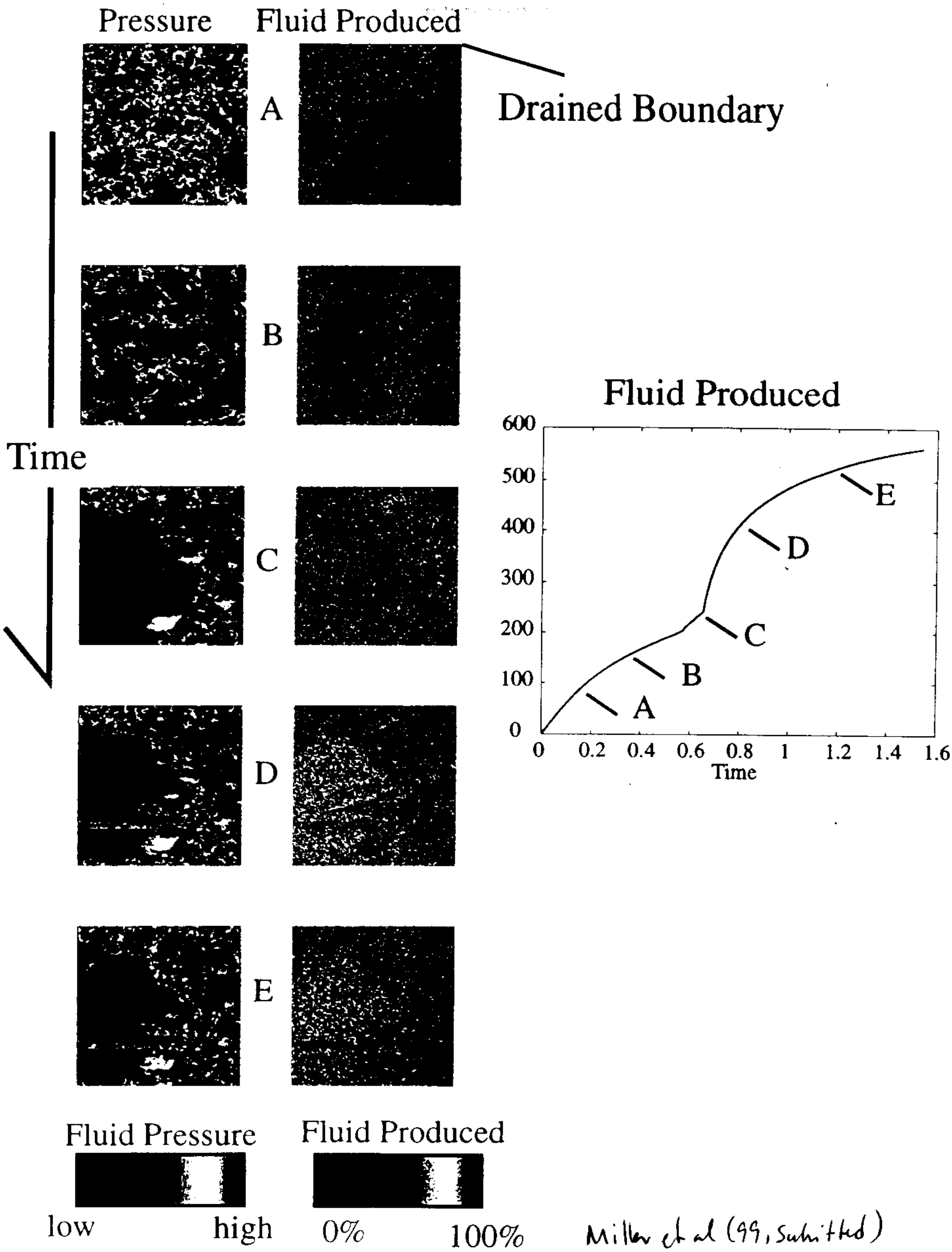
Miller et al (99, submitted)



Van der Zee, (97)

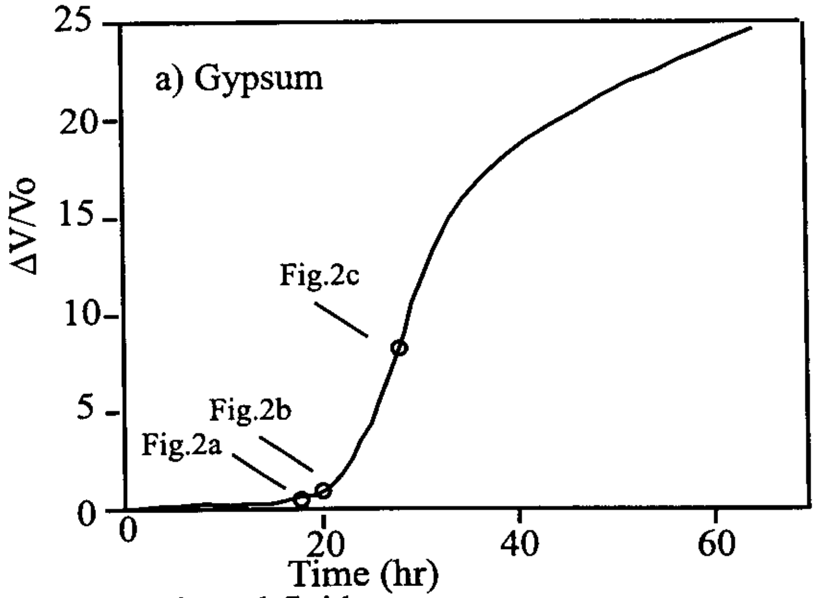


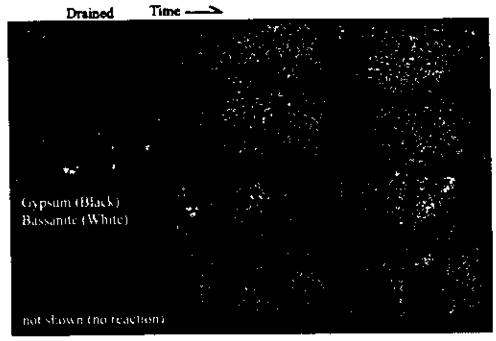
Model Results of Drained Boundary Connectivity and Fluid Production



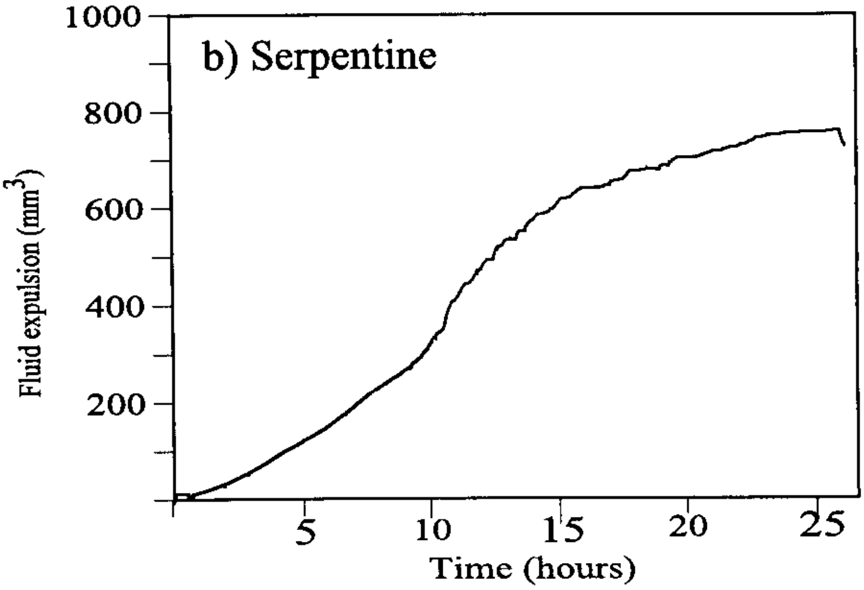
Miller et al (99, submitted)

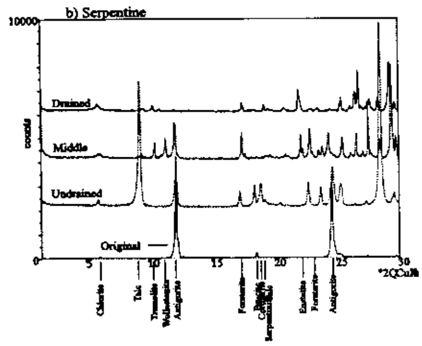
Comparison of Experiments and Model

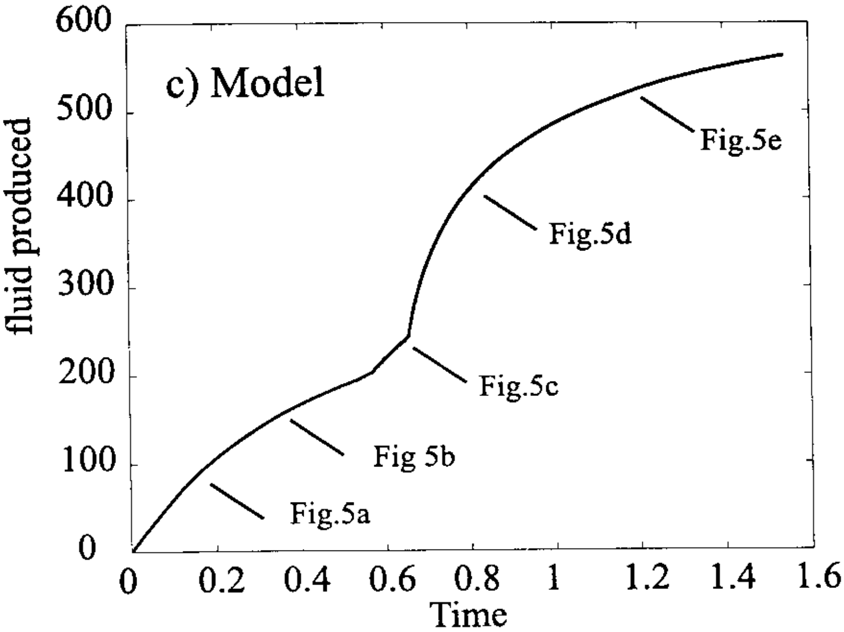


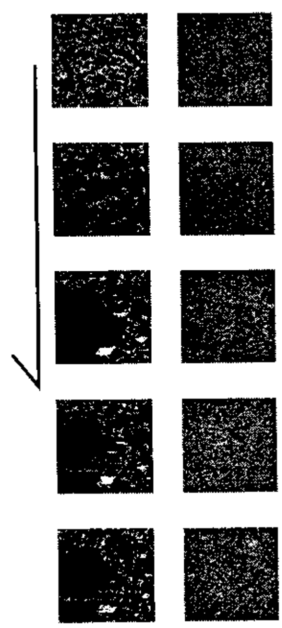


~1380mm^3 total fluid









Conclusions

Simple model for hydrofracture-controlled systems shows:

Self-organizing behavior.

Produces power -law statistics of cluster connectivity at the critical state.

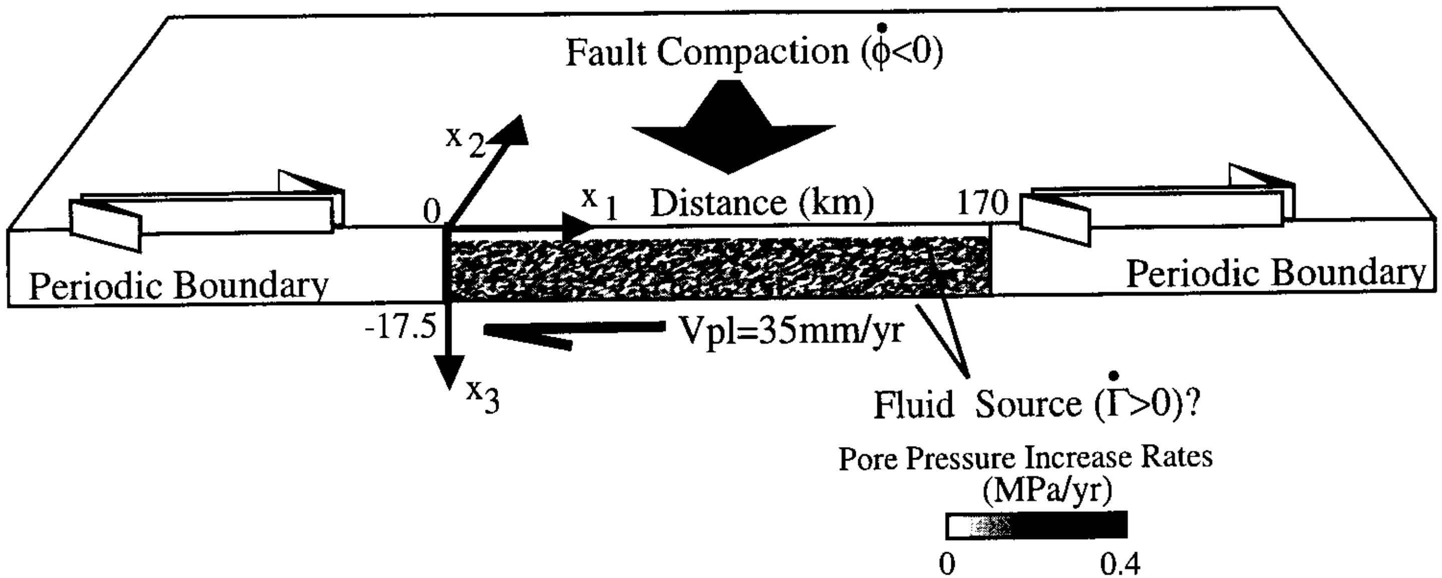
Large (sudden) fluctuations in pore pressure from connectivity with lower pressure regions.

Complex distribution of reaction products when coupled with kinetics of dehydration.

Good agreement with fluid expulsion curves of dehydration experiments in serpentine and gypsum.

Model applies to any kinetic system with positive Clapyron slopes.

3-D FLUID CONTROLLED EARTHQUAKE MODEL



Miller et al (JGR, 99)

Mohr-space dynamical system