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H4.SMR/1150 - 24

Fifth Workshop on Non-Linear Dynamics
and Earthquake Prediction

4 - 22 October 1999

Self Organized Criticality

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Forest-Fire Model

P. Bak et al., Phys. Let., A147, 297 (1992).

B. Drossel and F. Schwabl, Phys. Rev. Let., 69, 1629 (1992).

- Consider a square array of sites
- Choose a site at random

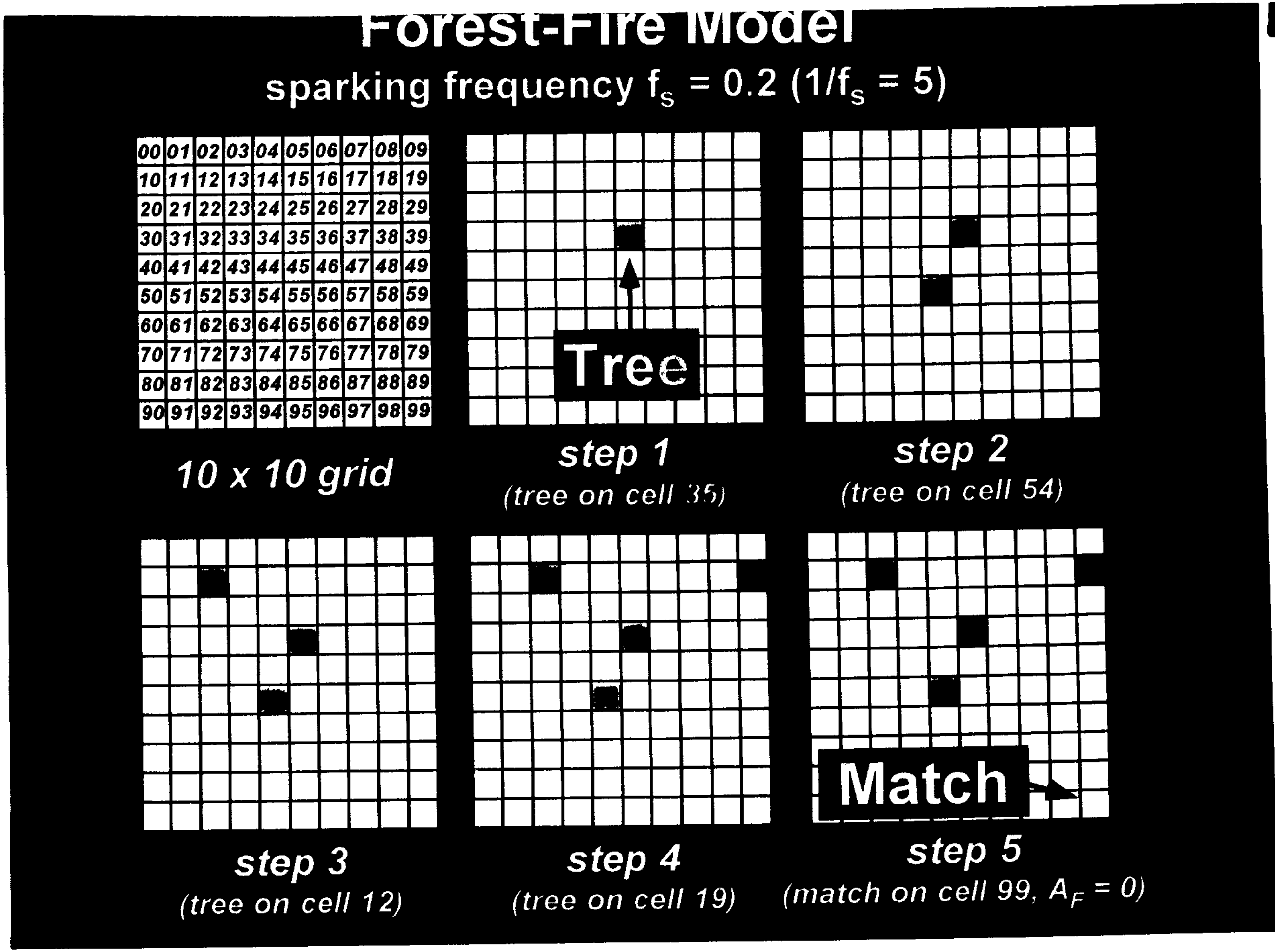
Either

(1) A tree is planted on the site if it is not occupied

or

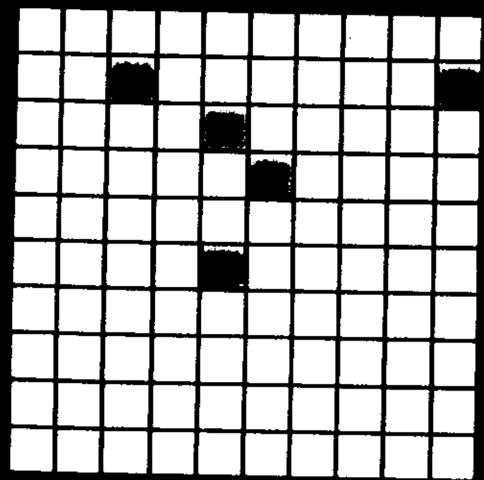
(2) A spark is dropped on the site, if the site has a tree that thee and all adjacent trees burn

- Choose a sparking frequency f, if f =1/500 do step (1) 499 times then do step (2)

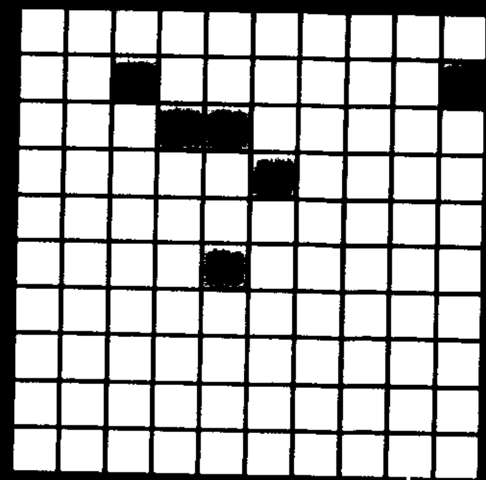

Forest-Fire Model
sparking frequency f_s = 0.2 (1/f_s = 5)
00 01 02 03 04 05 06 07 08 09
10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29
30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49
50 51 52 53 54 55 56 57 58 59
60 61 62 63 64 65 66 67 68 69
70 71 72 73 74 75 76 77 78 79
80 81 82 83 84 85 86 87 88 89
90 91 92 93 94 95 96 97 98 99
10 x 10 grid
Tree
step 1
(tree on cell 35)
step 2
(tree on cell 54)
step 3
(tree on cell 12)
step 4
(tree on cell 19)
Match
step 5
(match on cell 99, A_F = 0)

Forest-Fire Model

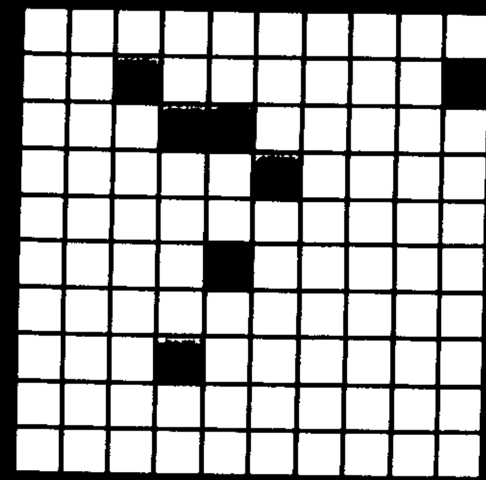
sparking frequency $f_s = 0.2$ ($1/f_s = 5$)



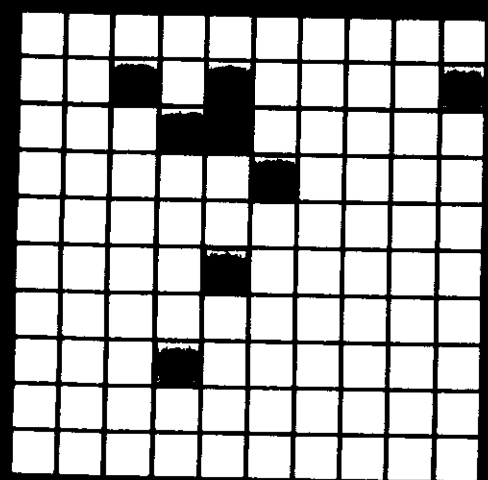
step 6
(tree on cell 24)



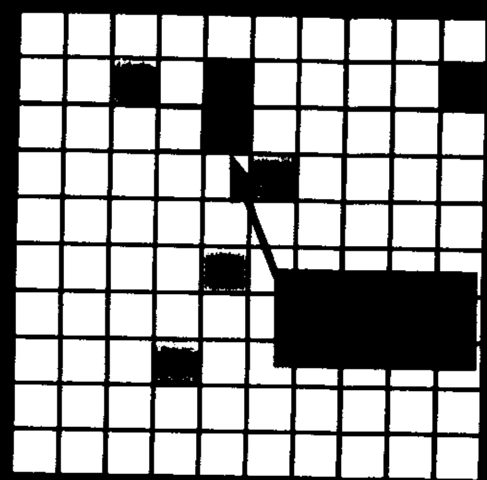
step 7
(tree on cell 23)



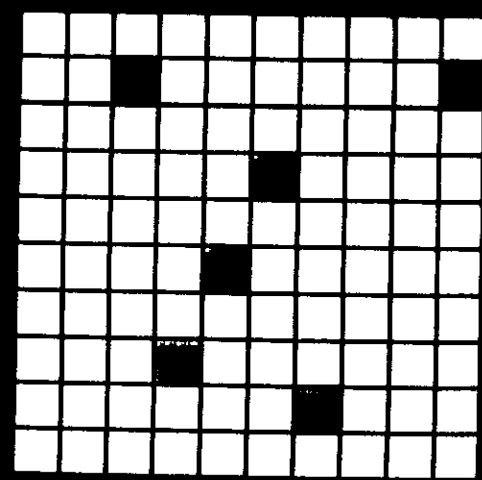
step 8
(tree on cell 73)



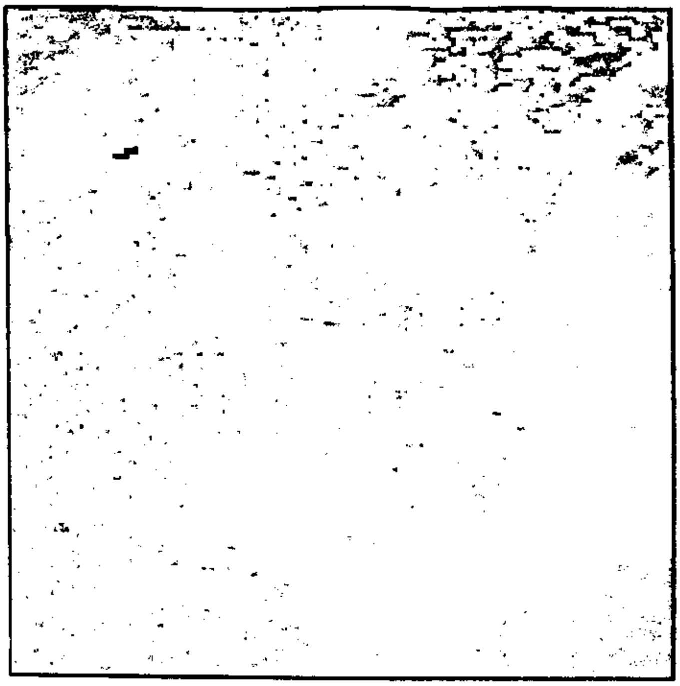
step 9
(tree on cell 14)



step 10
(match on cell 23, $A_F = 3$)

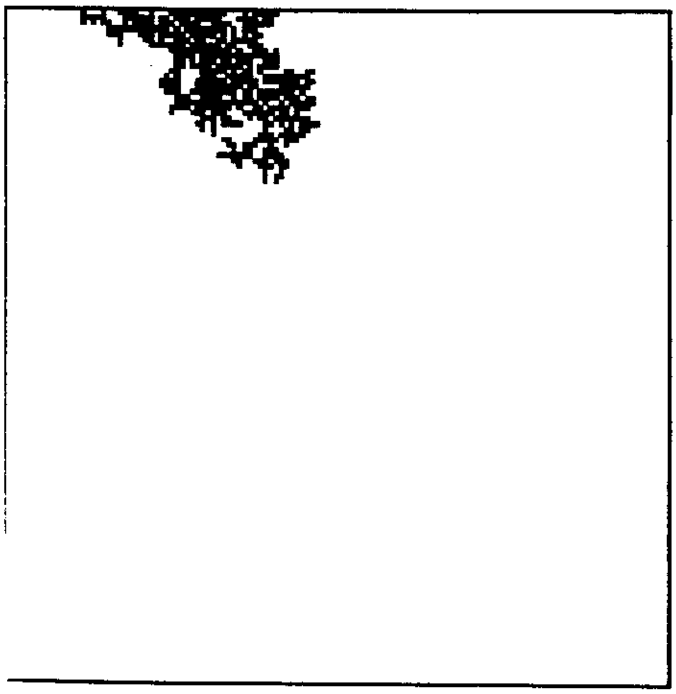


step 11
(tree on cell 86)

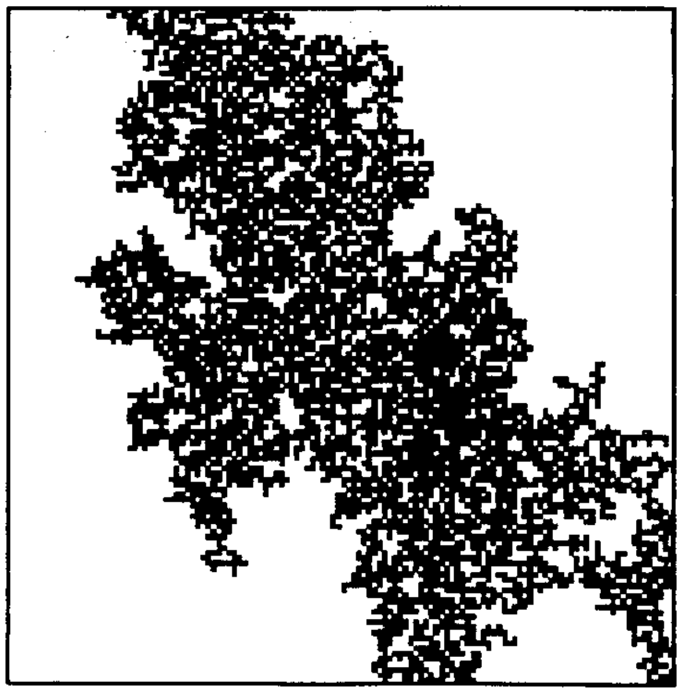


(a) A_F= 5 trees

(b) A_F = 51 trees



(c) A_F = 505 trees



(d) A_F = 5327 trees

Typical forest-fire model fires.

Grid = 128 x 128 cells. A_F = area of fire.

Sparking rate f = 1/2000.

L.FFM.4'

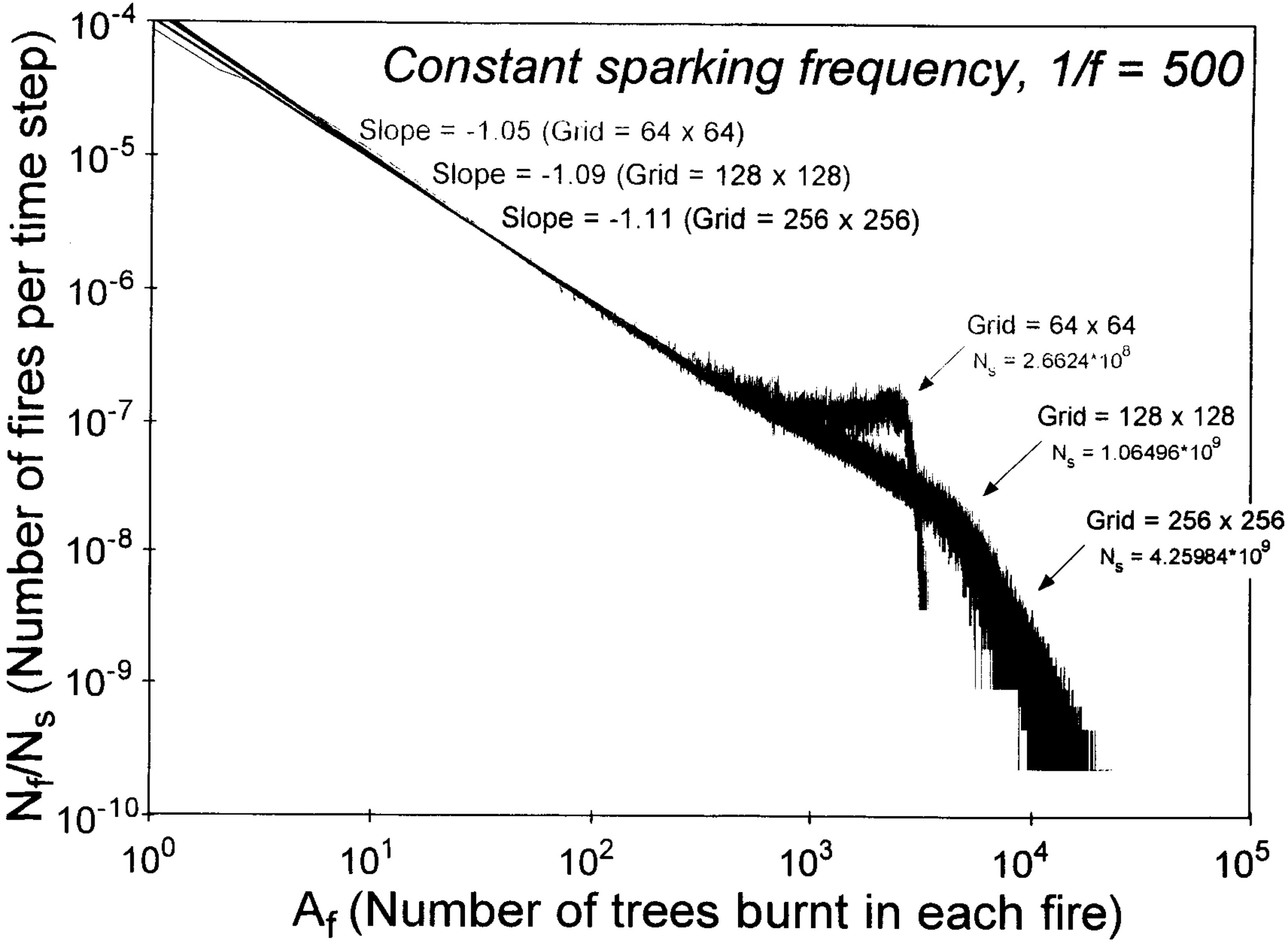
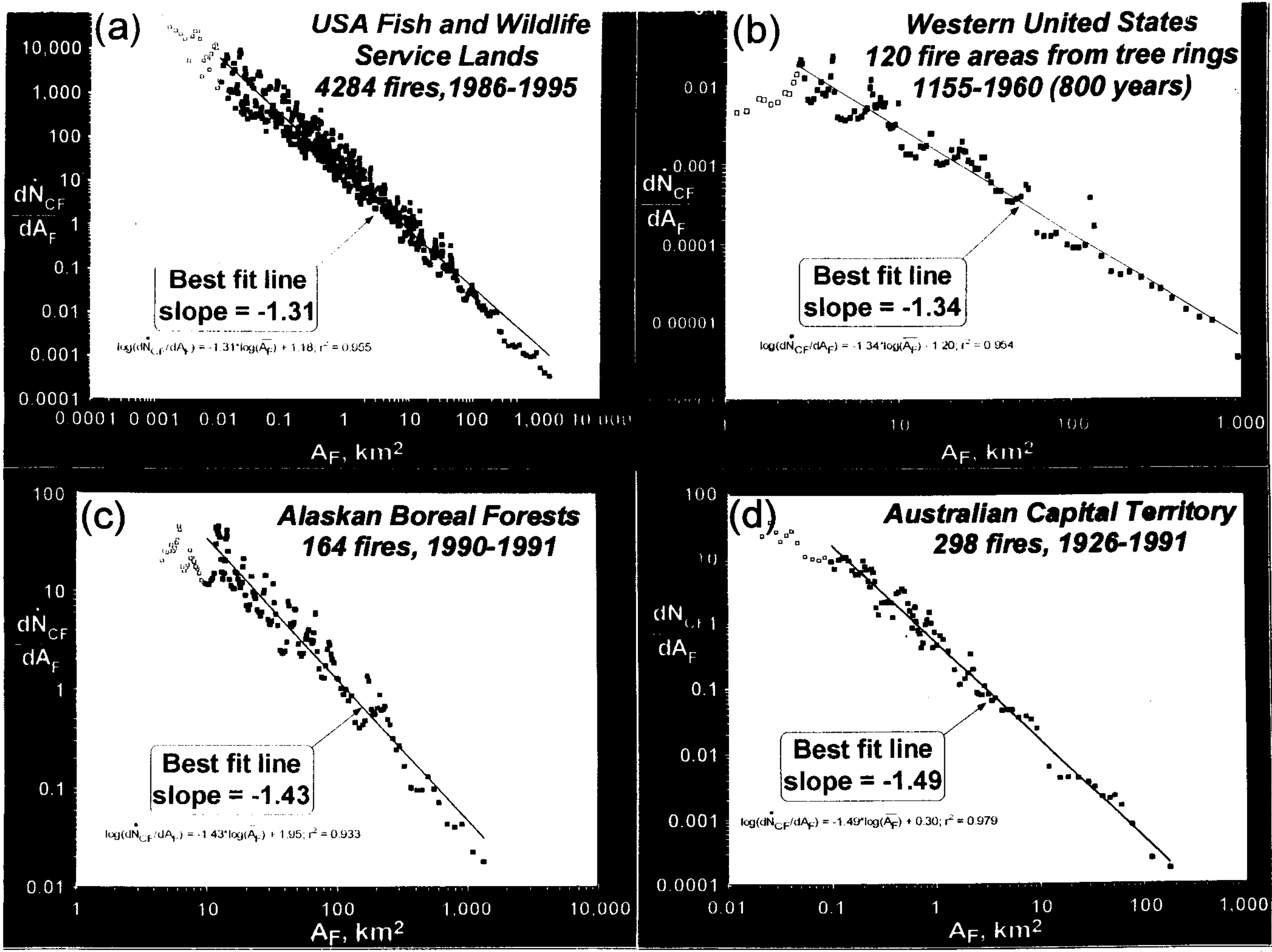


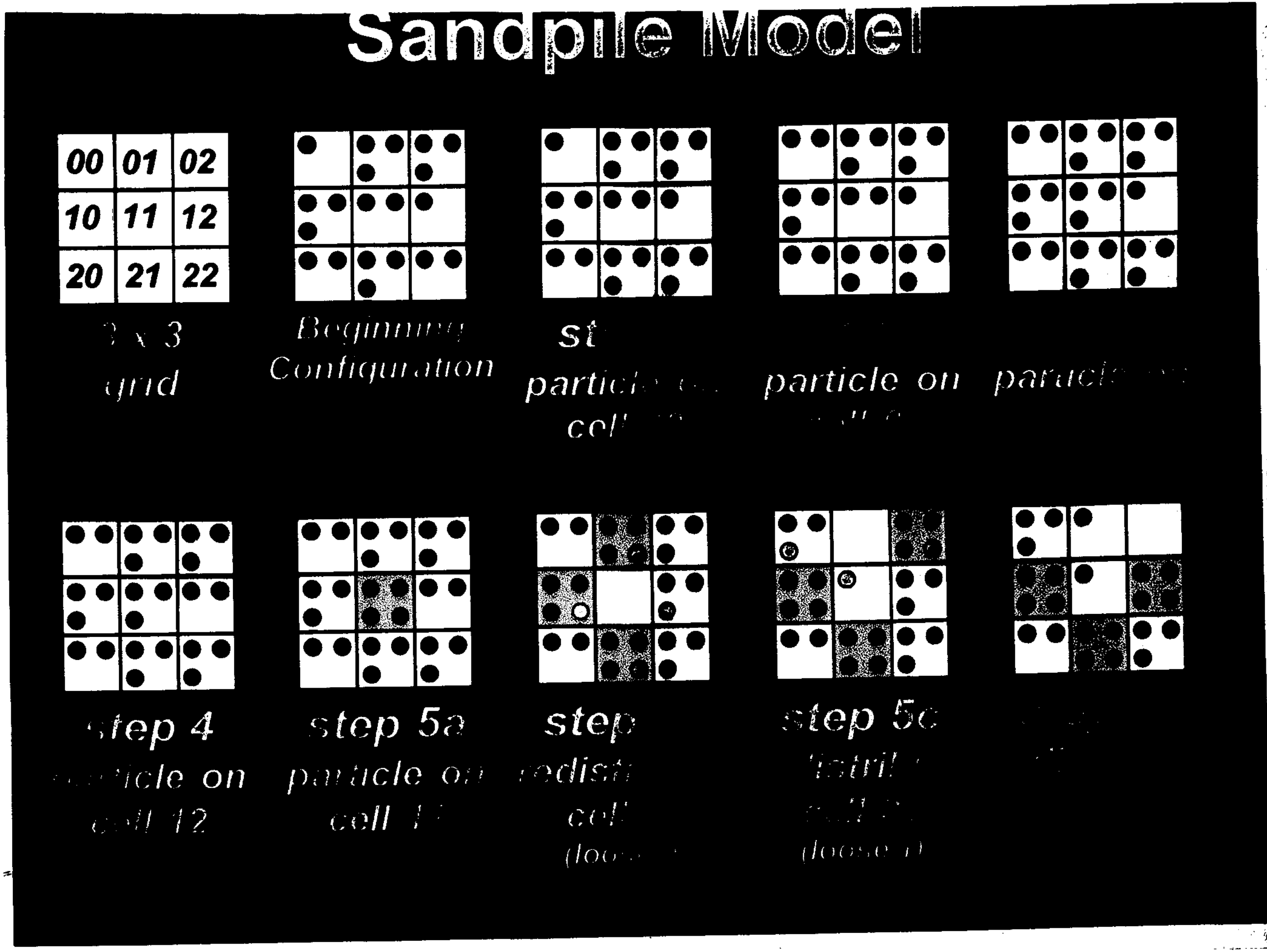
Figure 2. SOC Paper

SOC•FFM•5

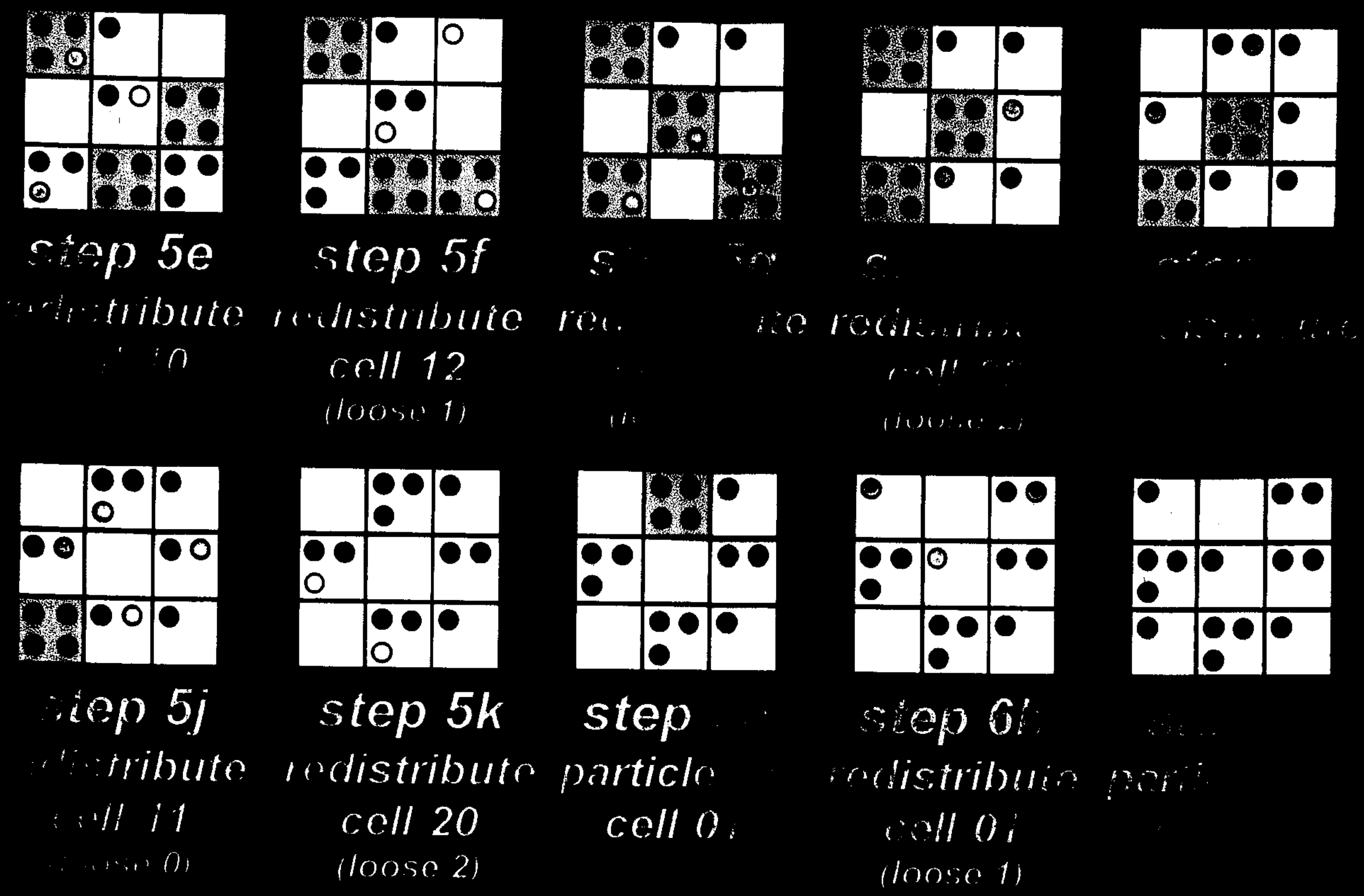

(a)
USA Fish and Wildlife Service Lands
4284 fires,1986-1995
Best fit line
slope = -1.31
log(dṄCF/dAF) = -1.31*log(AF) + 1.18; r² = 0.955
dṄCF/dAF
AF, km²
(b)
Western United States
120 fire areas from tree rings
1155-1960 (800 years)
Best fit line
slope = -1.34
log(dṄCF/dAF) = -1.34*log(AF) - 1.20; r² = 0.954
dṄCF/dAF
AF, km²
(c)
Alaskan Boreal Forests
164 fires, 1990-1991
Best fit line
slope = -1.43
log(dṄCF/dAF) = -1.43*log(AF) + 1.95; r² = 0.933
dṄCF/dAF
AF, km²
(d)
Australian Capital Territory
298 fires, 1926-1991
Best fit line
slope = -1.49
log(dṄCF/dAF) = -1.49*log(AF) + 0.30; r² = 0.979
dṄCF/dAF
AF, km²

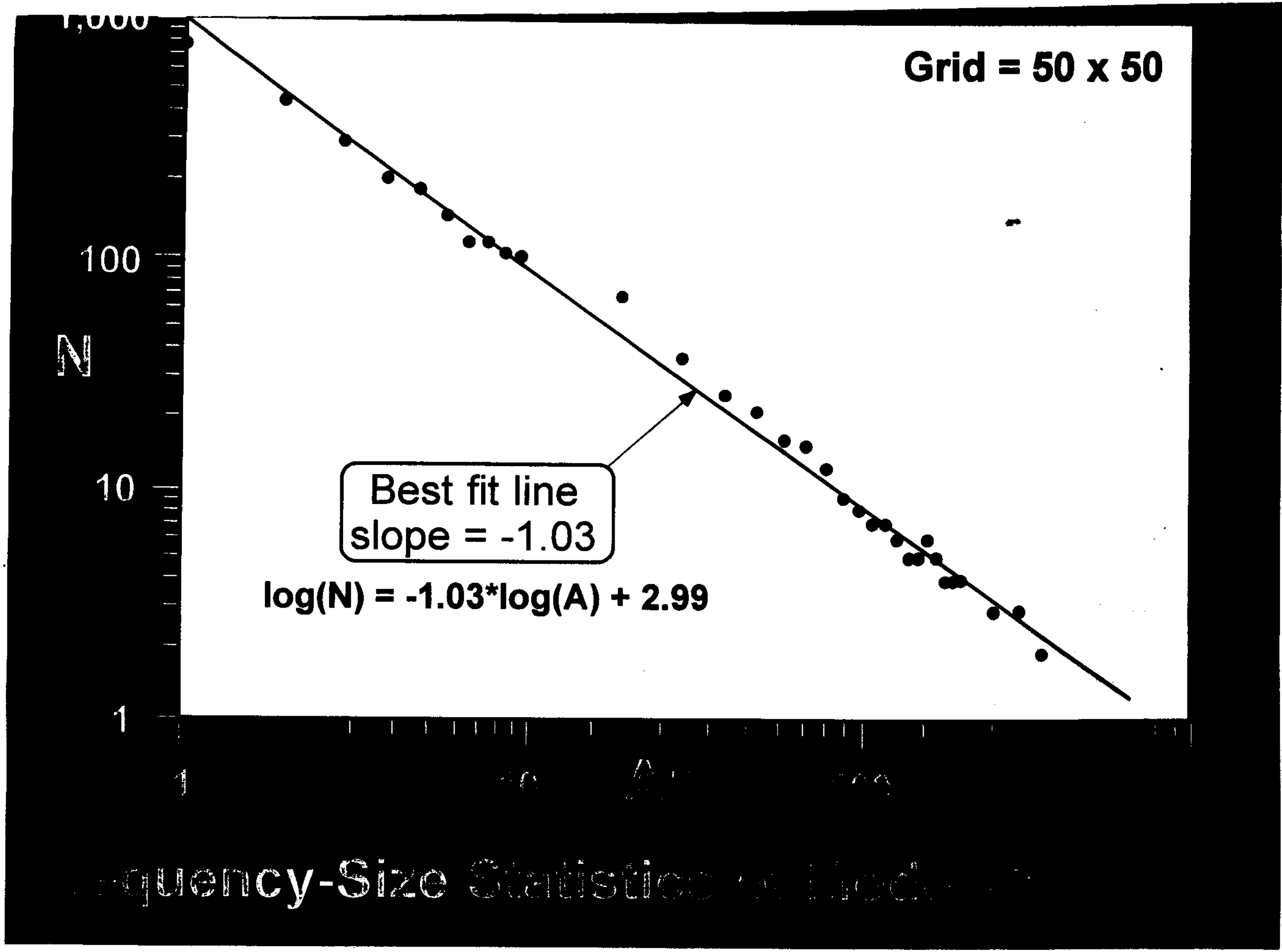
506·5M·1 [redacted]

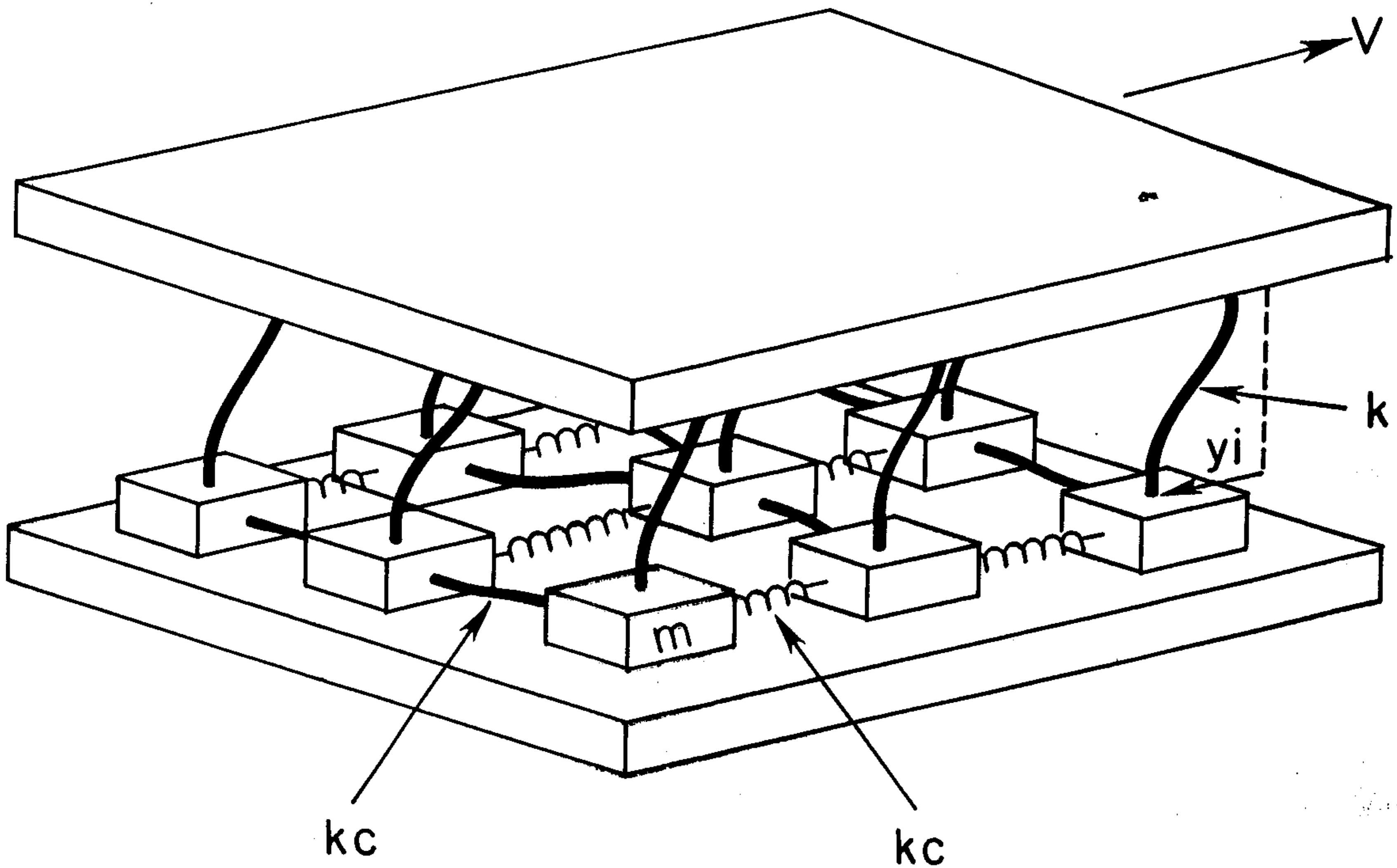
- *Square grid of n x n cells*
- *When cell has four particles, it is unstable and particles are redistributed to adjacent cells.*
- *After a redistribution, additional redistributions may be necessary.*


Sandpile Model
00 01 02
10 11 12
20 21 22
grid
Beginning Configuration
particle on
step 4
cell 12
step 5a
particle on
step
step 5c

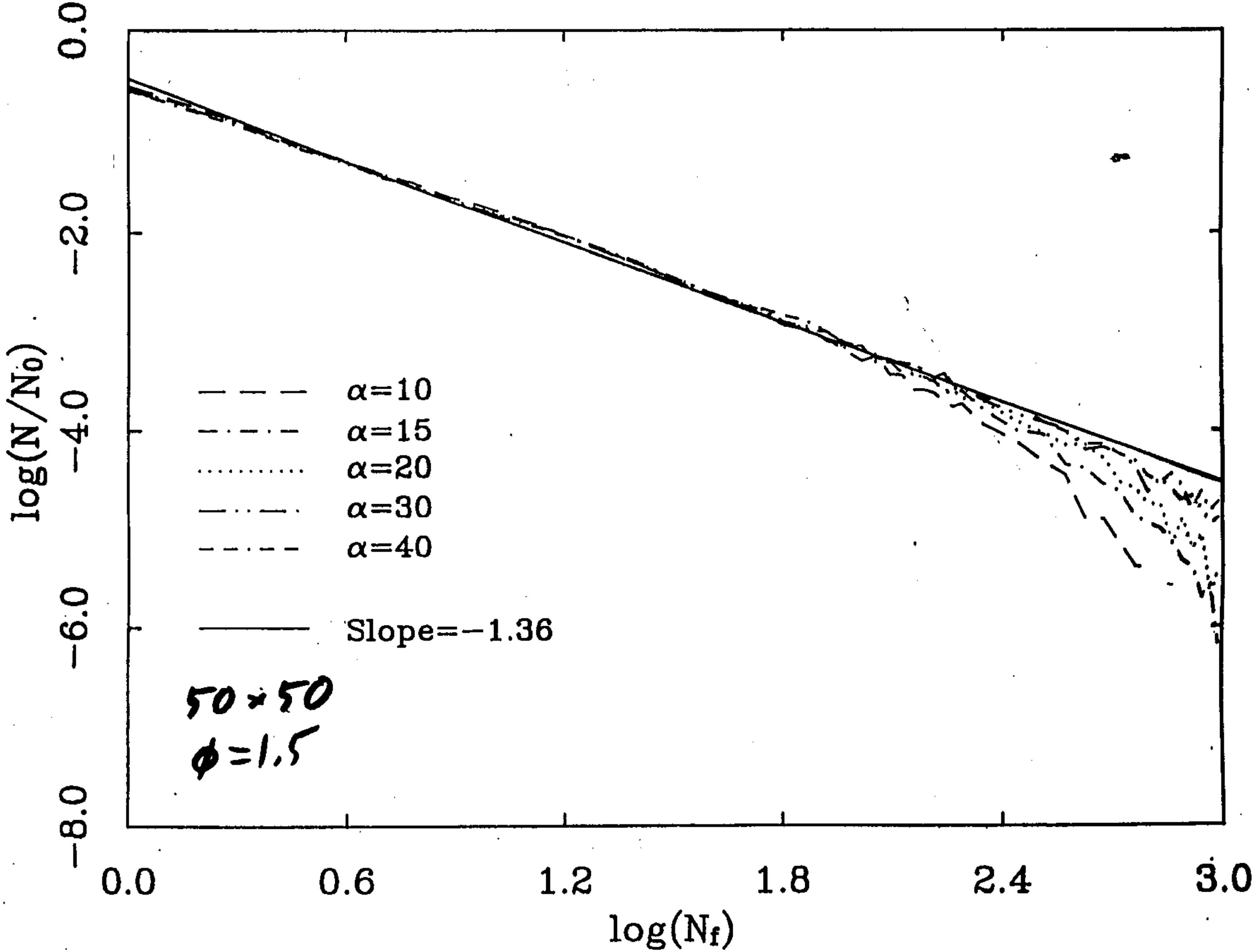
Sandpile Model



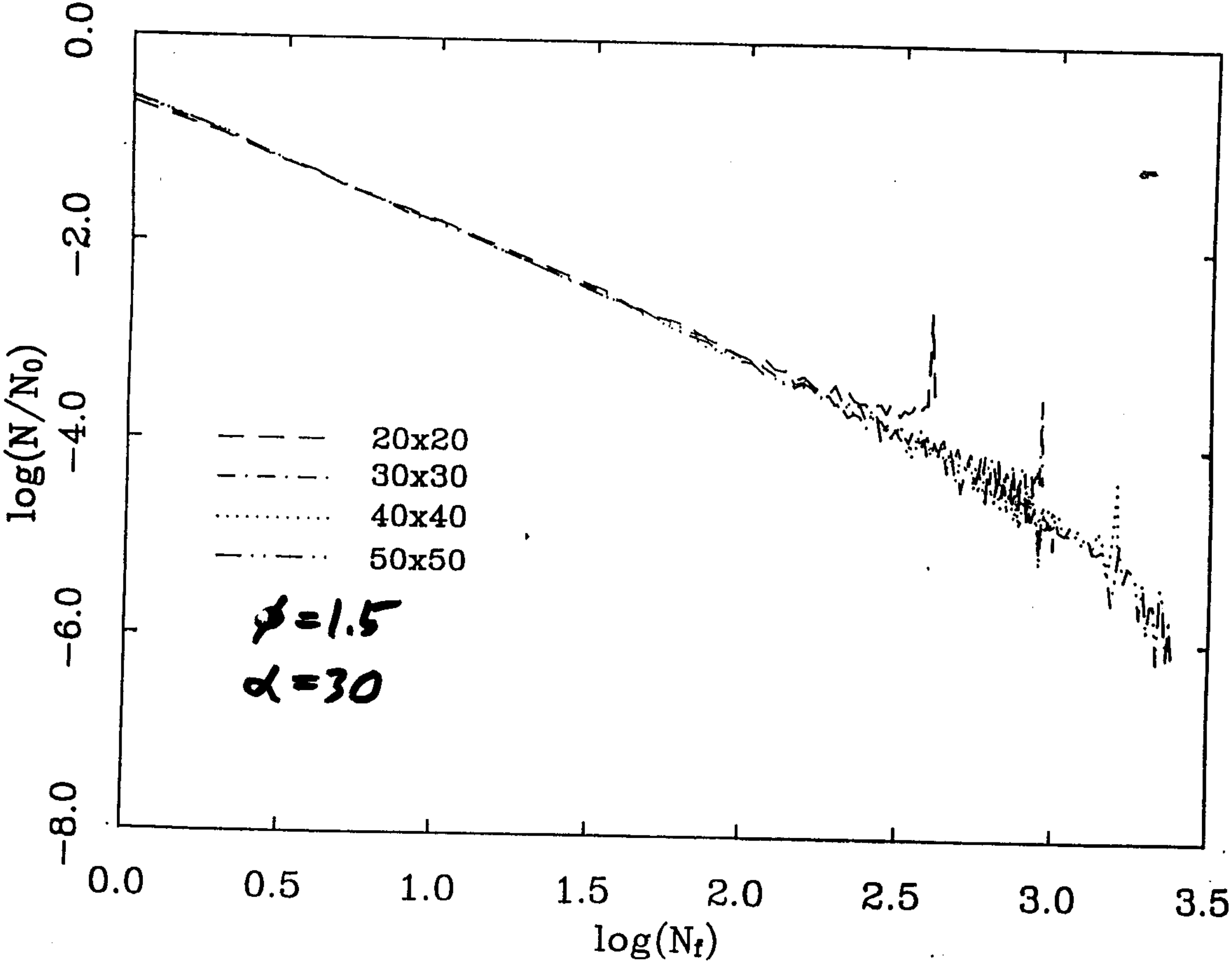

Grid = 50 x 50
Best fit line
slope = -1.03
log(N) = -1.03*log(A) + 2.99
100
10
1
N
quency-Size Statistic



Multiple slider-block model



[redacted] 50C·SBM.3



[redacted] SOC·SBM·4

Typical
slider
block
slip
events

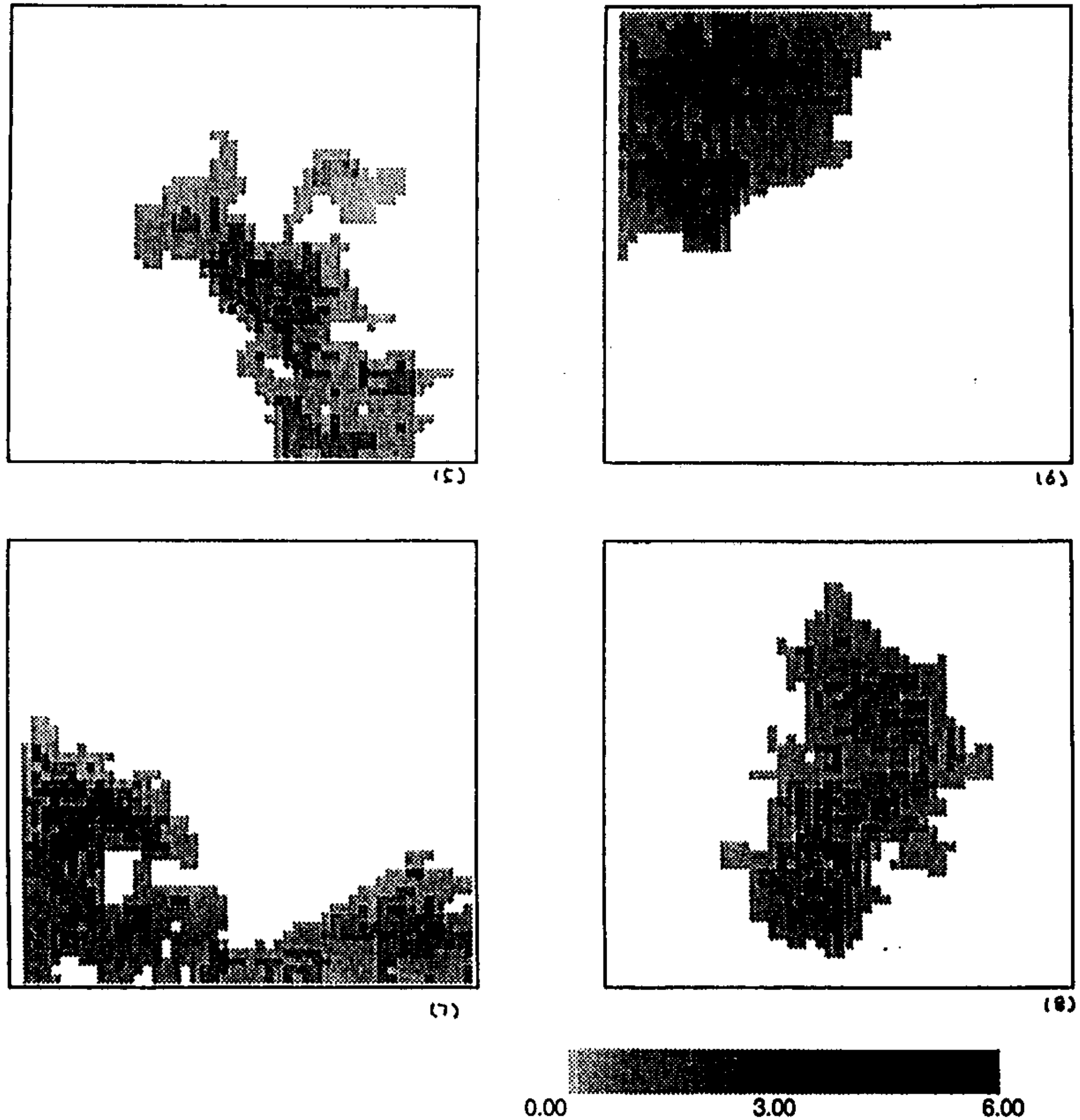


Figure 1b

[redacted] SOC.SDM.F

At each time step one tree is dropped.
What happens?
Four possibilities:

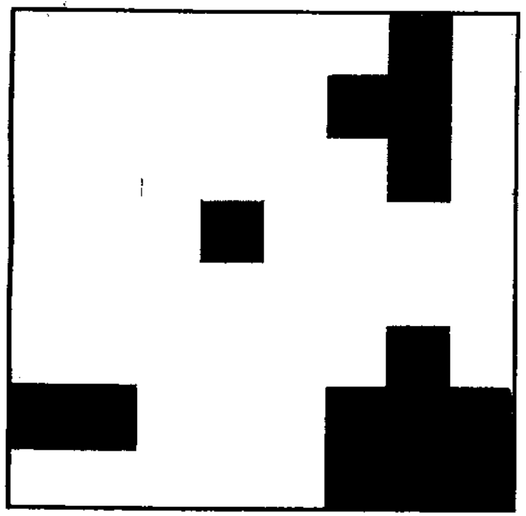
(1) Tree lands on cell with existing tree.
Nothing happens.

(2) Tree lands on empty cell with all adjacent cells empty.
A cluster of $A_c = 1$ tree is formed.

(3) Tree lands on empty cell adjoining one cluster with $A_c = n$ trees.
Original cluster grows by +1 ($A_c = n+1$).

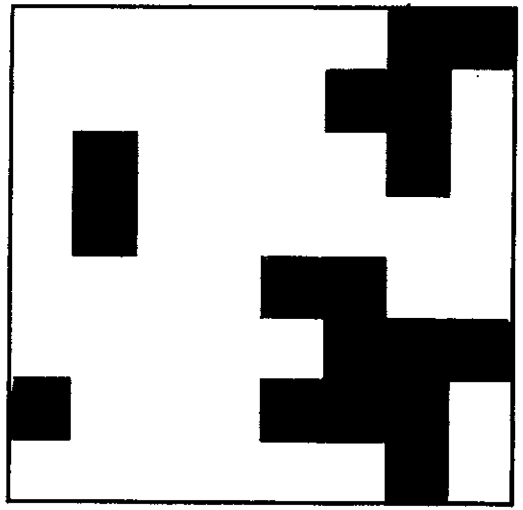
(4) Tree lands on empty cell that adjoins two clusters with $A_{c1} = n$ trees and $A_{c2} = m$ trees.
The two original clusters combine to form one cluster with $A_c = (n + m + 1)$ trees.

SMALL CLUSTERS EVOLVE TO BIGGER CLUSTERS



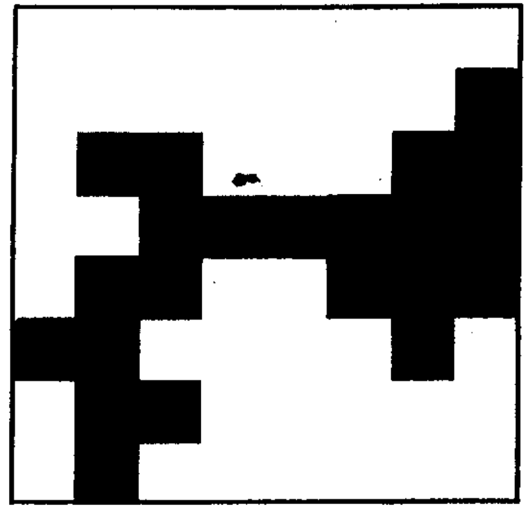
(a)

A new tree creates a new cluster



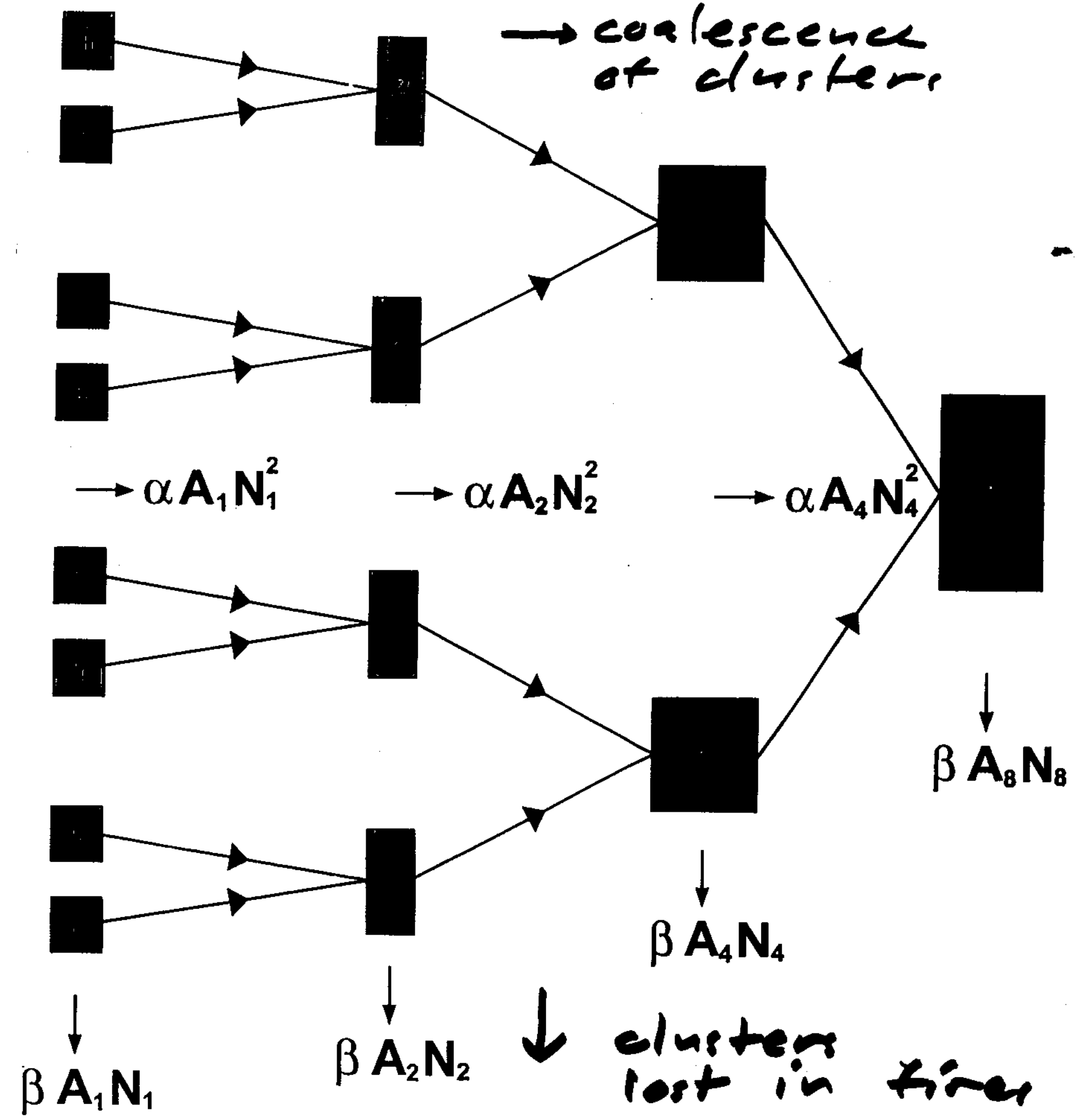
(b)

A new tree adds a tree to an existing cluster



(c)

A new tree connects two existing clusters


→ coalescence of clusters
→ $\alpha A_1 N_1^2$
→ $\alpha A_2 N_2^2$
→ $\alpha A_4 N_4^2$
↓ $\beta A_1 N_1$
↓ $\beta A_2 N_2$
↓ $\beta A_4 N_4$
↓ $\beta A_8 N_8$
↓ clusters lost in fires

SOC
CM
14

The number of clusters with 2^n particles is denoted by N_n.

$$\frac{dN_0}{dt} = C_0 - C_{0,1}N_0^2 - D_0N_0$$
$$\frac{dN_1}{dt} = \frac{1}{2}C_{0,1}N_0^2 - C_{1,2}N_1^2 - D_1N_1$$
$$\frac{dN_2}{dt} = \frac{1}{2}C_{1,2}N_1^2 - C_{2,3}N_2^2 - D_2N_2$$
$$\vdots$$
$$\frac{dN_n}{dt} = \frac{1}{2}C_{n-1,n}N_{n-1}^2 - C_{n,n+1}N_n^2 - D_nN_n$$

$$C_{n,n+1} = 2^n\alpha C_0 \qquad D_n = 2^n\beta C_0 \qquad \tau = C_0t$$

$$\frac{dN_0}{d\tau} = 1 - \alpha N_0^2 - \beta N_0$$

$$\frac{dN_1}{d\tau} = \frac{1}{2}\alpha N_0^2 - 2\alpha N_1^2 - 2\beta N_1$$
$$\frac{dN_2}{d\tau} = \frac{1}{2}2\alpha N_1^2 - 4\alpha N_2^2 - 4\beta N_2$$
$$\vdots$$
$$\frac{dN_n}{d\tau} = \frac{1}{2}2^{n-1}\alpha N_{n-1}^2 - 2^n\alpha N_n^2 - 2^n\beta N_n$$

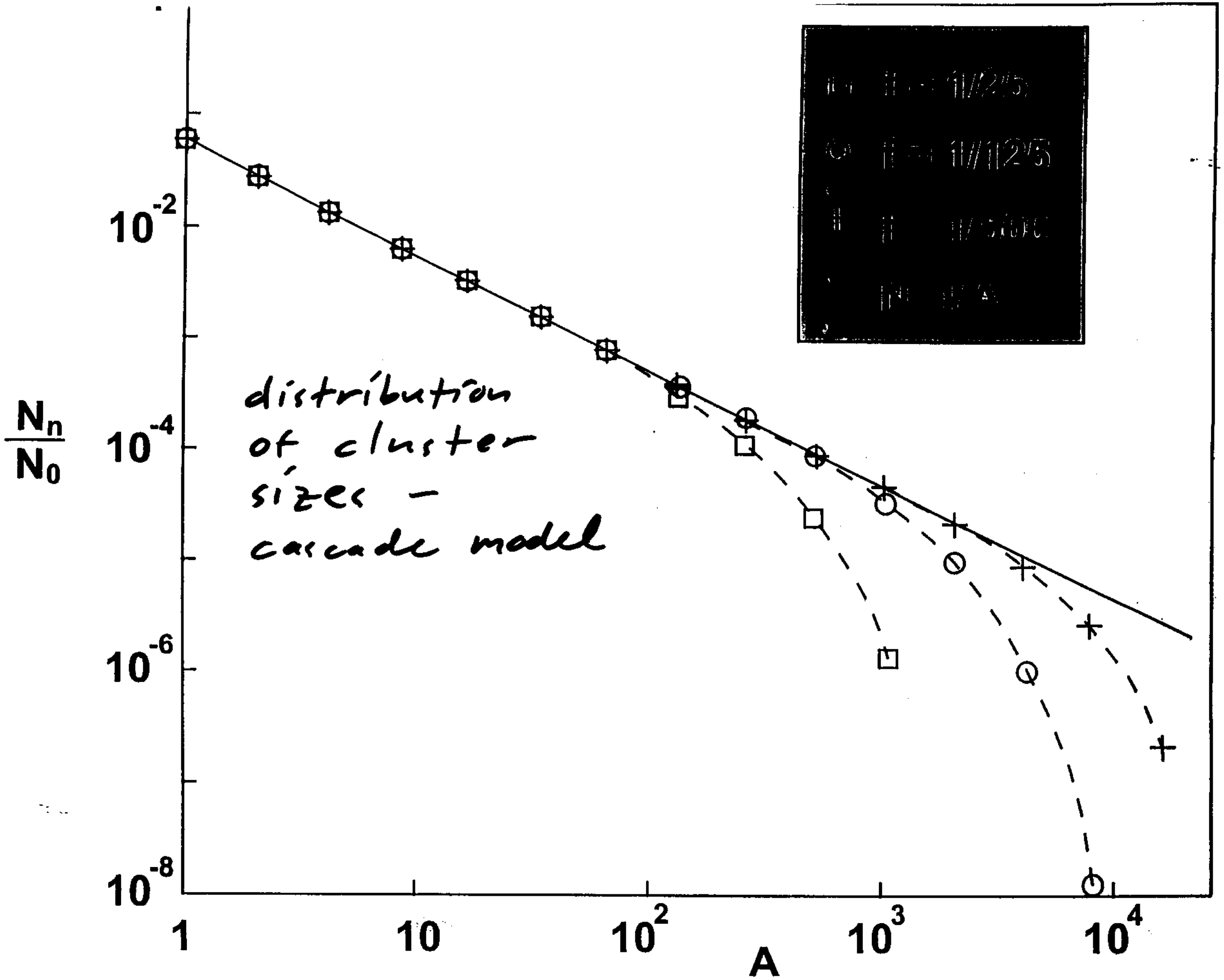
$$dN_n/d\tau = 0$$

$$N_0 = \frac{-\beta + (\beta^2 + 4\alpha)^{1/2}}{2\alpha}$$
$$N_1 = \frac{-\beta + (\beta^2 + \alpha^2 N_0^2)^{1/2}}{2\alpha}$$
$$N_2 = \frac{-\beta + (\beta^2 + \alpha^2 N_1^2)^{1/2}}{2\alpha}$$
$$\vdots$$
$$N_n = \frac{-\beta + (\beta^2 + \alpha^2 N_{n-1}^2)^{1/2}}{2\alpha}.$$

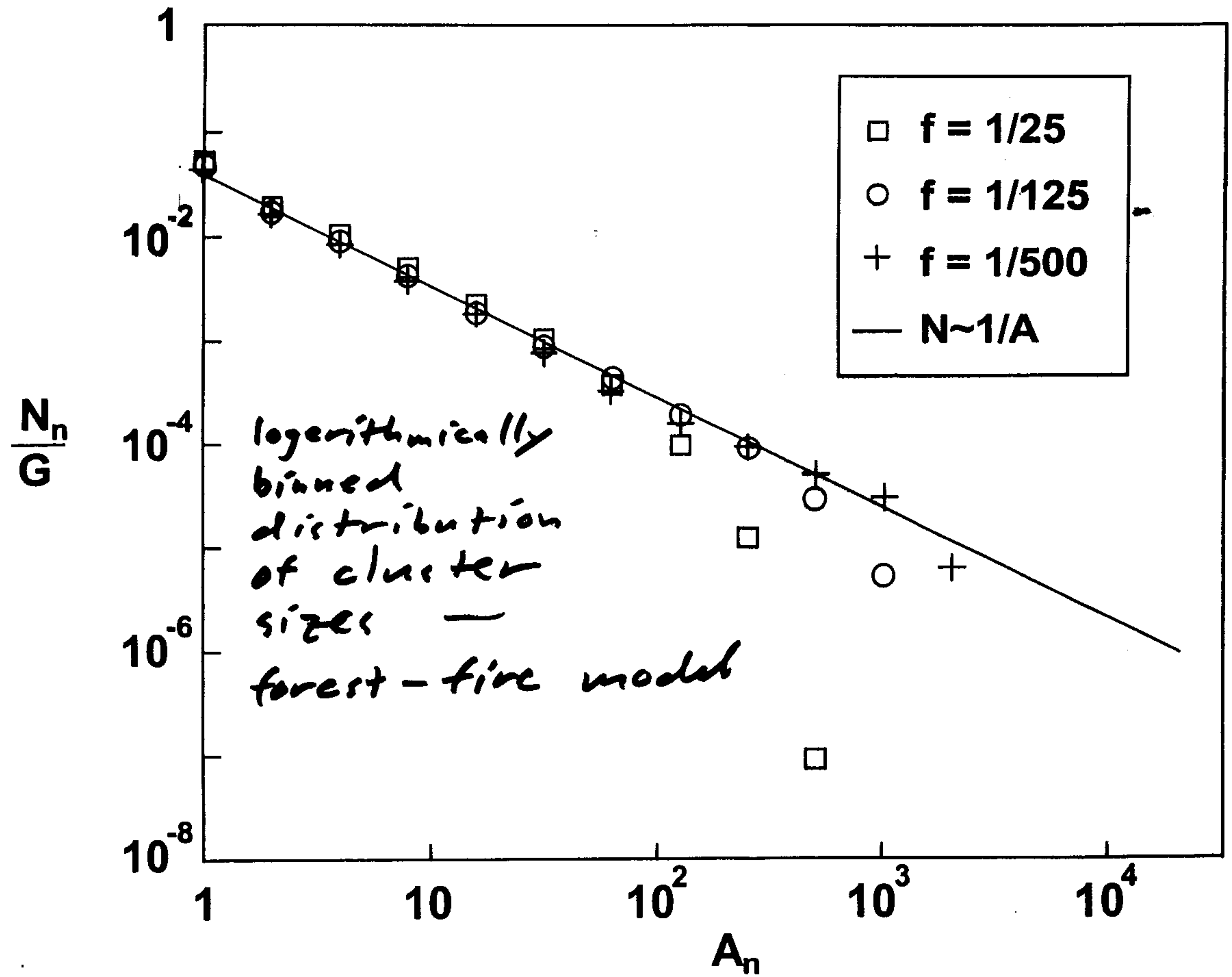
if $\beta \ll \alpha$.

$$N_0 = \frac{1}{\sqrt{\alpha}}, \quad N_1 = \frac{1}{2} N_0, \quad N_2 = \frac{1}{2} N_1, \quad \cdots, \quad N_n = \frac{1}{2} N_{n-1}$$

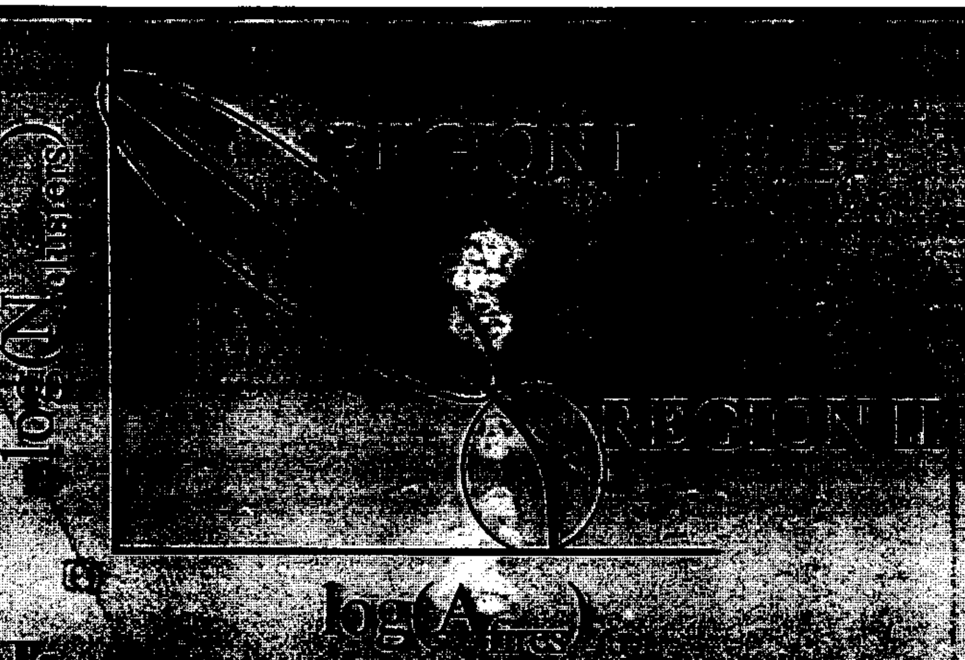
$$N_n = \frac{N_0}{A_n}.$$



distribution of cluster sizes – cascade model
$\frac{N_n}{N_0}$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
1
10
10^2
10^3
10^4
A



1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$\frac{N_n}{G}$
10
10^2
10^3
10^4
A_n
f = 1/25
f = 1/125
f = 1/500
N~1/A
logarithmically binned distribution of cluster sizes — forest-fire model



REGION I:

- **Multiplicati[illegible]**
- **Trees flow from small clusters to larger clusters.**
- **Small forest fires have little effect on cluster statistics in this region.**

$$N_{\text{clusters}} \sim (A_{\text{clusters}})^{-2}$$

equivalent to

$$N_{\text{fires}} \sim (A_{\text{fires}})^{-1}$$

REGION II:

- **Larger fires destroy the power-law relation of the clusters.**

$\rho = 1 - e^{-t}$ Critical point: $\rho = 0.59275$, $t = 0.898$

Forest fire model without fires

logarithmically binned number of clusters N_c of size $A_c = 2^n$ as a function of time

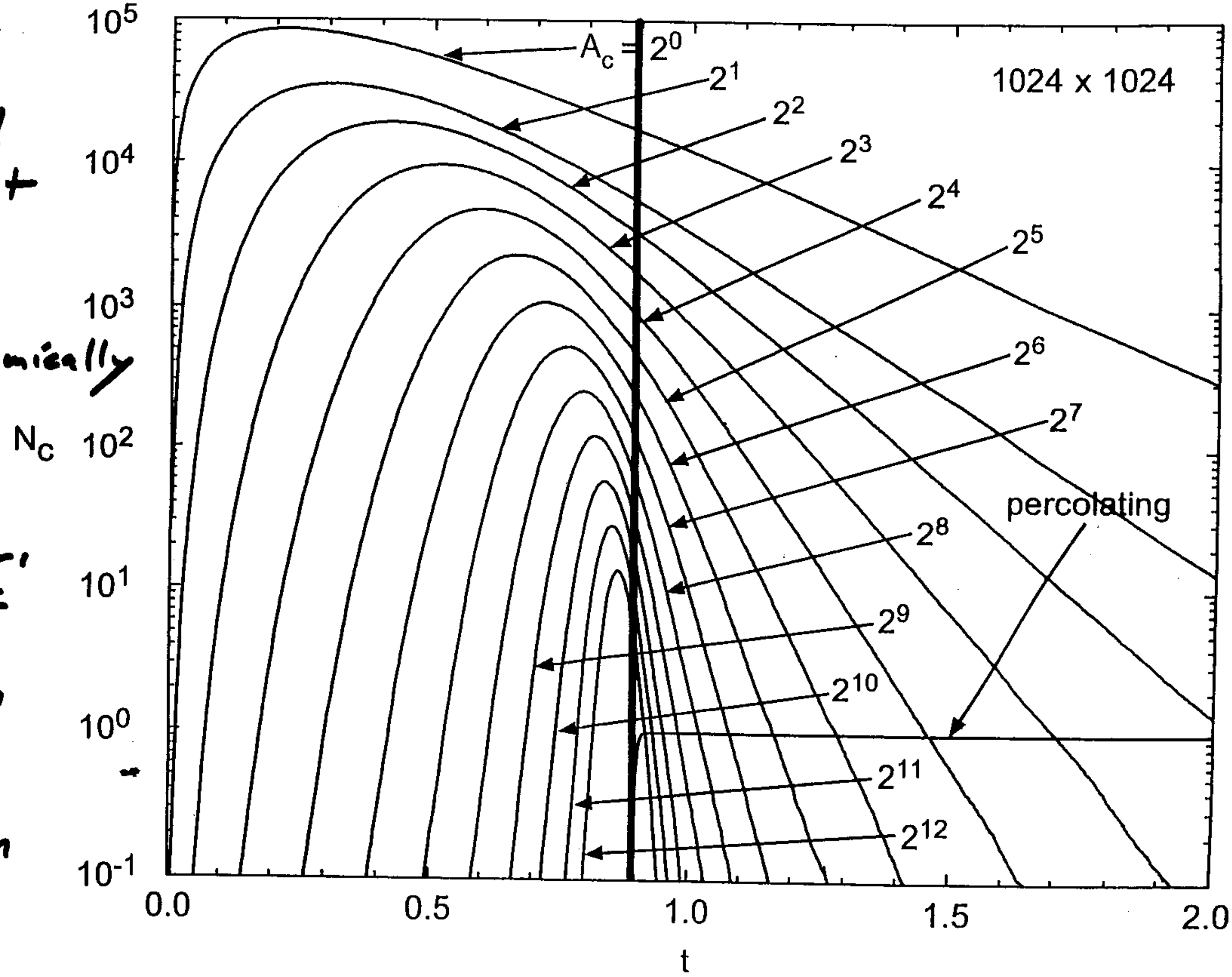


Fig 8.

Forest fire model without fires

logarithmically binned number of clusters

as a function of size A_c at various times

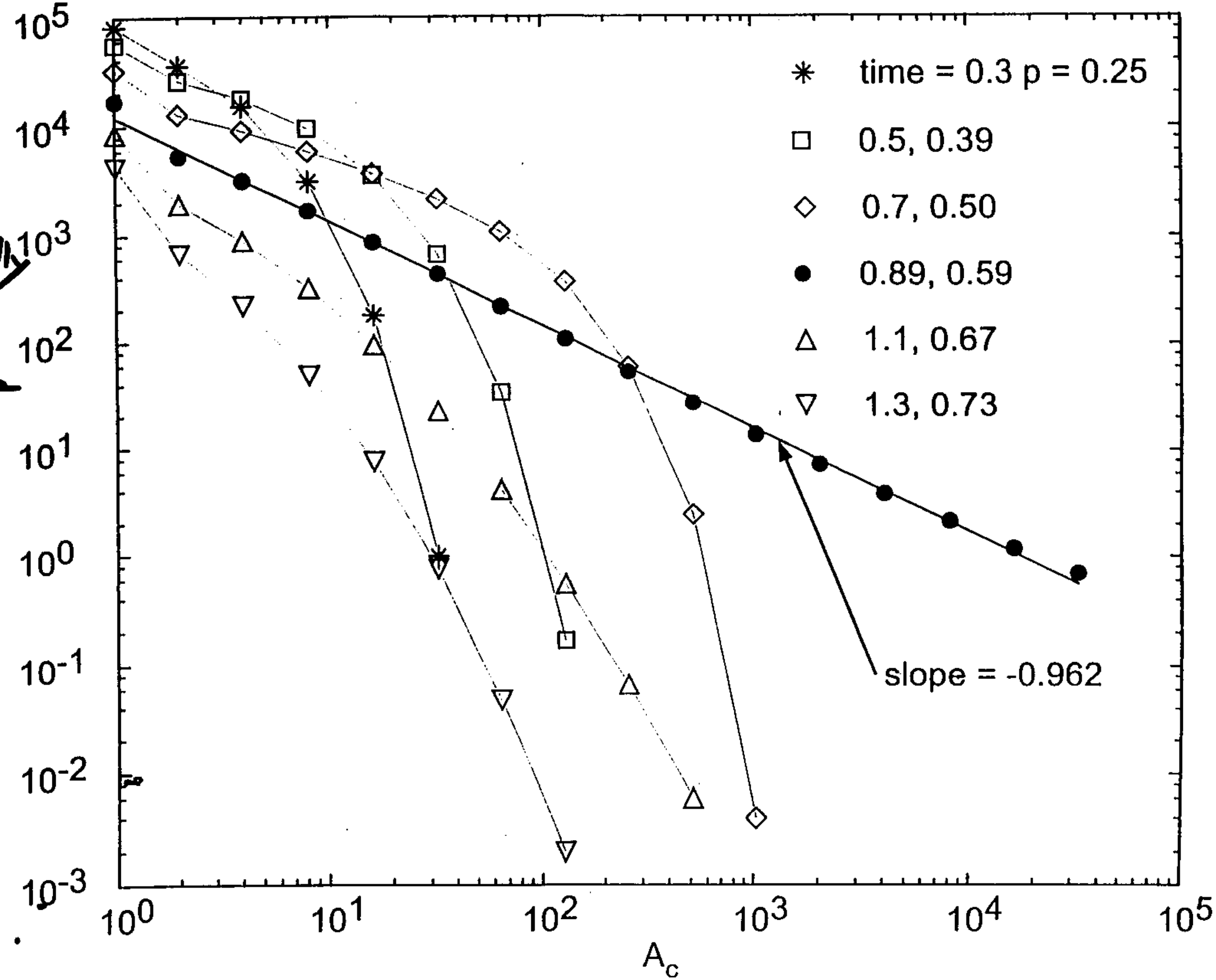


Fig 9.

Fisher exponent -2.055

EG
FS
1

FRACTAL CHARACTER OF EARTHQUAKES

N = number of earthquakes with magnitude greater than m and moment greater than M

r = radius of break

(the magnitude m is a measure of the surface wave amplitude at a specified distance from the source)

$$\log N = -bm + a$$

$$\log M = cm + d$$

$$M = \alpha r^3$$

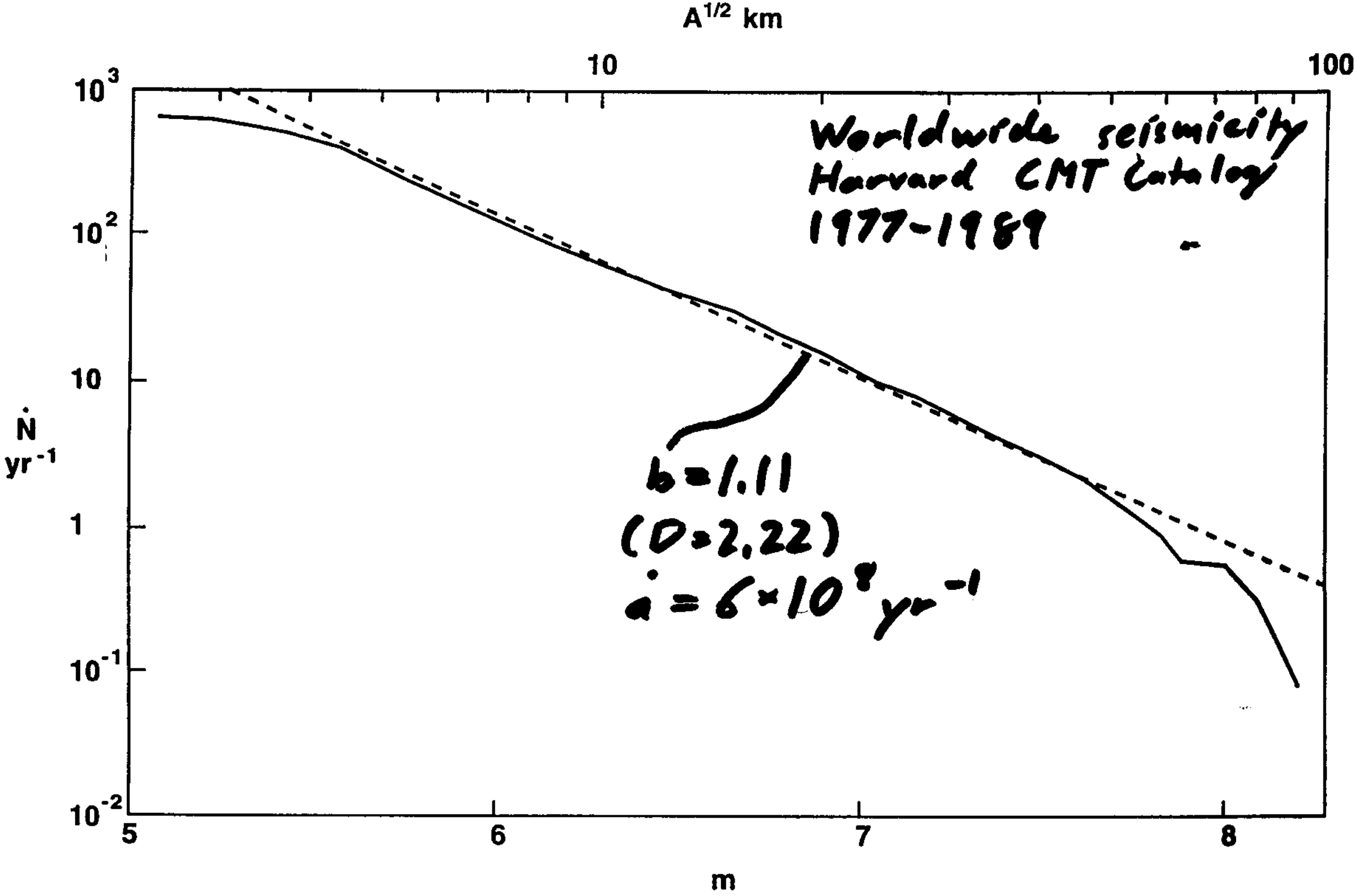
$$M \equiv \mu A S$$

$$\log N = -\frac{3b}{c} \log r + \beta$$

$$N \sim r^{-3b/c} \sim r^{-D}$$

$$D = \frac{3b}{c}$$

$$c = 1.5, \quad b = 0.85 \quad \Rightarrow \quad D = 1.7$$



DT/95-033

EQFS·2

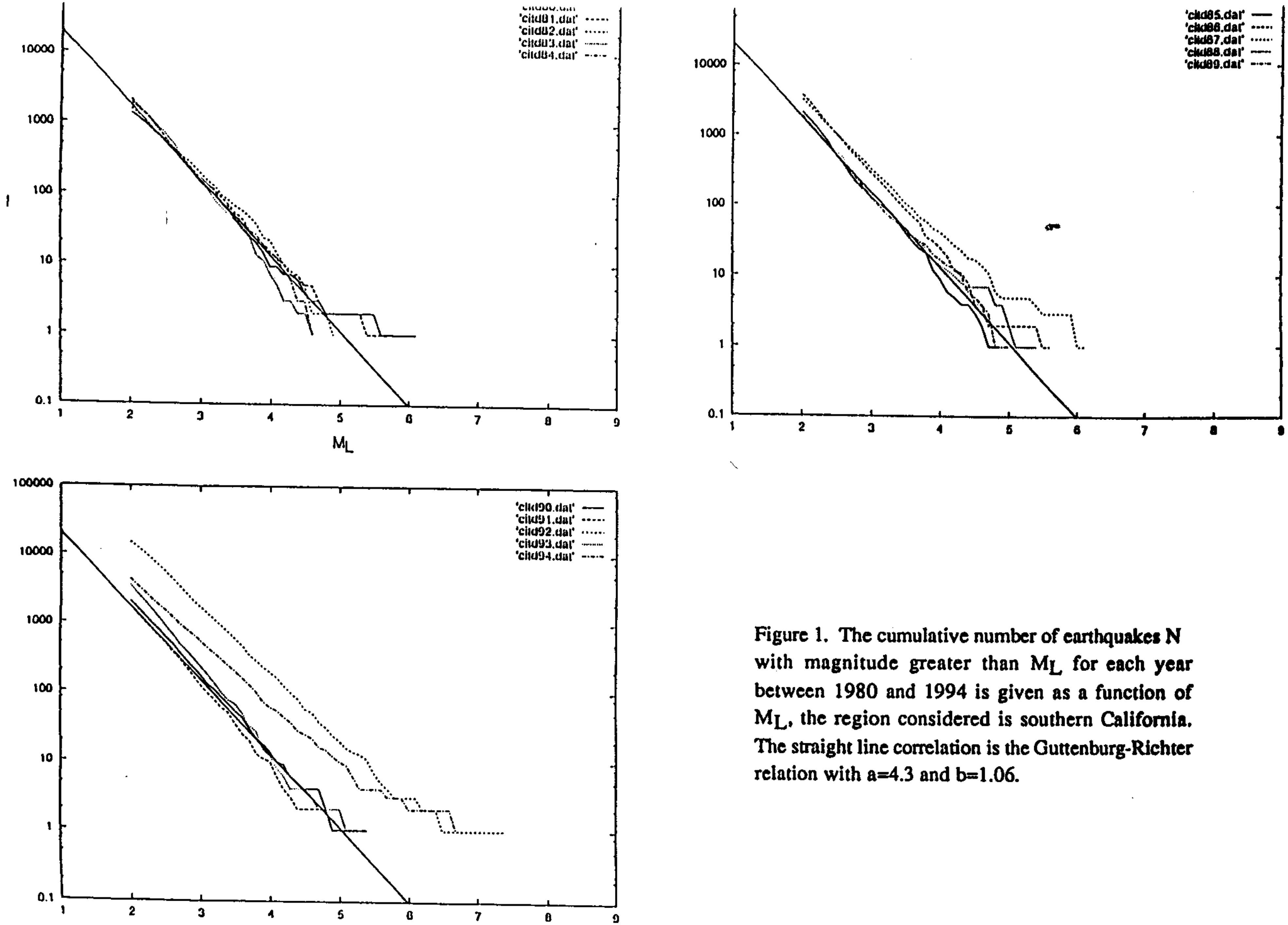
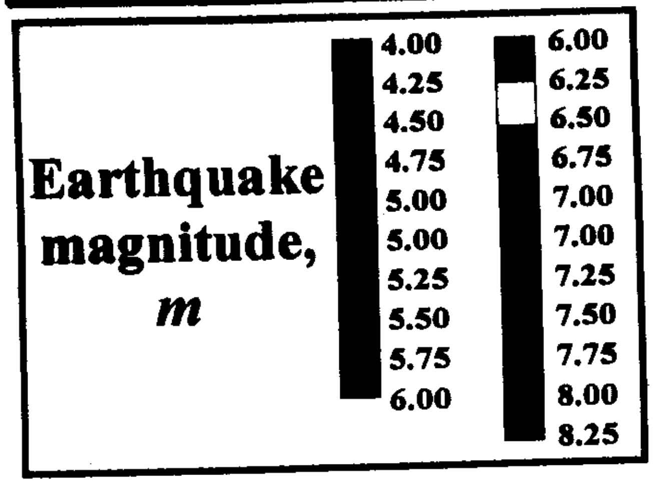


Figure 1. The cumulative number of earthquakes N with magnitude greater than M_L for each year between 1980 and 1994 is given as a function of M_L, the region considered is southern California. The straight line correlation is the Guttenburg-Richter relation with a=4.3 and b=1.06.

EQES-11

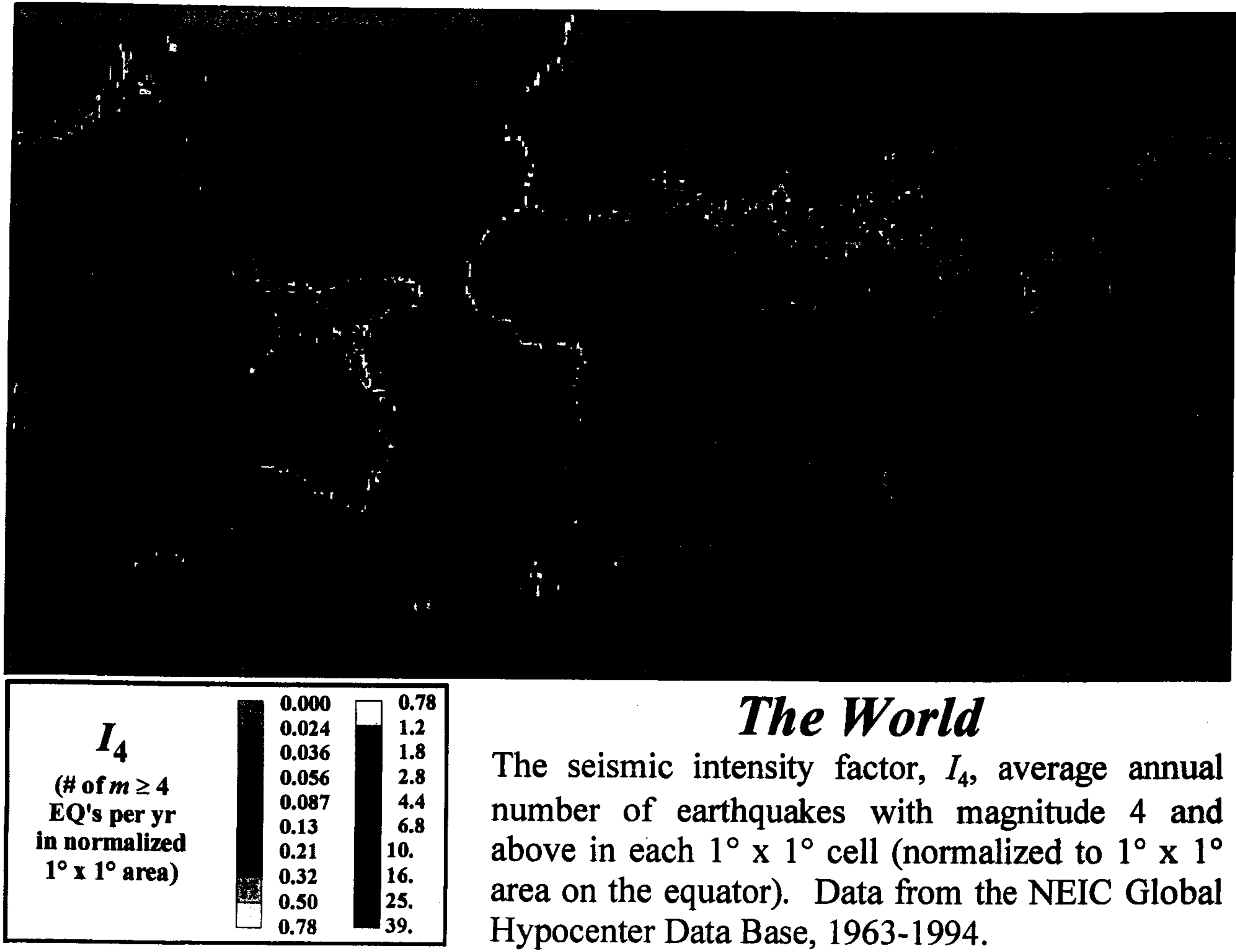
FS 12



The World

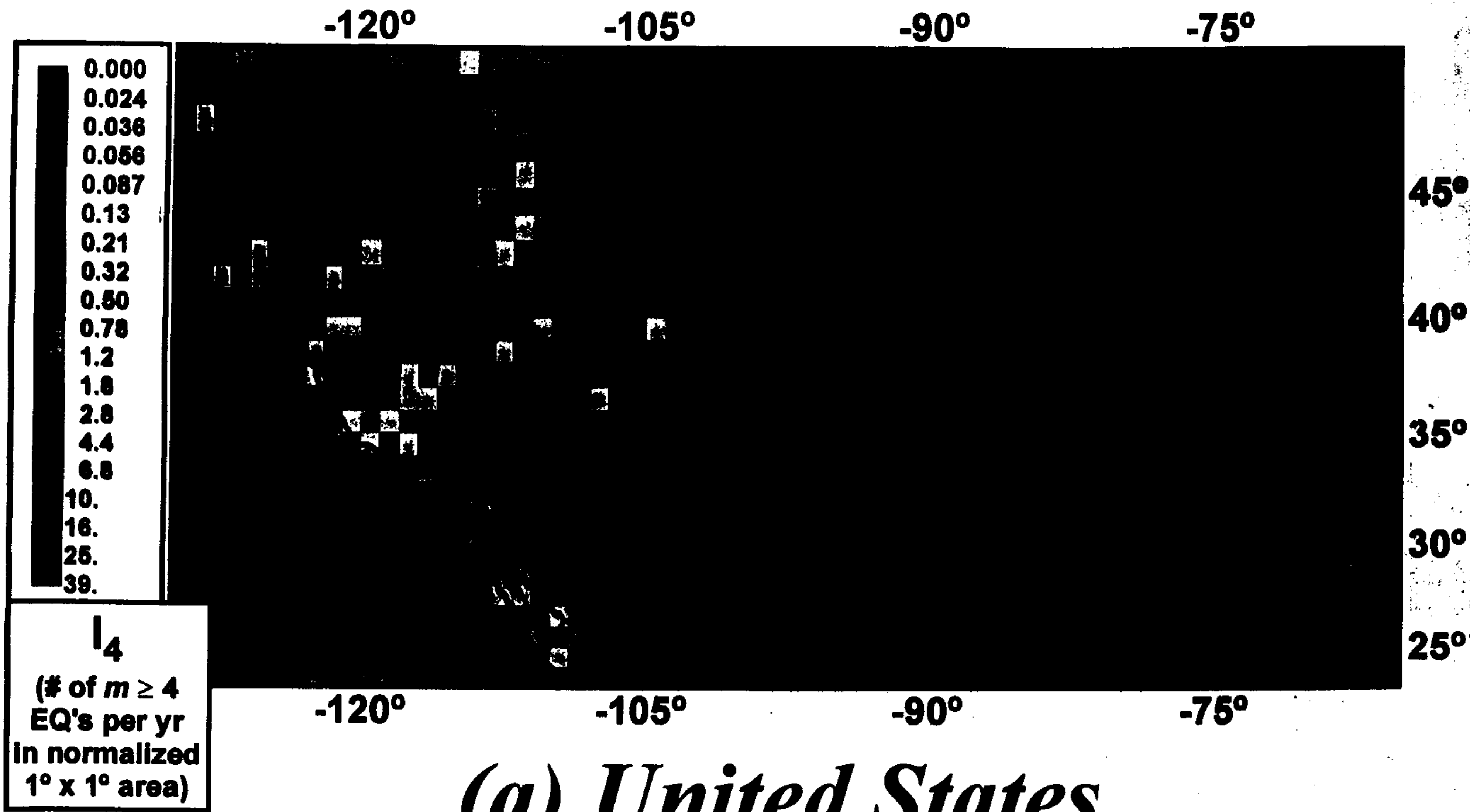
Maximum magnitude, *m*, of earthquakes in each 1° x 1° cell. Data from the NEIC Global Hypocenter Data Base, 1900-1997.

QFS 13



The World

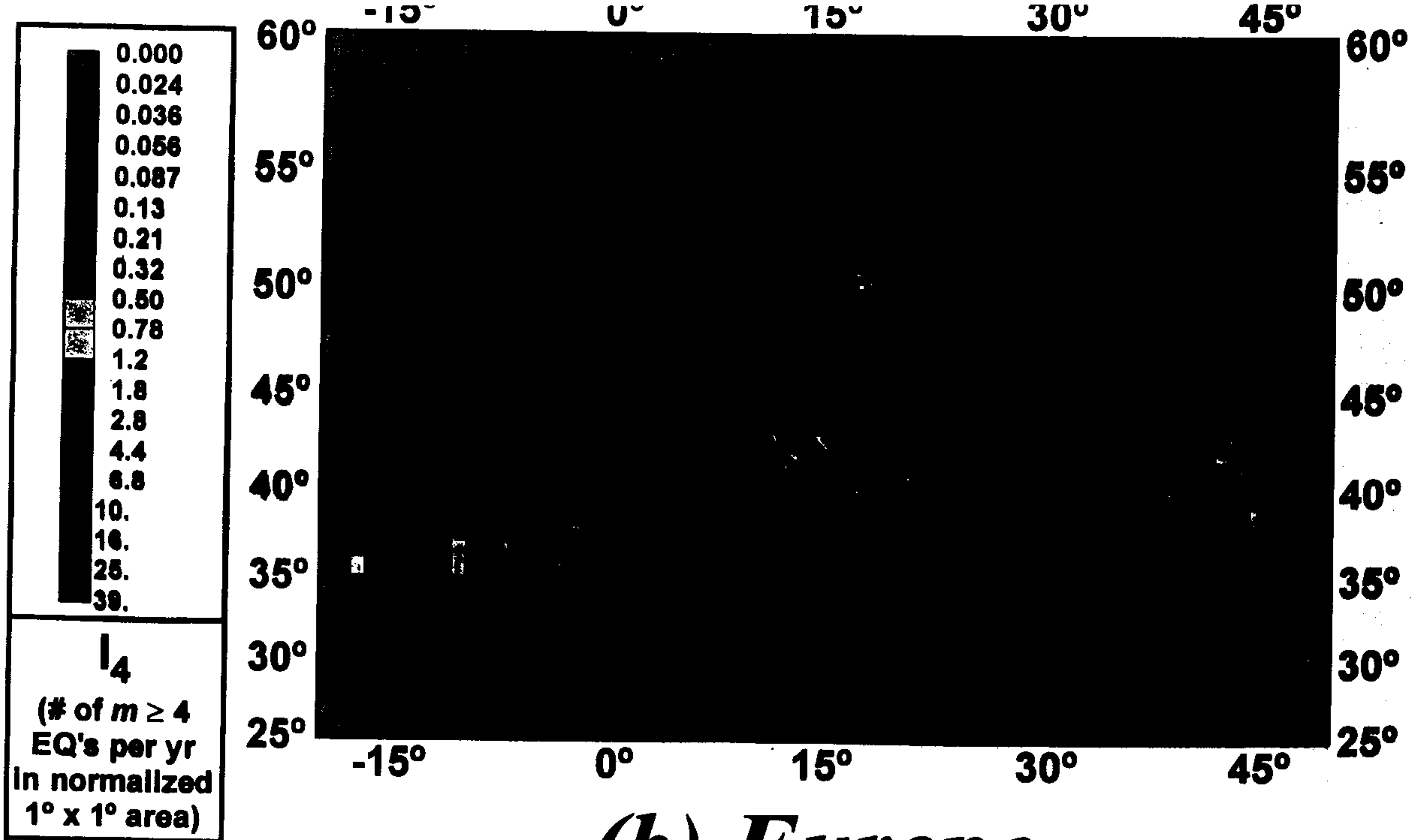
The seismic intensity factor, I_4, average annual number of earthquakes with magnitude 4 and above in each 1° x 1° cell (normalized to 1° x 1° area on the equator). Data from the NEIC Global Hypocenter Data Base, 1963-1994.



(a) United States

The seismic intensity factor, I_4, average annual number of earthquakes with magnitude 4 and above in each 1° x 1° cell (normalized to 1° x 1° area on the equator). Data from the NEIC Global Hypocenter Data Base, 1963-1994.

FIG. 14

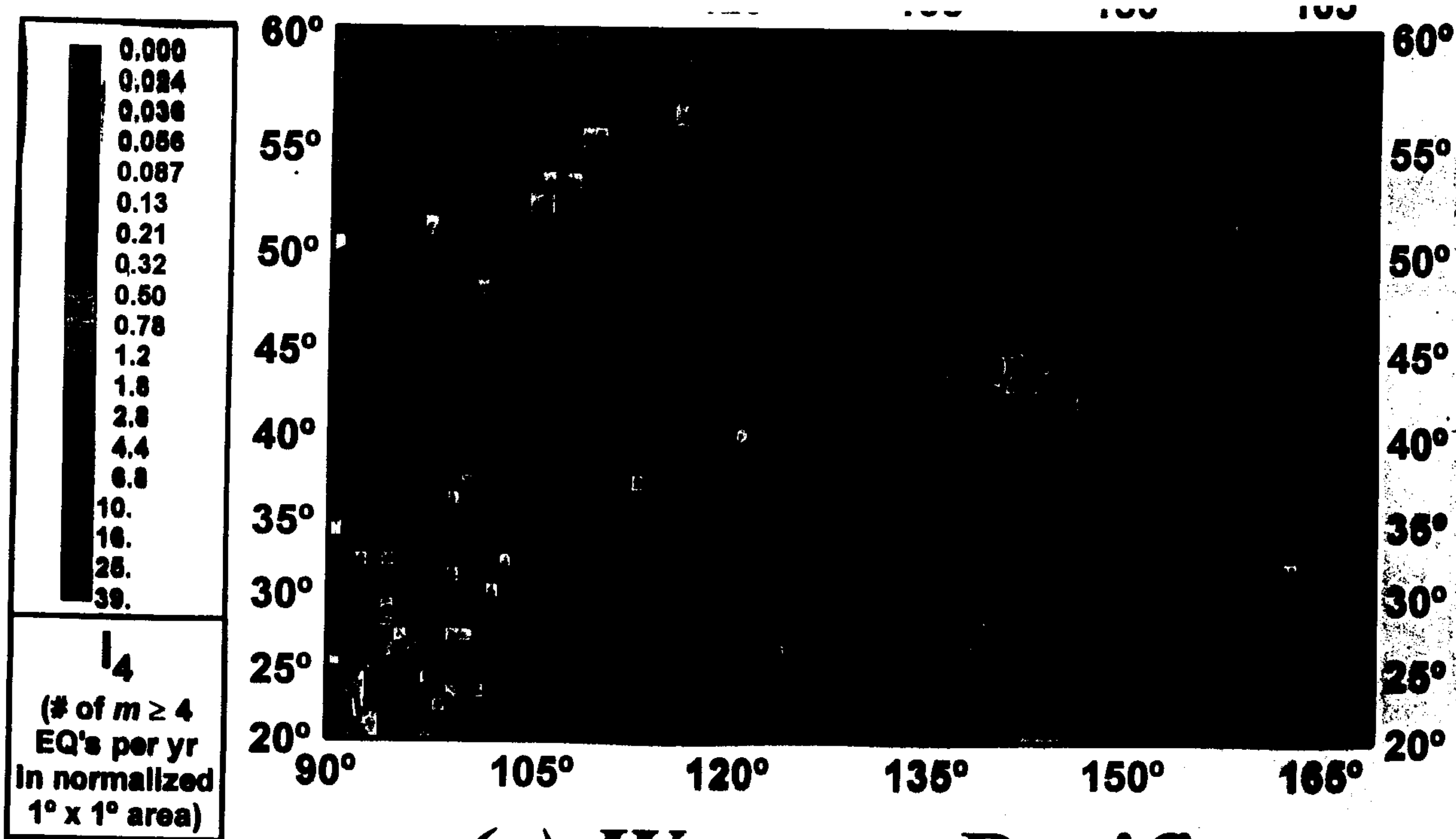


(b) Europe

The seismic intensity factor, I_4, average annual number of earthquakes with magnitude 4 and above in each 1° x 1° cell (normalized to 1° x 1° area on the equator). Data from the NEIC Global Hypocenter Data Base, 1963-1994.

GFS 15

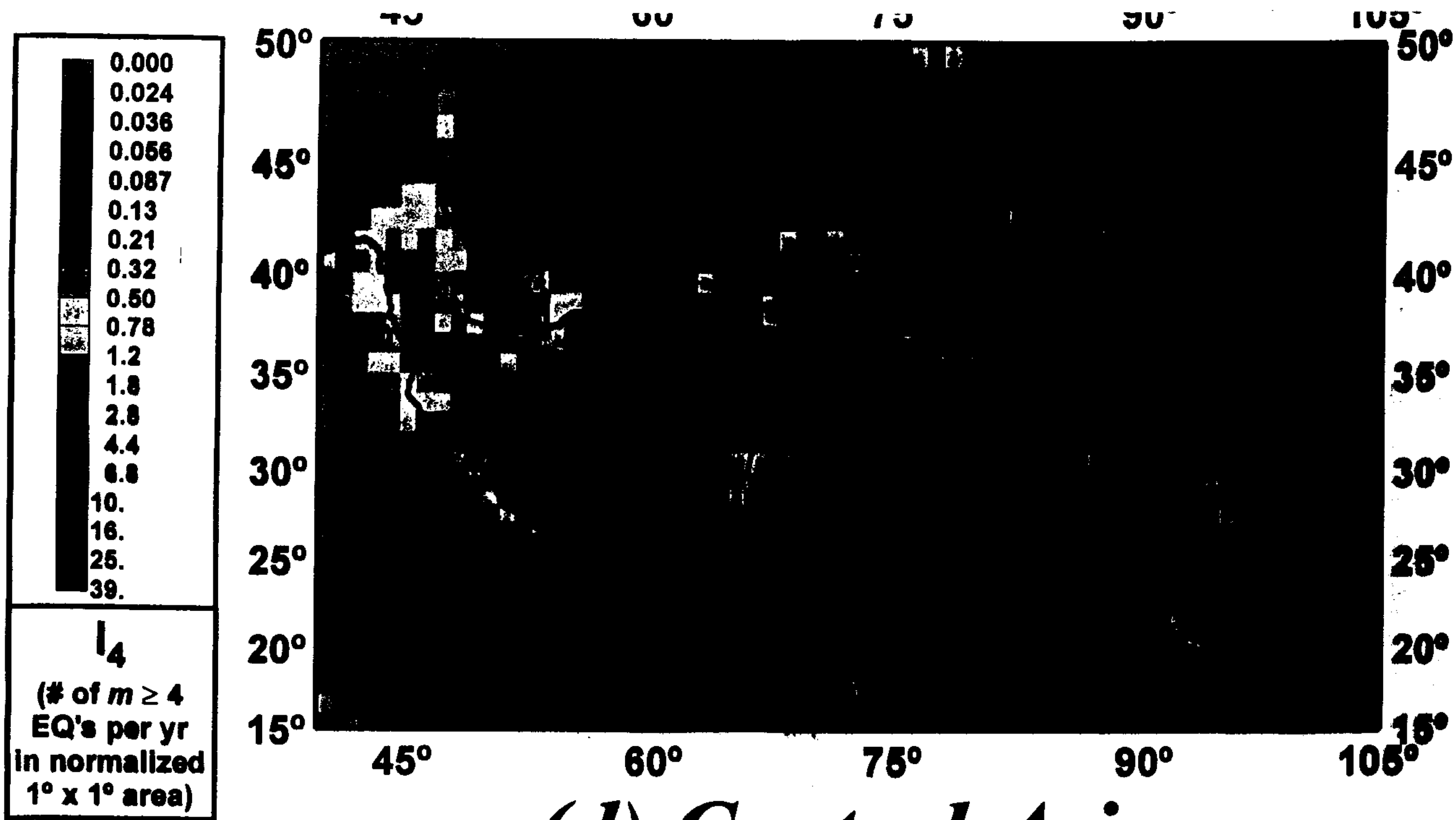
QFS 16



(c) Western Pacific

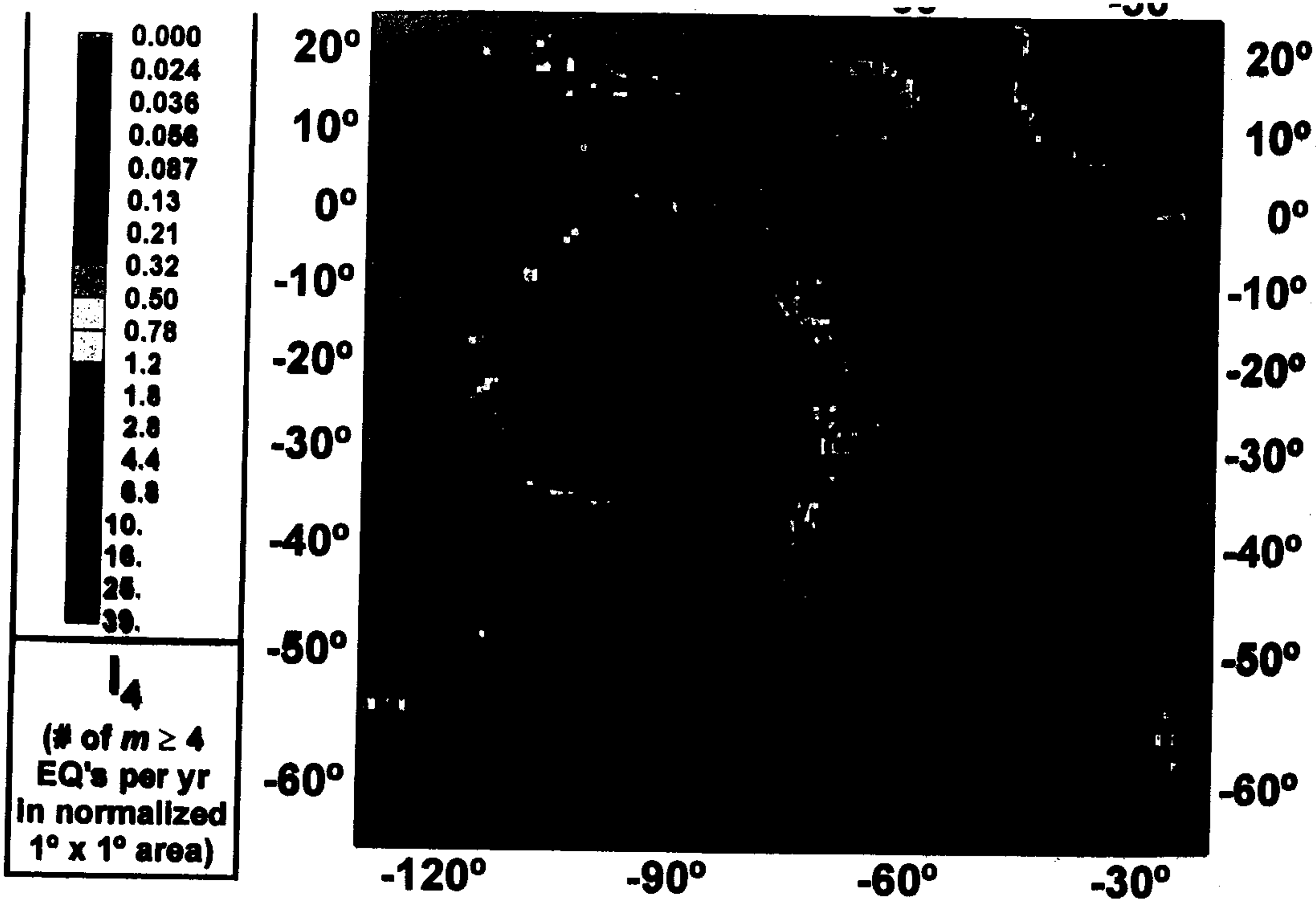
The seismic intensity factor, I_4, average annual number of earthquakes with magnitude 4 and above in each 1° x 1° cell (normalized to 1° x 1° area on the equator). Data from the NEIC Global Hypocenter Data Base, 1963-1994.

QFS 17



(d) Central Asia

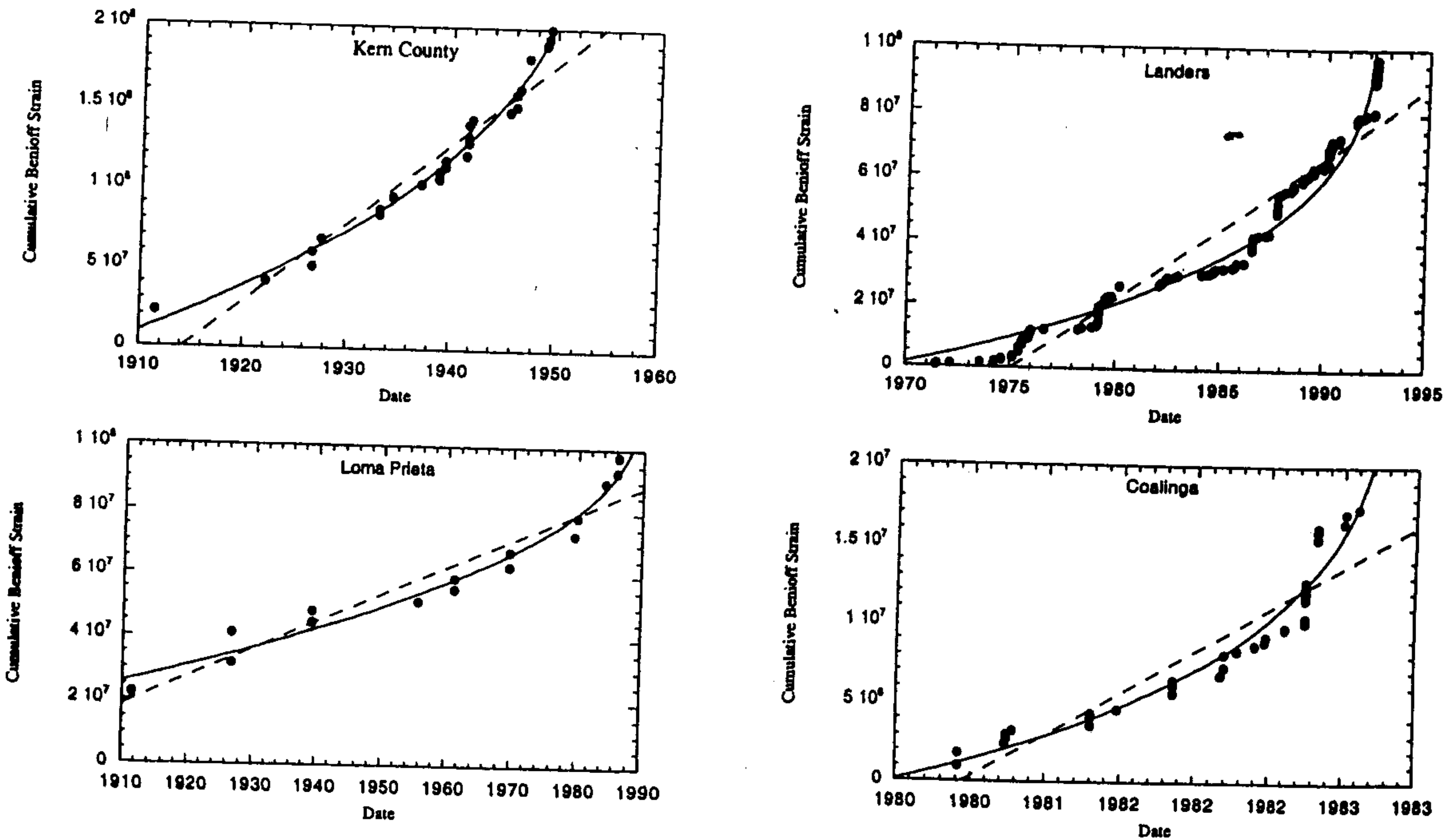
The seismic intensity factor, I_4, average annual number of earthquakes with magnitude 4 and above in each 1° x 1° cell (normalized to 1° x 1° area on the equator). Data from the NEIC Global Hypocenter Data Base, 1963-1994.



(e) South America

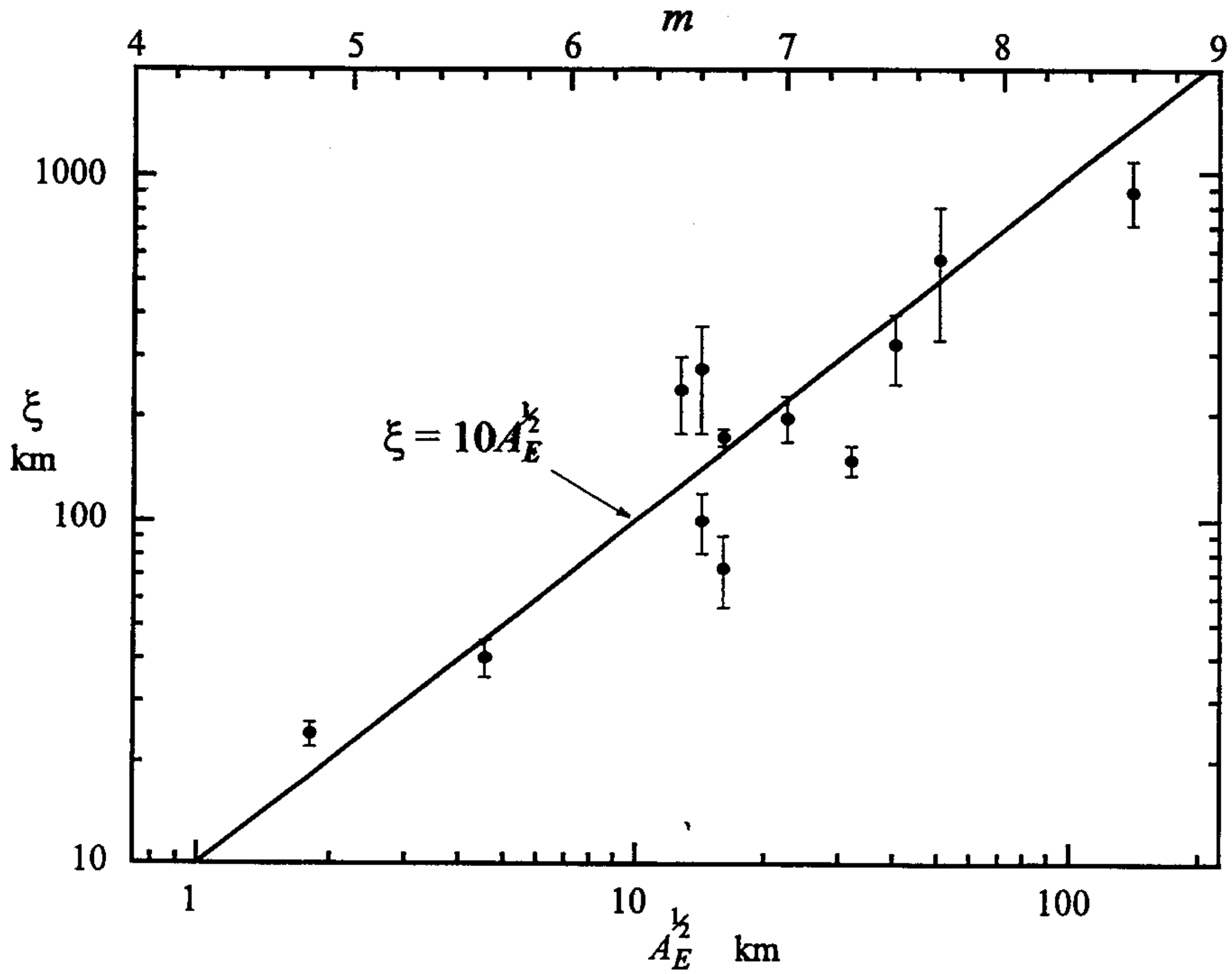
The seismic intensity factor, I_4, average annual number of earthquakes with magnitude 4 and above in each 1° x 1° cell (normalized to 1° x 1° area on the equator). Data from the NEIC Global Hypocenter Data Base, 1963-1994.

Increase in cumulative Benioff strain prior to several major California earthquakes



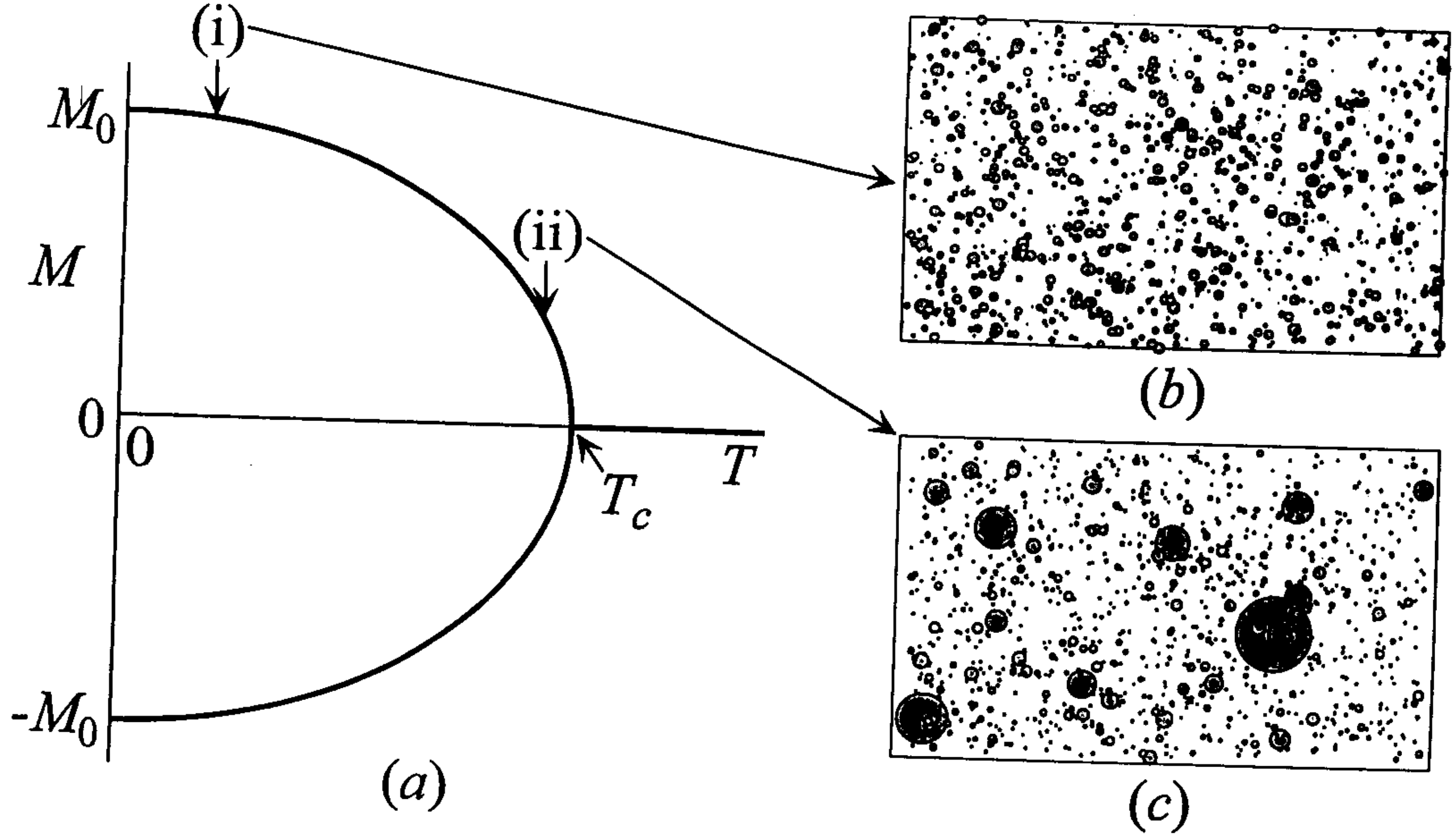
Bowman et al. JGR 103, 24,359 (1998)

[redacted]



Bowman et al. JGR 103, 24,359 (1998)

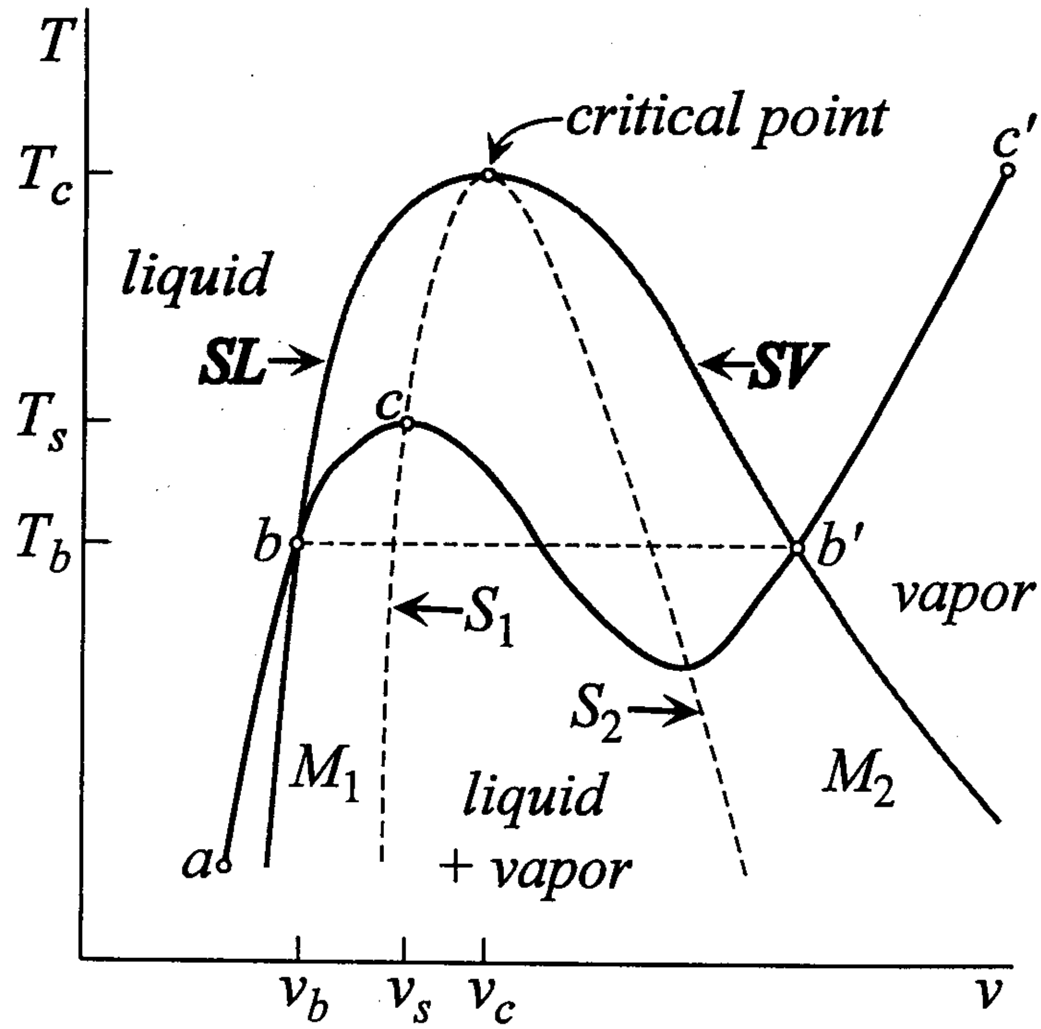
[redacted]



Magnetization M as a function of temperature according to the Ising model

Patches of negative magnitization

Figure 1, Rundle et al.



T vs v (1/ρ) phase diagram for water

Figure 2. Rundle et al.

Stress σ vs slip deficit ϕ during an earthquake cycle

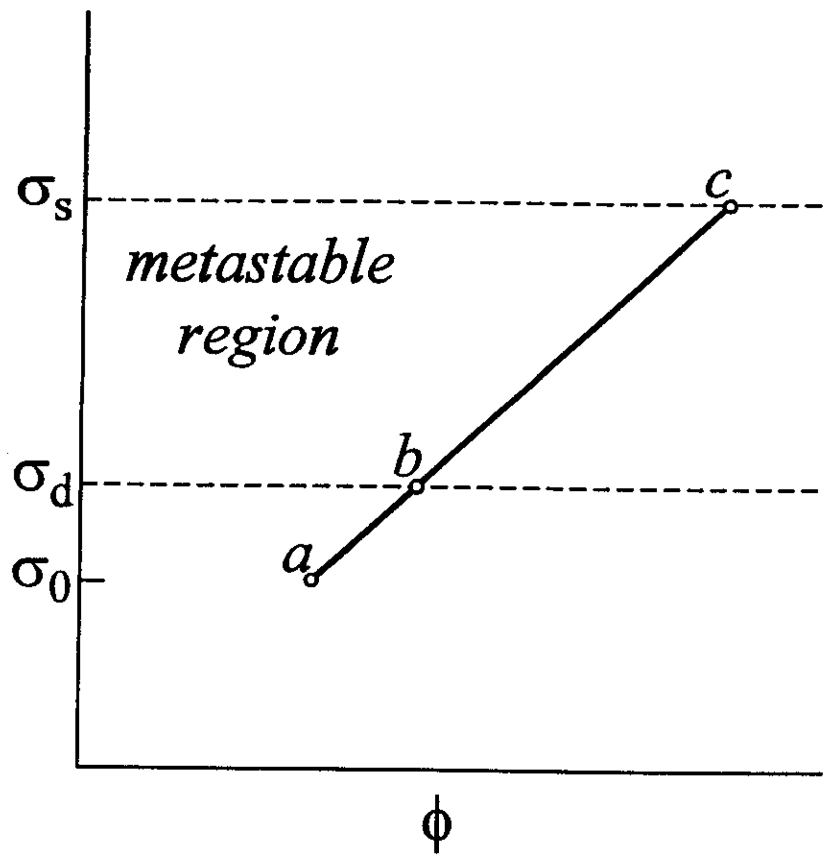


Figure 3. Rundle et al.

Illustration of seismic excitation

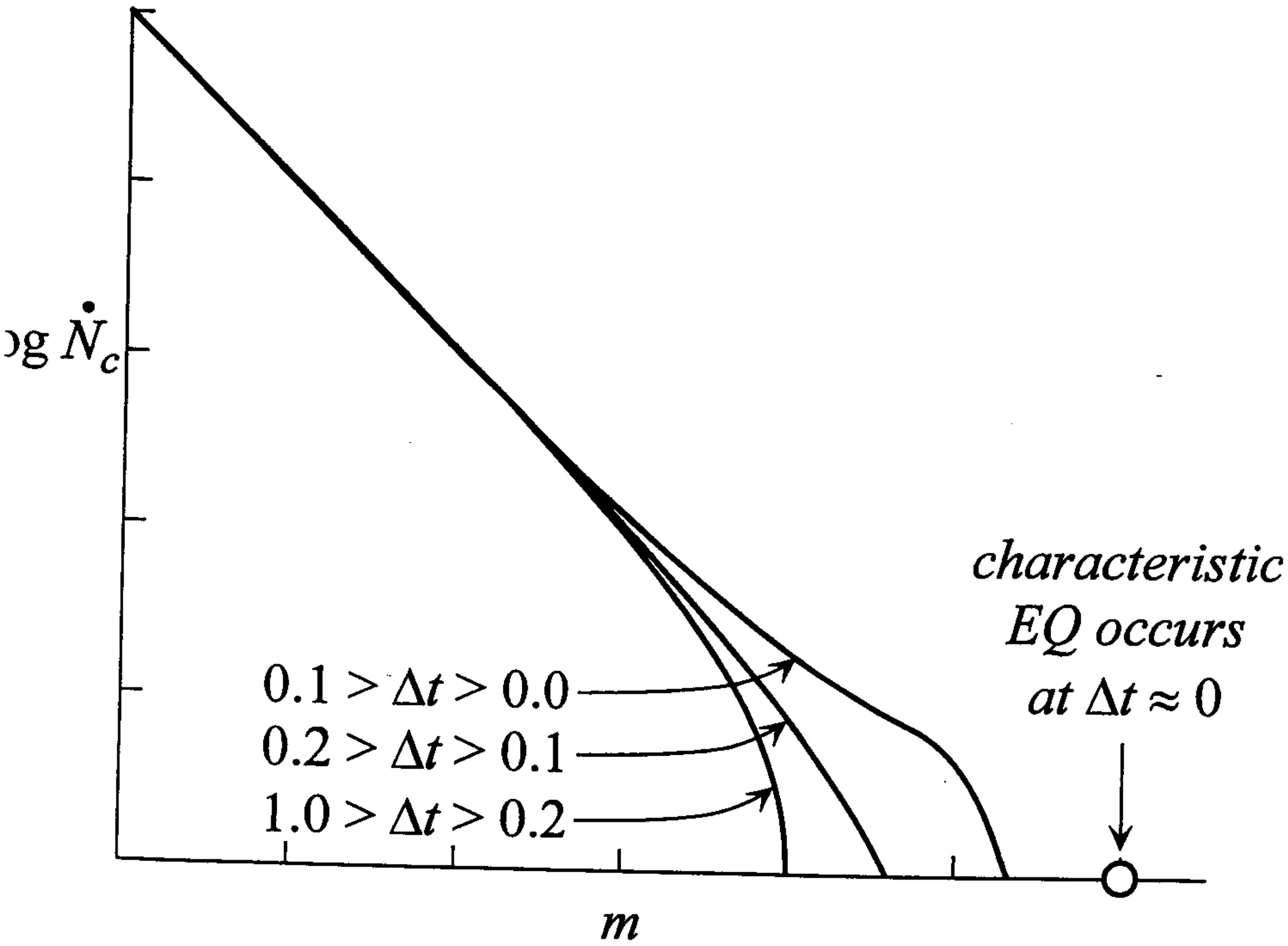


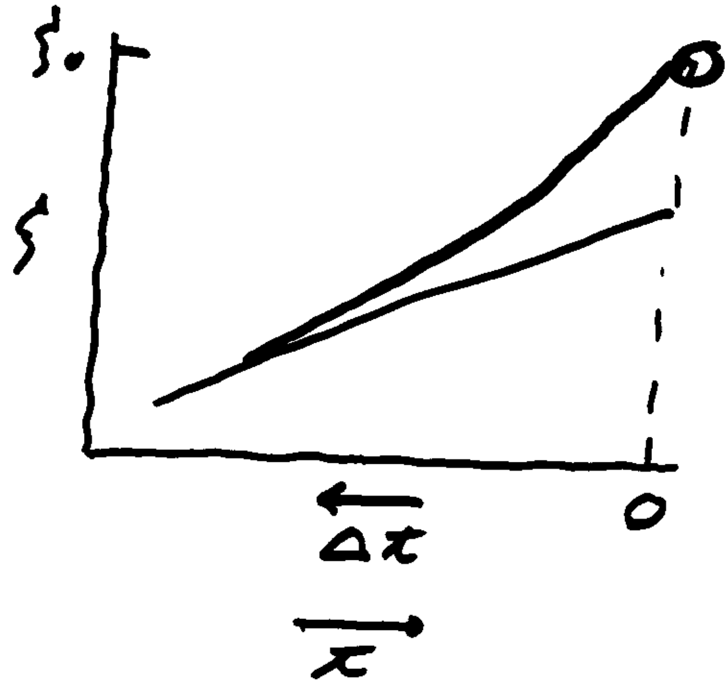
Figure 4. Rundle et al.

Observation

$$\xi(\Delta t) = \xi_0 - \beta\,(\Delta t)^s$$

ξ cumulative Benioff strain

$$\xi(t) = \sum_{i=1}^{N(t)} E_i^{1/2}$$



Theory

$$-K_c \nabla^2 \psi + K_L v \Delta t - \alpha \psi^2 = 0$$

slip deficit ϕ, slip s

$$\phi = vt - s, \qquad \psi = \phi_0 - \phi$$

time to failure $\Delta t = t_0 - t$

K_c ~ connector spring constant
K_L ~ loader spring constant
α ~ friction

$$\psi \sim (\Delta t)^{1/2}$$

$\nabla^2 \sim \frac{1}{r_c^2}$, r_c correlation length

$$r_c \sim \frac{1}{(\Delta t)^{1/4}}$$

$$\frac{dN}{dt} \sim \frac{1}{\Delta t}$$

$$\frac{dM}{dt} \sim \bar{\delta}\,\bar{A}\,\overline{\frac{dN}{dt}} \qquad \psi \sim \frac{\delta}{A}, \quad \delta A \sim \psi A$$

$$\frac{dM}{dt} \sim \psi r_c^4 \frac{dN}{dt} \sim (\Delta t)^{\frac{1}{2}} (\Delta t)^{-1} (\Delta t)^{-1} \sim (\Delta t)^{-\frac{3}{2}}$$

$$\frac{d\xi}{dt} \sim \left(\frac{dM}{dt}\right)^{\frac{1}{3}}, \quad \xi = \xi_0 - \int_{\xi}^{\xi_0} d\xi = \xi_0 - \int_0^{\Delta t} \frac{d\xi}{dt}\, d(\Delta t) = \xi_0 - C\Delta t^{1/2}$$