

**School on "Exploring the Atmosphere by
Remote Sensing Techniques"
18 October - 5 November 1999**

1151-19

"The ORA Experiment"

**D. Fussen
Institut d'Aeronomie Spatiale de Belgique
Brussels
Belgium**

Please note: These are preliminary notes intended for internal distribution only.

THE ORA EXPERIMENT

ACRONYM OF OCCULTATION
RADIOMETER

• SCIENTIFIC TEAMS AT THE
BELGIAN INSTITUTE FOR

SPACE AERODROMY , BRUSSELS

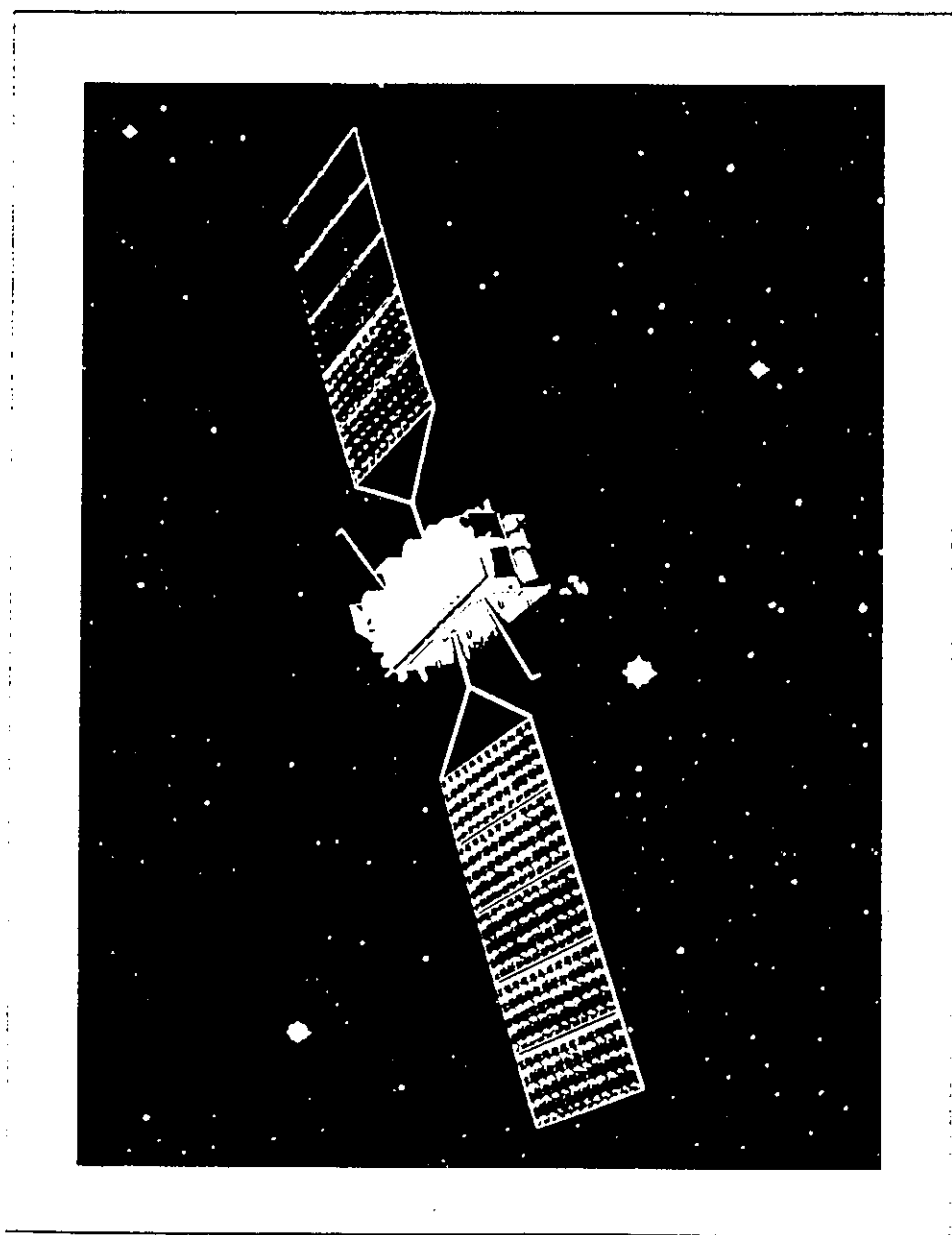
INSTRUMENT: Etienne ARIJS , Dennis NEVEJANS

DATA PROCESSING: Didier FUSSEN ,
Filip VANHELLEMONT , Christine BINGEN

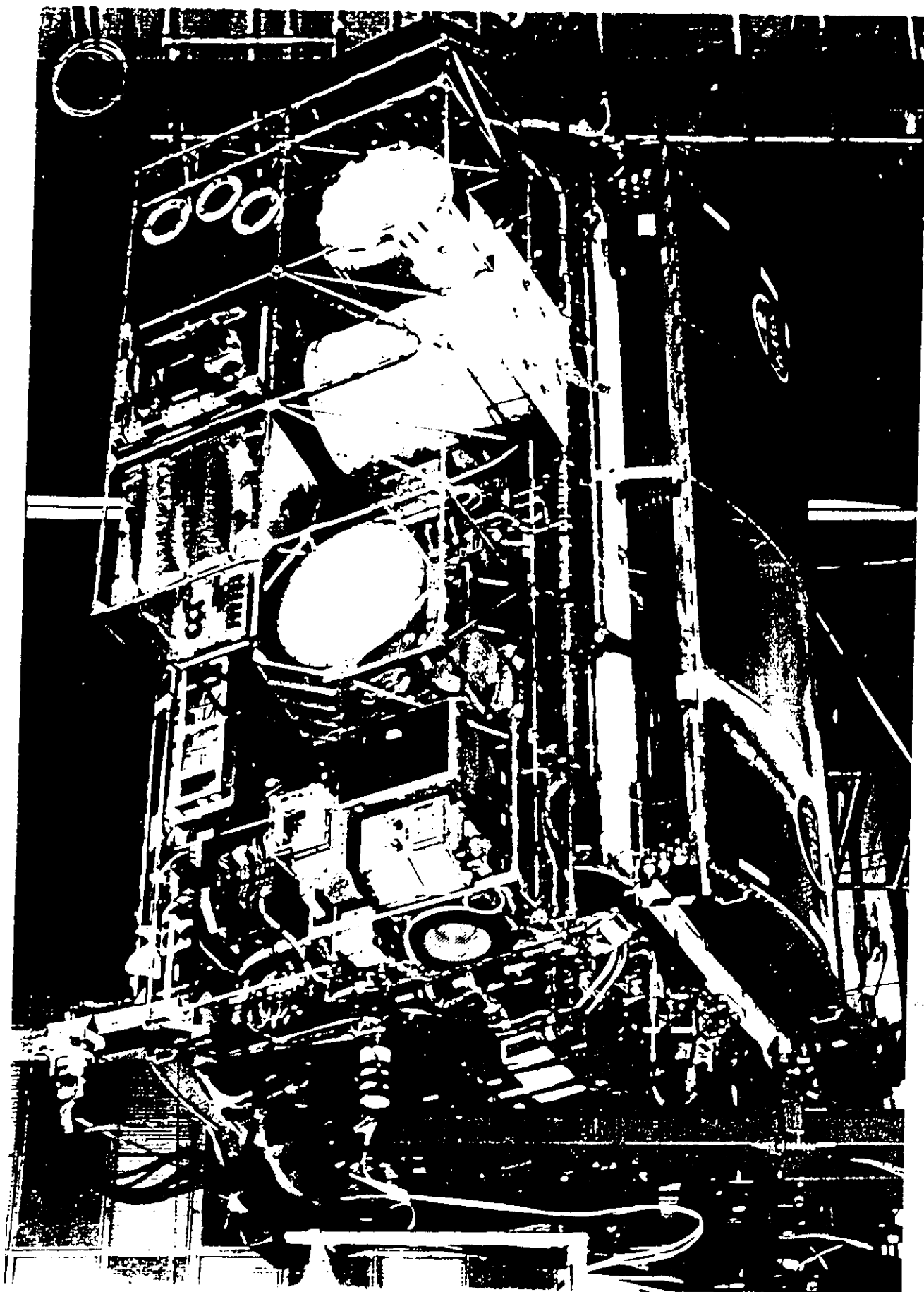
• SATELLITE : EUROPEAN
RETRIEVABLE
CARRIER

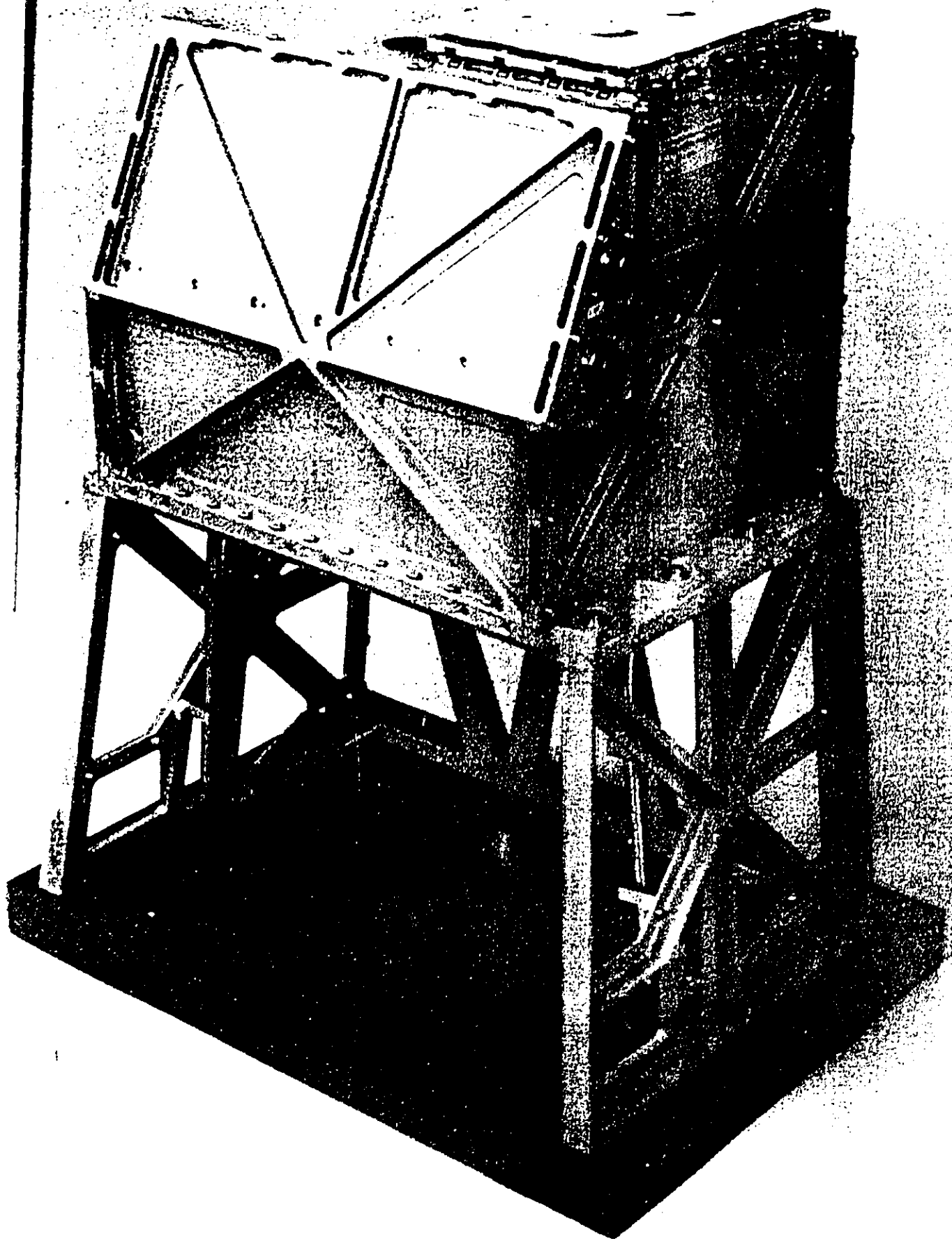
• TIME PERIOD : AUGUST 1992 - MAY 1993

(Sgd)



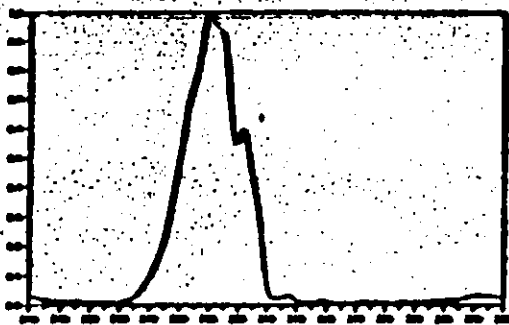
The Spacecraft



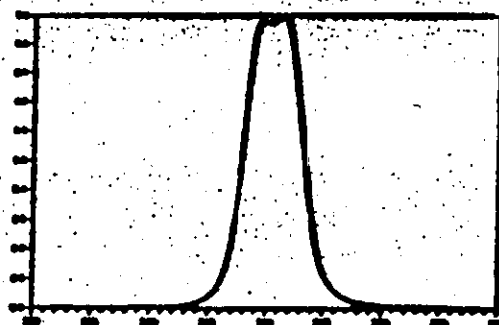




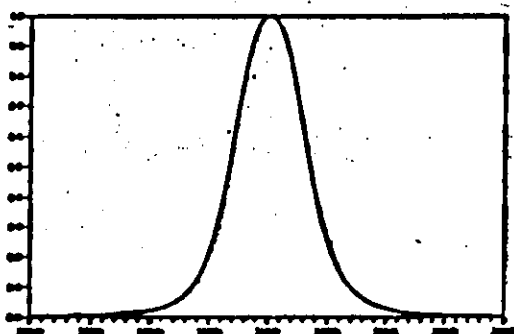
ORAdambda=338 nm transmission curve



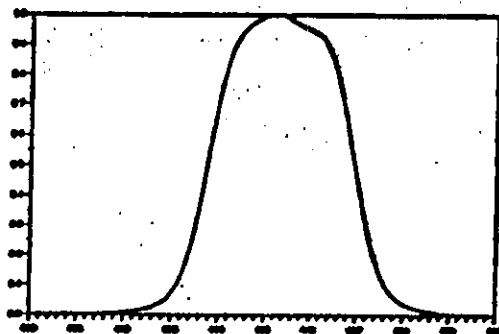
ORAdambda=340 nm transmission curve



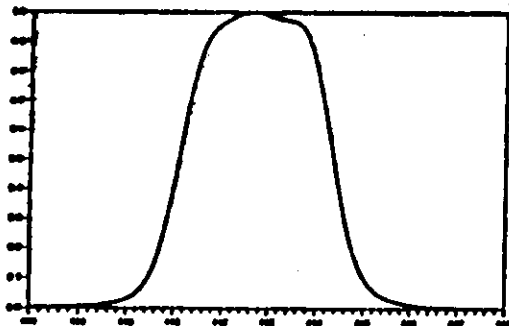
ORAdambda=385 nm transmission curve



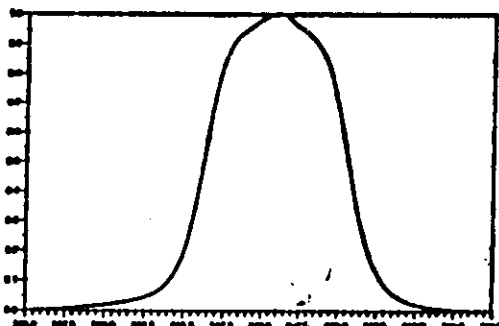
ORAdambda=435 nm transmission curve



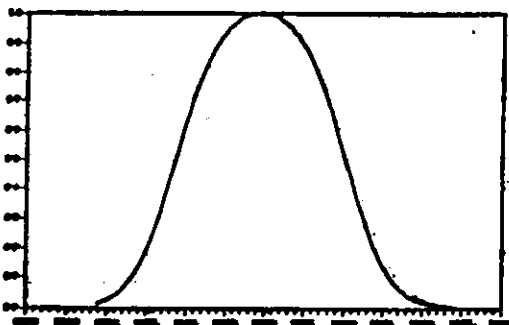
ORAdambda=442 nm transmission curve



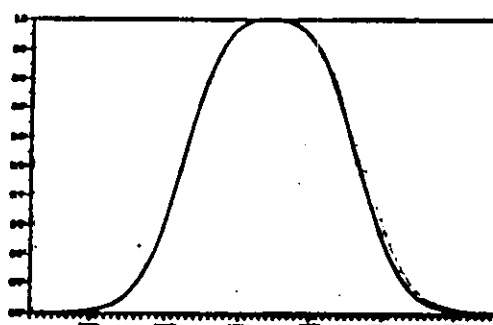
ORAdambda=600 nm transmission curve



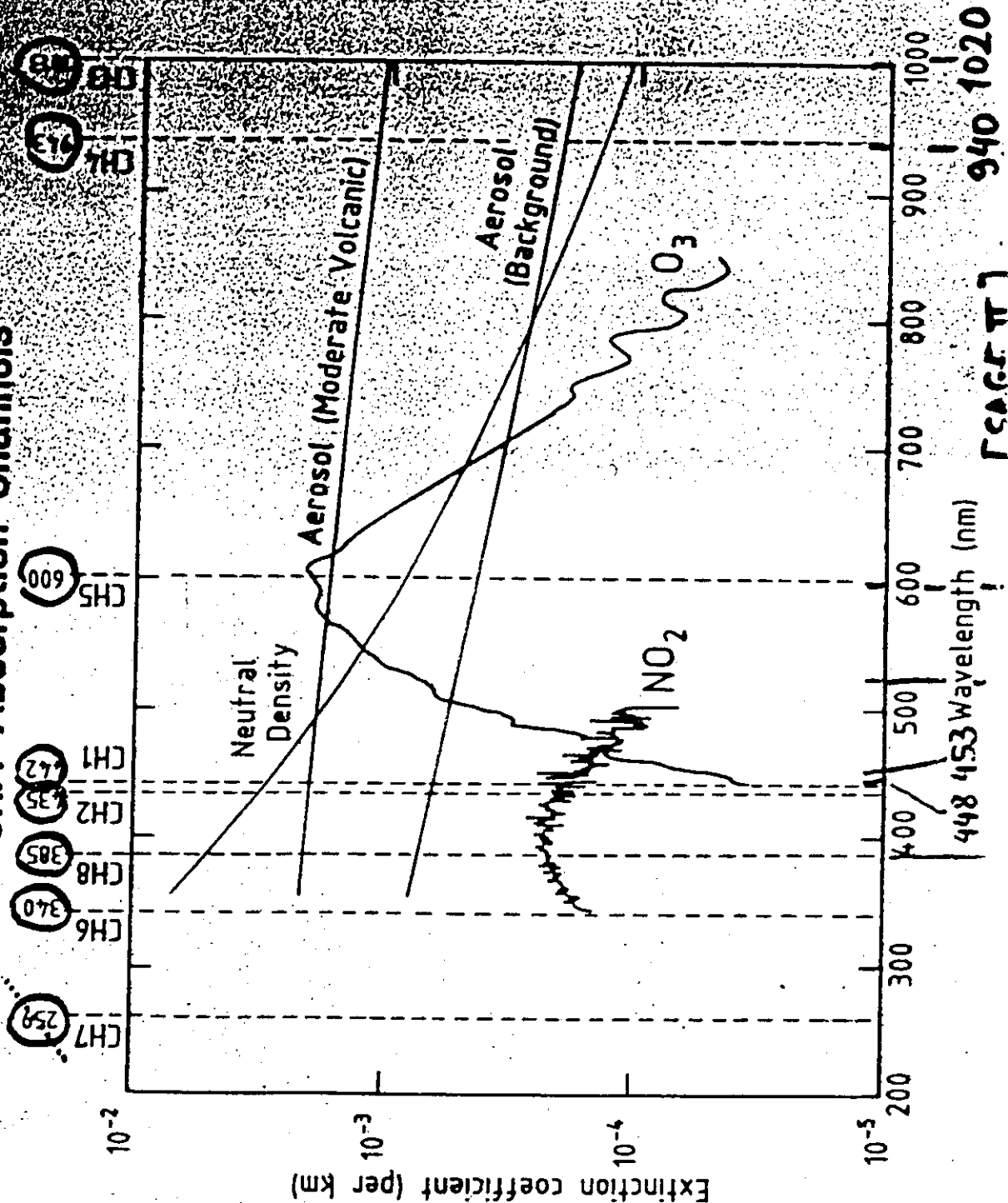
ORAdambda=943 nm transmission curve



ORAdambda=1012 nm transmission curve



ORA Absorption Channels



Page 7

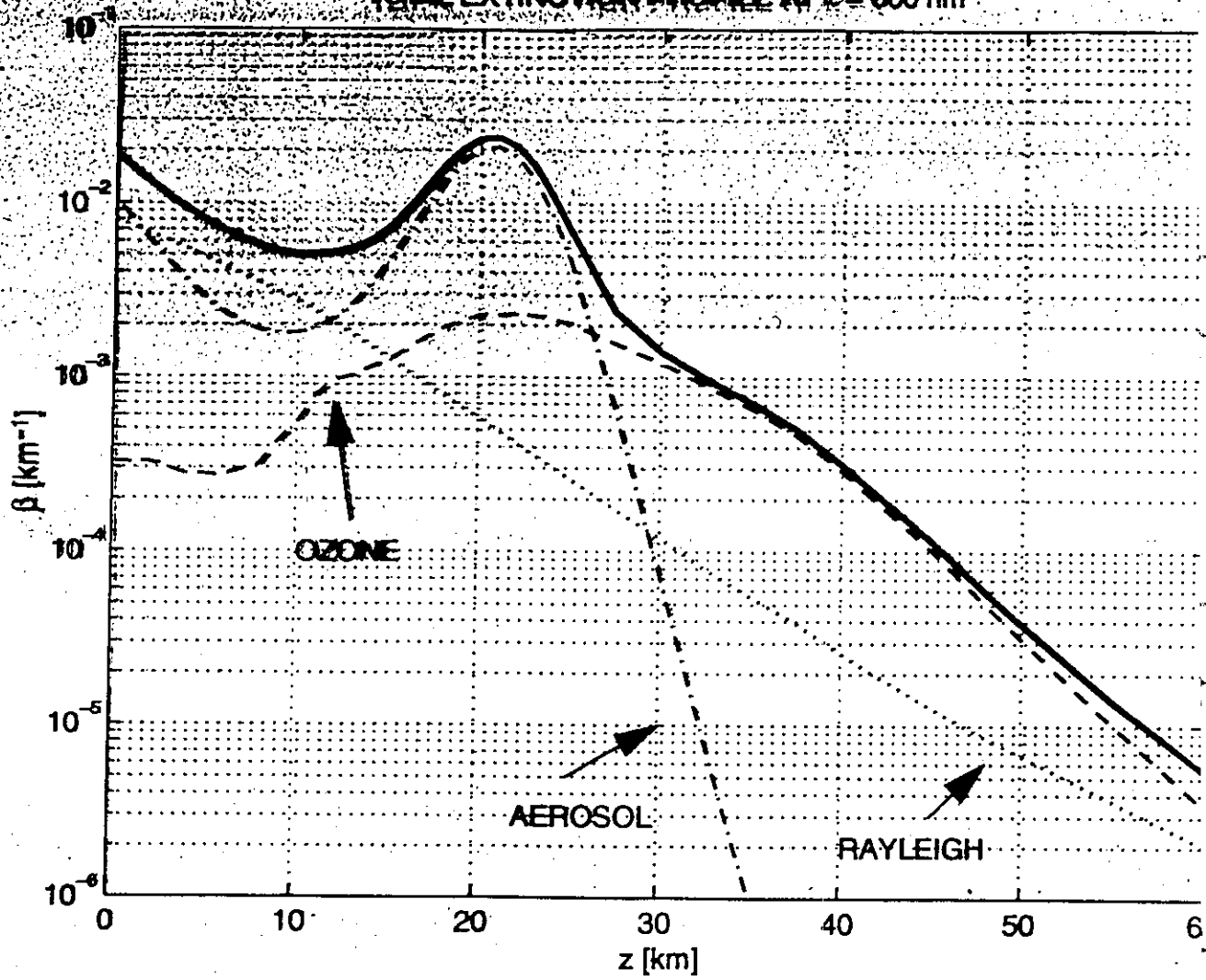
940 1020

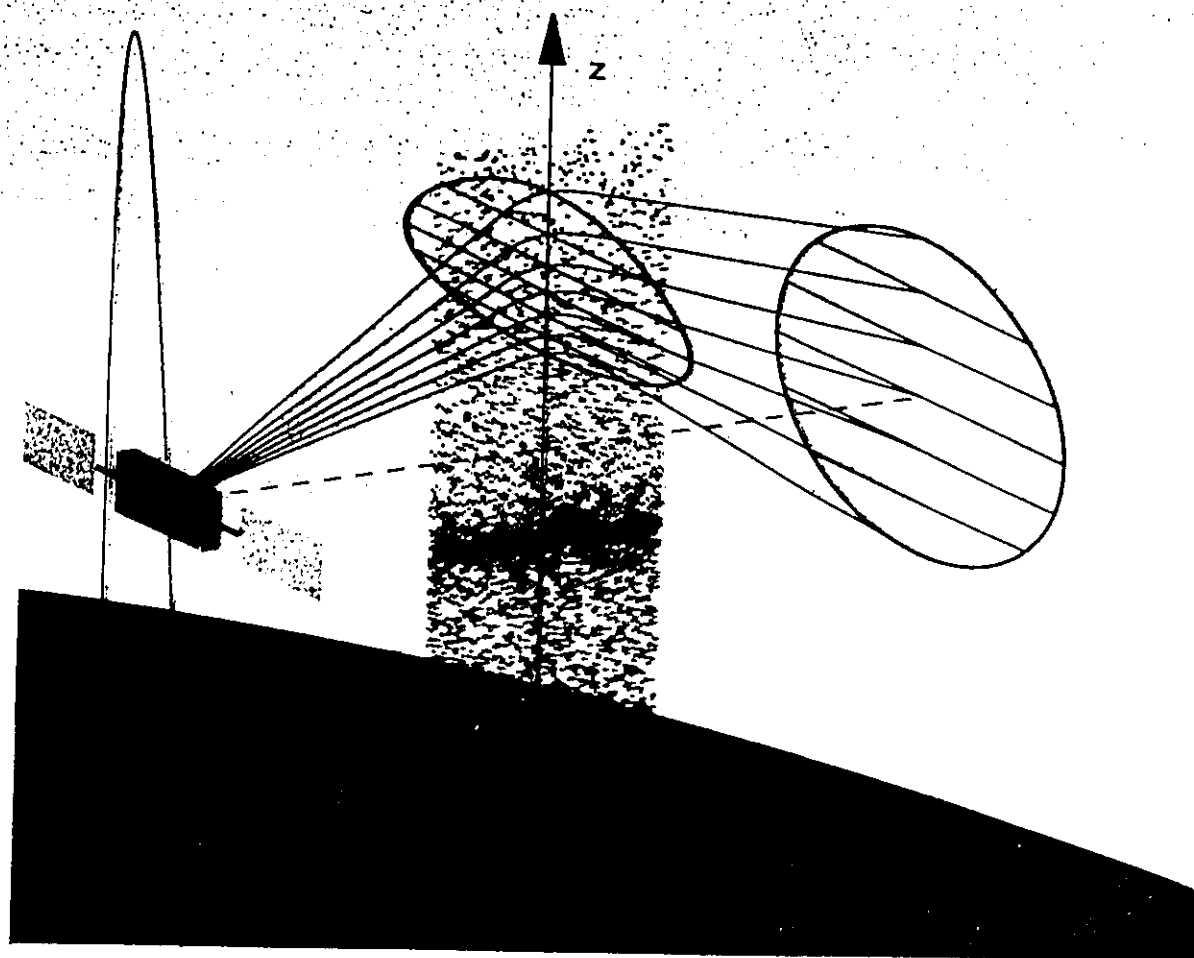
Wavelength (nm)

Extinction coefficient (per km)

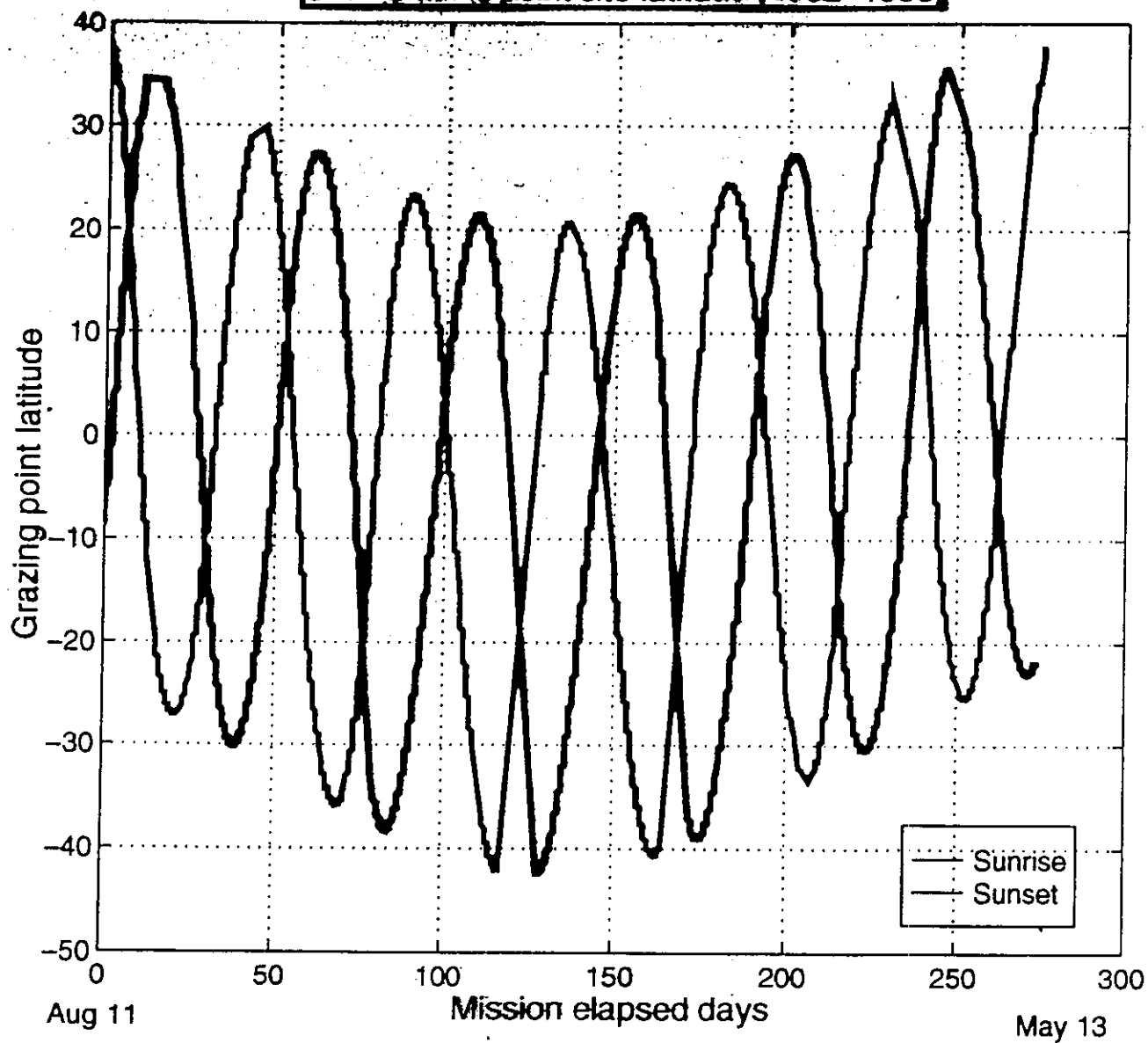
UP SUPERIEUR DOWN OPEN

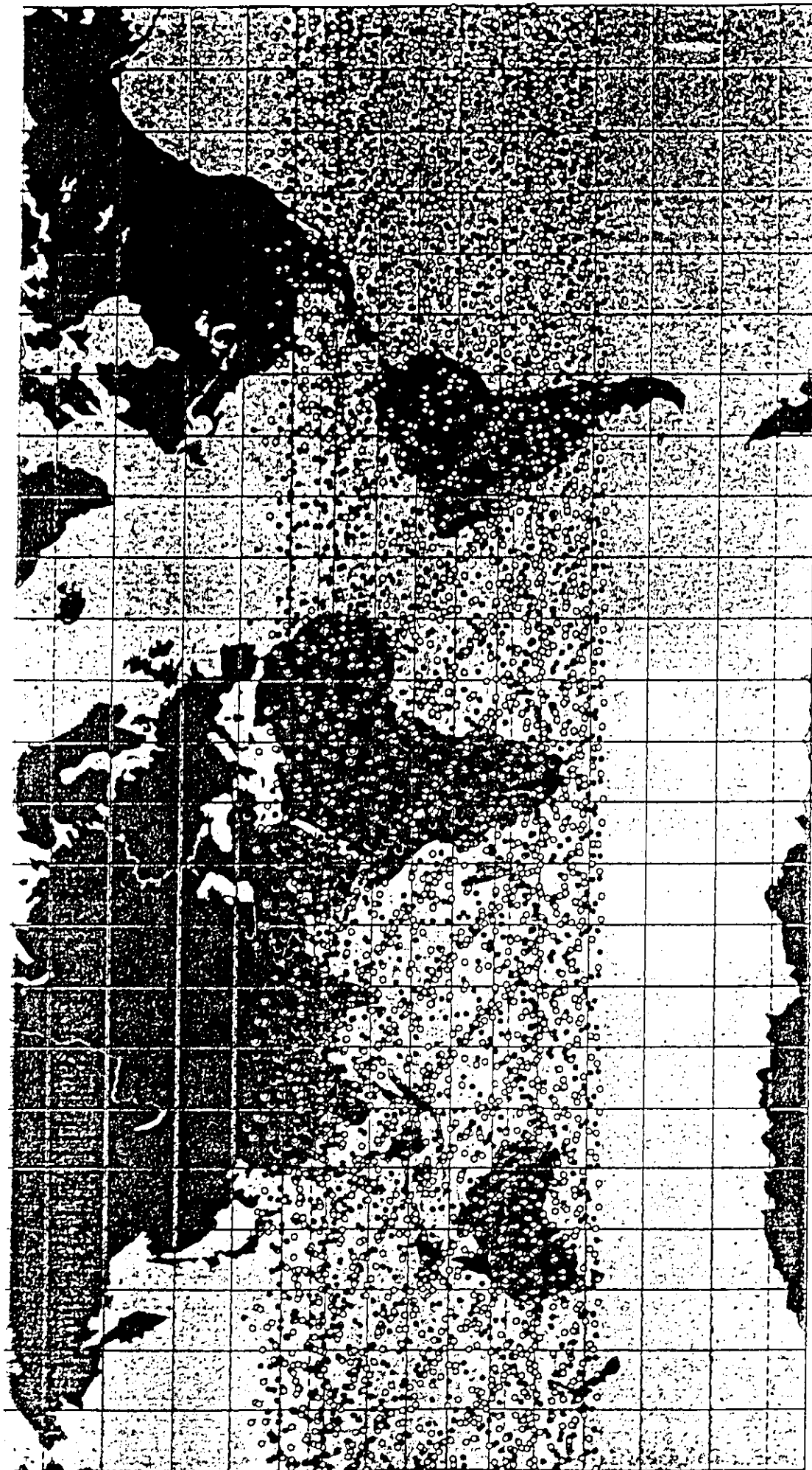
TOTAL EXTINCTION PROFILE AT $\lambda = 600$ nm



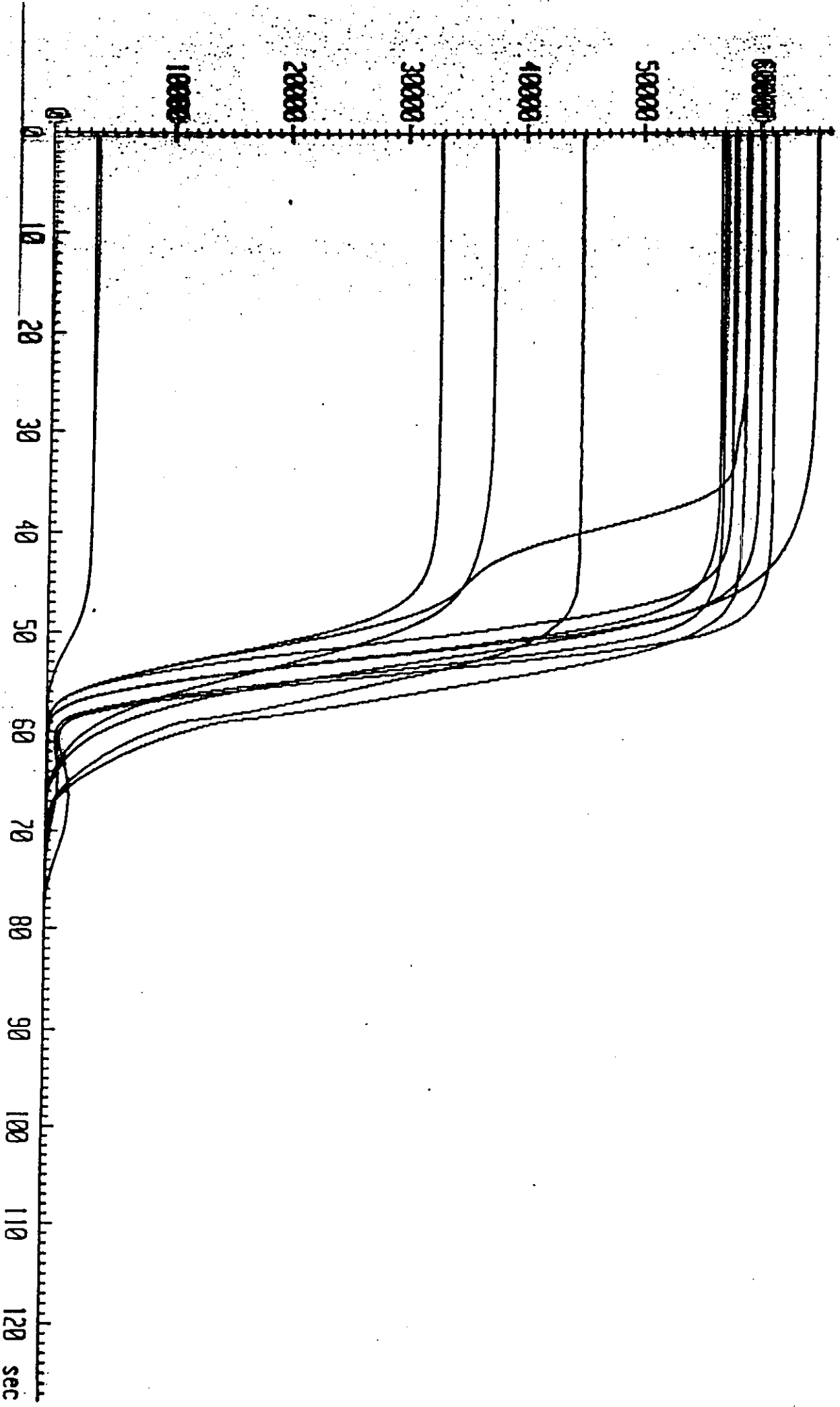


ORA grazing point site latitude (1992-1993)





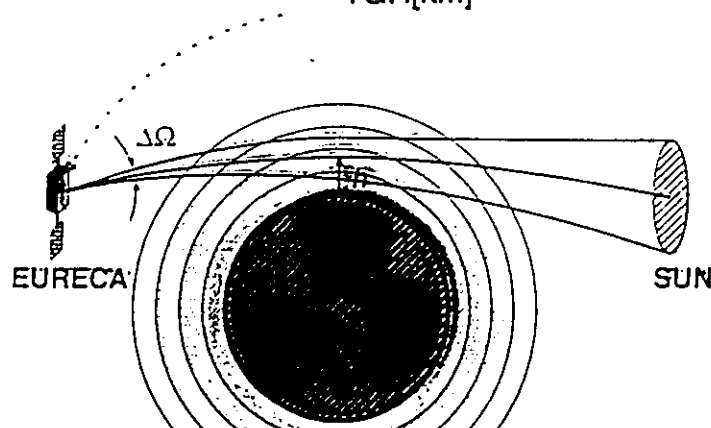
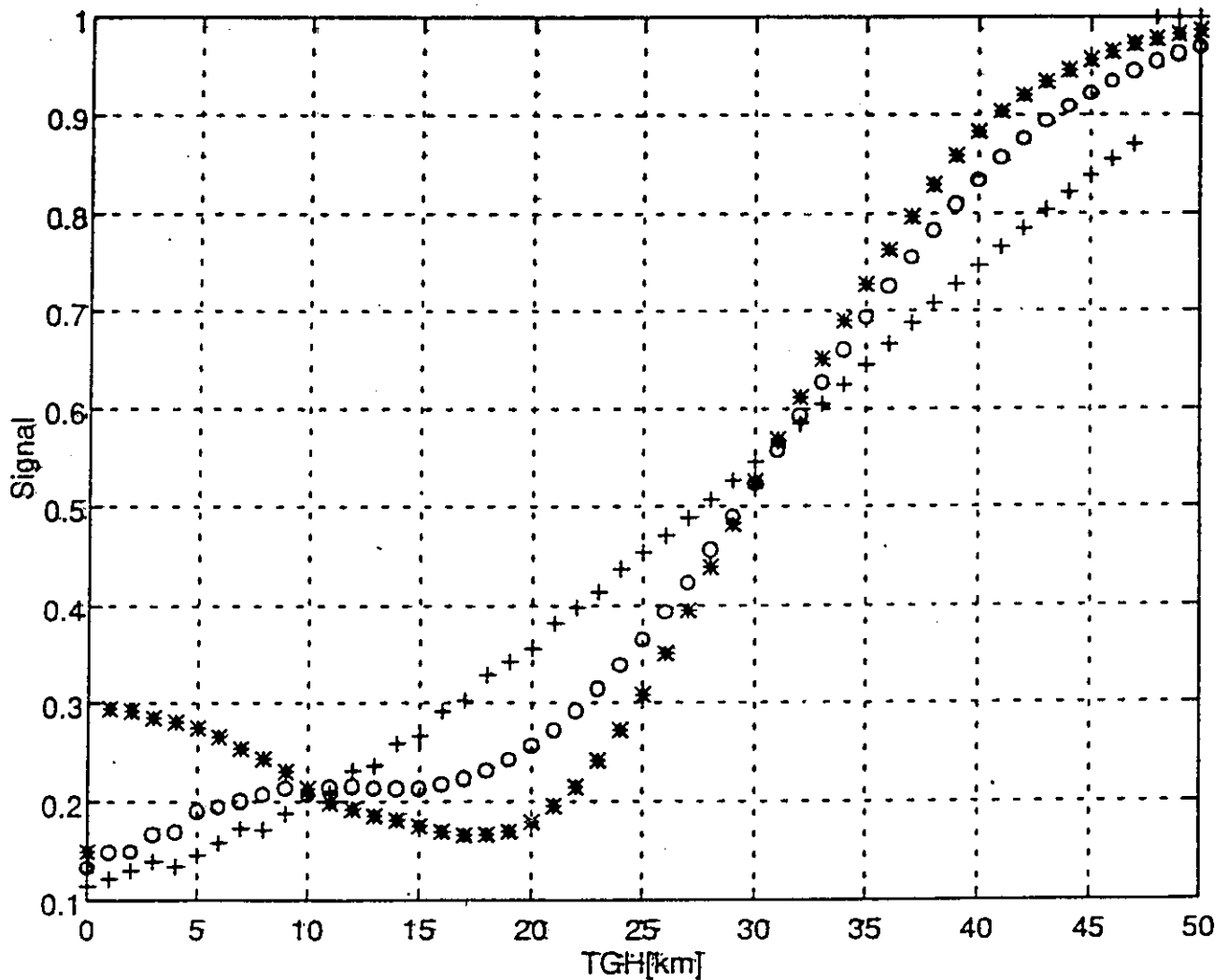
nr = 2352 , date is 10-oct-1992 16:37:09.9800282, type = SNAPSHOT
 channel : 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , 10 , 11 , 12 .
 Gain : 4.7, 5.6, 4.7, 4.0, 4.0, 13.3, 9.4, 13.3, 13.3, 5.6, 13.3
 Int time : 0.142, 0.142, 0.142, 0.142, 0.142, 0.142, 0.142, 0.142, 0.142, 0.142, 0.142



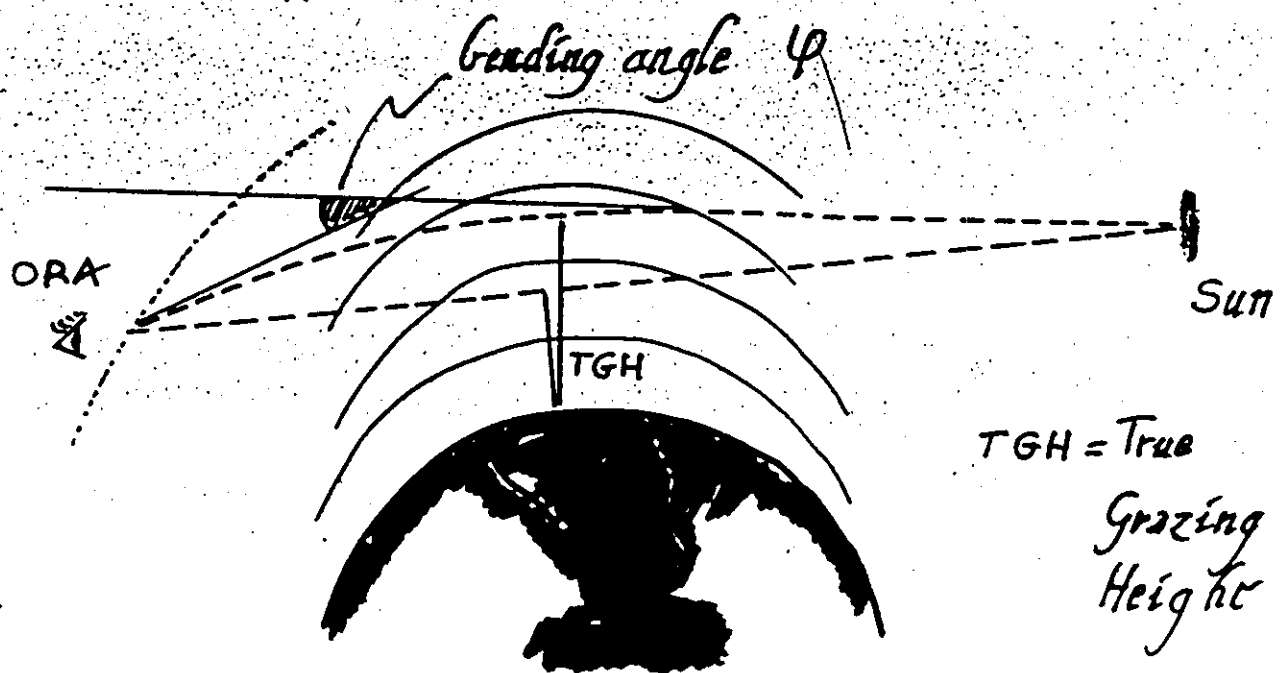
$$S_{\lambda}(k) = \int_{\Delta\Omega} W_{\lambda}(\Omega) e^{-\int_{s_1}^{s_2} \beta_{\lambda}(k(\Omega)) ds} d\Omega$$

$$\beta_{\lambda} = \beta_{\lambda}^{\text{Rayleigh}} + \beta_{\lambda}^{\text{Aerosol}} + \beta_{\lambda}^{\text{O}_3} + \dots$$

* punctual Sun O: true Sun +: double Sun



OPTICAL SEGMENTS



The optical path is the solution of

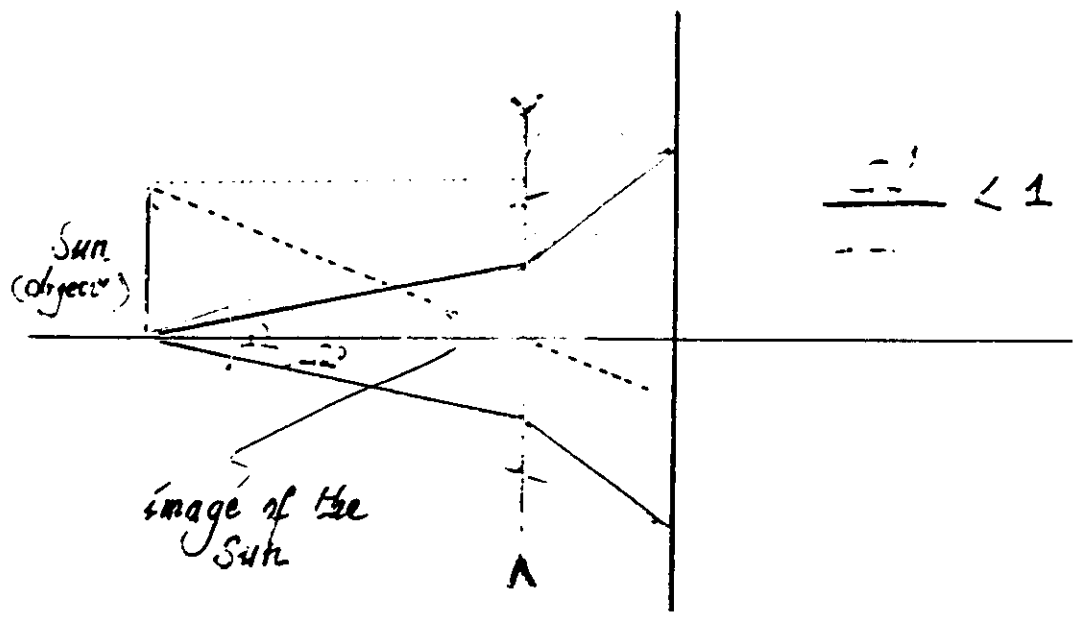
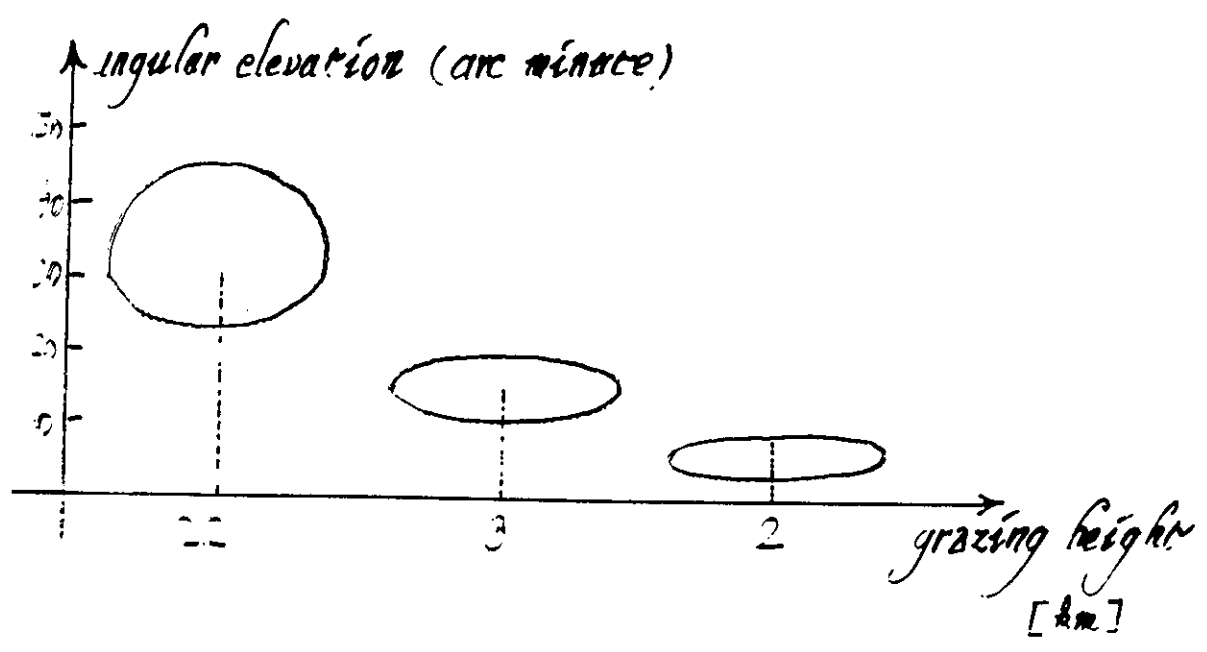
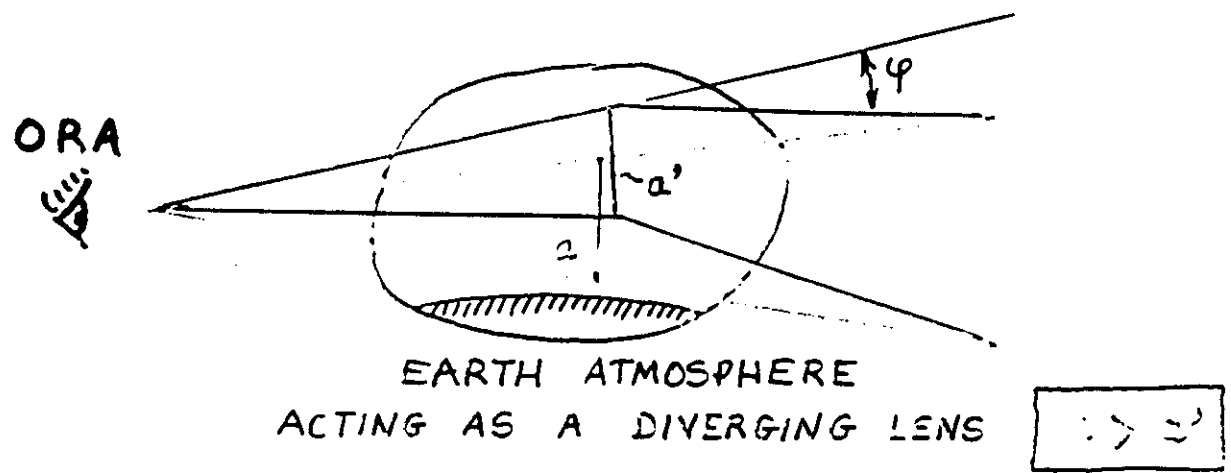
$$\frac{dr}{d\theta} = \pm \frac{r}{r_t n_t} \left[n(r)^2 r^2 - n_t^2 r_t^2 \right]^{\frac{1}{2}}$$

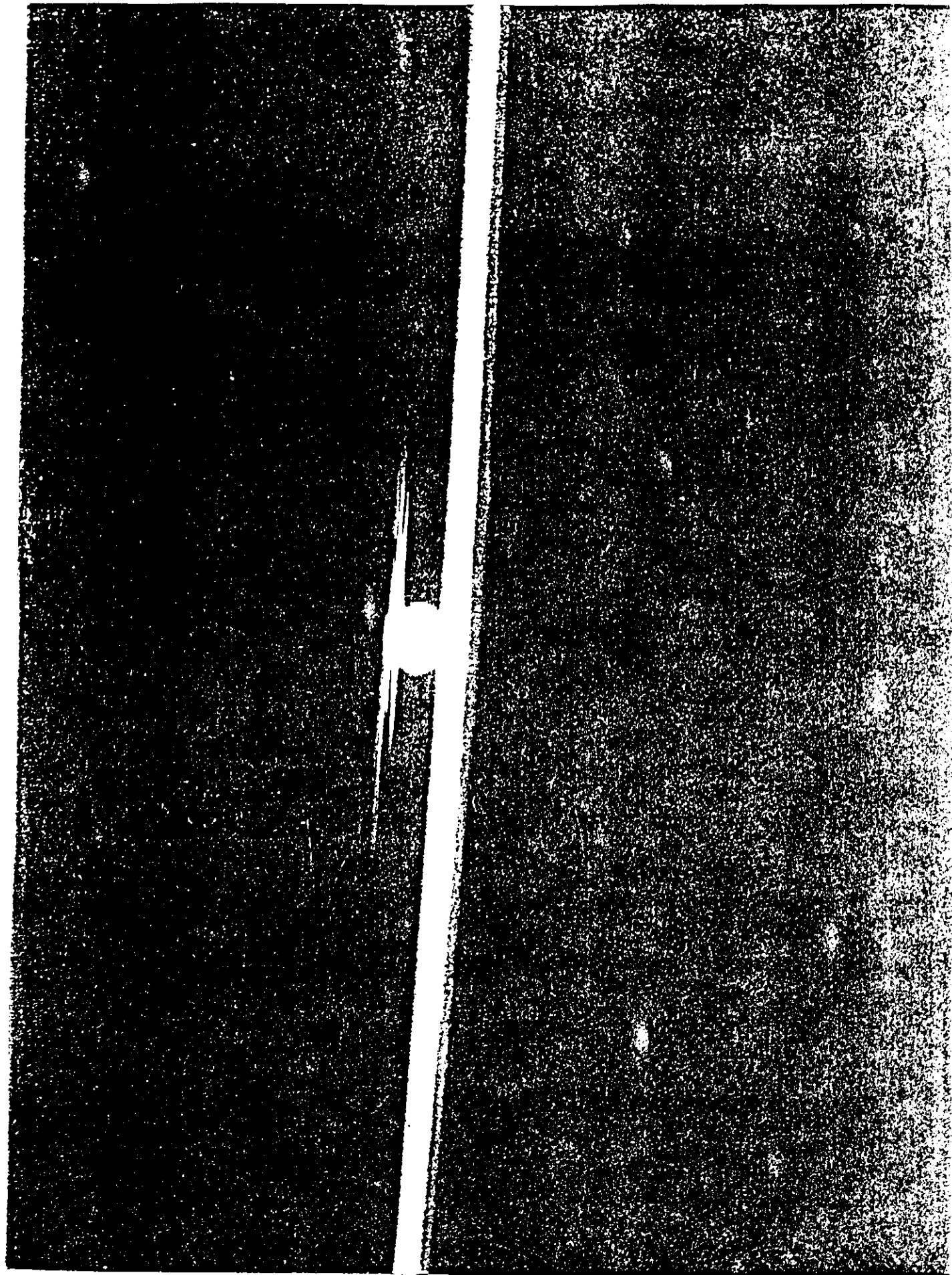
(Two points $\{r_{ORA}, \theta_{ORA}\}, \{r_{SUN}, \theta_{SUN}\}$)

⇒ Shooting method until convergence of

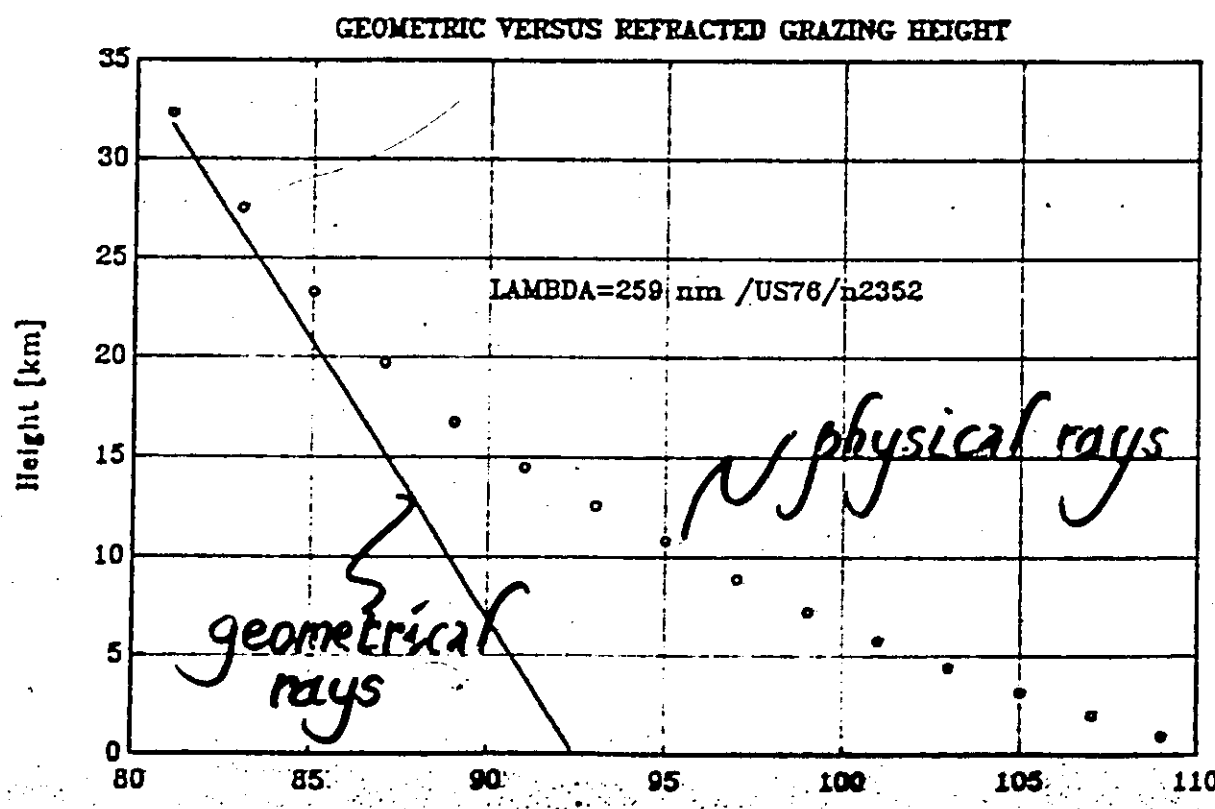
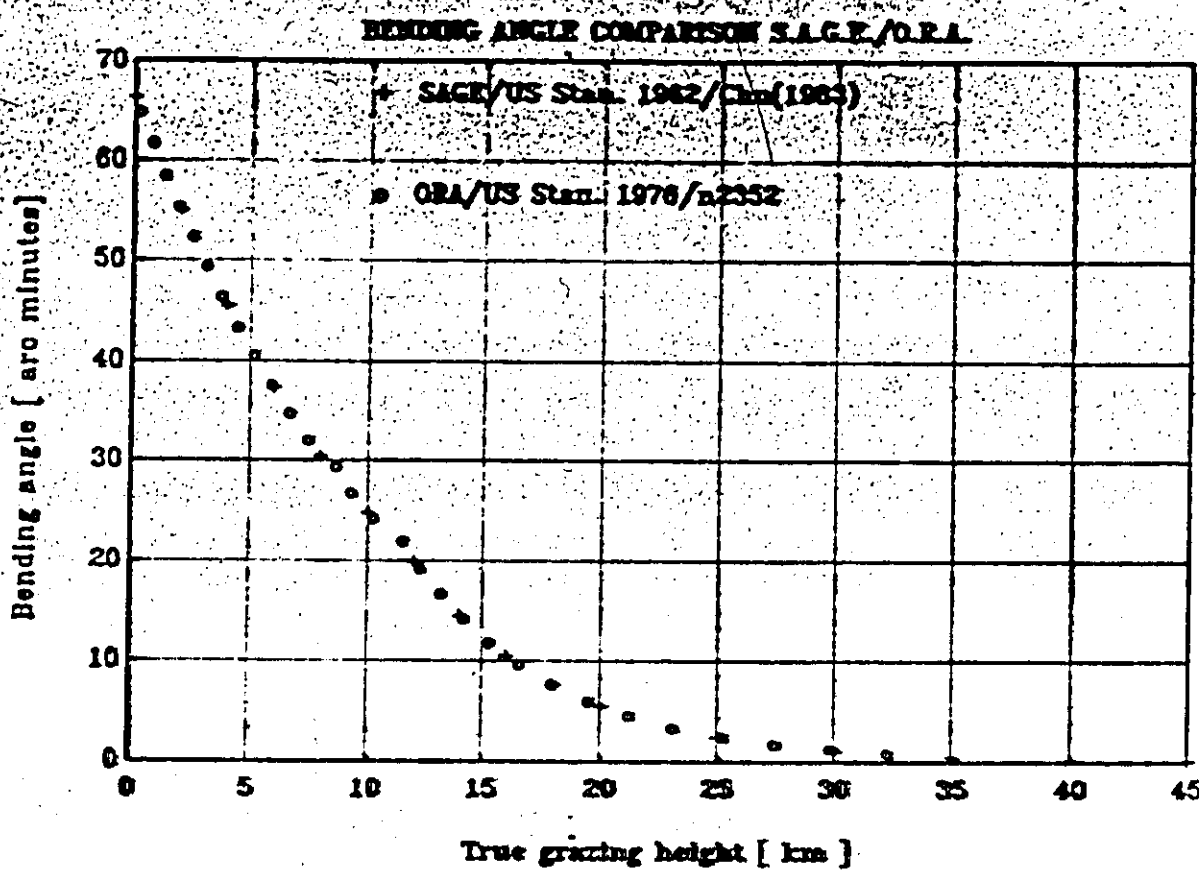
$$\begin{cases} TGH \\ \psi \\ S \end{cases}$$

Differential Refraction Effect



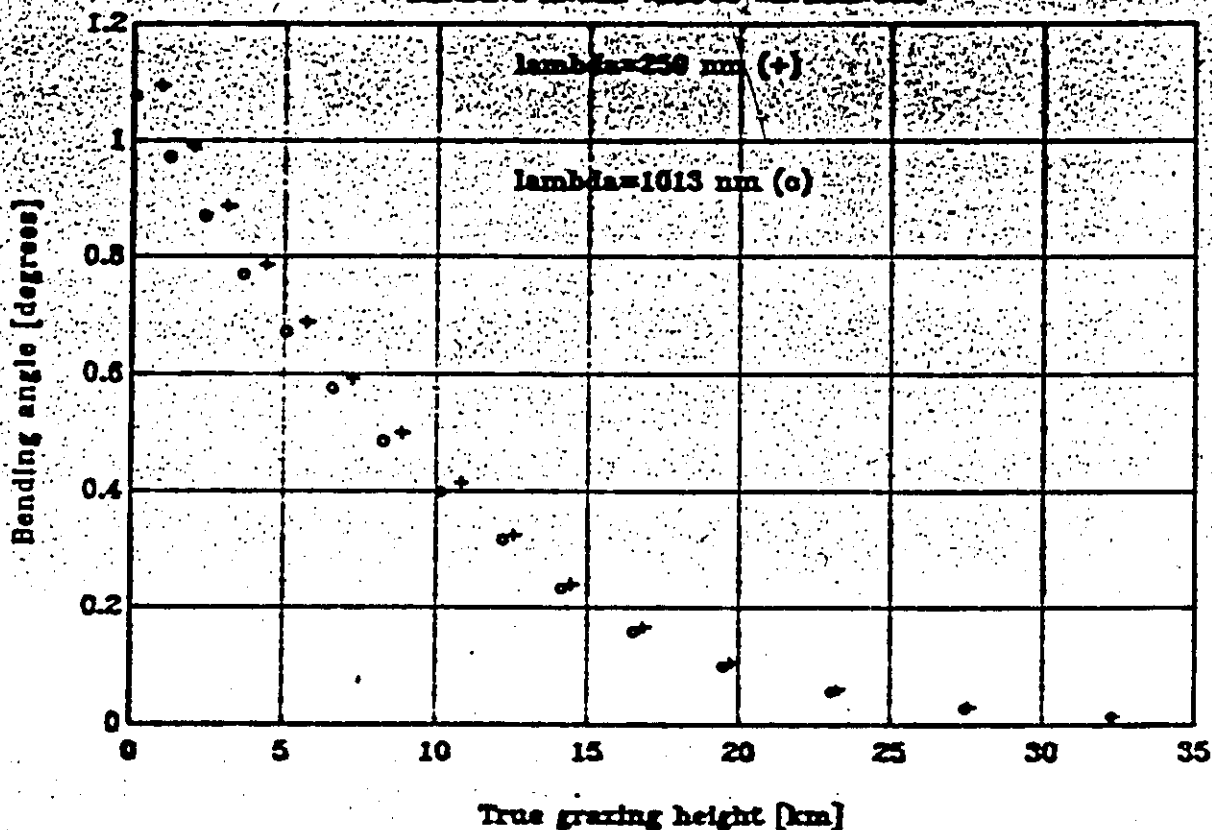


Effects of refraction

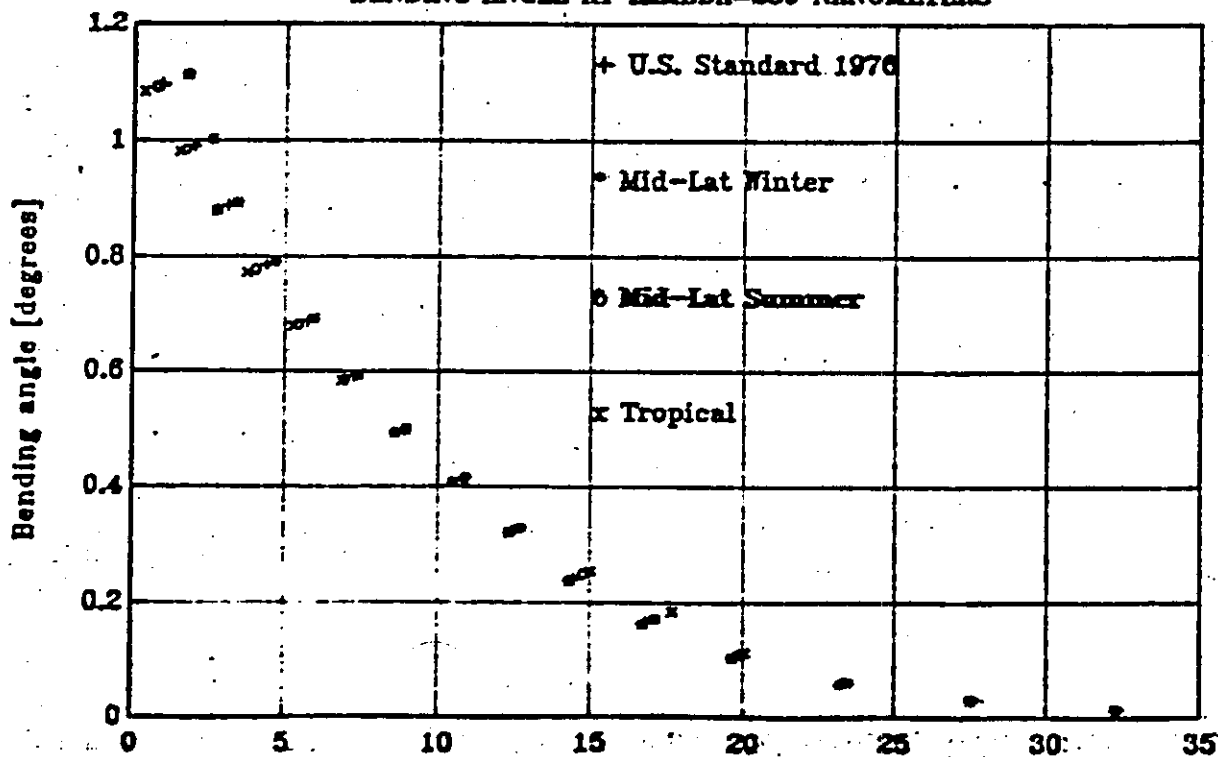


Wavelength and atmospheric dependencies

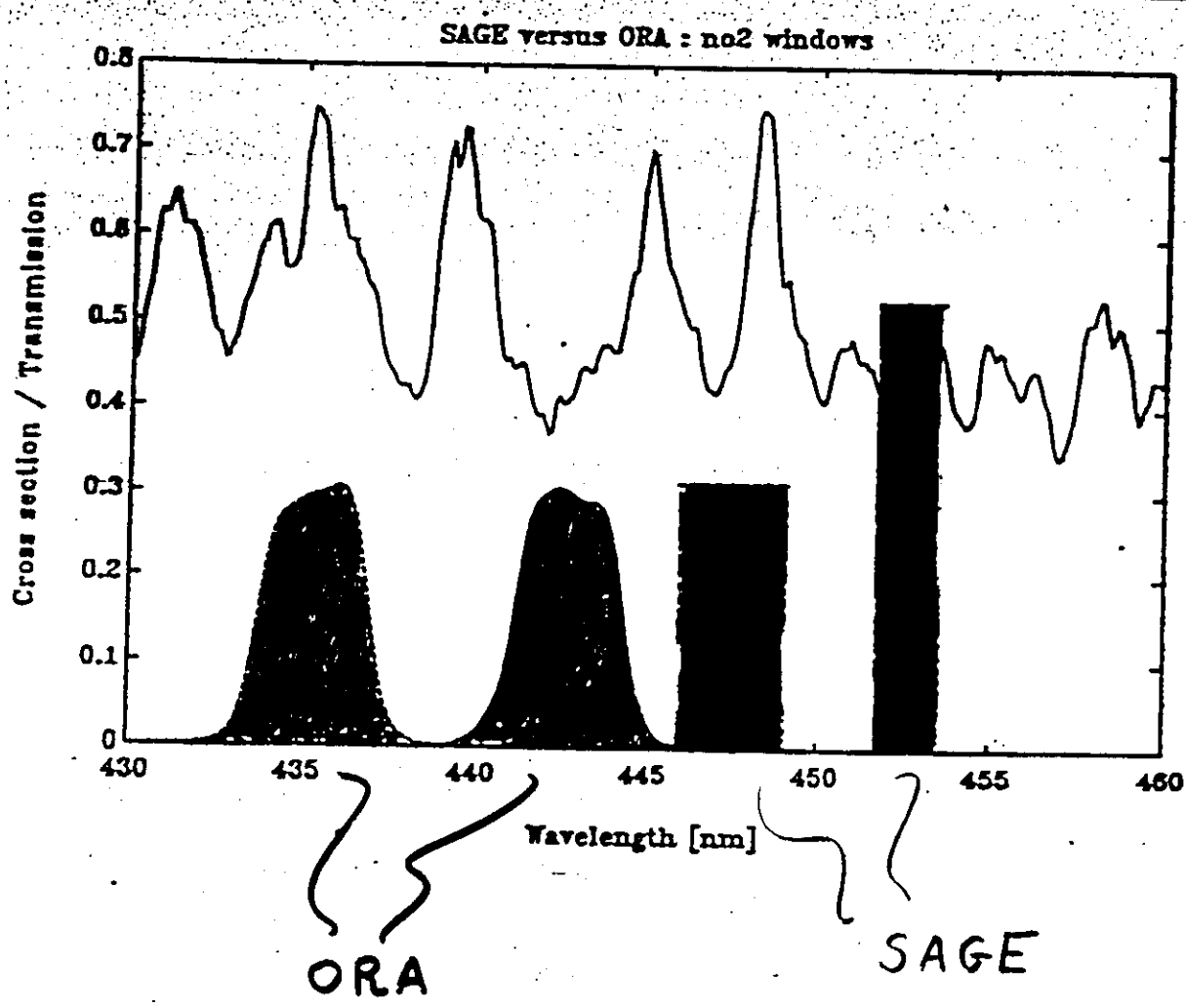
BENDING ANGLE VERSUS WAVELENGTH



BENDING ANGLE AT LAMBDA=259 NANOMETERS



NO₂ windows



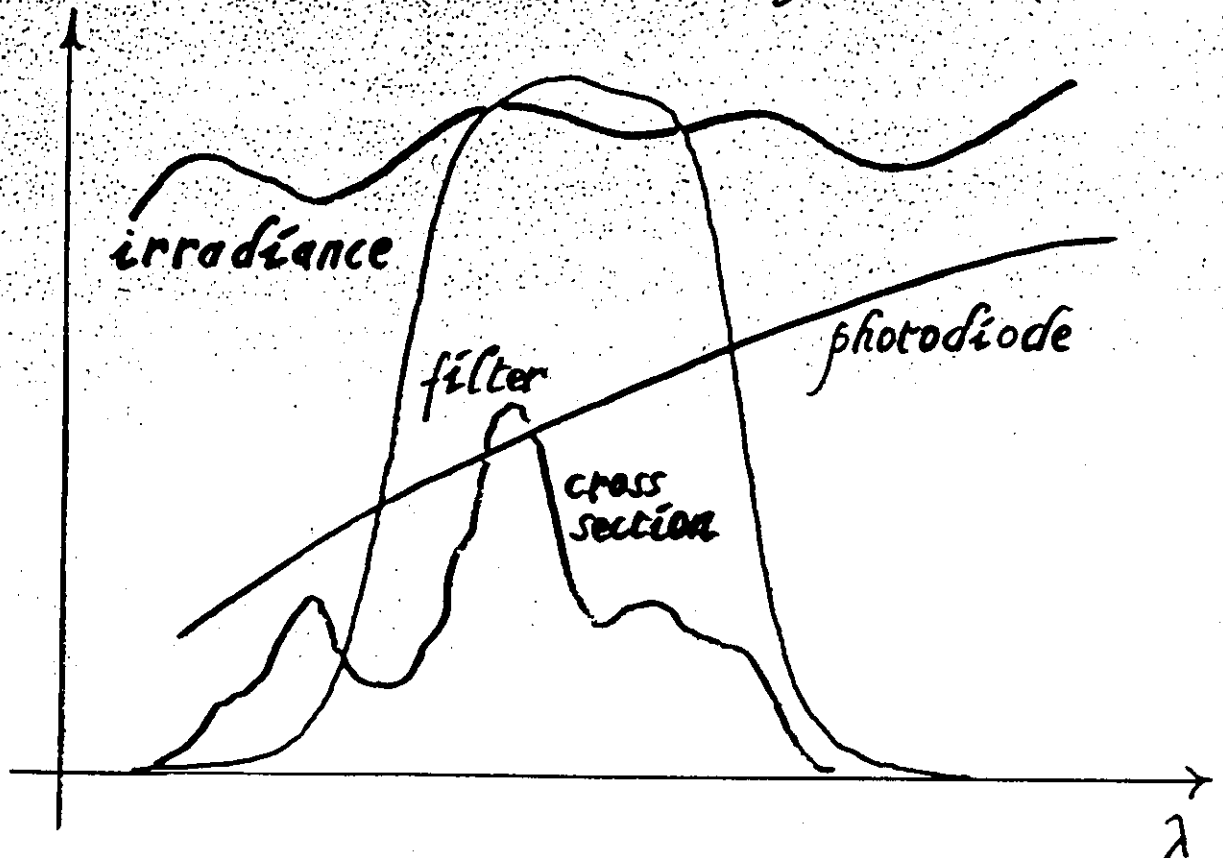
$$\frac{15}{91} = 1.35$$

$$\frac{15}{91} = 1.18$$

Total cross section convolutions

Define $\bar{\sigma}$ as: $\exp(-\bar{\sigma}p) = \int_{\lambda_1}^{\lambda_2} F(\lambda) \exp(-\sigma(\lambda)p) d\lambda$

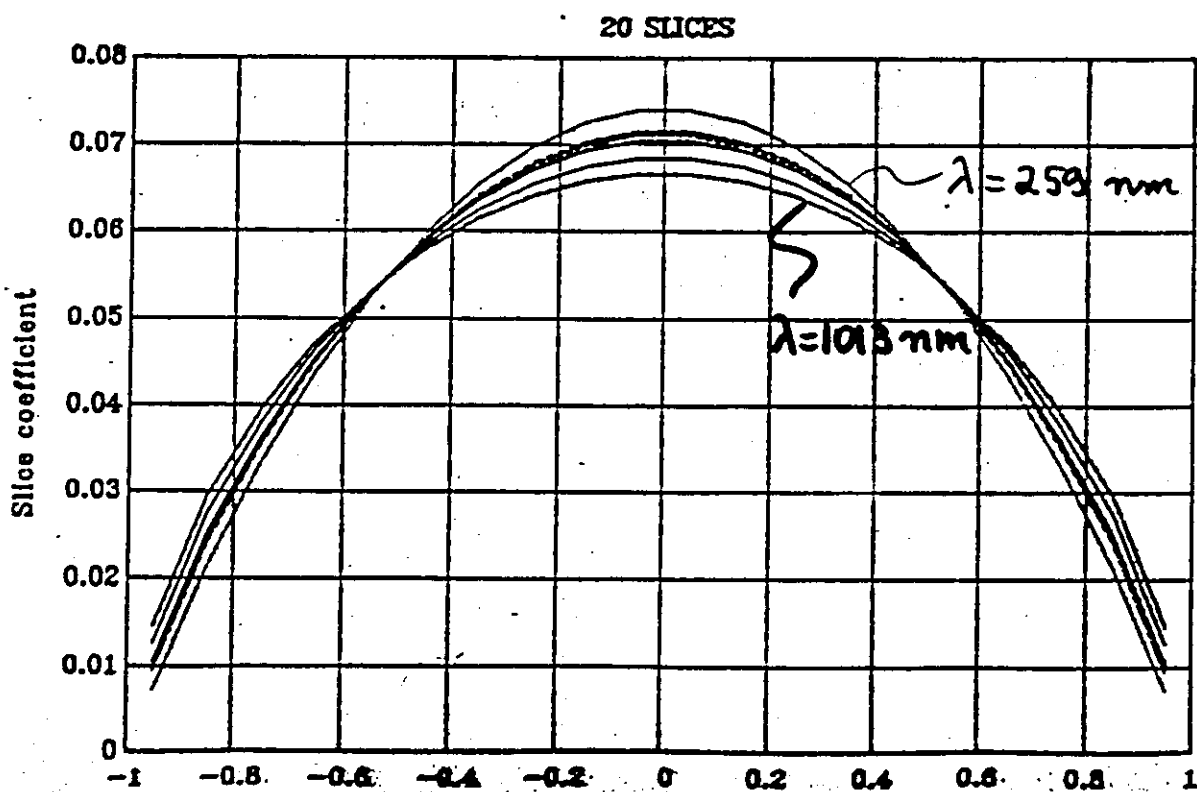
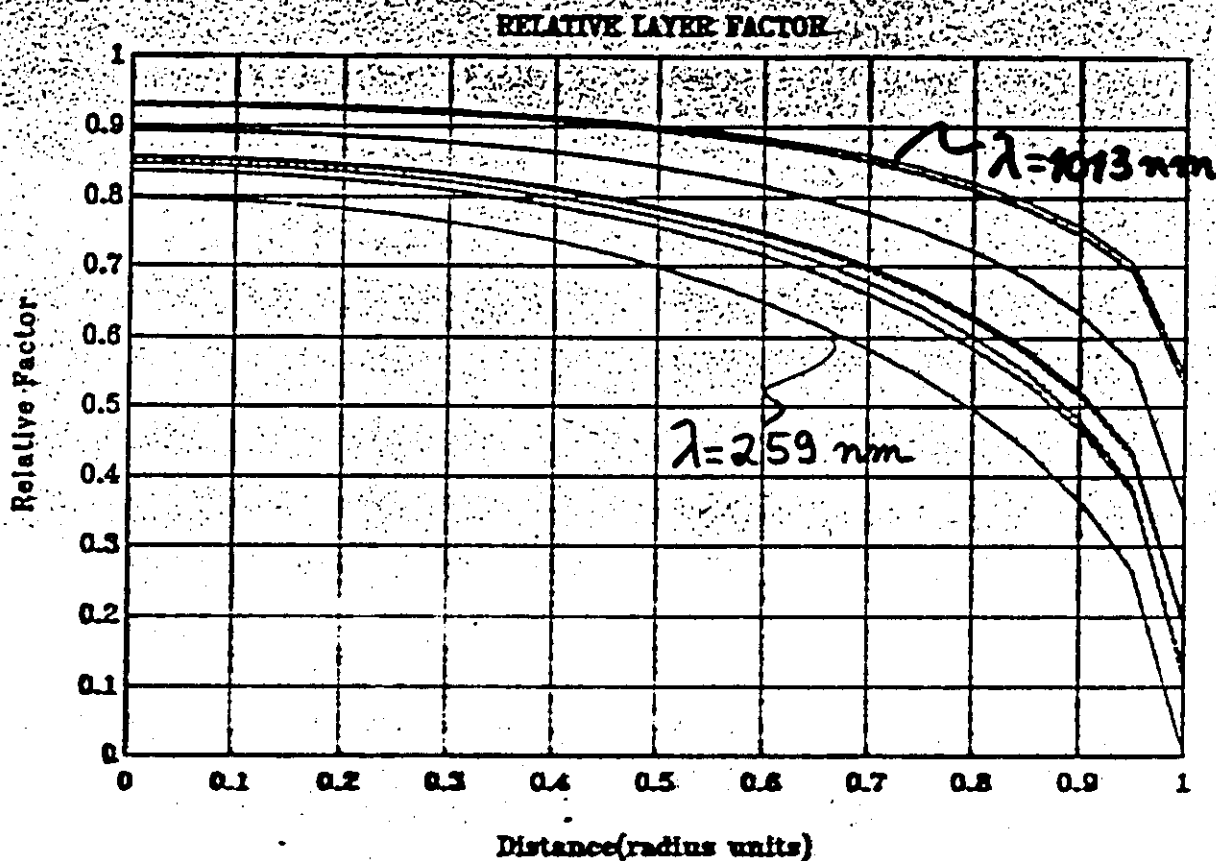
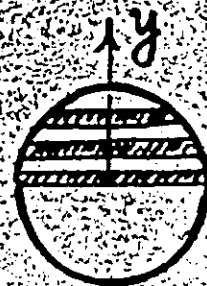
$$F(\lambda) = f(\lambda) \cdot \text{att}(\lambda) \cdot \text{photo}(\lambda)$$



$$\bar{\sigma} = \bar{\sigma}(p) = -\frac{1}{p} \ln \int_{\lambda_1}^{\lambda_2} F(\lambda) \exp(-\sigma(\lambda)p) d\lambda$$

$$\text{with: } \int_{\lambda_1}^{\lambda_2} F(\lambda) d\lambda = 1$$

SOLAR LIMB DARKENING



THE INVERSION ALGORITHM

$$S_{\lambda}(h) = \int_{\Delta\Omega} W_{\lambda}(\Omega) e^{-\int_{s_1}^{s_2} \beta_{\lambda}(h(\Omega)) ds}$$

$$\beta_{\lambda} = \beta_{\lambda}^{\text{Rayleigh}} + \beta_{\lambda}^{\text{Aerosol}} + \beta_{\lambda}^{O_2} + \dots$$

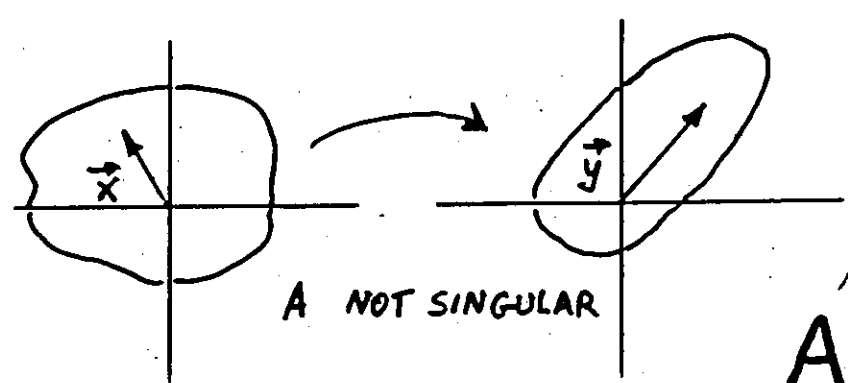
$\forall$ occultation: set of about 700 measurements $S_{\lambda}(h_i)$
 $\rightarrow$ choose 70 measurements $\sim$ number of atmospheric layers

The inversion problem is basically an ill-posed problem

no hope to retrieve more informations about $\beta(h)$ than the number of measurements. $\Rightarrow$ 'Spurious' fine structures.

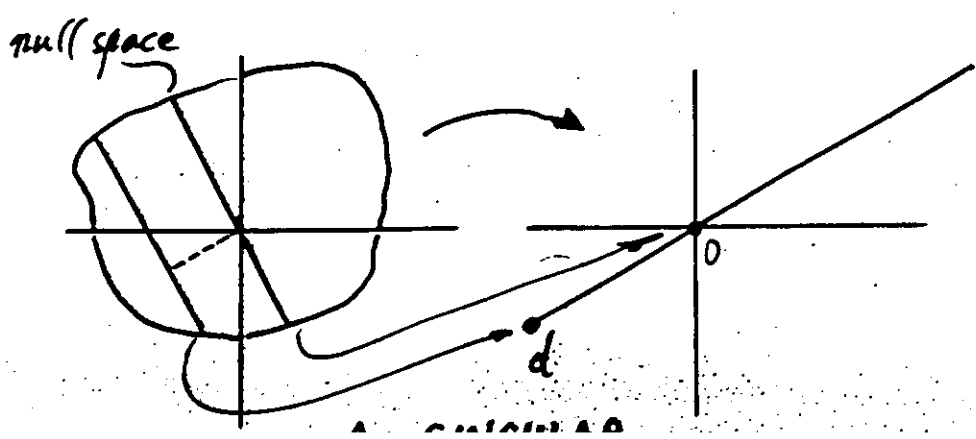
integration destroys information! (redundance does not help you...)

In the linear case, we have theorems...



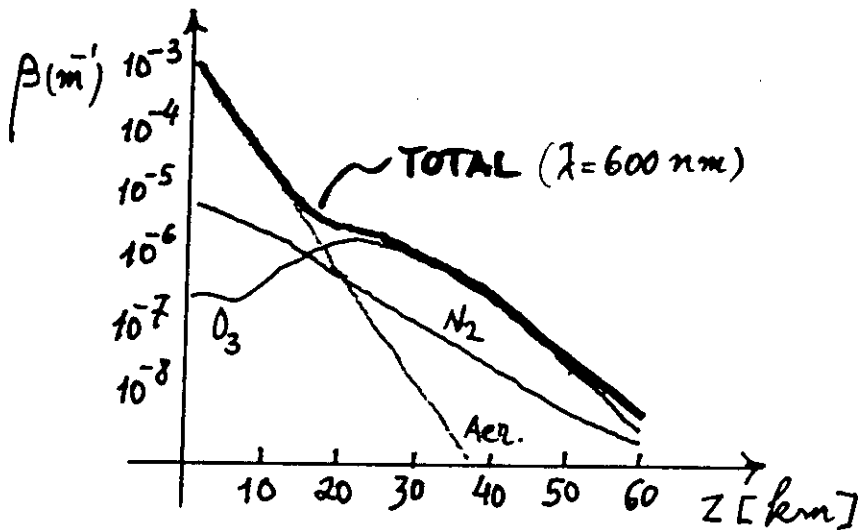
"measure"
 "something"
 "result"

$$A \vec{x} = \vec{y}$$



NATURAL ORTHOGONAL PROFILE EXPANSION

- 1) the information content of the best retrieved profile is not infinite!
- 2) iterative adjustment of the solution should be done globally
- 3) to preserve the constituent separation, the algorithm has to focus on small scale details

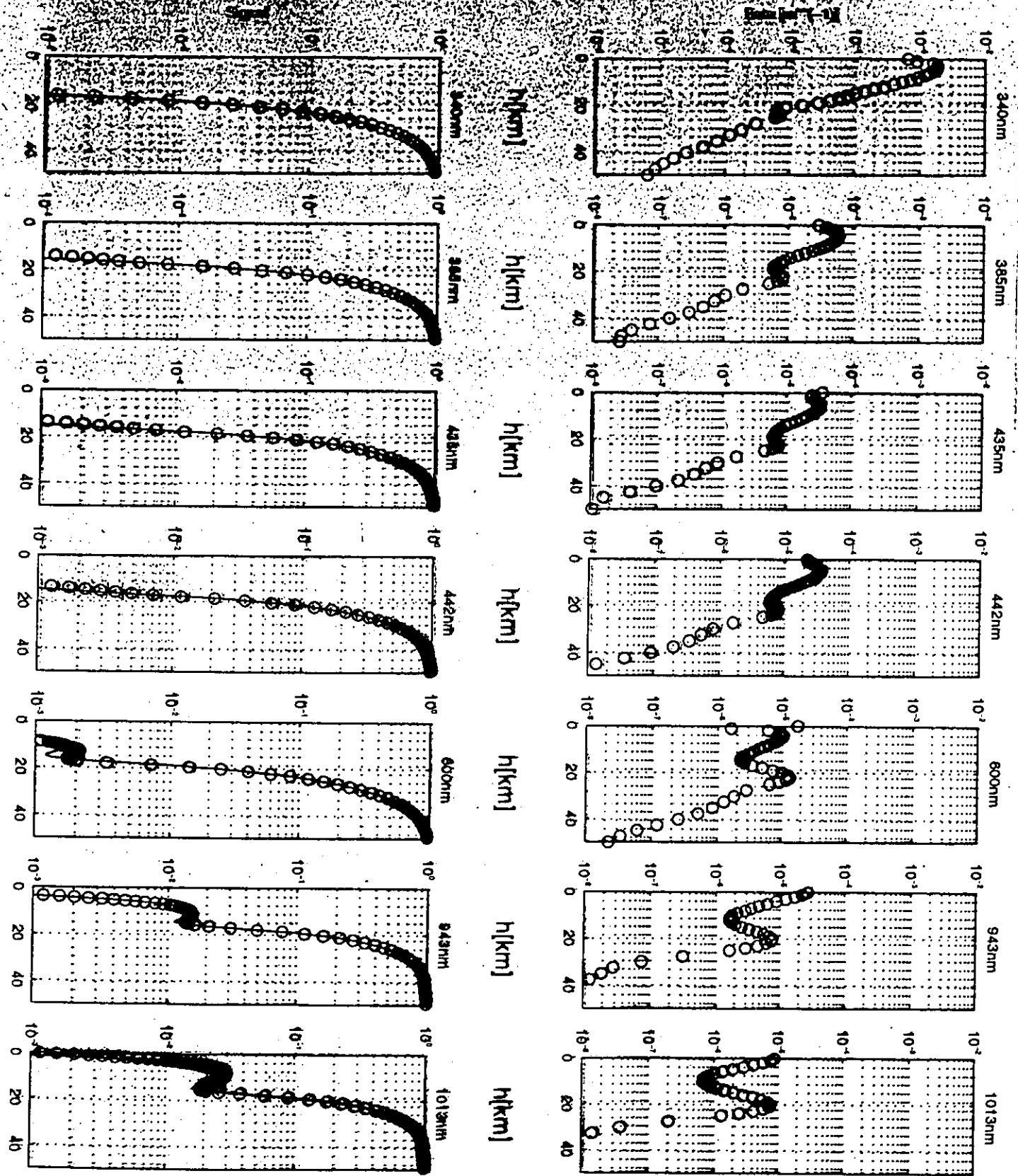


$$\beta(z) = \beta_0(z) \cdot \sum_{k=0}^N \alpha_k |\tilde{p}_k(z)\rangle$$

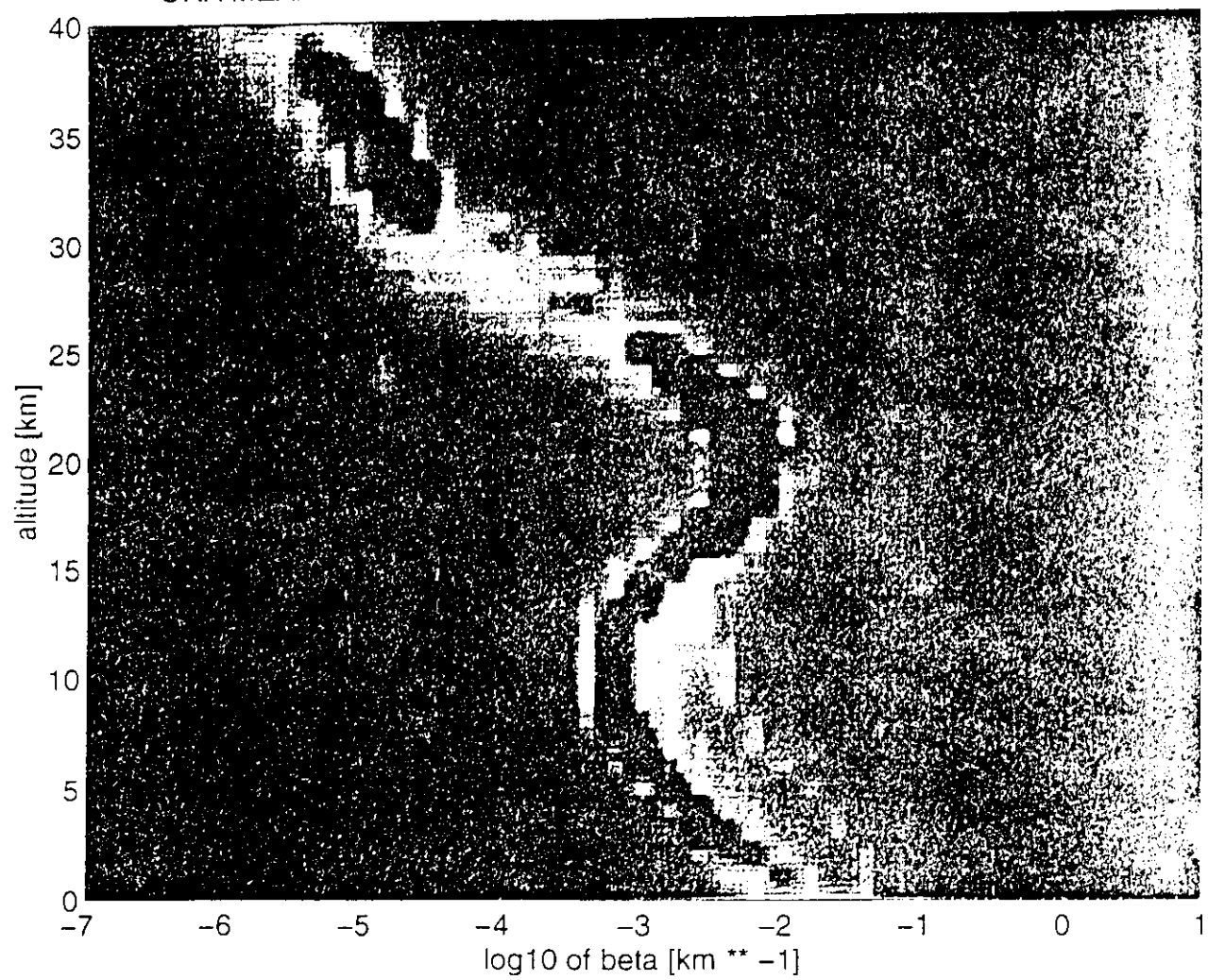
$$\langle \tilde{p}_k | \tilde{p}_l \rangle = \delta_{kl}$$

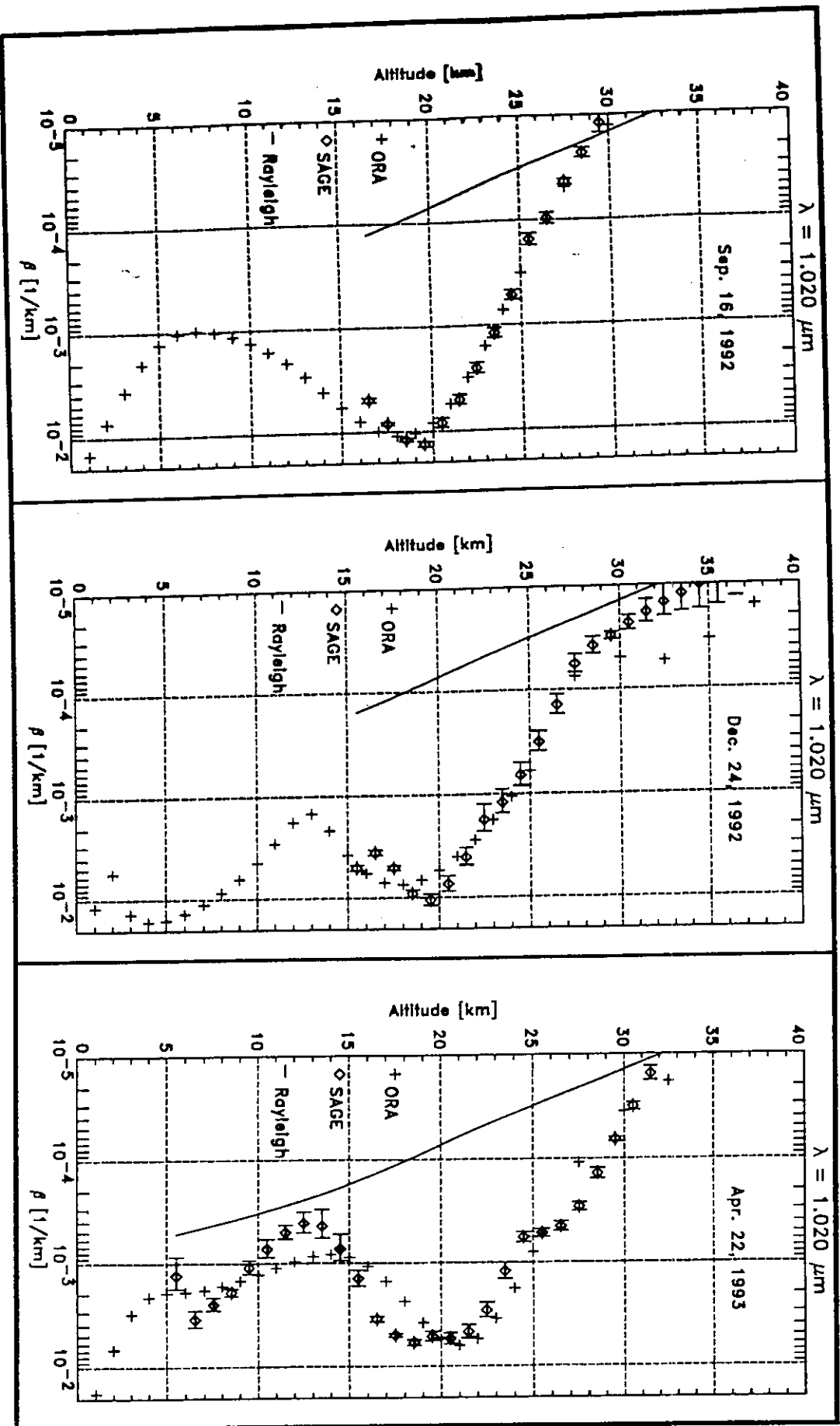
$\Rightarrow$ using $\beta_0(z)$ as the "measure" for the scalar product construct an orthogonal set of polynomials by STIELTJES procedure. Then solve by standard non-linear algorithm

$$\left\{ \chi^2 = \sum_i \left(\text{Signal}(i) - \text{Theory}(i; \alpha_0, \alpha_1, \dots, \alpha_N) \right)^2 \right\}_{\min}$$

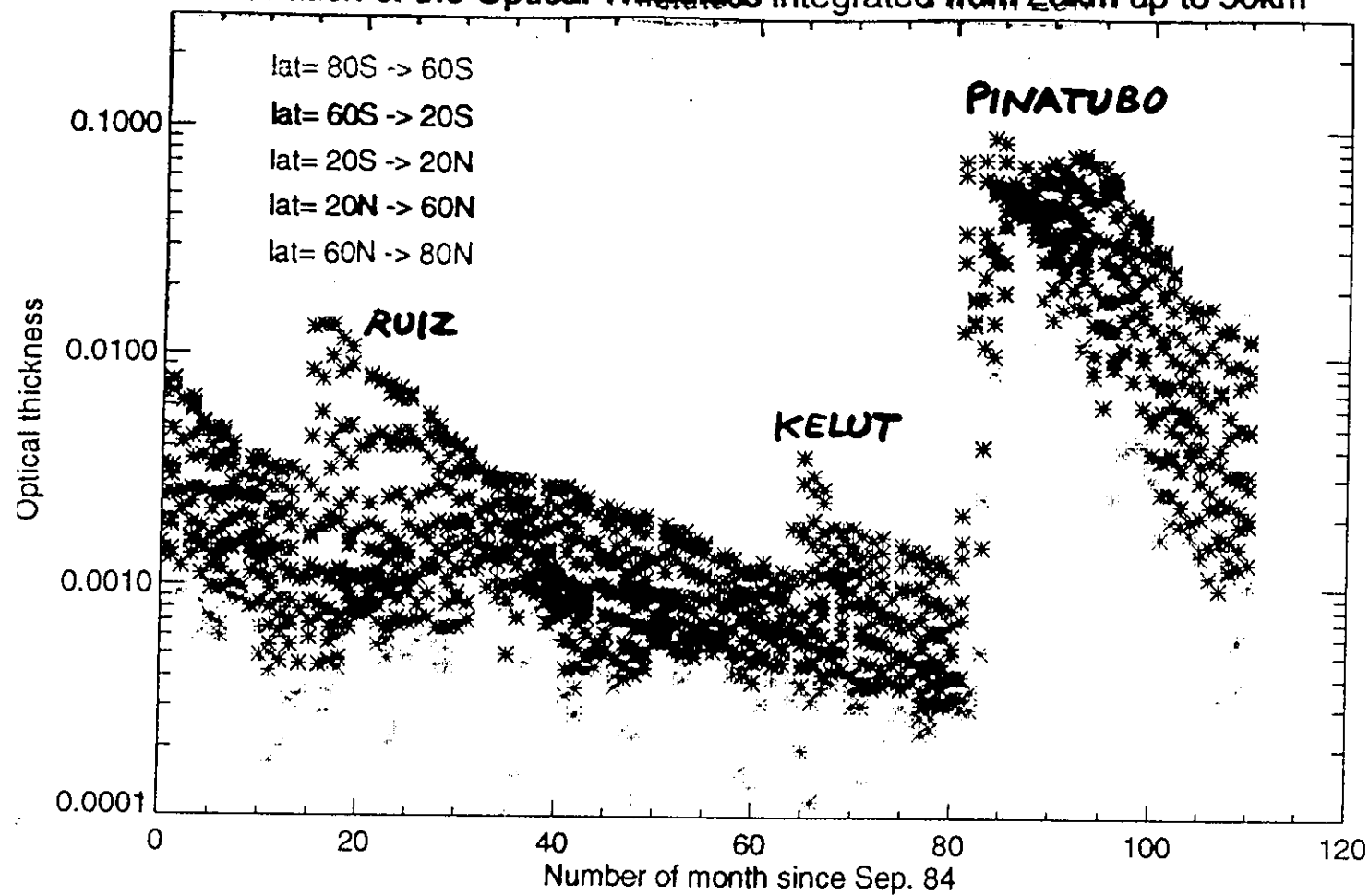


ORA MEAN TOTAL ABSORPTION PROFILE at 1013 nm (aug92-may93)

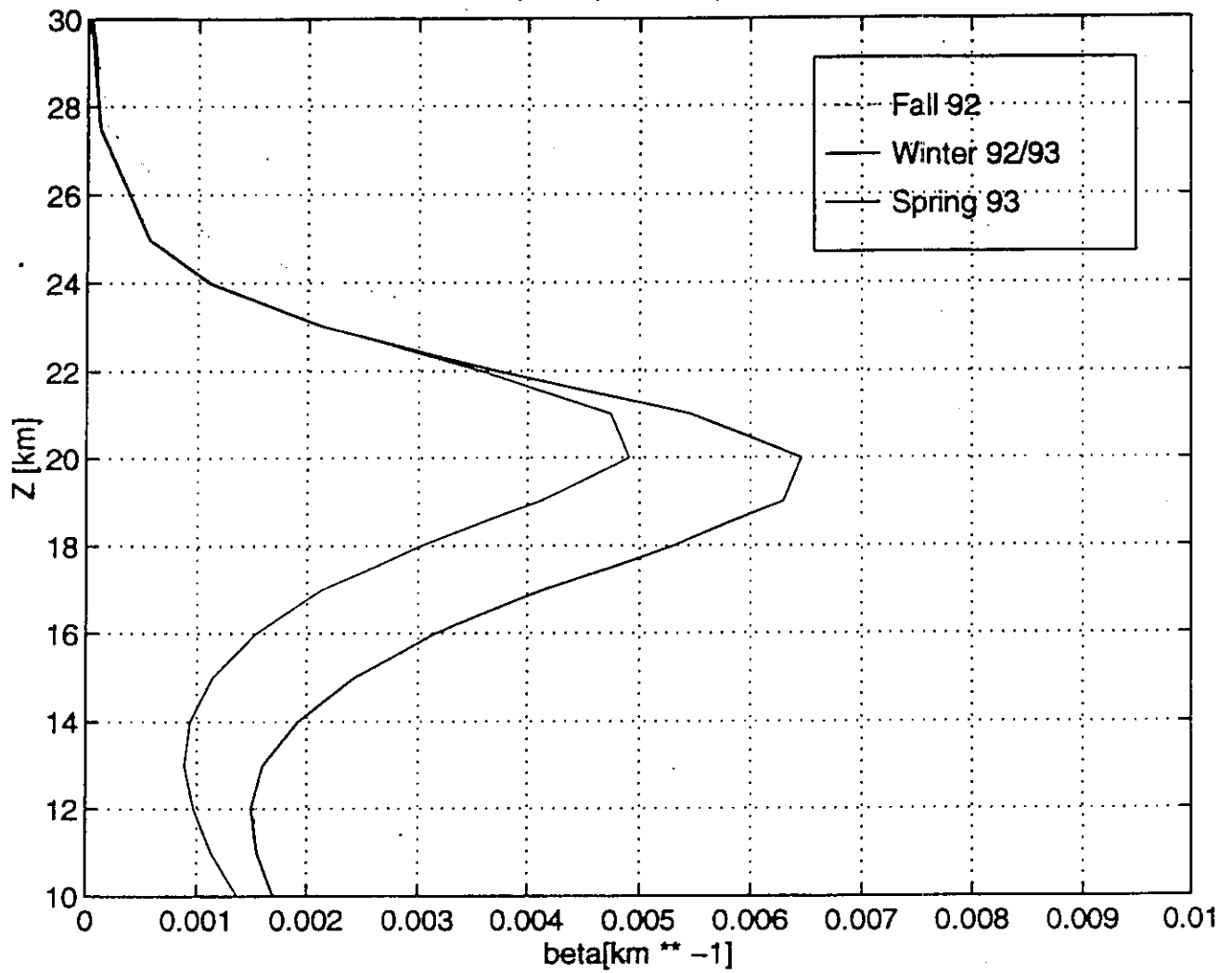


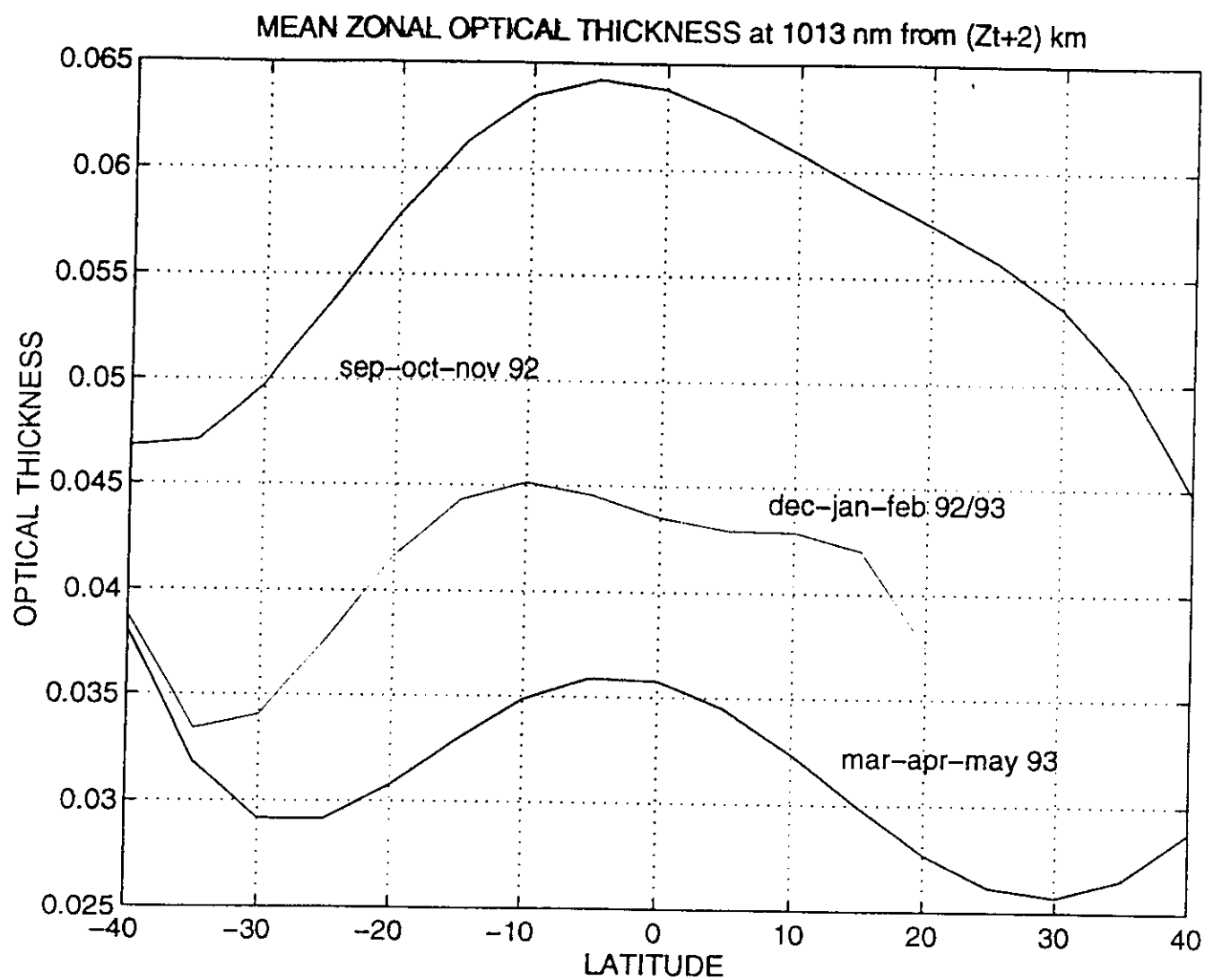


Variation of the Optical Thickness integrated from 20km up to 50km



ora mean total absorption (1013 nm) profiles / time evolution



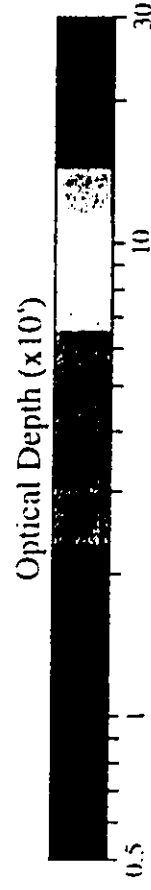


from Banded structures in stratospheric
aerosol distributions"

Trepte, Thomason & Kent

GRL, 21, p 2397 [Nov. 1, 1994]

SAGE, Aug 15 - Sep 24 (1993)



Main problem : Ill-conditioning (Ill-posedness)

Example :

$$\begin{bmatrix} 1 & 2 \\ 1 & 2.01 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 3.01 \end{bmatrix}$$

Exact solution :

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Now : introducing 'measurement error' (5 %)

$$\begin{bmatrix} 1 & 2 \\ 1 & 2.01 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.15 \\ 2.86 \end{bmatrix}$$

Solution :

$$\begin{bmatrix} 61.15 \\ -29 \end{bmatrix}$$

(4753 % error on the solution !)

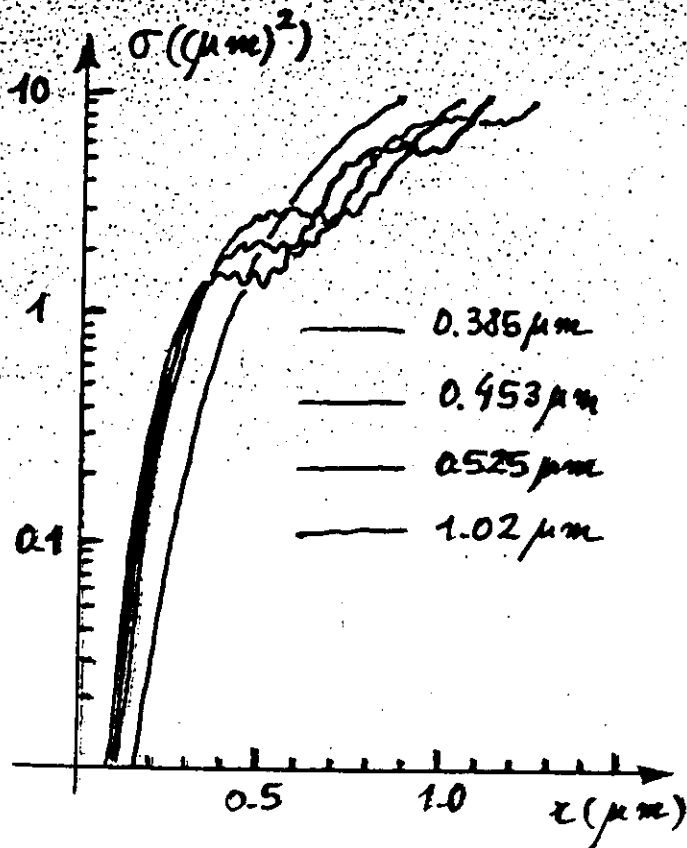
CONCLUSION :

enormous error magnification !

CAUSE :

**dependence between the columns and rows
of the linear system.**

Aerosols...



$$\delta = \int \sigma n ds$$

but

$$\sigma = \int_0^{\infty} \sigma(r) N(r) dr$$

Mie
cross section

Size distribution
function

REPLACE BY

$$\delta = \int \beta_{\lambda} ds$$

extinction
coefficient

From *ab initio* Mie calculations and observed unimodal size distribution, a reasonable wavelength dependence is

$$\beta_{\lambda} \sim \exp(c_0 + c_1 \lambda + c_2 \lambda^2)$$

$$\text{or } c_0 + c_1 \lambda + c_2 \lambda^2$$

$\Rightarrow$ try to determine c_0, c_1, c_2 by correlative measurements at $\lambda, \lambda_1, \lambda_2, \dots, \lambda_n$!

SPECTRAL INVERSION: Matrix equation

First: removal of Rayleigh extinction, using UKMO meteorological data.

Assumption: quadratic wavelength dependence for aerosols.

$$\beta_{\lambda}^A = c_0 + c_1(\lambda - \lambda_{1013}) + c_2(\lambda - \lambda_{1013})^2$$

$$\begin{bmatrix} \beta_{340}^{\text{NO}_2} & \beta_{340}^{\text{O}_3} & 1 & \Delta\lambda_{340} & (\Delta\lambda_{340})^2 \\ \beta_{385}^{\text{NO}_2} & \beta_{385}^{\text{O}_3} & 1 & \Delta\lambda_{385} & (\Delta\lambda_{385})^2 \\ \beta_{435}^{\text{NO}_2} & \beta_{435}^{\text{O}_3} & 1 & \Delta\lambda_{435} & (\Delta\lambda_{435})^2 \\ \beta_{442}^{\text{NO}_2} & \beta_{442}^{\text{O}_3} & 1 & \Delta\lambda_{442} & (\Delta\lambda_{442})^2 \\ \beta_{600}^{\text{NO}_2} & \beta_{600}^{\text{O}_3} & 1 & \Delta\lambda_{600} & (\Delta\lambda_{600})^2 \\ \beta_{1013}^{\text{NO}_2} & \beta_{1013}^{\text{O}_3} & 1 & \Delta\lambda_{1013} & (\Delta\lambda_{1013})^2 \end{bmatrix} \begin{bmatrix} x_{\text{NO}_2} \\ x_{\text{O}_3} \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \beta_{340} \\ \beta_{385} \\ \beta_{435} \\ \beta_{442} \\ \beta_{600} \\ \beta_{1013} \end{bmatrix}$$

or, for altitude z_i : $A_i \vec{x}_i = \vec{\beta}_i$

SPECTRAL INVERSION: Total Matrix

26 altitude levels

$$\begin{bmatrix} A_1 & 0 & \dots & 0 & \vec{s} \\ 0 & A_2 & & & \vec{s} \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & & \dots & A_{26} & \vec{s} \end{bmatrix} \cdot \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vdots \\ \vec{x}_{26} \\ x_s \end{bmatrix} = \begin{bmatrix} \vec{\beta}_1 \\ \vec{\beta}_2 \\ \vdots \\ \vec{\beta}_{26} \end{bmatrix}$$

with straylight $\vec{s} \sim \lambda^{-4}$

C_d : Measurement covariance matrix

C_v : A priori covariance matrix, with altitude smoothing for every constituent ($L \sim 5$ km):

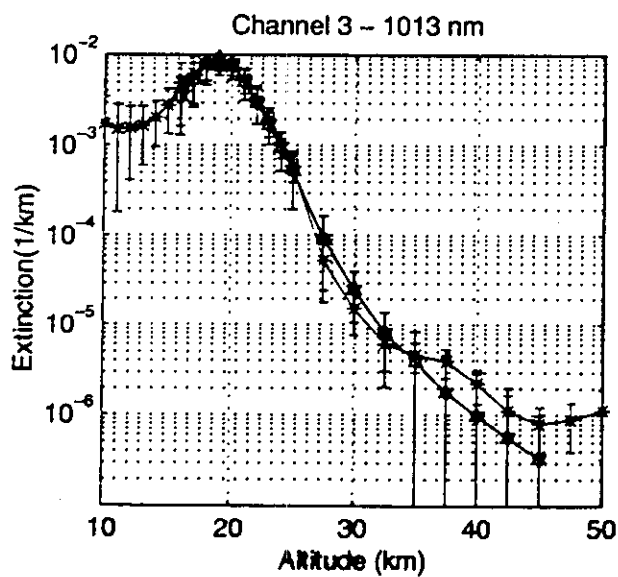
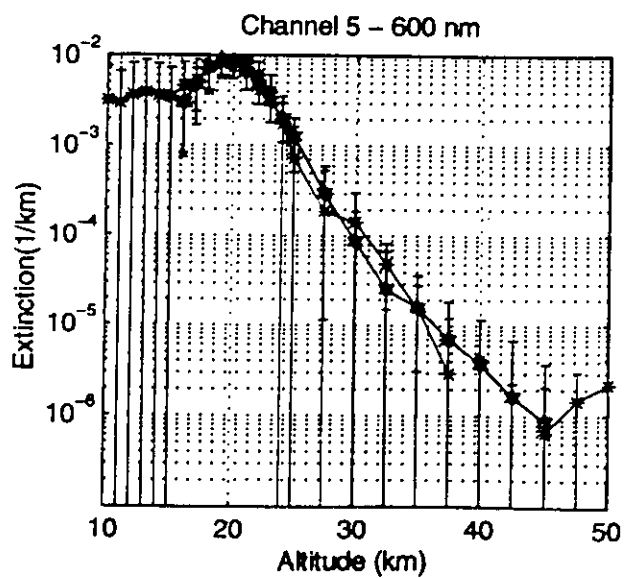
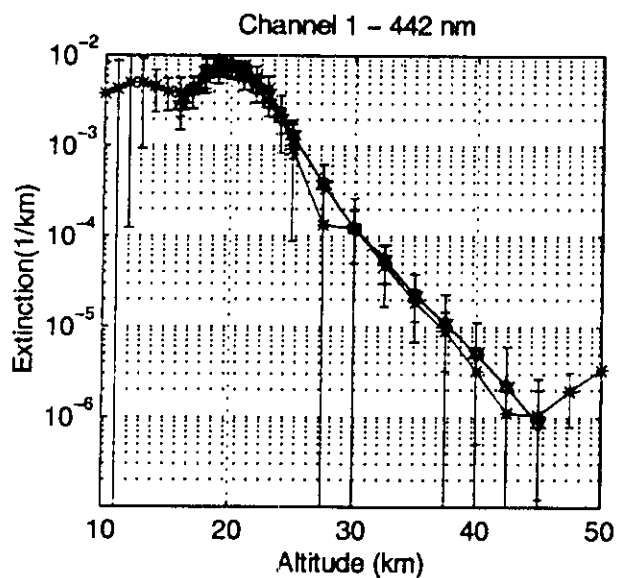
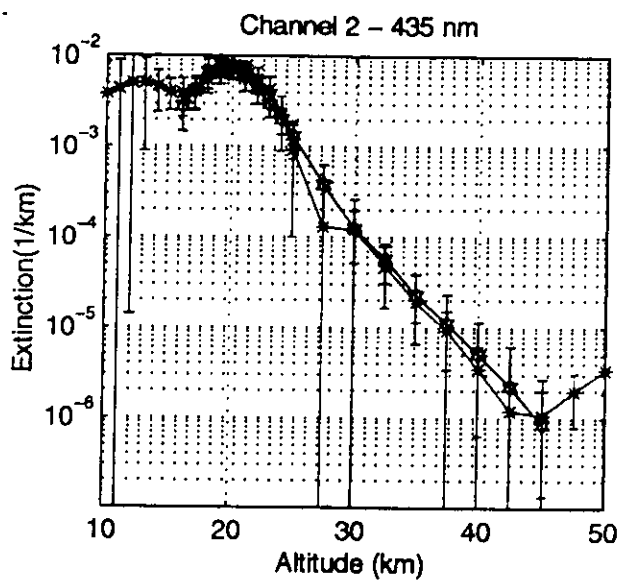
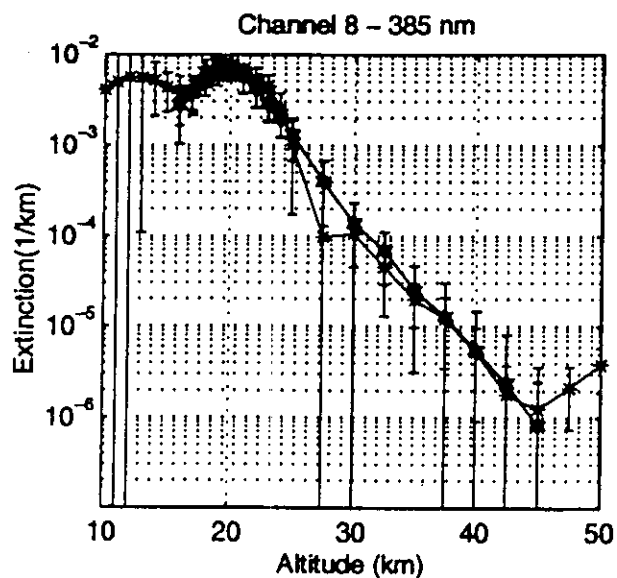
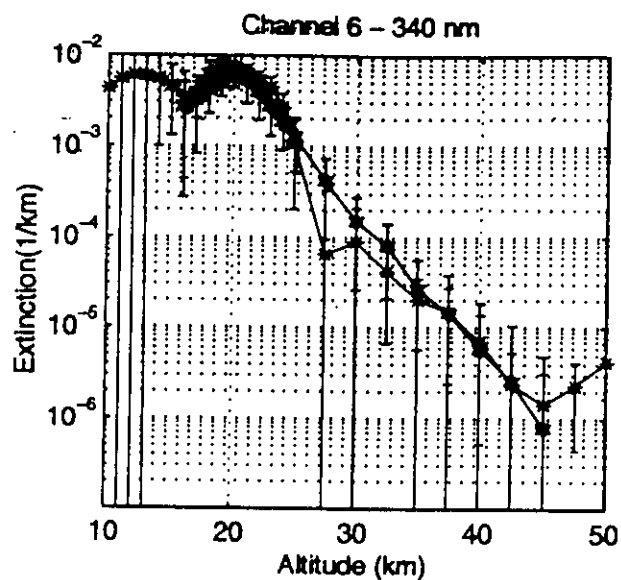
$$\sigma^2(z_i, z_j) = \sqrt{\sigma^2(z_i) \sigma^2(z_j)} \cdot \exp \left(- \left(\frac{z_i - z_j}{L} \right)^2 \right)$$

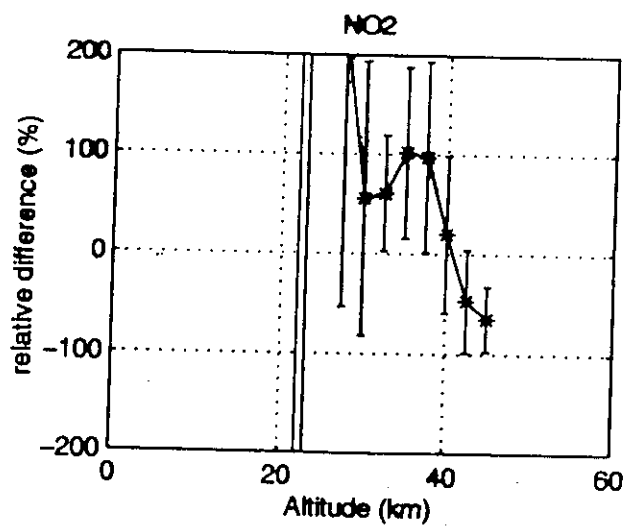
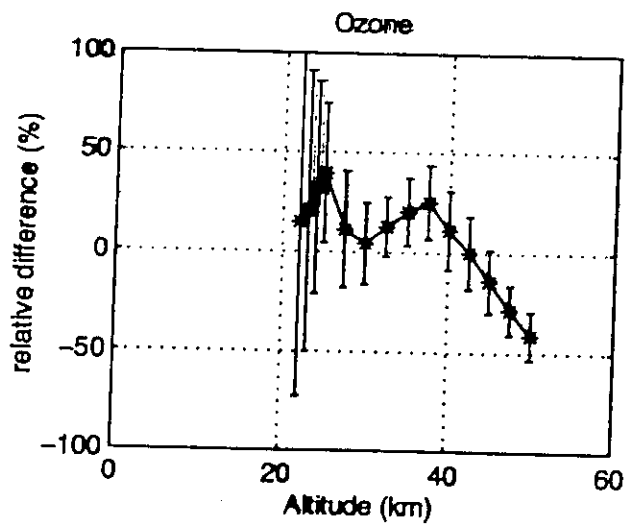
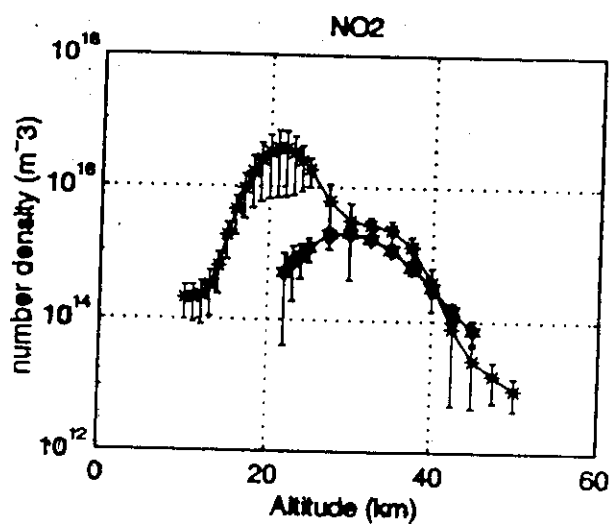
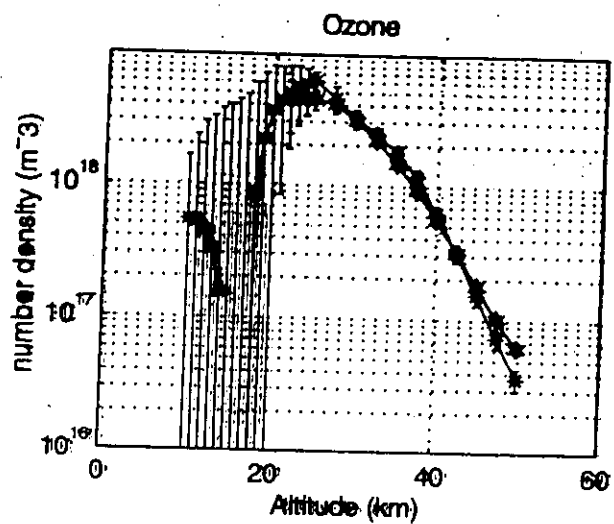
$\tilde{x}_t$: A priori solution (from climatology)

SPECTRAL INVERSION: General Least Squares

**Minimization of merit function
(assuming Gaussian noise):**

$$\begin{aligned} & \langle A_t \vec{x}_t - \vec{\beta}_t \mid C_d^{-1} \mid A_t \vec{x}_t - \vec{\beta}_t \rangle \\ & + \\ & \langle \vec{x}_t - \tilde{x}_t \mid C_v^{-1} \mid \vec{x}_t - \tilde{x}_t \rangle \end{aligned}$$

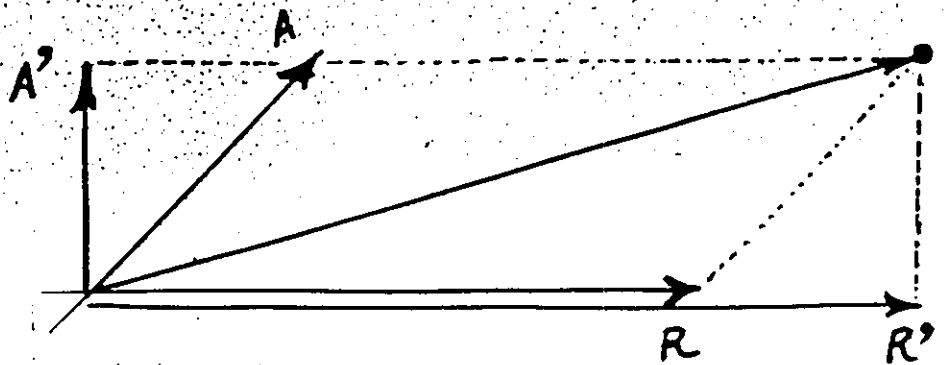




Kernels, orthogonality and more...

1. Rayleigh against aerosols.

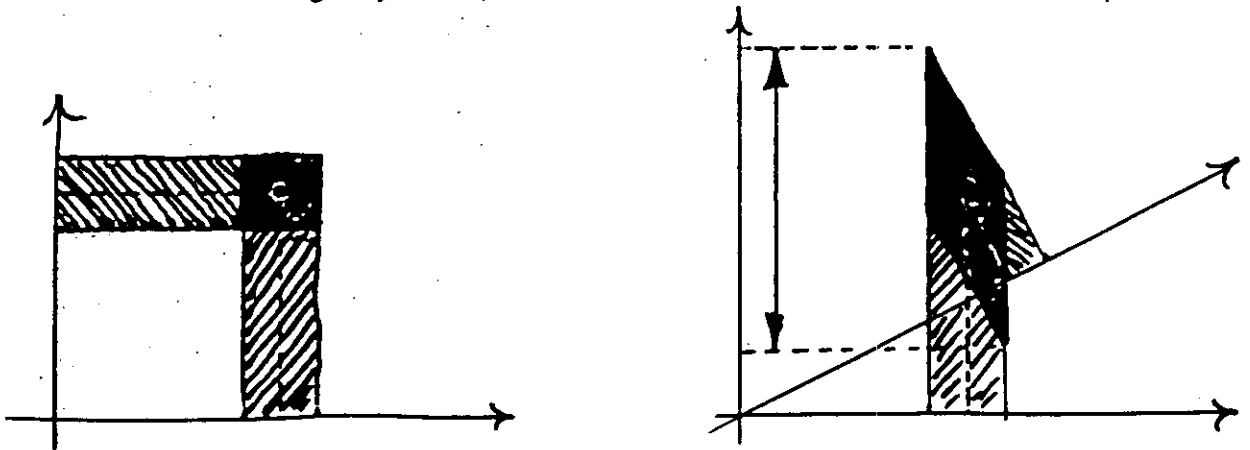
What if aerosol would behave as $1/\lambda^4$?



The direction of $\vec{A}$ is not known a priori.
It is a product of GOMOS!

$\Rightarrow$ some undetermination will persist...

$\Rightarrow$ noise amplification is unavoidable for skewed axes.



More fundamental ...

$$\beta(\lambda) = x_R \beta_R(\lambda) + x_{O_3} \beta_{O_3}(\lambda) + x_{NO_2} \beta_{NO_2}(\lambda) + \beta_{aerosol}(\lambda)$$

$$\text{with } \beta_{aerosol} = x_0 \beta_0(\lambda) + x_1 \beta_1(\lambda) + \dots$$

$$\chi^2 = \int_{\lambda_1}^{\lambda_2} (M(\lambda) - \beta(\lambda))^2 d\lambda$$

$$\frac{\partial \chi^2}{\partial x_i} = 0 \quad \forall_i$$

$$\begin{bmatrix} \int \beta_R^2 d\lambda & \int \beta_R \beta_{O_3} d\lambda & \int \beta_R \beta_{NO_2} d\lambda & \dots \\ \int \beta_{O_3}^2 d\lambda & \int \beta_{O_3} \beta_{NO_2} d\lambda & \dots & \\ \int \beta_{NO_2}^2 d\lambda & \dots & & \end{bmatrix} \begin{bmatrix} x_R \\ x_{O_3} \\ x_{NO_2} \\ \vdots \end{bmatrix} = \begin{bmatrix} \int \beta \beta_R d\lambda \\ \int \beta \beta_{O_3} d\lambda \\ \vdots \end{bmatrix}$$

KERNEL

$$K \vec{x} = \vec{t}$$

$$\vec{x} = K^{-1} \vec{t}$$

$$\text{and also } \Delta \vec{x} = K^{-1} \Delta \vec{t}$$

$$\text{Suppose } p(\lambda) = A/\lambda^4 + \underbrace{(B_0 + B_1\lambda + B_2\lambda^2)}_{\text{known by God only!}}$$

known by God only!

$$M(\lambda) = a/\lambda^4 + \underbrace{(b_0 + b_1\lambda)}_{\text{trial by some people}}$$

trial by some people

1) use of a 3 channel instrument $(\lambda_1, \lambda_2, \lambda_3)$

$$\left\{ \begin{array}{l} \frac{a}{\lambda_1^4} + b_0 + b_1\lambda_1 = \beta_1 \\ \frac{a}{\lambda_2^4} + b_0 + b_1\lambda_2 = \beta_2 \\ \frac{a}{\lambda_3^4} + b_0 + b_1\lambda_3 = \beta_3 \end{array} \right\} \Rightarrow \begin{array}{l} a = A + \epsilon_a B_2 \\ b_1 = B_1 + \epsilon_{b_1} B_2 \\ b_0 = B_0 + \epsilon_{b_0} B_2 \end{array}$$

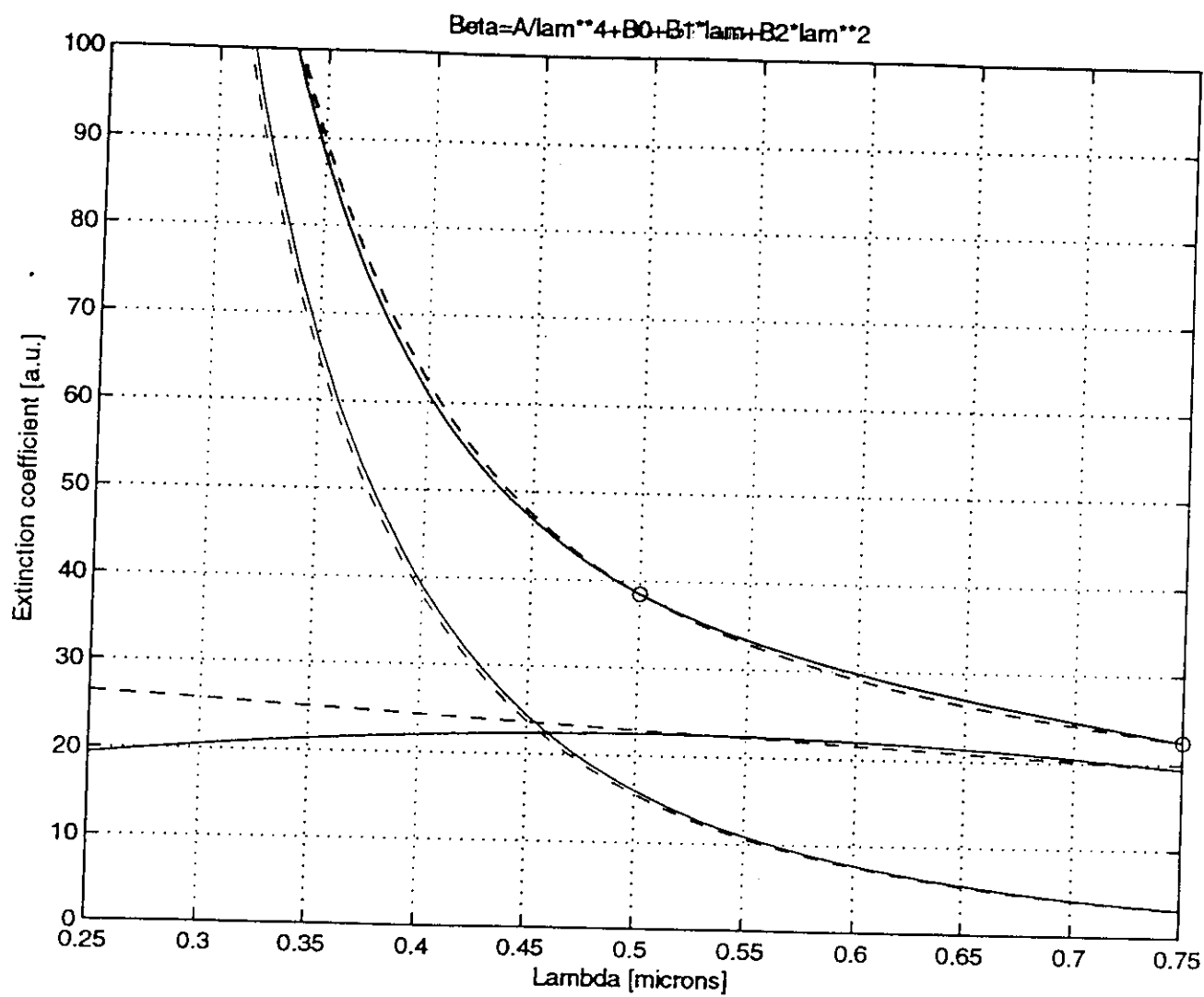
standard case: $A=1$ $B_0=10$ $B_1=50$ $B_2=-50$

B_2	a	b_0	b_1	$\bar{\Delta}(\%)$
0	1	10	50	0
-50	0.958	29.4	-12.5	.01
-100	0.916	48.8	-75.0	-3.9

① The value of the retrieved aerosol coefficients should be interpreted with caution!

$$\text{but } b_0 + b_1\lambda \approx B_0 + B_1\lambda + B_2\lambda^2$$

② Uncertainty for aerosols leads to uncertainty for air!



Where does the error come from?

1. C^{-1} is an "amplifier" and a "mixer" $C^{-1} \mathcal{B}$

C is a square symmetric matrix $\Rightarrow C \equiv U \Lambda U^T$

$$C^{-1} = U \Lambda^{-1} U^T$$

◆ $\rightarrow$ smallest eigenvalues are dominant

- (ill-conditioning by redundancy)
- rejection of small constituents

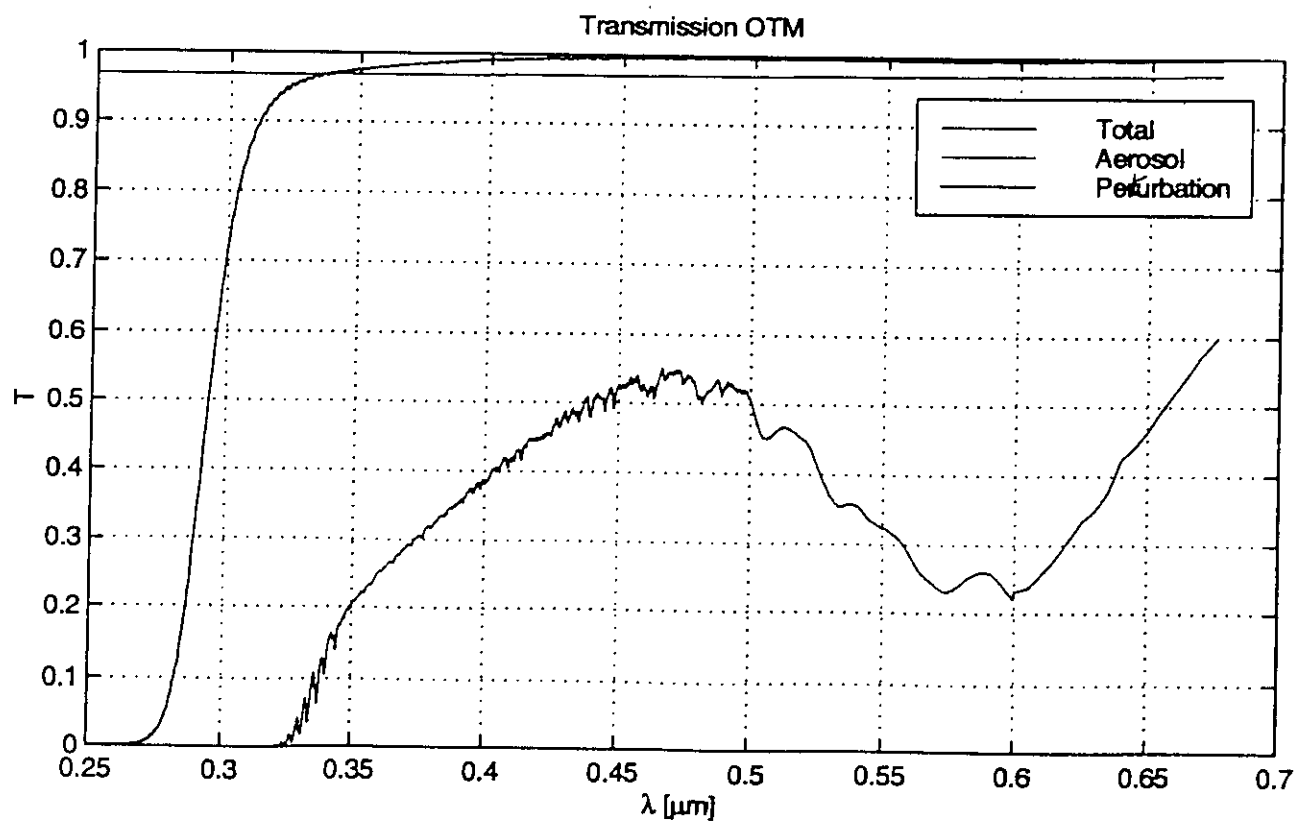
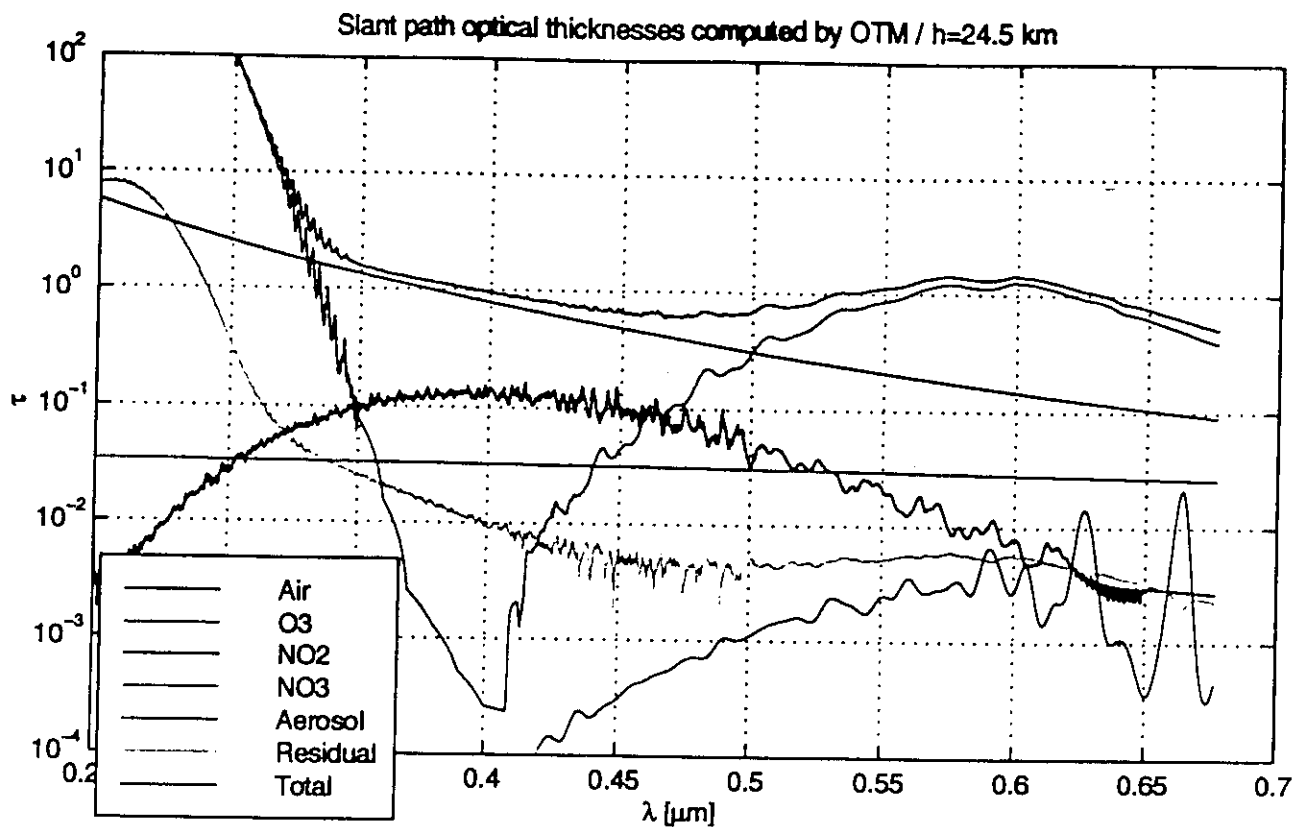
◆ $\rightarrow$ the more species, the smaller eigenvalues

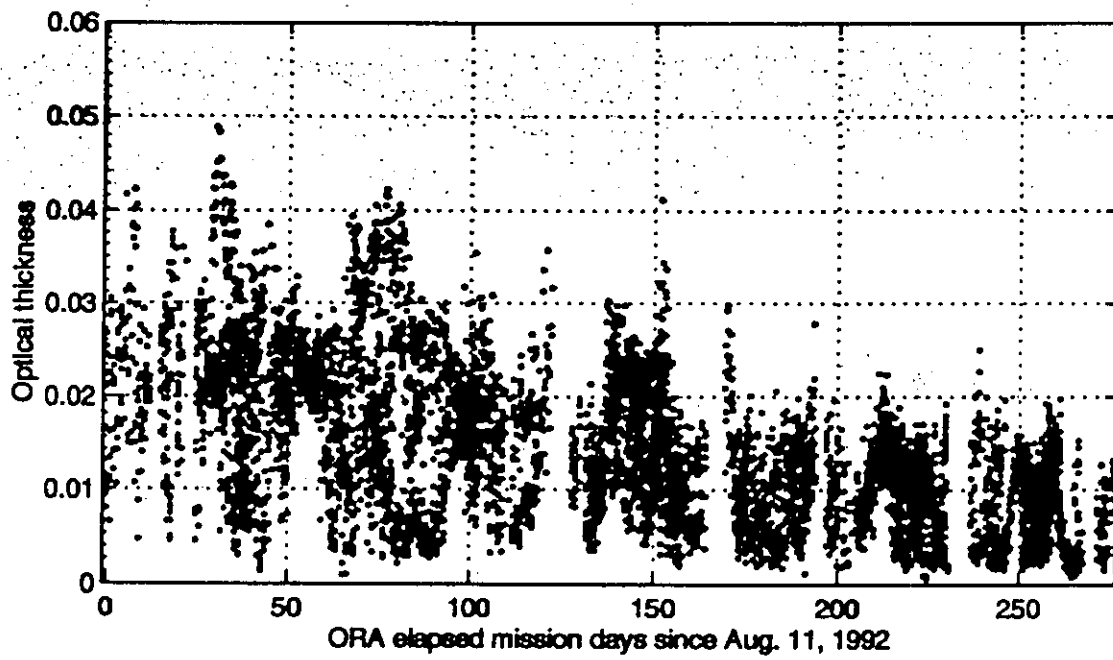
$$f_{m4} = \frac{1}{\lambda^4} \quad f_{m1} = \frac{3 \cdot 1}{\lambda} \quad f_0 = 16 \cdot 1 \quad f_1 = 32 \cdot \lambda \quad f_2 = 64 \cdot \lambda^2$$

BASIS and eigenvalues	Twomey criterion $r \approx (\lambda_{\min} / \lambda_{\max})^{\frac{1}{2}}$
$\{f_{m4}, f_{m1}\}$ 2385 52.5	14 %
$\{f_{m4}, f_0\}$ 2449 66.1	16 %
$\{f_{m4}, f_0, f_1\}$ 2409 147.9 1.03	2 %
$\{f_{m4}, f_0, f_1, f_2\}$ 2425 257 4.21 0.012	0.2 %

◆ $\rightarrow$ if C has important off-diagonal, C^{-1} will have too!
terms

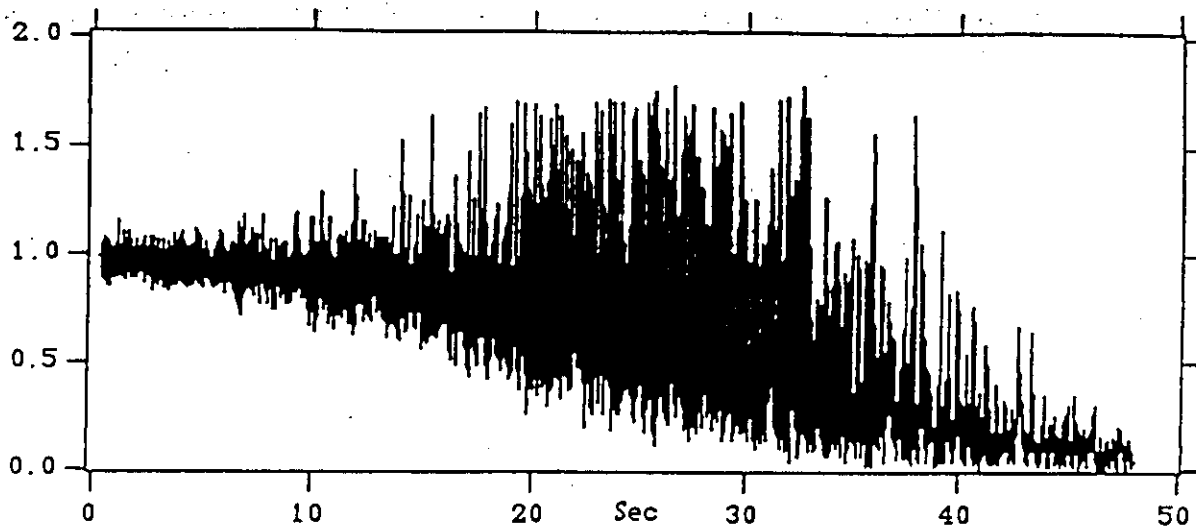
HOWEVER, C^{-1} IS SYMMETRIC AND NOT
RESPONSIBLE FOR ANY BIAS



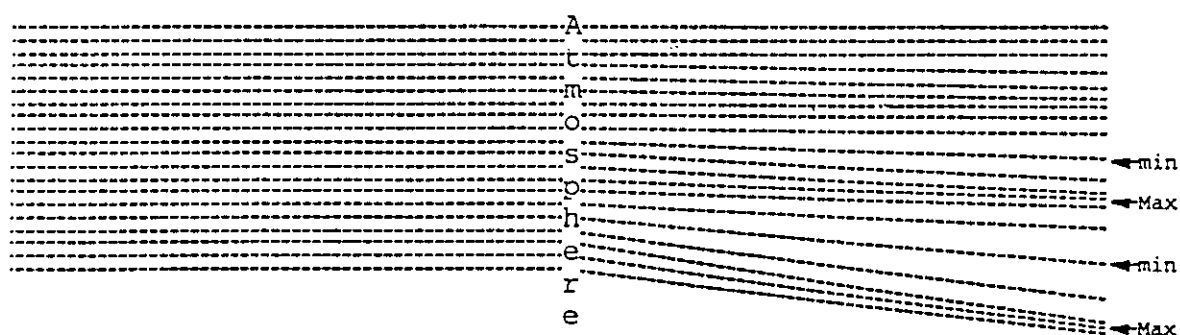


Total optical thickness @ $1.02 \mu\text{m}$

- SAGE II
- ORA



During this measurement, altitude of the tangent point varied (non linearly) between 43.8 and 16.7 km. This measurement contains noise (mainly from photon statistics) as well as the effect of absorption and aerosols and Rayleigh diffusion, however, the optical filter was chosen in order to limit these non refractive effects. (spherical symmetry). Within this simplified geometry the refractive phenomena arise from the vertical structure as sketched below.



As the rays go deeper within the atmosphere, the mean refraction angle increases as well as the distance between "consecutive" rays at the satellite position. This general increase produces the refractive dilution.

**Tomography of the Earth's atmosphere by the
spaceborne occultation radiometer ORA:
Spatial inversion algorithm**

Didier Fussen, Etienne Arijs, Fabienne Leclere, Dennis Nevejans and
Christine Bingen

Belgian Institute for Space Aeronomy, Brussels, Belgium

APPLIED OPTICS

37, p 3121-3127, 20 May 1998

**Validation of the ORA
spatial inversion algorithm with respect to the
Stratospheric Aerosol and Gas Experiment II data**

Didier Fussen, Etienne Arijs, Dennis Nevejans, Filip Van Hellemont, Colette Brogniez,
and Jacqueline Lenoble

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 103, NO. D7, PAGES 8455-8464, APRIL 20, 1998

**A critical analysis of the Stratospheric Aerosol and Gas
Experiment II spectral inversion algorithm**

Didier Fussen

Belgian Institute for Space Aeronomy, Brussels, Belgium