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MHD Stability Limits of Advanced Tokamak Regimes

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MHD Stability Limits of Advanced Tokamak Regimes

Presented by J. Menard

Autumn College on Plasma Physics
ICTP
Trieste, Italy
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What is an advanced tokamak?

The advanced tokamak concept
aims at simultaneously achieving:

- High $\beta_N = \beta_{(\%)} B_{(T)} a_{(m)} / I_{P(MA)}$ above the Troyon limit
- High bootstrap fraction $f_{BS} = I_{BS}/I_P$
- Steady state operation
- High energy confinement factor $H=\tau_E/\tau_{E\text{-}ITER89P}$

This talk will focus on theoretical/numerical
methods of optimizing the first 2 parameters
above continuously from low to high aspect ratio

Brief review of terminology/methodology

- Step 1: Compute equilibrium by solving Grad-Shafranov equation:

$$\mathbf{J}\times\mathbf{B} = \nabla p \quad \Rightarrow \quad \Delta^*\Psi = \mu_0 R J_\phi = -[\mu_0 R^2 dp/d\Psi + F dF/d\Psi]$$

  - $\Delta^*\Psi = R^2 \nabla\bullet(\nabla\Psi/R^2)$
  - Ψ is the poloidal flux function $= RA_\phi$
  - $p(\Psi)$ is the plasma pressure
  - $F(\Psi) = RB_\phi$ where B_ϕ is the toroidal magnetic field

  - Solve in flux coordinates where Ψ is a radial coordinate and is constant on the plasma boundary defined by:
    - $R=R_0+a\,cos(\theta + \delta\,sin\,\theta)$ $\quad$ R_0 = plasma major radius
    - $Z=Z_0+\kappa\,a\,sin\,\theta$ $\quad$ a = plasma minor radius
    - κ is plasma elongation (height/width)
    - δ is triangularity ($\delta > 0 \Rightarrow$ D-shaped plasma)

Brief review of terminology/methodology

- Step 1 has been routine for many years, but...

  - A semi-recent advance has been to constrain the current density profile to self-consistently match the neoclassical bootstrap current (here <A> denotes a flux surface average):

$$<\frac{J_\phi}{R}> = -p'\left(1 - \frac{<B_\phi^2>}{<B^2>}\right) + <\frac{1}{R^2}> \frac{<\vec{J}\cdot\vec{B}>_{BS+CD}}{<\vec{B}\cdot\nabla\phi>} \frac{<B_\phi^2>}{<B^2>}$$

  - This constraint can slow the calculation considerably since the bootstrap current - through the trapped particle fraction - depends on the magnetic field structure of the solution itself.

  - Since the bootstrap current is (to lowest order) proportional to the pressure gradient, the solution is now predominantly determined by the magnitude and shape of the pressure profile

Brief review of terminology/methodology

• Step 2: Compute ideal linear stability

• Energy principle ⇒ perturb equilibrium fluid elements with a small displacement ξ searching for displacement vectors which reduce the potential energy δW of the system.

• 1. High toroidal mode number/short radial wavelength ⇒ ballooning modes
1/n expansion reduces δW minimization to ODE.

• 2. Low to intermediate toroidal mode number ($n < 10$) ⇒ kink modes
  - need full 3D displacement,
  - can analytically remove parallel **b** and normal ($\mathbf{b}\times\nabla\psi$) components of ξ, a.k.a. Bineau reduction:
    - correct marginal stability
    - non-physical eigenvalues.

•All this is done in the PEST-II code.

Brief review of terminology/methodology

- Step 2: Also, these calculations have been routine for at least a decade, but...

  - A significant recent improvement (J. Manickam) to PEST-II is the ability to compute the number of unstable eigenvalues without searching for the eigenvalues themselves.

  - This allows for fast high-resolution scans of marginal stability with varied pressure (β) and/or stabilizing wall position

  - Both features are particularly useful for stability limit scans.

Question to Address:

- How are the ideal MHD stability properties of advanced high and low aspect ratio (spherical) tokamaks linked?

- Are there any stability invariants?
- Which parameters are not invariants?
- What are the optimal aspect ratios for normal and super-conducting reactors?

- What about neoclassical tearing modes?

Methodology:

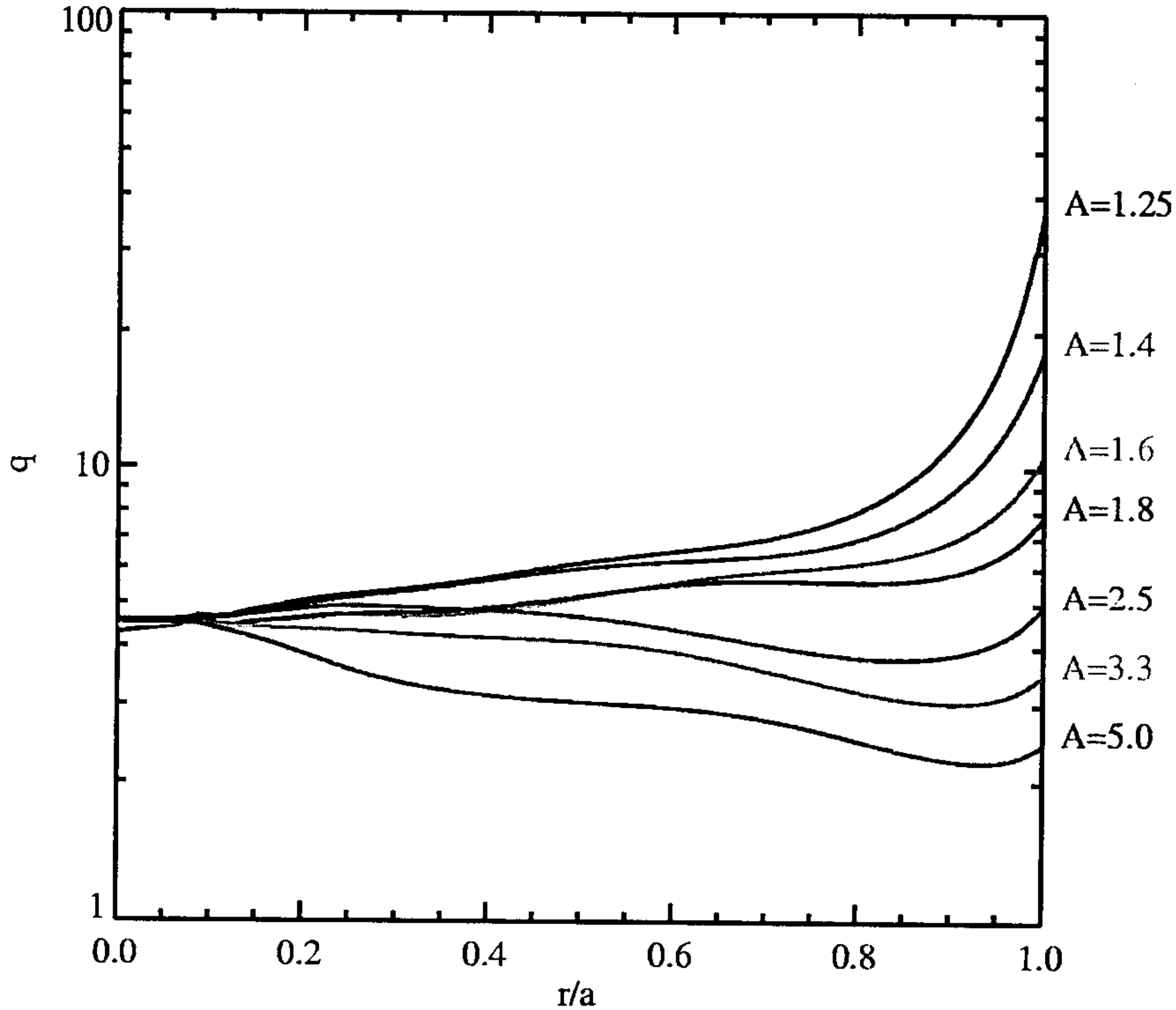
- For a range of aspect ratios, maximize β by optimizing the pressure profile and κ keeping the following fixed:
  - $\delta \approx 0.65$
  - $f_{BS} \approx 99\%$
  - Kink marginal wall position $r_{WALL}/a \geq 1.1$

- Must distinguish between wall+feedback stabilized scenarios and those with no wall
  - **HERE, WALL STABILIZATION IS ASSUMED TO WORK**

Codes Used:

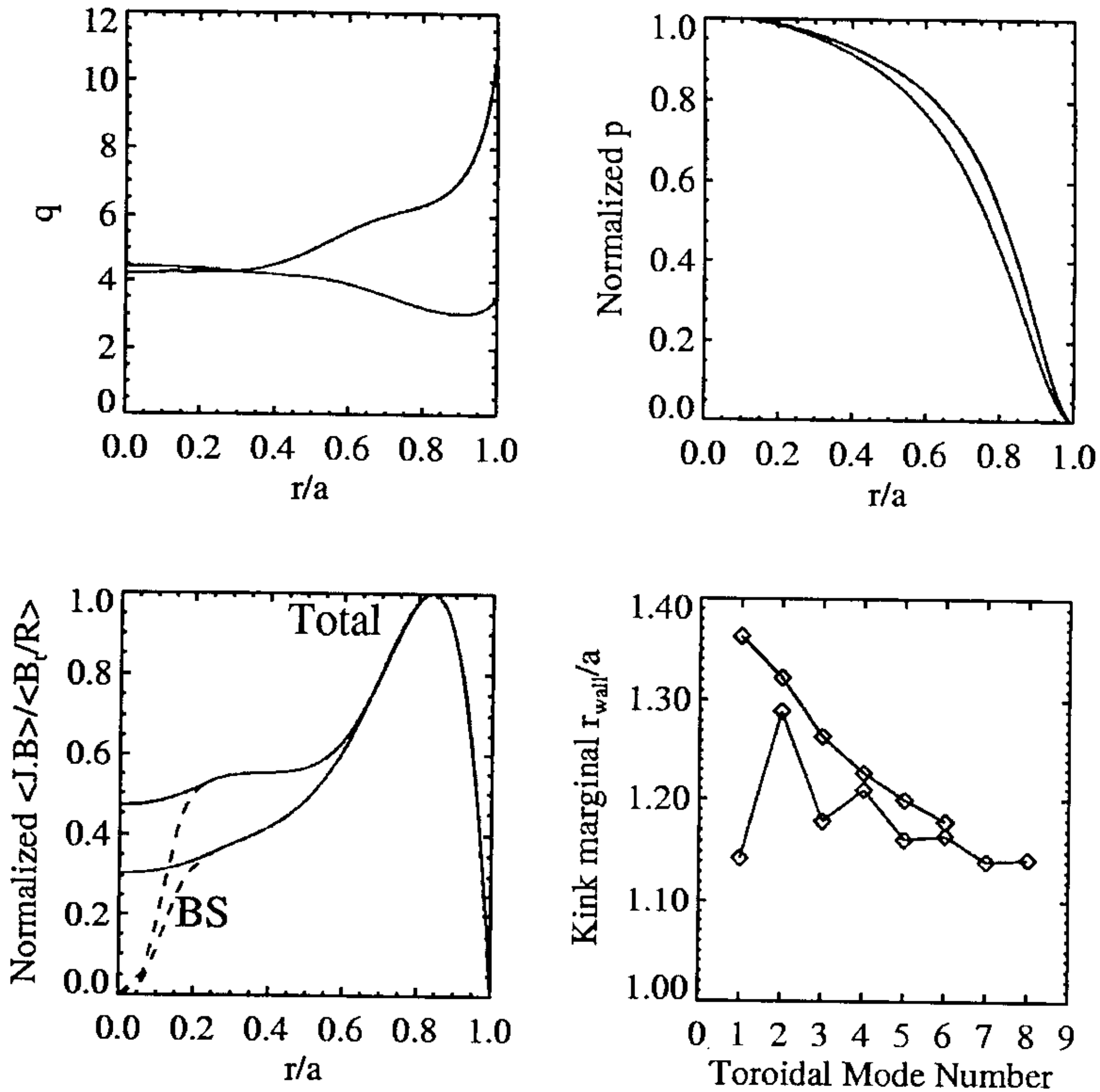
- JSOLVER - fixed boundary equilibria
- BALLOON & CAMINO - high-n ballooning
- PEST-II - low to intermediate-n kink modes

Reverse shear in q profile occurs for A>2



q profiles of optimized equilibria

Typical high and low-A optimal profiles



A=1.6

κ=3.6

δ=0.64

β=56%

β_N=8.2

A=3.3

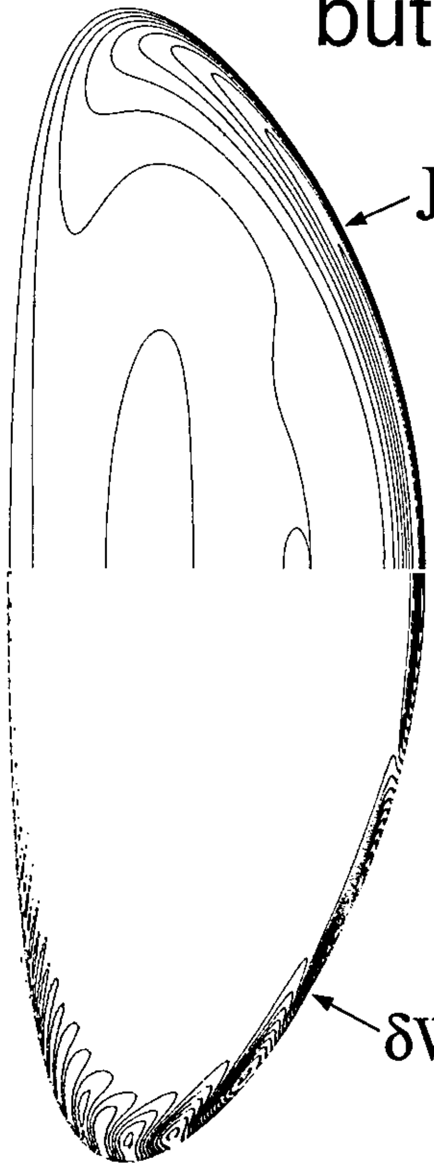
κ=2.5

δ=0.6

β=14%

β_N=6

Instability mode structure is ballooning-like but has large poloidal extent

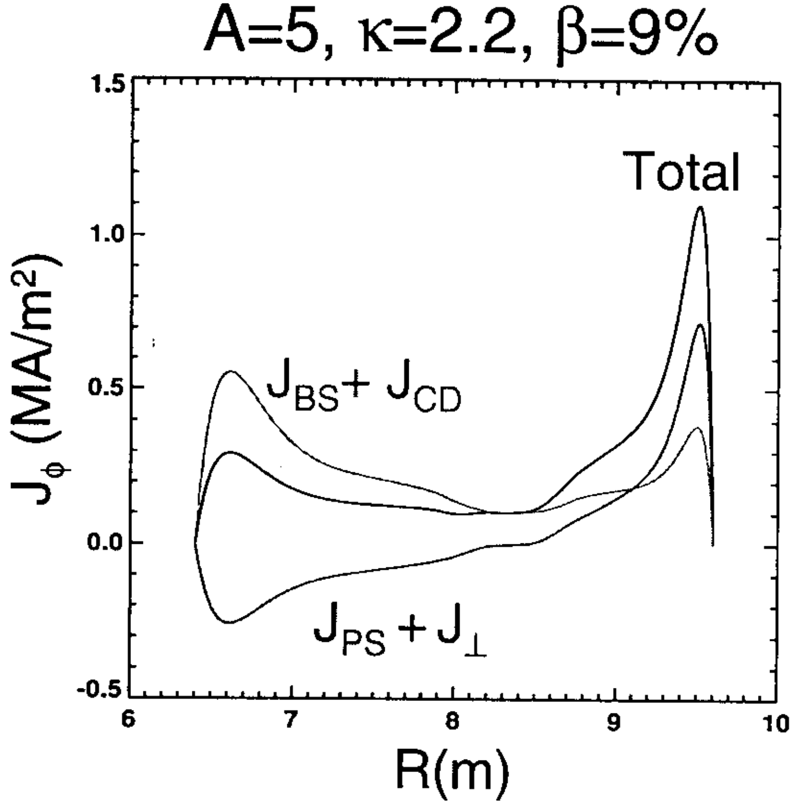


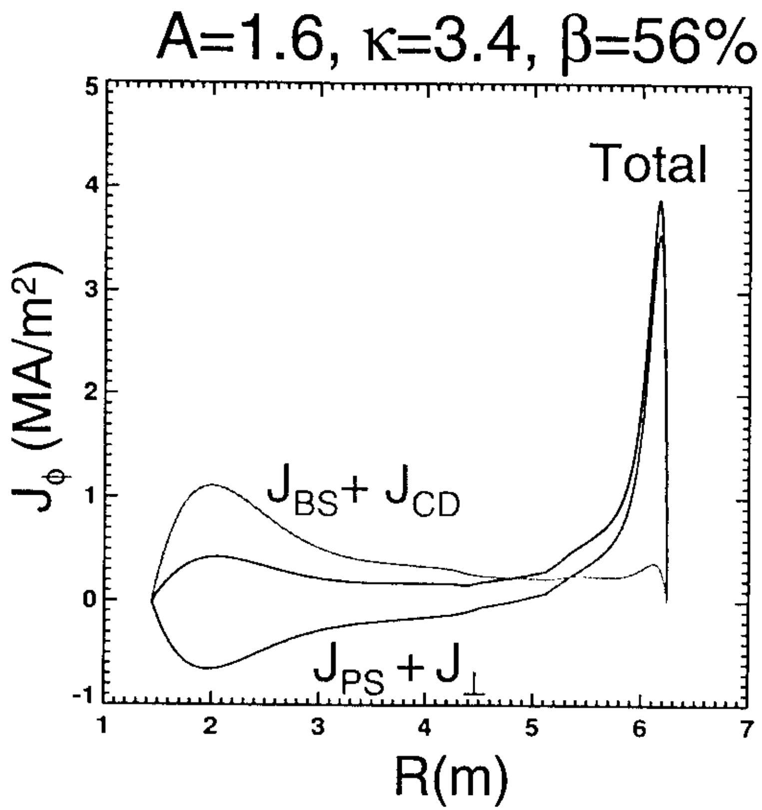
n=4 kink mode at A=1.4, β=40%

$$\delta W = \delta W_{Fluid} + \delta W_{Surface} + \delta W_{Vacuum}$$

$\delta W_F = 1/2\mu_0 \int dV \; |Q_\perp|^2$	Shear Alfven waves
$+ B^2 |\nabla \bullet \xi_\perp + 2\xi_\perp \bullet \kappa|^2$	Compressional Alfven
$+ \mu_0 \gamma p |\nabla \bullet \xi|^2$	Sound waves
$- 2\mu_0 (\xi_\perp \bullet \nabla p)(\kappa \bullet \xi^*_\perp)$	pressure driven instabilities
$- \mu_0 J_\parallel (\xi^*_\perp \times \mathbf{b}) \bullet Q_\perp$	current driven instabilities

High β equilibria with broad pressure profiles generate large, destabilizing, Pfirsch-Schluter currents





$$\vec{J_\perp} \cdot \nabla\phi = -p'(1 - \frac{B_\phi^2}{B^2})$$

$$\vec{J_{PS}} \cdot \nabla\phi = -p'(\frac{B_\phi^2}{B^2} - \frac{B_\phi^2}{< B^2 >}) \longleftarrow \text{Dominant}$$

Mode observed prior to ELM on DIII-D has similar structure
⇒ Is the high-β n=4 mode simply ELMy, or more destructive?

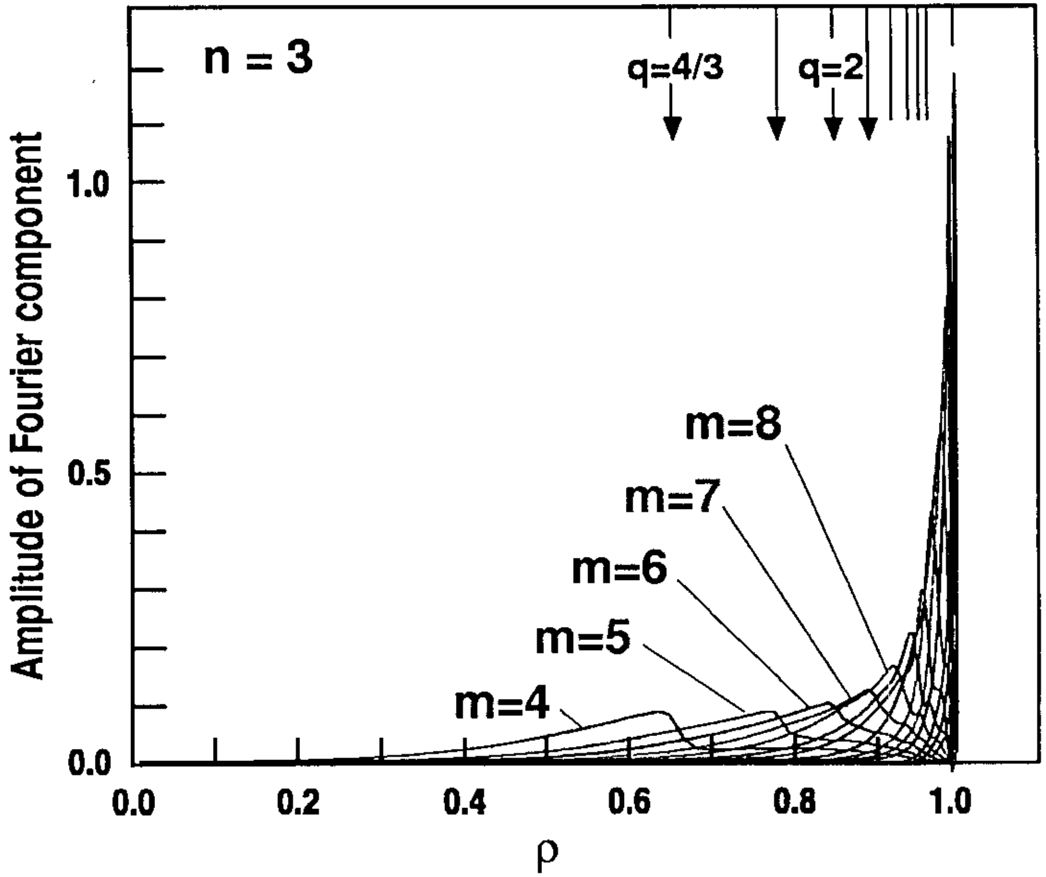


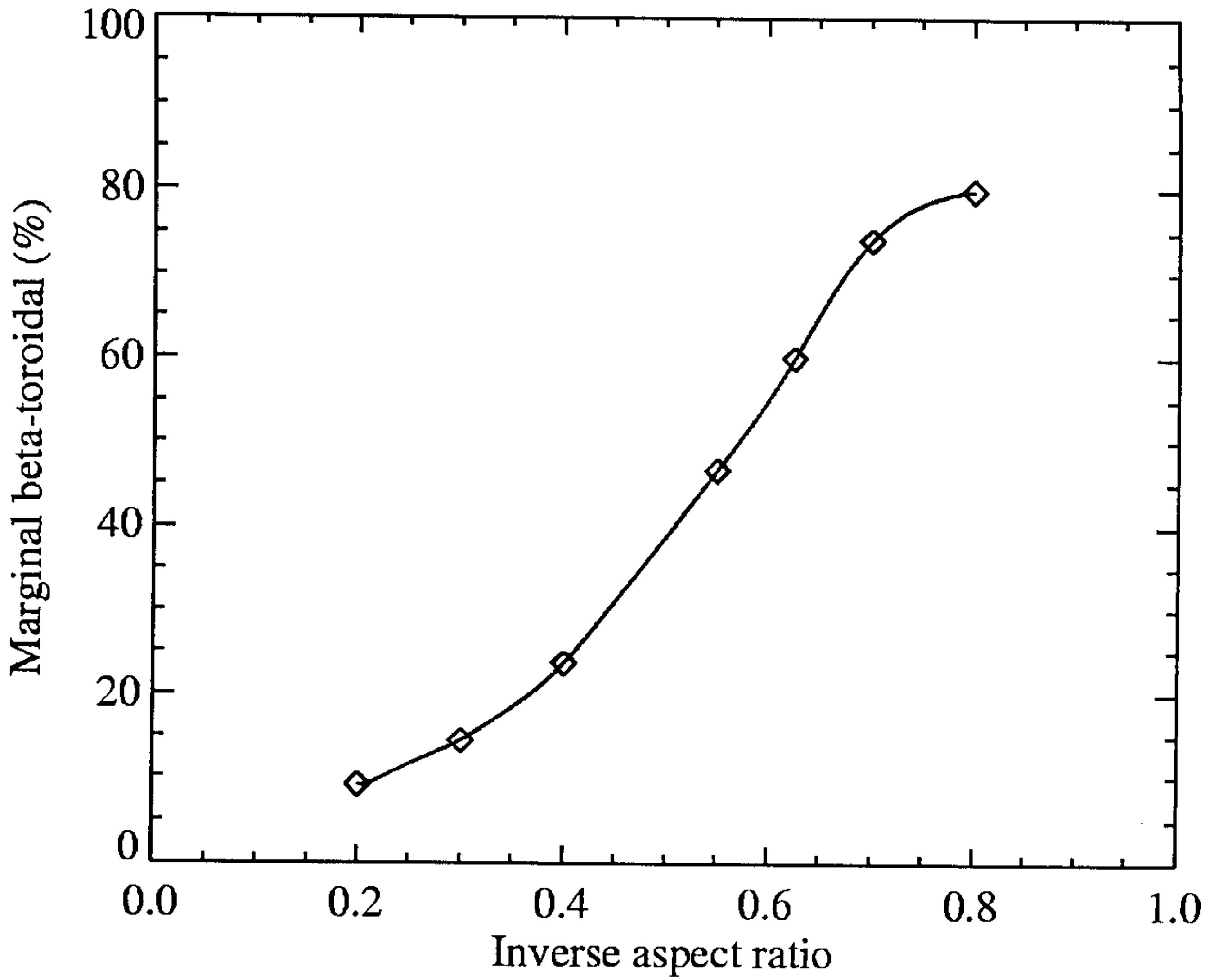
Fig. 1. *Radial structure of poloidal Fourier components of the unstable ideal n=3 mode, just before an ELM. Plotted is $\xi \bullet \nabla \Psi$, where ξ is the plasma displacement and ψ is the equilibrium poloidal flux.*

Data reproduced from: **DIII–D TECHNICAL BULLETIN** DIII-D National Fusion Facility

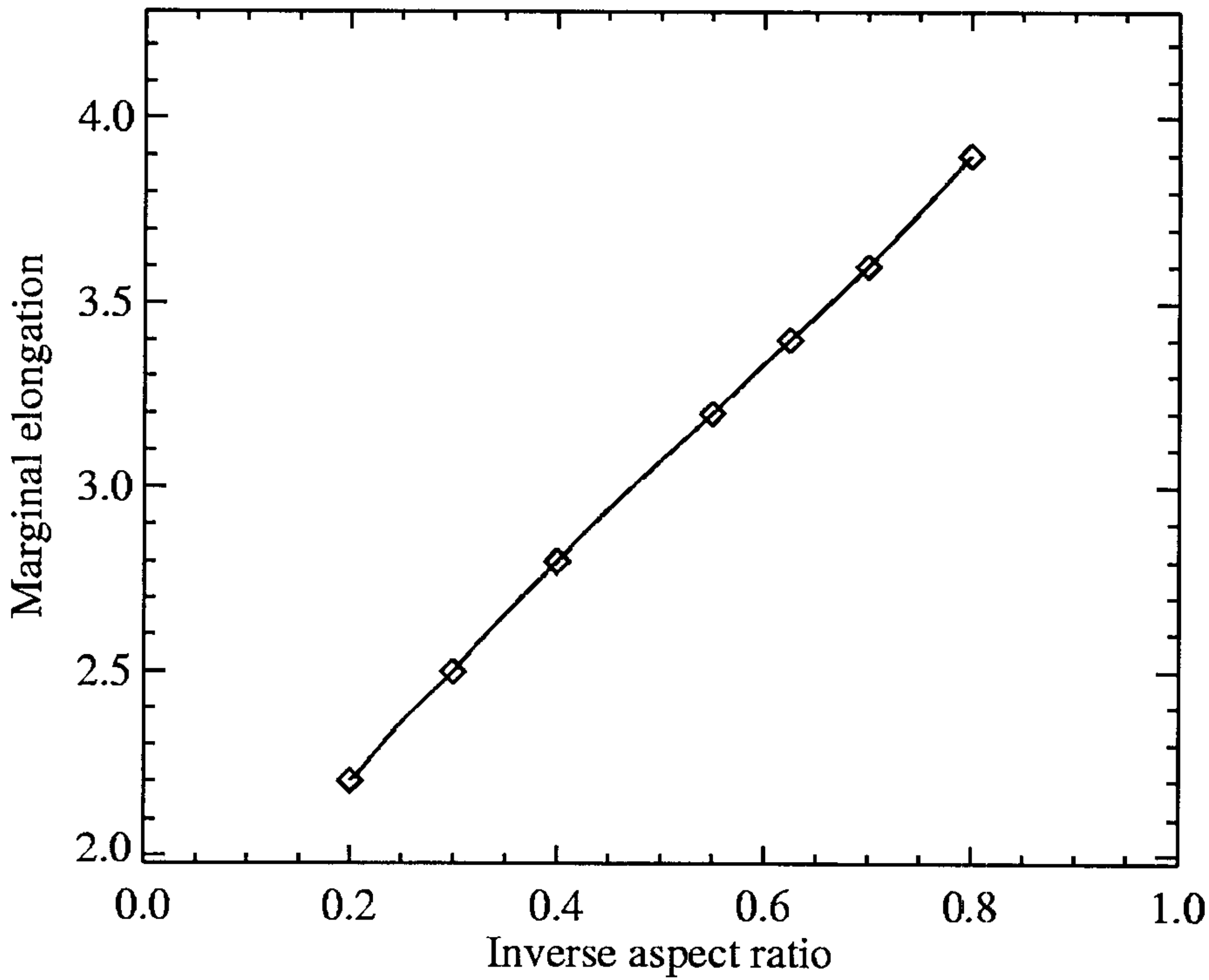
Number 2 March 25, 1999 A23064

Ideal MHD Kink-Peeling Modes and Their Relation to ELMs
A.D. Turnbull, GA

Maximum β saturates for ε > 0.7

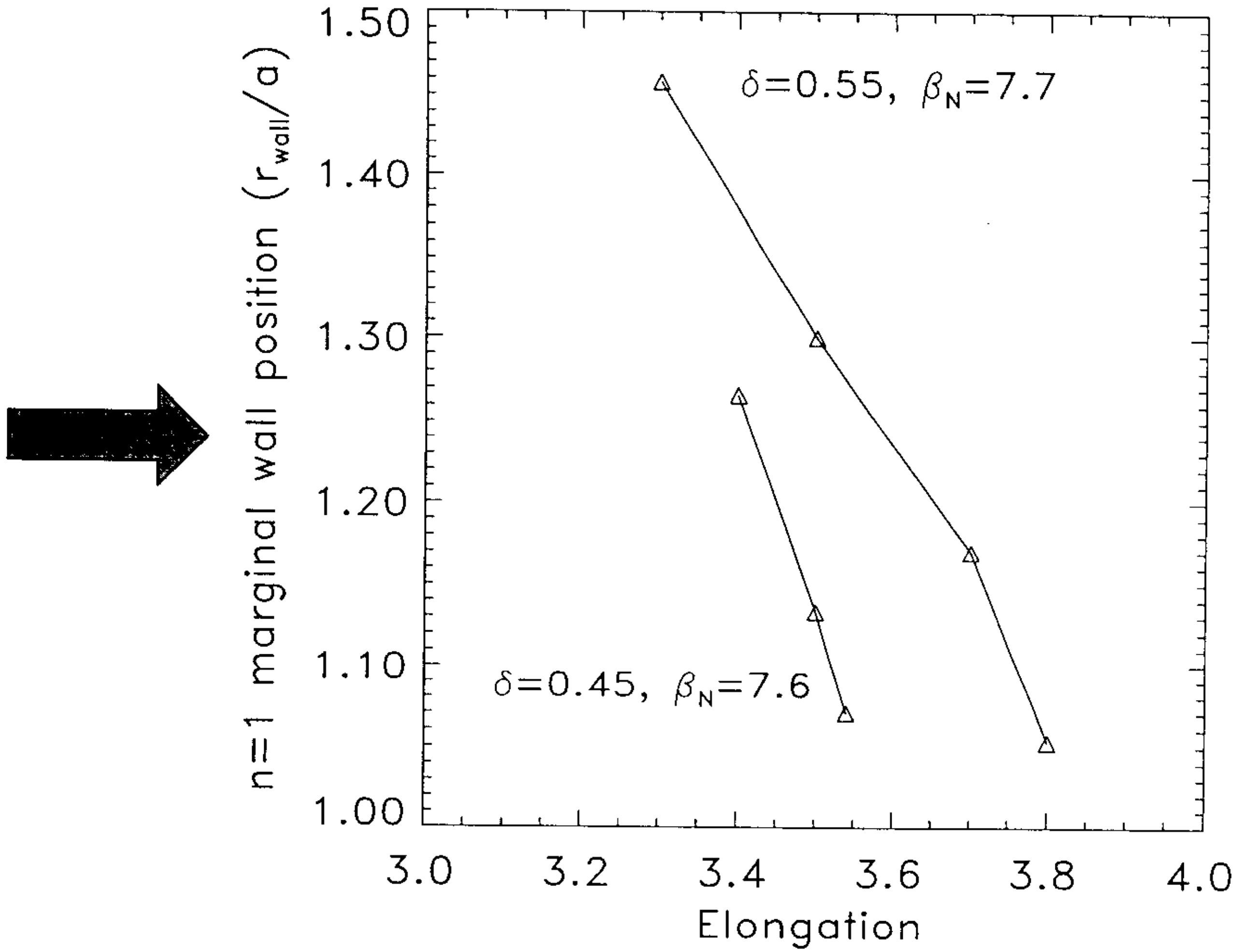


Max. elongation is a linear function of ε

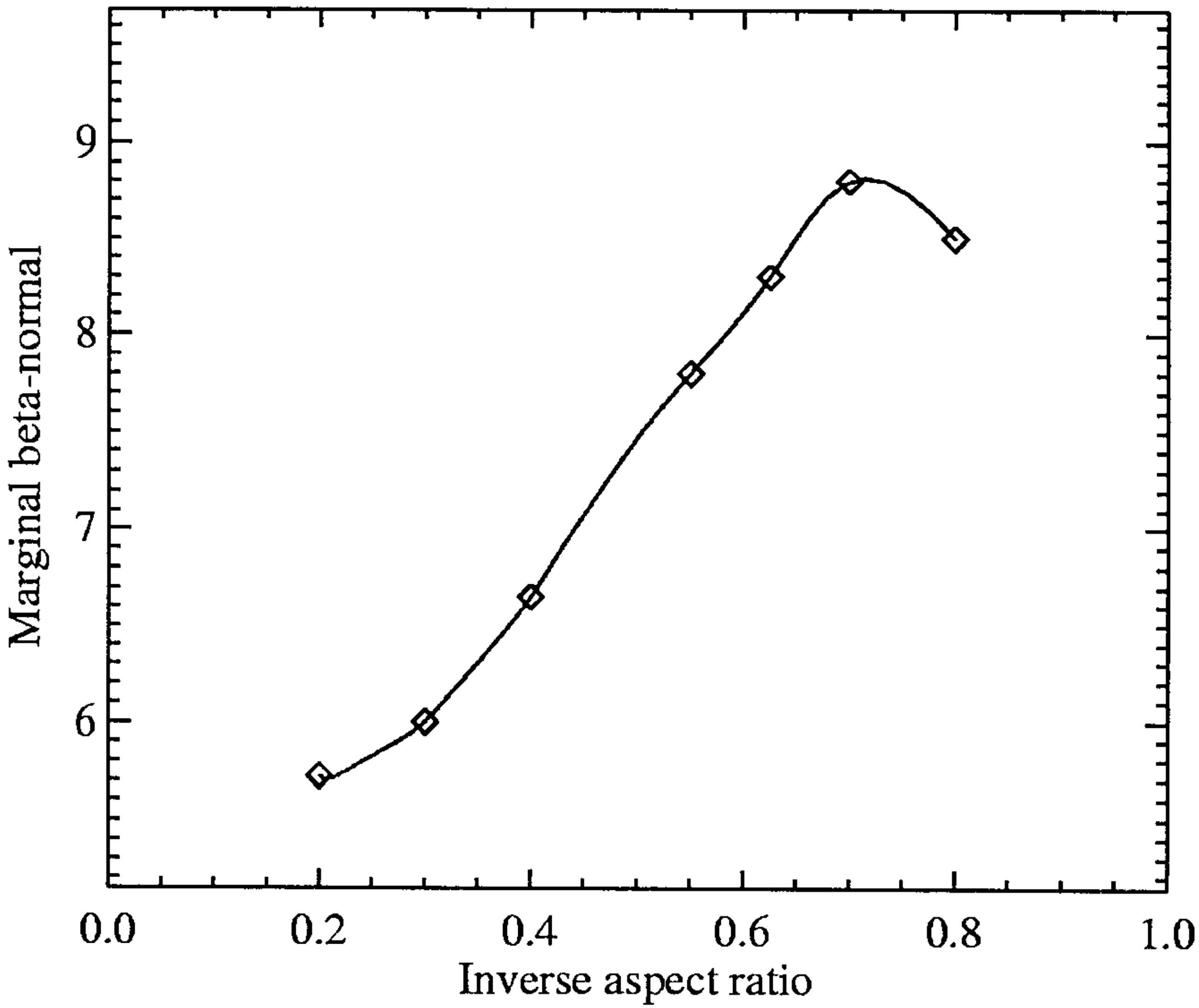


n=1 kink mode limits maximum κ

n=1 kink behavior for high-β A=1.4 equilibrium

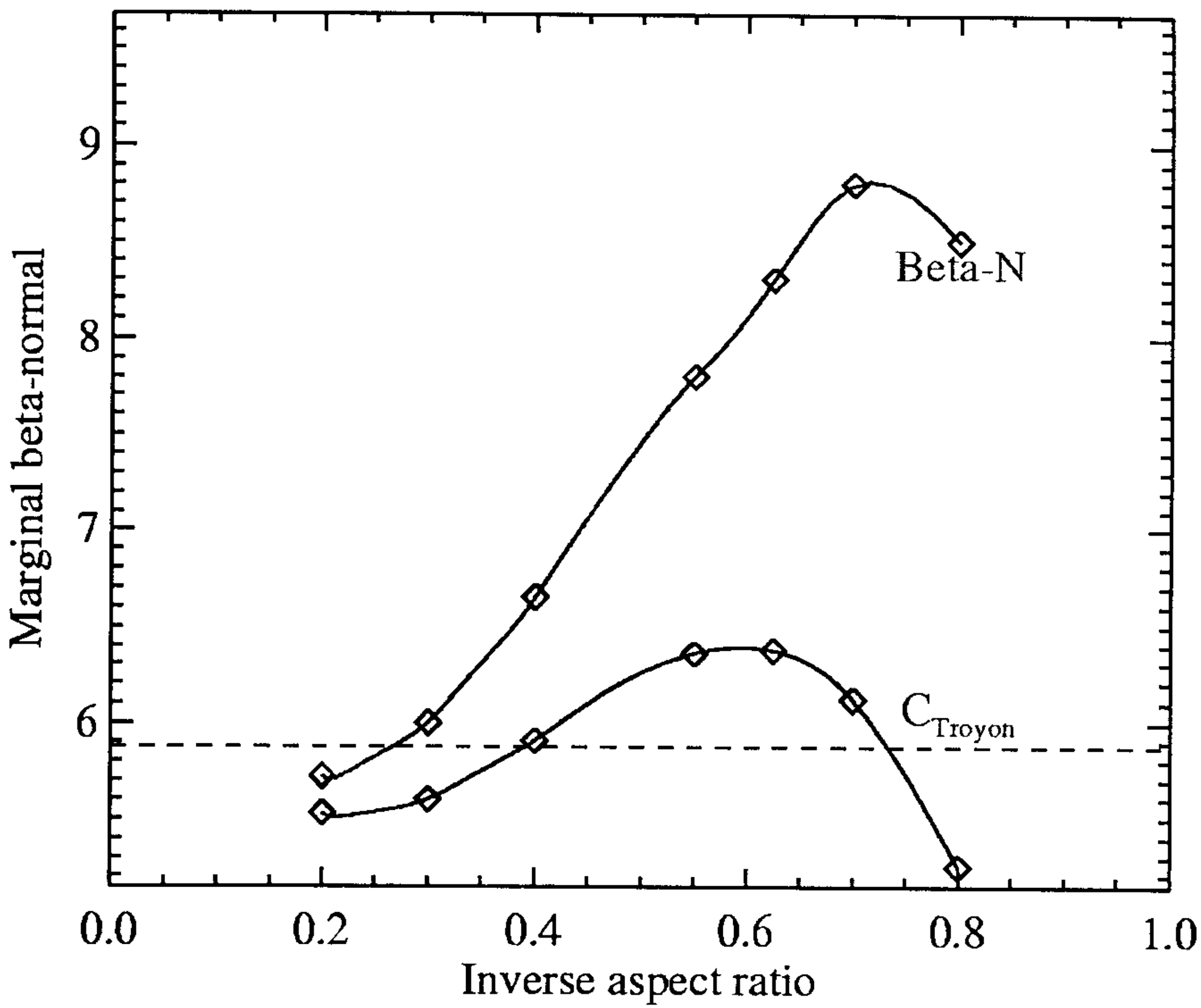


$\beta_N \equiv \beta_{t0} a B_{t0} / I_P$ increases 50% as $\varepsilon \rightarrow 1$



$$\beta_{t0} = \frac{2\mu_0 \langle p \rangle}{B_{t0}^2}$$

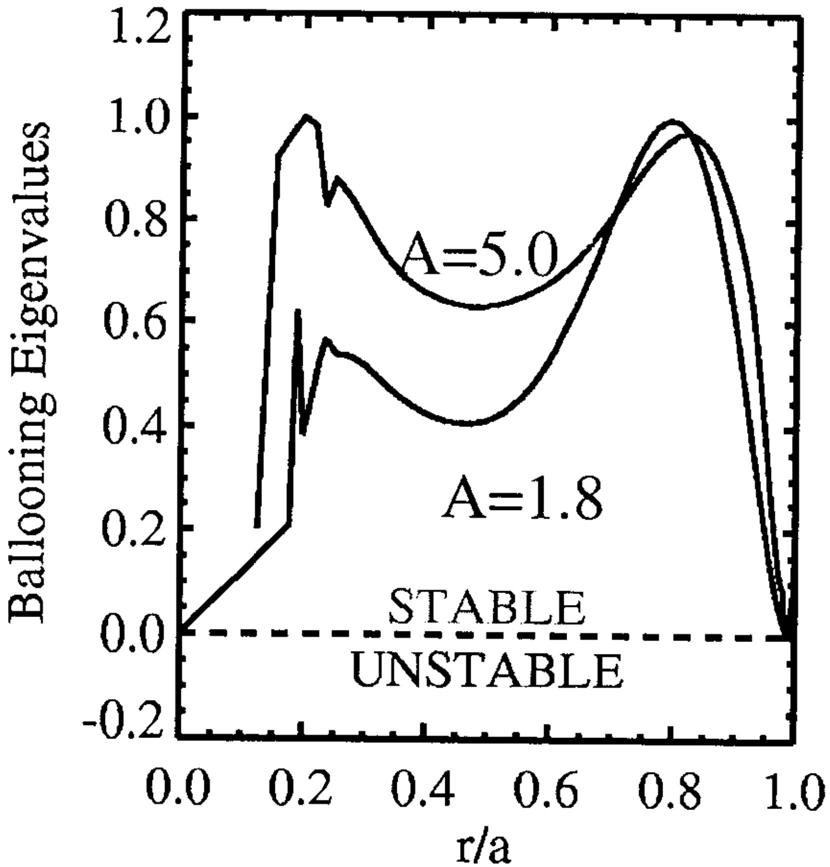
$C_T \equiv \langle\beta\rangle aB_{t0} / I_P$ is nearly invariant

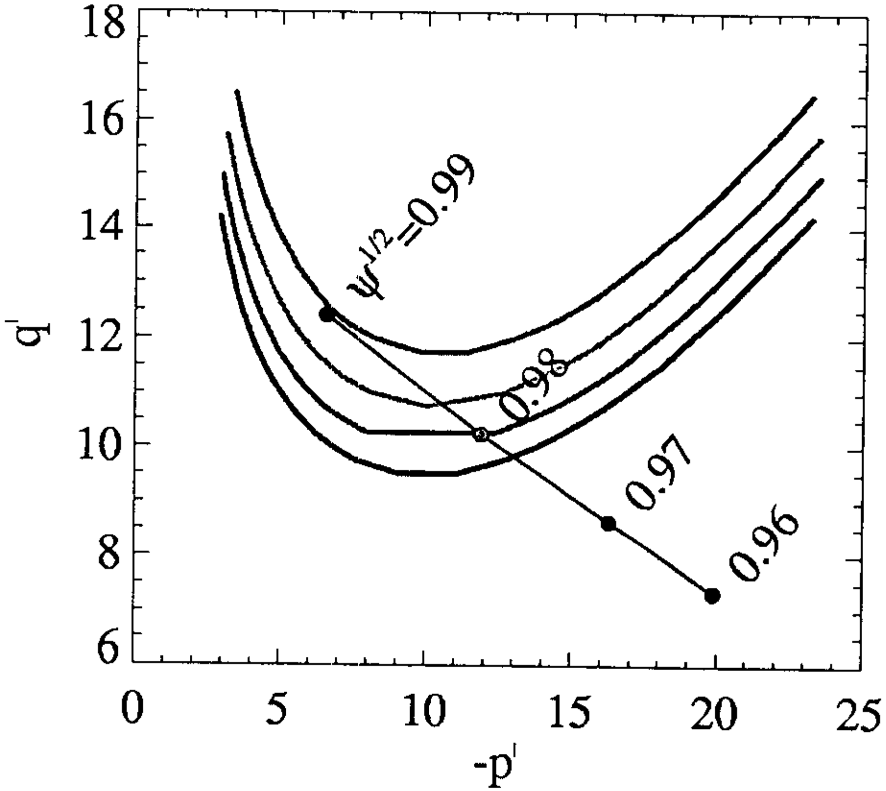


$$\langle\beta\rangle = \frac{2\mu_0\langle p\rangle}{\langle B^2\rangle}$$

This is Troyon's original definition

Ballooning stability limited by edge for all aspect ratios for $f_{BS} \approx 1$





Implications for optimal reactor A:

- $P_{fusion} \propto \beta^2 B^4 V_{plasma}$ where $\beta = 2\mu_0\langle p\rangle / B^2$
- Standard Troyon scaling: $\beta(\%) < \beta_N I_P(MA) / aB$

  $$\Rightarrow \quad \beta(\%) < 5\, \beta_N\, \varepsilon\, (1+\kappa^2)/2q^*$$

  - β_N = normalized β, ε = inverse aspect ratio a/R
  - κ = elongation, q^* = kink safety factor

- Self driven (bootstrap) current fraction:

  $f_{BS} = I_{BS}/I_P \approx C_{BS}\, \varepsilon^{1/2}\, \beta_P$

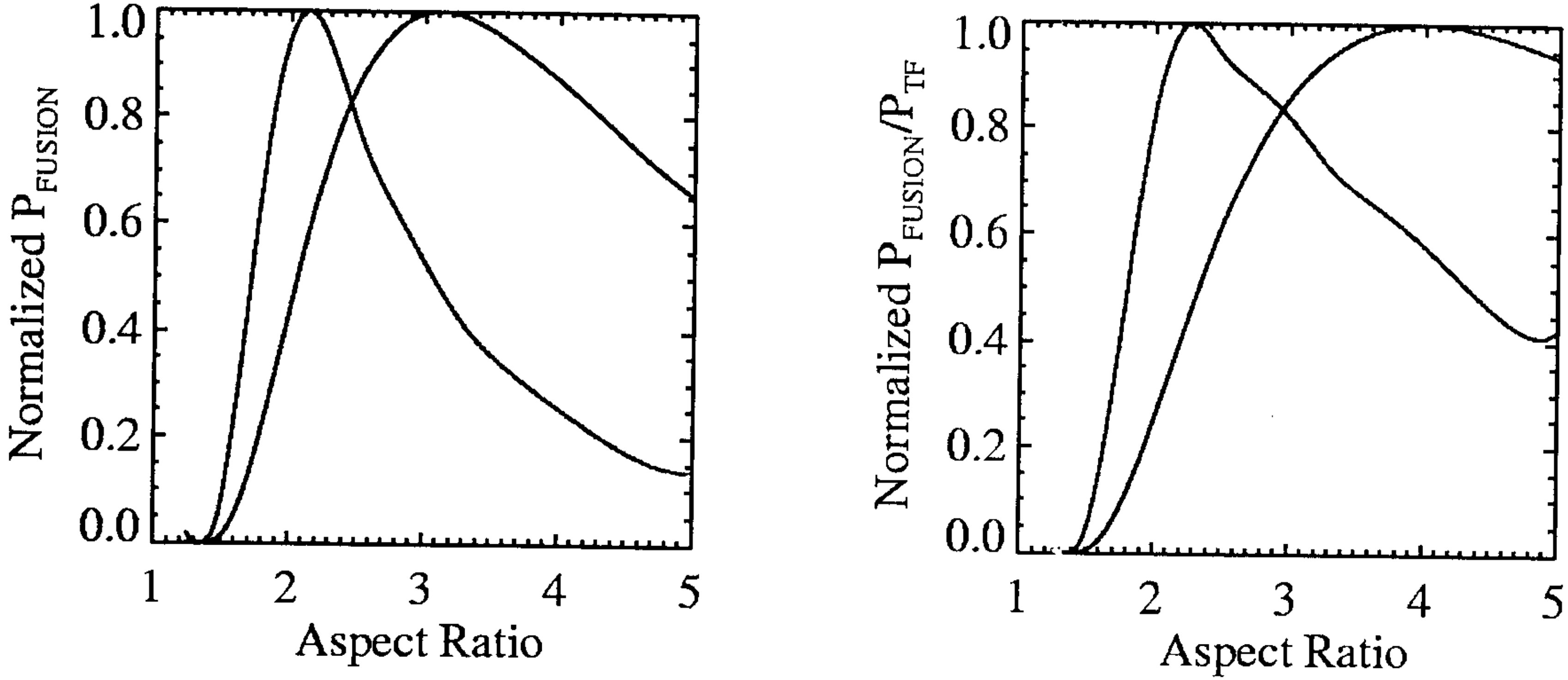
P_{fusion} scalings for fixed R_0

- Combining Troyon and BS scalings ⇒
  $\beta(\%) < \varepsilon^{1/2} C_{BS} (1+\kappa^2) (\beta_N)^2 / 8 f_{BS}$
- $B_{t0} = B_{MAX}(1 - \varepsilon - \Delta_{SHIELD}/R_0)$
  Δ_{SHIELD} = inboard shield thickness
- $V_{plasma} \propto R^3{}_0 \varepsilon^2 \kappa$
- $P_{TF\text{-}Coil} \propto B^2{}_{MAX} \varepsilon \kappa R_0$

How do P_{fusion} and P_{fusion}/P_{TF} vary with aspect ratio and inboard shield thickness?

Optimal A = 2-2.5 for $\Delta_{SHIELD}/R_0 = 0.25$

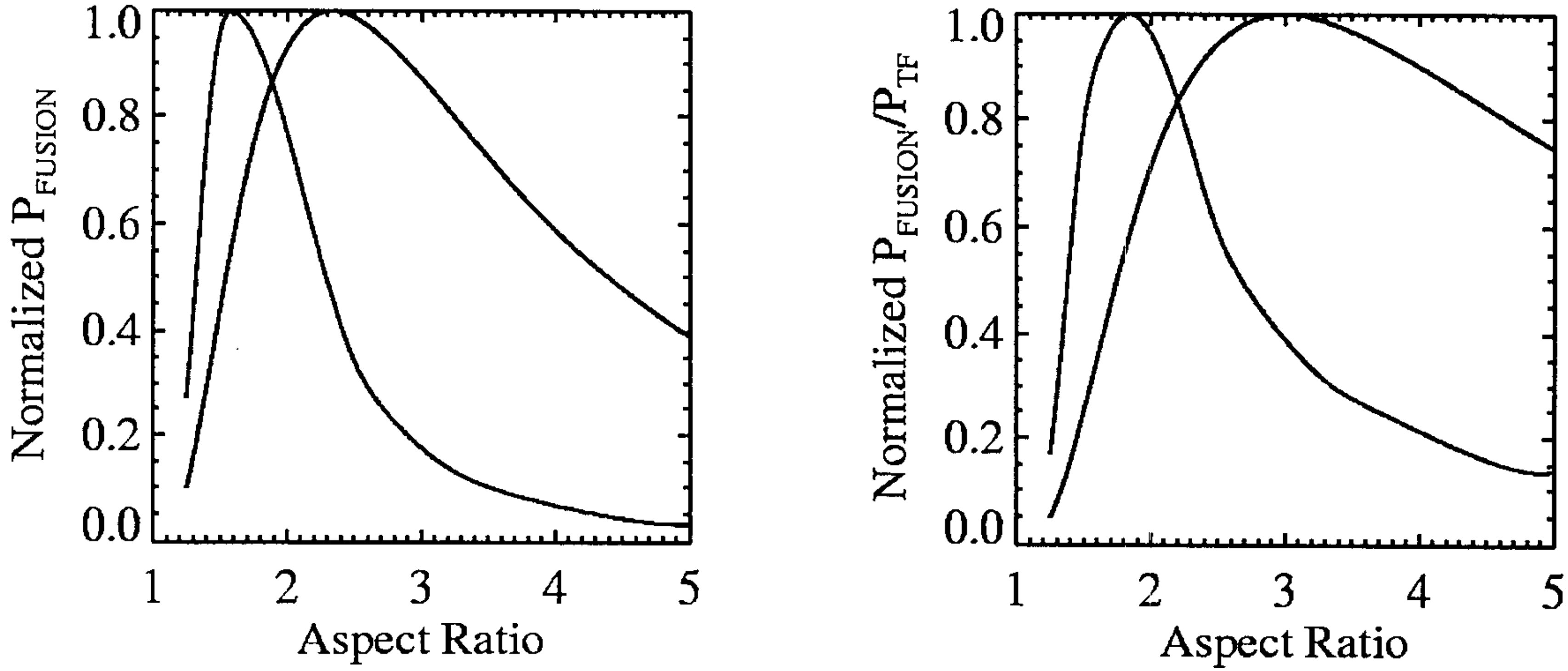
Red $\Rightarrow$ Stability optimized β_N and κ
Blue $\Rightarrow$ β_N and κ assumed constant



If β_N and κ are assumed to be constant, optimal A = 3-4

Optimal $A = 1.6\text{-}1.8$ for $\Delta_{SHIELD}/R_0 = 0$

Red ⇒ Stability optimized β_N and κ
Blue ⇒ β_N and κ assumed constant



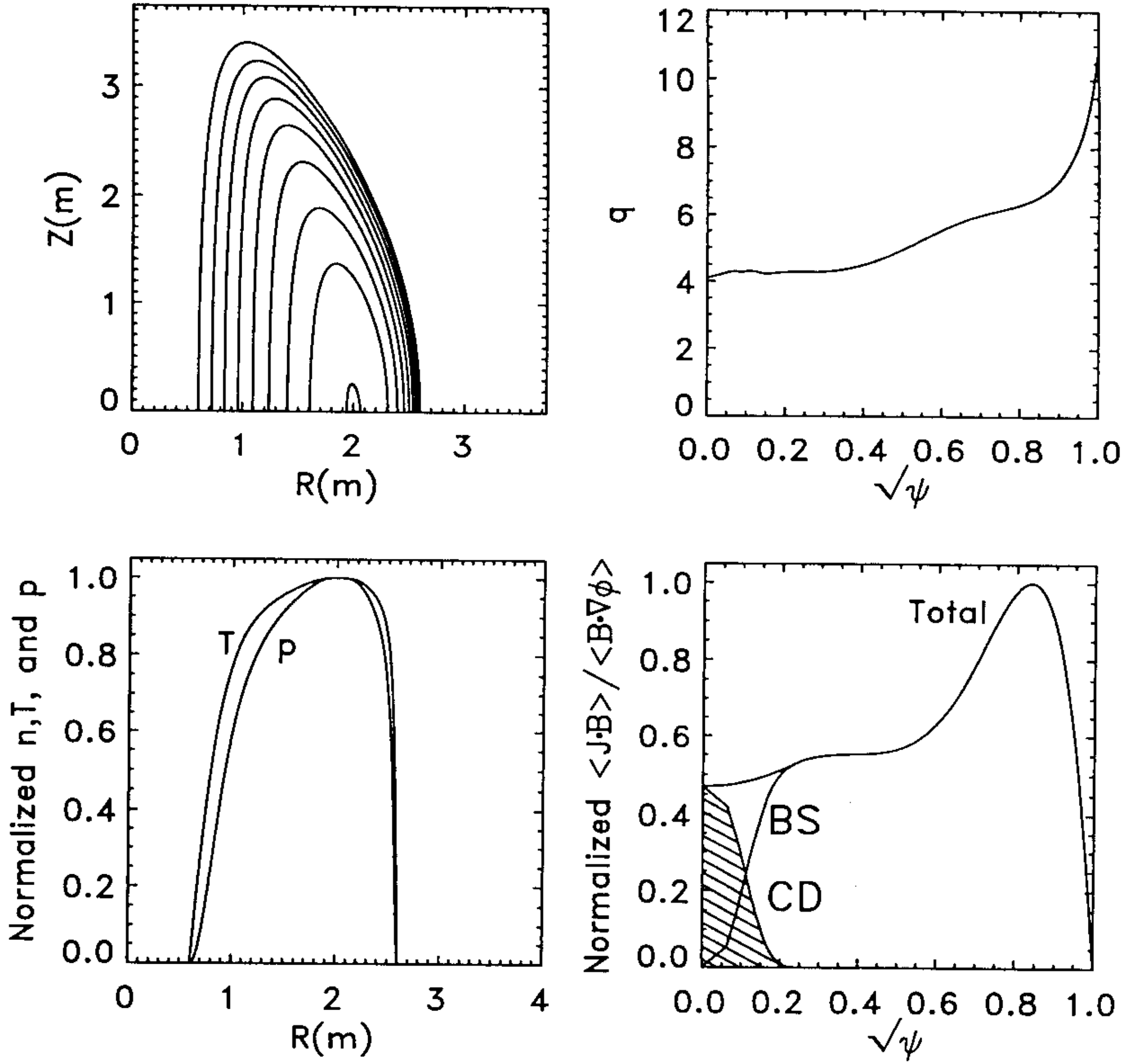
If β_N and κ are assumed to be constant, optimal $A = 2.5\text{-}3$

Optimization consistent with ARIES-ST reactor design

$A = 1.6$
$\kappa = 3.4$
$\delta = 0.64$

$\beta \;\; = 56\%$
$\beta_N = 8.2$
$f_{BS} = 99\%$
$I_P \; = 35MA$

$p(0)/\langle p \rangle = 1.4$



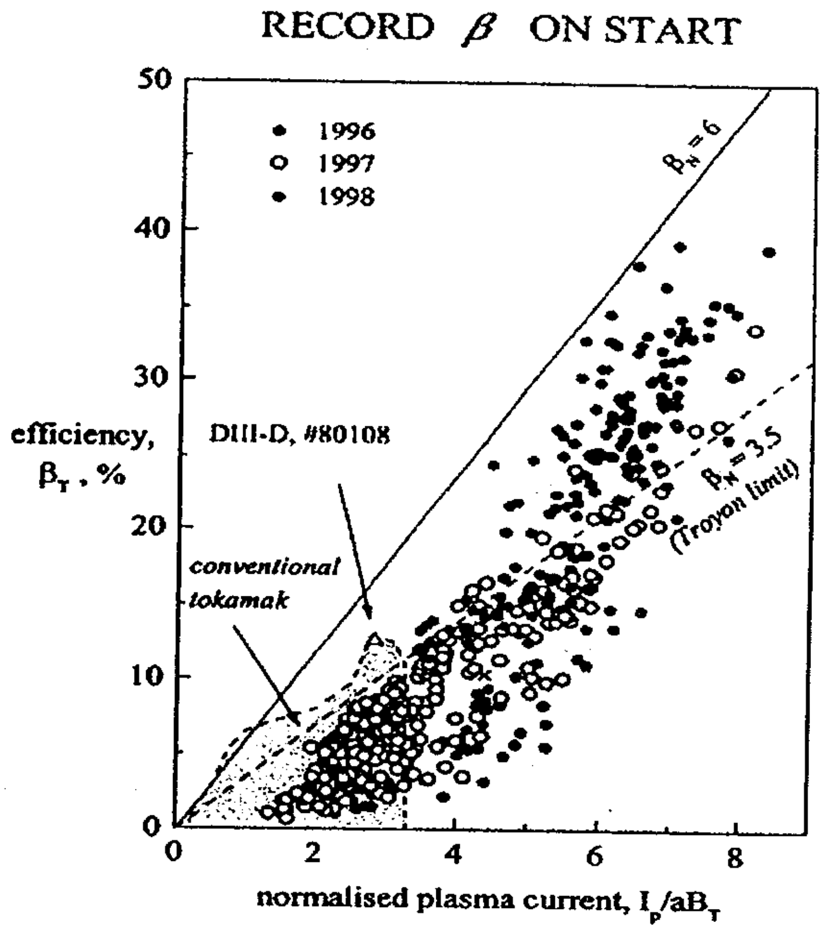
Again, several simultaneous MHD constraints determine optimal equilibria

- Maximum κ limited by n=0 mode and n=1 pressure driven kink mode
- High δ helps stabilize low-n pressure-driven external kink modes
- Maximum β_N limited by edge-localized ballooning modes

- High f_{BS} minimizes re-circulating power and is essential for economical reactor design
- $\beta \propto (1+\kappa^2)(\beta_N)^2$ for fixed f_{BS}, and maximum κ and β_N are aspect ratio dependent $\Rightarrow$ aspect ratio A=1.6 maximizes P_{fusion}/P_{TF}

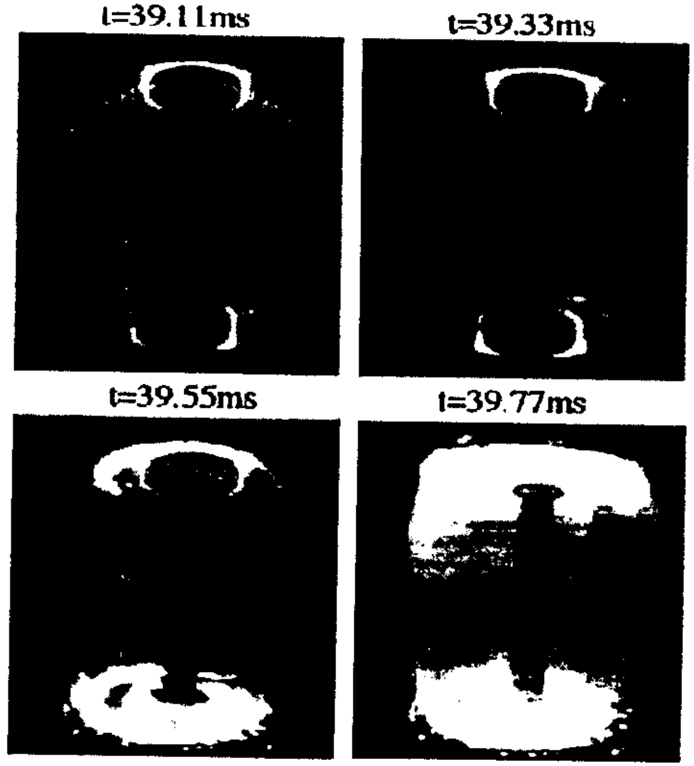
- Broad pressure profile places region of high pressure gradient near conducting wall to aid kink stabilization
- Wall stabilization and/or active feedback is essential for high β

START record β_N is 5-6 at high β



(Figure taken from the START web page)

High q operation may help the ST, as low $q^* \approx$ 2-3 discharges from START can terminate from n=1 external kinks



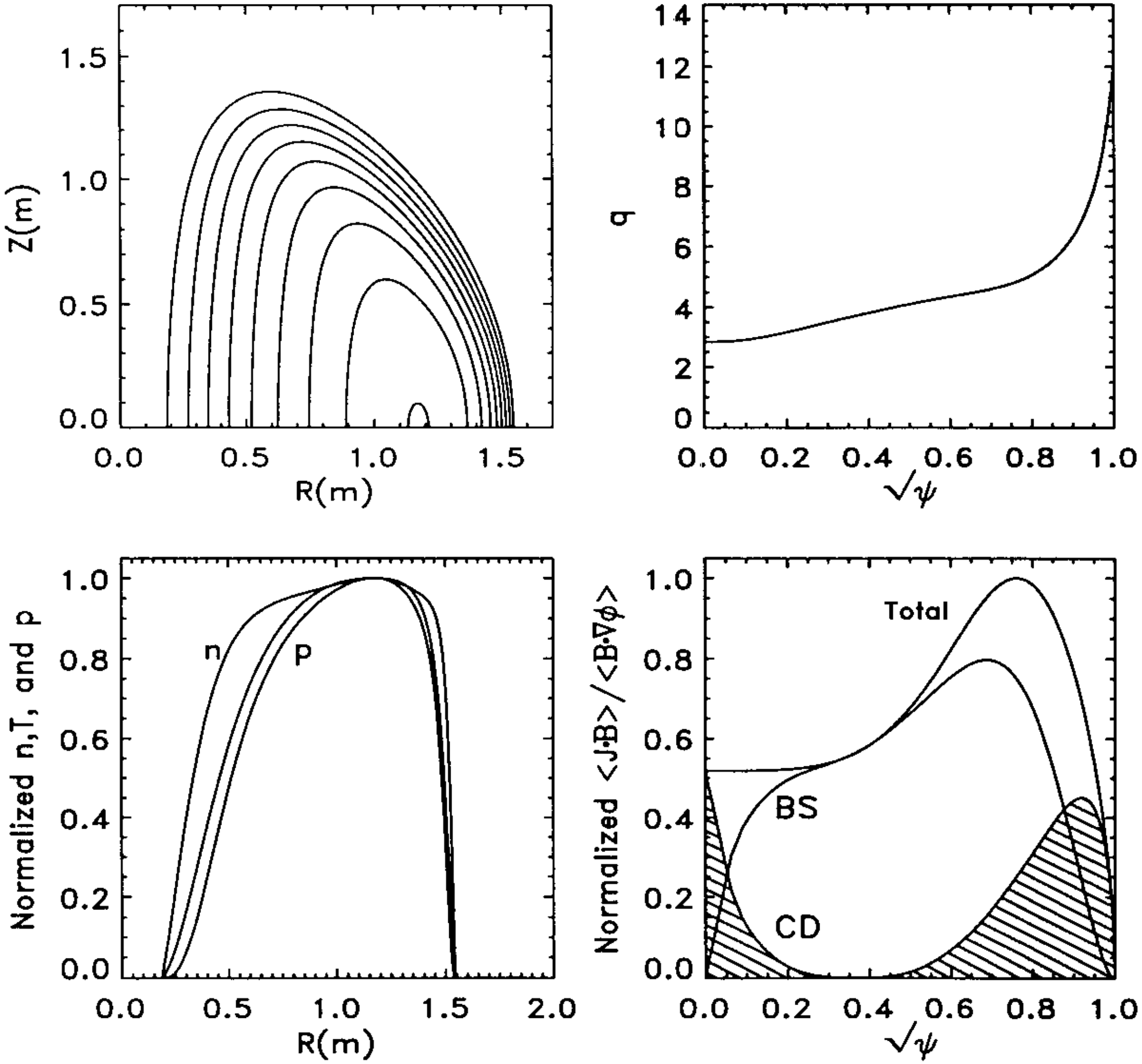
(Figure taken from D. Gates, et al., Phys. Plasmas, May 1998)

NSTX optimized case has $\beta_N = 8$

$R = 0.86$ m
$a = 0.67$ m
$k = 2.0$
$\delta = 0.45$
$A = 1.27$
$I_p = 1$ MA
$B_T = 0.3$ T
$\beta_N = 8.1$

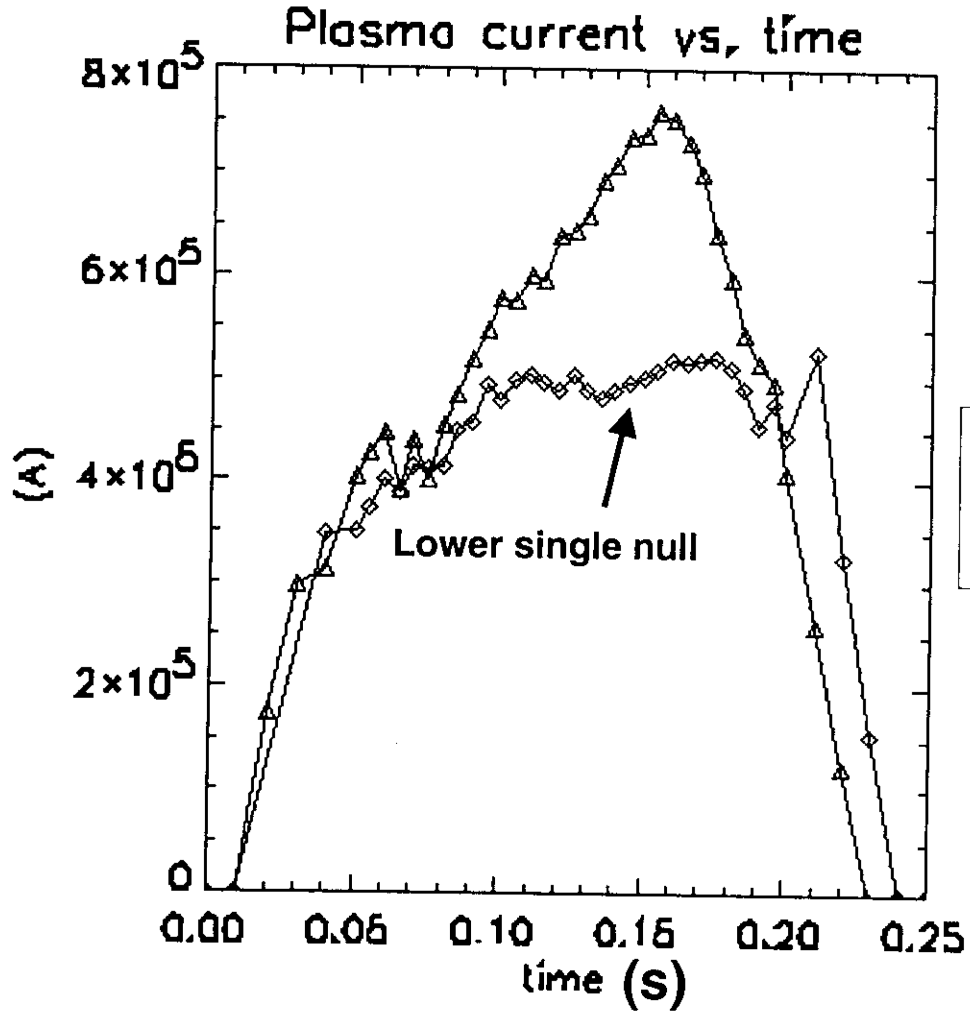
β = 40.4%
f_{BS} = 70%

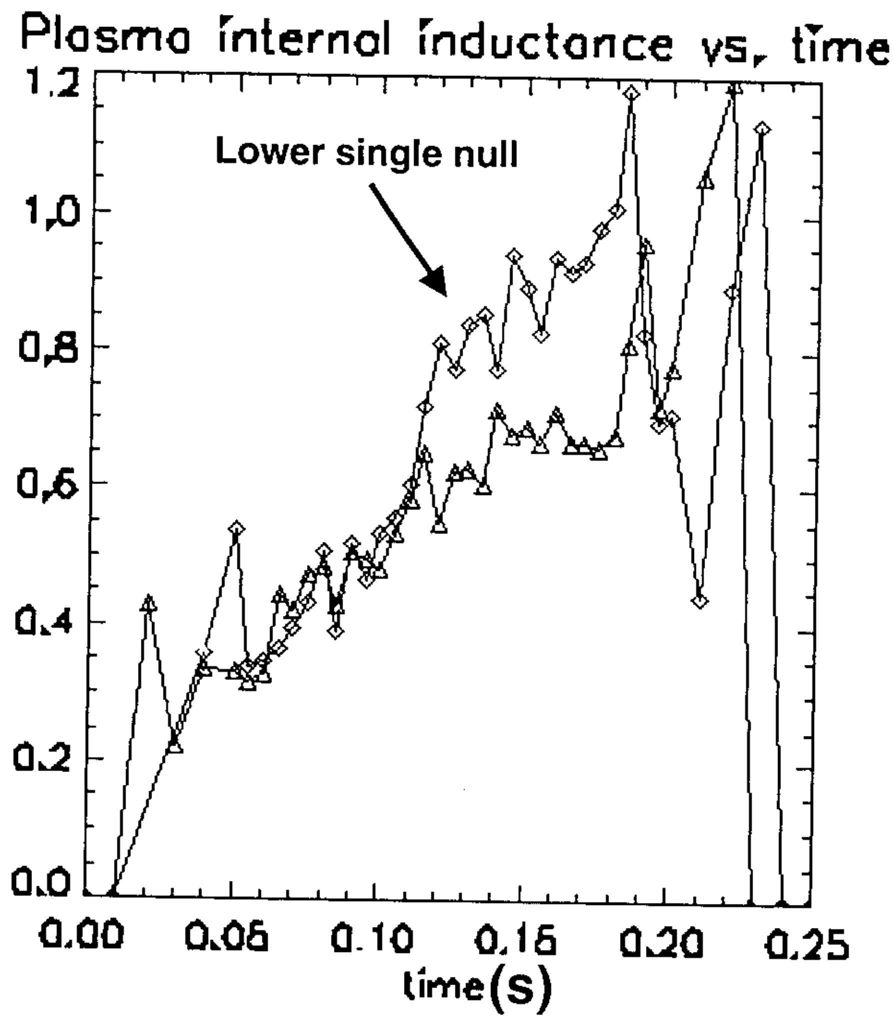
Ideally stable to ballooning and n=1-6 kinks with NSTX passive plate structure



This case has very low $l_i \approx 0.2$

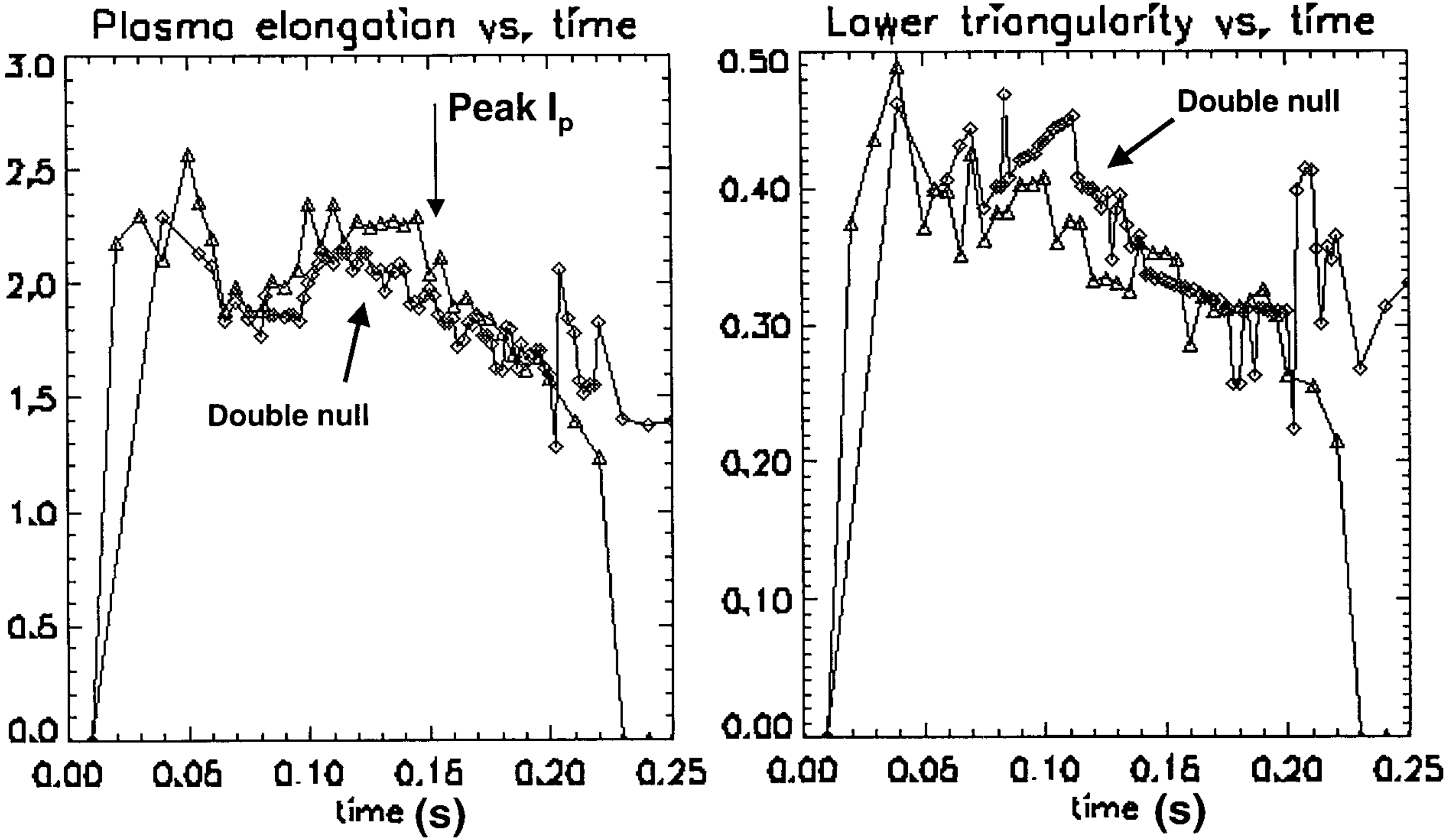
Internal inductance relatively low in 0.8 MA plasma





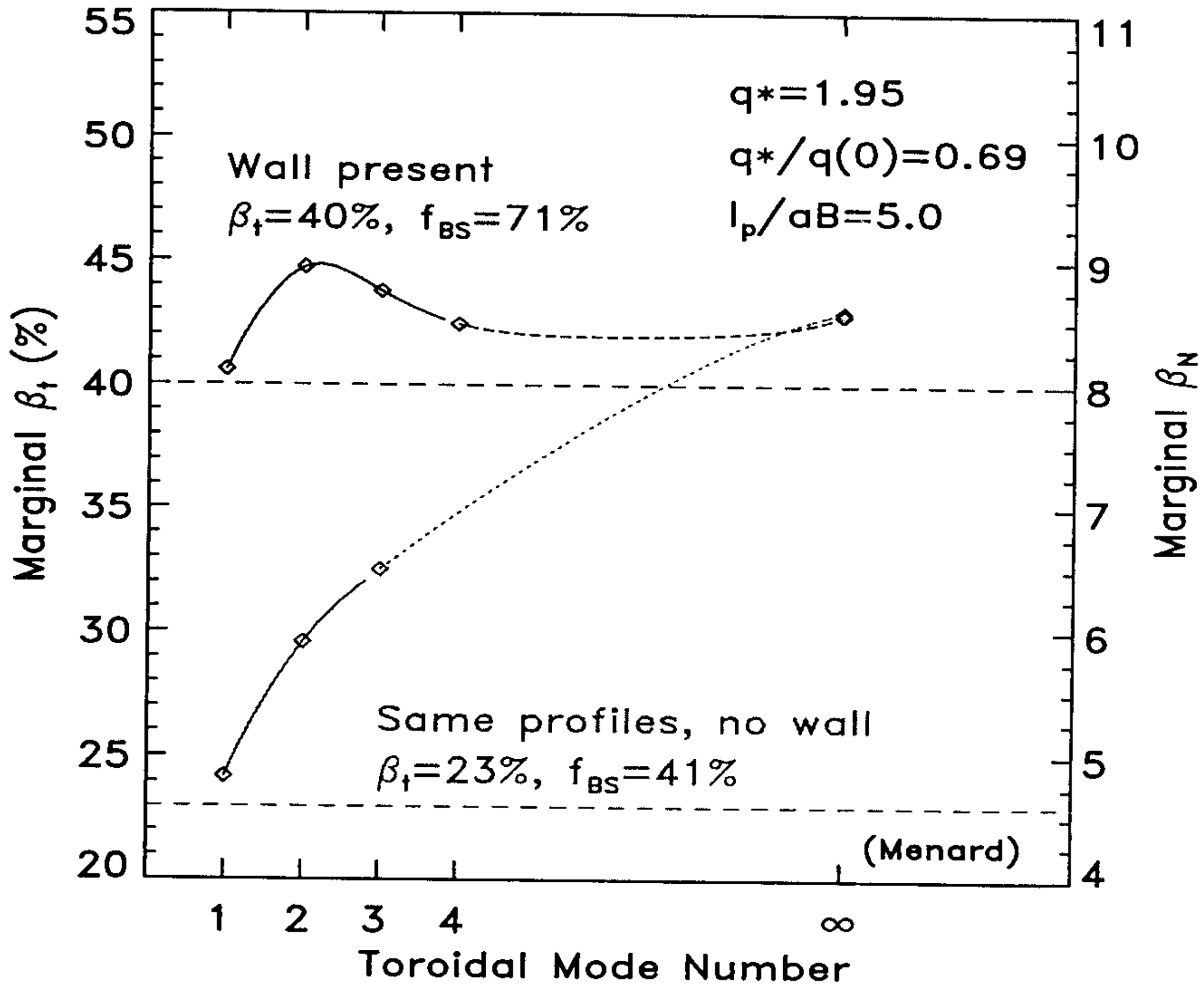
EFIT - S. Sabbagh

Elongation remains high in 0.8 MA plasma



EFIT - S. Sabbagh

β drops to 23% without wall stabilization
Can recover to 30% without wall with re-optimization



Operation above no-wall β limit slows plasma rotation in DIII-D

Can active feedback compensate?

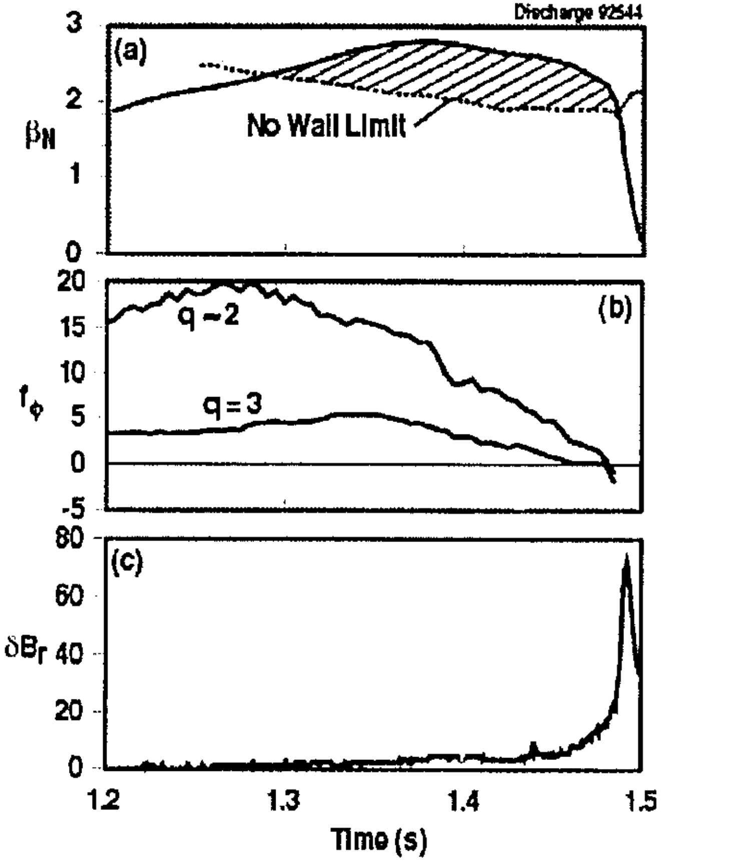


Fig. 1. Time history of a wall-stabilized discharge showing (a) normalized beta; (b) toroidal plasma rotation frequency at two radial locations; (c) and radial magnetic field at the wall due to the RWM.

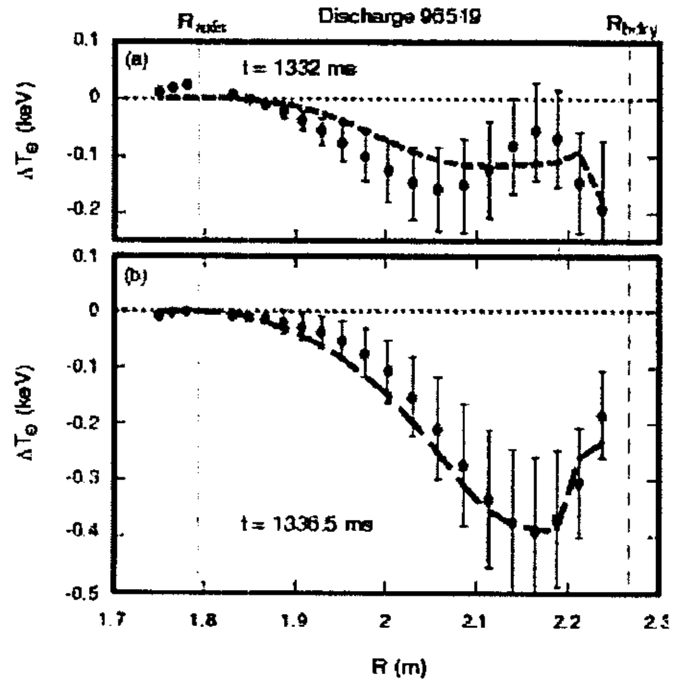


Fig. 2. Comparison of the radial profiles of the measured (data points) and predicted (dashed lines) electron temper-ature perturbation caused by the n=1 RWM in discharge 96519 The predicted change ΔT_e is determined from the displacement ξ by $\Delta T_e \propto \xi \cdot \nabla T_e$.

Data reproduced from:

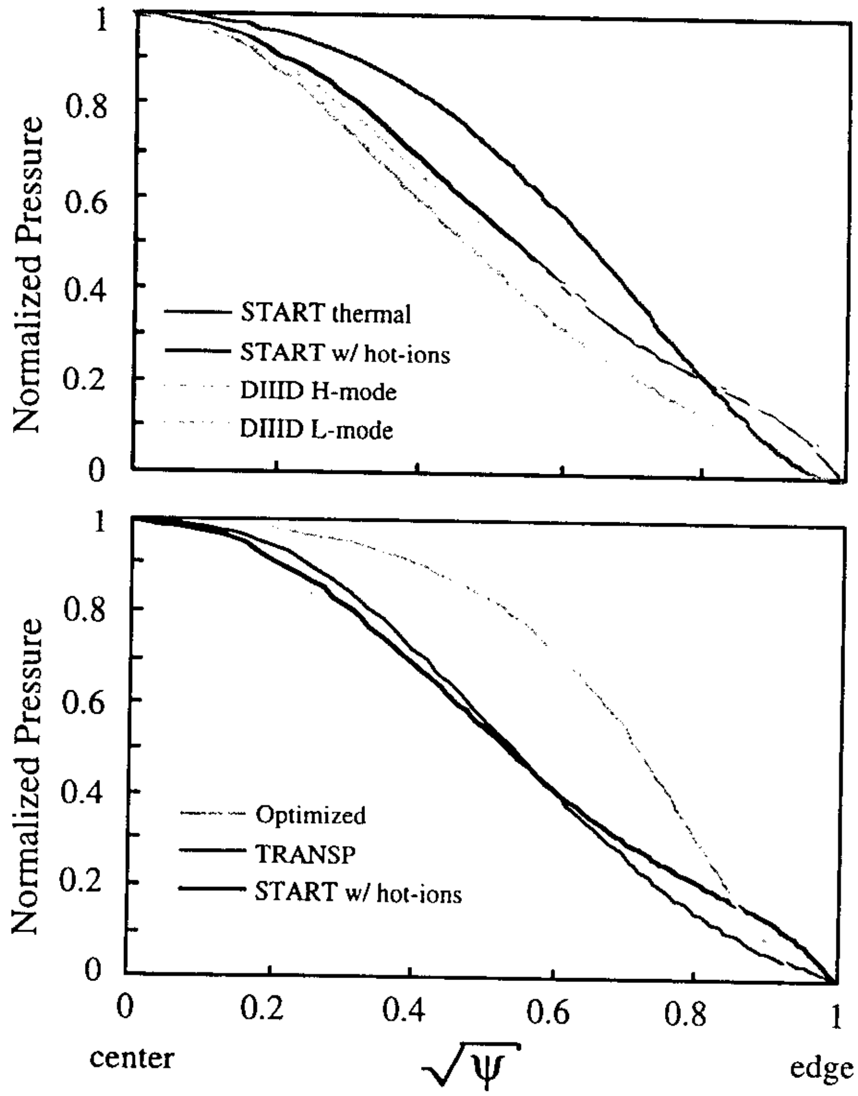
DIII–D TECHNICAL BULLETIN

Number 9 August 11, 1999 A23064

Observation of the Resistive Wall Mode

A.M. Garofalo, Columbia U.

Beam-heated plasma pressure profile may be incompatible with optimal stability properties



START thermal pressure profile looks very similar to optimized 40% β equilibrium pressure profile:

Off-axis auxiliary heating may yield optimal wall-stabilized kink stability ⇒ **use HHFW to broaden p profile?**

F. Paoletti, S. Sabbagh

Neoclassical Tearing Modes

(follows Kruger, et al., Phys. Plasmas, Vol. 5, No. 2, February 1998)

Island evolution: $$\frac{d}{dt} w = 1.22 \frac{\eta_{nc}}{\mu_0} \left(\Delta' + 4.6 \frac{D_{nc} + D_R}{w} \right)$$

(large island limit)

$\Delta' \sim -2m/r_s \propto m$ $\Rightarrow$ Instability favors low poloidal mode number m

$$D_{nc} = -\frac{1.5 f_t / f_c}{1 + 1.5 f_t / f_c} \frac{q\, dp/dq}{\langle B_\theta^2 \rangle / 2\mu_0}$$

$$D_R = D_I + (H - 1/2)^2,$$

$$D_I = E + F + H - 1/4,$$

$$E = C_1 (p_\psi / q_\psi^2) + C_2 (p_\psi / q_\psi)$$

$$F = C_3 (p_\psi / q_\psi)^2,$$

$$H = C_4 (p_\psi / q_\psi),$$

$$C_1 = \left(\frac{V_\psi}{2\pi} \right)^2 \left(\frac{I_\psi}{I} - \left\langle \frac{d}{d_\psi} \ln(R^2) \right\rangle \right),$$

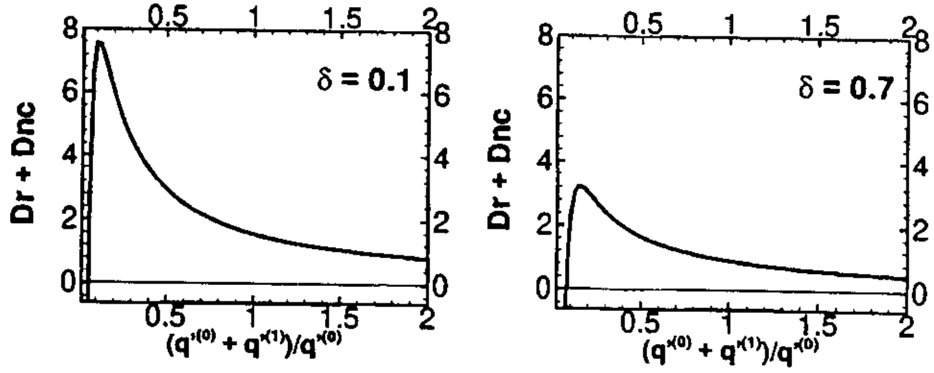
$$C_2 = \frac{V_\psi}{2\pi} \left(\frac{I}{\langle B^2 \rangle} \left\langle \frac{B^2}{|\nabla \psi|^2} \right\rangle - \frac{V_\psi}{2\pi q} \right),$$

$$C_3 = \left(\frac{V_\psi}{2\pi} \right)^2 \left(\left\langle \frac{R^2}{|\nabla \psi|^2} \right\rangle \left\langle \frac{B^2}{|\nabla \psi|^2} \right\rangle - I^2 \left\langle \frac{1}{|\nabla \psi|^2} \right\rangle^2 \right)$$

$$C_4 = \frac{V_\psi}{2\pi} I \left(\left\langle \frac{1}{|\nabla \psi|^2} \right\rangle - \frac{1}{\langle B^2 \rangle} \left\langle \frac{B^2}{|\nabla \psi|^2} \right\rangle \right),$$

Neoclassical Tearing Modes

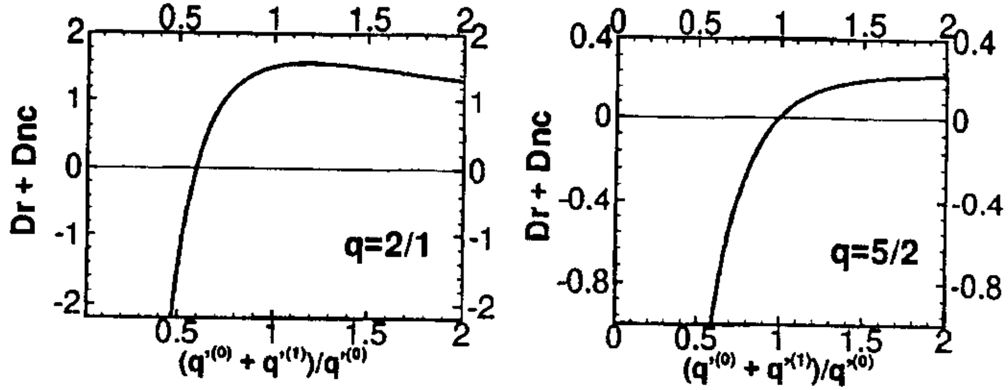
(from Kruger, et al., Phys. Plasmas, Vol. 5, No. 2, February 1998)



DIII-D β=3%

$$D_{\mathrm{nc}} \sim \frac{\sqrt{\epsilon}\beta_\theta}{\hat{s}}; \quad D_R \sim \frac{-\beta}{\hat{s}^2}$$

$$\hat{s} \equiv 2V/q(dq/dV)$$



Pegasus β=15%

Larger β at low A increases range of stable positive shear and decreases unstable drive

Summary

- MHD stability of high-β, high f_{BS} tokamak equilibria is a smoothly varying function of A
- κ and β_N increase significantly as $\varepsilon \rightarrow 1$
- Troyon's original definition $C_T \equiv \langle\beta\rangle a B_{t0}/I_P$ is a good stability invariant for these equilibria.
- Results validate the design point for ARIES-ST
- Wall stabilization is important for NSTX and the ST reactor concept - will it work?
- Neoclassical tearing mode scalings look favorable for low aspect ratio.