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The Madison Dynamo Experiment

C. FOREST

University of Wisconsin
Department of Physics
Madison, U.S.A.

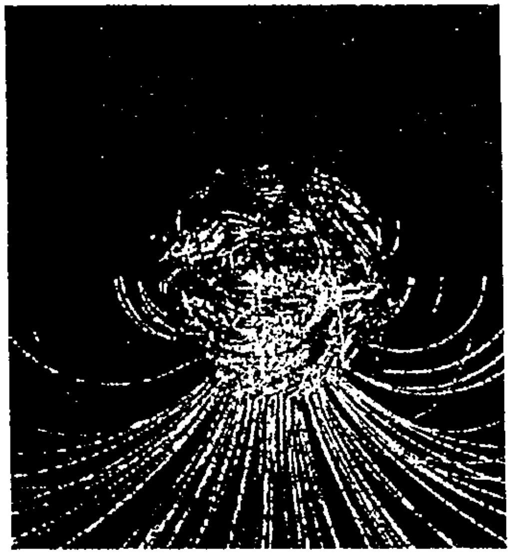
These are preliminary lecture notes, intended only for distribution to participants.

strada costiera, 11 - 34014 trieste italy - tel +39 0402240111 fax +39 040224163 - sci_info@ictp.trieste.it - www.ictp.trieste.it

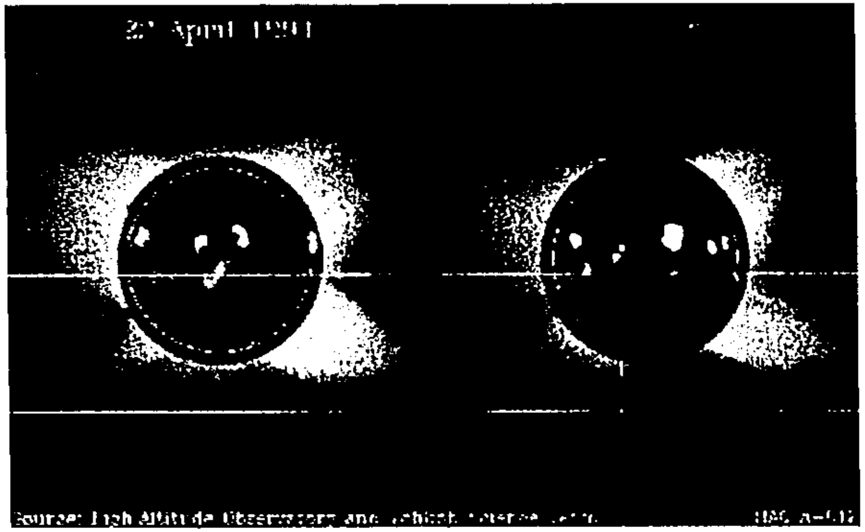
The Madison Dynamo Experiment

Cary Forest
University of Wisconsin
Department of Physics

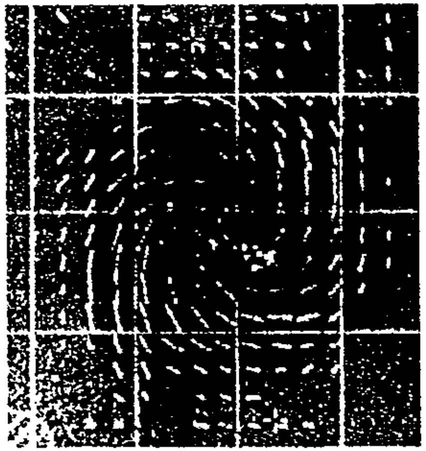
Planets, stars and perhaps the galaxy all have magnetic fields produced by dynamos



Glatzmaier Roberts simulation of geodynamo



Solar prominences and x-rays from sunspots

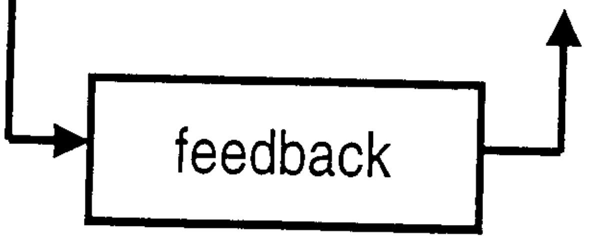


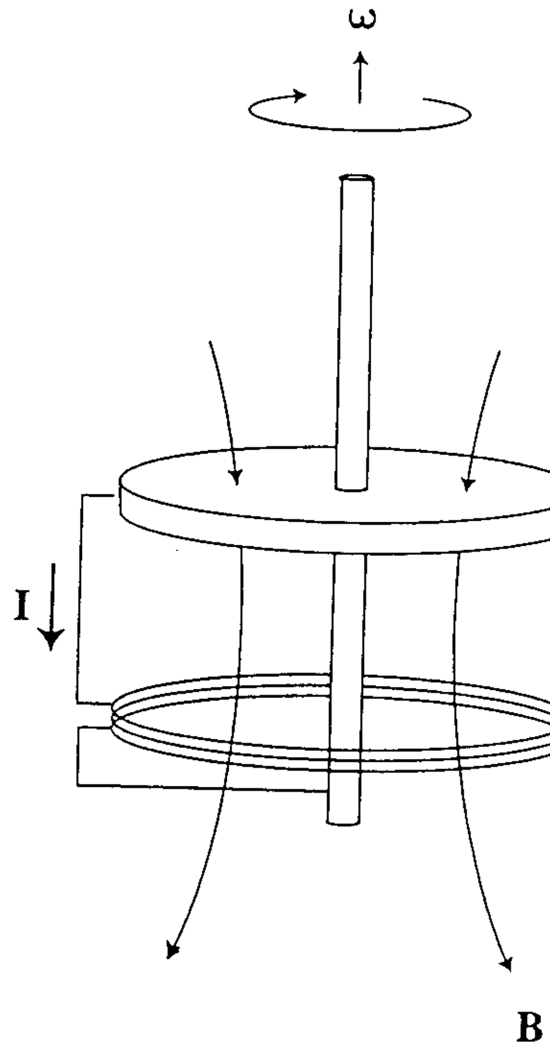
Magnetic fields observed in M83 spiral galaxy

Dynamos spontaneously generate magnetic energy from mechanical energy

- This is easy if you allow yourself the luxury of using insulators and solid conductors
- In a conductor, currents are generated by motion across a magnetic field

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$





In astrophysical dynamos the conductors are simply connected (no insulators) and can flow

- Plasmas or liquid metals
- Magnetohydrodynamics: systems are describe by two vector fields:
  - The magnetic field, is generated by electrical currents in the conducting fluid

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

→

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Induction — Resistive diffusion

-The velocity field evolves according to Navier-Stokes + electromagnetic forces

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)\mathbf{V} = \mathbf{J} \times \mathbf{B} - \nabla p + \nu\rho\nabla^2 V$$

Magnetic Reynolds number quantifies induction by V vs resistive diffusion through ohmic dissipation

- Dimensional analysis of magnetic induction equation
  - Characterisitic scale: a
  - Characteristic velocity: V_0
  - Characteristic time: $t = \mu_0 \sigma a^2$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\rightarrow \quad \frac{\partial \mathbf{B}}{\partial \hat{t}} = Rm \hat{\nabla} \times \hat{\mathbf{V}} \times \mathbf{B} + \hat{\nabla}^2 \mathbf{B}$$

$$Rm \equiv \mu_0 \sigma a V_0$$

In a large *Rm* system, magnetic field is frozen into moving fluid

- Compute flux change inside a closed loop moving with fluid

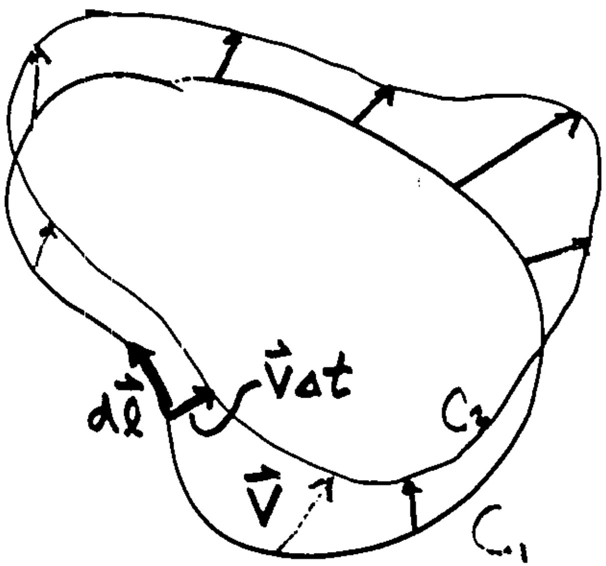
$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=\eta J$$

$$\eta \rightarrow 0 \Rightarrow \mathbf{E}=-\mathbf{V}\times\mathbf{B}, \quad \frac{\partial \mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}$$

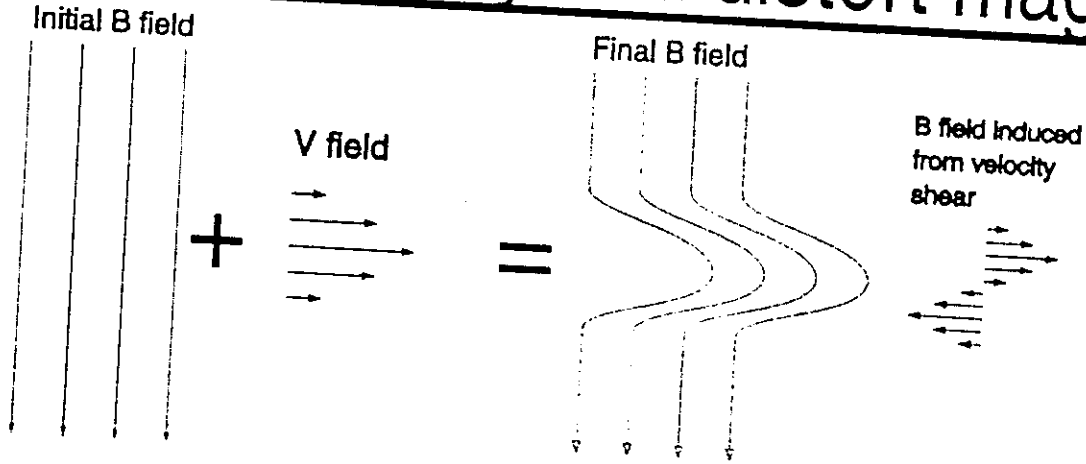
$$\frac{\partial \Phi}{\partial t}=\int_S \frac{\partial \mathbf{B}}{\partial t}\bullet d\mathbf{S}+\oint_C \mathbf{B}\bullet\frac{d\mathbf{S}}{dt}$$

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times\mathbf{V}\times\mathbf{B}, \quad \frac{\partial \mathbf{S}}{\partial t}=\mathbf{V}\times d\mathbf{l}$$

$$\frac{\partial \Phi}{\partial t}=\oint_C \mathbf{V}\times\mathbf{B}\bullet d\mathbf{l}+\oint_C \mathbf{B}\bullet\mathbf{V}\times d\mathbf{l}=0$$



Fluid flow can amplify and distort magnetic fields



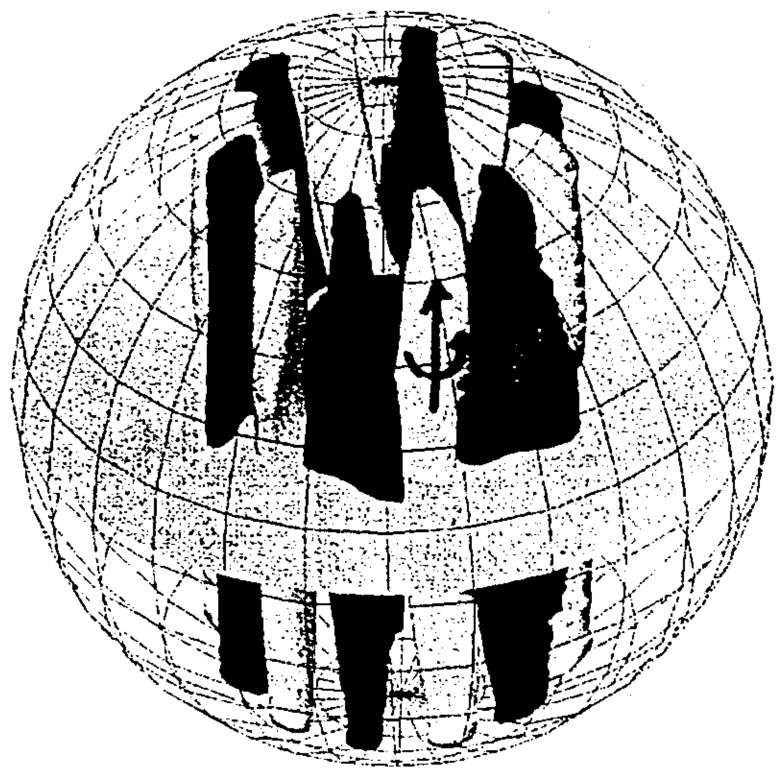
- In a fast moving, or highly conducting fluid, magnetic field lines are frozen into the moving fluid

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \quad (\to 0)$$

- Transverse component of field is generated and amplified
- Finite resistance leads to diffusion of field lines

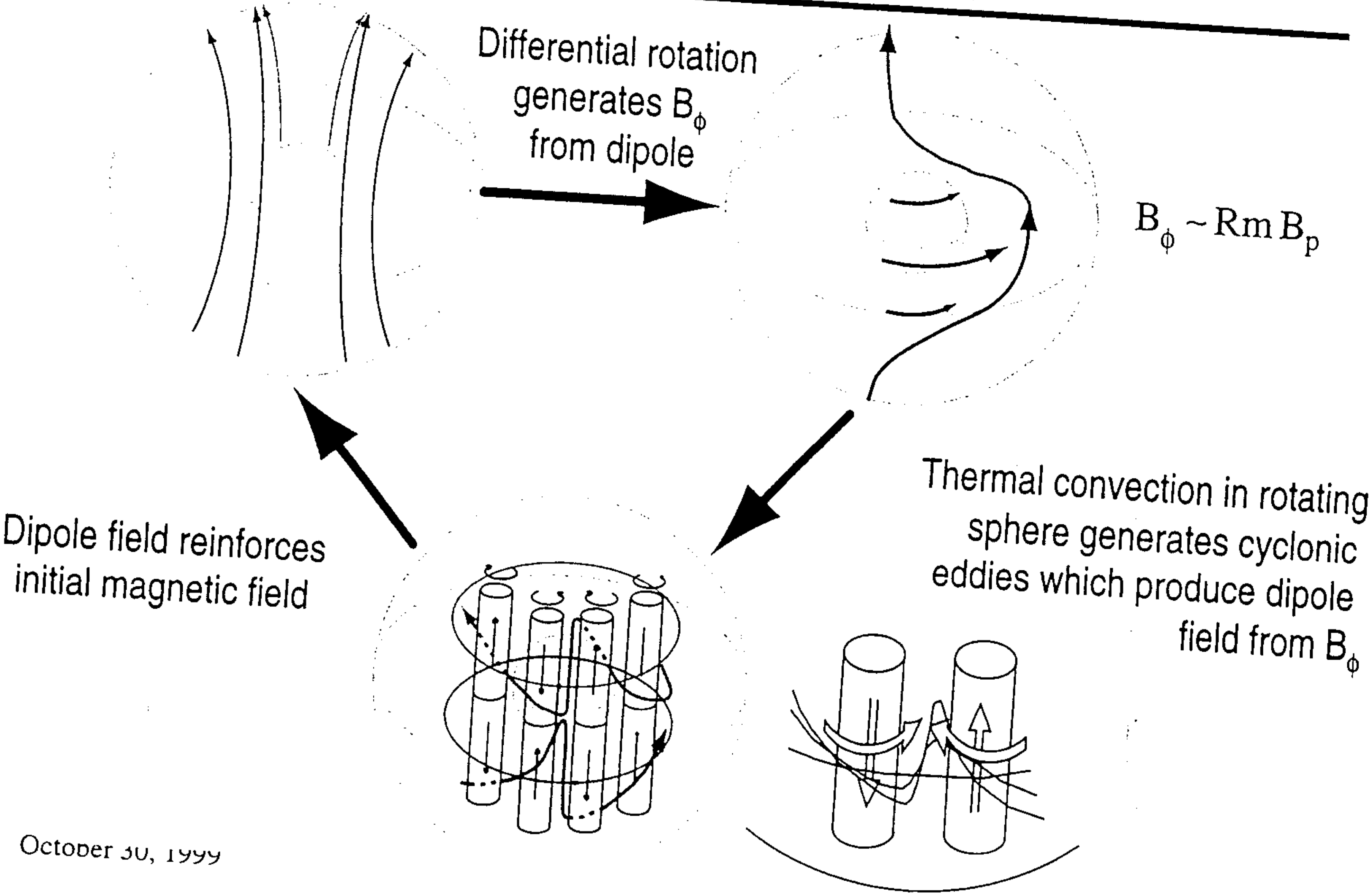
Thermal convection produces helical motion in conducting core of astrophysical systems

- Strong rotation imposes symmetry in z direction (Taylor columns)
- Eckman suction at tips induces axial flow along columns
- Inner core rotates faster (or slower) than planet, so differential rotation exists



Kageyama and Sato

Cartoon of a laminar geodynamo driven by convection and differential rotation



The kinematic dynamo problem

- Start with a sphere filled with a uniformly conducting fluid of conductivity σ, radius *a*, surrounded by an insulating region

- Find a velocity field $\mathbf{v}(\mathbf{r})$ inside the sphere, which leads to growing $\mathbf{b}(\mathbf{r},t)$

- Ignore the back-reaction of magnetic field on flow

The kinematic dynamo problem (continued)

- Transform to dimensionless variables:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\longrightarrow \frac{\partial \mathbf{B}}{\partial \hat{t}} = \hat{\nabla} \times \hat{\mathbf{V}} \times \mathbf{B} + \frac{1}{Rm} \hat{\nabla}^2 \mathbf{B} \qquad \mathrm{Rm} = \mu_0 \sigma a V_{max}$$

- Since its linear in B, use separation of variables:

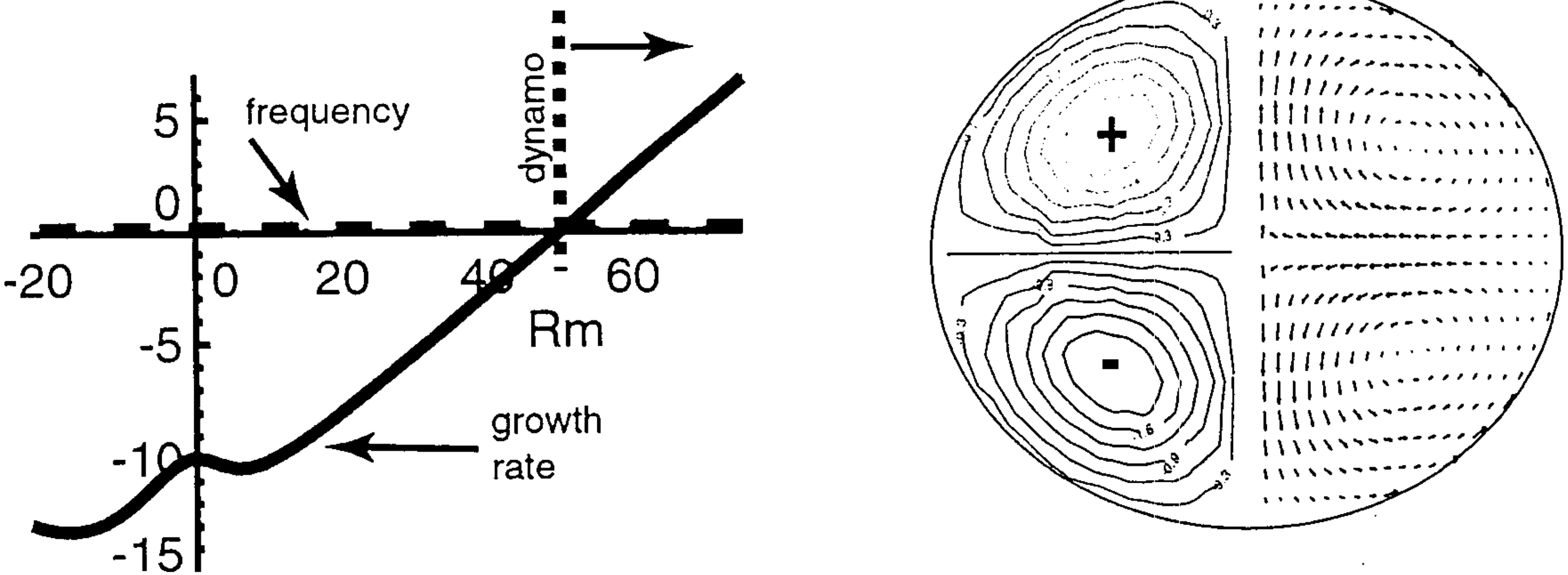
$$\mathbf{B}(\mathbf{r}, t) = \sum_i e^{\lambda_i t} \mathbf{B}_i(\mathbf{r})$$

- Solve eigenvalue equation for given $\mathbf{v}(\mathrm{r})$ profile

$$\lambda_i \mathbf{B} = \nabla \times \mathbf{V} \times \mathbf{B}_i + \frac{1}{Rm} \nabla^2 \mathbf{B}_i$$

Simple axisymmetric flows have been shown to be dynamos

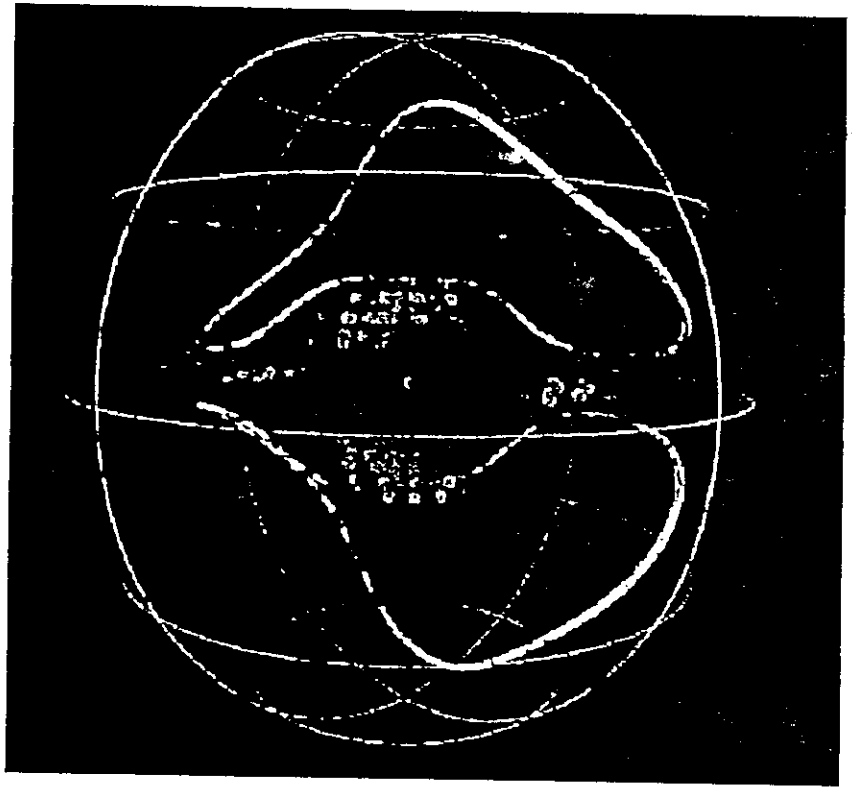
T2 S2



Axisymmetric flows do not violate Cowling's Theorem,
since B eigenmode is not axisymmetric

Eigenmodes are three dimensional

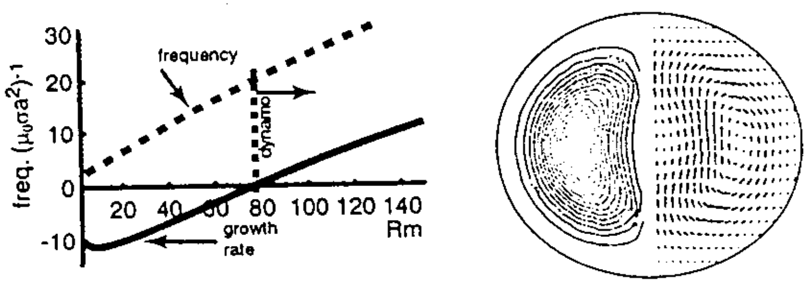
Electrical current stream lines



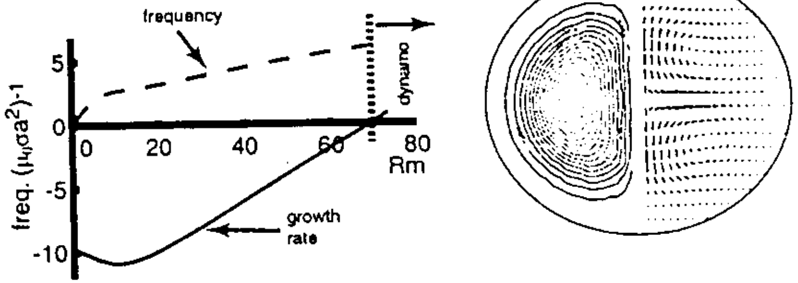
- Dipole points out at equator and rotates in laboratory frame
- Does not violate cowling's theorem

Growing magnetic fields are predicted for simple flow topologies in a sphere

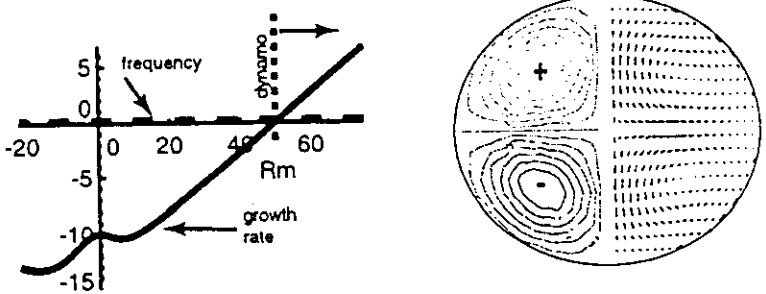
T1 S1



T1 S2



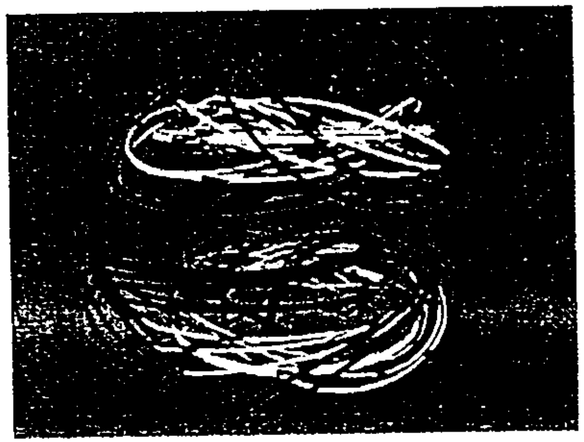
T2 S2



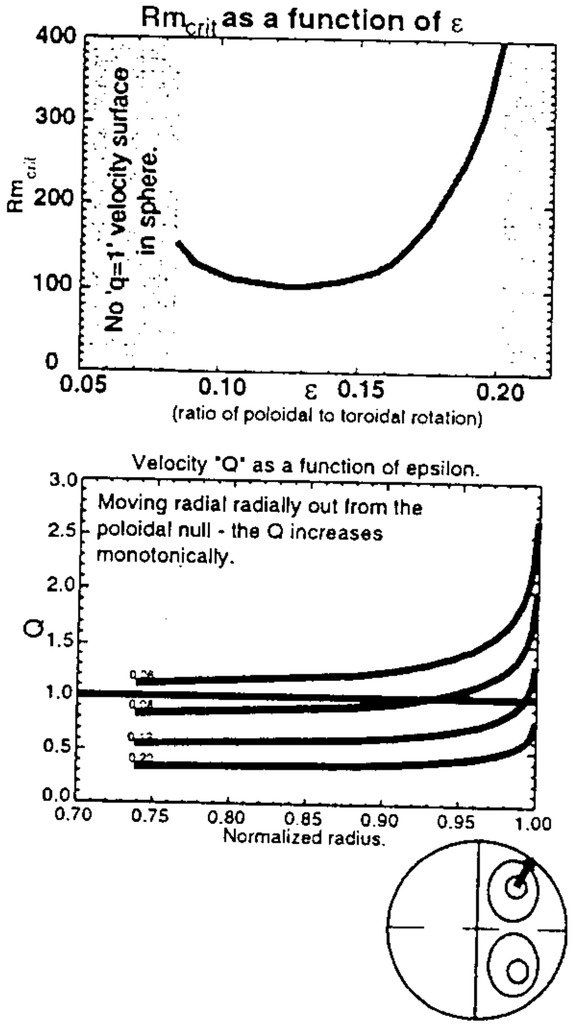
- Induction equation is solved numerically
- Flow fields are axisymmetric
- Growth rate depends upon

$$Rm = \mu_0 \sigma a V_{max}$$

conductivity X size X velocity



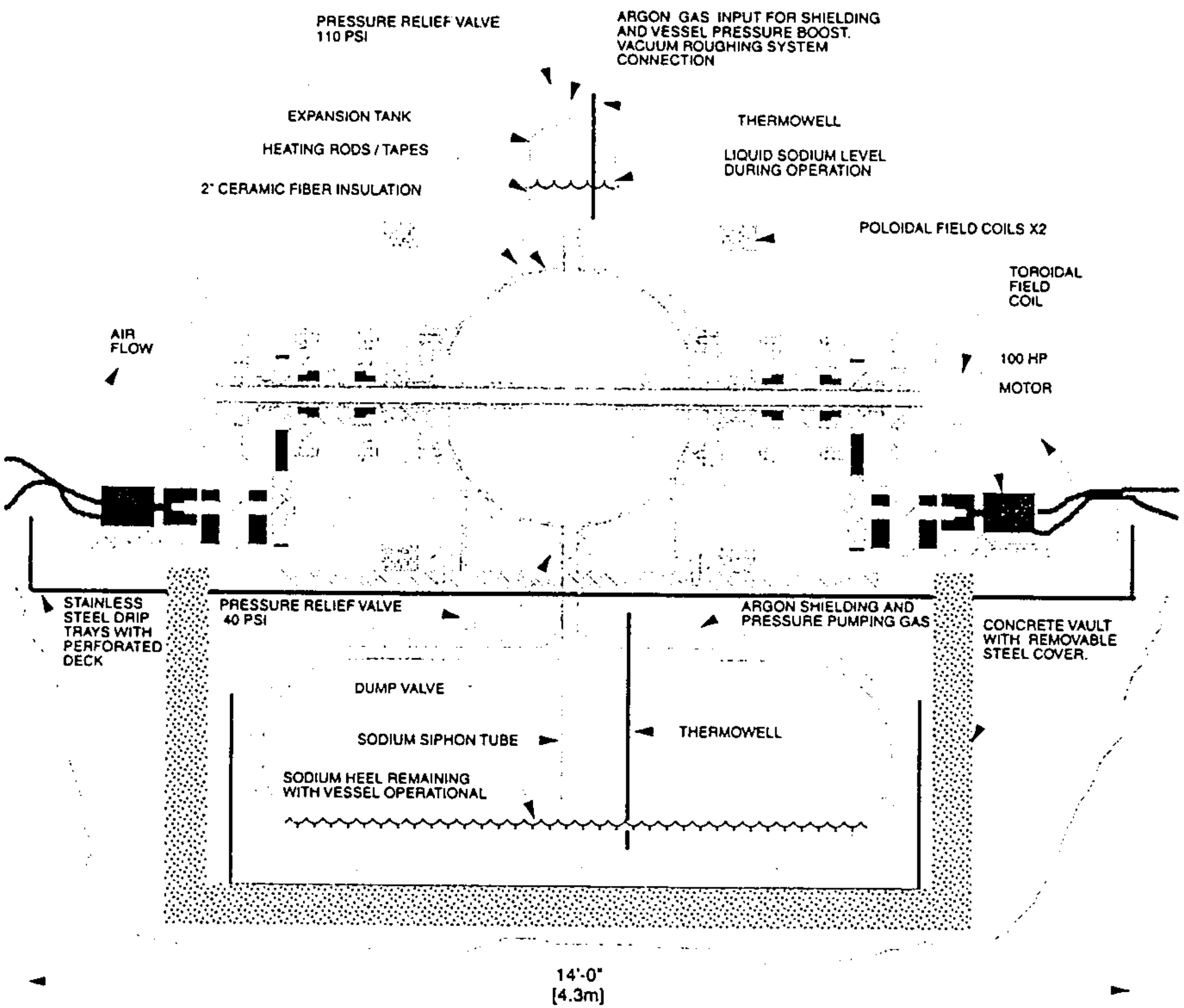
Dynamo solutions are sensitive to details of flow profiles

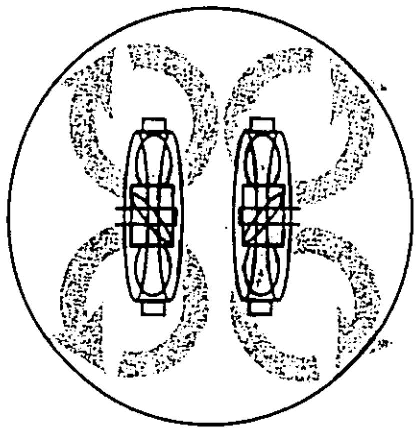


- Ratio of toroidal to poloidal flow affects dynamo solution

The dynamo experiment

- 1 m diameter
- 200 hp
- Sodium
- Rm=200





Why sodium?

- The control parameter is the magnetic Reynolds number $Rm = \mu_0 \sigma a V_{max}$

  conductivity X size X velocity

  - Quantifies relative importance of generation of B field by velocity and diffusion (decay) of B due to resistive decay of electrical currents
  - Must exceed critical value for system to self-excite
- Sodium is more conducting than any other liquid metal (melts at 100 C)
  - Rm = 120 for a=0.5 m, V_{max}=15 m/s

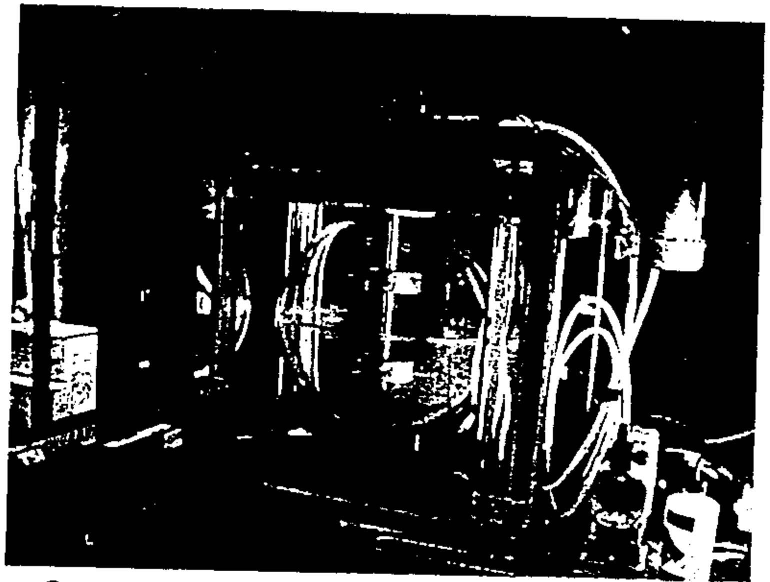
The dynamo experiment will address several dynamo issues experimentally

- Do dynamically consistent flows exist for kinematic dynamos?
- How does a dynamo saturate?
  - Role of Lorentz force on fluid velocity
- What role does turbulence play in a real dynamo?
  - Energy equipartition of velocity fields and magnetic fields
  - Enhanced electrical resistivity
  - Current generation

Water simulates aspects of molten sodium

	Sodium	Water
Temperature	$110°C$	$50°C$
viscosity	$0.65\times10^{-6}\ \mathrm{m^2\,sec^{-1}}$	$0.65\times10^{-6}\ \mathrm{m^2\,sec^{-1}}$
mass density	$0.925\ \mathrm{gm\,cm^{-3}}$	$0.988\ \mathrm{gm\,cm^{-3}}$
resistivity	$10^{-7}\ \Omega\mathrm{m}$	

$$\longrightarrow \quad Rm = \frac{\mu_0 a V}{\eta} = 4\pi\, a(m)\, V(m/s)$$



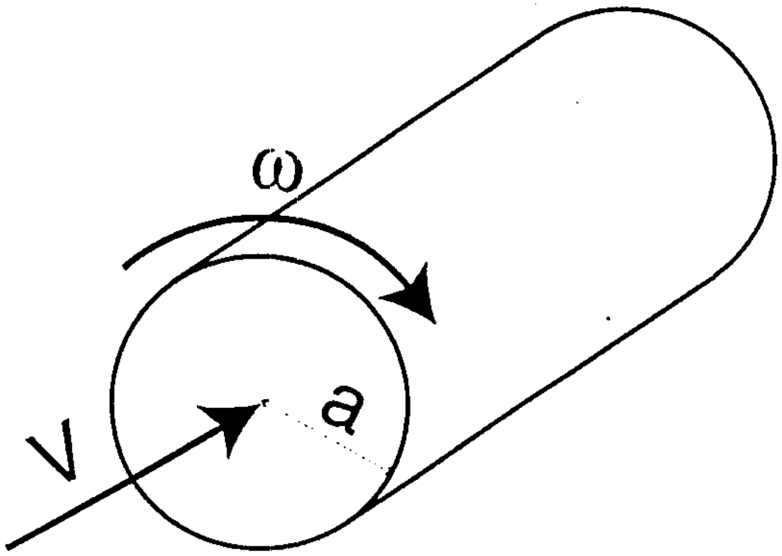
Small water experiment

- Water and sodium have similar hydrodynamic properties
- Water is transparent, allowing optical techniques can be used to measure flow velocity in dimensionally identical configurations
- Measured flows can be numerically tested for dynamo feasibility using MHD simulation

Simplest kinematic dynamo is a flow with twist devised by Ponomarenko

- 1 D flow with helicity (translation and twist)

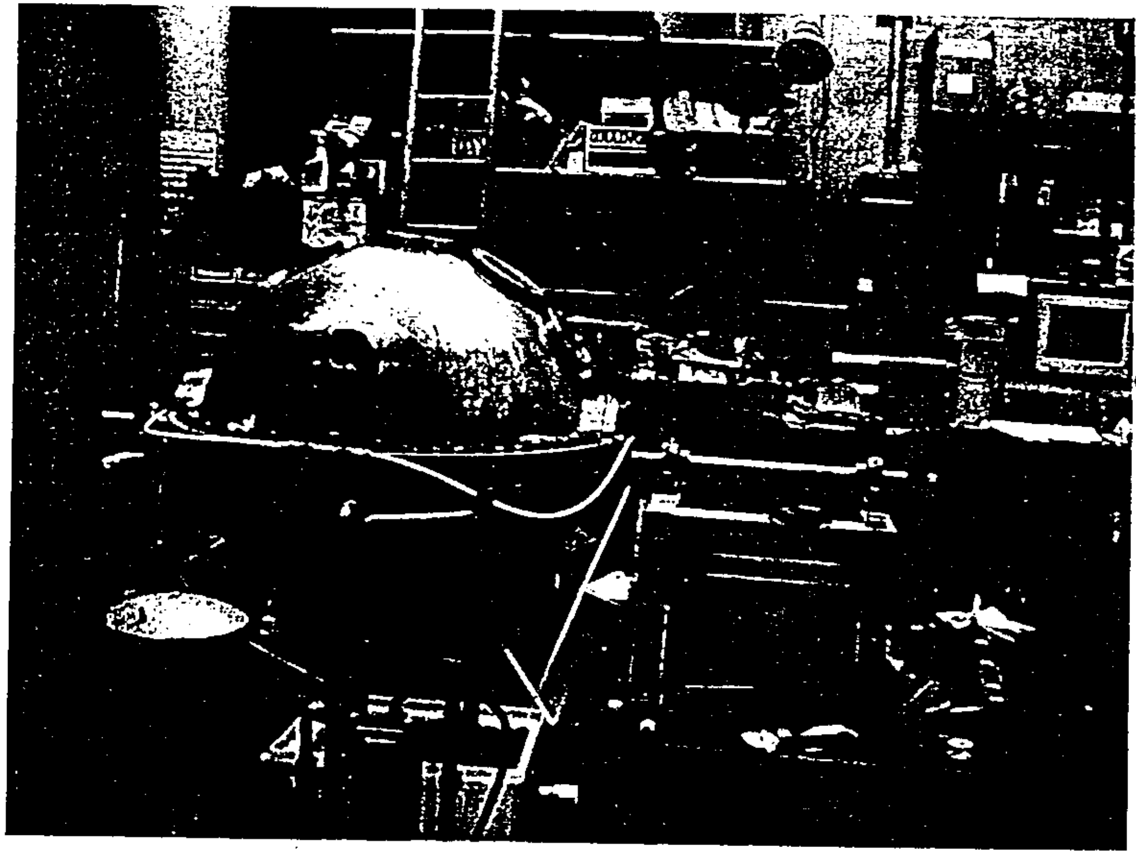
$$H = \mathbf{V} \bullet \nabla \times \mathbf{V}$$



- Unstable 2 D magnetic eigenmode
- Magnetic flux is generated at surface where large flow shear exists
- Critical Rm for excitation is low

$$Rm = \mu_0 \sigma \sqrt{V^2 + \omega^2 a^2}\, a \approx 15$$

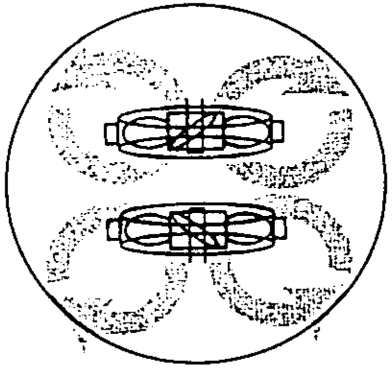
Water experiments are used to create flows and test technology

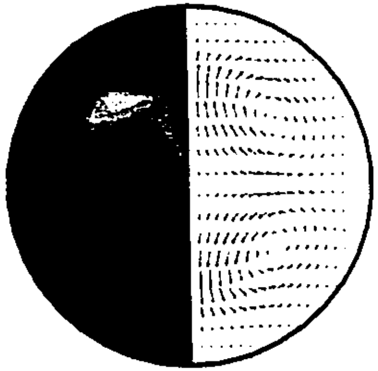


Full scale water experiment (100 Hp), radius=0.5 m

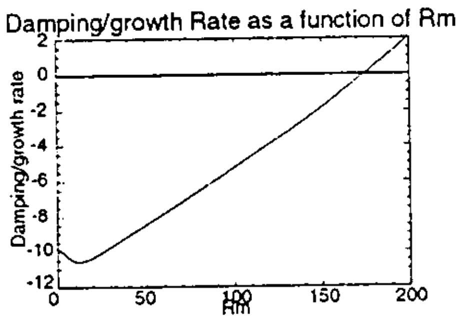
- Laser Doppler velocimetry is used to measure vector velocity field
- Measured flows are used as input to MHD calculation

Measured velocity fields have been produced in water, which are predicted to generate dynamos





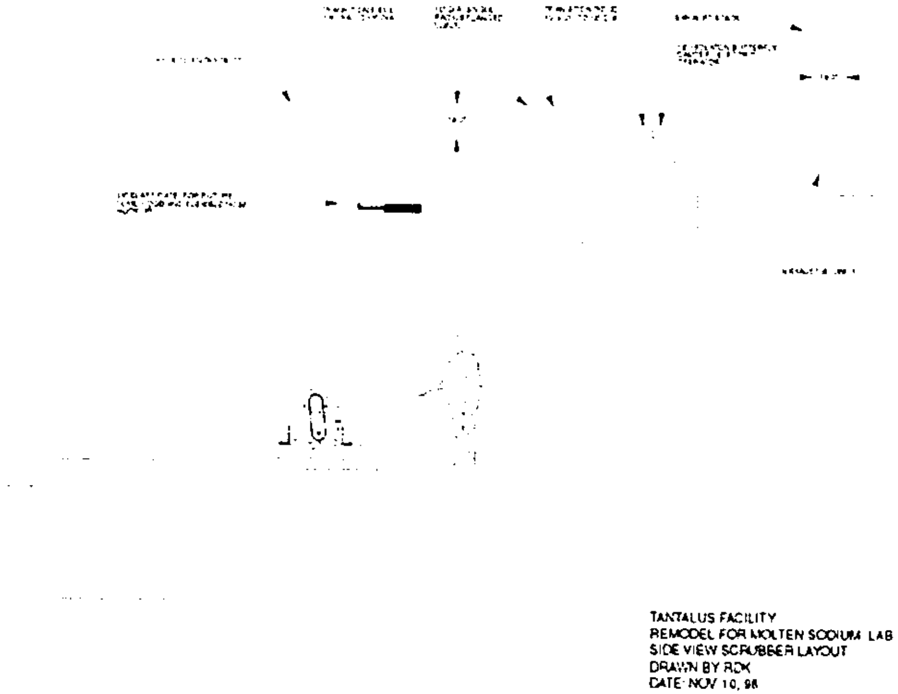
- Many impeller schemes have been tried
- Some extrapolate to dynamos

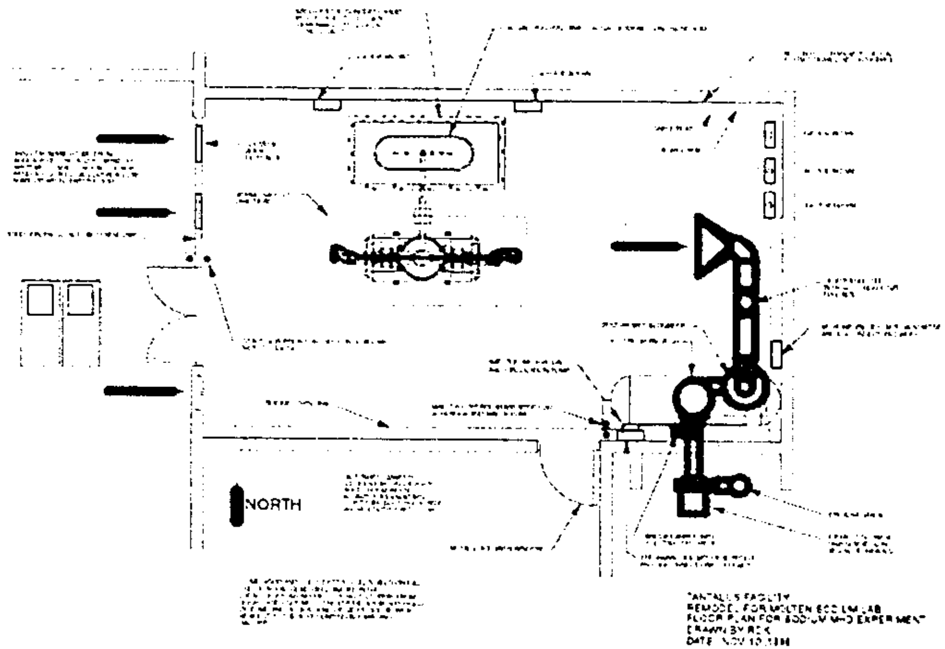


Sodium handling requires special facilities and careful engineering

- A new laboratory has been constructed for housing the dynamo experiment

LABORATORY LAYOUT

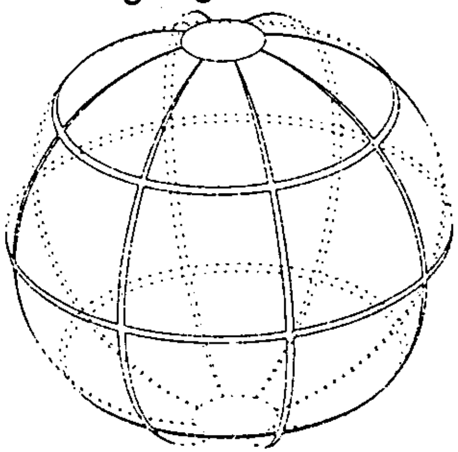


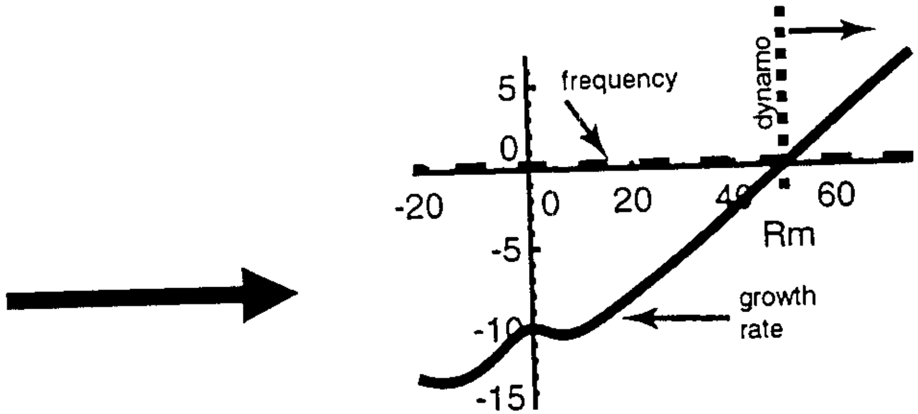


Experiment to characterize eigenmodes

- Use antenna to excite eigenmodes and measure damping rate
- When measured damping rate differs from predicted damping rate, turbulence may be implicated

m=4,n=8 antenna for driving eigenmodes





Remaining lectures

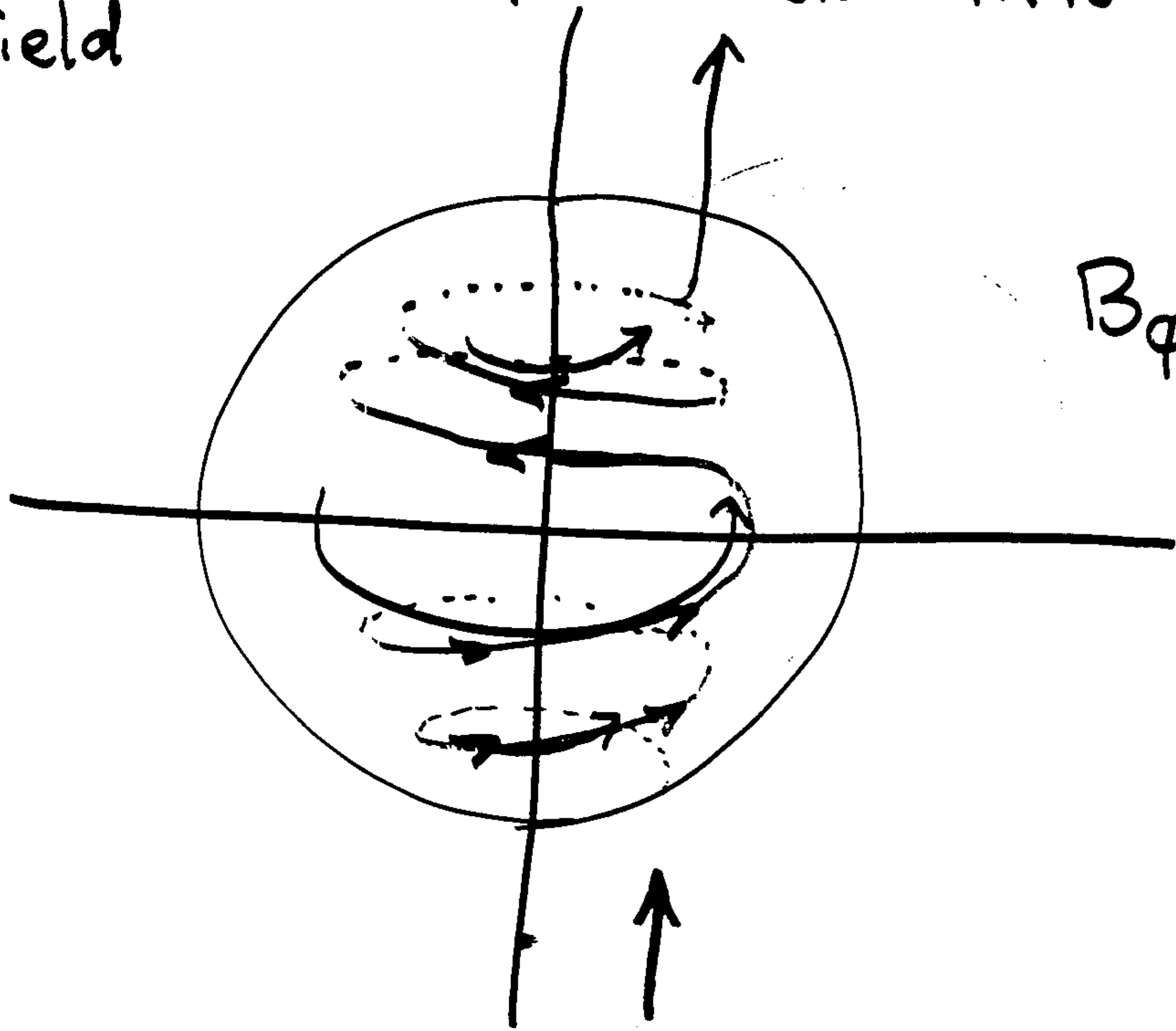
- Questions from dynamo which might be answered by an experiment
  - Eigenmode calculations
  - Mean-field electrodynamics
    - Anomalous resistivity
    - Alpha effect
  - MHD turbulence

- Review of previous experiments which have been done on dynamo questions
- New experiments to address these questions

Summary

- Water experiments have demonstrated flows which may lead to dynamo action
- Sodium laboratory facility is completed
  - Sodium experiment is under construction
  - Sodium operation should start in November
- Initial experiments will search for growing eigenmodes and evaluation of role of turbulence

Differential Rotation in a Sphere Stretches Dipole Field into toroidal Field



$B_\phi \sim Rm\, B_p$