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A Review of Dynamo Physics Questions and Experiments - I

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These are preliminary lecture notes, intended only for distribution to participants.

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A Review of Dynamo Physics Questions and Experiments (Part I)

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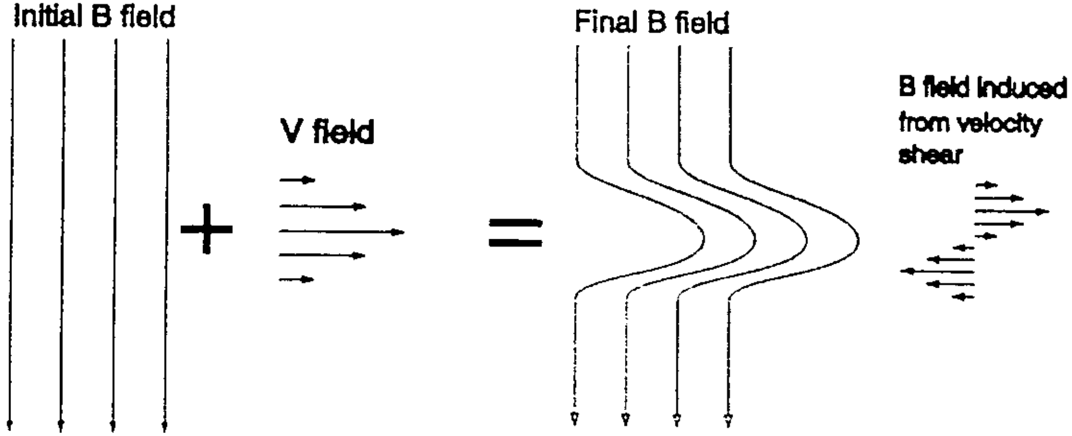
Issues of Dynamo Theory

- The MHD equations
- Frozen-flux and the magnetic Reynolds number
- Cowling's theorem
- Laminar dynamo solutions
- Mean-field electrodynamics
  - The a-effect, the b-effect
- MHD turbulence
  - Small scale dynamos
  - The Kraichnan-Irishnikov spectrum of MHD turbulence

-Nonlinear backreaction

Fluid Flow Can Amplify and Distort Magnetic Fields

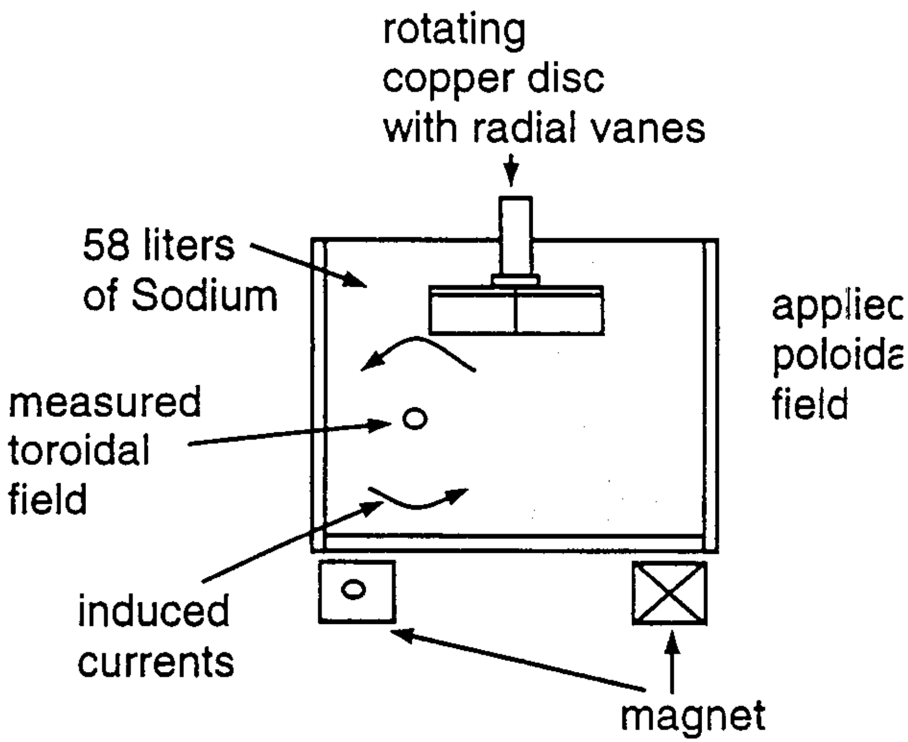


- In a fast moving, or highly conducting fluid, magnetic field lines are frozen into the moving fluid

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \quad (\text{last term} \to 0)$$

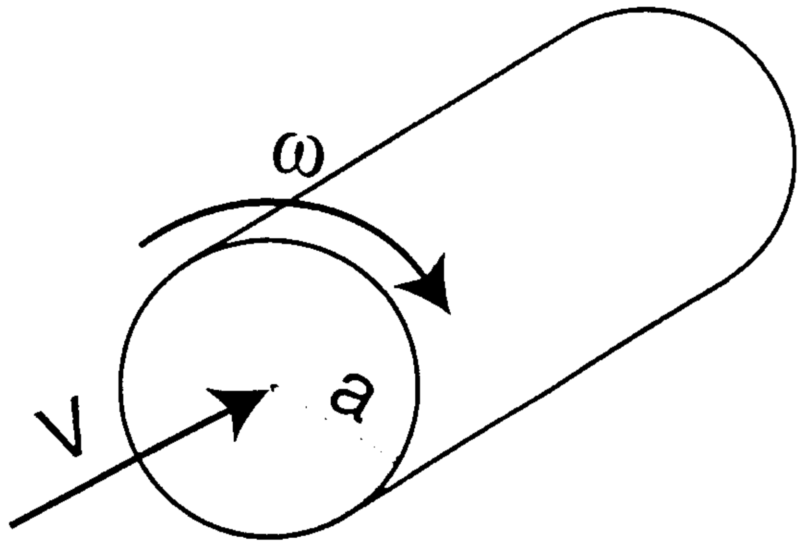
- Transverse component of field is generated and amplified
- Finite resistance leads to diffusion of field lines

Lehnert Experiment Tested Part of Ω-effect

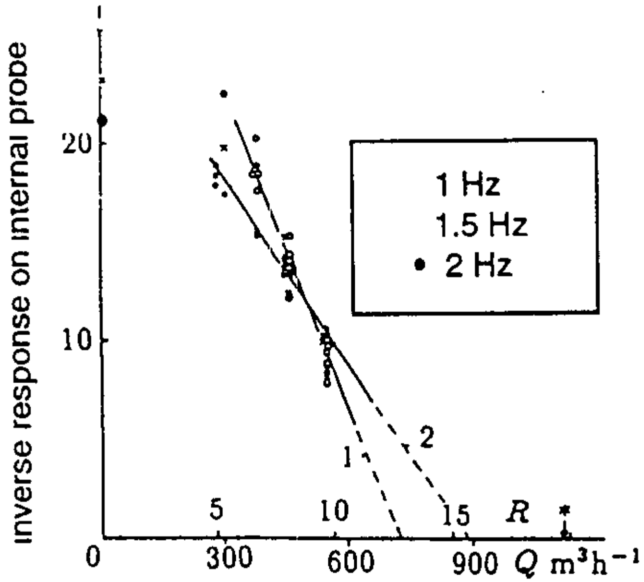


The kinematic Dynamo Problem in a Cylinder

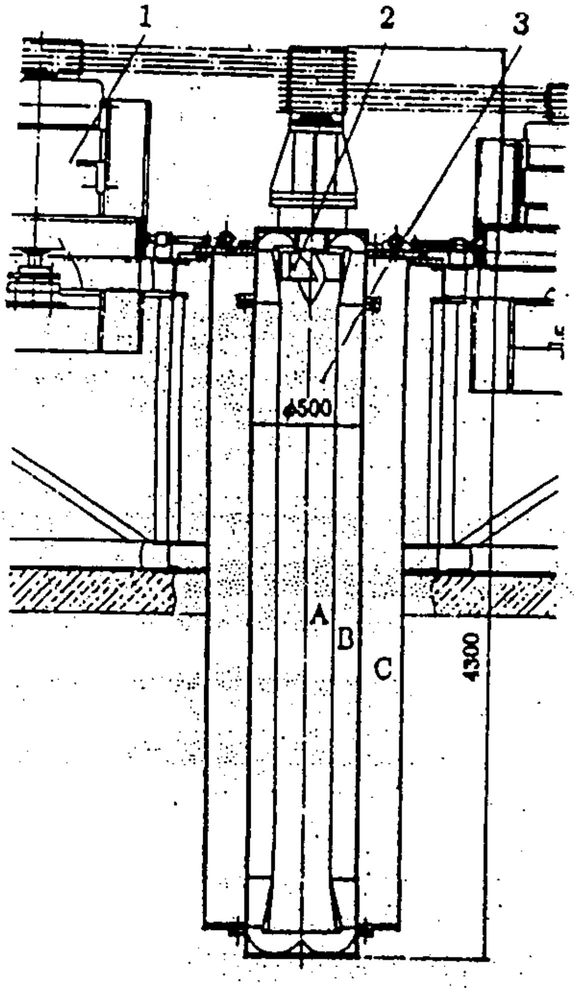
- Find a velocity field $\mathbf{v}(\mathbf{r})$, which leads to growing $\mathbf{b}(\mathbf{r},t)$
- Ponomarenko showed that the very simple flow of translation with twist leads to growing magnetic eigenmodes
- Low critical Rm

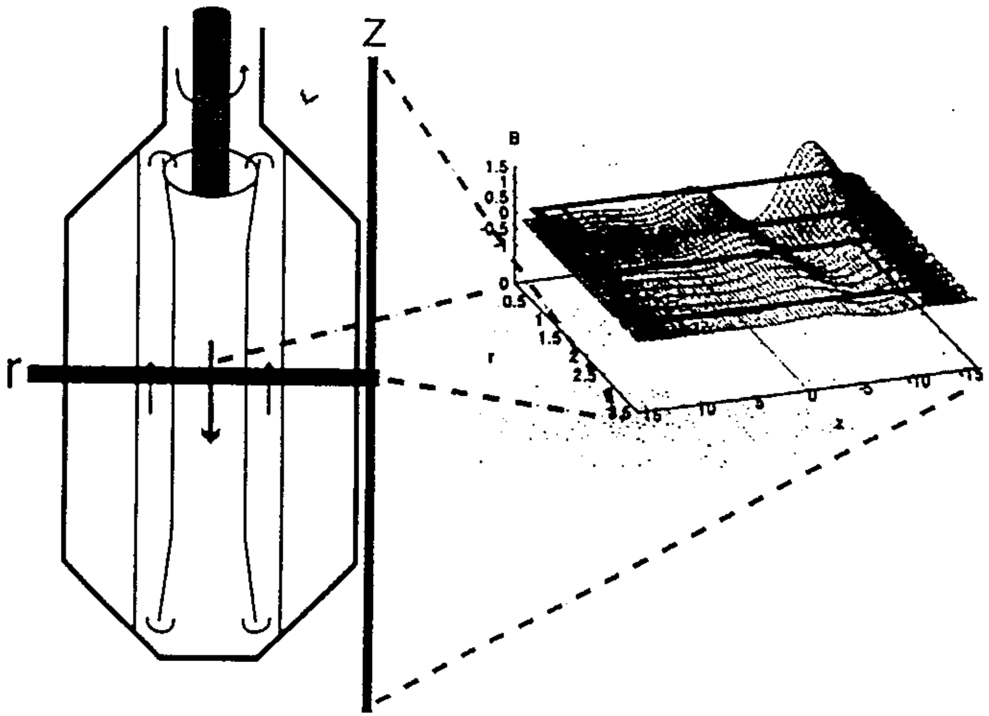


A 1987 Experiment in Riga Attempted a Finite Length Version of the Ponomarenko Dynamo

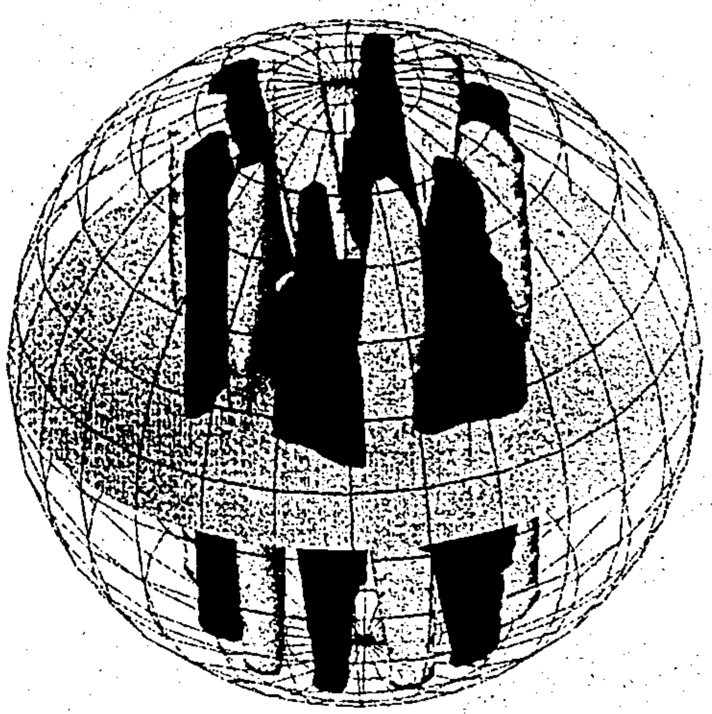


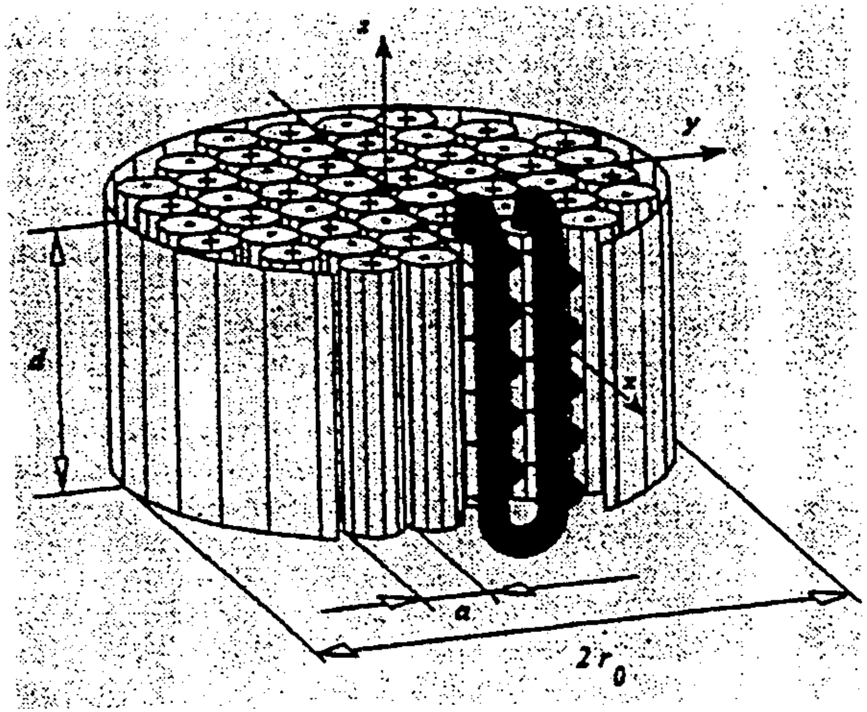
Second Generation Experiment Is Underway in Riga, Latvia.





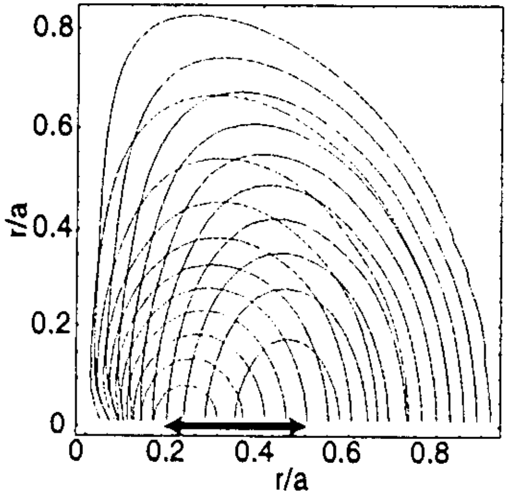
An Experiment in Karlsruhe Hopes to Simulate Aspects of Taylor Column Driven Dynamos



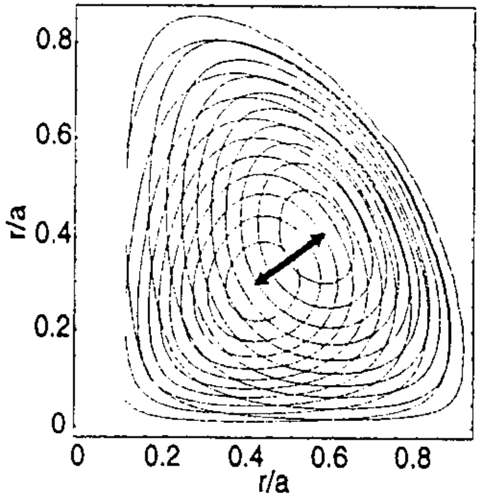


A NONLINEAR OPTIMIZATION USING BULLARD-GELLMAN FORMALISM IS USED TO DETERMINE DESIRABLE EXPERIMENTAL FLOW PROFILES

- **Example optimization of flow shape:**
  - peak position for toroidal velocity
  - null position for poloidal velocity stream function
  - ratio of toroidal to poloidal velocity
- **Simplex method nonlinear maximization of growth rate**

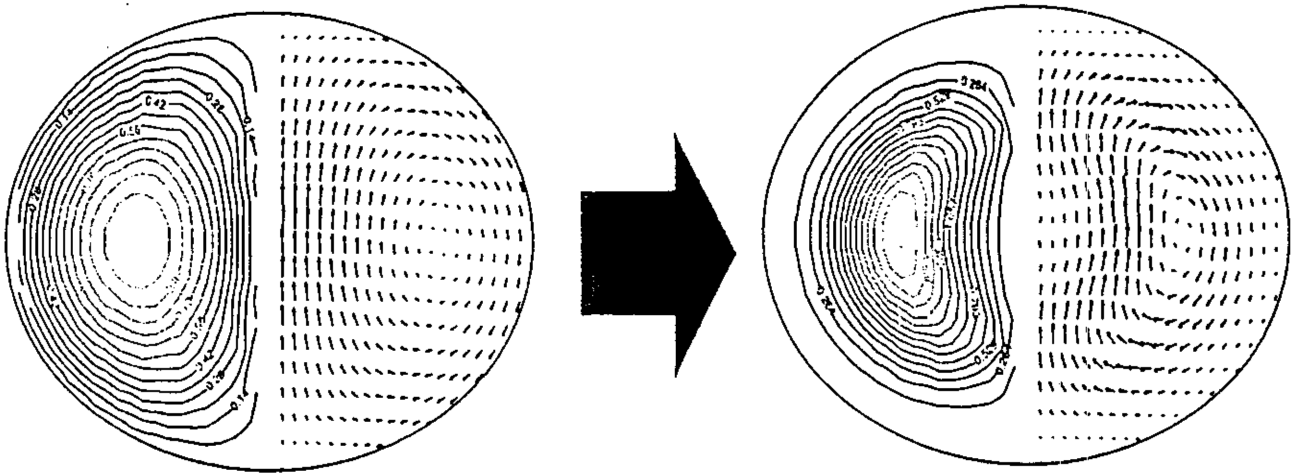


Contours of toroidal flow for extremum positions of numerical scan.



Stokes' functions for poloidal flow for extremum positions of numerical scan.

Optimized Solutions Have Low Critical Rm



Dudley & James t1s1 flow
$Rm_{crit} = 160$

Optimized t1s1 flow
$Rm_{crit}=79$

Flow Configuration	Dudley & James	Optimized
t1s1	150	79
t1s2	95	73
t2s2	55	47

Kinematic Dynamos

- Laminar flows which are kinematic dynamos possess flow helicity (recall Ponomarenko and spherical dynamos)
- Flows has a minimum Rm for self-excitation
- Do such flows exist in nature?

Nonlinearities in MHD Equations

- Induction equation by itself is linear in B
  - If V is given, linear solutions can be found (kinematic dynamo problem)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

- Induction equation is non-linear if V is affected by B
- Navier-stokes is nonlinear in V and JxB
  - V is naturally turbulent
  - JxB force modifying V we can call the backreaction

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla\right)\mathbf{V} = \mathbf{J} \times \mathbf{B} - \nabla p + \nu \rho \nabla^2 V$$

Fluctuations Can Generate Currents: Mean-field Electrodynamics

$$\mathbf{V} = \bar{\mathbf{V}} + \tilde{\mathbf{v}}, \quad \mathbf{B} = \bar{\mathbf{B}} + \tilde{\mathbf{b}}, \quad \bar{\mathbf{B}} = \langle \mathbf{B} \rangle$$

Separate induction equation int o

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} + \frac{1}{\mu_0 \sigma} \nabla^2 \bar{\mathbf{B}} = \nabla \times \bar{\mathbf{V}} \times \bar{\mathbf{B}} + \nabla \times \langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle$$

$$\frac{\partial \tilde{\mathbf{b}}}{\partial t} + \frac{1}{\mu_0 \sigma} \nabla^2 \tilde{\mathbf{b}} - \nabla \times \tilde{\mathbf{b}} \times \bar{\mathbf{V}} =$$

$$\nabla \times \tilde{\mathbf{v}} \times \bar{\mathbf{B}} + \nabla \times \left(\tilde{\mathbf{v}} \times \tilde{\mathbf{b}} - \langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \rangle \right)$$

Simplify to

$$\frac{\partial \tilde{\mathbf{b}}}{\partial t} = \nabla \times \tilde{\mathbf{v}} \times \bar{\mathbf{B}}, \Rightarrow \quad \tilde{\mathbf{b}}(t) = \int^{t} \nabla \times \tilde{\mathbf{v}}(t') \times \bar{\mathbf{B}} \, dt'$$

Fluctuations Can Generate Currents: Mean-field Electrodynamics (Continued)

Simplify to

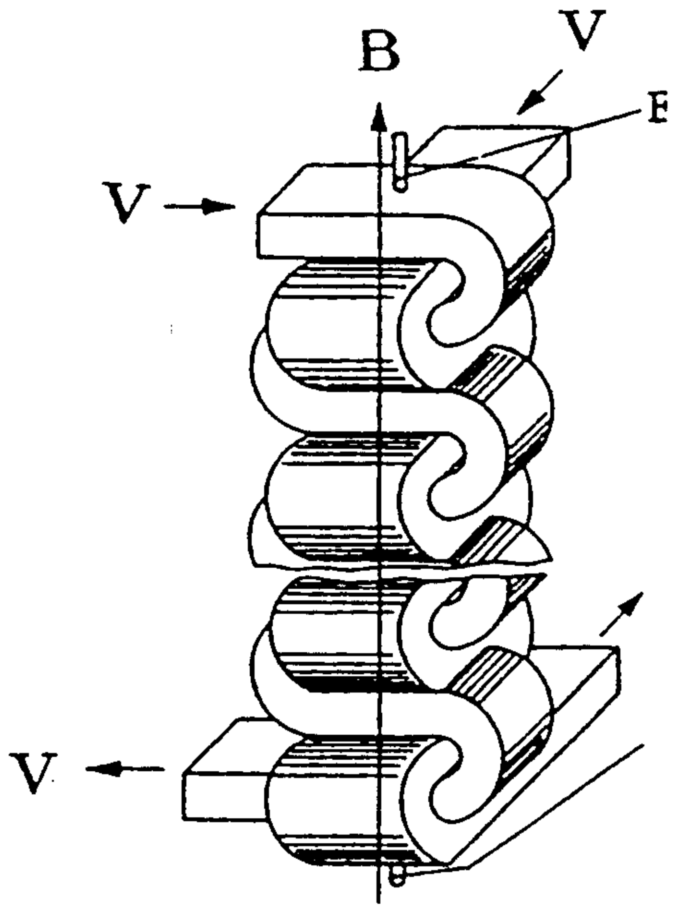
$$\left\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \right\rangle = \int^{t} \left\langle \tilde{\mathbf{v}} \times \nabla \times \left(\tilde{\mathbf{v}}(t') \times \overline{\mathbf{B}} \right) \right\rangle dt'$$

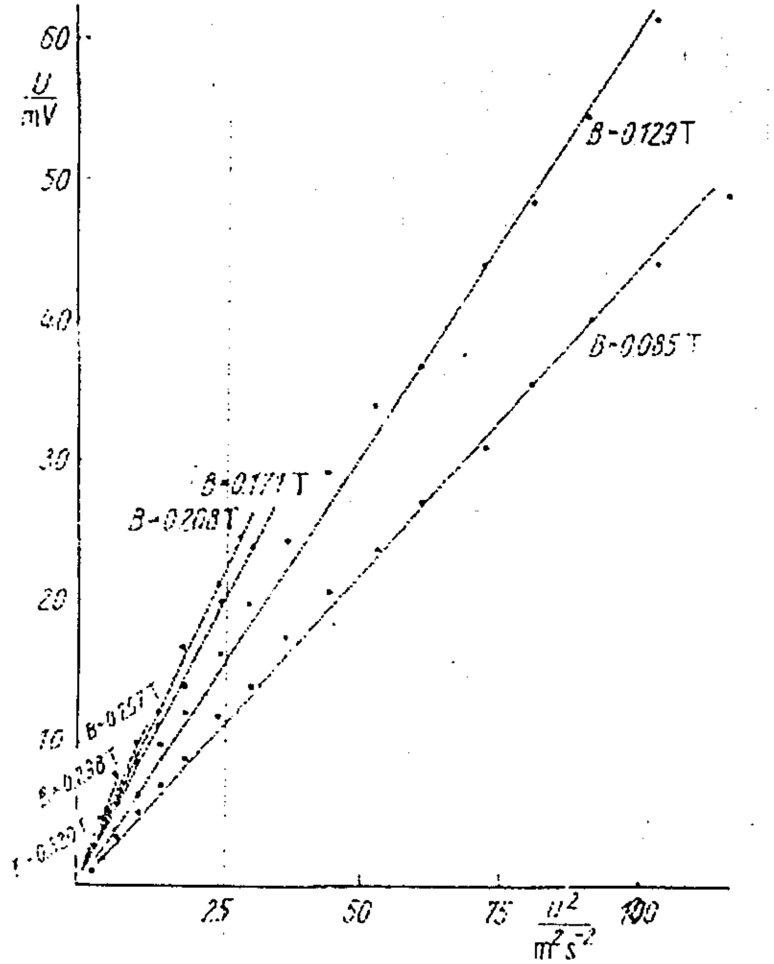
Assu min g isotropic, hom ogeneous turbulence

$$E = \left\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \right\rangle \approx \alpha \overline{\mathbf{B}} - \beta \nabla \times \overline{\mathbf{B}}$$

$$\alpha = \int_0^\infty \left\langle \tilde{\mathbf{v}}(0) \bullet \nabla \times \tilde{\mathbf{v}}(t') \right\rangle dt', \ \beta = \int_0^\infty \left\langle \tilde{\mathbf{v}}(0) \bullet \tilde{\mathbf{v}}(t') \right\rangle dt'$$

The α –Effect: Current Generated by Helical Fluctuations in the Direction of a Preexisting B





The β-effect: eddies move flux

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{V}\times\mathbf{B} + \alpha\mathbf{B} - \beta\nabla\times\mathbf{B}\right)$$

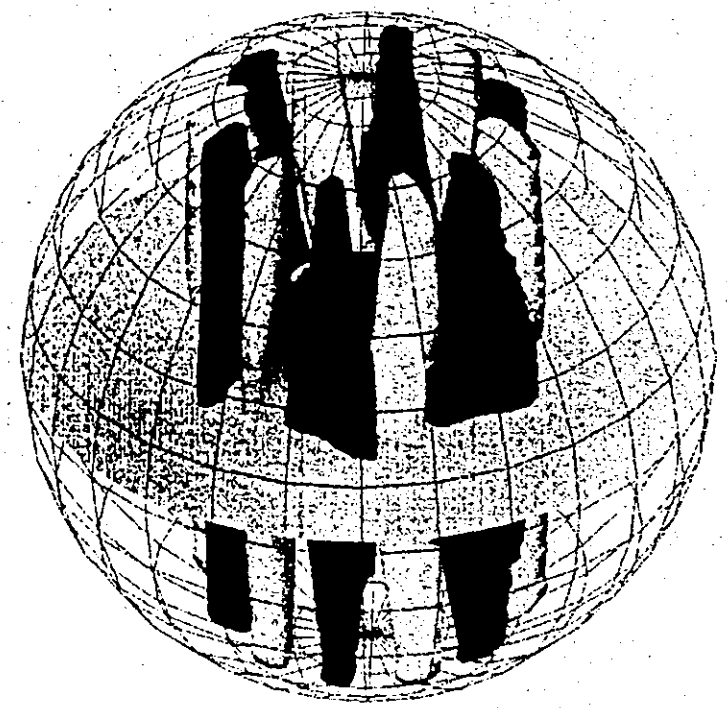
$$\Rightarrow \left(1 + \mu_0\sigma\beta\right)\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{V}\times\mathbf{B} + \alpha\mathbf{B}\right)$$

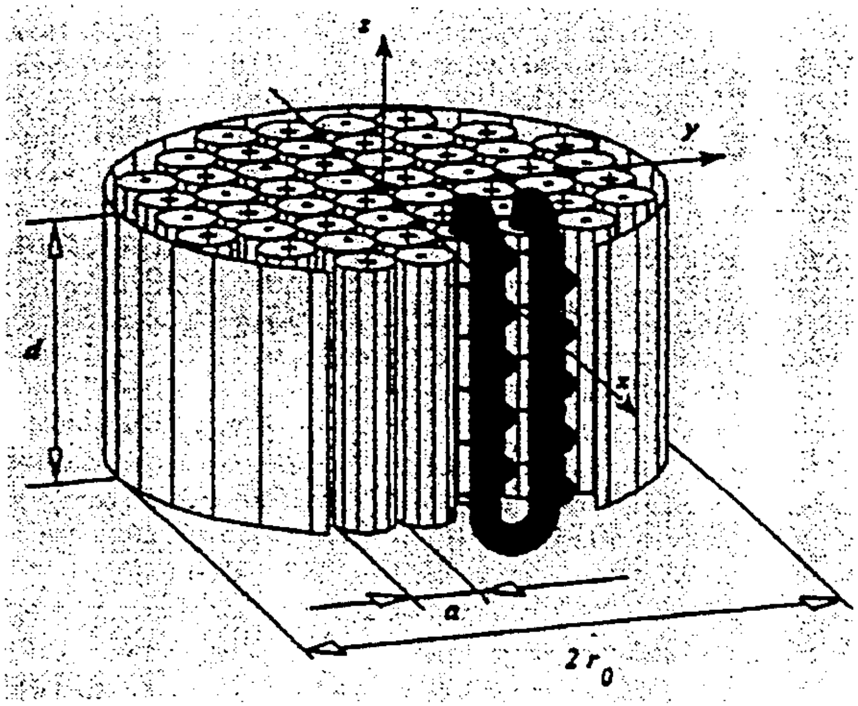
$$\Rightarrow \sigma_T = \frac{\sigma}{1+\mu_0\sigma\beta}, \quad \beta \approx \frac{\tau_{corr}}{3}\left\langle \mathbf{v}^2 \right\rangle$$

Will there be a turbulent modification to Rm-crit in experiments?

$$\Rightarrow Rm_T = \frac{Rm}{1+\mu_0\sigma\beta}$$

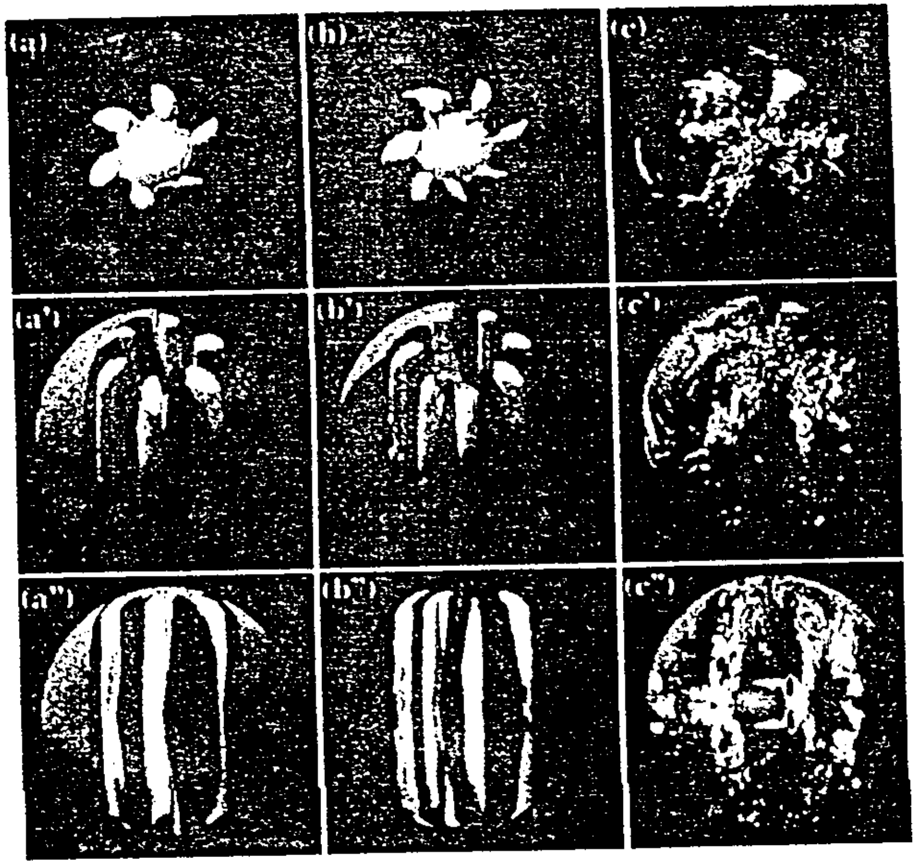
An Experiment in Karlsruhe Hopes to Simulate Aspects of Taylor Column Driven Dynamos





What Happens As the Magnetic Field Energy Grows?

- Simulations show a strong back-reaction when b=v
- Turbulent flows can be strongly modified



Nonlinear Effects in Mean-field Electrodynamics

- α-effect as derived is a kinematic treatment
- Treatment did not include backreaction of induced magnetic field on velocity fluctuations

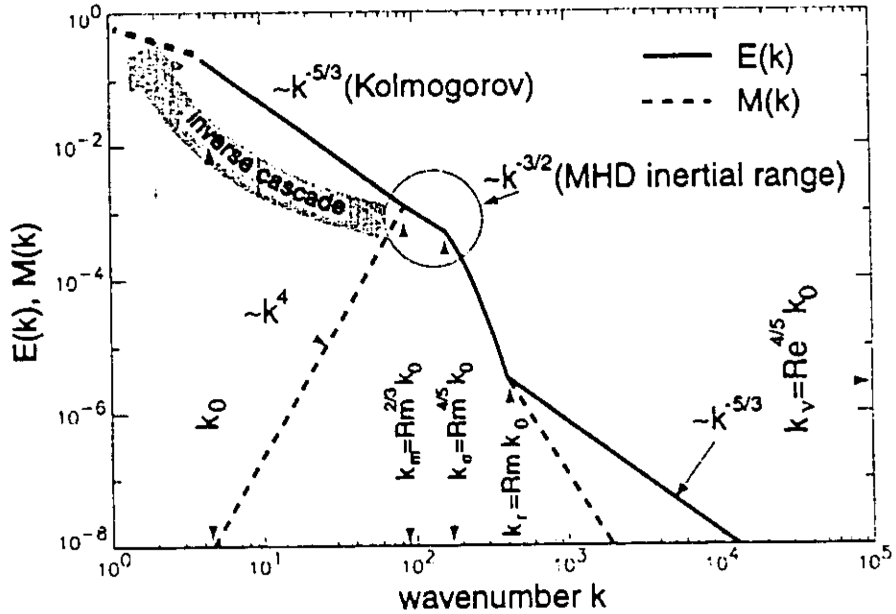
$$\mathbf{E} = \left\langle \delta \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \right\rangle + \left\langle \tilde{\mathbf{v}} \times \delta \tilde{\mathbf{b}} \right\rangle$$

$$\delta \mathbf{b} =$$

$$\delta \mathbf{v} =$$

$$\alpha \approx \frac{\tau_{corr}}{3} \frac{\left\langle \tilde{\mathbf{v}} \cdot \nabla \times \tilde{\mathbf{v}} \right\rangle}{1 + \dfrac{\sigma \tau_{corr}}{\rho} B^2}$$

MHD Turbulence Introduces Effects Beyond Scope of Laminar Dynamo Theory for Experiments to Study



- equipartition between magnetic and kinetic energy is predicted for small scales
- resistivity enhancement due to mixing of magnetic fields on small scales is predicted
- Currents can be generated by helical velocity fluctuations on small scales