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Tokamaks and Turbulent Transport

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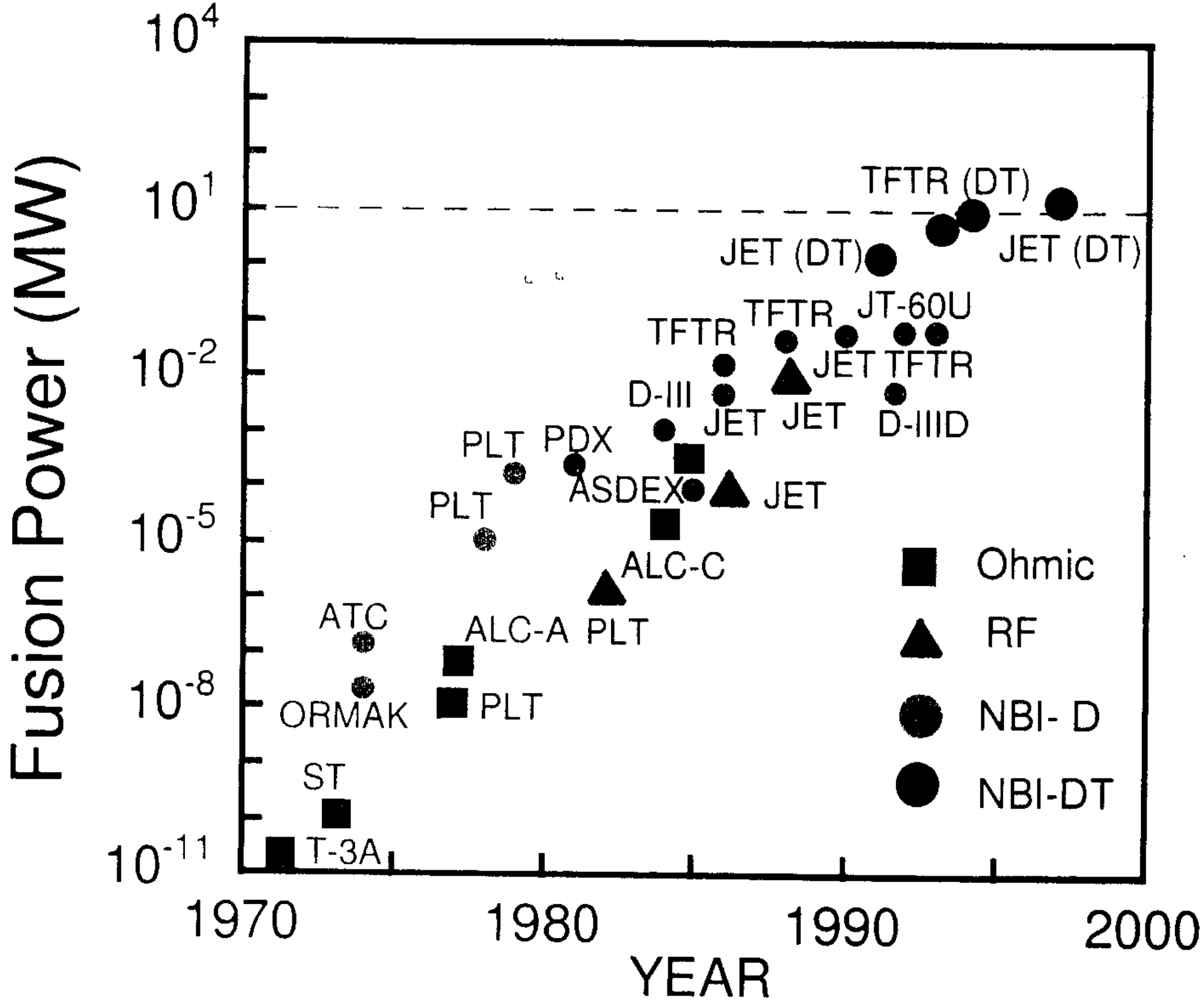
TOKAMAKS HAVE MADE EXCELLENT PROGRESS IN FUSION POWER



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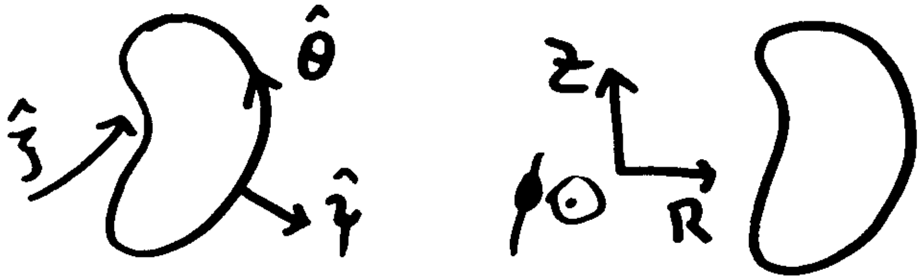
Tokamaks and Turbulent Transport

- Magnetohydrodynamic equilibrium
- Transport
  - Classical
  - Neoclassical
  - Turbulent
- Gyrokinetic equation, fundamental scaling
- Three old experimental puzzles
- Marginal stability model
- Importance of boundary conditions
- Pedestal (edge transport barrier) properties
- Electromagnetic pedestal stability/turbulence
- New puzzles

Magnetohydrodynamic Equilibrium

- Define coordinates (Ψ, ζ, θ), (R, Z, ϕ)



- Consider axisymmetric magnetic field

$$\mathbf{B} = B_T \hat{\zeta} + B_p \hat{\theta} = I \nabla\zeta + \nabla\zeta \times \nabla\theta$$

- Steady-state equation of motion:

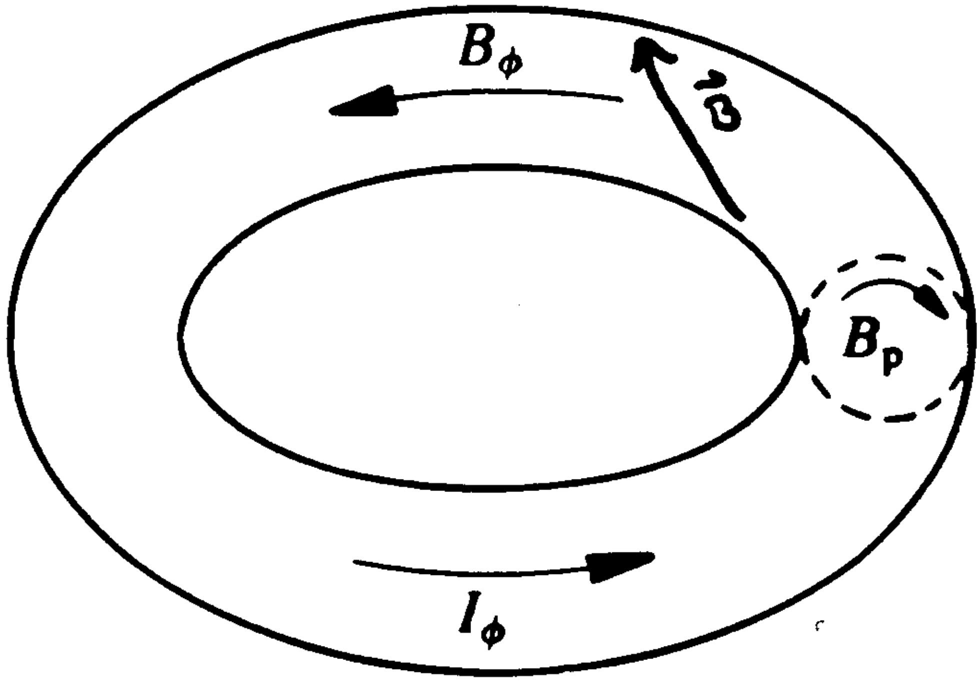
$$\mathbf{J} \times \mathbf{B} = \nabla p$$

- $\mathbf{B} \cdot \nabla p = 0 \rightarrow p = p(\Psi)$
- $\mathbf{J} = \nabla \times \mathbf{B}$ and $\mathbf{J} \cdot \nabla p = 0 \rightarrow I = I(\Psi)$.
- $\nabla\Psi \cdot (\mathbf{J} \times \mathbf{B} - \nabla p) = 0$ leads to Grad-Shafranov equation:

$$R^2 \nabla \cdot \left(R^{-2} \nabla\Psi\right) = -\mu_0 R^2 \frac{dp(\Psi)}{d\Psi} - I(\Psi) \frac{dI(\Psi)}{d\Psi}$$

- Nonlinear equation for Ψ, given $p(\Psi)$, $I(\Psi)$, boundary conditions.

Tokamak Configuration



- Strong toroidal magnetic field from external coils.
- Current flows in the plasma, generates (weaker) poloidal magnetic field.
- Resulting magnetic field lines wind around the torus helically.
- Pressure gradient in plasma balanced by $\mathbf{J} \times \mathbf{B}$ force.
- Mean free path along the magnetic field lines typically kilometers.
- Particles gyrate rapidly around field lines to zeroth order.

Cross-field Classical Transport is Weak

- Classical, cross-field collisional transport can be estimated as diffusion occurring from a random walk,

$$D \sim \frac{(\Delta x)^2}{\Delta t}$$

where $\Delta x \sim \rho$ and $\Delta t \sim 1/\nu$.

- Like-particle collisions lead to no particle diffusion. Heat diffusion is possible, with

$$\chi_{ii}^{cl} \sim \nu_{ii}\rho_i^2, \qquad \chi_{ee}^{cl} \sim \nu_{ee}\rho_e^2$$

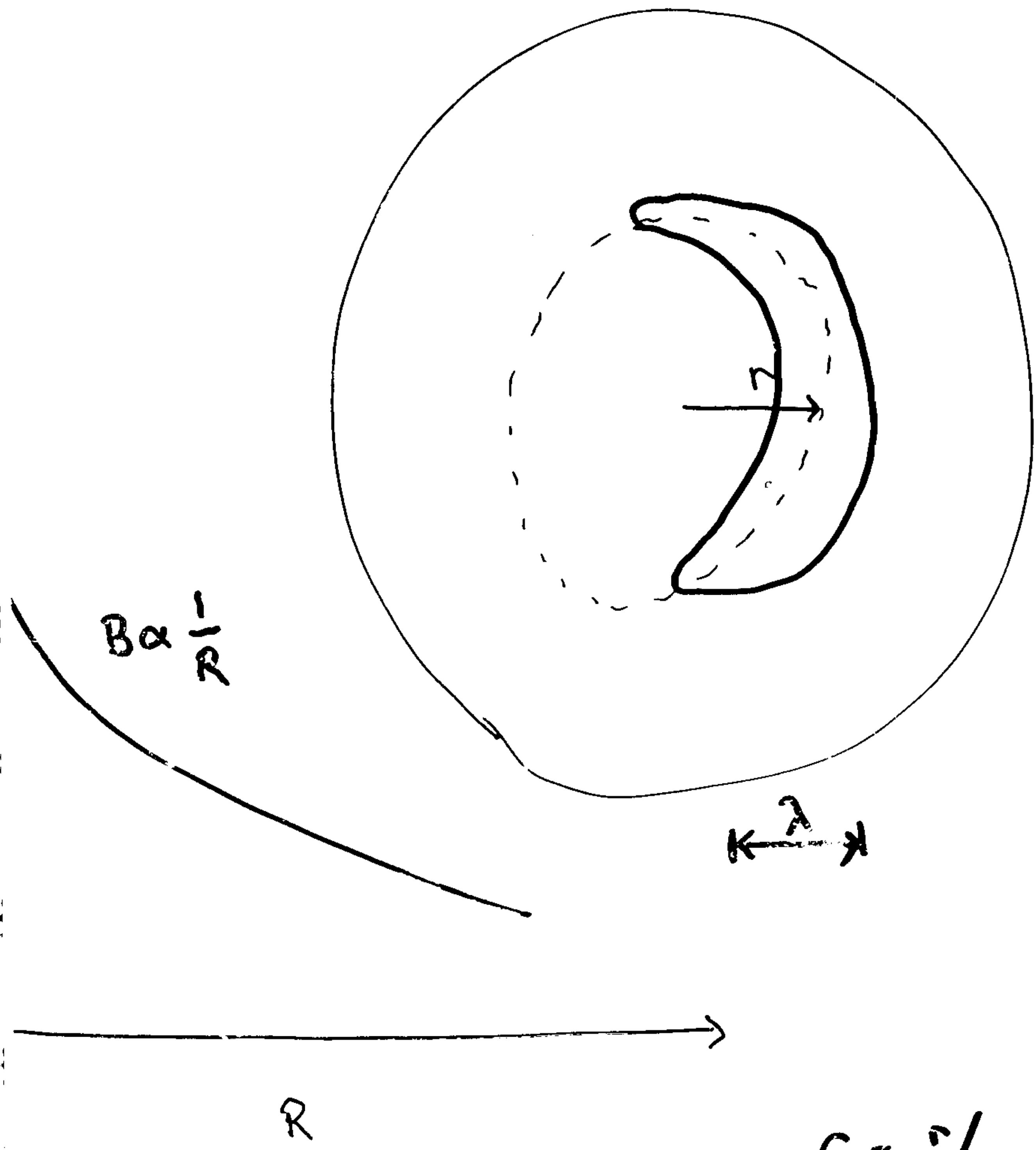
- Electron-ion collisions do lead to particle transport, so that

$$D^{cl} \sim \nu_{ei}\rho_e^2. \qquad \chi_{ei}^{cl} \sim \nu_{ei}\rho_e^2$$

- Total cross-field transport rates are small:
  - Electrons: $D \sim \nu_{ei}\rho_e^2$. $\chi_e \sim (\nu_{ee} + \nu_{ei})\rho_e^2$
  - Ions: $D \sim \nu_{ei}\rho_e^2$. $\chi_i \sim \nu_{ii}\rho_i^2$

- Observed cross-field transport is typically 10^3 larger.

Complicated Orbit in Tokamak



$\epsilon \equiv r/R$

"Neoclassical" Enhancement

- Long mean free path allows complicated orbits
- Energy, magnetic moment and toroidal angular momentum are conserved

$$E = \frac{mv_\parallel^2}{2} + \frac{mv_\perp^2}{2}, \qquad \mu = \frac{mv_\perp^2}{2B}$$

- Some particles will be reflected by magnetic field gradient

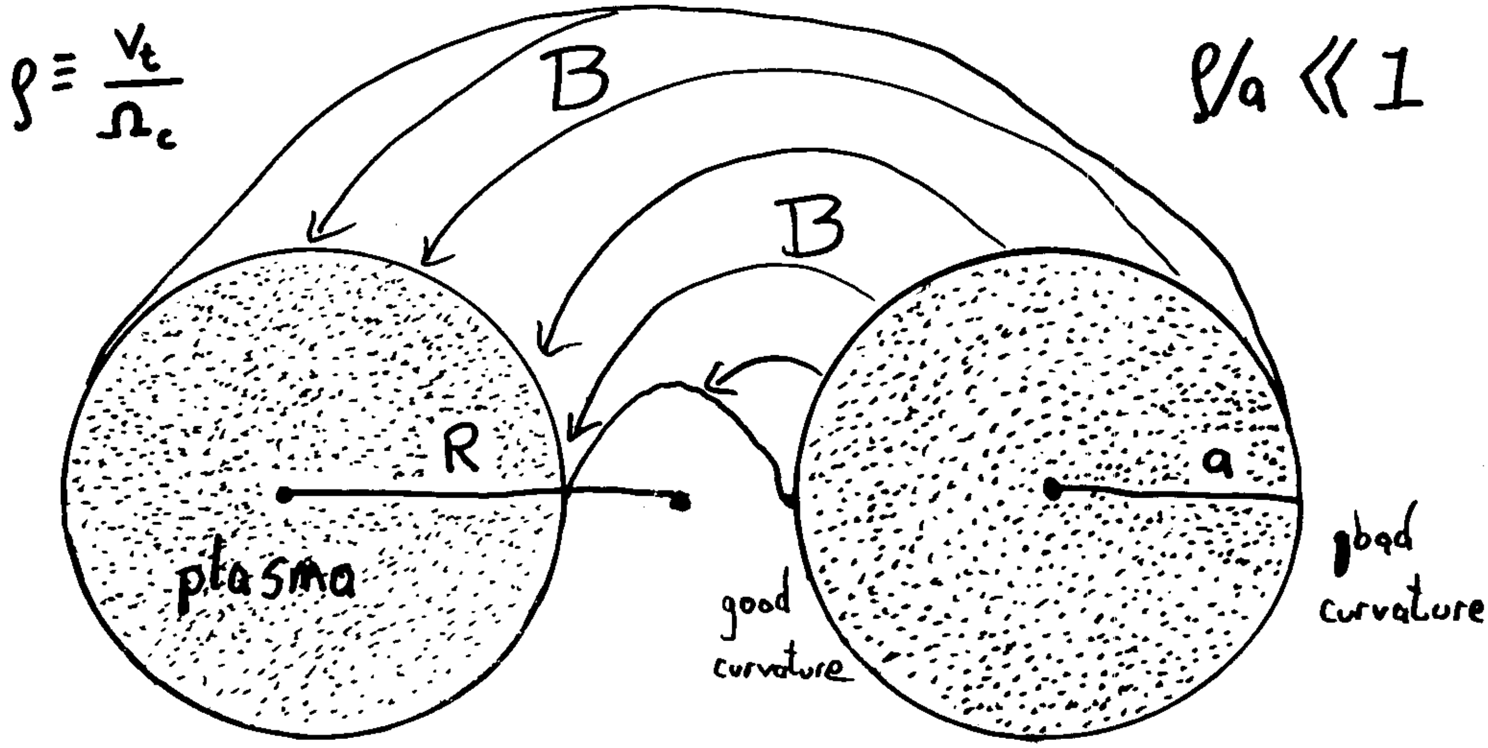
$$v_\parallel = \sqrt{\frac{2}{m}\left(E - \mu B\right)}$$

- Conservation of toroidal angular momentum yields $\lambda \sim \rho q/\sqrt{\epsilon}$, where $\epsilon = r/R$.
- Fraction of "trapped" particles $f_T \sim \sqrt{2\epsilon}$.
- Frequency of scattering between untrapped and trapped orbits $\nu_{\mathrm{eff}} \sim \nu/\epsilon$
- Classical diffusion enhanced

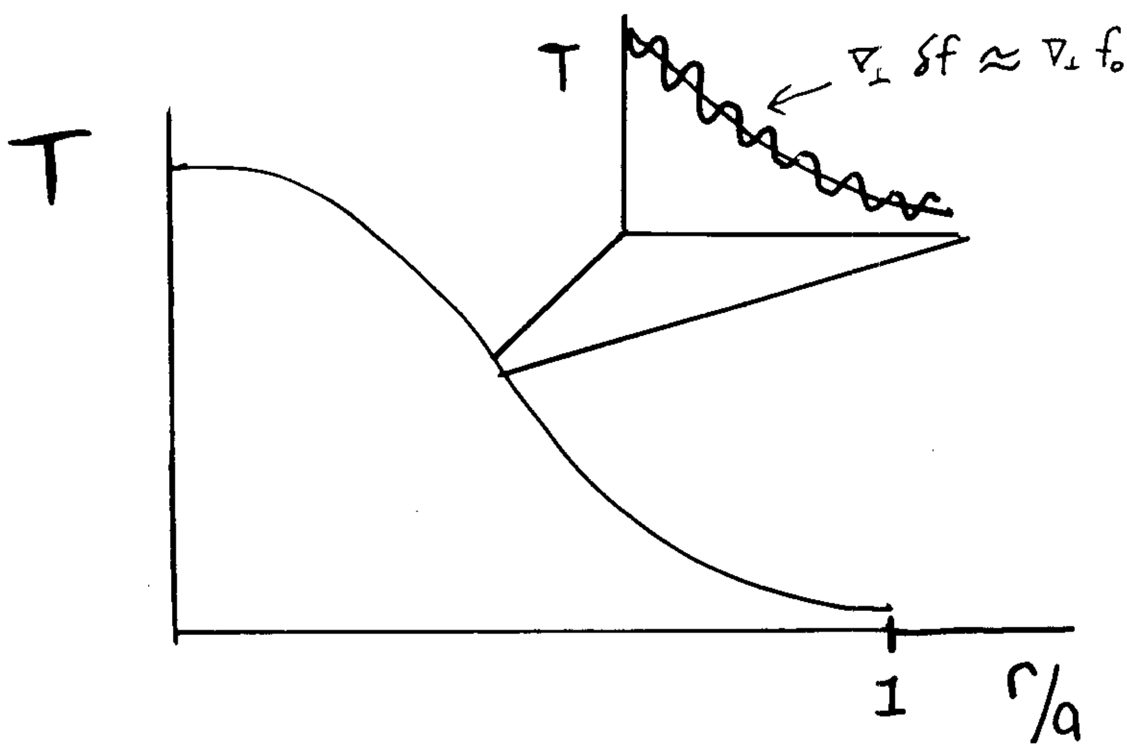
$$\chi, D \sim f_T \frac{(\Delta x)^2}{\Delta t} \sim \sqrt{\epsilon}\lambda^2 \nu_{\mathrm{eff}} \sim \nu \rho^2 \frac{q^2}{\epsilon^{3/2}}$$

- Too small, wrong radial dependence compared to experiment

Tokamak Confinement



- Metaequilibrium stable to "MHD" activity



- Fine-scale turbulence driven by gradients

Developing Understanding of Plasma Turbulence

- Experimental groups observe and measure fluctuating electric and magnetic fields in tokamak plasmas.
- Turbulence driven by strong gradients long suspected to cause fluctuations and "anomalous" heat loss.

1. Plasma instability identified in 1960's
   [Rudakov and Sagdeev, 1961; Coppi, Rosenbluth and Sagdeev, 1967]
2. WKB analysis appropriate for toroidal problem worked out in 1970's
   [Connor, *et al.*, 1979]
3. Comprehensive kinetic equation describing general low-frequency instabilities in magnetized plasma worked out in early 1980's
   [Frieman and Chen, 1982] ← nonlinear [Antonsen & Lane, 1980] ← linear
4. Analytical turbulence theory developed in 1980's
5. Numerical turbulence simulations developed in 1990's

Gyrokinetic Equation

- Gyrokinetic equation appropriate for small amplitude fluctuations with

$$\frac{\omega}{\Omega_c} \sim \frac{k_\parallel}{k_\perp} \sim \frac{\delta f}{f} \sim \frac{e\delta\Phi}{T} \sim \frac{\delta B}{B} \sim \frac{\rho}{L} \ll 1$$

- Gyrokinetic equation describes evolution of perturbed distribution function h. For $F_0 = F_0(\epsilon, \Psi)$:

$$\left(\frac{d}{dt} + v_\parallel \hat{\mathbf{b}} \cdot \nabla + i\omega_d + C\right) h = i\omega_*^T \chi - \frac{q}{m}\frac{\partial F_0}{\partial \epsilon}\frac{\partial \chi}{\partial t}.$$

- The total derivative is

$$\frac{dh}{dt} = \frac{\partial h}{\partial t} + \frac{c}{B}\{\chi, h\}.$$

- The drift frequency $i\omega_*^T = n_0 c\, \partial F_0/\partial\Psi$, where n_0 is the toroidal mode number of the perturbation and Ψ is the equilibrium poloidal magnetic flux enclosed by the magnetic surface of interest.

- The perpendicular drifts (curvature, grad-B) are

$$\omega_d = \mathbf{k}_\perp \cdot \mathbf{B}_0 \times \left(m v_\parallel^2 \hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}} + \mu \nabla B_0\right) / (m B_0 \Omega_c),$$

- The fields are represented by

$$\chi = J_0(\gamma)\left(\Phi - \frac{v_\parallel}{c} A_\parallel\right) + J_1(\gamma)\frac{v_\perp}{c}\frac{\delta B_\parallel}{k_\perp}.$$

Here, $\gamma = k_\perp v_\perp / \Omega_c$.

Gyrokinetic Equation Scaling

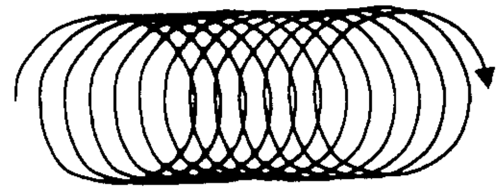
- Two length scales: ρ, R
- Two time scales: $R/v_t, \nu$
- Nonlinear terms ordered to be comparable to linear terms implies

$$\chi^{anom} \sim \frac{\rho^2 v_t}{R} f(\frac{R}{L_T}, \frac{R}{L_n}, \frac{\nu R}{v_t}, q, \epsilon, \ldots)$$

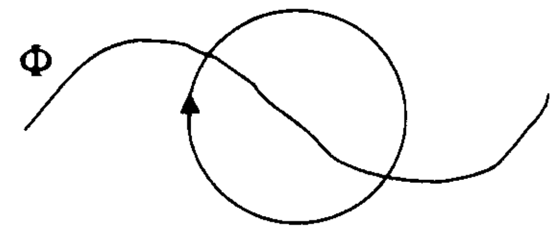
- Strongest obvious dependence is leading order $T^{3/2}$, which would imply an anomalous heat flux $Q \sim T^{5/2}$.

Gyrokinetic & gyrofluid methods of simulating turbulence

- Nonlinear gyrokinetic Eq. (1980's): possible to average over fast gyrofrequency & retain nonlinear dynamics



- With δf method (88-90's): ~10^5 speedup
- $d/dt\, f(\vec{x}, v_{\parallel}, v_{\perp})$: Solved by particle-in-cell (or as continuum f on grid with implicit/FFT) methods

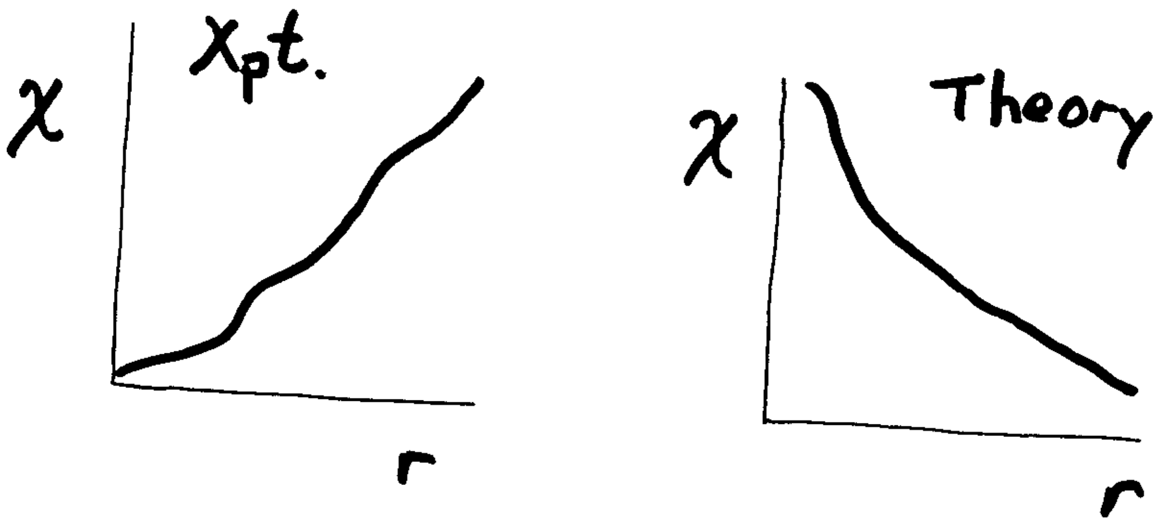


Arbitrary $k_{\perp}\rho$

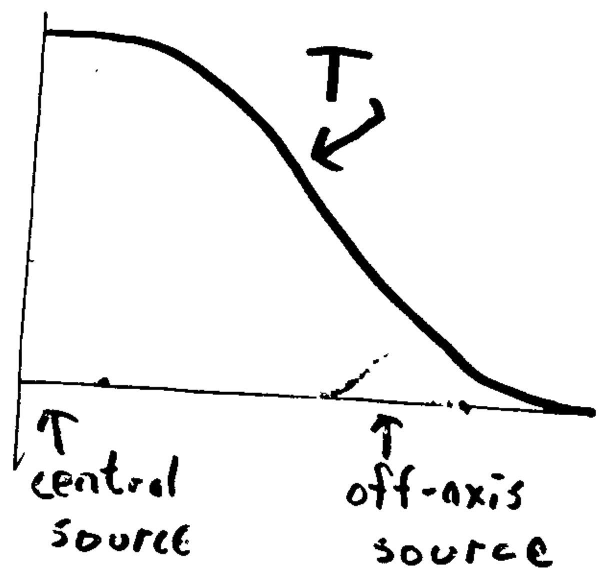
- Gyrofluid approach: moments over $v_{\parallel}$, $v_{\perp}$ --> reduced 3-D fluid equations for density, parallel flow, parallel and perpendicular pressures and heat fluxes. closure approximations to model wave-particle resonances. Landau-damping/mixing rate $v_t|k_{\parallel}|$ leads to nonlocal heat flux operator
- Gyrokinetics very important: more fundamental, but computationally more demanding. Gyrofluid approach can aid with more parameter scans, higher resolution, cross-check codes and physics. Gyrofluid closures may help extend MHD codes, has been applied to Alfvén turbulence in space, etc...

Three Old Experimental Puzzles

- Radial dependence of χ



- "Profile Consistency" (insensitivity of profiles to sources)



- "Non-local Transport:" central plasma temperature responds very rapidly to changes in edge conditions.

Resolution of Old Puzzles: "Marginal Stability"

- Particular functional form found by direct numerical simulations:

$$\chi^{anom} \sim \frac{\rho^2 v_t}{R} \left(\frac{R}{L_T} - \frac{R}{L_{T\text{crit}}} \right) f \left(\frac{L_n}{L_T}, \frac{\nu R}{v_t}, \ldots \right)$$

small

large, but weak variation

- Self-consistent χ naturally increases with radius.

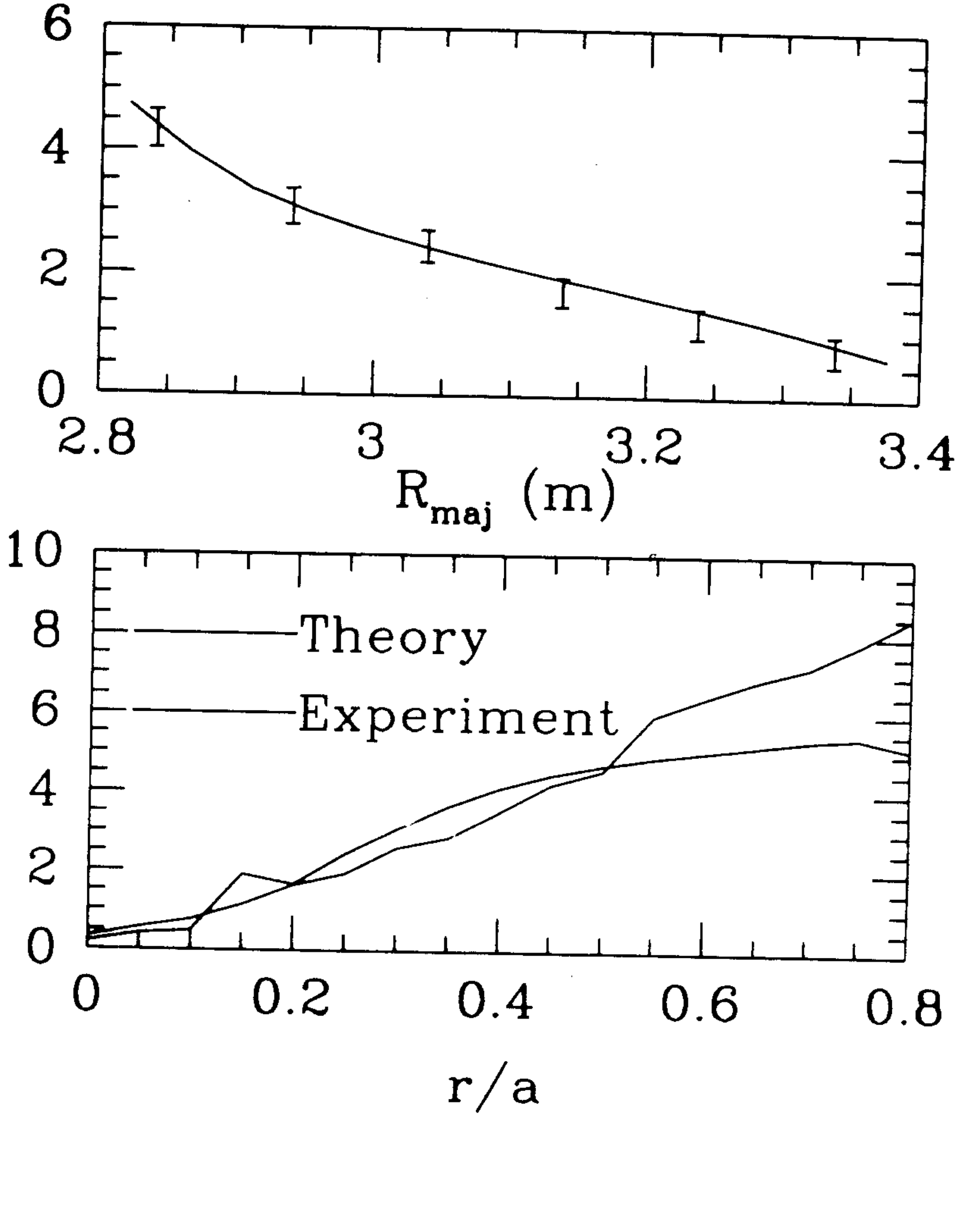
- Profile consistency explained by

$$\frac{R}{L_T} \sim \frac{R}{L_{T\text{crit}}}$$

- Fast response comes from

$$\tau_{\text{fast}} \sim \frac{1}{L_{T\text{crit}}/L_T - 1} \frac{R}{v_t}$$

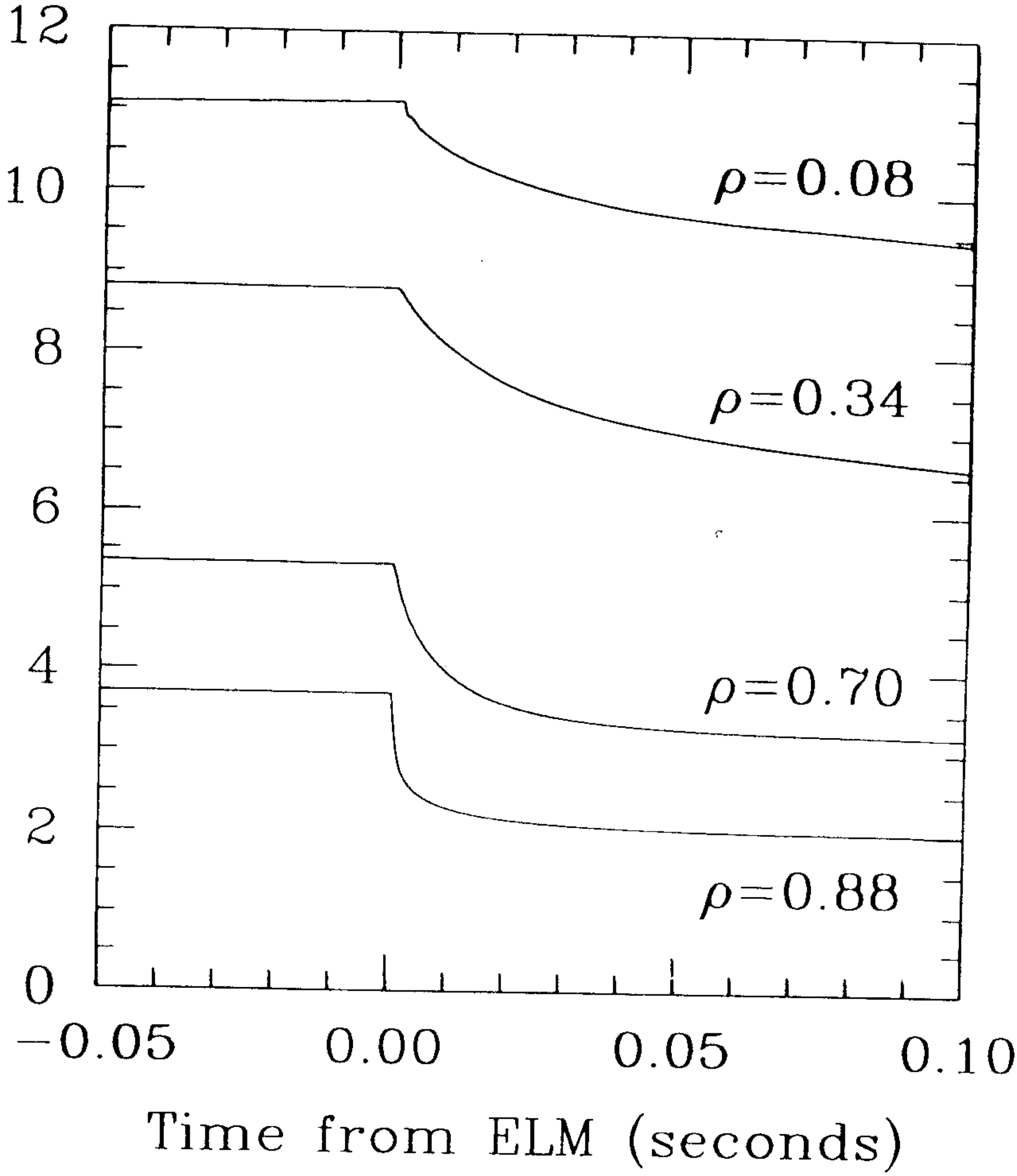
- Marginal stability predicts sensitivity of central temperature to edge conditions. Experimentally observed.



6
4
2
0
2.8
3
3.2
3.4
R_maj (m)
10
8
6
4
2
0
Theory
Experiment
0
0.2
0.4
0.6
0.8
r/a

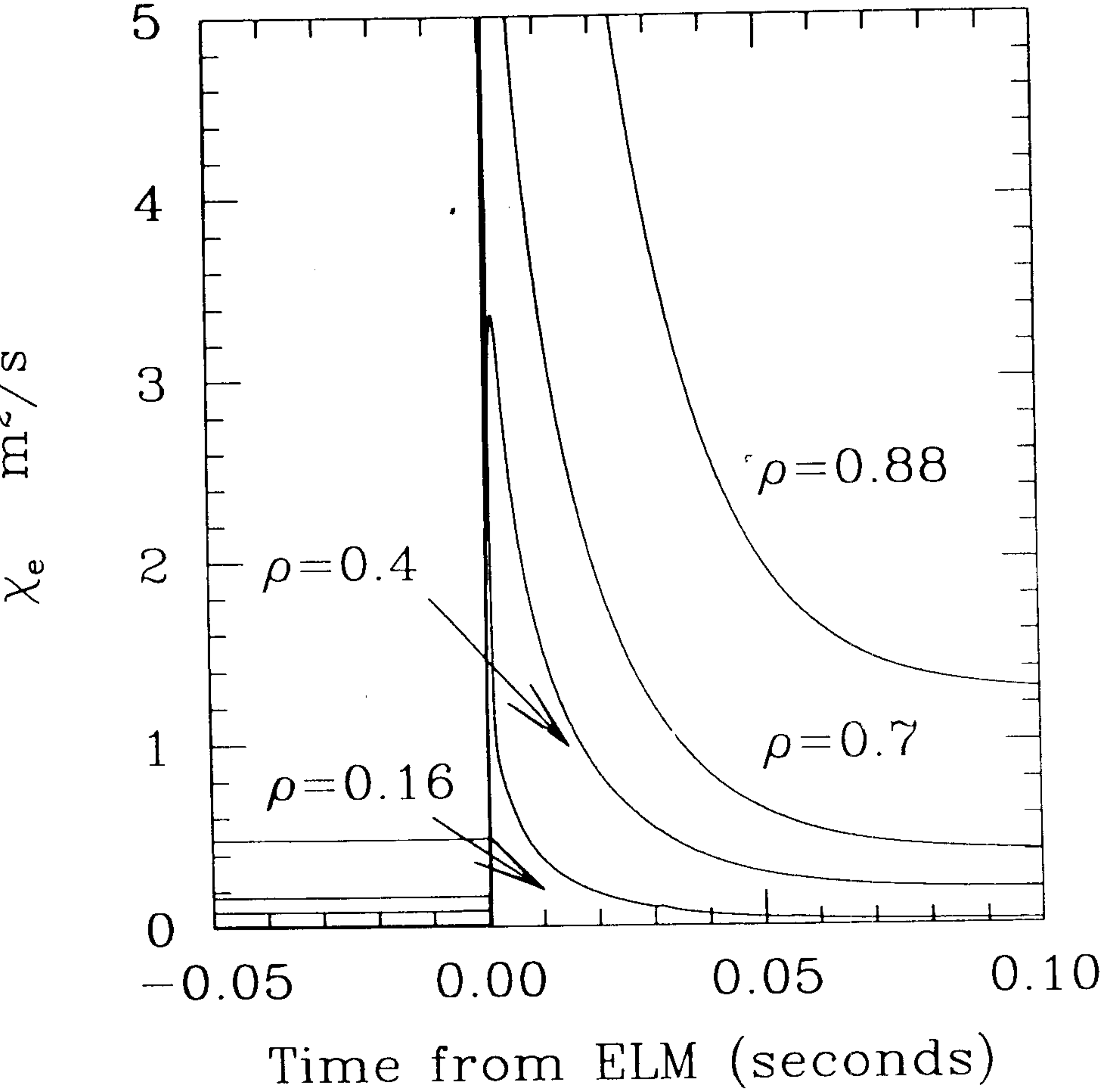
Transient Transport Experiments

- Large χ implies core temperature responds quickly to edge perturbations.
- Time scale predicted by theory depends on discharge conditions, but can be as fast as a few milliseconds.
- Edge perturbations from ELM's, cold pulses, L->H and H->L transitions can lead to fast response of core temperatures.
- Effects qualitatively explained by the local χ found from gyrofluid and gyrokinetic simulations.



12
10
8
6
4
2
0
ρ=0.08
ρ=0.34
ρ=0.70
ρ=0.88
−0.05
0.00
0.05
0.10
Time from ELM (seconds)

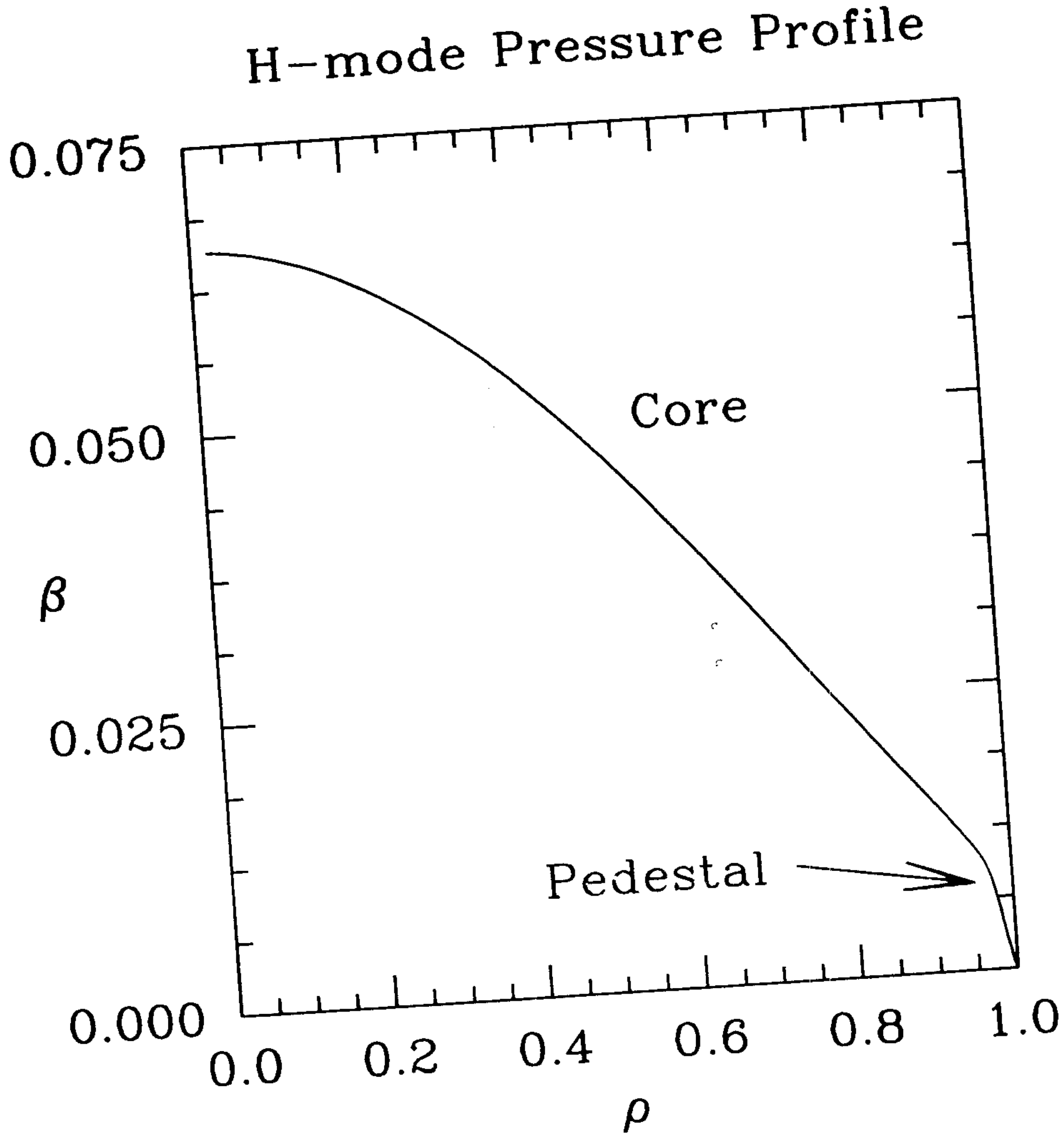
$\rho = r/a$



ρ=0.88
ρ=0.7
ρ=0.4
ρ=0.16
χe m²/s
0
1
2
3
4
5
−0.05
0.00
0.05
0.10
Time from ELM (seconds)

New Puzzles

- Differences among turbulence simulations remain.
- Influence of electromagnetic perturbations unknown. Can affect topology of perturbations, stability, *etc.*
- Electron thermal transport not completely understood, especially in Ohmic plasmas and in transport barriers.
- What physics governs pedestal formation, equilibrium, dynamics?
- What physics governs barrier formation, equilibrium, dynamics?

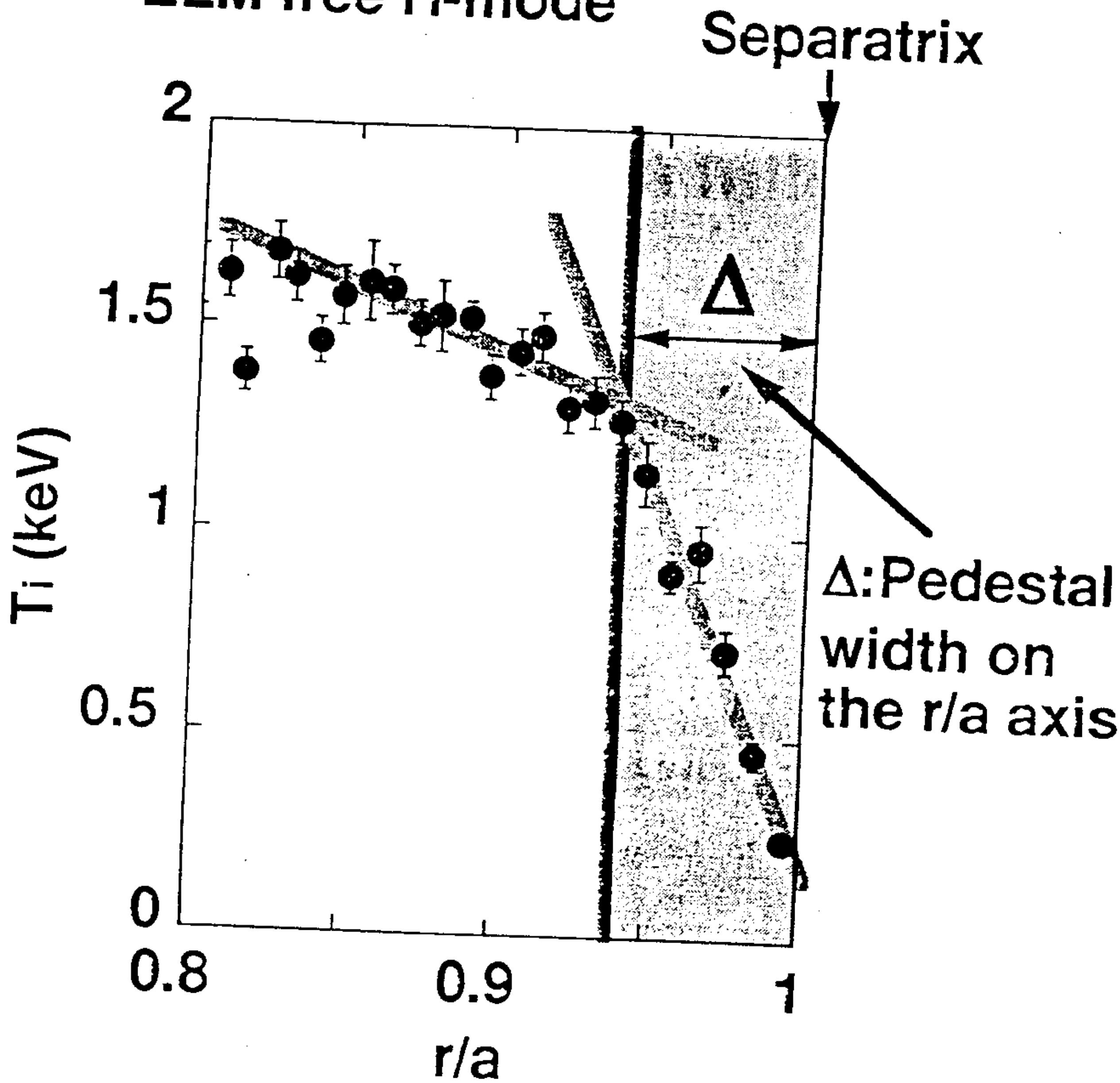

H-mode Pressure Profile
0.075
0.050
0.025
0.000
β
Core
Pedestal
0.0
0.2
0.4
0.6
0.8
1.0
ρ

Ion Temperature Pedestal Width by CXRS Diagnostic

JAERI

JT-60U

E024573 Ip=1MA, Bt=2.5T
t=6.0s
ELM free H-mode



Pedestal width for the ion temperature;

$$\Delta r = \Delta \times \text{(Minor Radius)}$$

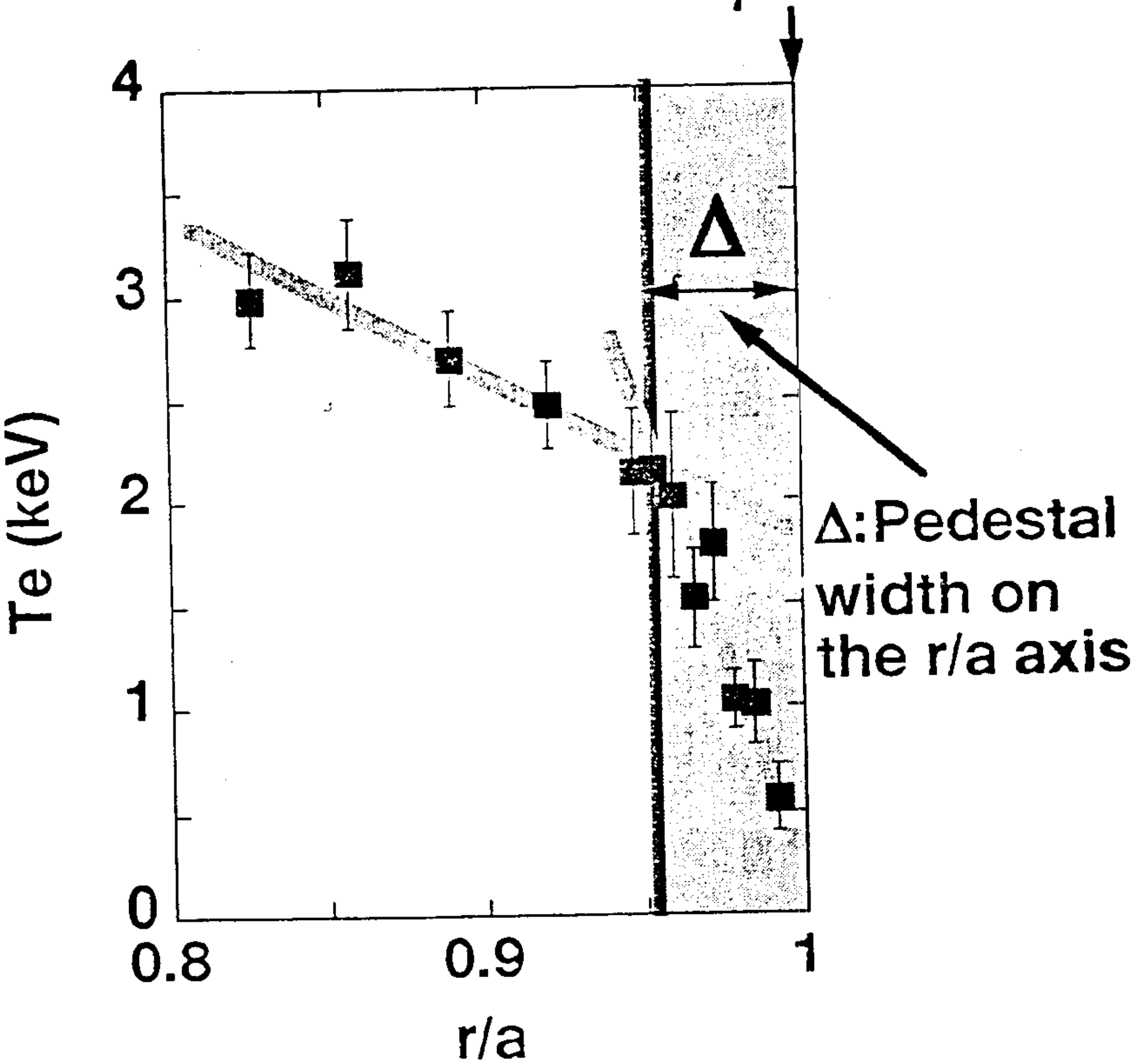
$$= (1 - 0.938) \times 90.6\text{cm} = 5.6\text{cm} (\pm 0.7\text{cm})$$

Electron Temperature Pedestal Width by Thomson Scattering Diagnostic

JAERI JT-60U

E023956 Ip=3.5MA, Bt=4T,
t=6.3s P_{abs}=23.7MW,H=1.53

ELM free H-mode



Pedestal width for the electron temperature;

$$\Delta r = \Delta \times \text{(Minor Radius)}$$

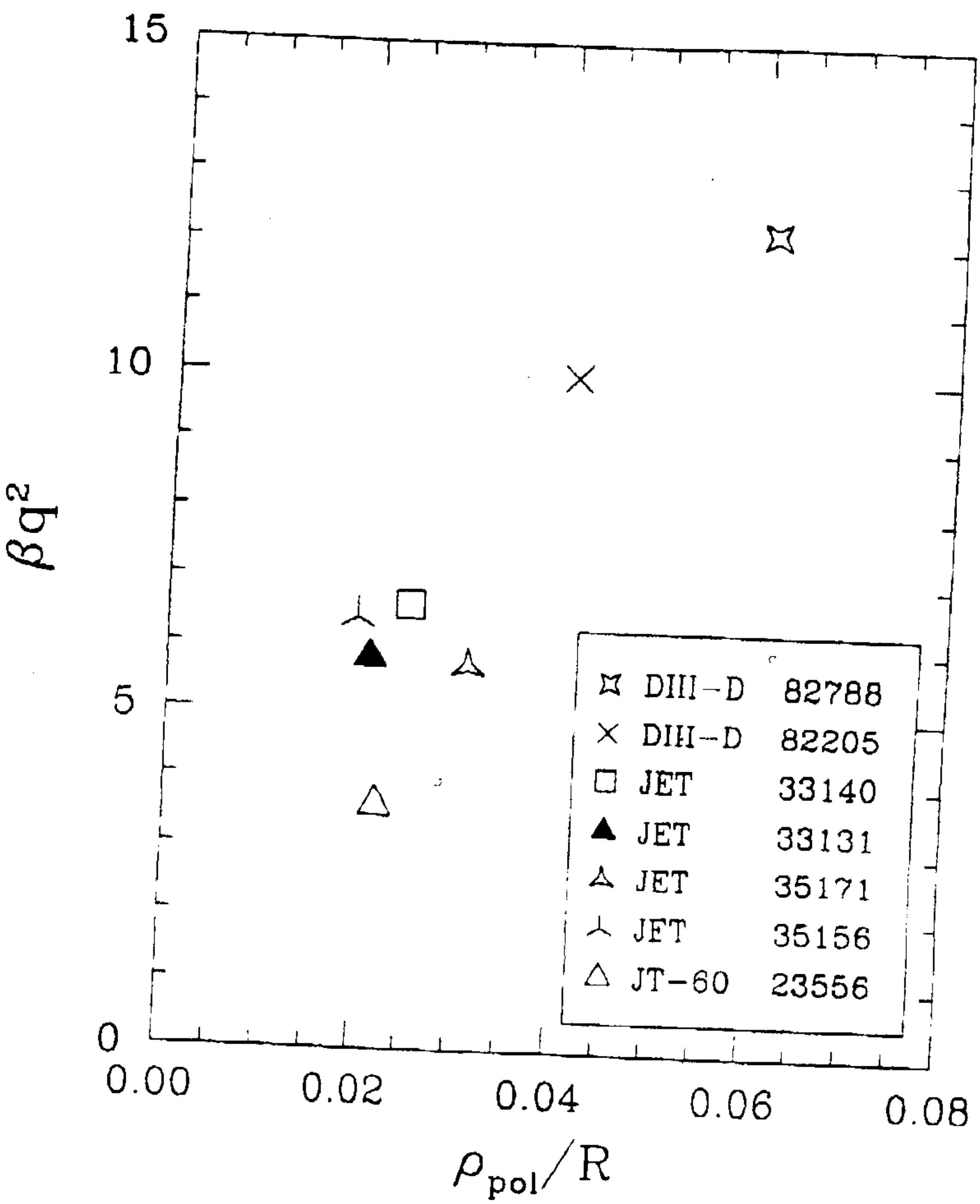
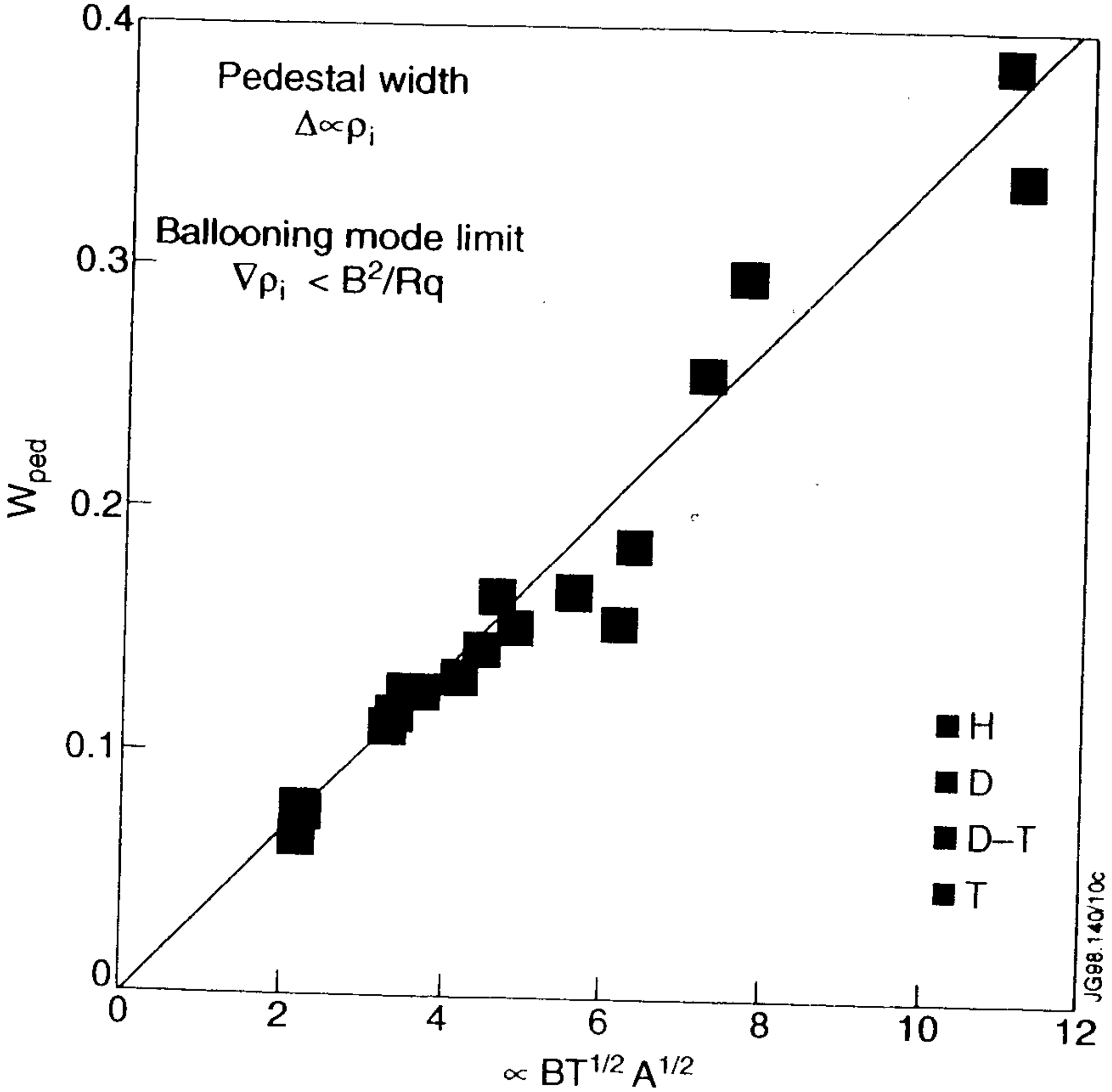


Figure 11

BETA versus RHO* for pedestal parameters (r/a=0.9) for ITER like RHO* scan discharges on DIIID, JET and JT-60.

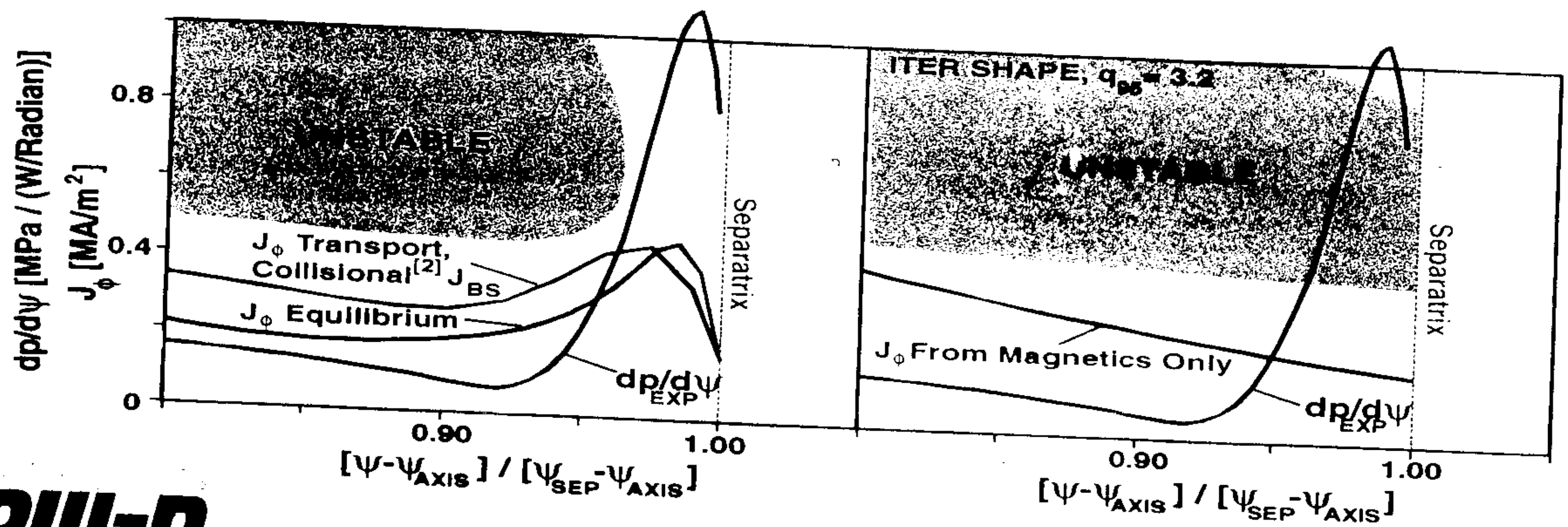
Scaling Of Pedestal Energy



- Pedestal energy ($\sim T_e(a) \times \int n_e d\ell_{edge}$) scales as if edge is at ideal ballooning limit over length $\sim\rho_i$

A MORE ACCURATE PLASMA CURRENT DENSITY MEASUREMENT WILL IMPROVE TESTING OF EDGE AND RESISTIVE STABILITY MODELS

- Edge localized modes are sensitive to details of edge current density
  - Measured edge P′ is consistent with ideal ballooning theory within uncertainties of present edge J measurements
  - A more accurate edge J measurement will improve testing of edge stability models and allow a more accurate projection of future devices
- Resistive stability is sensitive to details of current density gradient at rational q surfaces



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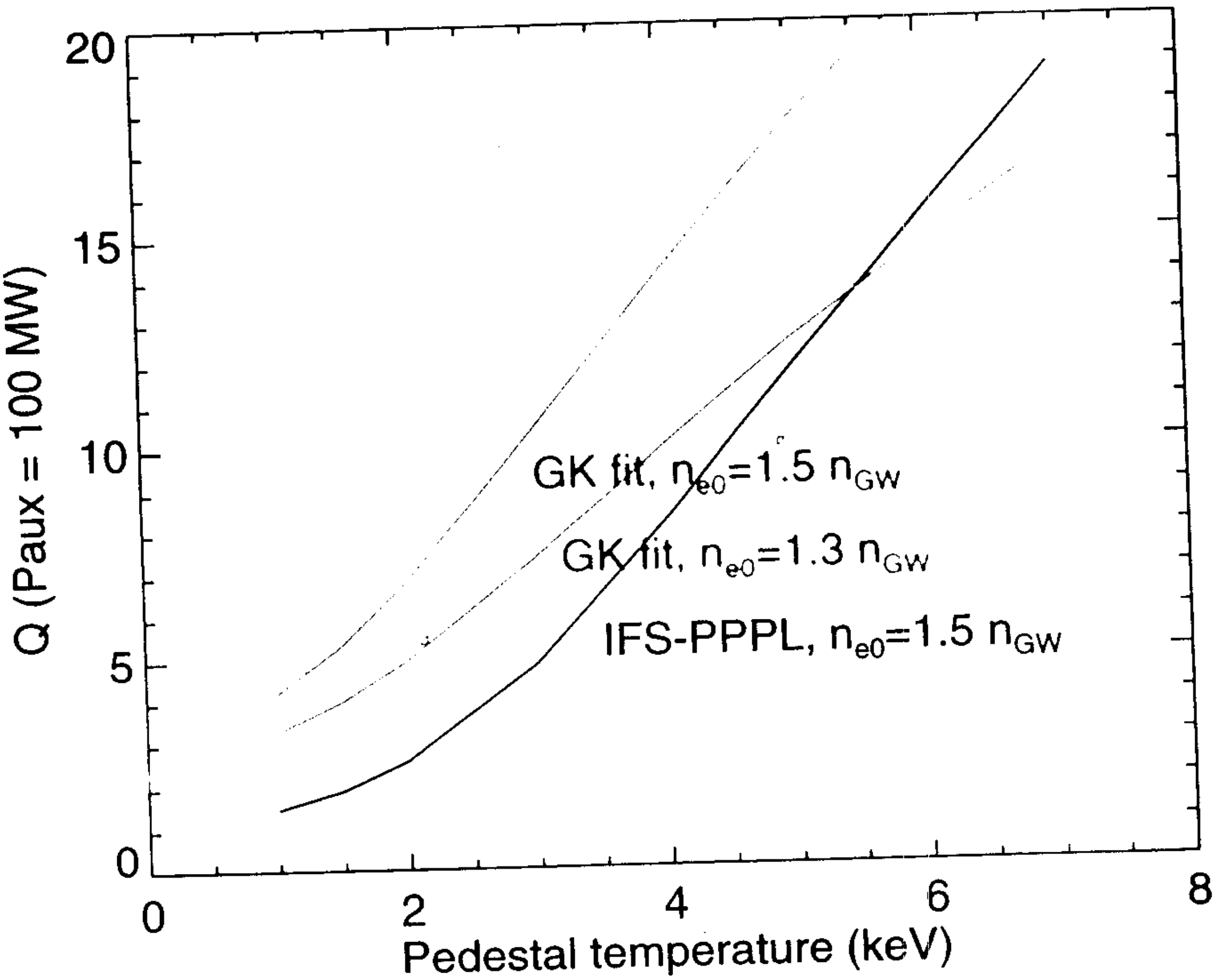
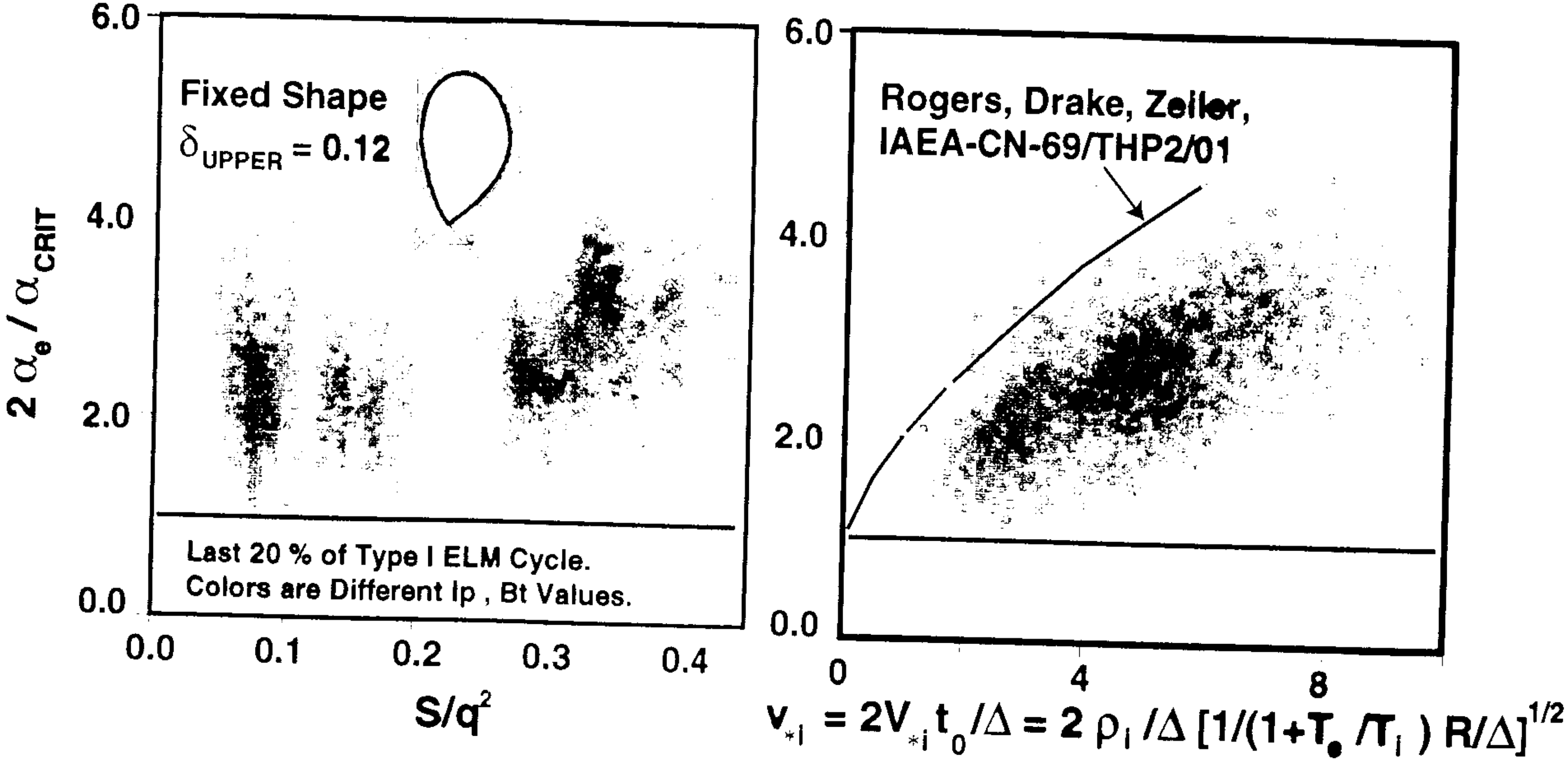
GENERAL ATOMICS



20
15
10
5
0
Q (Paux = 100 MW)
0
2
4
6
8
Pedestal temperature (keV)
GK fit, n_{e0}=1.5 n_{GW}
GK fit, n_{e0}=1.3 n_{GW}
IFS-PPPL, n_{e0}=1.5 n_{GW}

Edge Pressure Gradient May Exceed Infinite n Ballooning Limit Due to Small Barrier Width

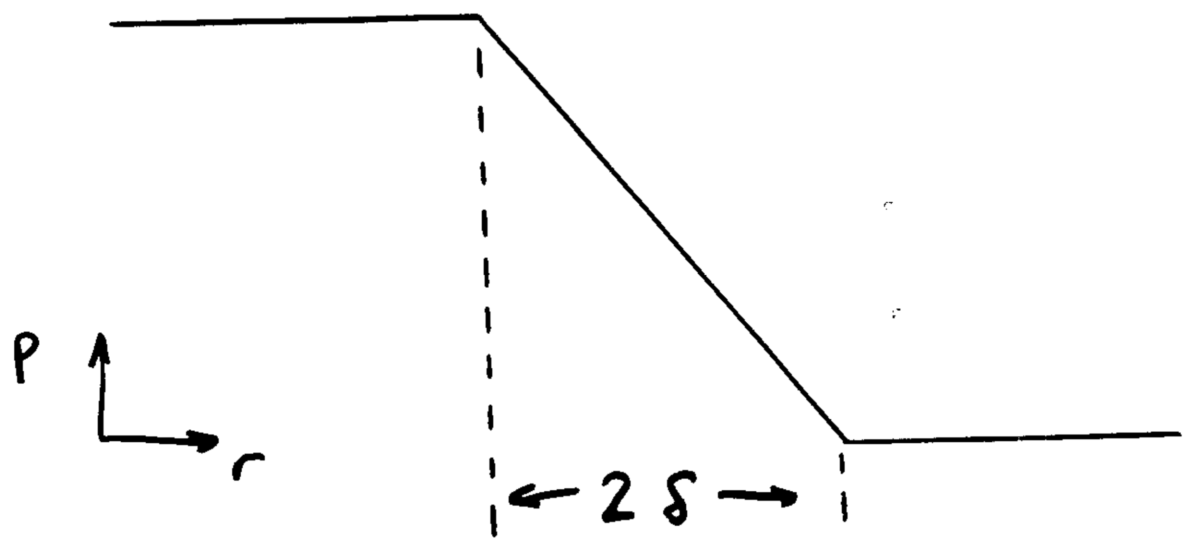
- Theory of Rogers, Drake and Zeiler predicts p' may exceed infinite n ballooning limit due effect of the small extent of the steep gradient region on long wavelength modes, and diamagnetic effect at short wavelength



T. Osborne

ω_{*i} can stabilize all modes because gradient is radially localized

- Essential physics in simple ramp gradient model:



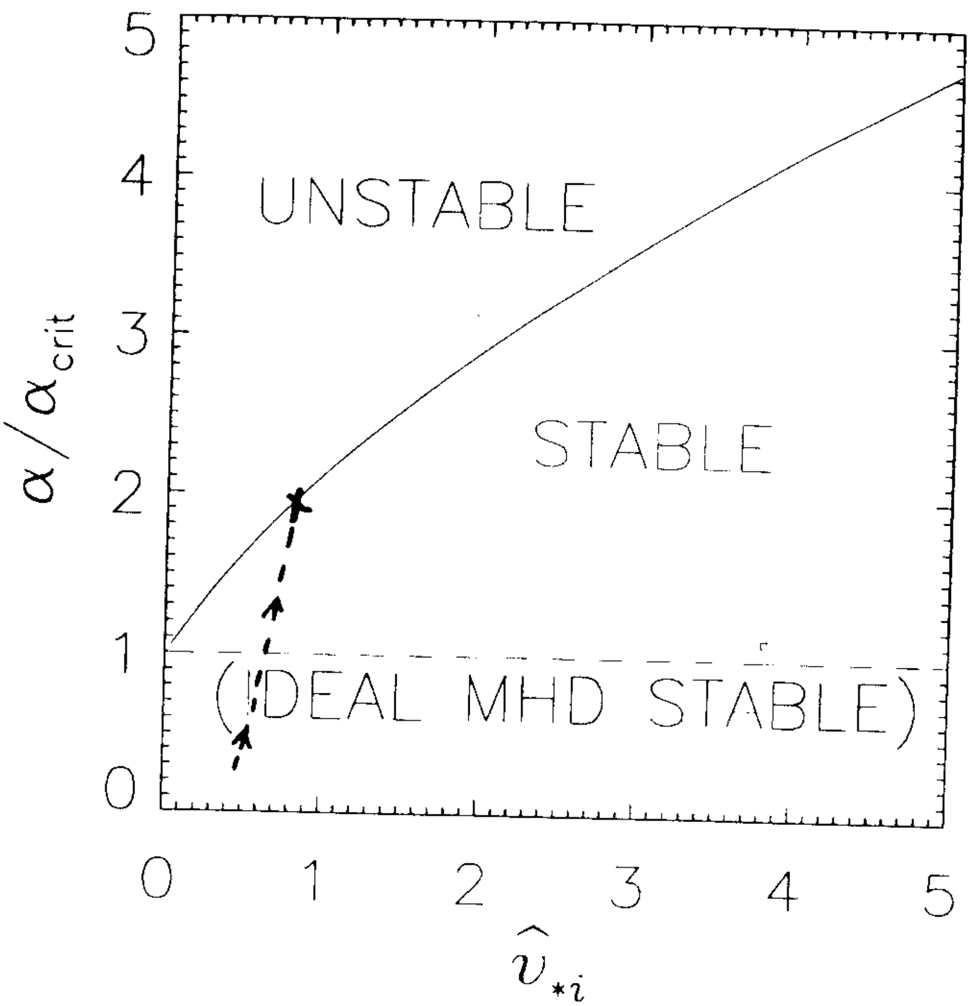
- Dispersion relation: $\omega(\omega + \omega_{*i}) = -\gamma_0^2$ with

$$\gamma_0^2 \simeq \left(\left[\frac{|k_\theta \delta|}{|k_\theta \delta| + 1} \right] \alpha - 1 \right) \frac{V_A^2}{q^2 R^2}$$

- Modes with $k_\theta \delta \ll 1$ always stable ($\alpha \sim 1$)

- Modes with higher $k_\theta \delta$ stable if $\omega_{*i} > 2\gamma_0$.

Modified Ideal Stability Boundary



- $\hat{v}_{*i} = V_{*i} t_0/\delta = \frac{\rho_i}{\delta}\left(\frac{T_i}{T_i+T_e}\frac{R}{2\delta}\right)^{1/2}$

- **Explains why pedestal exceeds ideal MHD boundary**
  - **in simulations**
  - **in experiments ?**

Schematic of Calculation

1. MHD equilibrium calculated
   - May include real magnet configuration
2. Heat/torque/particle sources $S_i(\rho)$ calculated
   - Models range from simple to Monte-Carlo
   - Uncertainties greatest near the walls
3. Diffusivities calculated from a physics model
   - Simple model based on prior simulations
   - More complex model based on prior nonlinear simulations together with linear calculations
   - Nonlinear simulations using fluid models
   - Nonlinear simulations using kinetic models
4. New $T_i(\rho), T_e(\rho), n(\rho)$, *etc.*, calculated
5. Repeat

Major complications:

- Boundary conditions significant, hard to calculate
- Transport barriers push physics understanding to the limit
- Electromagnetic effects on turbulence poorly understood

Significant progress is being made on these fronts...

New Puzzles

- Differences among turbulence simulations remain.
- Influence of electromagnetic perturbations unknown. Can affect topology of perturbations, stability, *etc.*
- Electron thermal transport not completely understood, especially in Ohmic plasmas and in transport barriers.
- What physics governs pedestal formation, equilibrium, dynamics?
- What physics governs barrier formation, equilibrium, dynamics?